

Article

Damage Detection in FGM Plates Using Modal Strain Energy and Gaussian Process Regression

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Abstract

The extensive use of composite materials, particularly Functionally Graded Materials (FGMs), across a wide range of industrial applications has generated a critical demand for robust and reliable structural health monitoring (SHM) and damage detection systems to ensure operational safety and long-term structural integrity. In this paper, a numerically validated comprehensive SHM framework is proposed for FGM plates through finite element investigations, based on modal strain energy analysis and employing isogeometric analysis (IGA) for accurate dynamic response evaluation. The proposed damage detection methodology exploits variations in modal strain energy characteristics induced by structural degradation. In the first stage, a damage detection index derived from modal strain energy distributions is used to efficiently and accurately identify the location of damage within the structure. In the second stage, the identified damage indices are employed as input features within a Gaussian-Process-Regression (GPR)-based framework, while the corresponding damage-severity levels are treated as outputs, enabling reliable damage quantification for FGM components. A series of numerical damage scenarios are systematically investigated to assess the performance of the proposed approach. The numerical results demonstrate the high predictive accuracy and effectiveness of the proposed methodology under the investigated damage scenarios and simulation conditions, highlighting its potential for vibration-based damage assessment of FGM plates.

Keywords: FGM; IGA; SHM; dynamic analysis; GPR



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1. Introduction

Damage detection in vibrating structures constitutes a critical issue, as even localized or minor defects may compromise structural integrity and induce significant changes in dynamic behavior. Consequently, the identification, detection, and quantification of structural damage have become paramount concerns in civil, aerospace, and mechanical engineering. This need is further intensified by the widespread use of advanced engineering systems such as aircraft, spacecraft, and wind turbines and by the aging of existing infrastructures, including bridges and towers, where ensuring structural reliability is essential to prevent catastrophic failures and loss of life.

Modern vibration-based damage identification (VBDI) techniques can be traced back to pioneering studies on offshore oil platforms conducted during the 1970s and 1980s.

Among the earliest contributions, Lifshitz and Rotem [1] measured the resonant frequencies and damping characteristics of quartz-particle-filled epoxy and polyester specimens. Damage was induced through static loading, while the dynamic response was continuously monitored throughout the loading process. The results demonstrated that damping was more sensitive to damage than changes in the dynamic moduli.

Structural damage is generally classified into linear and nonlinear categories. Linear damage refers to cases where the structure maintains linear elastic behavior after damage, and variations in modal properties arise primarily from changes in geometry or material characteristics. In contrast, nonlinear damage occurs when the post-damage response becomes nonlinear, as observed in fatigue cracks that open and close during vibration.

Within the broader framework of structural health monitoring (SHM), Dulieu-Barton [2], proposed a hierarchical classification consisting of five levels: (i) damage detection, (ii) damage localization, (iii) damage classification, (iv) damage assessment and quantification, and (v) prognosis of remaining useful life. In this hierarchy, non-model-based methods (NMBMs) are generally associated with the first two levels, while model-based methods (MBMs) are capable of addressing higher levels, particularly damage classification and assessment. The highest level, prognosis, typically requires fracture-mechanics-based approaches and remains beyond the scope of conventional modal analysis techniques.

In recent years, SHM has increasingly benefited from data-driven and machine-learning techniques capable of extracting damage-related information directly from measured structural responses. Unlike traditional model-based approaches, which rely on accurate numerical representations of the structure and sensitivity analyses, data-driven methods establish relationships between structural features and damage states through statistical learning. Recent developments include kernel-based regression models for predicting long-term structural responses under environmental variability, machine-learning techniques for vibration-response forecasting of bridge structures, anomaly detection methods for operational monitoring, and non-parametric learning approaches for short-term damage alarming under limited vibration measurements [3–7]. These methods have demonstrated promising capabilities in handling complex nonlinear relationships and noisy data, making them attractive tools for modern SHM applications. Nevertheless, their application to damage quantification in functionally graded material (FGM) structures remains relatively limited, particularly when coupled with physically meaningful vibration-based damage indicators [8].

During their service life, FGM plates may be subjected to different degradation mechanisms depending on the constituent materials, loading conditions, and environmental exposure. Typical damage mechanisms include localized cracking, fatigue-induced degradation, impact-related damage, local material deterioration, porosity or void-related defects, and degradation of the interface or transition region between dissimilar constituent materials. Such damage mechanisms can modify the local stiffness, mass distribution, and stress–strain characteristics of the structure and consequently affect its dynamic response. In particular, stiffness-related degradation generally produces changes in modal strain energy distributions and natural frequencies, whereas damage mechanisms involving crack opening and closing, interfacial effects, or other nonlinear phenomena may produce more complex changes in the vibration response. Therefore, the dynamic signature associated with a specific damage mechanism depends not only on its severity and location but also on its physical nature.

While vibration-based damage identification methods have shown considerable promise, it is important to note that natural frequencies and mode shapes can be significantly influenced by environmental and operational conditions, including temperature variations, humidity, and loading conditions [3–7]. These effects can mask or mimic

damage-induced changes, potentially leading to false positives or missed detections. The present numerical study is conducted under idealized conditions without considering such environmental variability; however, future experimental validation will need to address these effects, as discussed in the conclusions.

Functionally graded materials (FGMs) are sophisticated composite materials with smoothly varying mechanical and thermal properties throughout thickness. The combination of the beneficial qualities of metals and ceramics, such as toughness, machinability, and high resistance to heat and oxidation, is made possible by this continuous fluctuation, which also prevents acute material boundaries. The vibration behavior of FGM plates has been the subject of numerous investigations using various theoretical and numerical techniques. In order to increase the precision of vibration predictions, higher-order shear deformation theories have been used extensively. To illustrate the significance of higher-order kinematic assumptions, Neves et al. [9] created a quasi-3D higher-order shear deformation theory to examine the free vibration behavior of FGM plates. Vibration analysis of FGM constructions has also made extensive use of numerical methods like the finite element approach. In their research on the free vibration of functionally graded plates, Ferreira et al. [10] used sophisticated numerical formulations and showed how well higher-order models predicted natural frequencies. Isogeometric analysis (IGA) has become a potent numerical technique that connects structural analysis and computer-aided design in recent years. In contrast to traditional FEM, Hughes et al. [11] presented IGA as a framework that uses the Non-Uniform Rational B-splines (NURBS) basis functions to express both geometry and displacement fields, offering greater continuity and precise geometric representation.

Various analytical theories, including the Equivalent Single Layer (ESL) approach and three-dimensional (3D) elasticity theory, were presented and discussed in Ref. [12]. The dynamic behavior of a functionally graded piezoelectric plate using a finite element (FE) model based on First-Order Shear Deformation Theory (FSDT) was presented by Bendine et al. [13]. Tran, T. V. et al. [14] presented a static bending behavior of functionally graded (FG) sandwich spherical shells, which was analyzed using a triangular shell element that combines the Edge-based Smoothed Finite Element Method (ES-FEM) with the Mixed Interpolation of Tensorial Components (MITC) technique.

Magliacano et al. [15] propose a machine-learning-based approach for predicting the structural health of composite aircraft wing boxes. A comprehensive dataset was generated using FE simulations of a Cirrus-class carbon-fiber wing box. Material properties and damage descriptors are used as input features to train two regression models. The proposed machine-learning surrogate enables near-instant prediction of composite failure indices, significantly reducing the computational cost compared with conventional finite element analyses.

SHM and damage assessment have predominantly relied on analyzing a system's dynamic response and its evolution, an approach formally termed vibration-based damage identification (VBDI). This methodology exploits variations in dynamic signatures across the time, frequency, and modal domains, implemented either in isolation or through hybridized frameworks. Over the past few decades, the proliferation of VBDI techniques has led to a substantial body of literature, as comprehensively documented in the seminal reviews [16–18].

For FGM structures, damage identification has traditionally relied on model-based vibration analysis techniques. Yang et al. [19] investigated modal strain energy sensitivity for damage localization in functionally graded beams. Lu et al. [20] proposed a hybrid sensitivity matrix approach for damage identification in axially functionally graded beams. Dinh-Cong and Nguyen-Thoi [21,22] developed model-updating procedures for deterministic and probabilistic damage identification in FGM structures. More recently, Khatir et al. [23]

integrated machine learning and optimization techniques for damage assessment in FGM composite plates. Although these studies demonstrated the effectiveness of vibration-based indicators, most investigations focused either on damage localization or on direct machine-learning prediction without explicitly coupling a physically interpretable damage-sensitive indicator with a probabilistic regression framework for damage quantification.

Despite the significant progress achieved in vibration-based SHM, several challenges remain for damage assessment in FGM structures. First, accurate localization and reliable quantification of damage are often treated as separate tasks. Second, many machine-learning approaches require large datasets and provide limited physical interpretability of the predicted damage states. Third, the application of Gaussian Process Regression (GPR), which offers strong predictive capability together with uncertainty-aware learning, remains largely unexplored for damage quantification in FGM plates using modal strain energy-based indicators. Therefore, a unified framework capable of combining physics-based damage localization with data-driven damage quantification is still needed.

2. FGM Plates and NURBS Formulations

The Japanese researchers developed functionally graded materials (FGMs) with microscopically inhomogeneous properties in the mid-1980s [24]. FGMs are members of a composite material family. FGM plate is fabricated from two materials, for example, ceramic and metal, that vary smoothly and continuously through the thickness direction. The top surface $z = h/2$ is pure ceramic and the bottom surface $z = -h/2$ is pure metal. The distribution of FGMs is constructed as follows:

$$\begin{cases} E(z) = (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n + E_m \\ \nu(z) = (\nu_c - \nu_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n + \nu_m \end{cases} \quad (1)$$

where $E(z)$ and $\nu(z)$ are Young's modulus and Poisson's ratio at thickness coordinate z and n denotes the power-law exponent controlling the through-thickness variation in the material volume fraction, while E_c and E_m are the Young's moduli of ceramic and metal, respectively, and ν_c and ν_m are the corresponding Poisson's ratios.

Based on the displacement fields presented by Cuong Le et al. [25–27], the strains can be derived as

$$\begin{cases} \varepsilon = \begin{Bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} \end{Bmatrix}^T = \varepsilon_0 + z\kappa_1 + f(z)\kappa_2 \\ \gamma = \begin{Bmatrix} \gamma_{xz} & \gamma_{yz} \end{Bmatrix}^T = f'(z)\varepsilon_s \end{cases} \quad (2)$$

$$\varepsilon_0 = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}; \kappa_1 = -\begin{Bmatrix} w_{0,xx} \\ w_{0,yy} \\ 2w_{0,xy} \end{Bmatrix}; \kappa_2 = \begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix}; \varepsilon_s = \begin{Bmatrix} \beta_x & \beta_y \end{Bmatrix}^T \quad (3)$$

In these equations, ε_x , ε_y , and γ_{xy} are the normal and shear strain components, while κ_x , κ_y , and κ_{xy} are the curvatures. The terms ε_{xz} and ε_{yz} represent the transverse shear strains.

For an FGM plate, the stress–strain relation can be obtained from Ref. [25] as follows:

$$\begin{cases} \sigma = \begin{Bmatrix} \sigma_x & \sigma_y & \tau_{xy} \end{Bmatrix}^T = C(\varepsilon - \varepsilon^{th}) \\ \tau = \begin{Bmatrix} \tau_{xz} & \tau_{yz} \end{Bmatrix}^T = G\gamma \end{cases} \quad (4)$$

where

$$C = \frac{E(z)}{1+v(z)} \begin{bmatrix} \frac{1}{1-v(z)} & \frac{v(z)}{1-v(z)} & 0 \\ \frac{v(z)}{1-v(z)} & \frac{1}{1-v(z)} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}; G = \frac{E(z)}{2(1+v(z))} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

Here, σ_x and σ_y , denote the normal stresses, whereas τ_{xy} , τ_{xz} , and τ_{yz} represent the in-plane and transverse shear stress components associated with the adopted higher-order plate theory.

Substituting the higher-order forces yields the following:

$$\begin{aligned} \left\{ \begin{matrix} N_{ij} & M_{ij} & P_{ij} \end{matrix} \right\}^T &= \int_{-h/2}^{h/2} \sigma_{ij} \begin{Bmatrix} 1 & z & f(z) \end{Bmatrix}^T dz \quad ij : = x, y; \\ Q_{\alpha z} &= \int_{-h/2}^{h/2} \tau_{\alpha z} f'(z) dz \quad \alpha : = x, y \end{aligned} \quad (6)$$

Into Equation (6), the matrix form of stress–strain resultants of the FGM plate is obtained as

$$\hat{\sigma} = \begin{Bmatrix} N^u \\ M^u \\ P^u \\ Q^u \end{Bmatrix} = \begin{bmatrix} A^u & B^u & E^u & 0 \\ B^u & D^u & F^u & 0 \\ E^u & F^u & H^u & 0 \\ 0 & 0 & 0 & D^{us} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \kappa_1 \\ \kappa_2 \\ \varepsilon_s \end{Bmatrix} = \hat{D}_u \hat{\varepsilon} \quad (7)$$

$$(A^u, B^u, D^u, E^u, F^u, H^u) = \int_{-h/2}^{h/2} (1, z, z^2, f(z), zf(z), f^2(z)) C dz \quad (8)$$

$$D^{us} = \int_{-h/2}^{h/2} [f'(z)]^2 G dz \quad (9)$$

In these expressions, N , M , and P are the force, moment, and higher-order resultants, respectively. A^u , B^u , D^u , E^u , F^u , and H^u are the stiffness coefficients integrated through the thickness.

The B-spline basis function is constructed recursively by using the Cox–De Boor algorithm, starting with a polynomial of degree $p = 0$ as follows:

$$\begin{cases} p = 0, & N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \\ p \geq 1, & N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \end{cases} \quad (10)$$

where $\xi_i \in R$, p denotes the knot and the order of polynomial functions, respectively, $i = 1, 2, \dots, n + p + 1$ is the knot index and n is the number of basic functions. The value of the knot is taken from the knot vector $\Xi = \{\xi_1, \xi_2, \dots, \xi_{n+p+1}\}$.

A tensor product of two 1D B-splines with polynomial degrees of p and q is employed to establish the non-uniform rational basis spline (NURBS) functions as follows:

$$X(\xi, \eta) = \frac{N_i^p(\xi) M_j^q(\eta) w_{ij}}{\sum_{i=1}^n \sum_{j=1}^m N_i^p(\xi) M_j^q(\eta) w_{ij}} \quad (11)$$

where w_{ij} is the control weight. $N_i^p(\xi)$ and $M_j^q(\eta)$ are the univariate B-spline basis functions in the ξ and η directions, and p and q are the polynomial degrees.

Using the Non-Uniform Rational B-Splines (NURBS) approximation, the displacement field can be expressed as

$$u^h = \sum_{I=1}^{m \times n} N_I d_I \quad (12)$$

The NURBS basis function defined in Equation (11) is hereafter denoted by $R_I(\xi, \eta)$, where I represents the global control-point index.

$$u(\xi, \eta) = \sum R_I(\xi, \eta) q_I \quad (13)$$

where R_I denotes the NURBS basis functions associated with control point I , and q_I represents the vector of control-point degrees of freedom.

The displacement derivatives required for strain evaluation are obtained as

$$\begin{aligned} \frac{\partial u}{\partial x} &= \sum \frac{\partial R_I}{\partial x} q_I \\ \frac{\partial u}{\partial y} &= \sum \frac{\partial R_I}{\partial y} q_I \end{aligned} \quad (14)$$

Substituting these expressions into the strain–displacement relations given in Equations (2) and (3) yields

$$\varepsilon = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} & \varepsilon_{xz} & \varepsilon_{yz} \end{bmatrix}^T \quad (15)$$

Consequently, the strain field can be written in compact matrix form as

$$\varepsilon = B_I q_I \quad (16)$$

where B is the strain–displacement matrix assembled from the spatial derivatives of the NURBS basis functions.

Substituting Equation (10) into Equation (7), the matrix form of strain components can be rewritten as Equation (17).

$$\begin{aligned} \hat{\varepsilon} &= \sum_{I=1}^{m \times n} B_I d_I = \sum_{I=1}^{m \times n} \begin{bmatrix} (B_I^m)^T & (B_I^{b_1})^T & (B_I^{b_2})^T & (B_I^s)^T \end{bmatrix}^T \\ B_I^m &= \begin{bmatrix} N_{I,x} & 0 & 0 & 0 & 0 \\ 0 & N_{I,y} & 0 & 0 & 0 \\ N_{I,y} & N_{I,x} & 0 & 0 & 0 \end{bmatrix}; B_I^{b_1} = - \begin{bmatrix} 0 & 0 & N_{I,xx} & 0 & 0 \\ 0 & 0 & N_{I,yy} & 0 & 0 \\ 0 & 0 & 2N_{I,xy} & 0 & 0 \end{bmatrix} \end{aligned} \quad (17)$$

In this equation, B_I^m is the strain–displacement matrix containing derivatives of the NURBS basis functions and d_I is the displacement vector connected with control point I and R_I denotes the shape function as provided in [28].

From the principle of virtual displacement and the Garlekin weak form [25,26,29–31], the stiffness matrix and mass matrix are defined as follows

$$K = \int_{\Omega} B^T \hat{D} B d\Omega; M = \int_{\Omega} \tilde{R}^T m \tilde{R} d\Omega \quad (18)$$

Here, B_I denotes the strain–displacement matrix associated with the I -th control point and is constructed from the derivatives of the corresponding NURBS basis function. The matrix B appearing in the global finite element formulation represents the assembled strain–displacement matrix obtained by combining the contributions of all control points over the element. Thus, B_I is a local contribution, whereas B denotes the complete strain–displacement matrix used in the stiffness matrix formulation.

$$\begin{aligned} \tilde{R} &= \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix}; R_1 = \begin{bmatrix} N_I & 0 & 0 & 0 & 0 \\ 0 & 0 & -N_{I,y} & 0 & 0 \\ 0 & 0 & 0 & N_I & 0 \end{bmatrix} \\ R_2 &= \begin{bmatrix} 0 & N_I & 0 & 0 & 0 \\ 0 & 0 & -N_{I,y} & 0 & 0 \\ 0 & 0 & 0 & 0 & N_I \end{bmatrix}; R_3 = \begin{bmatrix} 0 & 0 & N_I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (19)$$

Here, K is the global stiffness matrix, M is the global mass matrix, and ω represents the natural frequencies. The stiffness matrix is derived from the strain energy formulation, while the mass matrix accounts for the kinetic energy of the FGM plate.

3. Validation

The convergence study is performed for simply supported square FGPs in terms of the number of nodes in FEM [32]. For IGA, NURBS order three is used to check the effectiveness of IGA compared with high-order NURBS in Ref. [33] of Aluminium (Al) and Alumina (Al_2O_3). The mechanical properties, of FGM plate in Figure 1, are provided in Table 1. Plates with the length-to-thickness ratios $a/h = 5$ and 10 are considered. Next, the values of the volume fraction exponent are taken as $V_f = 0, 0.5, 1, 4, 10$. The results are summarized in Table 2.

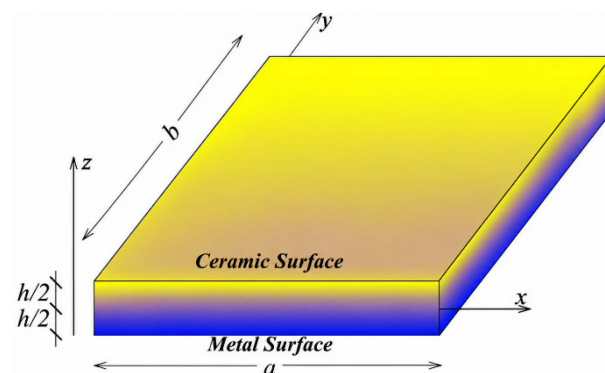


Figure 1. Functionally graded for simply supported square plate.

Table 1. Mechanical properties of the FGM components.

Material	Properties		
	$E [N/m^2]$	ν	$\rho [kg/m^3]$
Aluminium (Al)	70×10^9	0.30	2707
Alumina (Al_2O_3)	380×10^9	0.30	3800

Table 2. Comparison of the natural frequency parameter for simply supported square Al/ Al_2O_3 FG plates ($\bar{\omega} = \omega h \sqrt{\frac{\rho_c}{E_c}}$).

		V_f				
		0	0.5	1	4	10
$a/h = 5$	Present IGA (9×9)	0.2152	0.1841	0.1661	0.1397	0.1317
	FEM (9×9) [32]	0.2018	0.1726	0.1559	0.1332	0.1261
	FEM (17×17) [32]	0.2055	0.1757	0.1587	0.1356	0.1284
	Matsunaga [33]	0.2121	0.1819	0.164	0.1383	0.1306
$a/h = 10$	Present IGA (9×9)	0.0581	0.0493	0.0445	0.0383	0.0366
	FEM (9×9) [32]	0.056	0.0476	0.0429	0.0371	0.0354
	FEM (17×17) [32]	0.0567	0.0481	0.0434	0.0375	0.0359
	Matsunaga [33]	0.0577	0.0491	0.0442	0.0381	0.0364

Table 3 summarizes the five damage scenarios considered to evaluate the robustness of the damage indicator, while Table 4 reports the natural frequencies of the healthy FGM structure and those obtained for the five damaged scenarios.

Table 3. Damage locations and damage-severity levels considered in the numerical investigation.

Scenario	Damaged Element No.	Damage Location	Damage Rate (%)
1	4	Location 1	5–75% in increments of 2.5%
2	20	Location 2	5–75% in increments of 2.5%
3	41	Location 3	5–75% in increments of 2.5%
4	54	Location 4	5–75% in increments of 2.5%
5	55	Location 5	5–75% in increments of 2.5%

Table 4. Natural frequency of undamaged and damaged simply supported square Al/Al₂O₃ FG plates ($\bar{\omega} = \omega h \sqrt{\frac{\rho_c}{E_c}}$).

f	Undamaged	Damaged				
		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
1	0.0581	0.0581	0.0580	0.0580	0.0581	0.0580
2	0.1383	0.1382	0.1381	0.1382	0.1382	0.1382
3	0.1412	0.1411	0.1409	0.1411	0.1410	0.1410
4	0.1948	0.1944	0.1942	0.1948	0.1940	0.1936
5	0.1948	0.1948	0.1948	0.1948	0.1948	0.1948
6	0.2152	0.2150	0.2148	0.2151	0.2149	0.2149

The damaged elements were intentionally selected at different locations within the plate, including boundary, intermediate, and central regions, to evaluate the robustness of the proposed damage localization and quantification framework under varying spatial damage configurations.

In Table 3, “Damaged element No.” denotes the identifier of the computational element in the spatial discretization of the FGM plate and therefore defines the location at which the stiffness reduction is introduced. Each scenario represents a specific damage location, while 29 different damage-severity levels, ranging from 5% to 75% in increments of 2.5%, were considered at each location. Thus, the five scenarios correspond to 145 location–severity configurations in total.

For illustration, Figures 3–7 present selected damage cases for the five investigated locations. These cases are representative examples used to visualize the spatial distribution of the MSEcr indicator and the ability of the proposed approach to identify the damaged element. The complete range of damage-severity levels listed in Table 3 was nevertheless considered in the construction and evaluation of the GPR database.

4. Modal Strain Energy Change Ratio

The stiffness reduction in a structure composed of nel elements, considering the damage parameter $\delta_i (i = 1, 2, \dots, nel)$, is expressed by the following equation:

$$K_d = \sum_{i=1}^{nel} (1 - \delta_i) k_i^e \quad (20)$$

where K_d denotes the damaged stiffness matrix, k_i^e is the stiffness matrix of the i -th element and δ_i is the damage parameter ranging from 0 to 1; 0 corresponds to an intact element and 1 represents a fully damaged element.

It should be noted that the damage representation adopted in Equation (20) is an idealized linear damage model in which the elemental stiffness matrix is uniformly reduced by a scalar damage parameter. This formulation is appropriate for representing stiffness degradation under the assumption of linear elastic structural behavior; however, it has inherent limitations. In particular, it does not explicitly account for directional or

anisotropic stiffness degradation, nor does it reproduce complex damage mechanisms such as delamination, debonding, matrix cracking, or crack-breathing effects associated with nonlinear structural behavior. Therefore, the present formulation should be interpreted as a simplified representation of localized stiffness loss rather than a comprehensive model of all physical damage mechanisms that may occur in composite or functionally graded structures. More advanced damage formulations involving directional stiffness degradation, interface modeling, nonlinear constitutive behavior, or fracture mechanics would be required to represent such mechanisms explicitly. These extensions are beyond the scope of the present proof-of-concept study.

In practical service conditions, FGM plates may experience different forms of degradation, including localized cracking, fatigue-related damage, impact-induced damage, material deterioration, porosity or void-related defects, and degradation of interfaces or transition regions between dissimilar constituents. These mechanisms can affect the structural dynamic response in different ways by modifying the local stiffness, mass distribution, or strain–stress state. Consequently, changes in natural frequencies and modal strain energy distributions, and therefore in the MSE change ratio used in this study, may differ depending on the physical damage mechanism. For example, stiffness-dominated degradation can be reasonably represented by the present equivalent stiffness-reduction model, whereas crack breathing, interfacial damage, or other nonlinear mechanisms may produce more complex changes in the measured modal response.

Damage Localization Using Modal Strain Energy Change Ratio

After obtaining the natural frequencies and mode shapes from Equations (18) and (19), the modal strain energy for each element is computed. For an intact structure, the modal strain energy (MSE) for element j and mode i is given by:

$$MSE_{ij}^h = \frac{1}{2} (\phi_i^h)^T K_j \phi_i^h \quad (21)$$

For a damaged structure, the element stiffness matrix is reduced according to Equation (20), and the modal strain energy becomes

$$MSE_{ij}^d = \frac{1}{2} (\phi_i^d)^T K_j \phi_i^d \quad (22)$$

where i th is the mode number and j th is the element number. The superscript T denotes the vector transpose and ϕ_i represents the mode shape, while the subscripts h and d correspond to the healthy and damaged systems, respectively. K_j denotes the element stiffness matrix.

The modal strain energy change ratio ($MSEcr$) is proposed as an indicator for accurately identifying the damage location. It can be formulated based on the total strain energy of the structure, which is obtained by summing the modal strain energies MSE's of all elements, as expressed below:

$$MSEcr_{ij} = \frac{|MSE_{ij}^d - MSE_{ij}^h|}{MSE_{ij}^h} \quad (23)$$

$$MSEcr_{ij}^{\max} = \max_k \{MSEcr_{ik}\} \quad (24)$$

$$MSEcr_j = \frac{1}{m} \sum_{i=1}^m \frac{MSEcr_{ij}}{MSEcr_{ij}^{\max}} \quad (25)$$

The mean value of the $MSEcr$ indicator is calculated as follows:

$$meanMSEcr = \frac{1}{3} (MSEcr^{(3)} + MSEcr^{(6)} + MSEcr^{(10)}) \quad (26)$$

where $MSEcr^{(3)}$, $MSEcr^{(6)}$, and $MSEcr^{(10)}$ denote the modal strain energy change ratios computed using the first 3, first 6, and first 10 vibration modes, respectively. The superscript indicates the number of vibration modes considered and should not be interpreted as the element index.

The selection of 3, 6, and 10 vibration modes was introduced as a modal sensitivity study rather than as an arbitrary choice. These three modal subsets represent progressively increasing levels of modal information and allow the influence of modal truncation on the MSEcr-based damage localization to be examined. The first three modes provide a low-order modal representation, while six modes provide an intermediate level of information, and ten modes provide a broader representation of the structural dynamic response. The results presented for the five damage scenarios were compared for these three modal subsets. The comparison shows that increasing the number of modes generally improves the spatial concentration and stability of the MSEcr indicator, with the first ten modes providing the clearest and most consistent identification of the damaged element. The comparison shows that increasing the number of modes generally improves the spatial concentration and stability of the MSEcr indicator, with the first ten modes providing the clearest and most consistent identification of the damaged element. Therefore, the first ten modes were considered the most informative modal subset for damage localization. However, for the subsequent GPR-based damage-severity quantification, the mean MSEcr value obtained from the three modal subsets (first 3, first 6, and first 10 modes) was used as the principal input feature. This approach allows the GPR model to incorporate the information obtained from different levels of modal truncation while retaining a single damage-sensitive input variable. The choice was thus based on modal sensitivity and localization stability rather than on modal mass participation factors or an arbitrary numerical selection.

5. Methodology for Damage Rate Prediction

Once the damaged elements have been detected and localized using the MSEcr indicator, the subsequent step involves quantifying their damage severity. This entails determining the extent of degradation for each identified element. A dedicated methodology, namely Gaussian Process Regression (GPR), is then employed to accurately assess the damage level of the localized elements, ensuring precise and reliable damage quantification.

5.1. Problem Formulation

Structural damage leads to stiffness degradation, which directly affects the dynamic characteristics of structures. In this study, damage severity is quantified in terms of damage rate percentage, while the input feature is derived from vibration-based indicators, namely the mean value of the damage-sensitive index (MSEcr), which is computed from the modal strain energy derived from the identified mode shapes. These modal parameters are directly obtained from the modal analysis performed using the isogeometric formulation. The mode shapes are subsequently employed to compute the modal strain energy change ratio (MSEcr), which serves as the damage-sensitive feature for the regression model.

The objective is to learn a functional relationship:

$$\gamma = f(X) + o \quad (27)$$

where

$X \in \mathbb{R}$ represents the mean MSEcr value, $\gamma \in \mathbb{R}$ denotes the corresponding damage rate percentage, $f(\cdot)$ is an unknown nonlinear mapping and $o \sim \mathcal{N}(0, \sigma_o^2)$ is additive Gaussian noise.

This formulation corresponds to a supervised regression problem aimed at predicting the damage rate from vibration-based indicators.

5.2. Dataset Construction

For each element in the generated dataset, multiple measurements of the mean MSEcr value are available, while the associated damage rate remains constant.

Let

$$\Delta = \{(X_i, \gamma_i)\}_{i=1}^N \quad (28)$$

where N is the total number of samples obtained by aggregating all measurements corresponding to the considered damage scenarios. This strategy increases data density while preserving the damage characteristics associated with each damaged element.

The IGA model employed NURBS basis functions of order 3 with a 9×9 control point mesh. The first ten vibration modes were extracted for each damage scenario, from which the mode shape vectors φ were computed at each control point. The element stiffness matrices (K_j^e) were assembled following Equations (18) and (19). For each damage scenario, mode shapes corresponding to 3, 6, and 10 modes were extracted to evaluate the sensitivity of the MSEcr indicator, as illustrated in Figures 3a–7a.

To separate the effects of damage location and damage severity, multiple damage-severity levels were considered at each investigated damage location. Five damaged elements were investigated, namely Elements 4, 20, 41, 54, and 55. For each damaged element, the stiffness reduction was varied from 5% to 75% with an increment of 2.5%, resulting in 29 distinct damage-severity levels for each location. Therefore, the numerical database contains $5 \times 29 = 145$ location–severity configurations.

For each location–severity configuration, the modal strain energy change ratio (MSEcr) was evaluated using the first 3, first 6, and first 10 vibration modes. The mean MSEcr value was then calculated from these three modal configurations and used as the principal damage-sensitive input feature for the GPR model. The resulting database therefore preserves the variation associated with both damage location and damage severity rather than assigning a single severity level to each damaged element.

The use of multiple severity levels at every damage location enables the proposed regression model to learn the variation in the damage-sensitive indicator with increasing damage severity while reducing the possibility that the model merely recognizes a specific damage-location/severity combination. This dataset construction is particularly important for assessing whether the GPR model can generalize its severity prediction across different damaged elements.

The spatial resolution of the damage localization is governed by the adopted IGA discretization. In the present model, the FGM plate is discretized into 81 computational elements, and each element represents a distinct spatial location in the plate. Accordingly, the element numbers shown in Figures 3–7 are element identifiers rather than physical dimensions or coordinates. The damaged elements considered in this study are Elements 4, 20, 41, 54, and 55, as specified in Table 3. Therefore, the localization resolution of the proposed MSEcr-based approach corresponds to the spatial resolution of the adopted element discretization. It should be emphasized that the element size is determined by the geometric dimensions and the adopted IGA discretization; it should not be interpreted as a universal 1×1 cm resolution. A finer IGA discretization would provide a finer spatial localization resolution, at the expense of increased computational cost.

5.2.1. Scenario-Level and Leave-One-Location-Out Validation

To avoid information leakage between training and testing data, the samples were not randomly divided at the individual-sample level. Since multiple observations were generated for the same damaged element and damage severity, a random sample-wise split could place highly correlated observations from the same damage scenario in both the

training and testing sets, leading to an overestimation of the generalization capability of the GPR model.

Therefore, a leave-one-location-out cross-validation (LOLO-CV) strategy was adopted. The five investigated damage locations, corresponding to Elements 4, 20, 41, 54, and 55, were treated as independent groups. In each validation fold, all observations associated with one damaged element were completely excluded from model training and used exclusively for testing, while the observations associated with the remaining four damaged elements were used for training. This procedure was repeated five times so that each damage location was used once as an unseen test location.

Importantly, all damage-severity levels and associated modal configurations belonging to the held-out damage location were retained exclusively in the corresponding test set. Hyperparameter optimization was performed using only the training locations within each fold. The final performance was then evaluated on the completely unseen damage location. This validation strategy provides a more stringent assessment of the ability of the proposed GPR model to generalize to damage locations that were not represented during training.

To further reduce the risk of overfitting and improve model robustness, 5-fold cross-validation was employed during the training stage. The training dataset was randomly partitioned into five subsets. In each fold, four subsets were used for model training and the remaining subset was used for validation. The final GPR model was subsequently trained using the optimal hyperparameters identified through the cross-validation procedure.

To improve the clarity and reproducibility of the proposed methodology, Figure 2 illustrates the complete workflow adopted for dataset generation, preprocessing, training, validation, and damage-severity prediction. The flowchart summarizes the sequence from the isogeometric modal analysis and modal strain energy computation to the construction of the regression dataset and the subsequent GPR model development.

5.2.2. Generation of MSE_{cr} Variability for Each Damage Scenario

For each damage location, the complete set of 29 damage-severity levels, ranging from 5% to 75% in increments of 2.5%, was considered. The corresponding mean MSE_{cr} values obtained from the modal analyses were used to construct the machine-learning dataset. Thus, the numerical database comprised 145 location–severity configurations, corresponding to five investigated damage locations and 29 severity levels for each location.

To account for moderate input variability, a small Gaussian perturbation was introduced into the mean MSE_{cr} values used as inputs to the GPR model. A relative uncertainty level of 2% was adopted for this sensitivity analysis, while the corresponding damage-severity values were kept unchanged. This procedure was used to examine the sensitivity of the regression predictions to moderate variations in the damage-sensitive input while preserving the underlying location–severity configurations. The perturbed feature was generated as follows:

$$MSE_{cr}^* = MSE_{cr} \times (1 + o) \quad (29)$$

where MSE_{cr} is the original mean modal strain energy change ratio, MSE_{cr}^* is the perturbed value used as the GPR input, and ε is a random error term following a zero-mean Gaussian distribution: $o \sim \mathcal{N}(0, 0.02^2)$.

The complete database was subsequently evaluated using the leave-one-location-out cross-validation (LOLO-CV) strategy. Accordingly, the observations associated with one damaged element, including all 29 severity levels, were excluded from model training and used exclusively for testing in the corresponding validation fold. The five investigated damage locations were therefore evaluated as unseen locations rather than being randomly divided into training and testing samples.

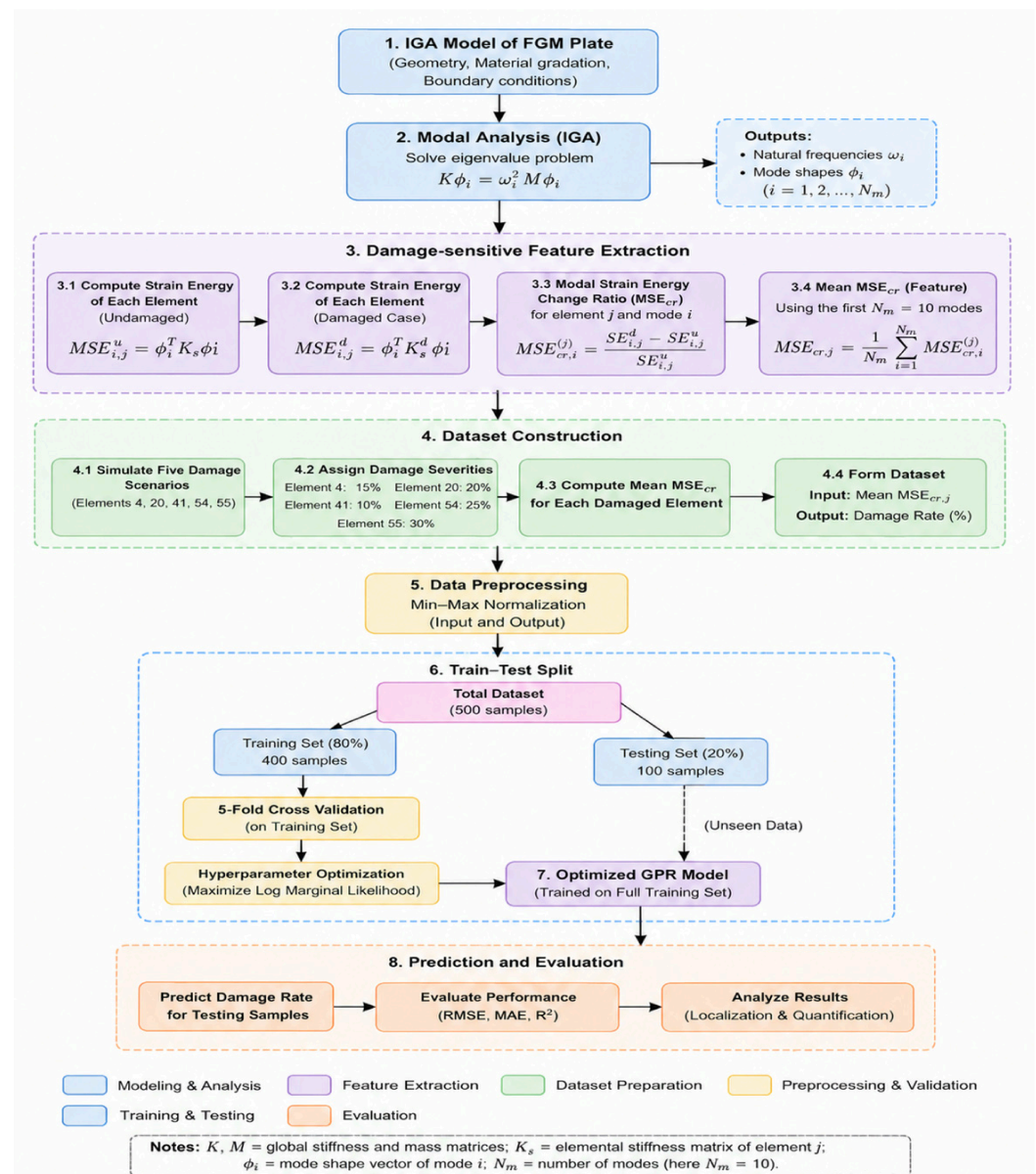


Figure 2. Workflow of the proposed damage-severity prediction framework, including modal analysis, modal strain energy change ratio (MSEcr) computation, dataset construction, preprocessing, GPR training, hyperparameter optimization, and damage rate prediction.

5.3. Data Normalization

To avoid scale dominance and to improve numerical stability, min–max normalization is applied independently to input and output. Since the regression model employs a single input feature (the mean MSEcr) and a single output variable (damage rate), min–max normalization was independently applied to both quantities to improve numerical stability and facilitate model training.

$$X_i^{(n)} = \frac{X_i - X_{\min}}{X_{\max} - X_{\min}}, \quad \gamma_i^{(n)} = \frac{\gamma_i - \gamma_{\min}}{\gamma_{\max} - \gamma_{\min}} \quad (30)$$

where $(\cdot)^n$ denotes normalized quantities. X_i is the i -th input value (mean MSEcr), $X_{\min}$ and $X_{\max}$ are the minimum and maximum values of the input feature in the training dataset, γ_i is the i -th output value (damage rate percentage), $\gamma_{\min}$ and $\gamma_{\max}$ are the minimum and maximum values of the output in the training dataset.

To assess the sensitivity of the GPR model to the choice of normalization technique, we conducted a comparative evaluation using three widely adopted scaling methods: Z-score standardization, robust scaling (using median and interquartile range), and min–max normalization. The evaluation was performed using the testing dataset with identical GPR hyperparameters to ensure comparability.

The results indicated that all three normalization techniques yielded comparable prediction accuracy, with min–max normalization providing slightly superior performance in terms of R^2 values (0.987 vs. 0.982 for Z-score and 0.984 for robust scaling). The Mean Squared Error (MSE) values were 0.0012, 0.0016, and 0.0014 for min–max, Z-score, and robust scaling, respectively. Given the single-input, single-output nature of the regression problem and the bounded range of the input feature (mean MSEcr values ranging from approximately 0.13 to 0.35), the choice of normalization strategy had limited impact on the final predictions.

Min–max normalization was ultimately selected due to its simplicity, interpretability, and slightly better performance. This approach preserves the relative relationships between data points while mapping the input and output variables to a $[0, 1]$ range, which facilitates stable GPR training.

5.4. Gaussian Process Regression (GPR)

Gaussian Process Regression is adopted as a non-parametric Bayesian learning framework, particularly suitable for small-to-moderate datasets and uncertainty-aware prediction.

A Gaussian process defines a distribution over functions:

$$f(X) \sim GP(m(X), cov(X, X')) \quad (31)$$

where $m(X)$ is the mean function (assumed zero without loss of generality). $cov(X, X')$ is the covariance (kernel) function.

5.5. Kernel Function

In this study, a squared exponential (RBF) kernel combined with a constant scaling term is employed:

$$cov(X, X') = \sigma_f^2 \exp\left(-\frac{(X - X')^2}{2\ell^2}\right) \quad (32)$$

where σ_f^2 controls the signal variance, and ℓ is the characteristic length scale governing function smoothness.

This kernel is well suited for capturing smooth nonlinear relationships between damage-sensitive indicators and damage severity.

5.6. Model Training

Given normalized training data $X = [X_1, \dots, X_N]^T$ and outputs y , the joint distribution of observed outputs and predicted values f , at a test point X , is

$$\begin{bmatrix} Y \\ f_* \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} COV + \sigma_n^2 I & cov \\ cov_*^T & cov(X_*, X_*) \end{bmatrix}\right) \quad (33)$$

where COV is the kernel matrix with entries $COV_{ij} = cov(X_i, X_j)$ and $cov_* = [cov(X_1, X_*), \dots, cov(X_N, X_*)]^T$.

The GPR model was implemented in Python 3.10 using the Scikit-learn library with a Constant Kernel $\times$ Radial Basis Function (RBF) kernel. The kernel hyperparameters were optimized automatically by maximizing the log-marginal likelihood during model training.

The optimized signal variance was $\sigma_f^2 = (1.84)^2$, the optimized characteristic length scale was $\ell = 0.61$, and the regularization parameter was fixed at $\alpha = 10^{-6}$.

5.7. Prediction and Uncertainty Quantification

The predictive distribution for $f^* f$, is Gaussian, with mean and variance

$$\begin{aligned}\mu_* &= \text{cov}_*^T (\text{COV} + \sigma_n^2 I)^{-1} \gamma \\ \sigma_*^2 &= \text{cov}(X_*, X_*) - \text{cov}_*^T (\text{COV} + \sigma_n^2 I)^{-1} \text{cov}\end{aligned}\quad (34)$$

The predictive mean μ_* , is used as the estimated damage rate, while σ_* , provides a quantitative measure of prediction uncertainty, enabling confidence interval construction.

5.8. Performance Evaluation

Model performance is evaluated using standard regression metrics.

- Mean Squared Error (MSE)

$$MSE = \frac{1}{N} \sum_1^N \left(\gamma_i - \hat{\gamma}_i \right)^2 \quad (35)$$

- Coefficient of Determination R^2

$$R^2 = 1 - \frac{\sum \left(\gamma_i - \hat{\gamma}_i \right)^2}{\sum \left(\gamma_i - \bar{\gamma} \right)^2} \quad (36)$$

Additionally, element-wise statistical indicators (mean, standard deviation, minimum, and maximum predicted damage rates) are analyzed to assess prediction convergence and robustness.

The proposed GPR-based framework offers several advantages. First, it enables the modeling of nonlinear relationships between vibration-based indicators and damage severity. It also demonstrates robust performance even with limited training data, which is a common situation in SHM applications. In addition, the approach provides explicit uncertainty quantification, thereby improving the reliability of damage assessment. Furthermore, the predictions are physically interpretable and consistent with monotonic evolution of damage.

These characteristics make the proposed method particularly suitable for vibration-based damage quantification in structural systems.

6. Results and Discussion

6.1. Damage Location and Detection

The results obtained from the application of the MSEcr indicator for damage detection in the FGM plate are presented in Figures 3–7. In these figures, subfigure (b) illustrates the actual location of the damage introduced in the structure. Subfigure (a) shows the evolution of the MSEcr indicator with respect to the number of vibration modes considered (3, 6 and 10 modes). In addition, subfigures (c) and (d) display the spatial distribution of the mean MSEcr values in two-dimensional (2D) and three-dimensional (3D) representations, respectively, providing a clearer visualization and interpretation of the damage localization.

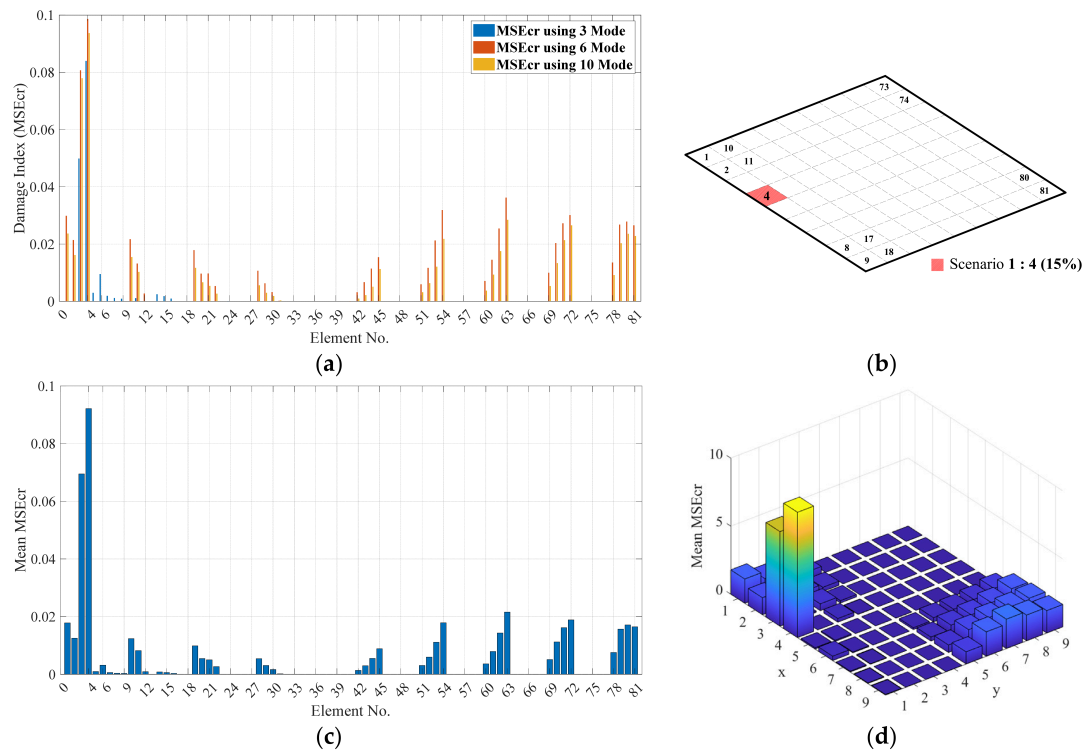


Figure 3. Scenario 1—(a) Damage detection with MSEcr using different numbers of modes, (b) position of damage in the plate (element 4, element size: 11.1 cm × 11.1 cm), (c) 2D damage detection using the mean of MSEcr and (d) 3D damage detection using the mean of MSEcr.

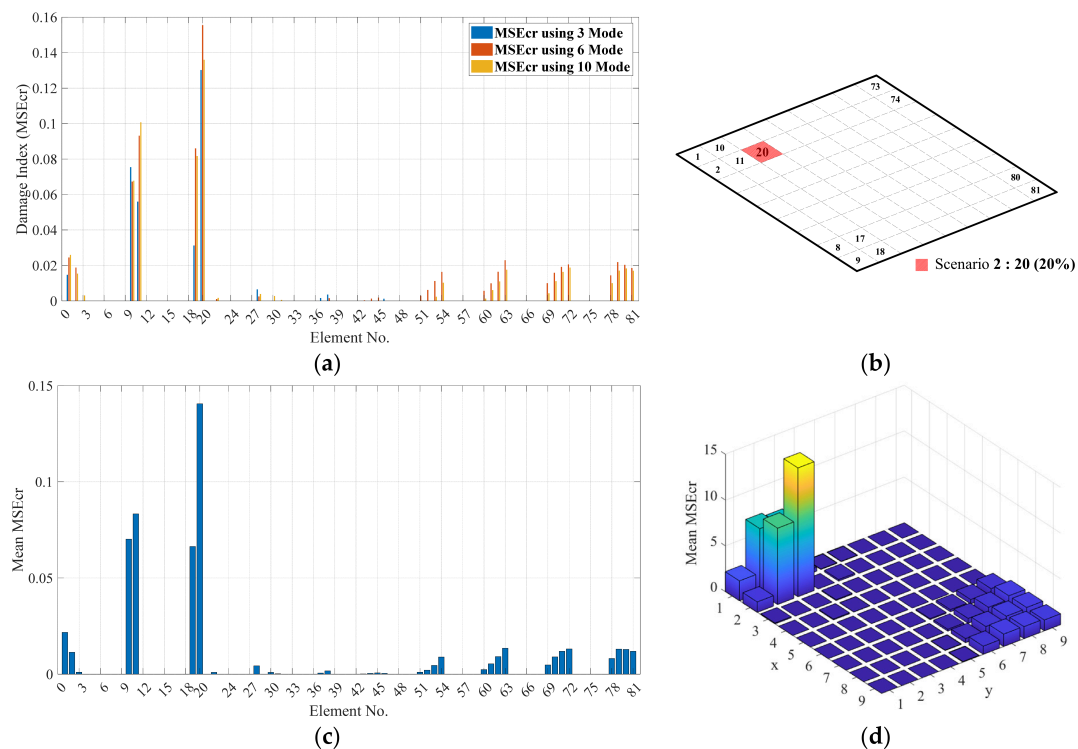


Figure 4. Scenario 2—(a) Damage detection with MSEcr using different numbers of modes, (b) position of damage in the plate (element 20, element size: 11.1 cm × 11.1 cm), (c) 2D damage detection using the mean of MSEcr and (d) 3D damage detection using the mean of MSEcr.

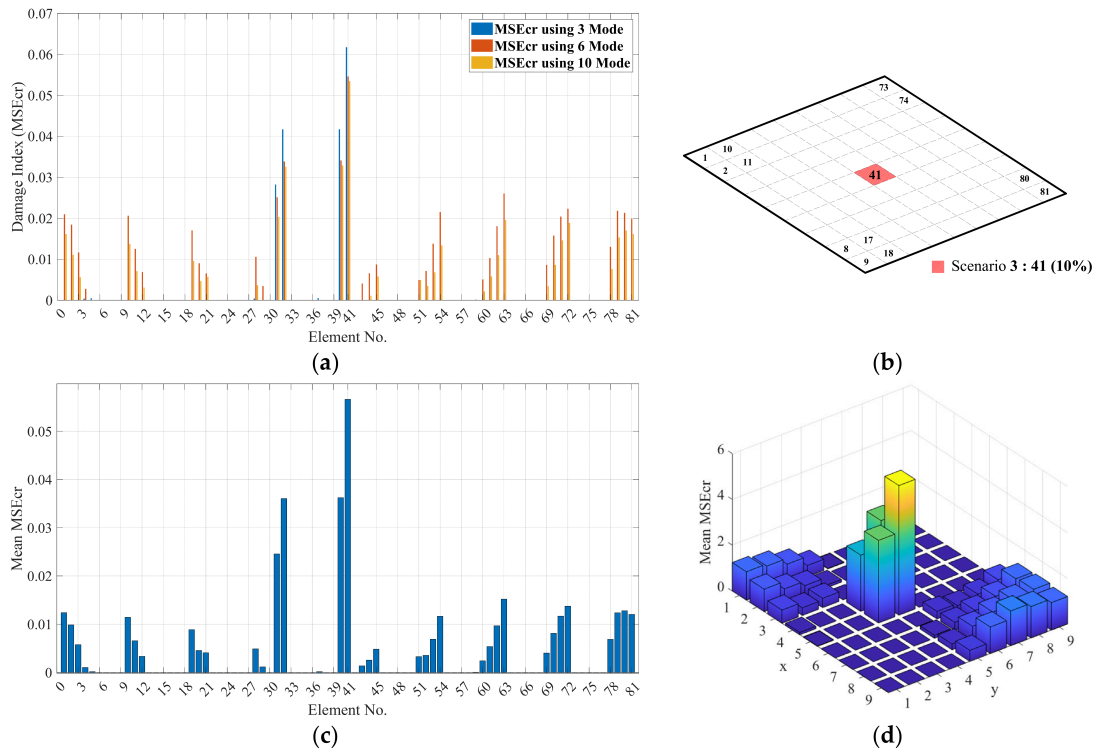


Figure 5. Scenario 3—(a) Damage detection with MSEcr using different numbers of modes, (b) position of damage in the plate (element 41, element size: 11.1 cm $\times$ 11.1 cm), (c) 2D damage detection using the mean of MSEcr and (d) 3D damage detection using the mean of MSEcr.

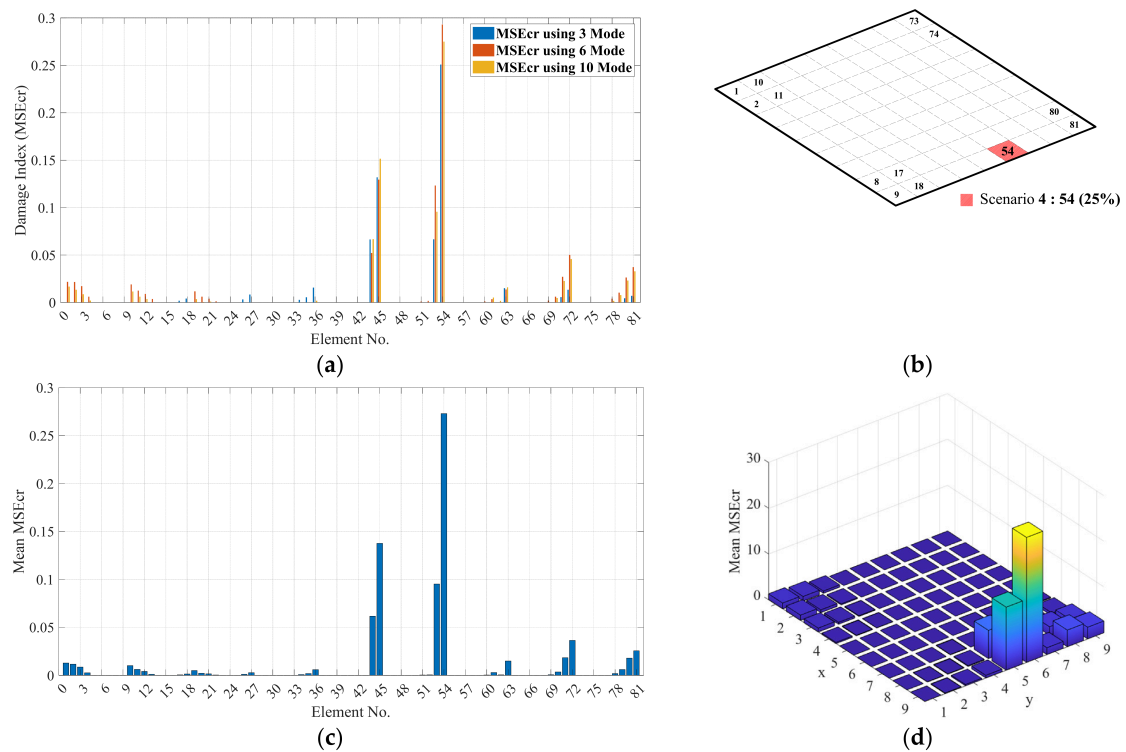


Figure 6. Scenario 4—(a) Damage detection with MSEcr using different numbers of modes, (b) position of damage in the plate (element 54, element size: 11.1 cm $\times$ 11.1 cm), (c) 2D damage detection using the mean of MSEcr and (d) 3D damage detection using the mean of MSEcr.

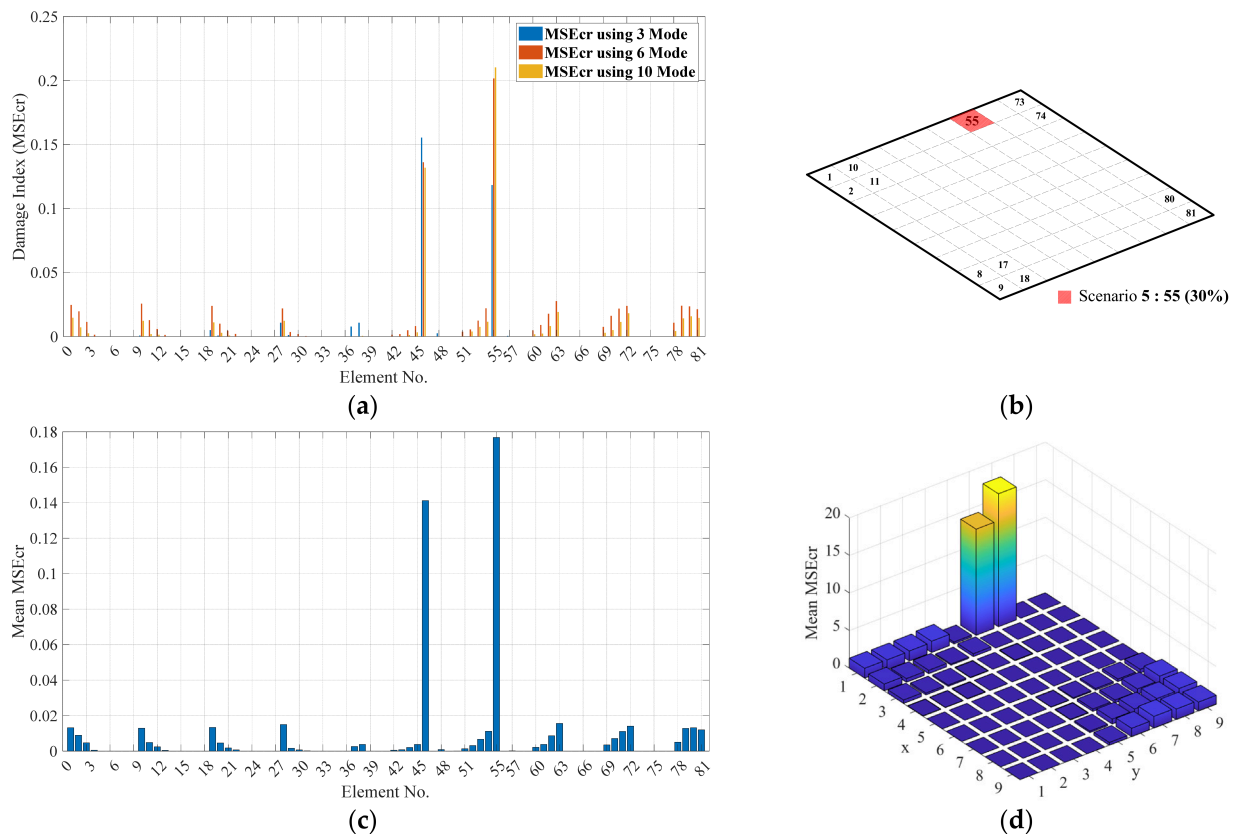


Figure 7. Scenario 5—(a) Damage detection with MSEcr using different numbers of modes, (b) position of damage in the plate (element 55, element size: 11.1 cm × 11.1 cm), (c) 2D damage detection using the mean of MSEcr and (d) 3D damage detection using the mean of MSEcr.

The results highlight the capability of the proposed method to accurately detect and localize structural damage. In particular, the damaged elements are characterized by significantly higher MSEcr values compared to the undamaged elements, which facilitates the clear identification of the damage location.

6.2. Damage Severity and Quantification

This subsection presents representative results obtained from the Gaussian-Process-Regression (GPR)-based framework for damage-severity prediction in the functionally graded plate. The complete numerical database comprises 145 location–severity configurations, obtained by considering five damaged elements and 29 damage-severity levels ranging from 5% to 75% in increments of 2.5%. The GPR model was evaluated using the leave-one-location-out cross-validation strategy described in Section 5.2.1.

For visualization purposes, only selected representative cases are presented in Figures 8–11 rather than plotting the complete set of 145 location–severity configurations. The selected cases are intended to illustrate the typical prediction behavior, convergence characteristics, prediction error, and distribution of the GPR estimates for the investigated damage locations. They should therefore be interpreted as illustrative examples of the overall numerical investigation and not as a complete graphical representation of the entire database.

The analysis uses the mean MSEcr value, obtained from the first 3, 6, and 10 vibration-mode configurations, as the principal input feature for the GPR model. The following figures provide complementary visual evidence of the relationship between the damage-sensitive indicator and the prescribed damage severity.

Figure 8 presents a representative comparison between the prescribed damage-severity values and the corresponding GPR predictions for selected cases from the five investigated damage locations. The examples were selected to provide a concise visualization of the agreement between the prescribed and predicted damage severity without displaying the complete set of 145 location–severity configurations. The figure illustrates the ability of the GPR model to reproduce the damage severity associated with the selected MSEcr inputs.

It should be emphasized that the cases shown in Figure 8 are representative examples and do not constitute the complete validation dataset. The full range of 29 severity levels at each of the five damage locations was considered in the numerical database and in the LOLO-CV procedure.

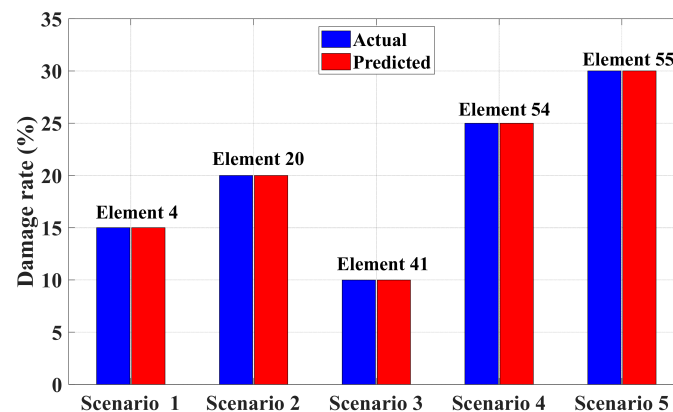


Figure 8. Representative comparison between prescribed and GPR-predicted damage-severity values for selected cases from the five investigated damage locations.

Figure 9 illustrates the convergence behavior of the GPR predictions for selected representative damage cases corresponding to the five investigated damage locations. The cases shown in the figure correspond to selected damage-severity levels and are presented to illustrate the stability and dispersion of the predictions around their respective prescribed values. The close clustering observed in these representative cases indicates that the GPR model can provide stable predictions for the corresponding MSEcr inputs.

To further examine the influence of data preprocessing on the GPR-based damage-severity prediction, three normalization techniques were investigated: Min–Max normalization, Z-score normalization, and robust scaling. The results, summarized in Table 5, indicate that all three normalization approaches provide comparable prediction performance. Min–Max normalization produced the highest coefficient of determination ($R^2 = 0.987$) and the lowest MSE (0.0012), followed by robust scaling ($R^2 = 0.984$, MSE = 0.0014) and Z-score normalization ($R^2 = 0.982$, MSE = 0.0016). The corresponding RMSE values were 0.0346, 0.0374, and 0.0400, respectively. These results indicate that the normalization strategy has a relatively limited influence on the prediction accuracy for the considered single-input/single-output regression problem. Because the mean MSEcr values are bounded within a relatively limited numerical range, all three normalization approaches preserve the underlying nonlinear relationship between the damage-sensitive indicator and damage severity. Accordingly, Min–Max normalization was retained for the subsequent GPR analysis because it provided the best overall performance among the three approaches.

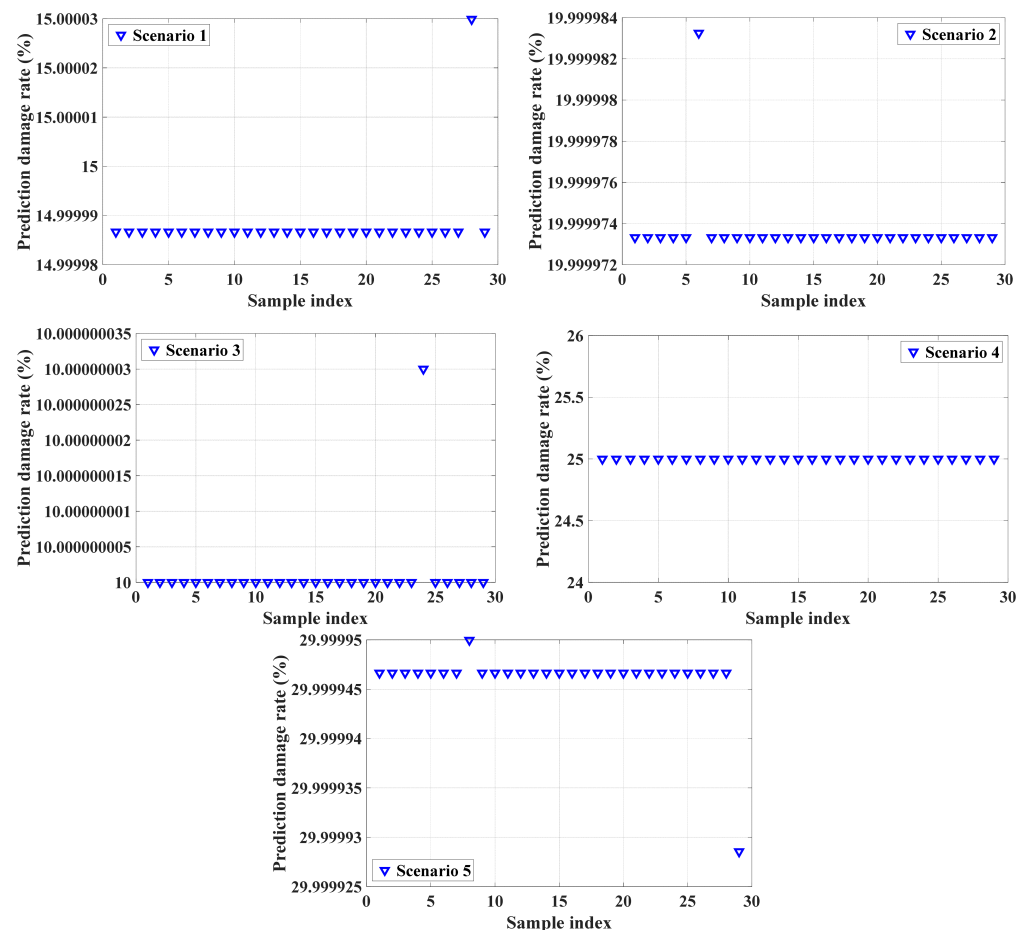
The differences among the three normalization techniques are relatively small, confirming that the proposed GPR formulation is not highly sensitive to the particular scaling strategy. Nevertheless, the slightly lower error and higher R^2 obtained with Min–Max normalization support its selection as the preprocessing method for the subsequent analyses.

Table 5. Performance comparison of different normalization techniques for GPR-based damage-severity prediction.

Normalization Method	MSE	RMSE	R ²
Min–Max	0.0012	0.0346	0.987
Z-score	0.0016	0.0400	0.982
Robust scaling	0.0014	0.0374	0.984
Selected Min–Max model	0.0012	0.0346	0.987

Figure 10 shows the prediction error as a function of the mean MSEcr input for the selected representative damage cases. The plotted errors provide a visual indication of the local prediction behavior and show no pronounced systematic trend within the illustrated cases. This suggests that, for these representative inputs, the GPR predictions remain closely distributed around the prescribed damage-severity values.

Figures 8–10 are intended as an illustrative visualization of the prediction-error behavior. The figure does not display the complete set of 145 location–severity configurations; the complete dataset was instead considered in the LOLO-CV evaluation.

**Figure 9.** Convergence of GPR-predicted damage severity for selected representative cases at the five investigated damage locations.

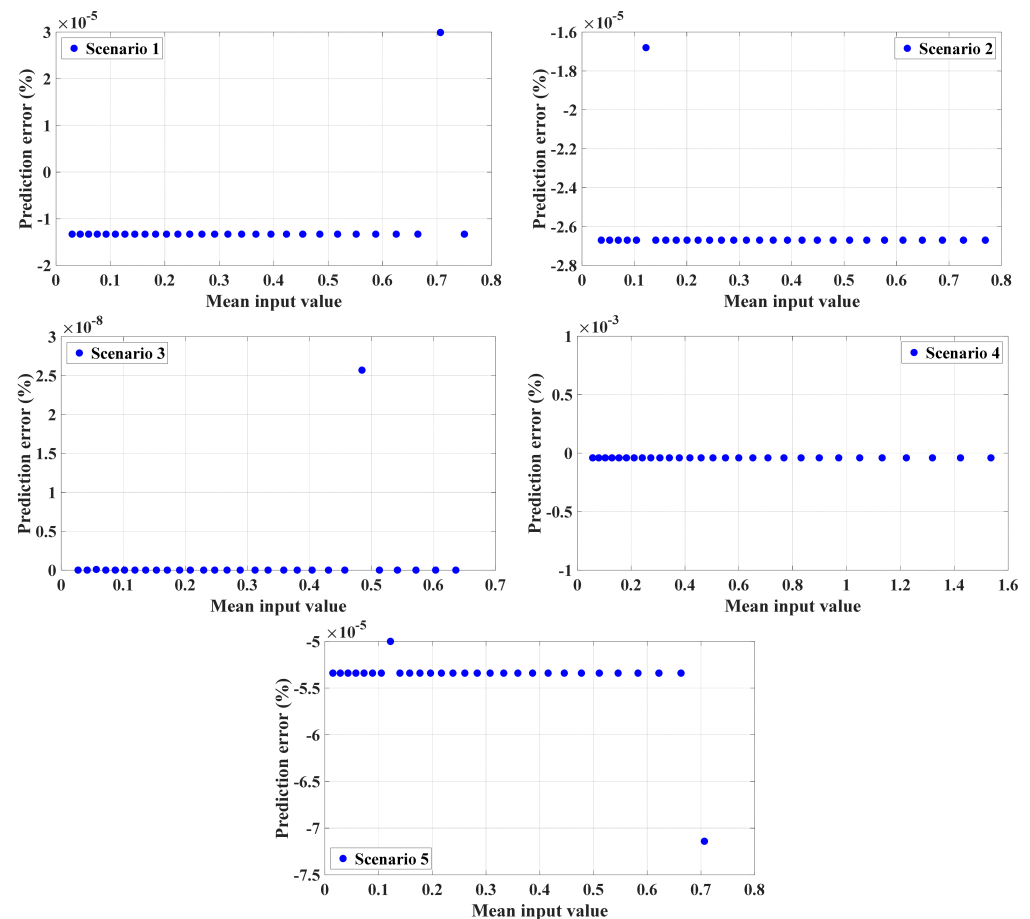


Figure 10. Prediction error as a function of the mean MSEcr input for selected representative damage cases.

Figure 11 illustrates the distribution of GPR-predicted damage severity for the selected representative cases associated with the five investigated damage locations. The distributions provide a visual indication of the prediction spread around the corresponding prescribed damage-severity values. The relatively narrow distributions observed for the illustrated cases indicate limited dispersion of the GPR predictions under the corresponding input conditions.

Taken together, Figures 8–11 provide complementary visual illustrations of the GPR prediction behavior, including agreement with prescribed damage severity, prediction convergence, prediction error, and prediction dispersion for selected representative cases. These examples demonstrate the typical behavior of the proposed regression framework for the investigated damage locations. The figures are intentionally limited to representative cases for clarity, while the complete set of location–severity configurations is incorporated in the numerical validation described in Section 5.2.1.

It should be noted that the present validation is performed within the numerical framework used to generate the dataset. Therefore, although the testing samples were not used during model training or hyperparameter optimization, the assessment does not fully eliminate possible model-form dependency. In particular, an independent finite element formulation, such as an Abaqus- or ANSYS-based model, could provide a stronger assessment of cross-model generalization. Such validation requires an independently implemented numerical model and is beyond the scope of the present study. Accordingly, the reported results should be interpreted as demonstrating the predictive capability of the proposed framework under the investigated numerical conditions rather than as a complete demonstration of model-independent robustness.

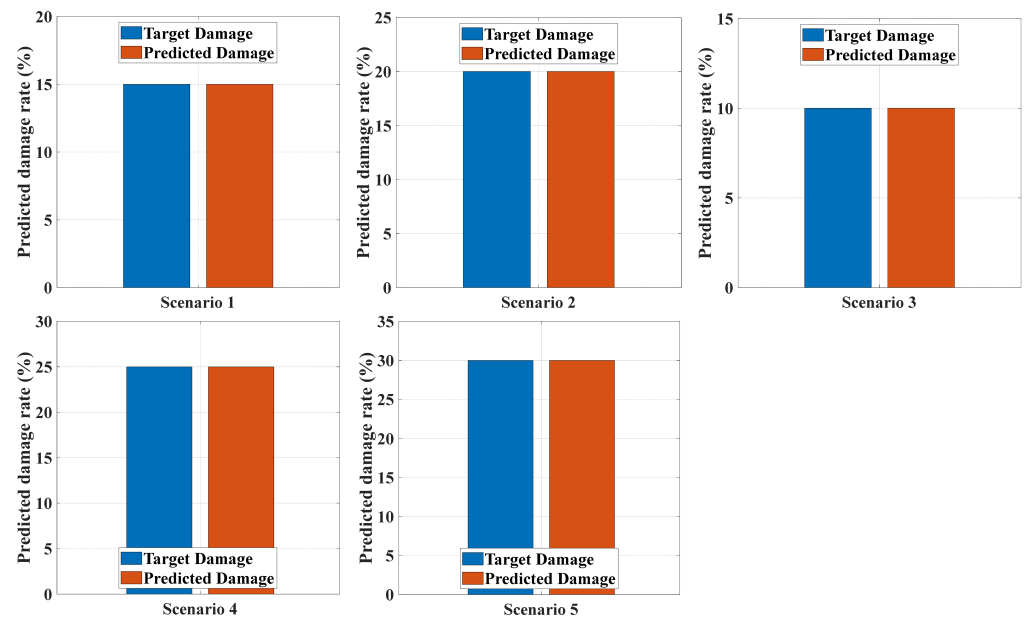


Figure 11. Distribution of GPR-predicted damage severity for selected representative cases from the five investigated damage locations.

6.3. Uncertainty and Robustness

The present numerical database includes inherent variability in the mean MSEcr values resulting from the adopted modal and IGA/NURBS configurations. In addition, the probabilistic GPR formulation provides predictive variance, allowing the uncertainty associated with the estimated damage severity to be assessed. The sensitivity analysis using 3, 6, and 10 vibration modes also provides an initial assessment of the influence of the available modal information. However, measurement noise, environmental effects, sensor uncertainty, and detailed modelling uncertainties were not explicitly considered and are therefore identified as limitations of the present numerical study and as directions for future experimental investigation.

6.4. Generality and Applicability of the Methodology

While the present validation is conducted on a simply supported square FGM plate, the proposed methodology is inherently general and applicable to plates with different dimensions, material combinations, and boundary conditions. The MSEcr indicator and GPR framework rely on modal strain energy distributions computed from mode shapes and stiffness matrices, which are fundamental structural properties independent of specific geometry or boundary conditions.

When applying the methodology to a different FGM plate configuration, the user must: (i) perform the isogeometric modal analysis for that specific structure; (ii) compute MSEcr values for the undamaged and damaged states; and (iii) retrain the GPR model using the corresponding dataset. The numerical results (natural frequencies and MSEcr values) would change with different configurations, but the fundamental relationship between damage-induced stiffness reduction and modal strain energy redistribution remains valid regardless of the specific structural parameters.

7. Conclusions

Extensive numerical investigations involving five damage scenarios were conducted to evaluate the effectiveness of the proposed framework for damage detection and quantification in FGM plates. The results demonstrated that the MSEcr is capable of accurately localizing damaged elements, while the GPR model successfully quantified damage severity using

the extracted MSEcr features. The predictions showed good agreement with the prescribed damage-severity values for the investigated numerical cases, indicating the potential of combining the physics-based MSEcr indicator with GPR for damage-severity assessment.

The proposed methodology offers a unified framework for damage localization and quantification within an IGA environment. By integrating modal strain energy analysis with Gaussian Process Regression, the framework benefits from both the physical interpretability of vibration-based indicators and the predictive capability of probabilistic machine learning. The results indicate that the proposed approach can serve as an effective proof-of-concept for vibration-based damage assessment of FGM plates under controlled numerical conditions.

Despite these promising results, several limitations should be acknowledged. These limitations can be grouped into three main categories.

The present investigation is based on a single IGA numerical formulation and the considered damage scenarios. Although the dataset was divided into independent training and testing subsets, both subsets originate from the same numerical modeling framework. Therefore, the present study does not provide a complete assessment of cross-model generalization or model-form uncertainty. An independent finite element implementation, such as Abaqus or ANSYS, would provide a stronger validation of the proposed framework and is therefore identified as an important direction for future work.

The damage representation adopted in the present study is based on an equivalent local stiffness reduction and therefore primarily represents damage whose dominant structural effect can be described through local stiffness degradation. Other physical damage mechanisms, such as crack breathing, impact-induced damage, material deterioration, and interfacial or delamination-type damage, may modify the modal response in different ways, including through directional or nonlinear effects. Such mechanisms may consequently affect the modal strain energy change ratio and the resulting damage-sensitive features differently from the stiffness-reduction scenarios considered here. Therefore, the present GPR model should not be interpreted as being directly transferable to all damage mechanisms without additional training and validation using damage-specific numerical or experimental data. Extending the framework to multiple physical damage mechanisms represents an important direction for future research.

The present investigation is limited to numerical simulations under idealized conditions. Experimental uncertainties, measurement noise, sensor uncertainty, operational variability, and environmental effects (such as temperature and humidity variations) were not considered. These factors can affect natural frequencies and modal properties, potentially influencing the performance of the proposed methodology. Consequently, the reported performance should be interpreted within the scope of the investigated numerical conditions, and further experimental investigation is required to assess the robustness of the proposed framework under realistic monitoring conditions.

Future research will focus on experimental validation using measured vibration data, independent finite element cross-validation, assessment of combined measurement and environmental uncertainties, investigation of multiple simultaneous damage scenarios, and evaluation of previously unseen damage locations and severity levels. Comparative studies involving alternative vibration-based damage indicators and machine-learning techniques will also be conducted to further assess the generalization capability and practical applicability of the proposed framework.

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E.M.; project administration, F.G. All authors have read and agreed to the published version of the manuscript.

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