

Article

Joint Geometry Controls Preferential Flow and Contaminant Transport in HDPE Geomembrane Composite Cutoff Walls

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Abstract

High-density polyethylene (HDPE) geomembrane composite cutoff walls are used to limit groundwater flow and contaminant migration, but panel joints can form localized transport pathways. The unresolved question is how predefined interlocking configurations differ in joint-controlled flow and transport. This component-scale study combined individual 20 g geotechnical centrifuge tests with local-scale, engineering-width, and three-dimensional centrifuge-apparatus numerical models to compare crescent-shaped, square, and triangular joint configurations. Two-dimensional local-scale models provided configuration-specific equivalent hydraulic conductivity and diffusion coefficients, which were transferred without calibration to a three-dimensional half-domain model of the centrifuge apparatus. Separate 6.0 m-wide two-dimensional calculations quantified the contribution of the joints and prescribed cracks to overall equivalent hydraulic conductivity. The crescent-shaped joint showed the smallest measured hydraulic-head reduction and the lowest local-scale equivalent hydraulic conductivity, whereas the triangular joint showed the largest values. The centrifuge Na^+ changes were within the stated instrumental error and were retained only as descriptive records. Extended 10 d constant-head analyses using the 0.40 m three-dimensional half-domain indicated that the maximum external-domain Na^+ concentration increased with hydraulic-head difference and that prescribed cracks accelerated the concentration response; differences among the 0.5–1.5 mm crack cases were small and non-monotonic at 10 d. The simulated concentration fields illustrate joint-controlled contaminant transfer into a common receiving-fluid domain at the component scale. The comparison provides a practical basis for joint detailing, defect control, and construction quality assurance in composite cutoff walls.



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1. Introduction

Groundwater contamination caused by industrial activities, mining operations, landfill leachate, and hazardous-waste disposal poses a major environmental challenge worldwide.

Once contaminants enter groundwater systems, they may migrate over considerable distances through aquifers, posing long-term risks to drinking water resources and ecological environments. Cutoff walls are therefore widely used for contaminated-land remediation and groundwater protection. Owing to their long service life, adaptability to complex geological conditions, and relatively low maintenance requirements, cutoff walls have become an indispensable component of contaminated land remediation and groundwater protection systems [1–4].

Among various cutoff technologies, high-density polyethylene (HDPE) geomembrane composite cutoff walls combine the extremely low hydraulic conductivity of geomembranes with the structural support and self-healing capacity of cement–bentonite or soil–bentonite backfills [5–8]. The incorporation of an HDPE geomembrane reduces the equivalent hydraulic conductivity of the composite barrier by several orders of magnitude compared with conventional slurry walls, thereby providing substantially improved hydraulic and contaminant barrier performance. Consequently, HDPE geomembrane composite cutoff walls have been increasingly employed in contaminated sites, municipal landfills, tailings storage facilities, and industrial waste containment projects where long-term environmental protection is required [9–12].

The hydraulic and transport behaviour of composite cutoff walls has therefore received considerable research attention. Previous studies have investigated the influence of material properties, hydraulic conductivity, diffusion characteristics, and coupled seepage–transport processes using analytical models, laboratory experiments, and numerical simulations [10–15]. Laboratory studies have demonstrated that the inclusion of HDPE geomembranes can significantly reduce seepage and contaminant migration, whereas numerical investigations have provided valuable insights into the coupled hydraulic and transport behaviour of multilayer barrier systems. More recently, analytical and numerical studies have considered consolidation in a buffer-layer–wall–aquifer system [12], multilayer barriers [14], and nonlinear adsorption [15]. Collectively, these studies have advanced the understanding and prediction of hydraulic and contaminant-transport behaviour in composite cutoff walls.

Despite these advances, the long-term effectiveness of composite cutoff walls depends not only on the intrinsic properties of the geomembrane and backfill materials but also on the integrity of panel joints. Because adjacent geomembrane panels must be connected during construction, panel joints inevitably introduce local discontinuities into an otherwise continuous hydraulic barrier. Field observations and laboratory investigations have shown that imperfections associated with joint construction, including incomplete sealing, geometric irregularities, and localized defects, may create preferential seepage pathways that significantly accelerate contaminant migration [16–22]. Consequently, panel joints are considered a potentially vulnerable component of HDPE geomembrane composite cutoff walls. Force-equilibrium modelling also addresses wall stresses [23]. Huang et al. [24] combined centrifuge testing and numerical analysis of defective composite walls, while Cao et al. [25] examined joint-defect leakage and the effects of backfill and loading. These studies establish joint leakage as a recognized problem. Related centrifuge work addresses barrier breakthrough [26]. The present question is how three predefined interlocking configurations compare in the same apparatus and how their local equivalent properties relate to the apparatus response.

Defect-geometry studies provide a useful comparison. Rowe and Fan [27,28] examine holes, slots, and rectangular defects beneath tailings, and Fan and Rowe [29] investigate seam leakage and subgrade conditions. These studies motivate comparison by aperture, path, and contact geometry. Recent analytical work addresses long defects [30], effective porosity and thermodiffusion [31], multilayer defective liners [32], and reactive wall–aquifer

transport [33]. Further developments include leakage uncertainty [34], composite-wall monitoring [35], defect influence range [36], and reaction-chain transport [37]. These contributions distinguish the present comparison of interlocking configurations from the wider tasks of predicting leakage uncertainty and field contaminant fate.

Conventional 1 g laboratory tests provide useful material data, but a reduced 1 g model cannot reproduce the enhanced-gravity hydraulic condition used here. Geotechnical centrifuge modelling applies enhanced gravity to impose a prescribed hydraulic loading in the scaled apparatus [38–43]. Accordingly, the centrifuge tests compare the investigated joints under a common 20 g hydraulic condition. Process-specific similitude limits for flow and contaminant transfer through the geometrically reduced manufactured aperture are stated in Section 2.2. The numerical calculations are assessed for consistency with the measured short-term centrifuge response after all model inputs are fixed.

Against this background, the present study compares the hydraulic and contaminant-transport responses of crescent-shaped, square, and triangular HDPE geomembrane panel joints using 20 g centrifuge modelling and multiscale numerical simulations. The scientific objectives are to: (1) compare the short-term hydraulic responses of the three investigated configurations; (2) determine configuration-specific equivalent hydraulic and diffusion parameters using explicit local models; (3) transfer these parameters to a three-dimensional centrifuge-apparatus model without calibration against the centrifuge measurements. The applied objectives are to evaluate the separate 6.0 m width effect and, as numerical sensitivity scenarios, the effects of prescribed hydraulic conditions and initial construction defects.

2. Experimental Programme

2.1. Centrifuge Modelling and Test Design

Panel joints can provide localized hydraulic connections through an otherwise low-permeability HDPE geomembrane barrier. Geotechnical centrifuge modelling imposed a common enhanced-gravity hydraulic condition on the scaled physical specimens. A reduced apparatus at 1 g would have a different hydraulic loading. The applicable hydraulic and transport scaling relationships are stated explicitly below.

Centrifuge tests were conducted for three predefined joint configurations (crescent-shaped, square, and triangular) and a joint-free control. The measurements were used to compare the short-term falling-head hydraulic response obtained from each joint test. NaCl was used as a tracer, and the endpoint Na^+ readings were retained as descriptive records for comparison with the stated instrument accuracy.

A centrifugal acceleration of 20 g was applied for 2 h. The conventional porous-medium seepage-time ratio, $t_m/t_p = 1/N^2$, requires model and prototype intrinsic permeability to be preserved. Because the manufactured joint aperture was geometrically reduced, this time relationship was not used to convert the 2 h joint-controlled seepage test to prototype duration. The ideal scaling requirements and their applicability are summarized in Table 1.

Table 1. Ideal model-to-prototype scaling relationships for centrifuge and conservative contaminant-transport analyses ($N = 20$).

Quantity	Definition/Unit	Ideal Scale $S_X = X_m/X_p$	$N = 20$	Applicability to This Study
Acceleration	g	N	20	Prescribed 20 g loading
Length	L	$1/N$	$1/20$	Applied geometrically
Volume	V	$1/N^3$	$1/8000$	Ideal geometric relationship
Concentration	C (mol/m ³)	1	1	Same nominal concentration scale
Molecular diffusion	D_0 (m ² /s)	1	1	$D_0 = 1.33 \times 10^{-9}$ m ² /s
Effective diffusion	D_e (m ² /s)	1	1	Ideal requirement for identical porosity and tortuosity

Table 1. Cont.

Quantity	Definition/Unit	Ideal Scale $S_X = X_m/X_p$	N = 20	Applicability to This Study
Darcy velocity	q (m/s)	N	20	Porous-medium requirement; ideal scaled slot gives $1/N$
Advective time	$t_a = L/(q/\epsilon)$	$1/N^2$	$1/400$	Ideal scaled slot gives 1
Diffusive time	$t_d = L^2/D_e$	$1/N^2$	$1/400$	Only under ideal diffusion assumptions
Péclet number	$Pe = qL/(\epsilon D_e)$	1	1	Not preserved for the ideal scaled slot
Dispersivity	α_L, α_T (m)	Required $1/N$	Required $1/20$	Not measured; zero only in the numerical model
Local molar-flux density	J (mol/(m ² ·s))	N	20	Porous-medium ideal relationship only
3D integrated mass rate	$\dot{M}$ (mol/s)	$1/N$	$1/20$	Not measured experimentally
3D cumulative mass	M (mol)	$1/N^3$	$1/8000$	Not measured experimentally

Note: The tabulated contaminant-transport ratios are ideal porous-medium requirements. For a two-dimensional plane-strain porous-medium model, the integrated rate per unit out-of-plane width scales as 1 and cumulative mass per unit width as $1/N^2$. These relationships do not demonstrate complete similitude of the manufactured joint.

The experiments were conducted using the geotechnical centrifuge facility at Tianjin University, which has a maximum rotation radius of 0.7 m and a maximum acceleration capacity of 200 g. The physical model was installed in a rigid aluminium container measuring $0.4 \text{ m} \times 0.4 \text{ m} \times 0.4 \text{ m}$. All internal joints of the container were sealed with silicone to prevent leakage during centrifuge operation.

2.2. Similitude Limits for Joint Flow and Contaminant Transport

For an ideal narrow slot, the fracture intrinsic permeability may be written as $\kappa_f = b^2/(12f)$, where b is hydraulic aperture and f is a geometric correction factor. If the same fluid and correction factor apply at both scales, $b_m/b_p = 1/N$ gives $\kappa_m/\kappa_p = 1/N^2$. With $g_m/g_p = N$ and $K = \rho g \kappa/\mu$, the hydraulic-conductivity ratio is $K_m/K_p = 1/N$. For a fully geometrically similar falling-head slot system, discharge and reservoir volume both scale as $1/N^3$, giving a characteristic fracture-flow time ratio $t_m/t_p = 1$.

For geometrically similar saturated porous media with identical pore fluid, porosity, tortuosity, and effective diffusion coefficient, $q_m/q_p = N$ gives $t_{a,m}/t_{a,p} = t_{d,m}/t_{d,p} = 1/N^2$ and preserves Pe . Local advective and diffusive molar-flux densities then scale as N , the three-dimensional boundary-integrated mass rate as $1/N$, and cumulative transported mass as $1/N^3$. Exact mechanical-dispersion similitude would additionally require $\alpha_m/\alpha_p = 1/N$.

Complete contaminant-transport similitude was not achieved in the present physical model. The ideal scaled-slot velocity ratio differs from the porous-medium requirement, physical dispersivity was not measured, and possible scale effects associated with printing tolerance, surface roughness, contact conditions, and deformation were not quantified. The centrifuge results are therefore used for comparison of the tested joint configurations.

Three representative joint geometries, namely crescent-shaped, square, and triangular configurations, were designed based on engineering prototypes using a geometric scale of 1:20. Owing to the extremely small dimensions of the scaled joints, with a minimum thickness of approximately 0.3 mm, stereolithography (SLA) 3D printing technology was employed to fabricate photopolymer-resin replicas of the HDPE joint geometries, with a nominal printer accuracy of $\pm 0.025 \text{ mm}$. The printed specimens exhibited smooth surfaces. The joint geometries are shown in Figure 1a.

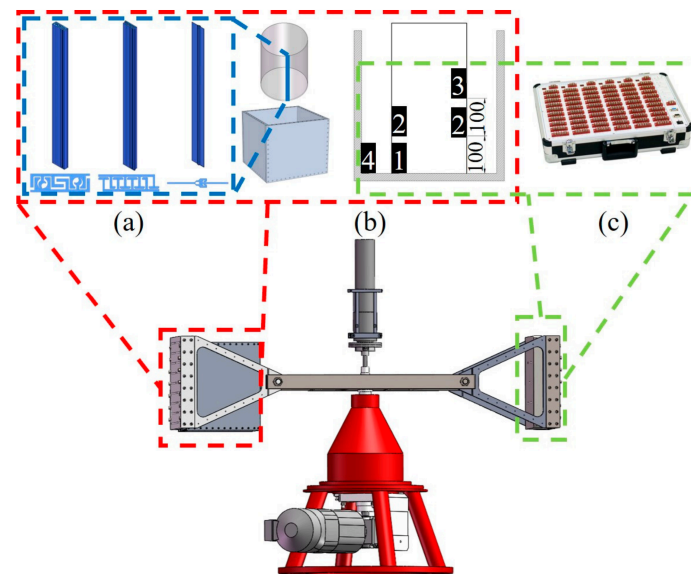


Figure 1. Experimental setup: (a) joint configurations and corresponding cross-sections; (b) cross-section of the model box showing pore-pressure-transducer locations (Points 1–4); and (c) centrifuge setup including the uT7160 strain-acquisition unit and counterweight system.

A custom-designed polymethyl methacrylate (PMMA) cylinder was used as the testing chamber to ensure that seepage occurred exclusively through the joint specimen. The cylinder had an internal radius of 0.10 m and a height of 0.40 m, with a prefabricated slot for joint installation. Each joint specimen was bonded into the slot using a waterproof adhesive and externally reinforced to maintain sealing integrity during centrifuge loading. Owing to the extremely low hydraulic conductivity of PMMA ($<1 \times 10^{-14}$ m/s), flow through the cylinder wall was considered negligible, ensuring that all seepage occurred through the joint region. This isolation was deliberate: it separates the intrinsic joint-controlled transfer response of the HDPE barrier component from site-specific attenuation processes in receiving soils. To reproduce the hydraulic boundary conditions beneath the prototype cutoff wall, a 0.10 m-thick sand layer was placed at the base of both the PMMA cylinder and the surrounding model box. The sand layer had a porosity of 0.41 and a hydraulic conductivity of 5×10^{-6} m/s, representing the permeable foundation layer beneath the cutoff wall.

2.3. Instrumentation and Tracer Selection

The hydraulic response was recorded continuously, whereas the tracer response was assessed using initial and endpoint measurements. Pore-water pressure was measured using five transducers with a range of 200 kPa and a resolution of 0.005 kPa. Four transducers were installed inside the PMMA cylinder at depths of 0.10 m, 0.20 m (two sensors), and 0.30 m, while one transducer was installed outside the cylinder at a depth of 0.10 m, as illustrated in Figure 1b. Pressure signals were recorded using a multi-channel acquisition system mounted on the centrifuge arm [19,24]. Na^+ was measured immediately before centrifugation and at the 2-h endpoint, rather than monitored continuously.

Sodium chloride (NaCl) was employed as a conservative tracer. The NaCl reagent (analytical grade) was supplied by Sinopharm Chemical Reagent Co., Ltd. Na^+ was measured using a BXS-Na portable meter with a display resolution of 0.01 pNa and a manufacturer-stated basic instrument error of ± 0.05 pNa. The numerical model treated Na^+ as nonreactive and used its free-fluid diffusion coefficient; adsorption, reaction, decay, precipitation, and concentration-dependent density were not included.

2.4. Preliminary Testing

Preliminary checks were conducted before centrifuge testing, including apparatus water-tightness verification, pore-pressure-transducer calibration under 1 g and centrifugal loading, and a response check of the Na⁺ meter.

(1) Water-tightness verification: The entire experimental system was filled with water and maintained under static conditions for 24 h. No measurable leakage was observed during the monitoring period, confirming that the sealing performance of the apparatus satisfied the requirements of the centrifuge tests.

(2) Calibration of pore pressure transducers under 1 g conditions: The pressure transducers were calibrated under normal gravitational conditions by establishing the relationship between hydraulic head and sensor output, as illustrated in Figure 2a,b. The transducers were successively positioned at water depths of 0, 0.2, 0.4, 0.6, 0.8, and 1.0 m, and the corresponding output signals were recorded using the static strain acquisition system. Each calibration measurement was repeated three times, and the mean value was used to establish the calibration relationship. A linear relationship between hydraulic head and sensor output was obtained:

$$y = 461.39x - 9.24 \quad (R^2 = 0.99921) \quad (1)$$

where x is the hydraulic head (m), and y is the output of the static strain acquisition system (mV).

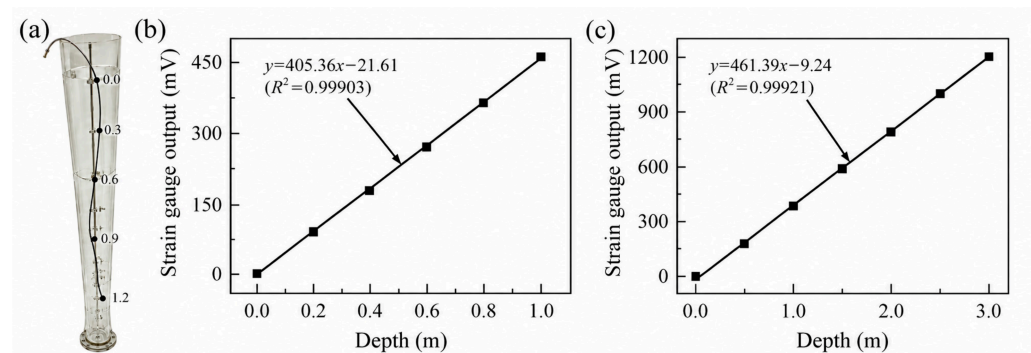


Figure 2. Preliminary tests and sensor calibration: (a) waterproofing test under 1 g conditions; (b) 1 g hydraulic-head calibration; and (c) 0~30 g centrifuge-loading calibration.

(3) Calibration under centrifugal conditions: To account for the influence of enhanced gravitational acceleration on sensor performance, an additional calibration was conducted under centrifuge loading (Figure 2c). The pressure transducers were fixed at a depth of 0.10 m while the centrifugal acceleration was gradually increased from 0 g to 30 g. Measurements were recorded at 0 g, 5 g, 10 g, 15 g, 20 g, 25 g, and 30 g, corresponding to equivalent hydraulic heads of 0, 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 m, respectively. Each loading condition was repeated three times, and the mean values were used to establish the calibration relationship:

$$y = 405.36x - 21.61 \quad (R^2 = 0.99903) \quad (2)$$

The resulting calibration equation accounts for the influence of the high-gravity environment on sensor response and was adopted for subsequent processing of the experimental measurements.

(4) Response check of the ion concentration meter: The meter was calibrated using 0.1 mol/L (1 pNa) and 0.001 mol/L (3 pNa) standard NaCl solutions. A 0.01 mol/L check

solution produced a reading of 2.01 pNa. This check confirmed the expected instrumental response but did not establish an uncertainty smaller than the manufacturer-stated basic error.

The preliminary checks established the operating response of the apparatus and sensors under the investigated conditions. Repeated sensor calibration was separate from the joint tests.

2.5. Model Preparation and Testing Procedure

Four centrifuge tests were conducted: one test for each of the crescent-shaped, square, and triangular joint configurations and one joint-free control test. Each test was operated at 20 g for 2 h. Each hydraulic curve represents one specimen and one test; the three sensor-calibration repeats are not replicate joint tests.

The experimental procedure consisted of the following steps:

- (1) Model assembly: The instrumented acrylic cylinder with the pre-installed joint specimen was positioned vertically at the centre of the model box. Signal cables from the pore pressure transducers were connected to the static strain acquisition system mounted on the opposite centrifuge arm. Prior to testing, all sensors and the data acquisition system were checked to ensure stable operation and reliable signal transmission, as shown in Figure 3.
- (2) Initial condition setup: The acrylic cylinder was filled with a 0.1 mol/L NaCl solution to a depth of 0.30 m. The surrounding model box was filled to a depth of 0.10 m with a prepared NaCl background solution, establishing an initial hydraulic-head difference of 0.20 m. The external Na^+ concentration was measured after filling and immediately before centrifugation and ranged from 34.7 to 36.3 mol/m³ among the tests.
- (3) Centrifuge operation: The centrifuge was gradually accelerated to 20 g and maintained at that acceleration for 2 h. Pore-water pressure was recorded continuously. Na^+ concentration was recorded immediately before centrifugation and at the end of the 2 h test.
- (4) Test termination: After completion of the centrifuge test, the rotor was decelerated gradually to avoid disturbance of the model. Final hydraulic head and Na^+ concentration measurements were then collected for subsequent analysis.

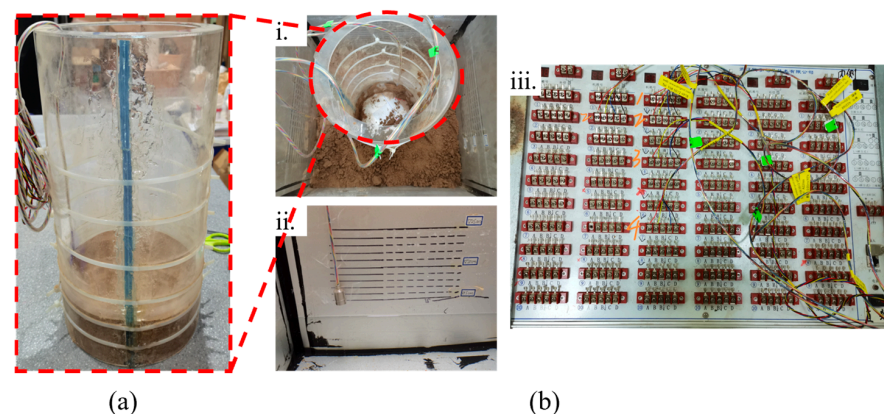


Figure 3. Experimental setup details: (a) acrylic cylinder with bonded joint; and (b) arrangement of pore-pressure transducers: (i) inside the cylinder, (ii) outside the cylinder on the inner wall of the model box, and (iii) connection to the strain-acquisition unit.

3. Numerical Investigation

3.1. Governing Equations and Hierarchical Framework

All numerical analyses were performed using COMSOL Multiphysics version 6.2. Three distinct model classes were used. First, two-dimensional plane-strain local-scale models (0.215 m wide and 0.600 m in the seepage direction) explicitly resolved the slurry backfill,

HDPE geomembrane, and three joint geometries. Separate stationary hydraulic and diffusion calculations produced configuration-specific K_{eq} and D_{eq} values. Second, two-dimensional engineering-width models (6.0 m wide and 0.600 m in the seepage direction) quantified the contribution of localized joints and prescribed cracks to the equivalent hydraulic conductivity across a practical panel width. Except for width, these models used the same material properties, corresponding barrier and joint cross-section, 1 Pa inlet, 0 Pa outlet, and no-flow lateral boundaries as the local-scale hydraulic models. Third, a time-dependent three-dimensional half-domain ($0.40 \text{ m} \times 0.20 \text{ m} \times 0.40 \text{ m}$) represented the centrifuge apparatus. The local-scale K_{eq} and D_{eq} values were transferred without calibration to its internal thin-barrier boundary. The engineering-width models constituted a separate width-effect analysis and did not exchange parameters with the three-dimensional model.

Figure 4 and Tables 2 and 3 summarize the geometry, purpose, and parameter links of the three model classes. Only K_{eq} and D_{eq} were transferred from the local-scale models to the three-dimensional centrifuge-apparatus model. The centrifuge response was not used to recalibrate these parameters or any other numerical input.

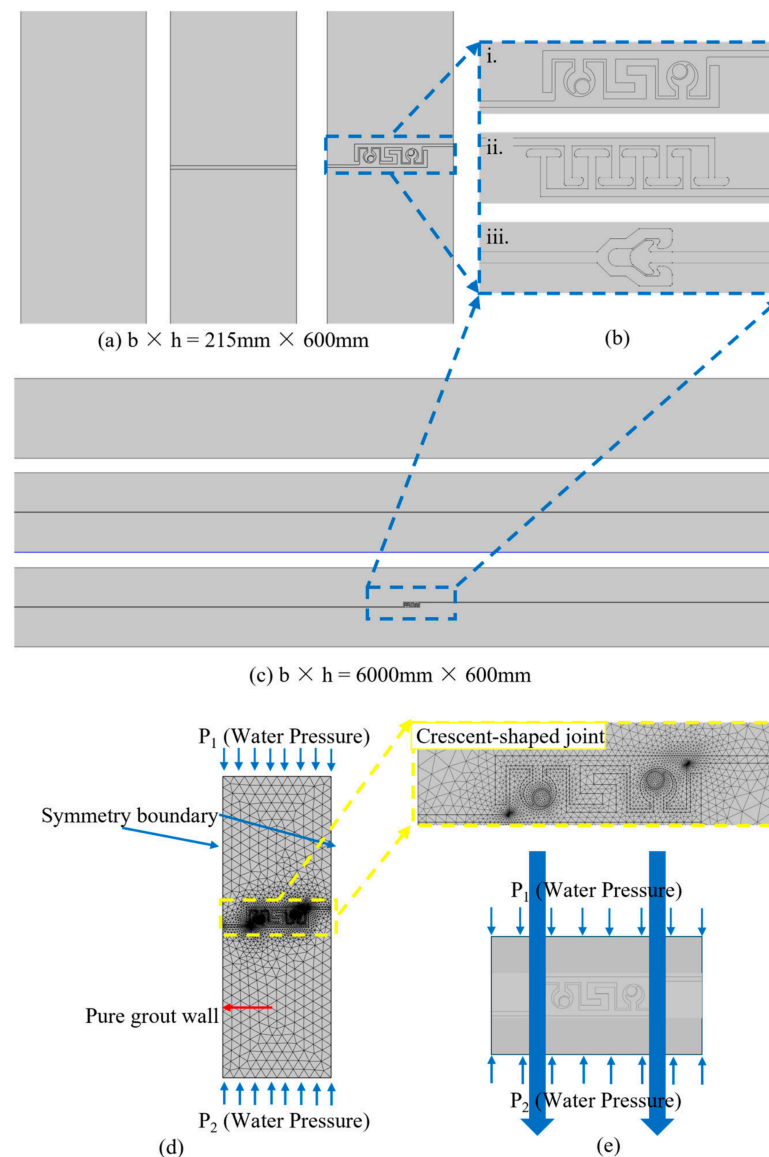


Figure 4. Numerical models used for local- and engineering-width analyses: (a) $0.215 \text{ m} \times 0.600 \text{ m}$ local-scale models; (b) representative joint geometries; (c) $6.0 \text{ m} \times 0.600 \text{ m}$ engineering-width models; (d) mesh refinement and initial pressure field; and (e) prescribed crack configuration.

The numerical configurations are summarized in Table 2. The three joint forms were predefined engineering configurations selected for comparison; they differ simultaneously in aperture, area, perimeter, corner geometry, and flow-path tortuosity. Consequently, the simulations compare complete configurations rather than isolate one geometric variable. Prescribed crack widths of 0.5, 1.0, and 1.5 mm were also examined as static numerical sensitivity cases; crack initiation, growth, and time-dependent opening were not modelled.

Saturated flow was described by mass conservation and Darcy's law, and solute transport was represented by advection with the total Darcy velocity and effective diffusion. The implemented equations and flux definitions were:

$$\frac{\partial(\varepsilon_p \rho)}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = Q_m \quad (3)$$

$$\mathbf{u} = -\frac{\kappa}{\mu} (\nabla p - \rho \mathbf{g}) \quad (4)$$

Solute transport in the saturated porous domains was written in the general form

$$\varepsilon_p \frac{\partial c}{\partial t} + \frac{\partial(\rho c_p)}{\partial t} + \nabla \cdot \mathbf{J} + \mathbf{u} \cdot \nabla c = R + S \quad (5)$$

$$\mathbf{J} = -(\mathbf{D}_D + D_e \mathbf{I}) \nabla c \quad (6)$$

where ε_p is porosity, ρ is fluid density, $\mathbf{u}$ is Darcy velocity, Q_m is a mass source, κ is intrinsic permeability, μ is dynamic viscosity, p is pore pressure, $\mathbf{g}$ is gravitational acceleration, c is aqueous concentration, c_p is sorbed-phase concentration, $\mathbf{D}_D$ is the mechanical-dispersion tensor, D_e is the effective diffusion coefficient, $\mathbf{I}$ is the identity tensor, and R and S are reaction and source terms. Density was fixed at 1000 kg/m³, porosity was time independent, and $Q_m = 0$, so $\partial(\varepsilon_p \rho)/\partial t = 0$. Adsorption, reaction, decay, solute sources, and concentration-dependent density were not included ($c_p = 0$ and $R = S = 0$). Mechanical dispersion was not activated ($\mathbf{D}_D = 0$ and $\alpha_L = \alpha_T = 0$).

Table 2. Numerical configurations and their relationship to the physical centrifuge tests.

Model Configuration	Geometry Description	Primary Purpose	Tested in Centrifuge
High-k slurry reference	Homogeneous 0.215 × 0.600 m slurry domain	Local-scale hydraulic upper-bound and diffusion reference	No
Continuous HDPE geomembrane	4.5 mm HDPE membrane without a joint	Local-scale intact-barrier reference	No
Crescent-shaped joint	Explicit 0.215 × 0.600 m local-scale joint geometry	Configuration-specific K_{eq} and D_{eq}	Yes
Square joint	Explicit 0.215 × 0.600 m local-scale joint geometry	Configuration-specific K_{eq} and D_{eq}	Yes
Triangular joint	Explicit 0.215 × 0.600 m local-scale joint geometry	Configuration-specific K_{eq} and D_{eq}	Yes
Engineering-width model	6.0 × 0.600 m 2D domain; corresponding barrier and joint cross-section	Panel-width contribution to overall K_{eq} ; numerical analysis only	No
Cracked joint model	Prescribed 0.5, 1.0, or 1.5 mm initial crack	Static numerical sensitivity	No

Note: A joint-free control specimen was included in the centrifuge programme but was not one of the local-scale numerical configurations listed here.

Table 3. Model hierarchy, dimensions, parameter transfer, and input provenance.

Model/Input	Dimensions or Representation	Purpose	Input Source and Transfer	Adjusted Using Centrifuge Response?
Local-scale model	2D plane strain; 0.215×0.600 m; explicit slurry, HDPE, and joint	Stationary K_{eq} and D_{eq} determination	Geometry and stated material inputs; K_{eq} and D_{eq} transferred to the 3D model	No
Engineering-width model	2D plane strain; 6.0×0.600 m; corresponding barrier and joint cross-section	Stationary panel-width and crack contribution to overall K_{eq}	Same constitutive inputs and boundary definitions as the corresponding 2D model; no transfer to the 3D model	No
Centrifuge-scale model	3D half-domain; $0.40 \times 0.20 \times 0.40$ m; cylinder $r = 0.10$ m, $h = 0.40$ m; 0.10 m sand layer	Falling-head comparison and 10-d constant-head sensitivity analyses	Receives local-scale K_{eq} and D_{eq} through thin barriers; remaining inputs from specimen and model settings	No
Joint boundary	$d_b = 0.003$ m; Thin Barrier and Diffusion Thin Barrier	Equivalent joint representation in the 3D model	$K_{eq} \rightarrow k_b$ through $k_b = K_{eq}\mu/(\rho g)$; $D_{eq} \rightarrow D_s$	No
Porous domain	Sand $K = 5.0 \times 10^{-6}$ m/s; $\varepsilon_p = 0.41$	3D flow and transport domain	Measured/specimen properties	No
Fluid/solute	$\rho = 1000$ kg/m ³ ; $\mu = 0.001$ Pa·s; $D_F = 1.33 \times 10^{-9}$ m ² /s	Fluid and transport properties	Standard value and cited literature input	No
Falling-head data	0.30/0.10 m water levels; 100 mol/m ³ internal; measured external background	Centrifuge-scale comparison	Experimental setup and pre-test measurements	No
Constant-head extension	$\Delta h = 0, 0.1, 0.2,$ or 0.3 m; 0–10 d output at 0.5 d intervals	Configuration, head-difference, and prescribed-crack sensitivity analyses	Same 3D geometry and transferred joint parameters	No
Storage/transport	Density and porosity constant; $\alpha_L = \alpha_T = 0$; no sorption, reaction, decay, or density feedback	Model assumptions	COMSOL settings	No
Study/solver	Stationary 2D models; time-dependent 3D model; segregated p and c ; direct linear solvers	Forward numerical solution	COMSOL settings; physics-controlled tolerances	No
Hydraulic records	Individual centrifuge-test records	Short-term model–experiment consistency comparison	Measured after all model inputs were fixed	Not applicable
Na ⁺ readings	Single initial and endpoint readings	Descriptive only	Measured after all model inputs were fixed	Not applicable

The implemented transport equation therefore reduced to

$$\varepsilon_p \frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (D_e \nabla c) \quad (7)$$

The local total contaminant molar-flux vector was defined as

$$\mathbf{J}_{tot} = c\mathbf{u} - (D_D + D_e\mathbf{I})\nabla c = c\mathbf{u} - D_e\nabla c \quad (8)$$

The outward molar-transfer rate through an outlet boundary Γ_{out} and the cumulative transported mass were

$$\dot{M}_{\text{out}} = \int_{\Gamma_{\text{out}}} \mathbf{J}_{\text{tot}} \cdot \mathbf{n} dA, \quad M_{\text{out}}(t) = \int_0^t \dot{M}_{\text{out}}(\tau) d\tau \quad (9)$$

Positive values denote outward transport. $\mathbf{J}_{\text{tot}}$ has units of $\text{mol}/(\text{m}^2 \cdot \text{s})$, while $\dot{M}_{\text{out}}$ has units of mol/s in three dimensions. For the two-dimensional models, the boundary integral is a molar rate per unit out-of-plane width, with units of $\text{mol}/(\text{m} \cdot \text{s})$.

A hybrid mesh with local refinement was used around the joint, geomembrane-slurry interfaces, and prescribed crack tips in the local-scale models. These definitions distinguish the pure diffusive quantity used for D_{eq} from the advective–diffusive total flux in Equation (8).

For the stationary local-scale hydraulic calculation, K_{eq} was defined from the boundary-integrated discharge Q as

$$K_{\text{eq}} = \frac{Q}{Ai} = \frac{QL}{A\Delta H} \quad (10)$$

$$i = \frac{\Delta H}{L} \quad (11)$$

where A is the outlet area per unit out-of-plane width, i is the hydraulic gradient, and L is the seepage length.

In the separate stationary diffusion calculation, advection was excluded and the boundary-integrated diffusive rate was:

$$\dot{M}_{\text{diff}} = \int_{\Gamma_{\text{out}}} (-D_e \nabla c) \cdot \mathbf{n} dA \quad (12)$$

The equivalent diffusion coefficient was:

$$D_{\text{eq}} = \frac{|\dot{M}_{\text{diff}}| L}{A \Delta c} \quad (13)$$

3.2. Local-Scale and Engineering-Width Models

The local-scale models were two-dimensional plane-strain domains measuring 0.215 m in width and 0.600 m in the seepage direction. Fixed pressures of 1 Pa and 0 Pa were prescribed on the inlet and outlet boundaries for the stationary hydraulic analyses, with no flow on the lateral boundaries. Concentrations of 1 mol/m^3 and 0 mol/m^3 were prescribed for the separate local-scale diffusion analyses, with zero lateral diffusive flux. The engineering-width models were 6.0 m wide and 0.600 m in the seepage direction. Apart from width, each engineering-width model used the same material properties, barrier and joint cross-section, fixed inlet and outlet pressures of 1 Pa and 0 Pa, and no-flow lateral boundaries as the corresponding local-scale hydraulic model. The model parameters and their applicable domains are summarized in Table 4.

The intact joint denotes the numerical reference configuration in which no crack or other discontinuity was intentionally introduced. The cracked cases contained prescribed initial widths of 0.5, 1.0, and 1.5 mm and used the Fracture Flow interface [44,45]. Crack initiation, propagation, and time-dependent changes in crack width were not simulated.

Table 4. Core material and transport parameters used in the numerical analyses.

Parameter	Symbol	Value	Unit	Applicable Model/Domain and Source
Sand hydraulic conductivity	K_{sand}	5.00×10^{-6}	m/s	3D centrifuge-scale model; laboratory measurement
Prescribed crack conductivity	K_{crack}	1.00×10^{-6}	m/s	Static crack scenarios; literature input
PMMA hydraulic conductivity	K_{PMMA}	1.00×10^{-14}	m/s	3D centrifuge-scale model; low-permeability representation
Fluid density	ρ	1000	kg/m ³	Local and 3D models; fixed standard value
Dynamic viscosity	μ	0.001	Pa·s	Local and 3D models; standard value
Na ⁺ free-fluid diffusion	D_F	1.33×10^{-9}	m ² /s	Transport models; literature value
Slurry porosity	n_{slurry}	0.20	–	Local-scale model; assumed value
Porous-domain porosity	ε_p	0.41	–	3D centrifuge-scale model; specimen property
High-k slurry hydraulic conductivity	K_{slurry}	1.00×10^{-7}	m/s	Local- and engineering-width hydraulic reference; prescribed upper-bound case [46]

3.3. Three-Dimensional Centrifuge-Scale Model

The centrifuge-apparatus model was a time-dependent three-dimensional half-domain measuring $0.40 \text{ m} \times 0.20 \text{ m} \times 0.40 \text{ m}$, corresponding to one half of the 0.40 m cubic physical container. A vertical symmetry plane was used. The PMMA cylinder had an internal radius of 0.10 m and a height of 0.40 m , and a 0.10 m sand layer was represented at its base. The external domain was a common receiving-fluid boundary used to compare joint configurations without introducing site-specific soil sorption, reaction, or dispersion.

(1) Joint representation and parameter transfer

The detailed joint geometry was not explicitly meshed in three dimensions. The joint was located on the corresponding internal boundary of the PMMA wall and implemented using COMSOL's Thin Barrier and Diffusion Thin Barrier features with physical thickness $d_b = 0.003 \text{ m}$. For each configuration, the local-scale K_{eq} value was converted to intrinsic permeability through $k_b = \frac{K_{\text{eq}} \mu}{\rho g}$, and D_{eq} was assigned as D_S . The transferred pairs (K_{eq} , D_{eq}) were $(1.85 \times 10^{-9} \text{ m/s}, 1.37 \times 10^{-11} \text{ m}^2/\text{s})$, $(2.08 \times 10^{-9} \text{ m/s}, 1.46 \times 10^{-11} \text{ m}^2/\text{s})$, and $(5.00 \times 10^{-9} \text{ m/s}, 1.98 \times 10^{-11} \text{ m}^2/\text{s})$ for the crescent-shaped, square, and triangular joints, respectively. The remainder of the PMMA wall was treated as impermeable.

(2) Initial and boundary conditions

The pressure head on the upper internal surface was updated using $H + \text{intop1}(dl.bndflux) * t / 0.0157 / 1000$, where 0.0157 m^2 is the upper-surface area of the half-cylinder and 1000 kg/m^3 is water density. This expression is a flux-feedback falling-head approximation rather than a strict cumulative-flux time integral. The remaining upper surfaces were at atmospheric pressure, three lateral surfaces were no-flow boundaries, and the fourth lateral surface was the vertical symmetry plane. The internal solution initially contained 100 mol/m^3 NaCl; the external initial concentration was assigned from the measured pre-centrifugation background value for the relevant test.

For solute transport, the internal and external fluid domains were initialized at 100 mol/m^3 and the measured pre-centrifugation background concentration, respectively. The two upper liquid surfaces were assigned Open Boundary conditions, the vertical half-domain plane was assigned Symmetry, and the remaining solute-transport boundaries were assigned No Flux. The PMMA wall was impermeable to solute except at the internal joint boundary, where the Diffusion Thin Barrier feature was applied.

(3) Transport and solver settings

In the porous fluid domains, the isotropic free-fluid diffusion coefficient was $D_F = 1.33 \times 10^{-9} \text{ m}^2/\text{s}$. The Millington–Quirk model used $\tau_F = \varepsilon_p^{-1/3}$, giving $D_e = \frac{\varepsilon_p}{\tau_F} D_F = \varepsilon_p^{4/3} D_F$

with $\varepsilon_p = 0.41$. This domain effective-diffusion relation is distinct from the configuration-specific D_{eq} assigned to the joint. Advection used the Total Darcy velocity field (dl/porous1). The three-dimensional model employed a time-dependent study, with pressure and concentration solved in separate segregated groups using direct linear solvers and physics-controlled tolerances. Results were stored at $t = 0, 0.2, \dots, 3.0$ h; these are output times rather than fixed internal time steps.

No parameter was calibrated against the centrifuge hydraulic-head or Na^+ measurements. The joint K_{eq} and D_{eq} values were obtained from the local-scale models, and the remaining inputs were fixed from specimen properties, geometry, measured initial conditions, or stated standard and literature values before the forward calculation. The centrifuge hydraulic-head records were used only for a short-term model-experiment consistency assessment under the prescribed 20 g physical condition.

3.4. Model-Experiment Consistency Assessment

(1) Assessment Strategy

The three-dimensional forward simulations were compared with the measured falling-head response under the prescribed test conditions. This assessment examined short-term consistency after all inputs had been fixed. The Na^+ readings were not used for parameter calibration, quantitative comparison, or the consistency assessment, because the observed changes were within the stated instrumental error.

The numerical model was a three-dimensional half-domain with a vertical symmetry plane, not a two-dimensional axisymmetric model. Figure 5 shows the current geometry and mesh. The detailed joint geometry was replaced at this scale by the internal-boundary equivalent joint representation described in Section 3.3.

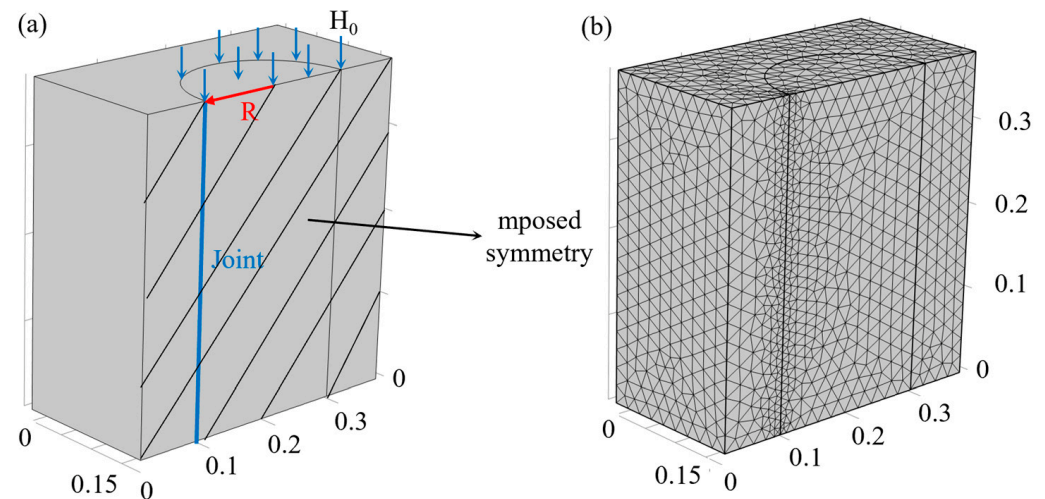


Figure 5. Three-dimensional half-domain numerical model of the centrifuge test: (a) geometry showing the internal joint location and vertical symmetry plane; and (b) finite-element mesh (dimensions in metres).

Parameter transfer was one-way, from the local-scale models to the three-dimensional model. K_{eq} and D_{eq} were not back-calculated from the centrifuge response. The 10-d constant-head sensitivity analyses reused the three-dimensional model and did not provide parameters to the centrifuge comparison.

The principal three-dimensional model settings, parameter sources, and roles in the model-experiment consistency assessment are summarized in Table 3.

The three-dimensional mesh also used local refinement near the equivalent joint boundary.

(2) Consistency Results

Figure 6 compares the simulated and measured hydraulic-head responses for the individual joint tests. After 2 h, the simulated reductions were 0.066, 0.073, and 0.148 m for the crescent-shaped, square, and triangular configurations, compared with observed reductions of 0.064, 0.070, and 0.145 m. The corresponding relative differences were 3.1%, 4.3%, and 2.1%. The simulations reproduced the configuration-specific ordering observed in the individual tests.

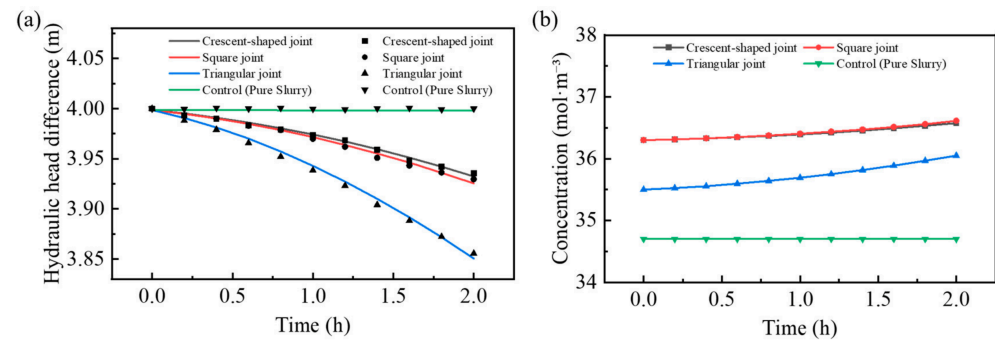


Figure 6. Experimental and numerical responses: (a) hydraulic-head difference and (b) Na⁺ concentration outside the cylinder. Solid lines denote numerical results, and symbols denote experimental measurements. The Na⁺ measurements in (b) are shown descriptively only and were not used for quantitative model–experiment assessment.

The simulated Darcy streamlines in Figure 7 converged toward the joint location at both 0 and 3 h, illustrating the principal hydraulic connection across the otherwise low-permeability PMMA wall.

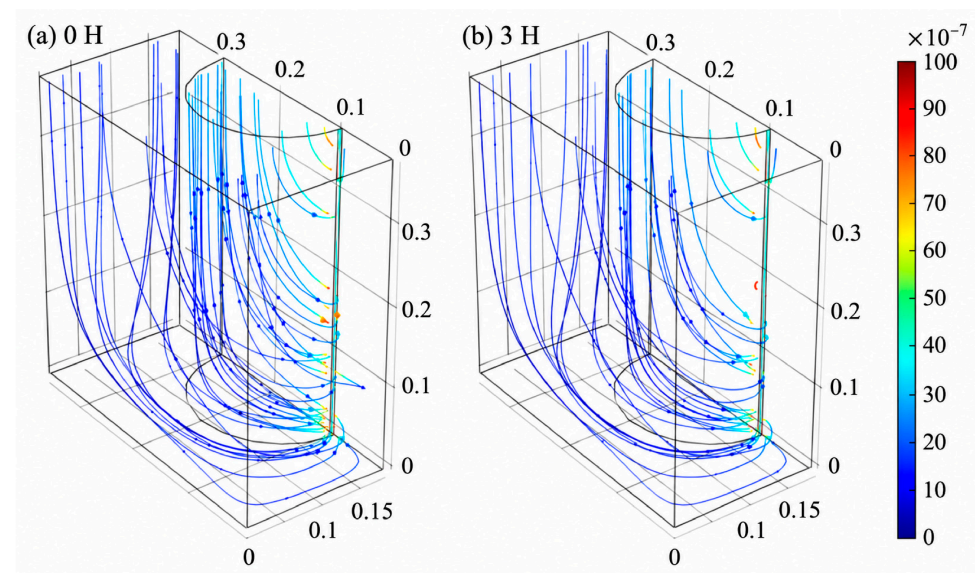


Figure 7. Simulated Darcy velocity streamlines for the crescent-shaped joint: (a) initial field and (b) field at 3 h (m/s).

The comparison provides a short-term hydraulic consistency assessment under the investigated falling-head conditions. The numerical concentration history is retained as a component-scale output for the common receiving-fluid domain.

3.5. Extended Constant-Head Numerical Analyses

The extended analyses used the same $0.40 \times 0.20 \times 0.40$ m three-dimensional half-domain, material properties, thin-barrier representation, and advection-effective-diffusion formulation. The falling-head boundary used for the centrifuge comparison was replaced by fixed hydraulic heads. The baseline head difference was 0.20 m. Additional cases used head differences of 0, 0.10, and 0.30 m. The 0 m case had equal hydraulic heads and therefore represented transport by the implemented effective-diffusion term without an imposed hydraulic gradient.

Three sets of 10-d calculations were performed: comparison of the crescent-shaped, square, and triangular joints with a continuous-HDPE-membrane numerical reference at $\Delta h = 0.20$ m; hydraulic-head-difference sensitivity for the crescent-shaped joint; and prescribed-crack sensitivity for the crescent-shaped joint with intact, 0.5, 1.0, and 1.5 mm cases. Results were stored at 0.5 d intervals. The reported scalar output is the maximum Na^+ concentration in the external domain; the accompanying concentration distributions are shown at 10 d. These are model-scale numerical sensitivity analyses and are not converted to prototype time or service life.

4. Results

4.1. Hydraulic Response of Different Joint Configurations

Figure 8 presents the hydraulic-head records from the individual centrifuge tests. After 2 h, the internal hydraulic-head reductions were 0.064, 0.070, and 0.145 m for the crescent-shaped, square, and triangular configurations, respectively. Thus, among the three investigated individual tests, the crescent-shaped configuration showed the smallest reduction and the triangular configuration the largest.

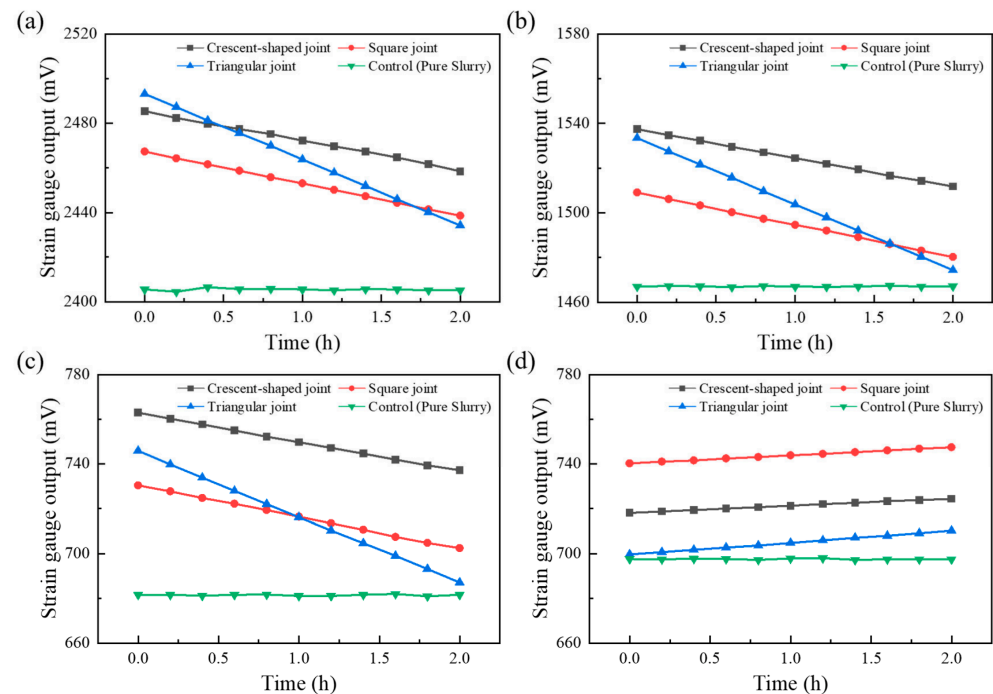


Figure 8. Time evolution of pore pressure at different monitoring points for various joint configurations: (a) 0.3 m depth; (b) 0.2 m depth; (c) 0.1 m depth; and (d) 0.1 m depth outside the cylinder.

The joint-free control showed negligible hydraulic-head variation. The observed external hydraulic-head increases were 0.016, 0.018, and 0.036 m for the crescent-shaped, square, and triangular tests, respectively. These measurements provide a model-scale hydraulic comparison under the prescribed falling-head conditions.

The crescent-shaped configuration was selected as the reference for the subsequent numerical sensitivity analyses because it showed the smallest hydraulic-head reduction among the investigated tests.

4.2. Contaminant Transport Response

The endpoint pNa readings are provided in Appendix A Table A1 as descriptive instrument records. The external solution was a prepared NaCl background solution, and its initial value was measured after filling and immediately before centrifugation.

Because the 0.01 pNa changes are within the manufacturer-stated ± 0.05 pNa basic error, the readings are not used for parameter calibration, quantitative comparison, or model consistency assessment.

4.3. Local-Scale Equivalent Parameters and Engineering-Width Hydraulic Results

(1) Hydraulic Equivalent Parameters

The local-scale equivalent hydraulic conductivities and the corresponding 6.0-m engineering-width results are summarized in Table 5. Differences were obtained among the three investigated configurations, but the limited configuration set does not establish a universal geometry-performance relationship.

Table 5. Hydraulic results for the local-scale and 6.0-m engineering-width intact configurations.

Configuration	Local Discharge/Width ($\times 10^{-16}$ m ² /s)	Local Ratio	Local K_{eq} ($\times 10^{-9}$ m/s)	6 m Discharge/Width ($\times 10^{-16}$ m ² /s)	6 m Ratio	6 m K_{eq} ($\times 10^{-9}$ m/s)
High-k slurry reference	358.0	100%	100.0	10,000.0	100%	100.0
Continuous HDPE membrane	0.048	0.01%	0.013	1.33	0.01%	0.013
Crescent-shaped joint	6.88	1.92%	1.85	8.32	0.08%	0.083
Square joint	7.41	2.07%	2.08	8.69	0.09%	0.087
Triangular joint	17.8	4.97%	5.00	19.4	0.19%	0.194

In the local-scale calculations, K_{eq} was 1.00×10^{-7} m/s for the high-k slurry reference and 1.30×10^{-11} m/s for the continuous HDPE reference. The crescent-shaped, square, and triangular joints gave 1.85×10^{-9} , 2.08×10^{-9} , and 5.00×10^{-9} m/s, respectively. The slurry value is a deliberately high-permeability upper-bound reference, not a representative design value for a completed cutoff wall. Water–cement–bentonite–standard-sand mixtures with measured hydraulic conductivities from 8.06×10^{-8} to 7.46×10^{-7} m/s have been reported [46]. All ratios relative to the high-k slurry reference are interpreted on that basis.

Among the three investigated configurations, the crescent-shaped joint had the lowest local-scale K_{eq} , followed by the square and triangular joints. The triangular value was approximately 2.7 times the crescent-shaped value.

In the separate 6.0 m engineering-width calculations, K_{eq} was 0.013×10^{-9} m/s for the continuous HDPE membrane and 0.083×10^{-9} , 0.087×10^{-9} , and 0.194×10^{-9} m/s for the crescent-shaped, square, and triangular joints, respectively. These deterministic two-dimensional results quantify how each localized joint contributes to the overall barrier width under the stated reference conditions.

In the local-scale static sensitivity analyses, prescribed cracks increased K_{eq} relative to the intact crescent-shaped reference. The values were 5.88×10^{-9} , 8.44×10^{-9} , and 1.11×10^{-8} m/s for initial crack widths of 0.5, 1.0, and 1.5 mm, respectively (Table 6).

Table 6. Hydraulic results for prescribed cracks in the local-scale and 6.0-m engineering-width crescent-shaped-joint models.

Configuration	Local Discharge/Width ($\times 10^{-16}$ m ² /s)	Local Ratio	Local K_{eq} ($\times 10^{-9}$ m/s)	6 m Discharge/Width ($\times 10^{-16}$ m ² /s)	6 m Ratio	6 m K_{eq} ($\times 10^{-9}$ m/s)
High-k slurry reference	358.0	100%	100.0	10,000.0	100%	100.0
Continuous HDPE membrane	0.048	0.01%	0.013	1.33	0.01%	0.013
Intact crescent-shaped joint	6.88	1.92%	1.85	8.32	0.08%	0.083
Crescent joint + 0.5 mm crack	21.3	5.95%	5.88	20.6	0.21%	0.206
Crescent joint + 1.0 mm crack	30.2	8.44%	8.44	30.8	0.31%	0.308
Crescent joint + 1.5 mm crack	37.8	10.56%	11.1	39.8	0.40%	0.398

For the corresponding 6.0-m engineering-width crescent-shaped-joint models, K_{eq} increased from 0.083×10^{-9} m/s for the intact case to 0.206×10^{-9} , 0.308×10^{-9} , and 0.398×10^{-9} m/s for prescribed crack widths of 0.5, 1.0, and 1.5 mm, respectively. These values describe static numerical width-effect scenarios only.

(2) Diffusion Equivalent Parameters

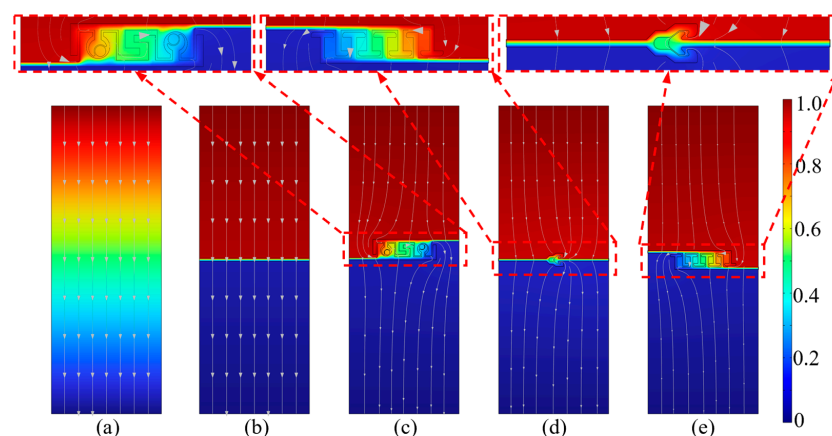
The local-scale equivalent diffusion coefficients are summarized in Table 7. The continuous HDPE reference reduced D_{eq} from 1.52×10^{-10} m²/s for the pure slurry wall to 1.20×10^{-11} m²/s.

Table 7. Local-scale boundary-integrated diffusive rates and equivalent diffusion coefficients.

Model	Boundary-Integrated Diffusive Rate per Unit Width (mol/(m·s))	Ratio	Equivalent Diffusion Coefficient (m ² /s)
Pure slurry	5.45×10^{-11}	100%	1.52×10^{-10}
Continuous HDPE membrane	4.29×10^{-12}	7.87%	1.20×10^{-11}
Crescent-shaped joint	4.90×10^{-12}	8.99%	1.37×10^{-11}
Square joint	5.22×10^{-12}	9.58%	1.46×10^{-11}
Triangular joint	7.10×10^{-12}	13.03%	1.98×10^{-11}

Among the three investigated configurations, D_{eq} was 1.37×10^{-11} , 1.46×10^{-11} , and 1.98×10^{-11} m²/s for the crescent-shaped, square, and triangular joints, respectively. The triangular value was approximately 1.45 times the crescent-shaped value.

Figure 9 shows the simulated concentration fields and total molar-flux streamlines for the local-scale effective-diffusion models.

**Figure 9.** Simulated concentration fields and total contaminant molar-flux streamlines in the local-scale models: (a) pure slurry wall; (b) continuous HDPE geomembrane; (c) crescent-shaped joint; (d) square joint; and (e) triangular joint.

4.4. Extended Constant-Head Sensitivity Analyses

4.4.1. Joint-Configuration Comparison

Figure 10 compares the three joint configurations and the continuous-HDPE-membrane numerical reference under a constant hydraulic-head difference of 0.20 m. The maximum Na^+ concentration in the external domain increased throughout the 10 d calculation. At 10 d, the values were 85.90, 94.26, and 99.30 mol/m^3 for the crescent-shaped, square, and triangular joints, respectively, compared with 8.89711 mol/m^3 for the continuous-membrane reference.

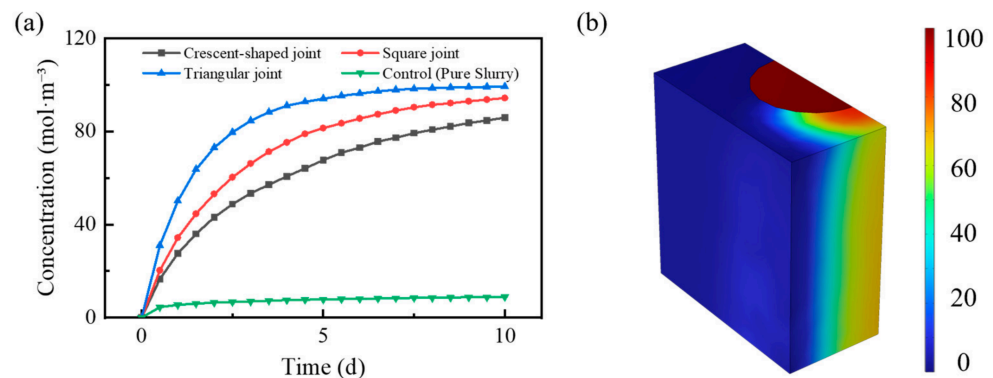


Figure 10. Constant-head numerical comparison of the three joint configurations and the continuous-HDPE-membrane reference: (a) maximum Na^+ concentration in the external domain versus time; and (b) Na^+ concentration field for the crescent-shaped joint at 10 d.

Within the evaluated configurations and parameters, the triangular joint produced the fastest concentration response and the crescent-shaped joint the slowest.

4.4.2. Effect of Hydraulic-Head Difference

Figure 11 compares constant hydraulic-head differences of 0, 0.10, 0.20, and 0.30 m for the crescent-shaped joint. At 10 d, the maximum external-domain Na^+ concentrations were 66.23, 82.84, 93.00, and 96.26 mol/m^3 , respectively. The 0 m case still exhibited transport through the effective-diffusion term, whereas increasing the imposed head difference increased the concentration response through the additional advective contribution.

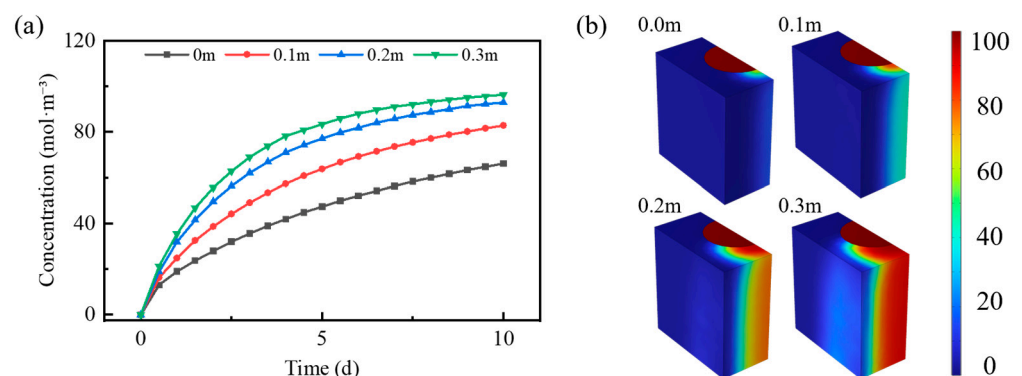


Figure 11. Effect of constant hydraulic-head difference on Na^+ transport: (a) maximum Na^+ concentration in the external domain versus time; and (b) Na^+ concentration fields at 10 d for hydraulic-head differences of 0, 0.1, 0.2, and 0.3 m.

The increment between 0.20 and 0.30 m was smaller than that between the lower-head cases as the reported maximum approached the 100 mol/m^3 internal concentration. These results describe the prescribed model-scale boundary conditions and are not extrapolated to field hydraulic regimes.

4.4.3. Effect of Prescribed Cracks

Figure 12 compares the intact crescent-shaped joint with prescribed 0.5, 1.0, and 1.5 mm cracks under the 0.20 m constant head difference. At 10 d, the maximum external-domain Na^+ concentration was 93.00 mol/m³ for the intact reference and 99.33, 99.43, and 99.33 mol/m³ for the 0.5, 1.0, and 1.5 mm cracks, respectively.

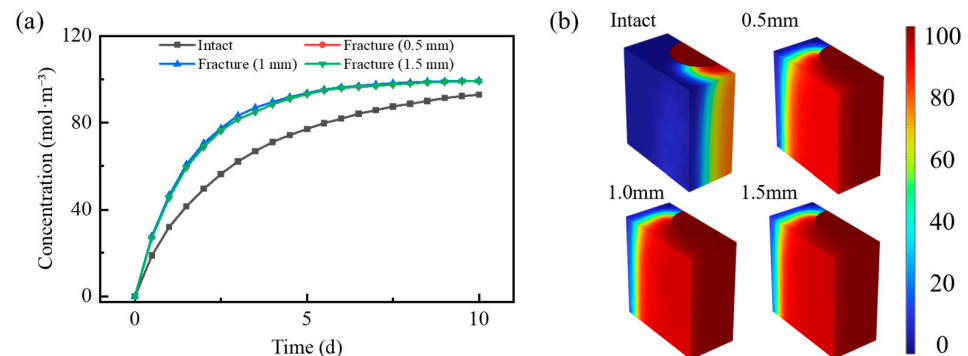


Figure 12. Effect of prescribed crack width on Na^+ transport through the crescent-shaped joint: (a) maximum Na^+ concentration in the external domain versus time; and (b) Na^+ concentration fields at 10 d for the intact reference and prescribed crack widths of 0.5, 1.0, and 1.5 mm.

The prescribed cracks accelerated the concentration rise relative to the intact reference. However, the three cracked cases were closely grouped and non-monotonic with crack width. The calculations therefore indicate that the presence of a crack affects contaminant transport under the evaluated conditions but do not establish a monotonic relationship with crack width.

5. Discussion

5.1. Configuration-Specific Hydraulic Response

Among the three investigated configurations, the crescent-shaped joint showed the smallest hydraulic-head reduction and the lowest local-scale K_{eq} , whereas the triangular joint showed the largest values. The local-scale fields suggest that differences in flow-path length and local resistance can contribute to this configuration-specific response. This is a numerical interpretation of the evaluated fields, consistent with the sensitivity of fracture flow to aperture and pathway geometry [44,45].

For the same reference area, length, and hydraulic loading, the triangular-to-crescent conductivity ratio of approximately 2.7 corresponds to a hydraulic-resistance ratio of about 0.37. Aperture-dependent permeability and resistance along the connected path provide a physical explanation for the configuration comparison. Defect and seam studies [27–29] and the joint-defect work of Cao et al. [25] similarly identify geometry and support/contact conditions as relevant. The relatively small crescent–square difference also makes measured manufacturing and installation tolerances important for subsequent repeatability assessment.

Minimum aperture, joint area, perimeter, corner geometry, and tortuosity varied simultaneously. The three configurations therefore cannot establish a universal geometry-performance law. A future systematic study could use normalized minimum aperture b_{min}/T , normalized joint area A_j/T^2 , normalized perimeter P_j/T , flow-path tortuosity L_{path}/T , compactness $4\pi A_j/P_j^2$, and normalized minimum internal angle θ_{min}/π , where T is a consistently defined wall thickness, and vary these descriptors independently.

5.2. Contaminant Transport and Strength of Evidence

The numerical formulation accounts for advection driven by Darcy flow and effective molecular diffusion. The calculated D_{eq} values and concentration fields describe joint-

controlled transfer across the barrier component into the common receiving-fluid domain. Mechanical dispersion, sorption, reaction, decay, and concentration-dependent density were not included because the purpose was to isolate the joint effect rather than reproduce a site-specific surrounding-soil attenuation system.

The joint-controlled receiving-fluid response differs from transport through the defective composite wall and its interfaces studied by Huang et al. [24]. K_{eq} and D_{eq} describe different processes and units; their reduction factors do not directly quantify the relative transported mass. In particular, a local concentration maximum can approach the source concentration while plume extent and cumulative transfer continue to differ. This helps explain the closely grouped cracked-case endpoints; earlier response and integrated flux would provide complementary evaluation quantities [30–33].

Experimentally, each joint test showed a pNa change of 0.01 within the manufacturer-stated ± 0.05 pNa basic error. The readings are therefore reported as descriptive records in Appendix A Table A1.

5.3. Applicability of the Multiscale Framework

The multiscale framework transferred K_{eq} and D_{eq} from the explicit local-scale joint models to the three-dimensional thin-barrier representation without calibration against the centrifuge measurements. All remaining inputs were fixed from the specimen, experimental setup, stated model assumptions, or cited properties before the forward calculation. The hydraulic-head records were then used only to assess short-term consistency under the prescribed 20 g condition.

This procedure separates parameter determination from the model-experiment consistency assessment. The centrifuge tests provide hydraulic-response data under the prescribed enhanced-gravity hydraulic condition; a reduced-scale 1 g apparatus would represent a fundamentally different condition. The separate 6.0 m engineering-width calculations are stationary width-effect analyses, and the 10 d constant-head calculations are component-scale sensitivity analyses using the 0.40 m three-dimensional half-domain.

5.4. Engineering Implications

Within the investigated configurations and assumptions, the results identify the joint as a location that warrants deliberate detailing, defect minimization, and inspection-based construction quality control. Available joint-defect studies and damage-detection practice likewise emphasize the importance of identifying discontinuities in geomembrane barriers [20,24,25].

The prescribed crack cases were static numerical sensitivity scenarios; they did not simulate crack initiation, propagation, ageing, or time-dependent opening. The 10 d constant-head calculations show that crack presence can accelerate the modelled concentration response, while the closely grouped 0.5–1.5 mm cases do not support a monotonic relationship between crack width and transport response. Engineering application still requires site-specific hydraulic heads, wall geometry, surrounding strata, material state, installation tolerances, inspection criteria, and monitoring plans.

A field extension would use measured joint-aperture and defect distributions, multiple-joint locations, actual strata, hydraulic boundaries, and chemistry-dependent material properties. Uncertainty propagation [34] provides a way to handle defect variability. Pressure and electrical monitoring of composite walls [35], defect influence-range analysis [36], and reaction-chain transport [37] provide specific routes for representing field geometry, material evolution, and groundwater chemistry. Independent leakage and concentration records would then evaluate the resulting wall-aquifer model.

Related material studies provide context for field extension: Diao et al. [47] linked cyclic deterioration of biomimetic cemented soil to pore-structure and hydraulic-

conductivity changes, while Zheng et al. [48] related the mechanical evolution of rapid-setting cement-based slurry to hydration and pore refinement.

6. Conclusions

This study combined individual 20 g centrifuge tests with a multiscale numerical framework to compare three predefined HDPE geomembrane joint configurations. Within the investigated conditions, the following conclusions are supported:

- (1) Among the three investigated configurations, the crescent-shaped joint showed the smallest 2-h hydraulic-head reduction and the lowest local-scale equivalent hydraulic conductivity, followed by the square and triangular joints. The triangular K_{eq} was approximately 2.7 times the crescent-shaped value.
- (2) In the separate local-scale numerical indices, the continuous HDPE reference reduced K_{eq} by approximately four orders of magnitude relative to the high-k slurry reference, while D_{eq} decreased by approximately one order of magnitude relative to the slurry-only diffusion reference. For the 6.0 m engineering-width models, K_{eq} was 0.013×10^{-9} m/s for the continuous membrane and 0.083×10^{-9} , 0.087×10^{-9} , and 0.194×10^{-9} m/s for the crescent-shaped, square, and triangular joints, respectively. The hydraulic ratios are explicitly referenced to the prescribed high-k slurry case.
- (3) In the 10-d constant-head extension of the three-dimensional model, the maximum external-domain Na^+ concentration increased with hydraulic-head difference. At $\Delta h = 0.20$ m, the 10 d values were 85.90, 94.26, and 99.30 mol/m³ for the crescent-shaped, square, and triangular joints, respectively, compared with 8.90 mol/m³ for the continuous-membrane reference. These values describe component-level transfer into the common receiving-fluid domain.
- (4) Prescribed 0.5, 1.0, and 1.5 mm cracks accelerated the concentration response relative to the intact numerical reference, but their 10-d maximum concentrations were closely grouped and non-monotonic. These results support the importance of defect minimization and inspection within the investigated configurations.
- (5) The local-scale equivalent parameters were transferred to the three-dimensional half-domain without calibration against the centrifuge measurements. The individual hydraulic records provide a short-term model-experiment consistency assessment under the prescribed 20 g test condition; the experimental Na^+ readings are retained as descriptive records.

Overall, the findings provide a component-scale basis for joint detailing and defect control. The comparison is limited to three predefined configurations, individual short-duration tests and the stated Na^+ measurement accuracy. Further work should combine replicate specimens, more sensitive tracer measurements and longer tests with matched-boundary checks of parameter transfer and a wider range of defect sizes and locations. Incorporating field variability, groundwater chemistry and material ageing, supported by independent monitoring, would extend the approach toward site-specific long-term assessment.

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Appendix A

Table A1. Descriptive Na⁺ readings before and after the 2 h centrifuge tests.

Joint Type	Initial pNa	Final pNa	Initial Na ⁺ (mol/m ³)	Final Na ⁺ (mol/m ³)
Crescent-shaped	1.44	1.43	36.3	37.1
Square	1.44	1.43	36.3	37.1
Triangular	1.45	1.44	35.5	36.3
Joint-free control	1.46	1.46	34.7	34.7

Note: Each value is a single instrument reading. The observed 0.01 pNa changes fall within the manufacturer-stated basic error of ± 0.05 pNa. The readings are retained as descriptive records. The initial values are measurements of the prepared external NaCl background solution after filling and before centrifugation.

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