

Article

Strain Energy-Based Calculation of Cracking Loads in Reinforced Concrete Members

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Abstract

The traditional methods for calculating the cracking loads of concrete members require the introduction of a semi-empirical inelastic influence coefficient of the section resistance moment to reflect the influence of sectional inelastic deformation development, and its value needs to be corrected according to the sectional characteristics and material properties. In contrast, emerging machine learning models for predicting the cracking loads of concrete members lack clear mechanical mechanisms, making their predictions difficult to interpret. Based on this, member cracking is reinterpreted as a change in the macroscopic performance of the member, and the strain energy variation before and after the fracture of plain concrete axially tensioned members is analyzed. A viewpoint is proposed that employs the incremental change in strain energy to characterize member cracking. With the internal tensile deformation representing the total strain energy of the member, a calculation method for the member cracking loads is established based on the critical condition that the second-order differential of this strain energy with respect to the member deformation equals zero. This method is applicable to both axially tensioned and flexural members of steel- and FRP-reinforced concrete that primarily undergo axial tensile deformation. The method is applied to analyze the cracking of axially tensioned and flexural members under different concrete strength grades, reinforcement types, and reinforcement ratios. It successfully explains the mechanism whereby the strain in the tension zone at member cracking exceeds the material ultimate tensile strain. Calculations verified against experimental data from four sets of tests on steel- or FRP-reinforced concrete beams demonstrate that the proposed method can be accurately applied to both normal- and high-strength concrete, as well as to both steel and FRP reinforcement, with a relative error of only 1% and a coefficient of variation of 0.12.

Keywords: fracture; cracking loads; reinforced concrete member; strain energy



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1. Introduction

In recent years, with the widespread application of high- and ultra-high-strength concrete, as well as new reinforcement materials such as fiber-reinforced polymers (FRPs) and eco-friendly recycled materials [1–9], the sectional stress and strain in steel- and FRP-reinforced concrete members under serviceability limit states have increased significantly. In some cases, the deformation and crack development of such concrete members may even exceed the limits specified in design codes, thereby affecting the normal serviceability of the structure. Consequently, the crack resistance of members at the serviceability stage has increasingly become the controlling factor for structural serviceability performance. By

accurately predicting crack development during the design phase, it is possible to fully leverage the performance advantages or environmental benefits of new materials while effectively controlling deformation and crack progression during service. Alternatively, based on prediction results, such new material members can be reasonably applied in scenarios with less stringent deformation requirements to ensure compliance with normal serviceability criteria. Based on different fracture mechanisms, traditional methods for analyzing concrete cracks can be divided into two categories. The first category is based on fracture mechanics theory, employing indicators such as the stress intensity factor K , energy release rate G , or strain energy density factor S to assess crack initiation and propagation. Typical applications include the double- K fracture criterion and the double- G fracture criterion proposed by Xu et al. and Zhao et al. [10–13]. The second category is based on the material mechanics theory, using indicators such as maximum tensile stress, maximum tensile strain, or maximum shear stress to evaluate material cracking. Typical applications include the crack resistance calculation methods based on member edge strain, proposed by Zhao et al. and Cheng et al. [14–16]. The first category focuses on the stress state at the crack tip. Its application requires solving the stress and strain fields around the crack tip, thus presenting difficulties for scenarios involving simultaneous propagation of multiple cracks. This category is more suitable for cases dominated by a single large crack, such as large dams in hydraulic structures. In contrast, the second category focuses on the stress state of the member section. It comprehensively reflects the effect of cracks through the average deformation of the cracked region [17]. Consequently, it can be flexibly applied to scenarios with simultaneous propagation of multiple cracks, such as slabs, beams, columns, and shear walls commonly found in building structure design. As a result, it has been widely adopted in building structure design codes worldwide, such as the Chinese code GB/T 50010-2010 [18], the European code Eurocode 2 [19], and the American code ACI 318 [20]. This study focuses on building structures and, therefore, is also based on the materials mechanics approach.

The key parameter in the material mechanics approach for analyzing member cracking is the inelastic influence coefficient of the section resistance moment. Experimental observations and studies on ordinary reinforced concrete members have shown that before reaching the commonly recognized “member cracking” state (i.e., “material cracking” on the member surface), a small number of fine cracks, difficult to detect with the naked eye, appear on their surface [21–23]. This phenomenon results in the calculated tensile stress at the edge of the member’s tension zone, derived from measured strain, being greater than the ultimate tensile stress of concrete. This phenomenon is generally considered to be caused by the inelastic deformation of concrete and is characterized by the inelastic influence coefficient of the section resistance moment (denoted as γ). Early research indicated that the inelastic influence coefficient γ of the section resistance moment for ordinary reinforced concrete members is related to material strength, sectional characteristics (section shape, section depth, reinforcement ratio), and loading conditions (load eccentricity) [14–16,24–26]. Current concrete structure design codes only consider the influence of the main factors, i.e., section shape and section depth [18]. Recent studies have shown that for concrete members with new reinforcement materials and high-strength concrete, the influencing factors and their weights on the inelastic influence coefficient γ of the section resistance moment are far more complex than those for ordinary concrete members. For ordinary reinforced concrete incorporating steel fibers, Gao et al. and Li et al. suggested that the value of γ should account for the thickness of the steel fiber reinforcement layer, the aspect ratio of the steel fibers, and the volumetric dosage [1,2]. For cases where FRP bars replace ordinary steel bars, Jia et al. believed that the value of γ should comprehensively consider the influence of concrete strength and FRP reinforcement ratio [3]. For cases where recycled

aggregates replace ordinary aggregates, Chen et al. proposed that the value of γ requires additional correction based on the depth of the compression zone [4]. For cases where reactive powder replaces ordinary cement, Zheng et al., Li et al. and Lu et al. argued that only the influence of the reinforcement ratio needs to be considered [5–7]. Overall, the inelastic influence coefficient γ of the section resistance moment used for calculating member cracking internal forces remains a semi-empirical and semi-theoretical parameter based on experimentation. Its value requires corresponding adjustments according to changes in member constituent materials. The focus of existing research has primarily been on its more detailed classification and refined valuation [27–30].

It is worth noting that, inspired by advances in artificial intelligence, some researchers have begun to explore the use of machine learning methods to analyze the complex cracking behavior of concrete members, aiming to eliminate errors arising from data regression bias in semi-empirical parameters. Talpur et al. compared the differences in accuracy of various machine learning models in predicting the mechanical properties of concrete by predicting the strength of concrete confined with sustainable natural FRP composites [31]. Ma et al. used an interpretable SHAP model to predict the cracking shear strength of reinforced concrete deep beams [32]. Liang et al. employed a Stacking ensemble model to predict the corrosion-induced cracking behavior of reinforced concrete members [33]. These machine learning models have successfully provided more accurate results under corresponding conditions compared to traditional methods. However, in contrast to traditional “mechanism-driven” methods, machine learning methods are inherently “data-driven” approaches. Their prediction results generally lack clear physical meaning, and the input–output process remains in a “black box”.

Indeed, although the commonly defined “member cracking” is not the state of the “first crack appearance” in the member (manifested by an inelastic influence coefficient γ of the section resistance moment greater than one) [15], this method has been well applied in engineering practice. This may be attributed to the fact that during the low-load stage, only very few and extremely fine microcracks are produced, and the cohesion between crack surfaces remains effective. Thus, the corresponding experimental load–displacement curve does not exhibit a distinct turning point, and the impact on concrete durability and stiffness is essentially negligible. Only when the load gradually approaches the “cracking load” and the crack width increases significantly (e.g., to 0.04–0.08 mm, as observed by Guo et al. [21]) does the corresponding experimental load–displacement curve show a clear turning point. At this stage, the intrusion of moisture and air becomes significantly easier, and the sectional stiffness is noticeably reduced, leading to macroscopic effects of decreased durability and stiffness due to “cracking”. This indicates that “member cracking” is not equivalent to a meso-level change like “material cracking,” but rather tends to be a macro-level change process reflected by alterations in member durability and stiffness. Accordingly, the appearance of visible cracks on the member surface or the inflection point in the load–deformation curve is generally adopted as the criterion for determining member cracking in experiments [15]. Based on this, the present study also employs the material mechanics approach focused on the overall sectional stress state, rather than the fracture mechanics method focused on local crack-tip stress conditions. From an energy perspective, the variation in energy before and after member cracking is analyzed based on the macroscopic mechanical behavior of reinforced concrete members. The critical state of member cracking is characterized by the strain energy. On the one hand, this approach avoids the complex classification of the inelastic influence coefficient γ of the section resistance moment based on sectional geometry and material mechanical properties, thereby eliminating the additional empirical coefficients. On the other hand, it also circumvents the lack of the mechanical basis in the prediction process, ensuring that

the results possess clear physical significance. Following this strain energy-based method, the cracking loads of plain and reinforced concrete members under uniaxial stress are analyzed. The mechanism whereby the strain within the tensile zone exceeds the material's ultimate tensile strain (i.e., the strain corresponding to the peak tensile stress) at member cracking is explained. The feasibility and accuracy of the proposed method are verified by using existing experimental results.

2. Characterizing Member Cracking Based on Strain Energy Increment

It is generally accepted that a plain concrete member under axial tension cracks along the plane perpendicular to the tensile force when its sectional stress reaches the material's peak stress [21]. The typical graph of the material's stress–strain curve (σ – ϵ curve) during this process is shown in Figure 1.

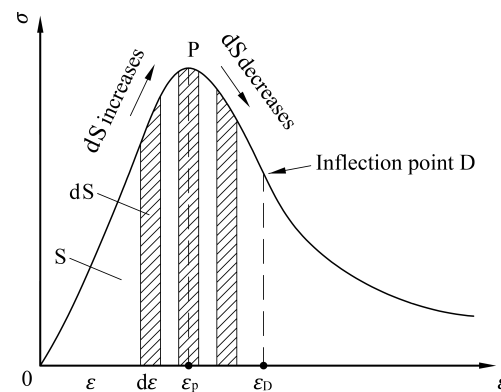


Figure 1. Concrete stress–strain curve under tension.

According to its definition, the strain energy density of the member equals the area S enclosed by the σ – ϵ curve and the ϵ -axis. Assuming the member is homogeneous and isotropic, the strain energy U of the member equals the product of the strain energy density S and the member volume V :

$$U(\epsilon) = V \int_{\epsilon} \sigma(\epsilon) d\epsilon \quad (1)$$

As illustrated, the variation in the strain energy increment, $\Delta(V \cdot dS)$, changes from increasing to decreasing near the peak stress point. Owing to the continuity of energy, the cracking point of a plain concrete member under axial tension corresponds to the condition of zero variation in its strain energy increment, i.e., $\Delta(V \cdot dS) = 0$. This process can be expressed as follows:

$$\frac{d^2 U(\epsilon)}{d\epsilon^2} = V \frac{d\sigma(\epsilon)}{d\epsilon} \quad (2)$$

At the cracking of the axially tensioned member, the member strain ϵ_{cr} exactly equals the concrete peak strain ϵ_p , i.e., $\epsilon_{cr} = \epsilon_p$, while the stress–strain curve has a zero slope:

$$\frac{d\sigma(\epsilon)}{d\epsilon} = 0, \quad \text{when } \epsilon = \epsilon_p \quad (3)$$

Consequently, at the cracking of the axially tensioned member, the second-order differential of the strain energy with respect to the member deformation equals zero:

$$\frac{d^2 U(\epsilon)}{d\epsilon^2} = 0, \quad \text{when } \epsilon = \epsilon_{cr} \quad (4)$$

It should be noted that the derivation is entirely and solely based on the experimental load–deformation curve. Provided that the adopted experimental curve is sufficiently

representative, the cracking results predicted by the above equation should also be sufficiently accurate.

The above conclusion also holds true for plain concrete and reinforced concrete members subjected to other loading conditions. Similarly, assume the member is homogeneous and isotropic, and consider the complete process wherein an arbitrary member is subjected to a system of external forces ΣF_i until cracking occurs. Note that the action of a bending moment can also be transformed into the action of a pair of forces (i.e., a force couple). The displacement generated within the member along the direction of each external force F_i is denoted as δ_i , the internal strain field of the member is denoted as ε , and the sum of the strain energies of the concrete and steel is denoted as ΣU_i . According to the principle of work and energy, the work performed by the external forces is transformed into the strain energy of the member; consequently, within any infinitesimal interval prior to cracking, the following relationship always holds:

$$\sum [(F_i)^{bf} d\delta_i] = d[\sum U_i(\varepsilon)]^{bf} \quad (5)$$

where the superscript “bf” denotes the state before the member cracks. During this phase, the external force F_i must be continuously increased to induce cracking; therefore, $(F_i)^{bf}$ continually increases, which in turn causes $d[\sum U_i(\varepsilon)]^{bf}$ to increase as well. Similarly, within the infinitesimal interval immediately preceding the instance of cracking, the following equation applies:

$$\sum [(F_i)^{cr} d\delta_i] = d[\sum U_i(\varepsilon)]^{cr} \quad (6)$$

where the superscript “cr” denotes the cracking state of the member. In the infinitesimal interval immediately after cracking, since cracking necessarily involves the formation of new cracks or the propagation of existing ones, this process inevitably results in energy dissipation in forms such as surface energy, acoustic emission, and optical emission (denoted collectively as ΣW_i). Consequently, part of the work done by the external forces is allocated to cover these energy expenditures. Moreover, given that the external force F_i remains constant at the instant of cracking, the following relationship is established:

$$\sum [(F_i)^{cr} d\delta_i] = d[\sum U_i(\varepsilon)]^{af} + d(\sum W_i) \quad (7)$$

where the superscript “af” denotes the state after the member has cracked. A comparison of Equations (6) and (7) reveals that

$$d[\sum U_i(\varepsilon)]^{af} < d[\sum U_i(\varepsilon)]^{cr} \quad (8)$$

which indicates that $d[\sum U_i(\varepsilon)]^{af}$ decreases in the infinitesimal interval immediately following cracking. In summary, the increment of the member’s strain energy, $d[\sum U_i(\varepsilon)]$, functions as an increasing quantity before cracking and as a decreasing quantity instantaneously after cracking. At the precise moment of cracking, it transitions from increasing to decreasing. Therefore, the cracking point corresponds to the extremum point of the strain energy increment $d[\sum U_i(\varepsilon)]$. That is, the cracking point can also be defined by the condition:

$$\frac{d^2[\sum U_i(\varepsilon)]}{d\varepsilon^2} = 0, \quad \text{when } \varepsilon = \varepsilon_{cr} \quad (9)$$

where the strain energy-strain relationships for the concrete and steel components can still be obtained from the load–deformation curves of the corresponding materials. Since the materials mechanics method generally attributes the material cracking to tensile strain or tensile stress, the load–deformation curves used here should be the experimental curves

obtained under pure tension conditions, and the material strain energy should be taken as a function of the normal strain in the tensile direction within the member. When the member is simultaneously subjected to significant shear forces or constraint effects, the load–deformation curves must be derived from new experimental curves that account for the effects of shear or constraints under the equivalent working conditions. This is necessary because these effects alter the material’s strain energy response—for instance, shear introduces shear strain energy, while confinement leads to a more complex strain energy composition under triaxial stress states. Theoretically, provided that the adopted load–deformation curve accurately reflects the variation of the strain energy with deformation in the member, the above method can still yield sufficiently accurate prediction results. This aspect will be addressed in future work.

For reinforced and FRP-reinforced concrete members primarily subjected to tensile deformation, the calculation procedure for cracking loads using the above strain energy method can be summarized as follows. Firstly, determine the tensile stress–strain curves of the concrete, steel reinforcement, or FRP reinforcement. Next, based on the sectional geometry, material layout, and material tensile stress–strain relationships, formulate the expression for the total strain energy of the member with the normal strain in the tensile direction serving as the independent variable. Finally, compute the second-order differential of the total strain energy of the member, and take the tensile strain obtained when this second-order differential equals zero as the cracking strain of the member. It is evident that the proposed calculation procedure requires only the fundamental constitutive relationship, i.e., the stress–strain curves of the constituent materials, thereby eliminating the need to introduce the empirical parameter such as the inelastic influence coefficient γ of the section resistance moment through specialized tests. It is important to note that commonly used design codes or manuals generally provide fitted formulas for the stress–strain curves of concrete and steel reinforcement based on experimental data. This allows the stress–strain constitutive relationship of the section materials required by the proposed method to be readily obtained, which will significantly simplify the application of the proposed method.

3. Cracking of Axially Tensioned Members of Steel-Reinforced Concrete

3.1. Case of Conventional Reinforcement Ratio

The deformation analysis of reinforced concrete members under axial tension by Guo et al. [22] indicates that the member does not crack immediately after the member strain ε exceeds the concrete peak strain ε_p . Conversely, the member’s bearing capacity N exhibits a short ascending segment before dropping rapidly, forming a small peak on the load–deformation curve (N – ε curve). Cracking of the member occurs only after the bearing capacity reaches this peak. That is, the strain at which the member cracks is greater than the strain at which the concrete cracks. This process is illustrated by curve G in Figure 2.

However, according to the viewpoint that characterizes member cracking by material cracking, the member cracks instantly when the concrete strain reaches the peak strain ε_p . The concrete immediately ceases to carry load, and the tension it carried is transferred to the reinforcement, causing an abrupt increase in the stress and strain of the reinforcement. The bearing capacity of the member remains unchanged throughout this process. It is noted that before cracking, the reinforcement deforms compatibly with the concrete, and the strain in the reinforcement is always equal to the strain in the concrete. After cracking, however, the reinforcement bears the entire tensile force as the concrete ceases to carry load, resulting in a sharp increase in the reinforcement’s strain. Compared to the total deformation during the loading process, the deformation accumulated in the reinforcement before cracking

is relatively small and can be neglected. Therefore, the incremental deformation of the member at the cracked section can be approximately expressed as:

$$\Delta \varepsilon = \frac{E_c A_c \varepsilon_p}{E_s A_s} \quad (10)$$

where E denotes the elastic modulus, A denotes the cross-sectional area, and the subscripts c and s denote concrete and steel reinforcement, respectively. This process is illustrated by curve M in Figure 2.

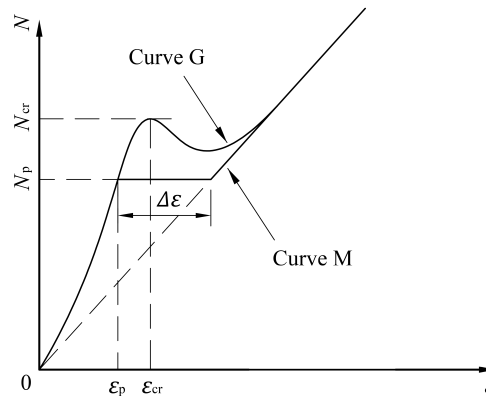


Figure 2. Load–strain curve of axially tensioned member based on classical method and experimental data.

The cracking process revealed by Guo et al. [22] can be reasonably explained by characterizing member cracking through strain energy. Prior to member cracking, the deformations of the concrete and steel reinforcement satisfy the plane-section assumption, and their longitudinal deformations are identical; therefore, we have:

$$\Sigma U_i = \int_0^\varepsilon (\sigma_s A_s + \sigma_c A_c) \times (L d\varepsilon) \quad (11)$$

where σ denotes stress, and L denotes the member length. At the instance of member cracking, the second-order differential of the strain energy equals zero, yielding:

$$d^2(\Sigma U_i) = \frac{\partial(\sigma_s A_s + \sigma_c A_c)}{\partial \varepsilon} \times (L d\varepsilon) = 0 \quad (12)$$

Given that the member length L and the cross-sectional areas A_s and A_c are non-zero, the above equation can be simplified to:

$$\rho \frac{\partial \sigma_s}{\partial \varepsilon} + (1 - \rho) \frac{\partial \sigma_c}{\partial \varepsilon} = 0 \quad (13)$$

where ρ represents the reinforcement ratio. Under conventional reinforcement conditions, the strain at member cracking is relatively small, with the steel deformation remaining in the elastic stage; thus the stress–strain relationship obeys Hooke’s law, always satisfying:

$$\frac{\partial \sigma_s}{\partial \varepsilon} = E_s \quad (14)$$

For steel reinforcement, E_s is always a positive constant. For concrete, when the strain is less than the peak strain, the stress–strain curve is in the ascending branch, and its slope is always greater than zero; when the strain is greater than the peak strain, the stress–strain curve is in the descending branch, and its slope is always less than zero. Consequently, the positive solution ε_{cr} for Equation (13) can only exist in the region where the strain is greater

than the peak strain, which explains the phenomenon that the member cracking strain ε_{cr} exceeds the material cracking strain. Furthermore, Equation (13) also indicates that the member cracking strain ε_{cr} is influenced by the reinforcement ratio ρ , which is consistent with the experimental results reported in references [34,35].

When the reinforcement is uniformly distributed along the width direction of the section (i.e., the section deformation satisfies the plane-section deformation assumption), the axial tensile cracking internal force for any section composed of this reinforced concrete can be solved according to Equation (13) by selecting a set of stress–strain curves for steel and concrete. The tensile stress–strain curve of concrete follows the expression suggested by Guo et al. [21]:

$$\frac{\sigma_c}{f_t} = \begin{cases} 1.2x - 0.2x^6, & x \leq 1 \\ \frac{x}{\alpha(x-1)^{1.7} + x}, & x \geq 1 \end{cases} \quad (15)$$

$$x = \frac{\varepsilon}{\varepsilon_p} \quad (16)$$

and the tensile stress–strain curve of steel follows the conventional tri-linear model [22]:

$$\sigma_s = E_s \varepsilon \quad (17)$$

Under conventional reinforcement conditions, the cracking strain ε_{cr} for a reinforced concrete axially tensioned member can be determined by substituting Equations (15)–(17) into Equation (13):

$$\rho \frac{E_s}{E_{t,p}} - (1 - \rho) \alpha \frac{0.7(x_{cr} - 1)^{1.7} + 1.7(x_{cr} - 1)^{0.7}}{[\alpha(x_{cr} - 1)^{1.7} + x_{cr}]^2} = 0 \quad (18)$$

where

$$x_{cr} = \frac{\varepsilon_{cr}}{\varepsilon_p} \quad (19)$$

Equations (18) and (19) are applicable to both normal-strength and high-strength concrete. In above equations: α is the parameter for the descending branch of the concrete tensile stress–strain curve, $E_{t,p}$ is the secant modulus of concrete corresponding to the peak tensile stress f_t , and ε_p is the strain of concrete corresponding to the peak tensile stress, given respectively by [22]:

$$\alpha = 0.312f_t^2 \quad (20)$$

$$E_{t,p} = \frac{(1.45 + 0.628f_t) \times 10^4}{1.2} \quad (21)$$

$$\varepsilon_p = 65 \times 10^{-6} f_t^{0.54} \quad (22)$$

The corresponding cracking internal force N_{cr} is:

$$N_{cr} = \left[\frac{E_s}{E_{t,p}} x_{cr} \rho + \frac{x_{cr}}{\alpha(x_{cr} - 1)^{1.7} + x_{cr}} (1 - \rho) \right] f_t A \quad (23)$$

where A represents the cross-sectional area of the member. Calculations were performed using the equations above for conventionally reinforced sections, with f_t ranging from 1.5 to 2.5 N/mm², approximately corresponding to concrete strength grades C20 to C50, covering essentially the normal-strength concrete range. The calculation results are shown in Figure 3.

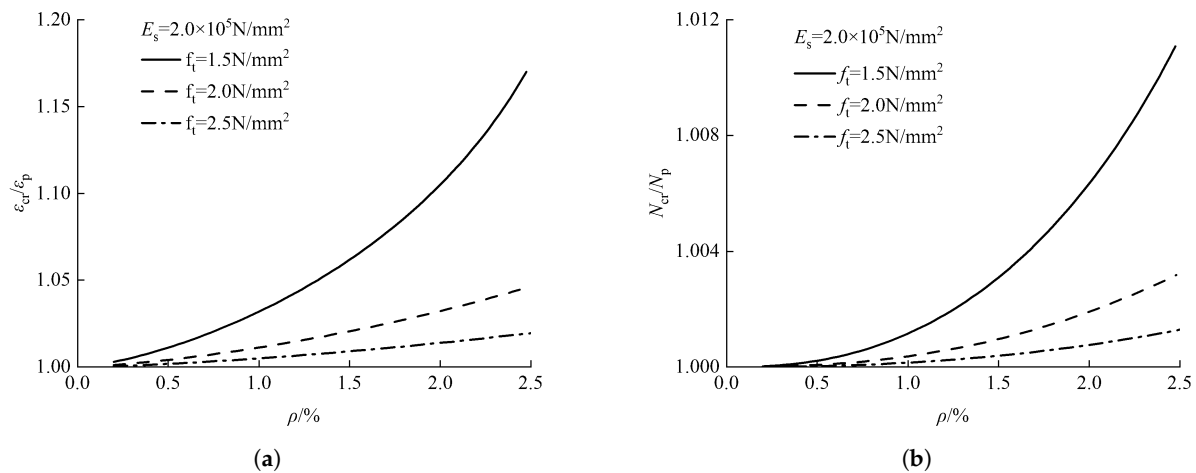


Figure 3. Relative cracking loads and strains of reinforced concrete axially tensioned members. (a) Relative strain ratios; (b) Relative load ratio.

The calculation results reflect the experimental phenomenon that “the member cracking strain exceeds the concrete cracking strain,” indicating the role of reinforcement in this strain enhancement phenomenon. Typically, the contribution of reinforcement to the increase in the cracking load of concrete members is relatively small, generally not exceeding 1%. Thus, calculating the cracking internal force based on the concrete tensile peak strain already provides sufficient accuracy. However, reinforcement can significantly increase the member’s cracking strain (e.g., by approximately 5% for concrete grade C30), which is clearly beneficial for delaying member cracking and enhancing crack resistance. Furthermore, this enhancement is particularly evident in members with lower concrete strength grades (e.g., the cracking strain can be increased by 5% to 15% for concrete grade C20).

3.2. Case of High Reinforcement Ratio

Analysis of the critical cracking condition for reinforced concrete members under axial tension, as expressed by Equation (13), indicates that the solution to this equation depends on the relative magnitudes of the slopes of the stress–strain curves for the concrete and steel. Under conventional reinforcement conditions, member cracking occurs shortly after the concrete reaches its peak tensile strain, while the steel remains elastic with a constant stress–strain curve slope. Since the slope of the concrete stress–strain curve reaches its minimum value at the inflection point of the descending branch (point D on the curve in Figure 1), an excessively high reinforcement ratio ρ may cause the left-hand side of Equation (13) to remain greater than zero when calculated using the concrete’s inflection point slope and the steel’s constant slope, resulting in no positive solution existing for the member during the steel’s elastic strain stage. This implies that cracking cannot occur while the steel deforms elastically. It can be inferred that cracking in such members must occur after the steel’s elastic deformation stage. At this stage, the concrete strain becomes extremely large, and the slope of its stress–strain curve approaches zero. The positive solution for Equation (13) will then be determined by the slope of the steel’s stress–strain curve. That is, a member with a high reinforcement ratio will crack at the point where the slope of the steel’s stress–strain curve equals zero. For mild steel, this point corresponds to the yield point of the steel. For hard steel (i.e., steel with no distinct yield point), this point corresponds to the fracture point of the steel. The critical transition state from conventional reinforcement to high reinforcement ratio occurs when the member strain ε reaches the concrete’s inflection point strain ε_D precisely at the moment of member cracking. The reinforcement ratio at this state is denoted as the inflection point reinforcement ratio ρ_D . When the reinforcement ratio

exceeds ρ_D , the member will maintain macroscopically uncracked behavior during the steel's elastic deformation stage, meaning its stiffness and durability remain essentially uncompromised, and its crack resistance is significantly enhanced.

The concrete's inflection point strain ε_D can be determined by locating the inflection point of the function, i.e., by setting the second derivative of the concrete stress with respect to strain equal to zero. Using the previously selected stress–strain relationships for concrete and steel, the following equation is obtained:

$$0.595x_D [\alpha(x_D - 1)^{1.7} + x_D] - (x_D - 1)[1.7 + 0.7(x_D - 1)] [1.7\alpha(x_D - 1)^{0.7} + 1] = 0 \quad (24)$$

$$\varepsilon_D = x_D \varepsilon_p \quad (25)$$

The inflection point reinforcement ratio ρ_D can be calculated as follows:

$$\rho_D \frac{E_s}{E_{tp}} - (1 - \rho_D) \alpha \frac{0.7(x_D - 1)^{1.7} + 1.7(x_D - 1)^{0.7}}{[\alpha(x_D - 1)^{1.7} + x_D]^2} = 0 \quad (26)$$

For common concrete sections, the calculation results are presented in Table 1.

Table 1. Inflection Point Reinforcement Ratio for Common Concrete Sections (%).

E_s ($\times 10^5$ N/mm ²)	f_t (N/mm ²)				
	1.0	1.5	2.0	2.5	3.0
2.0	1.32	2.95	5.14	7.81	10.87
2.1	1.26	2.82	4.91	7.46	10.40

E_s represents the elastic modulus of steel, f_t represents the peak tensile strength of concrete.

4. Cracking of Flexural Members of Steel- and FRP-Reinforced Concrete

4.1. Energy-Based Criterion for Critical Cracking State

Before the member cracks, the deformation of the flexural member's section satisfies the plane section assumption, and no relative slip occurs between the concrete and the reinforcement. Therefore, the sum of the strain energy of the concrete and steel, $\sum U_i$, equals the work done by the external moment M over the sectional rotation φ , thus:

$$\sum U_i(\varepsilon) = \int_0^\varphi M d\varphi \quad (27)$$

As the member approaches cracking, the development of inelastic deformation at the impending cracked section means the sectional deformation no longer strictly satisfies the plane-section assumption. However, extensive experimental measurements have demonstrated that the average strain along the tensile direction of the member still satisfies the plane-section assumption [22]. It is noted that the tensile stress–strain curve used in the proposed method is also derived from experimental measurements, where the deformation values are averages that inherently account for inelastic deformation. Therefore, the plane-section assumption remains applicable at the critical cracking state. According to Equation (9), the critical condition for flexural cracking is:

$$\frac{d^2 [\sum U_i(\varepsilon)]}{d\varepsilon^2} = \frac{\partial M}{\partial \varphi} = 0 \quad (28)$$

Consider a double-reinforced section with conventional reinforcement ratio, whose sectional dimensions and stress–strain distribution are shown in Figure 4.

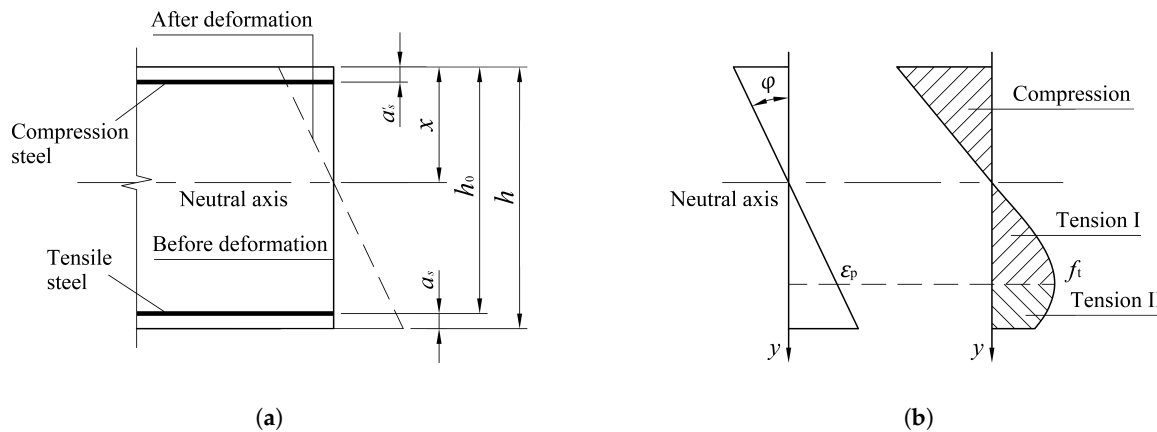


Figure 4. Sectional dimensions and stress–strain distribution of a doubly reinforced flexural member. (a) Sectional dimensions; (b) Sectional stress–strain distribution.

Since the sectional strain at member cracking is very small, both the compressive and tensile reinforcement remain in the elastic stage before cracking, and the stress–strain relationship of the concrete in the compression zone is essentially linear and can be simplified as a triangular distribution, namely:

$$\sigma_s^t = E_s \varphi (h - x - a_s) \quad (29)$$

$$\sigma_s^c = E_s \varphi (x - a_s') \quad (30)$$

$$\sigma_c^c = E_c \varphi y_c \quad (31)$$

where E_c is the secant modulus of concrete in compression, which can be considered slightly less than the initial tangent modulus E_{c0} during the loading process from zero to member cracking; y_c is the distance from the calculation point to the neutral axis when determining the compressive strain of the concrete in the compression zone; the superscripts c and t denote compression and tension, respectively.

The stress distribution of concrete in the tension zone, σ_c^t , is relatively complex. Since cracking occurs only after the concrete strain exceeds the peak tensile strain, the tension zone is divided into two parts: the region near the neutral axis where $\epsilon \leq \epsilon_p$ (Tension Zone I) and the region near the tensile edge where $\epsilon > \epsilon_p$ (Tension Zone II). Tension Zone I lies on the ascending branch of the concrete tensile stress–strain curve. According to the expression suggested by Guo et al. [21], the stress in this zone is given by:

$$\sigma_c^{t,I} = \left[1.2 \left(\frac{\varphi y_t}{\epsilon_p} \right) - 0.2 \left(\frac{\varphi y_t}{\epsilon_p} \right)^6 \right] f_t \quad (32)$$

Tension Zone II lies on the descending branch of the concrete tensile stress–strain curve, which exhibits a convex-to-concave shape. The expressions commonly suggested [21,36] are complex and inconvenient for calculation. We propose a straight-line simplification for this curve segment. A straight line drawn through the peak point P and the inflection point D of the curve will inevitably intersect the concave segment at a point E, as shown in Figure 5.

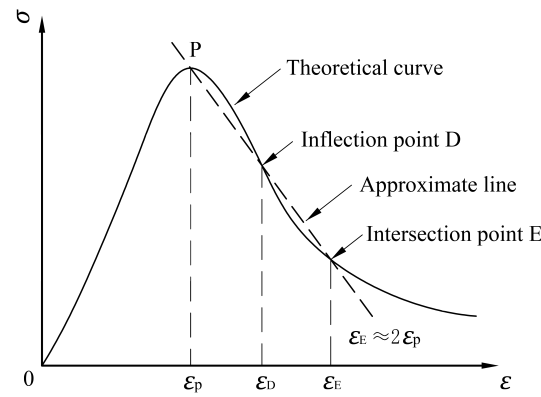


Figure 5. Schematic of the approximate line passing through the descending branch of the concrete stress–strain curve.

Analysis of existing experimental data and trial calculations reveals that the tensile strain at the member edge at cracking generally lies near point E. It is noted that within the strain range of Tension Zone II, the curvature of the original stress–strain curve is small, resulting in minor error when replaced by a straight line. Furthermore, the approximating line value is less than the curve value before the inflection point and greater than the curve value after the inflection point; averaging these two deviations can further reduce the error. Therefore, this straight line can be used to effectively approximate the descending branch of the concrete stress–strain curve. Applying normalization to the concrete stress–strain curve, with the ratio of strain to peak tensile strain as the x -axis and the ratio of stress to peak tensile stress as the y -axis, the shape of the descending branch depends solely on the concrete tensile peak stress f_t [21]. In this dimensionless coordinate system, the approximating line always passes through the point (1,1), i.e., the peak stress point. Its slope k depends on the curve shape, meaning the slope of the approximating line is also determined by the concrete tensile peak stress f_t . For the common range of f_t from 0.8 to 3.2 MPa, the k – f_t relationship is well approximated by a quadratic function. Fitting with a quadratic term yields:

$$k = -(0.0252f_t^2 + 0.1728f_t - 0.0752) \quad (33)$$

Comparing the slope estimated by the above equation with the slope calculated from the theoretical stress–strain curve, the fitted formula has a root mean square error of 0.001903 and a coefficient of determination of 0.9999, meeting the required accuracy. Therefore, the concrete stress in Tension Zone II can be simplified as:

$$\sigma_c^{t,II} = \left[k \left(\frac{\varphi y_t}{\varepsilon_p} - 1 \right) + 1 \right] f_t \quad (34)$$

Base on the force equilibrium conditions of the section, the equations for the forces are obtained as follows:

$$N_s^c + N_c^c = N_s^t + N_c^{t,I} + N_c^{t,II} \quad (35)$$

$$N_s^t = A_s E_s \varphi (h - x - a_s) \quad (36)$$

$$N_s^c = A'_s E_s \varphi (x - a'_s) \quad (37)$$

$$N_c^c = \int_0^x E_c \varphi y_c b dy_c = \frac{1}{2} b E_c \varphi x^2 \quad (38)$$

$$N_c^{t,I} = \int_0^{\frac{\varepsilon_p}{\varphi}} \left[1.2 \left(\frac{\varphi y_t}{\varepsilon_p} \right) - 0.2 \left(\frac{\varphi y_t}{\varepsilon_p} \right)^6 \right] f_t b dy_t = \frac{4}{7} f_t b \left(\frac{\varepsilon_p}{\varphi} \right) \quad (39)$$

$$N_c^{t,II} = \int_{\frac{\varepsilon_p}{\varphi}}^{h-x} \left[k \left(\frac{\varphi y_t}{\varepsilon_p} - 1 \right) + 1 \right] f_t b dy_t = f_t b \left[\frac{k}{2} (h-x)^2 \left(\frac{\varphi}{\varepsilon_p} \right) + (1-k)(h-x) + \left(\frac{k}{2} - 1 \right) \left(\frac{\varepsilon_p}{\varphi} \right) \right] \quad (40)$$

Similarly, base on the moment equilibrium condition of the section, the equation for the bending moment is obtained as follows:

$$M = M_s^t + M_s^c + M_c^c + M_c^{t,I} + M_c^{t,II} \quad (41)$$

$$M_s^t = A_s E_s \varphi (h-x-a_s)^2 \quad (42)$$

$$M_s^c = A'_s E_s \varphi (x-a'_s)^2 \quad (43)$$

$$M_c^c = \int_0^x y_c \times (E_c \varphi y_c b dy_c) = \frac{1}{3} b E_c \varphi x^3 \quad (44)$$

$$M_c^{t,I} = \int_0^{\frac{\varepsilon_p}{\varphi}} y_t \times \left\{ \left[1.2 \left(\frac{\varphi y_t}{\varepsilon_p} \right) - 0.2 \left(\frac{\varphi y_t}{\varepsilon_p} \right)^6 \right] f_t b dy_t \right\} = \frac{3}{8} f_t b \left(\frac{\varepsilon_p}{\varphi} \right)^2 \quad (45)$$

$$M_c^{t,II} = \int_{\frac{\varepsilon_p}{\varphi}}^{h-x} y_t \times \left\{ \left[k \left(\frac{\varphi y_t}{\varepsilon_p} - 1 \right) + 1 \right] f_t b dy_t \right\} = f_t b \left[\frac{k}{3} (h-x)^3 \left(\frac{\varphi}{\varepsilon_p} \right) + \frac{1-k}{2} (h-x)^2 + \left(\frac{k}{6} - \frac{1}{2} \right) \left(\frac{\varepsilon_p}{\varphi} \right)^2 \right] \quad (46)$$

Note that $f_t = E_{t,p} \varepsilon_p$, where $E_{t,p}$ is the secant modulus of concrete at the peak tensile stress. The $E_{t,p}$ and the initial tensile modulus of concrete E_t (the secant modulus corresponding to $0.5f_t$) satisfy the relationship [22]: $E_t = 1.2E_{t,p}$, and $E_t = (1.45 + 0.628f_t) \times 10^4$. According to the analysis by Guo et al. [22], the initial tensile modulus of concrete E_t is slightly less than the initial tangent modulus of concrete in compression E_{c0} . Furthermore, E_c during the loading process from zero to cracking can always be considered slightly less than E_{c0} . Therefore, it is approximately valid that $E_c = E_t$ and $E_c = 1.2E_{t,p}$. Consequently, the system of force Equations (35)–(40) can be simplified to:

$$\begin{aligned} & \left(\frac{3}{5} - \frac{k}{2} \right) \left(\frac{\varphi}{\varepsilon_p} \right) x^2 + \left[1.2 \frac{E_s}{E_t} \left(\frac{\varphi}{\varepsilon_p} \right) \left(\frac{A_s}{b} + \frac{A'_s}{b} \right) + h k \left(\frac{\varphi}{\varepsilon_p} \right) - k + 1 \right] x - \\ & 1.2 \frac{E_s}{E_t} \left(\frac{\varphi}{\varepsilon_p} \right) \left[\frac{A'_s a'_s}{b} + \frac{A_s}{b} (h-a_s) \right] - \frac{1}{2} k h^2 \left(\frac{\varphi}{\varepsilon_p} \right) + \left(\frac{3}{7} - \frac{k}{2} \right) \left(\frac{\varepsilon_p}{\varphi} \right) + h(k-1) = 0 \end{aligned} \quad (47)$$

The sectional bending moment M can be simplified to:

$$\begin{aligned} \frac{M}{f_t b} = & \left[\frac{k}{3} (h-x)^3 + \frac{2}{5} x^3 \right] \left(\frac{\varphi}{\varepsilon_p} \right) + 1.2 \frac{E_s}{E_t} \left[A'_s (x-a'_s)^2 + A_s (h-x-a_s)^2 \right] \left(\frac{\varphi}{\varepsilon_p} \right) + \\ & \left(\frac{k}{6} - \frac{1}{8} \right) \left(\frac{\varepsilon_p}{\varphi} \right)^2 + \frac{1-k}{2} (h-x)^2 \end{aligned} \quad (48)$$

Equation (47) is a quadratic equation in terms of the compression zone depth x with respect to the deformation φ . From this equation, the positive solution for x as a function of φ , denoted $x = x(\varphi)$, can be obtained. Substituting this into Equation (48), the expression for the sectional bending moment M as a function of the deformation φ , denoted $M = M(\varphi)$, can be derived. Finally, the critical deformation at member cracking, φ_{cr} , can be determined from Equation (28):

$$\frac{\partial M}{\partial \varphi} = 0, \quad \text{when } \varphi = \varphi_{cr} \quad (49)$$

4.2. Model Validation with Experimental Results

For flexural members of steel- and FRP-reinforced concrete with moderate reinforcement spacing and reasonable sectional geometry (i.e., deformations satisfying the plane-section assumption), the critical deformation φ_{cr} at member cracking and the corresponding depth x_{cr} of the compression zone can be calculated using Equations (47) and (48). Substituting φ_{cr} into Equation (48) yields the cracking moment M_{cr} of the member. To validate the effectiveness and predictive accuracy of the proposed method, four sets of experimental data on the cracking resistance of steel- or FRP-reinforced concrete beams are compared. The data are from tests conducted by Fang et al. [35], Jiang et al. [36], Sun et al. [37], and Zheng et al. [5], respectively. The experimentally measured values are compared with the calculated values from the original authors and the proposed method. These four comparative datasets cover a range from normal-strength to high-strength concrete and from steel to FRP reinforcement, which makes the data broadly representative.

4.2.1. Effect of Concrete Strength: Normal- to High-Strength

Fang et al. designed and tested 12 steel-reinforced concrete beams and additionally collected test data for 18 beams from the literature [35]. The cross-sectional widths of the beams ranged from 100 to 200 mm, and the heights ranged from 150 to 300 mm. The cube compressive strength f_{cu} of concrete ranged from 20.0 to 82.5 N/mm², covering both normal- and high-strength concrete. The reinforcement consisted of ordinary steel bars, with a reinforcement ratio ranging from 0.64% to 1.60%. Fang et al. proposed that the axial tensile strength f_t and the elastic modulus E_c of concrete be converted according to the following formulas [38]:

$$f_t = 0.395 f_{cu}^{0.55} \quad (50)$$

and

$$E_c = (1.45 + 0.628 f_t) \times 10^4 \quad (51)$$

Fang et al. primarily considered the influence of concrete strength on the cracking moment. Their calculation formula reads:

$$M_{cr} = \gamma_F W_0 f_t \quad (52)$$

where γ_F is the inelastic influence coefficient of the section resistance moment, which is adjusted for concrete strength (referred to as the calculated inelastic influence coefficient by Fang et al. [35]):

$$\gamma_F = \begin{cases} 1.55, & f_{cu} < 30 \text{ N/mm}^2 \\ 1.4, & 30 \text{ N/mm}^2 \leq f_{cu} \leq 60 \text{ N/mm}^2 \\ 1.1, & f_{cu} > 60 \text{ N/mm}^2 \end{cases} \quad (53)$$

and W_0 is the elastic section modulus of the transformed section at the tensile edge.

A comparison between the calculated cracking moments from the Fang method and the proposed method with the experimentally measured values is shown in Figure 6. The Fang method yielded an average error of 0.57 kN·m and a root mean square error of 0.75 kN·m, while the proposed method yielded an average error of 0.42 kN·m and a root mean square error of 0.87 kN·m. Comparing the ratio of calculated values to measured values, the Fang method had an average value of 0.96 and a coefficient of variation of 0.10, while the proposed method had an average value of 1.07 and a coefficient of variation of 0.13. It can be concluded that for both normal- and high-strength concrete, the proposed method agrees well with the experimental results, and its prediction accuracy is essentially equivalent to that of the Fang method. It is important to note that the proposed method requires only the fundamental constitutive relationship of the material, i.e., the

material's tensile stress–strain curve. In contrast to the Fang method, the proposed method eliminates the need to introduce a specially adjusted inelastic influence coefficient γ_F of the section resistance moment based on concrete strength, making it more straightforward for practical application.

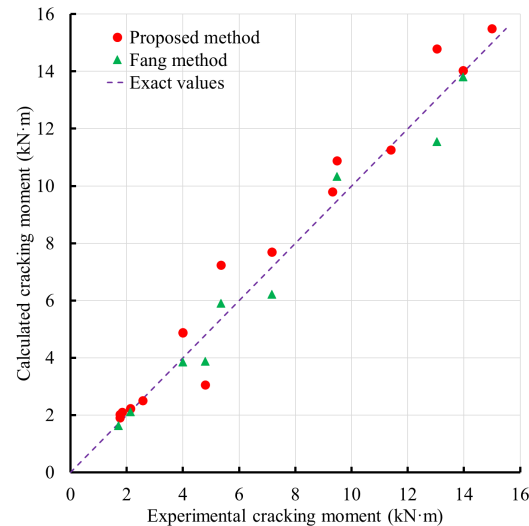


Figure 6. Comparison of measured and calculated cracking moments of normal- and high-strength concrete beams.

4.2.2. Effect of Concrete Strength: High-Strength

Jiang et al. designed and tested 11 steel-reinforced concrete beams [36]. The cross-sectional width of the beams was 180 mm, and the height was 210 mm. The concrete was high-strength C70 grade, with a characteristic axial compressive strength of 51.2 N/mm² and a characteristic axial tensile strength of 3.63 N/mm², both at a 95% assurance rate. The elastic modulus of the concrete E_c was 38.2 kN/mm². The reinforcement consisted of two groups of ordinary steel bars with yield strengths of 560.8 N/mm² and 429.1 N/mm², respectively. The elastic moduli of the steel bars were 201.1 kN/mm² and 199.1 kN/mm², respectively, and the reinforcement ratio ranged from 0.70% to 1.40%. According to Ref. [39], the coefficient of variation δ_c for the concrete tensile and compressive strengths is taken as 0.1. Therefore, the mean axial tensile strength and mean axial compressive strength used for checking the test data are given by the following formulas [18,39]:

$$f_m = \frac{f_k}{1 - 1.645 \delta_c} \quad (54)$$

where f denotes the tensile or compressive strength, the subscript m denotes the mean value and the subscript k denotes the characteristic value at a 95% assurance rate. Thus, the axial tensile strength f_t and axial compressive strength f_c used for verification are calculated as 4.34 N/mm² and 61.3 N/mm², respectively.

Similarly, Jiang et al. primarily considered the influence of concrete strength on the cracking moment. Their calculation formula reads:

$$M_{cr} = \gamma_I W_0 f_t \quad (55)$$

where γ_I is the inelastic influence coefficient of the section resistance moment, which is modified by the regression based on the concrete strength grade:

$$\gamma_I = 1.75 \times (1.1 - 0.015 f_{cu}^{2/3}) \quad (56)$$

where f_{cu} is the cube compressive strength of concrete, which can be converted according to Ref. [38]:

$$f_{cu} = \frac{f_c}{\alpha_{cl}} \quad (57)$$

where α_{cl} is the ratio of the axial compressive strength to the cube compressive strength of concrete, taken as 0.76 for C50 and below, 0.82 for C80, with linear interpolation for intermediate values. Thus, the cube compressive strength f_{cu} of concrete is calculated as 76.6 N/mm².

A comparison between the calculated cracking moments from the Jiang method and the proposed method against the experimentally measured values is provided in Table 2. The Jiang method yielded a mean error of 0.19 kN·m and a root mean square error of 0.49 kN·m, whereas the proposed method yielded a mean error of −0.34 kN·m and a root mean square error of 0.56 kN·m. Comparing the ratio of calculated values to measured values, the Jiang method had an average value of 1.02 and a coefficient of variation of 0.05, whereas the proposed method had an average value of 0.96 and a coefficient of variation of 0.05. It can be concluded that for high-strength concrete, the proposed method agrees well with the experimental results and achieves prediction accuracy very close to that of the Jiang method. Similarly, it is important to note that the proposed method requires only the fundamental constitutive relationship, i.e., the material's tensile stress–strain curve, while the Jiang method requires the introduction of an inelastic influence coefficient γ_I of the section resistance moment obtained from specialized tests, making the proposed method more straightforward for practical application.

Table 2. Cracking moments for high-strength concrete beams: measured vs. calculated.

$b \times h$ (mm)	h_0 (mm)	A_s (mm ²)	E_s (kN/mm ²)	M_{exp} (kN·m)	M_{Pro} (kN·m)	M_{Jiang} (kN·m)	$\frac{M_{Pro}}{M_{exp}}$	$\frac{M_{Jiang}}{M_{exp}}$
179 × 207	171	226	201.1	7.88	7.71	8.38	0.98	1.06
181 × 213	177	339	201.1	8.51	8.59	9.15	1.01	1.07
180 × 205	169	452	201.1	8.19	8.25	8.59	1.01	1.05
182 × 208	172	226	201.1	7.68	7.91	8.60	1.03	1.12
179 × 212	176	339	201.1	9.75	8.42	8.97	0.86	0.92
181 × 209	173	452	201.1	9.40	8.61	8.97	0.92	0.95
181 × 209	173	226	201.1	8.51	7.94	8.64	0.93	1.02
181 × 214	178	339	201.1	9.04	8.67	9.23	0.96	1.02
179 × 207	171	452	201.1	8.59	8.37	8.71	0.97	1.01
180 × 208	172	226	199.1	8.49	7.82	8.51	0.92	1.00
179 × 211	175	452	199.1	8.70	8.67	9.05	1.00	1.04

b and h represent the section width and height, respectively; h_0 represents the effective depth of the section; A_s represents the area of reinforcement; E_s represents the elastic modulus of steel; M_{exp} is the experimentally measured value; M_{Pro} and M_{Jiang} are the calculated values from the proposed method and the Jiang method, respectively.

4.2.3. Effect of Reinforcement Ratio: Steel Reinforcement in High-Strength Concrete

Sun et al. designed and tested eight steel-reinforced concrete beams [37]. The cross-sectional width of the beams was 120 mm, and the height was 250 mm. The concrete was high-strength concrete modified with reactive powder and steel fibers, with a cube compressive strength f_{cu} of 120 N/mm², an axial tensile strength f_t of 6.9 N/mm², and an elastic modulus E_c of 45.3 kN/mm². The reinforcement consisted of high-strength steel bars, with measured yield strengths ranging from 531 to 570 N/mm², an elastic modulus of 200.0 kN/mm², and a reinforcement ratio ranging from 0.87% to 16.35%.

Sun et al. primarily considered the influence of the reinforcement ratio on the cracking moment under conditions of high-strength concrete and high-strength steel reinforcement. Their proposed calculation formula reads:

$$M_{cr} = \gamma_S W_0 f_t \quad (58)$$

where γ_S is the inelastic influence coefficient of the section resistance moment, which is adjusted for the reinforcement ratio:

$$\gamma_S = \begin{cases} 1.33 + 12\rho, & \rho \leq 3.98\% \\ 1.81, & \rho > 3.98\% \end{cases} \quad (59)$$

where ρ is the reinforcement ratio.

A comparison of the calculated cracking moments from the Sun method and the proposed method against the experimentally measured values is shown in Figure 7. For the proposed method, the tensile stress–strain relationship for steel-fiber-reinforced concrete was still adopted from Equations (32)–(34) for ordinary concrete. The results indicate that the Sun method yielded a mean error of $-1.41 \text{ kN}\cdot\text{m}$ and a root mean square error of $1.89 \text{ kN}\cdot\text{m}$, whereas the proposed method yielded a mean error of $-0.23 \text{ kN}\cdot\text{m}$ and a root mean square error of $1.59 \text{ kN}\cdot\text{m}$. For the ratio of calculated values to measured values, the Sun method had an average value of 0.92 and a coefficient of variation of 0.09, whereas the proposed method had an average value of 0.99 and a coefficient of variation of 0.12. It is evident that for high-strength concrete modified with reactive powder and steel fibers under varying reinforcement ratios, the proposed method agrees well with the experimental results and achieves prediction accuracy essentially equivalent to that of the Sun method. Similarly, the proposed method requires only the fundamental constitutive relationship, i.e., the material's tensile stress–strain curve. In contrast to the Sun method, which requires correcting the inelastic influence coefficient γ_S of the section resistance moment based on the reinforcement ratio through specialized tests, the proposed method is more straightforward for practical application.

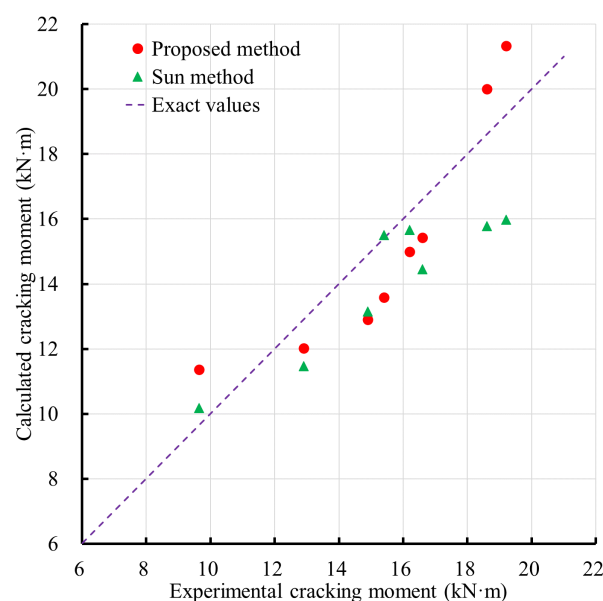


Figure 7. Comparison of measured and calculated cracking moments of high-strength concrete beams modified with reactive powder and steel fibers.

4.2.4. Effect of Reinforcement Ratio: FRP Reinforcement in High-Strength Concrete

Zheng et al. designed and tested eight GFRP-reinforced concrete beams. The cross-sectional width of the beams was 150 mm, and the height was 280 mm. The concrete was high-strength concrete modified with reactive powder and steel fibers. The compressive strength f_c of the 100 mm × 100 mm × 300 mm prism was 102.28 N/mm², the axial tensile strength f_t was 10.19 N/mm², and the tensile elastic modulus E_c of the concrete was 48.1 kN/mm². The GFRP bars had diameters of 5.5 mm, 12 mm, and 14 mm, with tensile strengths of 1159 N/mm², 990 N/mm², and 836 N/mm², respectively. The elastic moduli of the GFRP bars were 49.4 kN/mm², 47.6 kN/mm², and 50.0 kN/mm², respectively. The reinforcement ratio ranged from 0.13% to 2.69%. Two beams with a reinforcement ratio below the minimum longitudinal reinforcement ratio were excluded. Six beams with reinforcement ratios ranging from 0.61% to 2.69% were selected for validation.

Zheng et al. primarily considered the influence of the reinforcement ratio on the cracking moment of beams under conditions of high-strength concrete and FRP reinforcement. Their proposed calculation formula reads:

$$M_{cr} = \gamma_Z W_0 f_t \quad (60)$$

where γ_Z is the inelastic influence coefficient of the section resistance moment, which is adjusted for the reinforcement ratio:

$$\gamma_Z = 1.1 + 6\rho \quad (61)$$

where ρ is the reinforcement ratio.

A comparison of the calculated cracking moments from the Zheng method and the proposed method against the experimentally measured values is provided in Table 3. For the proposed method, the tensile stress–strain relationship for steel-fiber-reinforced concrete was also adopted from Equations (32)–(34) for ordinary concrete. The results indicate that the Zheng method yielded a mean error of −1.00 kN·m and a root mean square error of 1.32 kN·m, whereas the proposed method yielded a mean error of −1.00 kN·m and a root mean square error of 1.63 kN·m. For the ratio of calculated values to measured values, the Zheng method had an average value of 0.96 and a coefficient of variation of 0.04, whereas the proposed method had an average value of 0.96 and a coefficient of variation of 0.05. It is evident that for high-strength GFRP-reinforced concrete modified with reactive powder and steel fibers under varying reinforcement ratios, the proposed method agrees well with the experimental results and achieves prediction accuracy very close to that of the Zheng method. Similarly, the proposed method requires only the fundamental constitutive relationship, i.e., the material's tensile stress–strain curve. Again, in contrast to the Zheng method, which requires correcting the inelastic influence coefficient γ_Z of the section resistance moment based on the reinforcement ratio through specialized tests, the proposed method is more straightforward for practical application.

Table 3. Cracking moments for GFRP-reinforced concrete beams: measured vs. calculated.

$b \times h$ (mm)	h_0 (mm)	A_s (mm ²)	E_s (kN/mm ²)	M_{exp} (kN·m)	M_{Pro} (kN·m)	M_{Zheng} (kN·m)	$\frac{M_{Pro}}{M_{exp}}$	$\frac{M_{Zheng}}{M_{exp}}$
150 × 280	249	226	47.6	24.00	24.12	22.91	1.00	0.95
150 × 280	230.5	452	47.6	24.00	24.30	23.85	1.01	0.99
150 × 280	230.5	565	47.6	24.00	24.44	24.33	1.02	1.01

Table 3. Cont.

$b \times h$ (mm)	h_0 (mm)	A_s (mm ²)	E_s (kN/mm ²)	M_{exp} (kN·m)	M_{Pro} (kN·m)	M_{Zheng} (kN·m)	$\frac{M_{Pro}}{M_{exp}}$	$\frac{M_{Zheng}}{M_{exp}}$
150 × 280	230.5	678	47.6	27.00	24.58	24.81	0.91	0.92
150 × 280	228.5	769	50.0	27.00	24.70	25.22	0.91	0.93
150 × 280	228.5	923	50.0	27.00	24.89	25.89	0.92	0.96

b and h represent the section width and height, respectively; h_0 represents the effective depth of the section; A_s represents the area of GFRP reinforcement; E_s represents the elastic modulus of GFRP; M_{exp} is the experimentally measured value; M_{Pro} and M_{Zheng} are the calculated values from the proposed method and the Zheng method, respectively.

4.2.5. Discussion of Results

As mentioned above, the four sets of beam tests encompass normal-strength and high-strength concrete, as well as ordinary steel bars, high-strength steel bars, and FRP reinforcement. The proposed strain energy-based method demonstrates good agreement with the experimentally measured values across all these conditions. Comparing the ratio of the measured value to the calculated value, for all four sets of test data, the average ratio is 1.01, with a coefficient of variation of 0.12. These results indicate that the calculated results from the proposed method are generally very close to the experimental results, although the coefficient of variation is slightly large. This discrepancy arises from two main reasons. On the one hand, the experimental data for concrete cracking inherently exhibit significant scatter, with the coefficient of variation ranging from 0.16 to 0.10 as the concrete strength grade varies from C25 to C80 [39]. On the other hand, the axial tensile strength of concrete, f_t , which significantly influences the calculation of the cracking moment, is partly derived, potentially amplifying the error. Overall, the deviation between the calculated values and experimental values remains within an acceptable range. The overall accuracy is satisfactory. This demonstrates the feasibility of the strain energy-based method for calculating the cracking loads of members.

It is worth noting that the calculation error of the proposed method is significantly larger for the following two cases than that observed in the aforementioned four sets of tests: the ultra-high-strength reinforced concrete beams with the concrete cube compressive strength exceeding 150 N/mm² tested by Wang et al. [29], and the recycled aggregate concrete beams using construction waste tested by Kong et al. [8]. For the former case (Wang et al.), the discrepancy arises because the concrete in the tensile zone remains in the elastic deformation stage without entering the inelastic deformation stage at cracking for ultra-high-strength concrete beams [29]. Consequently, the stress–strain distribution at the cracked section does not satisfy the assumptions of the proposed method, as illustrated in Figure 4, leading to considerable deviation in the calculated results. This indicates that the proposed method is not suitable for scenarios involving brittle fracture, such as cracking in ultra-high-strength concrete beams. For the latter case (Kong et al.), the increased error may be attributed to the numerous initial microcracks in the recycled aggregate compared to conventional natural aggregate, which reduce its deformation resistance. Consequently, the deformation characteristics under load differ significantly from those of conventional aggregate concrete. Thus, noticeable deviations occur in the calculated results when the proposed method still employs the stress–strain relationship based on conventional concrete. This indicates that the proposed method must select an appropriate stress–strain relationship according to the properties of the constituent materials. Specifically, the existing constitutive relationships are applicable only to normal-strength and high-strength concrete incorporating conventional natural aggregates or modified with steel fibers or reactive powder.

4.3. Model Assessment via Inelastic Deformation Analysis

Based on the critical deformation φ_{cr} and the corresponding depth x_{cr} of the compression zone at member cracking obtained from the proposed strain energy method, the ultimate strain of concrete at the tensile edge is given by:

$$\varepsilon_{cr}^t = \varphi_{cr}(h - x_{cr}) \quad (62)$$

The ratio of the ultimate strain at the tensile concrete edge to the concrete peak tensile strain, $\varepsilon_{cr}^t/\varepsilon_p$, reflects the development of sectional inelastic deformation and is closely related to the calculation of the cracking moment. Classical theories for cracking of concrete members [22,34] uniformly adopt a value of 2.0 for this ratio. Fang et al. [35] suggested that this ratio decreases with increasing concrete strength, recommending values of 1.5 for normal-strength concrete and 1.1 for high-strength concrete. In fact, the ultimate strain of the concrete at the tensile edge is influenced by concrete strength, reinforcement quantity and arrangement [22,34,35]. Neglecting these factors or considering only the concrete strength may be incomplete. A singly reinforced rectangular beam section, typical in laboratory studies, is selected, with dimensions $b \times h = 150 \times 300$ mm, the effective depth $h_0 = 270$ mm, and reinforcement of HRB400. The strain ratio is calculated for: high reinforcement ratio ($\rho = 1.40\%$), economical ratios ($\rho = 1.13\%$ and 0.89%), low ratio ($\rho = 0.50\%$), and the minimum ratio ($\rho = 0.35\%$). The results are presented in Table 4.

Table 4. Ratio of concrete ultimate strain at tensile edge to peak tensile strain $\varepsilon_{cr}^t/\varepsilon_p$.

ρ	f_t (C20~C30)								f_t (C30~C50)					f_t (C50~C80)			
	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.7	3.9	4.1
1.40%	8.82	4.96	3.62	3.24	2.95	2.73	2.55	2.40	2.28	2.17	2.08	2.00	1.94	1.88	1.78	1.69	1.63
1.13%	4.95	3.60	2.93	2.71	2.53	2.39	2.27	2.16	2.08	2.00	1.93	1.87	1.82	1.77	1.69	1.62	1.57
0.89%	3.59	2.92	2.53	2.38	2.26	2.16	2.07	2.00	1.93	1.87	1.82	1.77	1.73	1.69	1.62	1.57	1.52
0.50%	2.55	2.28	2.09	2.01	1.94	1.88	1.83	1.78	1.74	1.70	1.66	1.63	1.60	1.57	1.53	1.48	1.45
0.35%	2.31	2.11	1.96	1.90	1.85	1.80	1.75	1.71	1.68	1.64	1.61	—	—	—	—	—	—

ρ represents the reinforcement ratio; f_t represents the axial tensile stress of concrete (N/mm²); the entries marked with “—” indicate sections where under-reinforced failure occurred and thus are omitted.

Classical cracking theories for concrete members are primarily based on experimental results from normal-strength concrete members, roughly corresponding to concrete grades C30 to C50. The average strain ratio $\varepsilon_{cr}^t/\varepsilon_p$ within this range is analyzed: 2.09 for the economical ratio group, 1.98 for the group combining economical and lower ratios, and 2.04 for the full set of reinforcement ratios. The calculated results are in close agreement with the classical theory. Furthermore, the results demonstrate that the strain ratio decreases with increasing concrete strength and increases with the quantity of reinforcement. This trend aligns with the experimental observations that “concrete becomes more brittle with increasing strength” and that “reinforcement enhances the crack resistance of concrete”.

Furthermore, classical cracking theories for concrete members also consider the influence of section height on the development of inelastic deformation. By introducing the section height h into the calculation formula of the inelastic influence coefficient γ of the section resistance moment, these theories reflect the relationship between the member size and the concrete cracking resistance. For rectangular sections, a typical fitted formula for the inelastic influence coefficient γ is given by [22]:

$$\gamma = 1.75 \times \left(0.7 + \frac{120}{h}\right) \quad (63)$$

where h is in millimeters.

In the proposed method, the influence of member size on the concrete cracking load is implicitly reflected through the coupling effect of the plane-section assumption and the concrete stress–strain relationship. For members with different section heights, the strain distribution characteristics at the extreme tensile fiber vary with size according to the plane-section assumption. Larger section heights lead to a more pronounced extent by which the strain in the edge region exceeds the strain corresponding to the peak stress of the concrete. The post-peak characteristics of the concrete stress–strain relationship (i.e., the stress decay law after the peak strain) translate this size-dependent strain variation into differences in stress. These stress differences are subsequently reflected in the calculation of the total strain energy of the member, thereby ultimately manifesting as the influence of the size effect.

For the common range of concrete strength grades, the cracking moments of reinforced concrete beams with different section heights are calculated using the proposed method. The corresponding inelastic influence coefficient γ is then back-calculated. The curve of γ vs. the section height h , obtained from the proposed method, is compared with the typical fitted formula curve in Figure 8. Owing to the significant scatter inherent in the experimental data for this parameter [22], the results calculated by the proposed method also show a relatively wide distribution. Nevertheless, the variation trend aligns fundamentally with the fitted formula results. Moreover, the proposed method similarly captures the influence of member size on the cracking resistance of concrete beams.

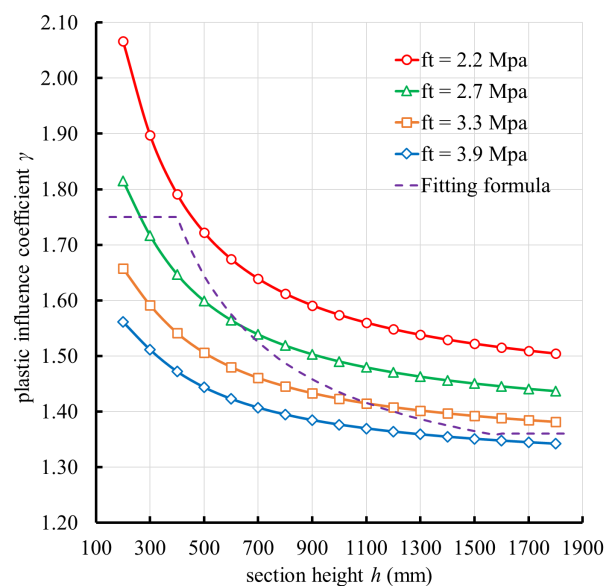


Figure 8. Variation of inelastic influence coefficient γ with section height h .

4.4. Sensitivity Analysis of Key Influencing Parameters

Based on the calculation procedure of the proposed method, the main factors influencing the cracking moment of flexural members are the sectional dimensions, effective depth, concrete strength grade, and the reinforcement ratio of steel or other reinforcement. The effective depth primarily reflects the influence of the concrete cover thickness and the arrangement of the reinforcement. The concrete strength grade primarily reflects the influence of the concrete tensile strength, tensile elastic modulus, and tensile stress–strain constitutive relationship. The reinforcement ratio primarily reflects the influence of the cross-sectional area of the reinforcement bars. From a practical perspective, efficiently improving the cracking resistance of a member under given working conditions is of greater concern. Therefore, this sensitivity analysis focuses on the influence of the concrete strength grade and the steel reinforcement ratio on the cracking moment. These two pa-

rameters are directly adjustable by the designer under given design conditions to enhance cracking resistance.

Again, the commonly used singly reinforced rectangular beam section in tests is selected: $b \times h = 150 \times 300$ mm, with the effective depth $h_0 = 270$ mm, and steel reinforcement of HRB400. Using the proposed method, the variation of the cracking moment with concrete strength grade is calculated for reinforcement ratios of 0.3%, 0.7%, 1.1%, and 1.4%. Similarly, the variation of the cracking moment with reinforcement ratio is calculated for concrete tensile strengths of 2.2 N/mm², 2.7 N/mm², 3.3 N/mm², and 3.9 N/mm². The results are shown in Figures 9 and 10. It can be observed that the cracking moment increases with either the concrete strength grade or the reinforcement ratio. A positive correlation exists between these two parameters and the cracking moment.

Figure 9 indicates that when the reinforcement ratio is below 1.1% (i.e., within the economical and low reinforcement ratio ranges), the cracking moment exhibits an approximately linear relationship with the concrete tensile strength. When the reinforcement ratio exceeds 1.1% (i.e., within the high reinforcement ratio range), the cracking moment initially increases at a lower rate with increasing concrete tensile strength within the range of lower concrete strength grades. As the concrete tensile strength increases into the range of higher concrete strength grades, the cracking moment gradually recovers to an increasing rate comparable to that observed under economical and low reinforcement ratios. Therefore, for beams with lower concrete strength grades and higher steel reinforcement ratios, the cracking moment is less sensitive to the concrete strength grade. Consequently, solely increasing the concrete strength grade is not the most efficient measure for enhancing the cracking resistance of the member under such conditions.

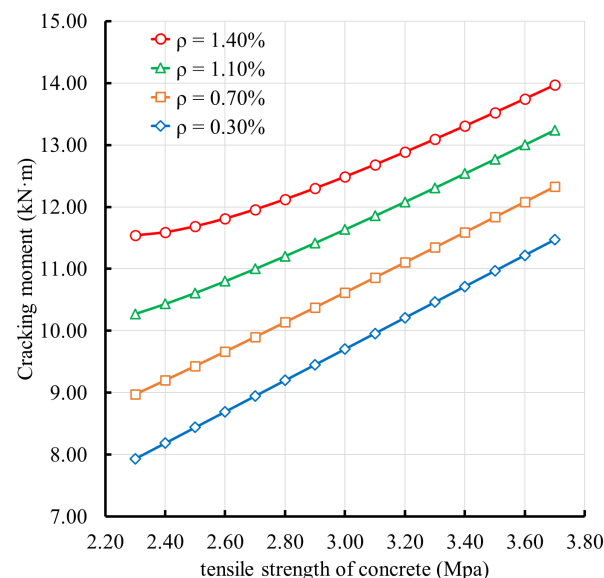


Figure 9. Variation of beam cracking moment with concrete tensile strength for different reinforcement ratios.

Figure 10 indicates that when the concrete tensile strength exceeds 2.7 N/mm² (i.e., within the higher concrete strength ranges), the cracking moment exhibits an approximately linear relationship with the steel reinforcement ratio. When the concrete tensile strength is below 2.7 N/mm² (i.e., within the lower concrete strength range), the cracking moment initially increases linearly with increasing reinforcement ratio within the low and economical reinforcement ratio ranges. As the reinforcement ratio increases into the high reinforcement ratio range, the cracking moment increases significantly faster. Therefore, for beams with higher reinforcement ratios and lower concrete strength grades, the cracking moment is

highly sensitive to the reinforcement ratio. Consequently, increasing the steel reinforcement ratio is an effective measure for enhancing the cracking resistance of the member under such conditions.

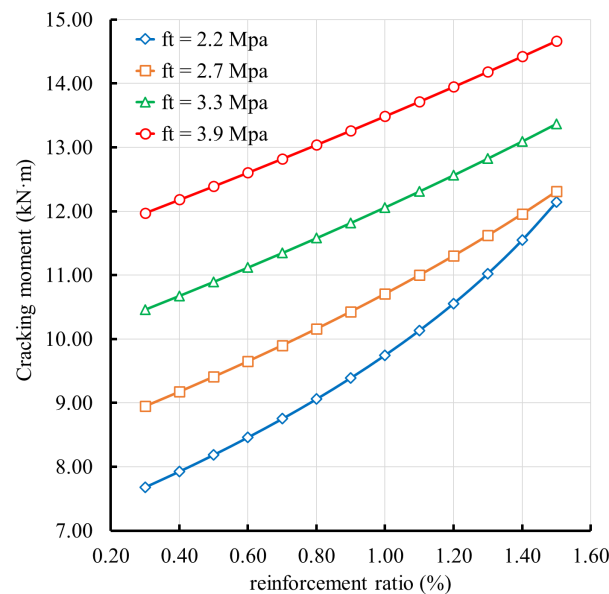


Figure 10. Variation of beam cracking moment with reinforcement ratio for different concrete strength grades.

In summary, within the range of economical reinforcement ratios ($\rho = 0.7\sim 1.1\%$) and normal- to high-strength concrete ($f_t = 2.7\sim 3.9 \text{ N/mm}^2$), the beam cracking moment always exhibits a linear relationship with both the concrete strength grade and the steel reinforcement ratio. The cracking resistance of the member can therefore be effectively enhanced by increasing either the concrete strength grade or the steel reinforcement ratio. However, when the concrete strength grade is relatively low and the steel reinforcement ratio is high, the contribution of the reinforcement to the sectional cracking resistance substantially exceeds that of the concrete. Therefore, under such conditions, increasing the steel reinforcement ratio is more effective for improving the cracking resistance of the member than increasing the concrete strength grade.

5. Conclusions

This paper proposes a strain energy-based method for calculating the cracking loads of steel- and FRP-reinforced concrete members, applicable to axially tensioned members and ordinary flexural members where tensile deformation is the primary deformation mode. The method establishes an expression for the total strain energy of the member using the axial tensile deformation as the independent variable. The critical condition for cracking is defined as the point where the second-order differential of this total strain energy with respect to the axial deformation equals zero. Based on this condition, the deformation at cracking and the corresponding cracking loads are determined. Compared to traditional calculation methods, the proposed approach requires only basic sectional geometrical parameters and the fundamental stress–strain constitutive relationship of the materials. It eliminates the need for introducing additional coefficients to describe the development of sectional inelastic deformation, such as the inelastic influence coefficient γ of the section resistance moment, making it simpler to apply. In contrast to emerging machine learning approaches, the proposed method possesses a clear physical basis and a transparent calculation process, ensuring the results are fully credible. In the derivation process, the constituent materials of the member are assumed to be homogeneous and

isotropic. However, since the concrete stress–strain constitutive relationship adopted in this method is entirely based on experimental measurements, the results inherently incorporate the effects of material inhomogeneity and anisotropy. Therefore, the aforementioned assumption does not hinder the application of the method to materials like concrete, which are inhomogeneous and not strictly isotropic.

Calculations verified against experimental data from four sets of tests on steel- and FRP-reinforced concrete flexural members demonstrate that the proposed method is fully applicable to both normal- and high-strength concrete, as well as to both ordinary steel and FRP reinforcement. All results agree well with the experimentally measured values, with a relative error of only 1% and a coefficient of variation of 0.12 for the calculated-to-measured ratio. However, verification against experimental data for flexural members made with recycled aggregate concrete and ultra-high-strength concrete indicates that the proposed method is not applicable to these two types of concrete. For recycled aggregate concrete, the reduction in deformation resistance leads to a tensile stress–strain curve that does not match the curve based on conventional natural aggregate concrete used in this method. For ultra-high-strength concrete, the section remains in the elastic deformation stage at cracking, which contradicts the sectional inelastic deformation state assumed in the proposed method.

The proposed method is applied to analyze the cracking loads of axially tensioned and flexural members of steel-reinforced concrete. The results indicate that for axially tensioned members, when the reinforcement area exceeds a critical value (termed the inflection point reinforcement ratio ρ_D in this study), the member exhibits no significant loss of macroscopic performance, such as durability and stiffness, before the steel reinforcement yields or fractures in tension. Under this condition, the cracking resistance of the member increases severalfold. For flexural members, within the range of economical reinforcement ratios and normal- to high-strength concrete grades, the cracking resistance can be effectively enhanced by increasing either the concrete strength grade or the reinforcement ratio. However, when the concrete strength grade is relatively low and the reinforcement ratio is high, increasing the reinforcement ratio is more effective for improving the cracking resistance.

Owing to its foundation in traditional mechanics of materials, the proposed method can be readily integrated into existing structural design procedures. Furthermore, since it does not rely on any empirical coefficients that characterize the development of sectional inelastic deformation (such as the inelastic influence coefficient γ of the section resistance moment), there is no need to recalibrate such parameters through new tests for novel structural forms or materials. As a result, the proposed method not only avoids the complexity and uncertainty inherent in empirical parameters but also significantly enhances the adaptability of the cracking design approach to various new structures and materials.

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