

Article

Visualization Study on Construction Disturbance of Drainage Board Sleeve Pile Shoes

Junzhi Lin , Bojun Zhang, Zelong Liang, Hongming Chen, Zonglin Yang, Yan Tang and Yan Du

The College of River and Ocean Engineering, Chongqing Jiaotong University, Chongqing 400074, China; 622230960192@mails.cqjtu.edu.cn (B.Z.)

* Correspondence: junzhi_lin@cqjtu.edu.cn

Abstract

One of the key indicators of the foundation soil consolidation is the smear effect brought on by the insertion of a Prefabricated vertical drain (PVD), which also smears the extent of disturbance. Prior research primarily examined the impact of the diameter of the Prefabricated vertical drain sleeves, ignoring the impact of pile shoe size on smear effect. The penetration process of pile shoes of varying sizes in layered soils was simulated using transparent soil model experiments, and Particle Image Velocimetry (PIV) technology was used to visualize and assess the soil disturbance caused by the pile shoes. Theoretical and experimental data are used to suggest and analyze the correction coefficients for the geometric characteristics of pile shoes using the Mohr–Coulomb criterion and reaming theory. The study’s findings demonstrate that transparent soil and the PIV method can successfully capture the dynamic evolution of the “inverted cone” in the smeared area, which is consistent with the theory of cylindrical pore expansion’s prediction. The horizontal disturbance range will increase as the equivalent radius of the pile shoes increases, and it is 4.5d for pile shoes with an equivalent radius of 1.5 mm and 5d for pile shoes with an equivalent radius of 2.0 mm. The discontinuity of the soil layer interface will be made worse by pile shoes with a high equivalent radius, making the phenomenon of stress concentration more noticeable. Its quantitative analysis demonstrates the reasonableness of the correction factor λ , which offers a trustworthy tool to quantify the perturbation effect of the pile shoe size. A correction factor λ is proposed so that the error between the corrected theoretical value and the test value is less than 5%.

Keywords: transparent soil; prefabricated vertical drain pile shoe; disturbance range; revision of reaming theory



Academic Editors: Biao Hu and Eugeniusz Koda

Received: 14 August 2025

Revised: 26 October 2025

Accepted: 18 November 2025

Published: 20 November 2025

Citation: Lin, J.; Zhang, B.; Liang, Z.; Chen, H.; Yang, Z.; Tang, Y.; Du, Y. Visualization Study on Construction Disturbance of Drainage Board Sleeve Pile Shoes. *Buildings* **2025**, *15*, 4195. <https://doi.org/10.3390/buildings15224195>

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Because of its benefits in terms of ease of construction, resistance to material corrosion, and structural stability, a drainage board is frequently utilized in foundation soil treatment projects in coastal locations. For a shore protection project in Bangladesh, Shu [1] and colleagues employed a drainage board in conjunction with the pile load pre-compaction method for marine sedimentary soft soil with a high moisture content. Dong Lei [2] and colleagues confirmed that the drainage board piled-load pre-compaction technology greatly reduced the construction time for the port project and successfully increased the soft soil consolidation rate.

Notably, the smear effect caused by sleeve penetration during the installation of drainage board has a direct impact on the foundation treatment’s ultimate outcome. The

result is a decrease in the soil's strength and coefficient of permeability in a certain area surrounding the drainboard (the smear zone).

In order to determine the link between the diameter d_s of the smeared area and the equivalent diameter of the sleeve, d_m , or the equivalent diameter of the shaft, d_m , the current state of research on the extent of smearing mostly relies on experimental and theoretical investigations [3]. Based on the theory of cylindrical hole expansion, Zhang Yiping [4] and others used the Mohr–Coulomb yield criterion to determine that the extent of the smeared zone is directly related to the degree of crowding. They also proposed a method for rationally choosing the extent of the smeared zone based on the type of shaft, the soil quality, and even the construction method. Using circular pore expansion theory, Sathanantha and colleagues [5] calculated the smeared zone's extent as $d_s = (2-4)d_m$. In order to characterize the disturbed zone surrounding the sleeve, Ghandeharioon and others [6] applied the elliptical cavity expansion theory to an example. The results validated the suggested method of determining the extent of the smeared zone, and the value of $d_s = 3.1d_m$ was found to be close to the value proposed by Sathanantha and others. The installation of prefabricated vertical drainage boards was then experimentally studied by Ghandeharioon and others [7]. The results were numerically analyzed to determine that $d_s = 3.5d_m$ and that the dimensions of the coated zones decreased as the effective in situ stress increased. Using soil samples from a soft clay site in Ballina, NSW, Indraratna and others [8] developed a radial consolidation analytical model to examine the impact of soil disturbance during the installation of vertical drainage devices on the permeability and compressibility of soft clay soils, yielding $d_s = 6.3d_m$. Digital image correlation techniques used in laboratory modeling studies by S. Prabavathy and others [9] produced $d_s = (4-6)d_m$, and the diameter of the smeared region decreases as the installation rate increases. Recent experimental research on smearing range is displayed in Table 1.

Table 1. Smear zone parameter values.

Reference	Diameter of Coating Area	Methodologies
Sathanantha et al. [5]	$d_s = (2-4)d_m$	Inverse analysis
Ghandeharioon et al. [6]	$d_s = 3.1d_m$	Inverse analysis and indoor testing
Ghandeharioon et al. [7]	$d_s = 3.5d_m$	
Indraratna et al. [8]	$d_s = 6.3d_m$	Indoor testing
S. Prabavathy et al. [9]	$d_s = (4-6)d_m$	Indoor testing
Sathanantha et al. [5]	$d_s = (2-4)d_m$	Inverse analysis

The link between sleeve diameter and application range is the primary focus of existing research, which also analyzes siltation characteristics [10–14] and bending features [15–20], largely neglecting the impact of drainage board pile shoes on the application range. In actuality, the drainage board pile shoe plays a significant role in determining the range of applications, and its size is typically more than the sleeve's diameter. Using the standard 100 mm × 4 mm drainage board as an example, the sleeve diameter is around 127 mm, but the pile shoe length can reach 255 mm [21], which is roughly twice the sleeve diameter. This size discrepancy suggests that the pile shoe's effect cannot be disregarded.

Using the Mohr–Coulomb yield criterion and the theory of circular pore expansion, a shape factor is proposed to correct the application range of drainage board with circular stick pile shoes. The effects of their length and cross-section equivalent radius on the application range are further examined. This paper focuses on the visualization of the insertion process of drainage board with circular stick pile shoes (the most widely used). Additional research is carried out into how the application range is affected by length and section equivalent radius.

2. Overview of Visualization Tests

2.1. Test Equipment

A model box, a drainage board insertion tool, a high-speed CCD camera, a laser, a computer, and a drainage board model with a circular cross-section pile shoe make up the Drainage Board Pile Shoe Insertion Test System (Figure 1).

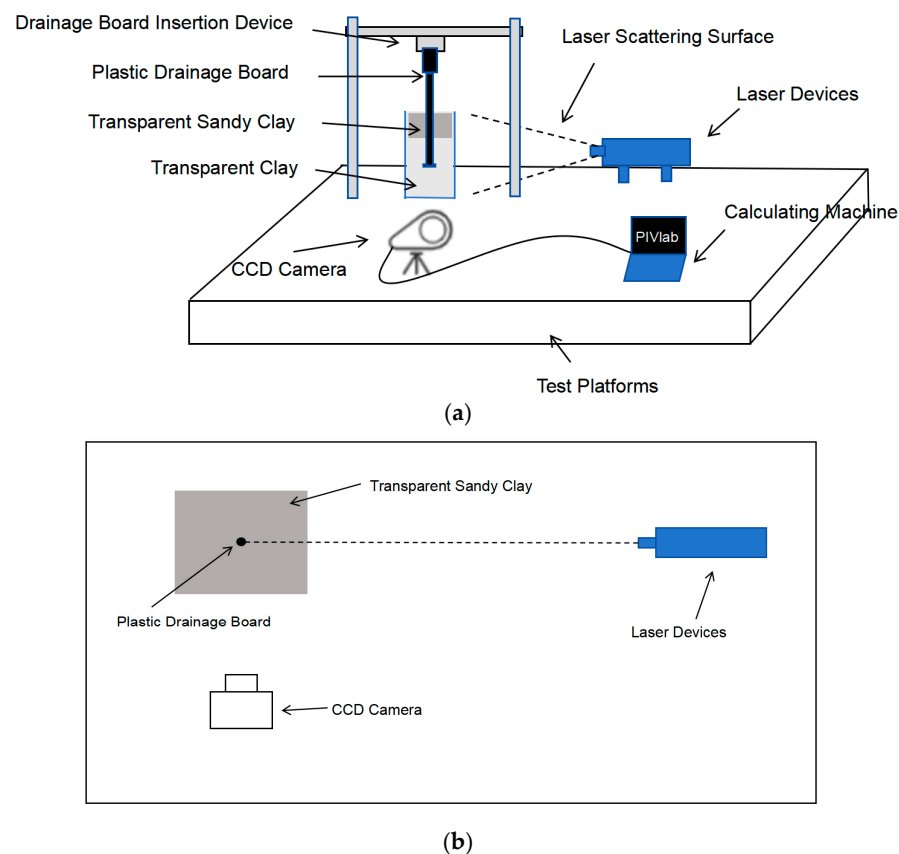


Figure 1. Effective diagram of the test model system: (a) Model test layout; (b) Drainage board pile shoe insertion layout plan.

The model box is $130 \text{ mm} \times 130 \text{ mm} \times 250 \text{ mm}$, with an open top and walls and a bottom plate that are 6 mm thick. The drainage board insertion device is made up of two parts: an embedding device and a fastening device. The high-speed CCD camera was positioned at 1.2 m immediately in front of the model box, and the voltage was adjusted to maintain a constant pile shoe insertion rate of 1 mm/s throughout the test. The computer control system was in charge of the image acquisition and processing, and the PIVlab 3.02 and Tecplot 360 EX 2022 R1 software were utilized for the image analysis. The laser was an integrated laser generator that could create a sheet light source and create a sheet scattering field in the transparent soil.

The transparent soil specimen is exposed to the laser's sheet light source during the test, which interacts with the tracer particles to create a dynamic scattering field. A high-speed CCD camera records the scattering field's changes in real time during the pile-boot insertion process, and the images are subsequently post-processed using the PIVlab and Tecplot software to enable visualization and analysis of the process.

This experiment created two types of circular drainage board pile shoe models based on a 1:20 ratio, in accordance with the standard size of engineering [22,23]. The first displays the precise design specifications (Table 2). The model is constructed using the popular

fused deposition 3D printing process and is composed of two parts (Figure 2): the pile shoe and the cylindrical drainage board sleeve.

Table 2. Parameters of cylindrical drainage sheet pile shoes.

Pile Shoe Category	Sleeve Length	Sleeve Diameter	Pile Shoe Length	Stockbroker	Pile Shoe Equivalent Radius
I	100	6	10	10	1.5
II	100	6	10	10	2.0

Note: Unit mm.

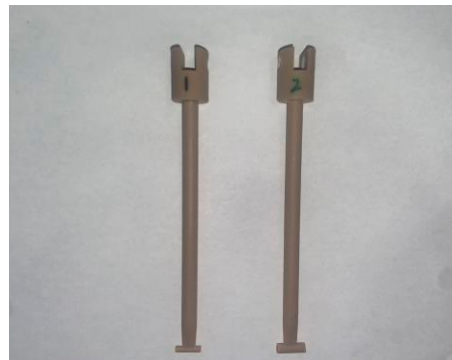


Figure 2. Model drawing of pile shoe of cylindrical drainage sheet.

2.2. Preparation and Test Materials

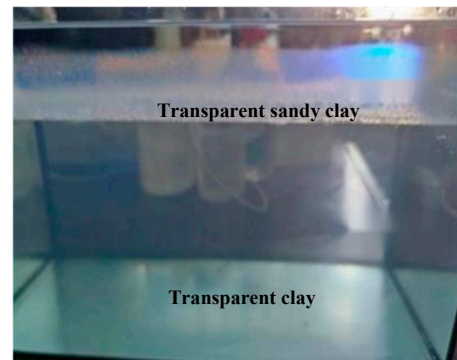
Foundation soil treatment is primarily concentrated in coastal or port areas, where the soil is largely composed of high-organic-content silty clay. This silty clay exhibits extremely high porosity, resulting in an extremely weak soil body with very low inherent structural strength. Therefore, the composite translucent soil specimens were made using the layering approach, and this experiment mimics the “upper sand and lower clay” layered soil [24]. The visualization research of the drainage board pile shoe insertion test may be realized by simulating natural clay and sandy soils using various transparent soil materials. Transparent soil attributes are comparable to those of natural soils. The transparent sand skeleton for the sandy soil was made up of 0.5–1 mm quartz sand particles; the transparent clay skeleton was made up of a 3.5% concentration of Laponite RD gel (combined with PSP tracer particles) and Laponite RD powder.

Transparent sandy soil pore liquid was prepared in this experiment using 3# and 15# white oil, based on the existing study results. First, the lowest transparent clay layer was made by filling the model box with deionized water, adding Laponite RD powder, and stirring to create a gel. The gel was then allowed to stand for a full day. The top layer of transparent sand soil is then prepared by washing and drying fused silica sand particles and alternating layers of a 1:9 volume ratio of 3# white oil and 15# white oil mixture of pore liquid. Each time, the first layer is poured into the pore liquid, followed by a layer of sand that is 2 mm thick. This process is repeated until the desired thickness is achieved, all while maintaining a constant temperature of 25 °C [22].

Table 3 displays the characteristics of the configured and finished layered transparent soil material, whereas Figure 3 displays the created transparent soil material.

Table 3. Parameters of transparent earth materials.

Style	ρ ($\text{g}\cdot\text{cm}^{-3}$)	E (MPa)	ν	φ_0	k ($\text{cm}\cdot\text{s}^{-1}$)	c0 (kPa)	e
Sandy soil	1.52	40	0.3	30°	/	0	0.65
Clayey soil	1.03	0.8	0.4	11°	8.1×10^{-7}	0.55	1.46

**Figure 3.** Layered transparent soil.

2.3. Test the Procedure and Program

This test is based on the common types of drainage board pile shoes used in the project, and it was designed to conduct insertion tests under six different working conditions. For example, in the case of a transparent sand layer and a transparent clay layer with a fixed height, cylindrical drainage board pile shoes were inserted at different depths: 2d, 3d, and 6d (d is the diameter of the sleeve/mm). For the specific working condition settings refer to Table 4.

Table 4. Study conditions for insertion of cylindrical drainage board pile shoes.

Working Condition	Pile Shoe Category	Sandy Soil	Clayey Soil	Insertion Depth
Condition 1	I	30	110	2d
Condition 2	I	30	110	3d
Condition 3	I	30	110	6d
Condition 4	II	30	110	2d
Condition 5	II	30	110	3d
Condition 6	II	30	110	6d

Note: Unit mm, d is the sleeve diameter.

The cylindrical drainage board pile shoe model and the drainage board insertion device were connected after the upper sand and lower clay composite transparent soil specimen was configured using the previously described method. The cylindrical drainage board pile shoe model was placed in the middle of the model tank, and the top of it was connected to the drainage board insertion device via the connecting piece. The drainage board insertion device has two parts: a fixed device and an embedded device. The laser is on the left side of the model box, and the high-speed CCD camera is positioned in front of the board at a constant rate of 1 mm/s. Make sure to isolate the external light source with shading measures prior to the test. Once the test has begun, insert the drainage boards at a uniform speed of 1 mm/s while recording the entire process with a high-speed CCD camera. Once the boards are in place, stop loading and filming.

3. Test Results and Analysis

3.1. Validation of Tests

The pile-less boot cylindrical sleeve was used as a research example in order to confirm the accuracy and dependability of the small-size model built in this paper using transparent soil material and PIV technology. The test data were normalized, compared, and analyzed with the model test results of the circular hole expansion theory and the research results of Ni [25], and the analysis's findings are displayed in Figure 4. The vertical coordinate in the figure shows the ratio u/d of the horizontal displacement u of the soil from the center point to the diameter d of the sleeve, while the horizontal coordinate shows the ratio r/d of the distance r from the sleeve to the diameter d of the sleeve.

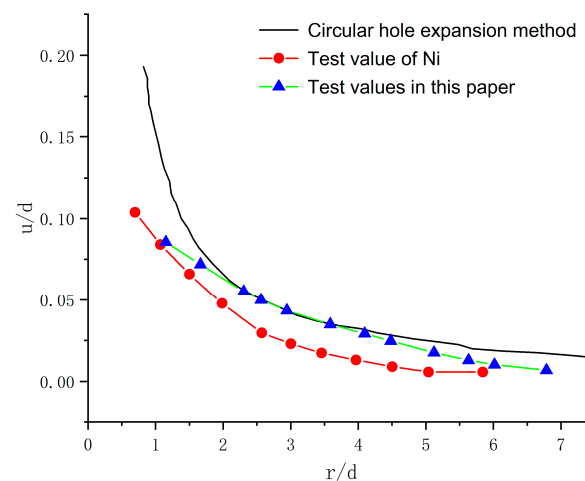


Figure 4. Horizontal displacement comparison verification curve.

All curves exhibit a monotonically declining trend of u/d as r/d increases, as seen in Figure 4. The experimental values in this paper are consistently lower by roughly 14% when compared to the theoretical solution for circular hole expansion, with the stabilization zone occurring at $2 < r/d < 4$ and the largest discrepancy occurring in the $r/d < 2$ near-field region. The experimental data are in high agreement with those of Ni [25]. However, because the experimental and theoretical circumstances differ, there is some variation in the outcomes. This could be because the laser penetrated the soil to irradiate the sleeve with reflections, and the pile shoes were modeled in this study using high-precision brown material. This affected the scattering field imaging effect, which is why the test values were lower than those of the circular hole expansion theory.

In conclusion, the accuracy and dependability of the small-scale model developed in this work using transparent earth materials and PIV technology are excellent, and it can offer solid experimental support and technological assistance for further relevant research.

3.2. Evaluation of How Various Pile Shoes Affect the Scope of Use

When the No. 1 pile shoe, which has an equivalent radius of 1.5 mm, enters the formation soil, the displacement vectors of the surrounding soil are as shown in Figure 5. The morphology is similar to the “inverted cone” which is near the “funnel shape” that Zhang Yiping examined using the theory of expansion of cylindrical holes combined with the Mohr-Coulomb yield criterion. It is evident that as the insertion depth increases, the horizontal smearing range of the upper sandy layer gradually decreases and the horizontal disturbance range of the clay layer gradually increases. Near the “funnel shape” examined by Zhang Yiping [4] and others, the morphology resembles an “inverted cone”. As illustrated in Figure 5a, the pile shoes penetrated for $2d$. The bouncing action of the

weak clay layer beneath the sand layer caused the sandy soil at the pile's lateral end to be displaced upward; the force transfer from the sand layer affects the clay layer beneath the sand layer, causing a considerable range of disturbance. As seen in Figure 5b, pile shoes pierced the 3D soil layer interface. Extrusion stacking made the sandy soil thick and decreased its disturbance range, while the soil displacement was mostly focused at the pile side, which causes plastic strain in the clay layer, which in turn causes the disturbance range to expand vertically to the range of 9d and increase horizontally to 5d. As shown in Figure 5c, when the pile shoes penetrated the clay layer for 6d, the displacement characteristics of the clay soil at the pile end and the sandy soil on the pile side remained essentially unchanged, while the disturbed range of the clay layer increased, particularly the vertical displacement of the soil body, to 10d. An explanation of how drainage board pile shoes were introduced into sandy soil at the start of the primary compressive deformation. As the pile shoes passed through the soil interface, the insertion depth increased and the clay soil body sheared continuously.

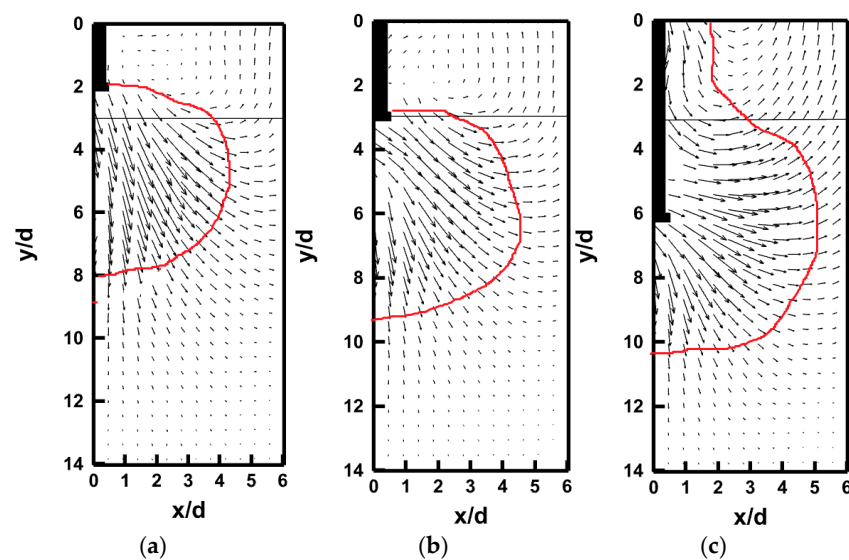


Figure 5. Vector map of soil displacement in the formation of pile shoe No. 1: (a) pile driving depth 2d; (b) pile driving depth 3d; (c) pile driving depth 6d.

When the second pile shoes into the layered soil with an analogous radius of 2.0 mm, the displacement vectors of the surrounding soil are shown in Figure 5. In Figure 6a–c, the radial disturbance ranges are approximately 5d, 5d, and 5d, respectively, which are comparable to the soil disturbance characteristics of No. 1 pile shoe, which are 4d, 4.5d, and 4.5d. As the penetration depth increases, the radial disturbance of the soil body is similar to that of No. 1 pile shoe and gradually decreases. However, the ranges of horizontal radial disturbances are expanded. Similar to the trend of the changes obtained from the tests conducted by Tehrani [26] and others, Figure 6a shows that the soil displacement magnitude rapidly decreases when the pile penetrates into the sandy soil layer, Figure 6b shows that the soil displacement interface is affected by a large amount and begins to decrease, and Figure 6c shows that the soil displacement is also dominated by the vertical displacement when the pile shoe penetrates into the clay layer. As the pile shoes got closer to the lower clay layer, the shear strain started to decrease in relation to the upper sandy layer. This could be because of the discontinuity in the soil near the soil interface and the relatively low compaction of the upper and lower layers at the interface, which caused the soil to move radially.

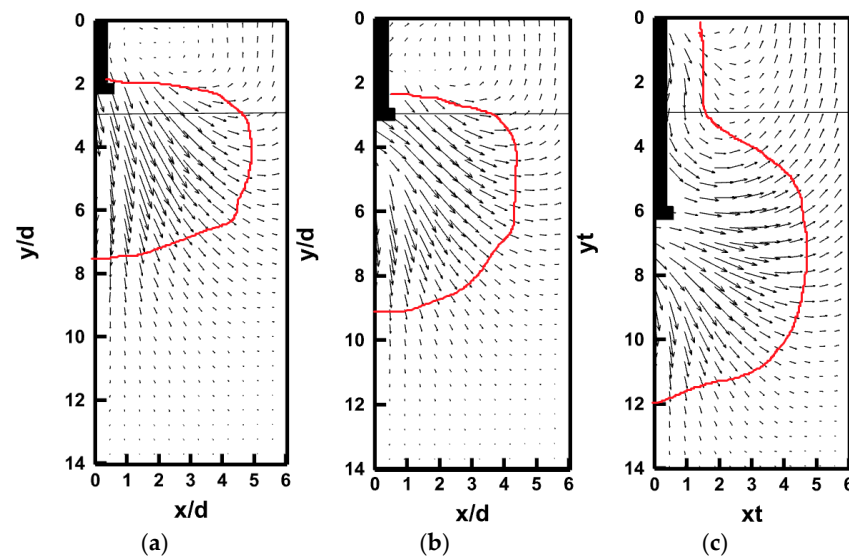


Figure 6. Vector map of soil displacement in the formation of pile shoe No. 2: (a) Pile driving depth 2d; (b) Pile driving depth 3d; (c) Pile driving depth 6d.

Although the equivalent radius of the two types of pile shoes are not equal, the displacement vector diagrams of the two types of pile shoes penetrating into the soil layers reveal a “inverted cone” perturbation pattern in their penetration behaviors, which is consistent with the cylindrical hole expansion theory’s prediction of the “funnel shape”. Although the horizontal disturbance range of pile shoe No. 1 is less, the vertical displacement of the clay layer is greater, suggesting a larger shearing action on the deeper clay; with a more uniform distribution of plastic strain in the clay layer and a more noticeable soil discontinuity at the interface, pile shoe No. 2 has a stable horizontal disturbance range of 5 d because of its greater radius and faster attenuation of displacement in the sandy soil layer. This indicates that the vertical disturbance range is positively correlated with the design of the equivalent radius of the pile shoe, and that a larger contact area and specific geometry lead to more obvious stress concentration phenomena. The vertical displacements in the clay layer are similar in depth, and the difference is primarily reflected in the horizontal stress transfer mechanism: No. 1 pile shoe is shear dominated, and No. 2 pile shoe is more significant than No. 1 pile shoe under the composite effect of extrusion-shear. The stress concentration phenomena is more noticeable due to the bigger contact area and particular shape.

4. Remaining Theoretical Analysis Taking Pile Shoes into Account

The classical cylindrical pore expansion theory [27–31] states that the plastic region ($r \leq R_p$) is modeled as a simplified elasto-plastic model using the Mohr–Coulomb criterion, and the elastic region ($r > R_p$) satisfies the generalized Hooke’s law under the assumptions of the theory of small deformations. It is also assumed that the soil body satisfies the following fundamental assumptions: (1) soil is a uniform, isotropic, and ideally robust substance; (2) the procedure of installing drainage board pile shoes is axisymmetric; (3) the plastic region’s volume remains constant (no shear expansion benefit); (4) the soil body’s elastic deformation as a result of the redistribution of stress is disregarded, and it can be presumed that the plastic region’s elastic deformation is equivalent to the elastic deformation of the elastic region side of the elastic-plastic region intersection line.

The columnar hole expansion problem’s computational model is displayed in Figure 7, where R_0 is the sleeve’s radius, R_u is the circular hole’s initial hole diameter, r_1 is the hole’s radius during the circular hole’s expansion process, r is the hole’s expansion radius, R_p is

the radius of the plastic zone, P_u is the limiting expansion pressure, σ_r and σ_θ are the soil body's radial and tangential stresses, respectively.

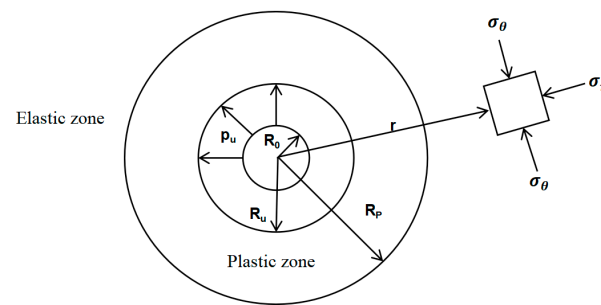


Figure 7. Expansion sketch of cylindrical hole.

4.1. A Generalized Solution to Hooke's Rule Based on the Notion of Circular Hole Expansion

4.1.1. Fundamental Equations

Equilibrium equations:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1)$$

geometric equation:

$$\varepsilon_r = -\frac{\partial u_r}{\partial r} \quad (2)$$

$$\varepsilon_\theta = -\frac{u_r}{r} \quad (3)$$

The arithmetic equation: ε_r is the radial strain, ε_θ is the tangential strain, and u_r is the radial displacement.

The elasticity eigenstructure equation (generalized Hooke's law):

$$\varepsilon_r = \frac{1 - \nu^2}{E} \left(\sigma_r - \frac{\nu}{1 - \nu} \sigma_\theta \right) \quad (4)$$

$$\varepsilon_\theta = \frac{1 - \nu^2}{E} \left(\sigma_\theta - \frac{\nu}{1 - \nu} \sigma_r \right) \quad (5)$$

4.1.2. Elastic Solution for Circular Hole Dilatation

When the initial stress field is not considered, the stress field and displacement field in the elastic region can be obtained from the boundary conditions: $r = r_1$, $\sigma_r = p$, $r = \infty$, $\sigma_r = 0$:

$$\sigma_r = p \left(\frac{r_1}{r} \right)^2 \quad (6)$$

$$\sigma_\theta = -p \left(\frac{r_1}{r} \right)^2 = -\sigma_r \quad (7)$$

$$u_r = \frac{1 + \mu}{E} \sigma_r r \quad (8)$$

When the initial stress field p_0 is considered, the stress field and displacement field in the elastic region can be obtained from the boundary conditions: $r = r_1$, $\sigma_r = p$, $r = \infty$, $\sigma_r = p_0$:

$$\sigma_r = (p - p_0) \left(\frac{r_1}{r} \right)^2 + p_0 \quad (9)$$

$$\sigma_\theta = -(p - p_0) \left(\frac{r_1}{r} \right)^2 + p_0 \quad (10)$$

$$u_e = \frac{1+\nu}{E}(p-p_0)\left(\frac{r_1}{r}\right)^2 r \quad (11)$$

4.2. Mohr-Coulomb Solution Based on Circular Hole Expansion Theory

4.2.1. Displacement Solution for Plastic Zone

For Mohr–Coulomb materials, the material yield expression is:

$$(\sigma_r - \sigma_\theta) = (\sigma_r + \sigma_\theta) \sin \varphi_0 + 2c_0 \cos \varphi_0 \quad (12)$$

In the equation: φ_0 , c_u are the angle of internal friction and cohesion of the soil, respectively.

Yielding of the pore wall at $r = R_p$ starts to occur when the pore pressure reaches the critical pressure p_c . At this point, the critical expansion pressure at which the pore wall begins to yield is:

$$p_c = c_0 \cos \varphi_0 \quad (13)$$

The plastic zone continues to expand outward when the pressure within the circular hole surpasses the critical expansion pressure. The stress field in the plastic zone at this moment is determined by Equation (1) and the boundary conditions $r = r_1$ and $\sigma_r = p$:

$$\sigma_r = (p + c_0 \cot \varphi_0) \left(\frac{r_1}{r}\right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \quad (14)$$

$$\sigma_\theta = \frac{1 - \sin \varphi_0}{1 + \sin \varphi_0} \left[(p + c_0 \cot \varphi_0) \left(\frac{r_1}{r}\right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \right] - \frac{2c_0 \cos \varphi_0}{1 + \sin \varphi_0} \quad (15)$$

At $r = R_p$, $\sigma_r = \sigma_p = p_c$, the radial displacement at the intersection of the elastic and plastic zones can be obtained from Equations (6)–(8):

$$u_p = \frac{1+\nu}{E} \sigma_p R_p = \frac{1+\nu}{E} p_c R_p \quad (16)$$

Substituting (14) into the above equation yields the plastic radial displacement field:

$$u_p = \frac{1+\nu}{E} R_p \left[(p + c_0 \cot \varphi_0) \left(\frac{r_1}{R_p}\right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \right] \quad (17)$$

Assuming that the total volume change after expansion of the circular hole is equal to the sum of the volume changes in the elastic and plastic zones, the:

$$\pi(R_u^2 - R_0^2) = \pi R_p^2 - \pi(R_p - u_p)^2 + \pi(R_p^2 - R_u^2) \Delta \quad (18)$$

In the equation: Δ —average volume of the plastic zone;

Ignoring u_p^2 , expanding Equation (18) has:

$$1 + \Delta - \left(\frac{R_0}{R_u}\right)^2 = \left(\frac{R_p}{R_u}\right)^2 \Delta + 2u_p \frac{R_p}{R_u^2} \quad (19)$$

Substituting Equation (17) into Equation (19) yields:

$$1 + \Delta - \left(\frac{R_0}{R_u}\right)^2 = \frac{2(\nu+1)}{E} \left(\frac{R_p}{R_u}\right)^2 \left[(p + c_0 \cos \varphi_0) \left(\frac{R_u}{R_p}\right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \right] + \left(\frac{R_p}{R_u}\right)^2 \Delta \quad (20)$$

From Equation (7), there is $\sigma_r = \sigma_\theta$ for $r = R_p$, which can be obtained by substituting it into Equation (12):

$$\sigma_p = c_0 \cos \varphi_0 \quad (21)$$

Substituting Equation (20) into Equation (14) yields:

$$c_0 \cos \varphi_0 = (p + c_0 \cot \varphi_0) \left(\frac{r_1}{r} \right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \quad (22)$$

Substituting Equation (22) into Equation (20) yields:

$$1 + \Delta - \left(\frac{R_0}{R_u} \right)^2 = \left(\frac{R_p}{R_u} \right)^2 \Delta + 2 \left(\frac{R_p}{R_u} \right)^2 \frac{1 + \nu}{E} c_0 \cos \varphi_0 \quad (23)$$

Calculate the radius of the plastic zone R_p :

$$R_p = R_u \frac{\sqrt{\frac{E}{2(1+\nu)} \left[1 + \Delta - \left(\frac{R_0}{R_u} \right)^2 \right]}}{c_0 \cos \varphi_0 + \frac{E}{2(1+\nu)} \Delta} \quad (24)$$

The most remaining pressure in the hole:

$$p_u = c_0 (\cos \varphi_0 + \cot \varphi_0) \left(\sqrt{\frac{\frac{E}{2(1+\nu)} \left[1 + \Delta - \left(\frac{R_0}{R_u} \right)^2 \right]^2}{c_0 \cos \varphi_0 + \frac{E}{2(1+\nu)} \Delta}} \right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \quad (25)$$

Substituting $p = \sigma_r$ and $r_1 = R_p$ into Equation (8) gives the displacement field in the elastic region as:

$$u_r = \frac{(1 + \nu) c_0 \cos \varphi_0}{E} \left(\frac{R_p}{r} \right)^2 r \quad (26)$$

Substituting Equations (25) and (26) into Equation (17) yields the plastic zone displacement:

$$u_p = R_u \sqrt{\frac{(1 + \nu) \left[1 + \Delta - \left(\frac{R_0}{R_u} \right)^2 \right]}{2E \left(c_0 \cos \varphi_0 + \frac{E}{2(1+\nu)} \Delta \right)}} \cdot \left[c_0 (\cos \varphi_0 + \cot \varphi_0) \left(\frac{r_1}{R_u} \right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \right] \quad (27)$$

The preceding procedures are repeated and changed to obtain the solution for the displacement field, the stress field in the plastic zone, and the stress field in the elastic zone while taking into account the initial stress situation in the soil.

4.2.2. Adjusted Plastic Zone Solution Taking the Pile Shoe's Shape into Account

The variations in the soil disturbance capacity of pile shoes with various geometries can be seen by taking into account the impact of the pile shoe length a and the equivalent radius b of the pile shoe cross-section on the plastic zone. Figure 8 illustrates the introduction of a correction factor.

$$\lambda = 1 + k \cdot \frac{a}{b} \quad (28)$$

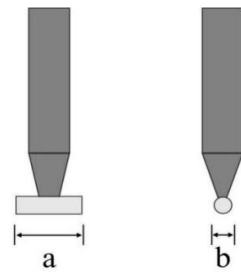


Figure 8. Stick pile shoe with circular cross-section.

The equation is as follows: a is the pile shoe's length, b is the pile shoe cross-section's equivalent radius, and k is the correction factor, which is found to be $k = 0.512$.

The radius of the plastic zone following rectification, taking into account the impact of pile shoes, is:

$$R'_p = R_u \cdot \frac{\sqrt{\frac{E}{2(1+v)} \left[1 + \lambda \Delta - \left(\frac{R_0}{R_u} \right)^2 \right]}}{c_0 \cos \varphi_0 + \frac{E}{2(1+v)} \lambda \Delta} \quad (29)$$

Horizontal to elastic and plastic zone displacements of the cylindrical pile shoe soil are:

$$u_r = \frac{(1+v)c_0 \cos \varphi_0}{E} \left(\frac{R_p}{r} \right)^2 r \quad (30)$$

$$u'_p = \frac{R_u}{6} \sqrt{\frac{(1+v) \left[1 + \lambda \Delta - \left(\frac{R_0}{R_u} \right)^2 \right]}{2E \left(c_0 \cos \varphi_0 + \frac{E}{2(1+v)} \lambda \Delta \right)}} \cdot \left[c_0 (\cos \varphi_0 + \cot \varphi_0) \left(\frac{r}{R_u} \right)^{\frac{2 \sin \varphi_0}{1 + \sin \varphi_0}} - c_0 \cot \varphi_0 \right] \quad (31)$$

4.2.3. Theoretical Model Validation

Table 5 illustrates that the parameters of the experimental model and the theoretical model and soil parameters agree. When the section equivalent radius b and the pile shoe length a are substituted into the correction factor (28), $\lambda = 2.71$ is obtained. Replace the radius of the modified plastic zone (29) with $R'_p = 3.152$ mm, where $r < R'_p$ is the plastic zone and $r > R'_p$ is the elastic zone, when $\Delta = 0.015$ [32].

Table 5. Theoretical model validation parameters.

Style	a (mm)	b (mm)	R _u (mm)	R ₀ (mm)	c (kpa)	φ (°)	E (Mpa)	v
I	10	3	3	3	0.55	11	0.8	0.4
II	10	4	3	3	0.55	11	0.8	0.4

The normalized horizontal displacement from two sets of cylindrical pile shoe insertion tests based on transparent soil testing was compared with the theoretical model values to confirm the theory's viability. The extension of the radius of the plastic zone is represented by $R'_p > R_p$, and Figure 9 illustrates that the error between the corrected theoretical values and both sets of test values is less than 5%. This suggests that the pile shoe geometry greatly increases the degree of soil smearing. The figure shows that the displacement values from the transparent soil test correlate well with the theoretical values, suggesting that the updated model is rather accurate.

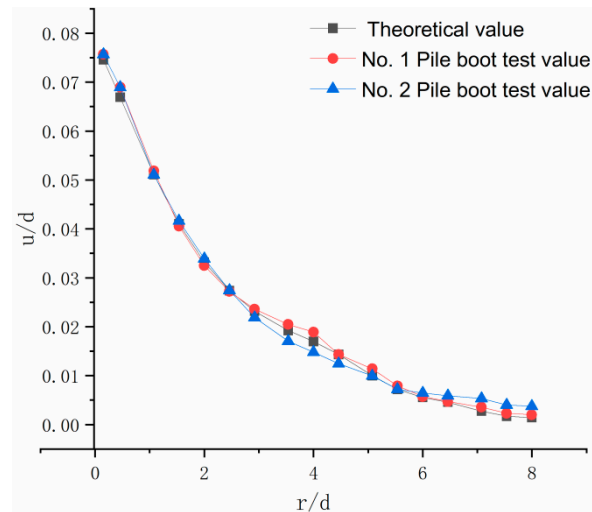


Figure 9. Comparison of theoretical predictions and experimental data.

4.3. Quantitative Validation of the Correction Coefficient λ Versus the Perturbation Range

By adding five sets of pile shoe geometry parameters (length a and section equivalent radius b) and computing the soil displacement u/d , the theoretical model's accuracy and the validity of the correction coefficients are confirmed. Table 6 displays the precise results, and the soil parameters are in agreement with Table 5.

Table 6. Values of different pile shoe sizes and correction coefficients.

Groups	a (mm)	b (mm)	λ
①	10	1.5	4.413
②	10	2.0	3.560
③	10	3.0	2.707
④	5	1.5	2.707
⑤	15	3.0	3.560

Figure 10 illustrates how the five data sets were normalized in order to compare the horizontal displacement in the horizontal direction.

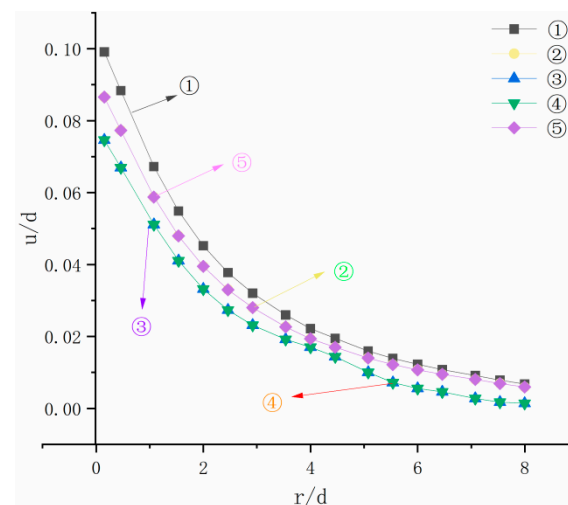


Figure 10. Comparison of normalized horizontal displacement data for different groups.

When comparing group 1 to group 4, it is demonstrated that when b is certain, increasing a will increase λ , and u/d will grow in the same case of r/d ; in the same case of a , increasing b in groups 1, 2, and 3 will decrease λ , and u/d will drop in the same case of r/d . It demonstrates that the impact of pile shoe geometry (a/d) on displacement can be accurately reflected by the correction coefficient λ . Because groups 3 and 4 have the same λ , u/d is likewise the same, and the curves overlap; similarly, groups 5 and 2 share the same λ , causing u/d to be the same and the curves to overlap. It demonstrates that λ dominates the displacement u regardless of the individual parameters a and b , demonstrating the reasonableness of the correction coefficient λ .

5. Conclusions and Outlook

Three findings can be obtained from the research of drainage sheet casing pile shoes penetrating into layered soils using a model test system based on transparent soil materials and PIV technology:

1. It is confirmed that the morphology of the smeared area (“inverted cone”) is closely related to the layering characteristics of the soil, which is consistent with the prediction of “funnel shape” by the theory of cylindrical pore expansion. Transparent soil and PIV technology can work together to effectively capture the dynamic process of construction disturbance (smearing).
2. From an experimental perspective, it was confirmed that the amount of soil coating in layered soils is influenced by the variation in the equivalent radius of the pile shoes of the drainage board casing. Because of its smaller contact area, the No. 1 pile shoe with an equivalent radius $b = 1.5d$ has a horizontal disturbance range of $4.5d$; in contrast, the No. 2 pile shoe with a 33% larger equivalent area has a horizontal disturbance range of $5d$. However, the larger contact area and the particular geometry result in the more noticeable stress concentration phenomenon, the displacement of the sandy soil layer attenuates more quickly, the discontinuity of the soil at the interface is more noticeable, and the distribution of plastic strain in the clay layer is more uniform. The stress concentration effect will be more noticeable due to the increased contact area and particular geometry.
3. The correction coefficient of the pile shoe geometric parameters is proposed based on the Mohr–Coulomb criterion and reaming theory. The correction coefficient λ is then used to determine the radius of the plastic zone and the displacement value of the plastic zone after correction. In order to determine the theoretical solution taking into account the smearing range of the pile shoe shape, a comparison and analysis of the theoretical model and the test model reveal that the results of the two are more consistent with one another, the error between the theoretical value and the test value is less than 5%, and the radius of the plastic zone after correction is larger than that of the traditional model. Taking into account the geometry of the pile shoes, the theoretical solution of the smearing range is found. Then, by adding five different sets of pile shoe geometry parameters, the correction coefficient λ is quantitatively verified. It is discovered that the correction coefficient λ can accurately reflect how the pile shoe geometry (a/d) affects the displacement and that the pile shoe geometry can increase the range of the soil coating; the displacement u is dominated by λ , which is independent of a and b , demonstrating the reasonableness of the correction coefficient.
4. By using sophisticated visualization tools, this work effectively closes the gap between drainage board construction and conventional reaming theory. Not only did it highlight the crucial impact of pile shoes that had been previously disregarded, but more significantly, it offered a quantitative correction method that was confirmed via experimentation (the λ coefficient). This makes it possible for engineers to more

accurately and scientifically take into consideration the consequences of construction disturbances in real-world designs, which eventually leads to optimized design, improved construction quality, and financial gains. It has substantial application value in engineering.

Author Contributions: Writing—original draft, B.Z.; Supervision, J.L.; Project administration, J.L.; Funding acquisition, J.L.; Formal analysis, Z.L.; Investigation, H.C.; Resources, Z.Y.; Validation, Y.T.; Software, Y.D.; All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Shu, Z.M.; Nie, K.C.; Wang, H. Application of Plastic Drainage Board Combined with Surcharge Method in Revetment Project of Mirsarai. *Port Waterw. Eng.* **2020**, *576*, 159–163. [CrossRef]
2. Dong, L. Application of Preloading Method with Plastic Drainage Plate in the Port Engineering Treatment of Soft Foundation. Master's Thesis, Dalian University of Technology, Dalian, China, 2016. Available online: <https://kns.cnki.net/KCMS/detail/detail.aspx?dbname=CMFD201702&filename=1017701533.nh> (accessed on 1 January 2025).
3. Deng, Y.B. Analytical Theory and Finite Element Analysis for Consolidation of Soft Soils by Vertical Drains. Master's Thesis, Zhejiang University, Hangzhou, China, 2013. Available online: <https://kns.cnki.net/KCMS/detail/detail.aspx?dbname=CFD1214&filename=1013147722.nh> (accessed on 1 January 2025).
4. Zhang, Y.P.; Wang, W. Study on Disturbance Effect on Soft Soil Ground During Vertical Drain Construction. *J. Zhejiang Univ. (Eng. Sci.)* **2006**, *40*, 1239–1243.
5. Sathananthan, I.; Indraratna, B.; Rujikiatkamjorn, C. Evaluation of Smear Zone Extent Surrounding Mandrel Driven Vertical Drains Using the Cavity Expansion Theory. *Int. J. Geomech.* **2008**, *8*, 355–365. [CrossRef]
6. Ghandeharioon, A.; Indraratna, B.; Rujikiatkamjorn, C. Analysis of Soil Disturbance Associated with Mandrel-Driven Prefabricated Vertical Drains Using an Elliptical Cavity Expansion Theory. *Int. J. Geomech.* **2010**, *10*, 53–64. [CrossRef]
7. Ghandeharioon, A.; Indraratna, B.; Rujikiatkamjorn, C. Laboratory and Finite-Element Investigation of Soil Disturbance Associated with the Installation of Mandrel-Driven Prefabricated Vertical Drains. *J. Geotech. Geoenviron. Eng.* **2012**, *138*, 295–308. [CrossRef]
8. Indraratna, B.; Perera, D.; Rujikiatkamjorn, C.; Kelly, R. Soil Disturbance Analysis Due to Vertical Drain Installation. *Proc. Inst. Civ. Eng.-Geotech. Eng.* **2015**, *168*, 236–246. [CrossRef]
9. Prabavathy, S.; Rajagopal, K.; Pitchumani, N.K. Investigation of Smear Zone Around PVD Mandrel Using Image-Based Analysis. *Int. J. Geosynth. Ground Eng.* **2021**, *7*, 94. [CrossRef]
10. Palmeira, E.M.; Gardoni, M.G. The Influence of Partial Clogging and Pressure on the Behaviour of Geotextiles in Drainage Systems. *Geosynth. Int.* **2000**, *7*, 403–431. [CrossRef]
11. Lu, X.Q.; Li, H.; Liu, F.Y. Experimental Study of Waste Slurry Treated by a PHD-PVD Vacuum Preloading Method Combined with Electro-osmosis. *J. Civ. Environ. Eng.* **2024**, *46*, 129–135. [CrossRef]
12. Haegeman, W.; Impe, W.F.V. Filtration Performance Testing of Geotextiles for Vacuum Consolidation Drains. *Geosynth. Int.* **1999**, *6*, 41–51. [CrossRef]
13. Shi, L.T.; Li, C.X.; Yang, Y. Analytical Solution for Consolidation of Soils with Vertical Drain Considering Time-and Depth-Dependent Well Resistance Under Vacuum and Multi-Stage Surcharge Preloading. *Chin. J. Rock Mech. Eng.* **2023**, *42*, 3755–3766. [CrossRef]
14. Li, C.H.; Wu, H.; Cheng, G.Y. Comparative Experimental Study on Soft Soil Reinforcement by Vacuum Preloading with Different Band Drains. *Rock Soil Mech.* **2022**, *43*, 2819–2827. [CrossRef]
15. Huang, D.Z. Research on the Performance of the Prefabricated Vertical Drains in Reinforcement Dredger Fill Sludge in Vacuum Preloading. Master's Thesis, Jiangxi University of Science and Technology, Ganzhou, China, 2016. [CrossRef]
16. Bao, S.; Guo, L.; Dong, Z.; Zhou, R.; Zhou, S.; Chen, J. Clogging Behaviors in PVD and Their Evaluation Criteria for Dredged Mud Slurry. *Water* **2023**, *15*, 4206. [CrossRef]

17. Chai, J.-C.; Miura, N.; Nomura, T. Effect of Hydraulic Radius on Long-Term Drainage Capacity of Geosynthetic Drains. *Geotext. Geomembr.* **2004**, *22*, 3–16. [[CrossRef](#)]
18. Lawrence, C.A.; Koerner, R.M. Flow Behavior of Kinked Strip Drains. In Proceedings of the Symposium on Geosynthetics for Soil Improvement at the ASCE Convention, Nashville, TN, USA, 9 May 1988.
19. Miura, N.; Park, Y.; Madhav, M.R. Fundamental Study on Drainage Performance of Plastic-Board Drains. *Doboku Gakkai Ronbunshu* **2010**, *481*, 31–40. [[CrossRef](#)]
20. Ying, S.; Cheng, P.S. Effects of Bended Plastic Drainage Plates on Consolidation Caused by Vacuum Preloading. *Chin. J. Rock Mech. Eng.* **2011**, *30*, 3633–3640.
21. Yue, J.S.; Li, J. Measures to Solve “Returning Belt” in Driving of Plastic Drain Board. *Port Waterw. Eng.* **2018**, *2*, 194–197+210. [[CrossRef](#)]
22. Wu, Y.D.; Chen, J.M.; Zhou, Y.F. Distribution and Basic Characteristics of New Transparent Clay. *Chin. J. Geotech. Eng.* **2020**, *42*, 141–145. [[CrossRef](#)]
23. Deng, J.B. Application of Reusable pile shoes in Drainage Board Construction. *Pearl River Water Transp.* **2023**, *19*, 10–12. [[CrossRef](#)]
24. Wu, M.L.; Fu, Y.Q.; Liu, C. Model Tests on the Penetration Characteristics of Jacked Pile in Transparent Granular Soil. *Build. Sci.* **2022**, *38*, 49–55. [[CrossRef](#)]
25. Ni, Q.; Hird, C.C.; Guymier, I. Physical Modelling of Pile Penetration in Clay Using Transparent Soil and Particle Image Velocimetry. *Géotechnique* **2010**, *60*, 121–132. [[CrossRef](#)]
26. Tehrani, F.S.; Arshad, M.I.; Prezzi, M.; Salgado, R. Physical Modeling of Cone Penetration in Layered Sand. *J. Geotech. Geoenviron. Eng.* **2018**, *144*, 04017101. [[CrossRef](#)]
27. Lu, Q.; He, B.; Xiao, L. Experimental Study on CPT Penetration in Layered Soil Based on Transparent Soil. *Chin. J. Geotech. Eng.* **2022**, *44*, 2215–2224.
28. Tian, Y. Refined Consolidation Theory for Soft Soils Improved by Prefabricated Vertical Drain. Ph.D. Thesis, China University of Geosciences, Beijing, China, 2022. Available online: <https://kns.cnki.net/KCMS/detail/detail.aspx?dbname=CDFDLAST2024&filename=1023827012.nh> (accessed on 1 January 2025).
29. Jiang, M.J.; Shen, Z.J. Expansion of Cylindrical Cavity of Materials with Strain-Softening Behaviour. *Chin. J. Geotech. Eng.* **1995**, *4*, 10–19.
30. Gong, X.N. *Soil Plastic Mechanics*; Zhejiang University Press: Hangzhou, China, 1990.
31. Zhou, H.; Yuan, J.R.; Liu, H.L. Model Test of Rectangular Pile Penetration Effect in Transparent Soil. *Rock Soil Mech.* **2019**, *40*, 4429–4438. [[CrossRef](#)]
32. Liu, Y.H.; Chen, Z.Y.; Peng, Z.J. Analysis of Pile Driving Effect of Precast Tubular Pile Using Cylindrical Cavity Expansion Theory. *Rock Soil Mech.* **2007**, *10*, 2167–2172. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.