

Article

Research on Wind Resistance Optimization Method for Cable-Stiffened, Single-Layer Spherical Reticulated Shell Based on QPSO Algorithm

Ying Zhao ¹, Guohan Chen ² , Shushuang Song ³, Mingyao Huang ², Tianhao Zhang ^{4,5} , Pengcheng Li ^{2,*} and Gang Xiong ²

¹ Shanghai Technical Institute of Electronics & Information, Shanghai 201411, China; zhaoying@stiei.edu.cn

² School of Civil Engineering, Chongqing University, Chongqing 400045, China; 202216021196t@stu.cqu.edu.cn (G.C.); 202316131092t@stu.cqu.edu.cn (M.H.); xionggang76@cqu.edu.cn (G.X.)

³ School of Civil Engineering, Lanzhou University of Technology, Lanzhou 730050, China; songss@lut.edu.cn

⁴ Department of Architecture, Meijo University, Nagoya 468-8502, Japan; zhang-th@iis.u-tokyo.ac.jp

⁵ Institute of Industrial Science, The University of Tokyo, Tokyo 153-8505, Japan

* Correspondence: lipengcheng@cqu.edu.cn

Abstract: This study proposes an improved mixed-variable quantum particle swarm optimization (QPSO) algorithm capable of optimizing both continuous and discrete variables. The algorithm is applied to the wind resistance optimization of a cable-stiffened, single-layer spherical reticulated shell (SLSRS), optimizing discrete variables like member dimensions and cable dimensions alongside continuous variables such as cable prestress. Through a computational case study on an SLSRS, the optimization results of the proposed QPSO method are compared with other optimization techniques, validating its accuracy and reliability. Furthermore, this study establishes a mathematical model for the wind resistance optimization of cable-stiffened SLSRSs and outlines the wind resistance optimization process based on the mixed-variable QPSO algorithm. The optimization of these structures reveals the strong stability and global search capabilities of the proposed algorithm. Additionally, the comparison of section optimization and shape optimization highlights the significant impact of the shell shape on steel usage and costs, underscoring the importance of shape optimization in the design process.

Keywords: spherical reticulated shell; prestressed cable; wind load; structural optimization; quantum particle swarm optimization



Citation: Zhao, Y.; Chen, G.; Song, S.; Huang, M.; Zhang, T.; Li, P.; Xiong, G. Research on Wind Resistance Optimization Method for Cable-Stiffened, Single-Layer Spherical Reticulated Shell Based on QPSO Algorithm. *Buildings* **2024**, *14*, 2474. <https://doi.org/10.3390/buildings14082474>

Academic Editor: Binsheng (Ben) Zhang

Received: 17 July 2024

Revised: 2 August 2024

Accepted: 5 August 2024

Published: 10 August 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The cable-stiffened, single-layer spherical reticulated shell (SLSRS) is a type of pre-stressed spatial structure system composed of rigid members and flexible cables. This structure combines the advantages of reticulated shells, such as economical steel use, aesthetic appearance, and open transparency, while overcoming the shortcomings of reticulated shells in terms of shear strength and stability by introducing prestressed cables [1]. Bulenda et al. [2] conducted a stability analysis of cable-stiffened reticulated shells using the finite element method and found, through parametric analysis, that this type of structure was highly sensitive to structural imperfections. Cai et al. [3] further performed elastic-plastic buckling analysis of cable-stiffened reticulated shells and discussed the impact of asymmetric loads on the stability of the shell based on finite element simulations. Additionally, Li et al. [4,5] studied the effects of key factors such as node stiffness and cable prestress on the stability of prestressed, cable-stiffened reticulated shells. Many scholars have also analyzed the static performance of cable-stiffened reticulated shells [6–8] and investigated various improved forms of cable-stiffened reticulated shells, including hexagonal grid cable-stiffened spherical reticulated shells [9,10] and cylindrical reticulated shells [10].

As a large-span roof structure, the lightweight and low stiffness characteristics of a reticulated shell make it prone to significant vibrations and deformations under wind loads, classifying it as a wind-sensitive structure [11]. Numerous scholars have conducted wind tunnel tests on roof structures to study their wind-induced response and wind resistance performance, including space grid roofs [12], suspended cable roofs [13], and orthogonal cable-net membrane structures [14]. Numerical simulations have also been employed to investigate the dynamic response characteristics, load-bearing capacity, and collapse mechanisms of double-layer spherical reticulated shells [15], plate-conical reticulated shells [16], single-layer cylindrical reticulated shells [17], and Kiewitt-type spherical reticulated shells [18]. However, although there have been studies on the dynamic response of cable-stiffened reticulated shells, most of the research has primarily focused on their seismic performance [19–21], while studies on the wind resistance performance of cable-stiffened reticulated shells remain relatively limited. Figure 1 displays the model of the cable-stiffened reticulated shell, which includes the nodes, cables, members, and struts of the shell.

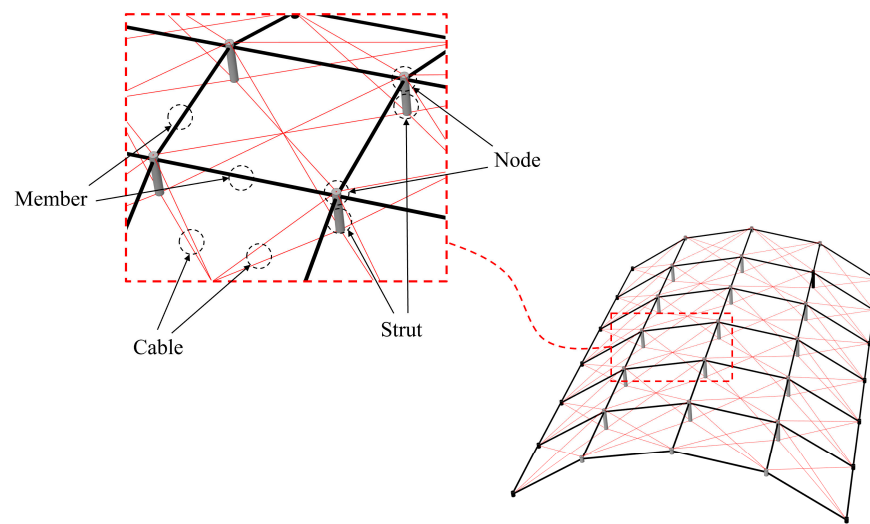


Figure 1. Model of cable-stiffened reticulated shell.

The research on wind resistance optimization has predominantly focused on high-rise buildings and tall structures [22]. In contrast, there is relatively less research on the wind resistance optimization of large-span roof structures. This can be categorized into aerodynamic shape optimization [23] and performance-based structural design optimization [24]. Aerodynamic shape optimization aims to enhance the wind resistance performance of structures by altering their shape, employing optimization algorithms such as genetic algorithms [25] and CFD-based gradient optimization algorithms [26]. In contrast, performance-based structural design optimization focuses on optimizing structural design parameters, such as materials [27], member cross-sectional sizes [28], and topological forms [28], to achieve economic, safe, and functional goals. Kociecki et al. [29,30] proposed a two-stage optimization method based on genetic algorithms and applied this method for the cross-sectional and topological optimization of a free-form steel reticulated shell, resulting in a lighter and more economically efficient shell design. Hsaine [31] proposed various hybrid optimization strategies based on the deformation-progressive structure optimization method, aiming to reduce structural costs. These strategies were found to be effective in optimizing the wind resistance of reticulated shells by adjusting parameters such as the curvature radius, number of members, number of nodes, and cross-sectional dimensions.

As a structure highly sensitive to wind loads, the cable-stiffened SLSRS is at risk of dynamic instability under strong winds. Research on wind resistance optimization methods for this type of structure is relatively limited.

This study, based on the particle swarm optimization (PSO) algorithm [32,33], proposed a quantum particle swarm optimization (QPSO) algorithm capable of handling the mixed optimization of continuous and discrete variables involved in optimizing cable-stiffened SLSRSs. The optimization design program was proposed using MATLAB in conjunction with ANSYS APDL, and the feasibility and effectiveness of the proposed optimization method were verified through typical computational cases. On this basis, a mathematical model for the wind resistance optimization of cable-stiffened SLSRSs was established. Subsequently, wind resistance section optimization and shape optimization studies were conducted on cable-stiffened SLSRSs.

2. QPSO Algorithm Based on Mixed Variables

2.1. QPSO Algorithm: Basic Principles

The PSO algorithm is known for its simplicity, strong adaptability, and few parameters. However, it still has limitations, such as the inability to guarantee convergence to the global optimal solution. Building on the PSO algorithm, the QPSO algorithm introduces the principles of quantum mechanics into the PSO algorithm [34]. This integration addresses the issue of incomplete coverage of the search space in the PSO algorithm and transforms it into a probabilistic PSO algorithm with significant potential.

In the search space of the traditional PSO algorithm, the state of each particle is determined by both the particle's position and velocity. However, in the quantum space, since the position and velocity of a particle cannot be simultaneously determined, the state of the particle must be described using the wave function $\psi(X, t)$ from quantum mechanics, where $X = (x, y, z)$ represents the three-dimensional spatial position vector of the particle. In a three-dimensional space, the wave function of a particle $\psi(X, t)$ can be expressed as Equation (1), where Q represents the probability density function.

$$\int_{-\infty}^{\infty} |\psi|^2 dx dy dz = \int_{-\infty}^{\infty} Q dx dy dz = 1 \quad (1)$$

In the quantum space, the motion of particles satisfies the Schrödinger equation, as displayed in Equation (2), where $\hbar$ is the reduced Planck constant, and $\hat{H}$ is the Hamiltonian operator. The Hamiltonian operator can be expressed as Equation (3), where m is the mass of the particle, and $V(X)$ is the potential field in which the particle resides.

$$i\hbar \frac{\partial}{\partial t} \psi(X, t) = \hat{H} \psi(X, t) \quad (2)$$

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(X) \quad (3)$$

Based on the analysis of the convergence behavior of particles in the PSO algorithm, it can be inferred that there is a necessary local attractor point p_i acting as an attractive potential for the particles. Therefore, a potential well can be constructed at the local attractor point p_i , with the potential energy function given in Equation (4), where $Y = X - p_i$, and γ is a positive proportional coefficient. Therefore, the Hamiltonian operator $\hat{H}$ can be further expressed as Equation (5).

$$V(X) = -\gamma \delta(X - p_i) = -\gamma \delta(Y) \quad (4)$$

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dY^2} - \gamma \delta(Y) \quad (5)$$

The Schrödinger equation for particles in the potential well δ and the wave function $\psi(Y)$ are displayed in Equations (6) and (7), respectively.

$$\frac{d^2\psi}{dY^2} + \frac{2m}{\hbar^2} [E + \gamma\delta(Y)]\psi = 0 \quad (6)$$

$$\psi(Y) = \frac{1}{\sqrt{L}} e^{-\frac{|Y|}{L}}, \quad L = \frac{\hbar^2}{m\gamma} \quad (7)$$

The corresponding probability density function $Q(Y)$ and cumulative distribution function $F(Y)$ can be expressed as Equations (8) and (9), where E and L represent the particle energy and the characteristic length of the potential well p_i , respectively.

$$Q(Y) = |\psi(Y)|^2 = \frac{1}{L} e^{-\frac{2|Y|}{L}} \quad (8)$$

$$F(Y) = e^{-\frac{2|Y|}{L}} \quad (9)$$

The wave function $\psi(Y)$ actually describes the probability of the particle at the local attractor point p_i , and the particle position needs to be further randomly simulated using the Monte Carlo method. The particle position equation is finally obtained as Equation (10), where u is a random variable uniformly distributed over $[0, 1]$. Since the position of particle i changes with time t and to ensure that the particle can converge, the characteristic length L must also change with time. Therefore, L is $L(t)$ as a function of time t , and Equation (10) can be written as Equation (11).

$$X = p \pm \frac{L}{2} \ln \left[\frac{1}{u} \right] \quad (10)$$

$$X_i(t+1) = p_i(t) \pm \frac{L_i(t)}{2} \ln \left[\frac{1}{u_i(t)} \right] \quad (11)$$

The characteristic length L essentially represents the search range of the particle. A larger L value indicates a larger search range for the particle. However, an excessively large L value may cause the particle to diverge during the search process, making it difficult to converge to the global optimal solution; conversely, an overly small L value may cause the particle to fall into a local optimum. Therefore, to determine L reasonably, the average of the optimal positions of the individual particles is introduced, referred to as the mean best position $mbest$. The expressions for L and the particle position update for the QPSO algorithm are displayed in Equations (12) and (13):

$$L_i(t) = 2\beta \cdot |mbest_i(t) - X_i(t)| \quad (12)$$

$$X_{i,j}(t+1) = p_{i,j}(t) \pm \beta \cdot |mbest_j(t) - X_{i,j}(t)| \cdot \ln \left[\frac{1}{u_{i,j}(t)} \right] \quad (13)$$

where $X_{i,j}(t+1)$ and $X_{i,j}(t)$ represent the j -th dimension coordinates of the i -th particle at time $(t+1)$ and t , respectively, $p_{i,j}(t)$ represents the attractor of particle i at time t during convergence, β is the contraction–expansion factor, $mbest_j(t)$ represents the mean of the optimal positions of all particles at time t , and $u_{i,j}(t)$ is a uniformly distributed random number over $[0, 1]$. When $u_{i,j}(t) > 0.5$, the sign on the right side of Equation (13) is positive; otherwise, it is negative. The convergence attractor $p_{i,j}(t)$ can be expressed as Equation (14), where φ is a random number uniformly distributed over $U(0, 1)$, $P_{i,j}(t)$ is the individual optimal position of particle i , and $G_j(t)$ represents the global optimal position of the population.

$$p_{i,j}(t) = \varphi \cdot P_{i,j}(t) + [1 - \varphi] \cdot G_j(t) \quad (14)$$

To increase the randomness of particle motion and avoid local convergence, this study replaces the single random parameter φ with two random parameters: φ_1 and φ_2 . For minimization problems, the smaller the objective function value, the better the corresponding fitness value. Therefore, the individual best position $P_i(t)$ of particle i can be determined using Equation (15), where $P_i(t)$ represents the individual best position of particle i at time t , $X_i(t)$ represents the current position of particle i at time t , $P_i(t-1)$ represents the individual best position of particle i at time $(t-1)$, $F(X_i(t))$ represents the fitness value of particle i at time t , and $F(P_i(t-1))$ represents the fitness value of the individual best position of particle i at time $(t-1)$.

$$P_i(t) = \begin{cases} X_i(t), & \text{if } F(X_i(t)) < F(P_i(t-1)) \\ P_i(t-1), & \text{if } F(X_i(t)) \geq F(P_i(t-1)) \end{cases} \quad (15)$$

The global best position $G(t)$ of the population is determined using Equations (16) and (17), where g represents the particle number with the smallest fitness value among the individual best positions, $\min F(P_i(t))$ represents the smallest fitness value among all individual best positions, N represents the total number of particles in the population, arg represents the particle number corresponding to the fitness value, and $P_i(t)$ represents the individual best position with the smallest fitness value at time t .

$$g = arg \min_{1 \leq i \leq N} F(P_i(t)) \quad (16)$$

$$G(t) = P_g(t) \quad (17)$$

In Equation (13), the contraction–expansion factor β is used to adjust the particle speed in the algorithm, controlling the convergence rate of the algorithm. In this study, β decreases linearly with time t . Moreover, the value of $mbest_j(t)$ is obtained by averaging the coordinates of all individual best positions. The expressions for β and $mbest_j(t)$ are given in Equations (18) and (19), where D represents the search dimension.

$$\beta = 0.5 + 0.5 \times \frac{T-t}{T} \quad (18)$$

$$mbest_j(t) = \frac{1}{N} \sum_{i=1}^N P_{ij}(t) = \left(\frac{1}{N} \sum_{i=1}^N P_{i1}(t), \frac{1}{N} \sum_{i=1}^N P_{i2}(t), \dots, \frac{1}{N} \sum_{i=1}^N P_{iD}(t) \right) \quad (19)$$

The basic flowchart of the proposed QPSO algorithm is displayed in Figure 2. The optimization steps are given as follows:

- Step 1: Set the population size N , maximum number of iterations T , and search dimension D ; randomly initialize the positions of the particles in the population;
- Step 2: Calculate the fitness value of each particle;
- Step 3: Find or update the individual best positions and the global best position based on the fitness values of the particles;
- Step 4: Calculate the convergence attractor $p_{i,j}(t)$, contraction–expansion factor β , and mean best position $mbest$ according to Equations (15), (18), and (19), respectively;
- Step 5: Update the position of each particle according to Equation (13);
- Step 6: If the function reaches the maximum number of iterations or the iteration error is less than the threshold, stop the algorithm, and output the optimal solution; otherwise, return to Step 2.

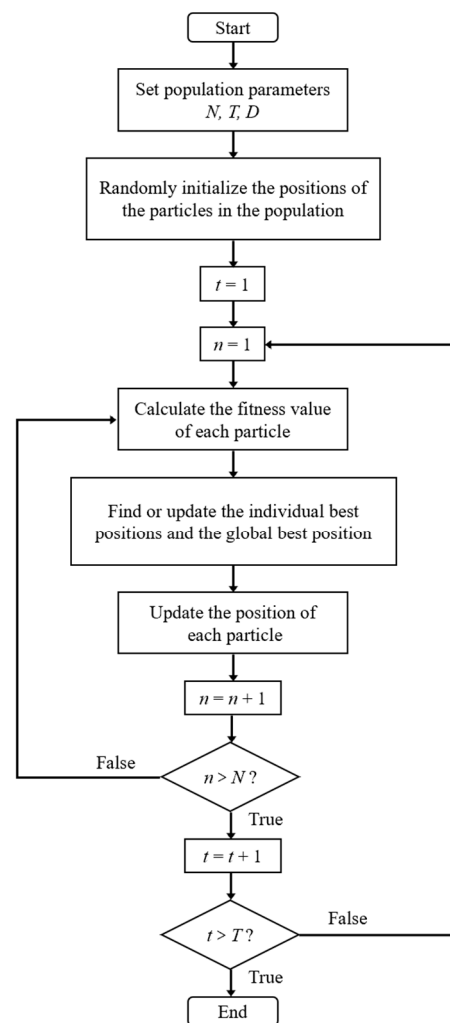


Figure 2. Flowchart of QPSO algorithm.

2.2. Handling of Mixed Variables

While the QPSO algorithm demonstrates superior performance compared to traditional PSO algorithms in optimizing problems in continuous variables, it falls short in addressing optimization problems in mixed spaces of discrete and continuous variables. In practical engineering structures, a significant number of design variables are either discrete or a mix of discrete and continuous variables. Particularly for cable-stiffened SLSRSs, the selection of member cross-sections and cable cross-sections must meet the specifications of the steel component. All these cross-section design variables are discrete. In contrast, variables such as the initial prestress corresponding to prestressed cables and the node coordinates corresponding to the shell shape are continuous. Therefore, handling mixed variables is crucial. This study improves on the continuous QPSO algorithm [35] to develop a mixed-variable QPSO algorithm.

In processing mixed variables, the approach taken involves temporarily treating discrete variables as continuous variables in each optimization iteration round and updating the positions according to Equation (13). Subsequently, for the updated values of the discrete variables in the continuous state, they are compared to a series of discrete candidate values, and the closest discrete value is selected as the actual value of the discrete variable after that round of optimization. Since the values in the discrete cross-section database lack

regularity, mapping is required for the optimized discrete variables. The specific mapping function is expressed as follows:

$$\Delta x_i^{(t)} = |x_i^{(t)} - x_j| \quad (20)$$

where $\Delta x_i^{(t)}$ represents the absolute difference between the continuous and candidate discrete values, $x_i^{(t)}$ is the continuous value of the discrete variable, and x_j represents the given discrete candidate values in the section database. $\Delta x_i^{(t)}$ is arranged from small to large, and the database size corresponding to the minimum value is selected as the actual value of the discrete variable.

This value selection strategy does not follow the traditional rounding-up method but adopts a more refined approach of approximating and matching the updated continuous variable values with the given discrete set. This accelerates the convergence of the entire optimization process, effectively obtaining the optimal solution for the problem.

2.3. Particle Initialization and Boundary Methods

When using the QPSO algorithm for structural optimization, the first step involves randomly initializing the positions of the population particles. The particle initialization can be performed according to the following expression:

$$X_{i,j}(t) = X_j^L + r \cdot (X_j^U - X_j^L) \quad (21)$$

where r represents a random number following a uniform distribution $U(0, 1)$, and X_j^U and X_j^L denote the upper and lower limits of the j -th dimension of the particle, respectively.

During the process of updating particle positions, there may be cases where individual particles exceed the constraint boundaries. To maintain the stability of the population size and the effectiveness of the search, it is essential to address such boundary-crossing occurrences. The approach taken is to implement a fly-back strategy, which involves forcibly relocating particles that exceed the boundaries back to a random position within the feasible domain. This ensures that the particles remain within the valid solution space and continue to participate in the optimization search process.

2.4. Constraint Methods

In the field of structural optimization design, many problems are generally function optimization problems with constraints, while the QPSO algorithm is an unconstrained optimization method. For constrained optimization problems, penalty function methods are commonly used. This study adopts the dynamic penalty function method, where a penalty function is added to the objective function to penalize the occurrence of infeasible solutions, thus transforming constrained structural optimization problems into unconstrained optimization problems. According to the rules of the dynamic penalty function, the fitness function expression can be constructed as follows:

$$F(X_i(t)) = f(X_i(t)) + r^t \left(\sum_{k=1}^{n_g} [G_k(X_i(t))]^2 \right) \quad (22)$$

where $F(X_i(t))$ is the fitness value of particle i at the t -th iteration, $f(X_i(t))$ is the objective function value of particle i at the t -th iteration, r is the penalty factor taking 2.0, $G_k(X_i(t))$ is the constraint violation of particle i at the t -th iteration, and n_g is the number of constraints. The constraint violation $G_k(X_i(t))$ is calculated as:

$$G_k(X_i(t)) = \max(g_k(X_i(t)), 0), \quad k = 1, 2, \dots, n_g \quad (23)$$

where $g_k(X_i(t))$ is the constraint redundancy of particle i at the t -th iteration, and $g_k = V_k - V_t$ is the computed value of the constraint at the t -th iteration, with V_t being the allowable value of the constraint variable.

2.5. Parallel Computing Methods

Although the QPSO algorithm has a high convergence speed and produces good quality solutions, the computational complexity of the algorithm increases with higher dimensions or larger population sizes. This can result in excessively long running times, impacting the practical application of the algorithm. To address this issue, this study utilizes the Parfor parallel structure, assigning each particle in each iteration to different CPU threads for computation. The results from different threads are then combined to update the global best solution and individual best solutions. This parallelized QPSO algorithm significantly reduces the running time without compromising its convergence and solution quality, thereby improving its performance.

3. Algorithm Validation

3.1. Case Study Introduction

To ensure the accuracy and effectiveness of the improved QPSO algorithm proposed in this study, the Kiewitt-type (K6) SLSRS case study [36] was selected for optimization design. The structure is displayed in Figure 3. The reticulated shell has a span of 70 m, a height of 20 m, and a corresponding height-to-span ratio of 1/3.5. There are 10 rings of member, a total of 930 members, and 331 nodes in the shell. The members of the shell are all circular steel pipes with a yield strength of 235 MPa, a design strength value of 215 MPa, an elastic modulus of 2.06×10^5 MPa, a density of 7850 kg/m^3 , and a Poisson's ratio of 0.3. Welded hollow spherical nodes are adapted for the shell nodes, and perimeter three-way fixed hinge supports are employed. All nodes of the shell are subjected to a vertically downward concentrated load of 35.5 kN.

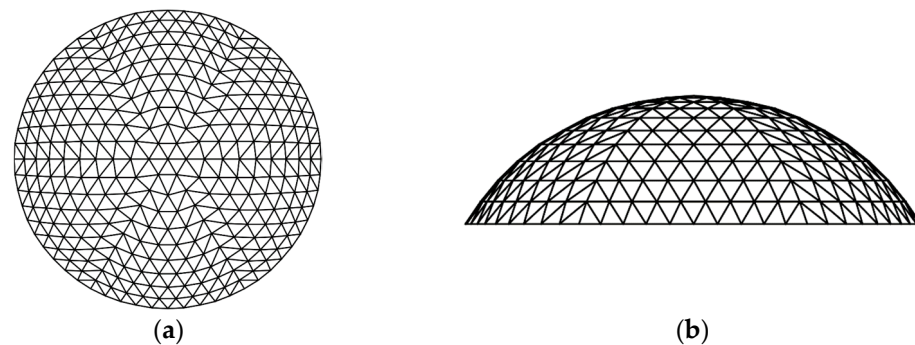


Figure 3. K6 single-layer spherical reticulated shell [36]. (a) Plan view. (b) Elevation view.

3.2. Optimization Process

During the cross-sectional optimization of the K6 SLSRS, the member section dimensions are considered as the design variables to be optimized. The members of the K6 SLSRS are classified into three categories based on their spatial positions and directions: circumferential members, radial members, and oblique members. Circumferential members are those forming multiple concentric circles around the center, radial members extend radially outward from the center, and the remaining members are considered oblique members. The members are divided into 10 circles from the innermost section to the outermost. To reduce the number of design variables, the shell members are grouped into nine categories based on the structural loading characteristics, as listed in Table 1. Each group of members is assigned a specific section dimension, resulting in a total of nine design variables. It is important to note that the member section dimensions are discrete variables rather than continuous variables. In this case, a seamless steel pipe section library [36] consisting of 32 different steel pipe sectional dimensions and areas is used, as listed in Table 2.

Table 1. Section set of members.

Set	Member	Set	Member
1	1–5 ring circumferential member	6	6–10 ring oblique member
2	6–9 ring circumferential member	7	1–3 ring radial member
3	10 ring circumferential member	8	1–5 ring radial member
4	2–3 ring oblique member	9	6–10 ring radial member
5	4–5 ring oblique member		

Table 2. Allowable section library of K6 reticulated shell members.

No.	Dimensions	Sectional Area (cm ²)	No.	Dimensions	Sectional Area (cm ²)
1	$\phi 83 \times 4$	9.93	17	$\phi 127 \times 4.5$	17.32
2	$\phi 83 \times 4.5$	11.10	18	$\phi 89 \times 7$	18.03
3	$\phi 95 \times 4$	11.44	19	$\phi 133 \times 4.5$	18.17
4	$\phi 89 \times 4.5$	11.95	20	$\phi 140 \times 4.5$	19.16
5	$\phi 83 \times 5$	12.25	21	$\phi 95 \times 7$	19.35
6	$\phi 102 \times 4$	12.32	22	$\phi 146 \times 4.5$	20.00
7	$\phi 95 \times 4.5$	12.79	23	$\phi 152 \times 4.5$	20.85
8	$\phi 89 \times 5$	13.19	24	$\phi 102 \times 7$	20.89
9	$\phi 102 \times 4.5$	13.78	25	$\phi 127 \times 5.5$	20.99
10	$\phi 114 \times 4$	13.82	26	$\phi 140 \times 5.5$	23.24
11	$\phi 89 \times 5.5$	14.43	27	$\phi 114 \times 7$	23.53
12	$\phi 121 \times 4$	14.70	28	$\phi 114 \times 8$	26.64
13	$\phi 127 \times 4$	15.46	29	$\phi 133 \times 8$	31.42
14	$\phi 133 \times 4$	16.21	30	$\phi 146 \times 9$	38.74
15	$\phi 121 \times 4.5$	16.47	31	$\phi 152 \times 10$	44.61
16	$\phi 102 \times 5.5$	16.67	32	$\phi 159 \times 10$	46.81

Furthermore, the minimum weight of the shell structure, serving as the optimization objective for the shell, is mathematically expressed as displayed in Equation (24):

$$\min W = \sum_{i=1}^n \rho_i A_i l_i \quad (24)$$

where W is the structure weight, n is the total number of members, ρ_i is the member density, A_i is the member sectional area, and l_i is the member length.

Optimizing the member dimensions minimizes the weight of the shell structure, as the objective involves satisfying various constraints. Building upon existing research [37], the constraints utilized in this study include member strength constraints, member stability constraints, member slenderness ratio constraints, and node displacement constraints. The strength of shell member can be calculated as follows:

$$\sigma_i = \frac{N_i}{A_i} + \frac{\sqrt{M_{xi}^2 + M_{yi}^2}}{\gamma_m \cdot W_i} \leq f \quad (25)$$

where σ_i is the member axial stress i , N_i is the member axial force, M_{xi} and M_{yi} are the moments about the x - and y -axes of the same cross-section, W_i is the net section modulus of the member, γ_m is the plastic development coefficient for the cross-section (taken as 1.15), and f is the design strength of the steel, which is 215 MPa.

The stability of compression members can be assessed using the following equations:

$$\sigma_w = \frac{N_i}{\varphi A_i f} + \frac{\beta M_i}{\gamma_m W_i \left(1 - 0.8 \frac{N_i}{N_{Ei}}\right) f} \leq 1.0 \quad (26)$$

$$M_i = \max\left(\sqrt{M_{xA}^2 + M_{yA}^2}, \sqrt{M_{xB}^2 + M_{yB}^2}\right) \quad (27)$$

$$\beta = \beta_x \cdot \beta_y \quad (28)$$

$$\beta_x = 1 - 0.35\sqrt{N/N_E} + 0.35\sqrt{N/N_E}(M_{2x}/M_{1x}) \quad (29)$$

$$\beta_y = 1 - 0.35\sqrt{N/N_E} + 0.35\sqrt{N/N_E}(M_{2y}/M_{1y}) \quad (30)$$

where σ_w is the stability stress of the compression member, φ is the overall stability factor, and M_i is the calculated bending moment value. M_{xA} , M_{yA} , M_{xB} , and M_{yB} represent the moments about the x - and y -axes of the member, β is the equivalent moment coefficient, N_E is the Euler critical force, and $N'_{Ei} = \pi^2 EA / (1.1\lambda^2)$.

The node displacement constraint and member slenderness ratio constraint are, respectively, expressed as follows:

$$\delta_{\max} \leq [\delta] \quad (31)$$

$$\lambda = \frac{l_{0i}}{r_i} \leq [\lambda] \quad (32)$$

where $\delta_{\max}$ is the maximum calculated displacement of the shell, $[\delta]$ is the allowable displacement, which is taken as $L/400$ [38], l_{0i} is the member calculated length, r_i is the sectional radius of gyration, and $[\lambda]$ represents the allowable slenderness ratio of the shell components, set at 150 for compressed and bent components and 250 for tension and bending components [38].

Adopting the mathematical model for optimizing shell sections described above, an optimization design was performed on a K6 SLSRS in the case study. The iterative process curve of the shell weight was obtained, as displayed in Figure 4. The optimization curve demonstrates rapid convergence within the first 50 iterations, reaching the optimal solution at the 309th iteration with a weight of 42.833 t. The maximum strength stress ratio of the optimized shell is 0.550, and the corresponding stability stress ratio cloud diagram is also displayed in Figure 4, where the stability stress ratio typically represents the ratio of the actual stress in the member to its buckling stress. It can be observed that the maximum stability stress ratio is 0.969, indicating the crucial role of stability in controlling the optimization design of the shell. The maximum deflection of the shell is 2.906 cm, and the maximum slenderness ratio of the bars is 146.02, satisfying all constraints.

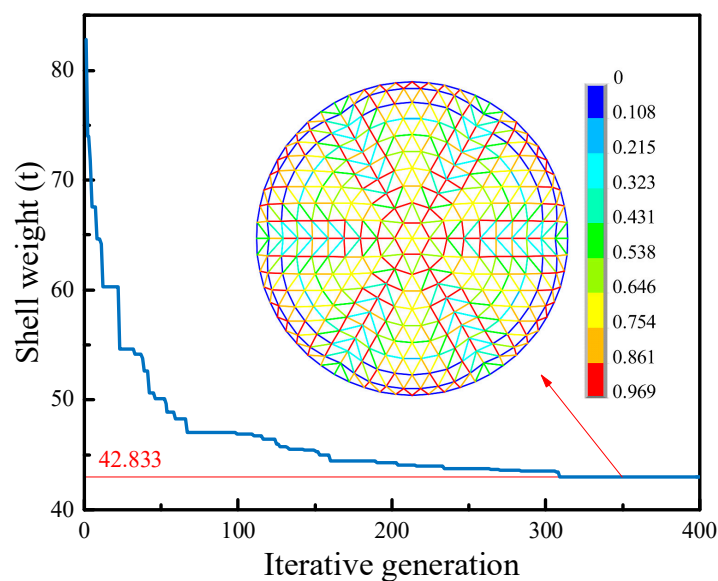


Figure 4. The iterative process curve of the reticulated shell weight and the contour of the stable stress ratio of the optimized SLSRS.

3.3. Results Comparison and Validation

The feasibility and reliability of the QPSO algorithm were verified by comparing its computational results with those of other algorithms from the existing literature [36,39,40]. Table 3 lists the optimization results of the case study using the QPSO algorithm and other optimization methods. It is observed that the optimization results obtained in this study were consistent with the results from the plant growth simulated algorithm based on a limited growth space and parallel search (GSL&PS-PGSA) [36], yielding a consistent shell weight of 42.833 t. Additionally, the same cross-sectional dimensions were selected for different member sets in both cases. The results from the simulated plant growth-genetic hybrid algorithm (SPGG-HA) [39], improved niche genetic algorithm (INGA) [40], and genetic algorithm (GA) [40] yielded shell weights of 42.906 t, 45.099 t, and 46.527 t, respectively. These alternative optimization methods utilized 0.08 t, 2.27 t, and 3.70 t more steel compared to the QPSO algorithm proposed in this study. This indicates that the developed QPSO algorithm minimizes the steel usage for the shell, demonstrating the higher accuracy and reliability of this optimization method.

Table 3. Assessing the optimized outcomes of the K6 reticulated shell.

Member Set	Sectional Area (cm ²)				
	QPSO	GSL&PS-PGSA [36]	SPGG-HA [39]	INGA [40]	GA [40]
1	12.32	12.32	13.78	13.0	13.0
2	11.44	11.44	11.44	12.0	14.0
3	9.93	9.93	9.93	12.0	15.0
4	13.82	13.82	13.82	15.0	15.0
5	13.82	13.82	13.82	15.0	15.0
6	13.82	13.82	13.82	14.0	14.0
7	13.82	13.82	11.44	18.0	16.0
8	13.82	13.82	11.44	14.0	19.0
9	13.82	13.82	11.44	11.0	11.0
Shell weight (t)	42.83	42.83	42.91	45.10	46.53

4. Cable-Stiffened SLSRS Wind Resistance Optimization

4.1. Structural Model and Load

The introduction of prestressed cables can significantly enhance the dynamic performance of SLSRSs [20]. However, optimizing cable-stiffened SLSRSs involves a more complex set of variables than those for traditional SLSRSs. These variables include discrete variables such as section dimensions and continuous variables such as cable prestress. The improved mixed-variable QPSO algorithm proposed in this study is well suited for optimizing such structures. A study on the wind resistance optimization of cable-stiffened SLSRSs was conducted, as displayed in Figure 5. The span of the shell was, uniformly, 60 m, with a rise of 10 m, resulting in a span–rise ratio of 1/6. The shell was evenly divided into 18 grids along the span. Except for the boundary members, all other members of the shell were equal, with a member length of 3.5 m. The members were made of seamless Q235 steel tubes of type $\phi 203 \times 8$, complying with the design specification [41]. The struts were made of seamless Q235 steel tubes of type $\phi 168 \times 8$ with a length of 700 mm. The sectional area of the cables was 302.18 mm², with a prestress of 200 MPa applied. The yield strength of the shell members and struts was 2.35×10^8 Pa, and the elastic modulus was 2.06×10^{11} Pa. The prestressed cables were made of galvanized steel strands of grade 1670 [42], with a tensile strength of 1.67×10^9 Pa and an elastic modulus of 1.6×10^{11} Pa. Both the steel and cables had a density of 7850 kg/m³ and a Poisson's ratio of 0.3. The structural damping ratio was 0.02. Considering that the roof was non-accessible, the live load was taken as 0.5 kN/m². The shell nodes were assumed to be fully rigidly connected, and the self-weight of the nodes was considered as 10% of the self-weight of the shell. The boundary conditions were set as three-directional hinged supports around the perimeter,

with a support height of 20 m. The structure was located in a Class B site [43], with a basic wind pressure of 0.55 kN/m^2 and a basic wind speed of 30 m/s at a height of 10 m .

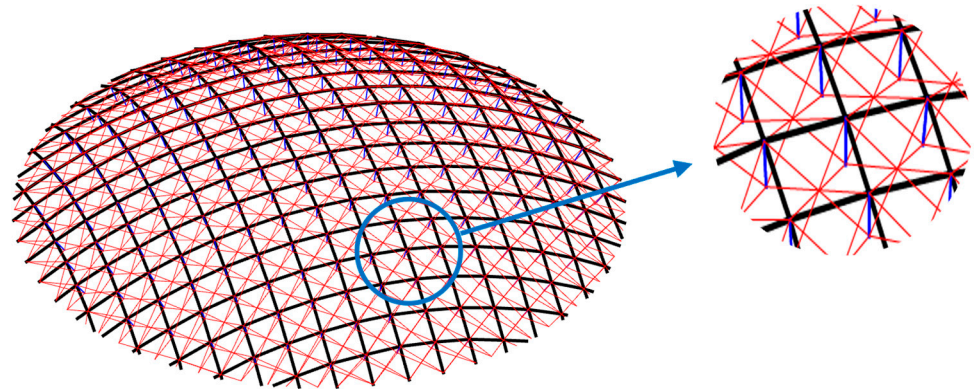


Figure 5. Cable-stiffened SLSRS model.

Due to the complexity of calculating the wind-induced vibration response of the structure using the time-history analysis method, the equivalent static wind method was commonly used in practical engineering wind resistance design to calculate wind loads on the structure. For convenience in engineering applications, the equivalent static wind load calculation formula [43] used in this study for wind resistance optimization design is given as follows:

$$w_k = \mu_s \mu_z w_0 + \mu_d \mu_z w_0 \quad (33)$$

where w_k is the standard value of the wind load, w_0 is the basic wind pressure, μ_z is the wind pressure height variation coefficient, μ_s is the wind load shape coefficient, and μ_d is the equivalent wind pressure coefficient of fluctuating wind effects. The method for determining the wind load shape coefficient μ_s is provided in the design specification [43], as displayed in Figure 6 and Table 4.

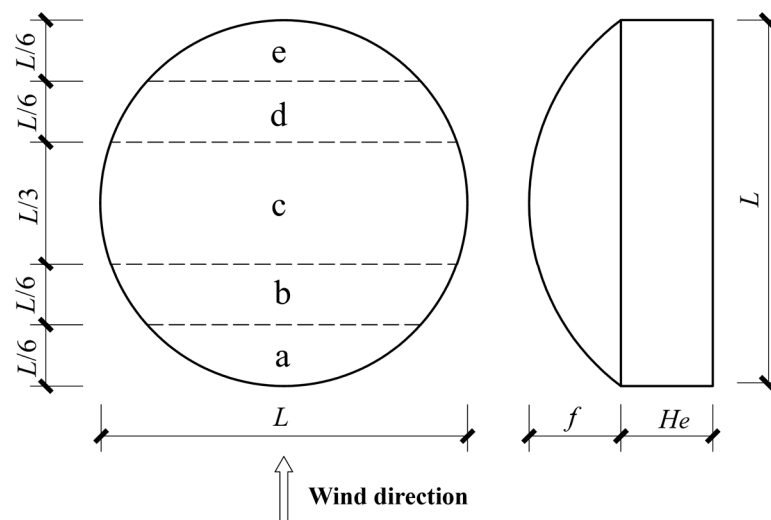


Figure 6. Spherical roof-wind load shape coefficient partition [43].

Table 4. Spherical roof-wind load shape coefficient values [43].

f/L	Roof Partition				
	a	b	c	d	e
1/4	−0.2	−0.9	−1.3	−0.8	−0.4
1/6	−0.4	−0.9	−1.0	−0.8	−0.4
1/8	−0.6	−0.9	−1.0	−0.8	−0.5

Appendix B of the design specification [43] provides values for the equivalent wind pressure coefficient of fluctuating wind effects, μ_d , for several typical roof structures. For SLSRS structures, as displayed in Figure 7, the equivalent wind pressure coefficient of fluctuating wind effects, μ_d , can be determined using Equations (34)–(36), where μ_p is the peak equivalent wind pressure coefficient of wind-induced fluctuating responses, with values listed in Table 5, f^* is the reduced frequency, L is the shell span, and n is the first-order natural frequency of the structure. It can be observed that the fluctuating wind load on the shell is related to the natural frequency, span, span–rise ratio, and basic wind pressure of the shell. Therefore, when optimizing the section or shape of the cable-stiffened SLSRS, changes in the member section or geometric shape will inevitably lead to changes in the natural frequency of the structure, which, in turn, will affect the fluctuating wind load on the structure. Thus, during the wind resistance optimization of the cable-stiffened SLSRS, the wind load should be updated in each iteration.

$$\mu_d = \pm \mu_p \quad (34)$$

$$f^* = nL/U \quad (35)$$

$$U = 40\sqrt{w_0} \quad (36)$$

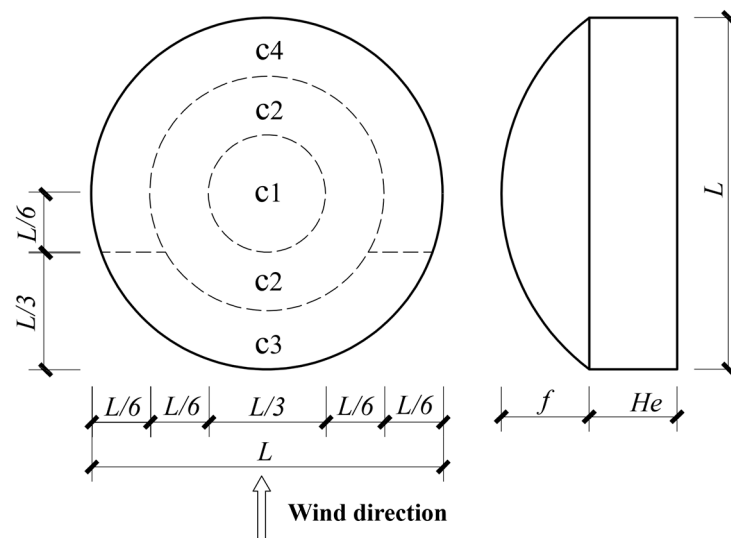


Figure 7. Spherical reticulated shell and roof partition [43].

Table 5. Peak equivalent pressure coefficients of wind-induced fluctuating response of spherical roof [43].

Roof Partition	$f/L=1/4$	$f/L=1/6$	$f/L=1/8$
c1	$-0.04f^* + 1.2$	$-0.01f^* + 0.9$	$-0.02f^* + 1.1$
c2	$-0.03f^* + 0.9$	$-0.03f^* + 0.8$	$-0.02f^* + 0.9$
c3	$-0.02f^* + 0.7$	$-0.02f^* + 0.7$	$-0.02f^* + 1.0$
c4	$-0.03f^* + 0.8$	$-0.02f^* + 0.7$	$-0.01f^* + 0.7$

Furthermore, it is worth noting that under the action of fluctuating wind loads, the vibration of the shell includes both upward and downward vibrations. When the shell vibrates upward, the equivalent wind pressure coefficient of fluctuating wind effects, μ_d , takes a negative value, indicating that the equivalent static wind load is an upward wind suction. Conversely, when the shell vibrates downward, the equivalent wind pressure coefficient μ_d takes a positive value, indicating that the equivalent static wind load is a downward wind pressure [44]. Therefore, when calculating the equivalent static wind load on the shell, both conditions should be considered as described in Equation (34). Consequently, this study designed a total of six conditions controlled by wind load, considering

both upward suction and downward pressure as the two different fluctuating wind load conditions, as listed in Table 6.

Table 6. Conditions of load combinations [45].

Condition	Load Combination
1	$1.0 \times \text{permanent load} + 1.5 \times \text{wind load (upward suction)}$
2	$1.0 \times \text{permanent load} + 1.5 \times \text{wind load (downward pressure)}$
3	$1.3 \times \text{permanent load} + 1.5 \times 0.7 \times \text{live load} + 1.5 \times \text{wind load (upward suction)}$
4	$1.3 \times \text{permanent load} + 1.5 \times 0.7 \times \text{live load} + 1.5 \times \text{wind load (downward pressure)}$
5	$1.3 \times \text{permanent load} + 1.5 \times \text{live load} + 1.5 \times 0.6 \times \text{wind load (upward suction)}$
6	$1.3 \times \text{permanent load} + 1.5 \times \text{live load} + 1.5 \times 0.6 \times \text{wind load (downward pressure)}$

4.2. Optimization Mathematical Model

This study employed the proposed mixed-variable QPSO algorithm to conduct wind resistance optimization studies on cable-stiffened SLSRSs. The optimization included both section optimization and shape optimization. It is noteworthy that there are some classical methods for calculating the optimal shell shape, like the minimum bending moment method and thin shell theory. The former method minimizes the internal bending moments of the shell under external loads by optimizing its geometric shape, and the latter solves for the optimal shape of the shell based on classical thin shell mechanics through mathematical modeling and analytical methods. However, these methods are usually applied under the assumption of geometric linearity. Moreover, they cannot account for the introduction of prestressed cables well and are not effective enough for optimizing variables such as cable prestress, which makes them unsuitable for the wind resistance optimization of complex cable-stiffened SLSRSs. In contrast, the QPSO algorithm proposed in this study enabled the effective optimization of the different sections and the shell shape under wind loads.

It is important to note that in the shape optimization conducted in this study, the variables affecting the macroscopic surface shape of the shell were optimized rather than the node coordinates of the shell structure. This was because the shell had a large number of node coordinates, and directly using them as optimization variables would have increased the complexity of the optimization. Furthermore, the node coordinates of the shell structure were not randomly distributed but were determined by the macroscopic surface shape of the shell. Therefore, this study converted the optimization variables from node coordinates to macroscopic surface shape parameters, thereby simplifying the optimization problem. The macroscopic surface shape parameters selected in this section were the rise and strut length of the cable-stiffened SLSRS.

The optimization mathematical model included three aspects: design variables, objective function, and constraints. When conducting wind resistance section optimization and shape optimization for cable-stiffened, single-layer spherical reticulated shells, the mathematical models for both optimizations had the same objective function and constraints, differing only in the design variables. For both section optimization and shape optimization, the design variables included the member section dimension A_b , cable section dimension A_c , strut section dimension A_s , and cable prestress P . Additionally, the shape optimization included two more design variables: the shell rise P and the strut length L_s . Considering the large number of members in the cable-stiffened SLSRS and the impracticality of individually optimizing each member in practical engineering scenarios, this study grouped the shell members. Based on the distribution pattern of node displacements under wind loads, which decrease from the central area of the shell to the supports around the [21], the members were divided into four groups, as displayed in Figure 8. During the optimization process, each type of member was assigned the same section size, resulting in a total of four design variables for the shell members. The other design variables were treated as a single group.

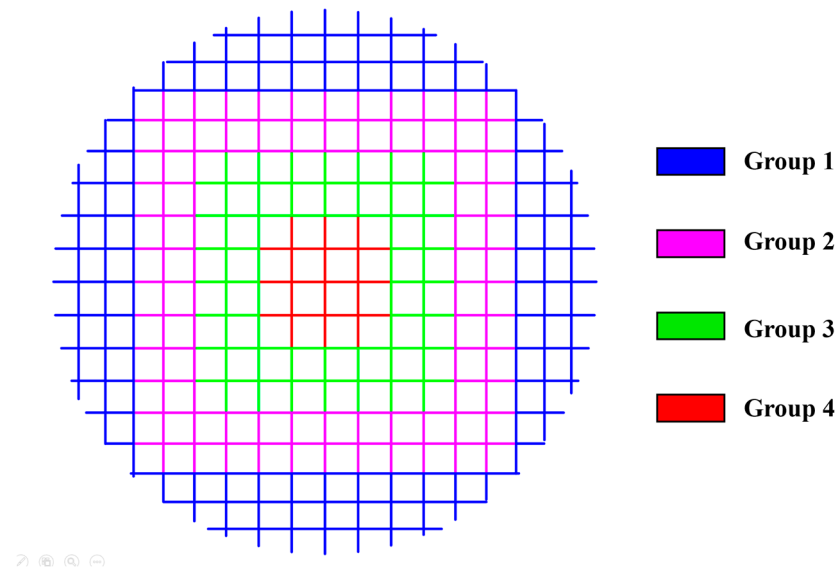


Figure 8. Grouping of SLSRS members.

In this study, the wind resistance section optimization and shape optimization of the cable-stiffened SLSRSs were both aimed at minimizing the construction cost. Ignoring the impact of the nodes, the expression for the construction cost of the shell is given by Equation (37). Since the densities of the steel and cables are the same, the optimization objective can be simplified from the construction cost to the volume of the shell, as displayed in Equation (38), where F is the construction cost of the shell; V is the volume of the shell; C_1 and C_2 are the unit weight prices of the steel members and prestressed cables; m , n , and r are the numbers of members, cables, and struts; ρ_{bi} , ρ_{cj} , and ρ_{sq} are the densities of the members, cables, and struts; V_{bi} , V_{cj} , and V_{sq} are the volumes of the i -th member, j -th cable, and q -th strut; and η is the ratio of the unit cost of the cables to that of steel, which is taken as 1.5 in this study.

$$\min F = C_1 \left(\sum_{i=1}^m \rho_{bi} V_{bi} + \sum_{q=1}^r \rho_{sq} V_{sq} \right) + C_2 \sum_{j=1}^n \rho_{cj} V_{cj} \quad (37)$$

$$\min V = \sum_{i=1}^m V_{bi} + \sum_{q=1}^r V_{sq} + \eta \sum_{j=1}^n V_{cj} \quad (38)$$

In addition, when optimizing the wind resistance of SLSRSs, it is necessary to fully consider various constraints. The constraints include member strength constraints, member stability constraints, member slenderness ratio constraints, and node displacement constraints, as displayed in Equations (25)–(32). For prestressed cables, the strength can be calculated according to Equation (39), where σ_c is the strength stress of the cable, N_c is the axial force of the cable, and f_c is the design strength value of the cable. The design strength value of the cable should be controlled within 40% to 55% of the ultimate tensile strength of the cable [42]. Therefore, this study takes the design strength value of the cable as 668 MPa, which is 40% of the ultimate tensile strength of 1670 MPa.

$$\sigma_c = \frac{N_c}{A_c} \leq f_c \quad (39)$$

The proposed mathematical optimization model includes strength constraints, stability constraints, displacement constraints, and slenderness ratio constraints, making it a multi-constraint optimization problem. Due to the inconsistent dimensions of various constraints, this can result in large differences in constraint values, which may affect the convergence of the algorithm. Therefore, it is necessary to normalize and remove dimensions from

these constraints. Additionally, to ensure the safety and reliability of the structure, the safety factor of the structure in practical engineering is generally taken as 0.7 to 0.8. Hence, this study adopted a conservative safety factor of 0.7 for the shell members, and the corresponding constraints are given in Equations (40)–(44). Besides the aforementioned constraints, this study also imposed constraints on other parameters of the cable-stiffened SLSRS. The section size variables of the members and struts used the same allowable discrete set of sections, as listed in Table 7. The discrete set of cable section size variables included 25 specifications of steel strands, as listed in Table 8. Furthermore, the upper and lower limits of the cable prestress values were set to 0 and 668 MPa, respectively. The upper and lower limits of the shell rise were set to 15 m and 7.5 m, respectively, which were 1/4 and 1/8 of the shell span. The upper and lower limits of the strut length were set to 1.75 m and 0.7 m, respectively, which were 1/2 and 1/5 of the shell member length.

$$\sigma_i / f - 0.7 \leq 0 \quad (40)$$

$$\sigma_c / f_c - 0.7 \leq 0 \quad (41)$$

$$\sigma_w / f - 0.7 \leq 0 \quad (42)$$

$$\delta / [\delta] - 1.0 \leq 0 \quad (43)$$

$$\lambda / [\lambda] - 1.0 \leq 0 \quad (44)$$

Table 7. Sectional allowable discrete set of members and struts [41].

No.	Dimension (mm)	Sectional Area (cm ²)	No.	Dimension (mm)	Sectional Area (cm ²)	No.	Dimension (mm)	Sectional Area (cm ²)
1	$\phi 60 \times 3.5$	6.21	16	$\phi 95 \times 5$	14.14	31	$\phi 140 \times 8$	33.18
2	$\phi 63.5 \times 3.5$	6.60	17	$\phi 102 \times 5$	15.24	32	$\phi 146 \times 8$	34.68
3	$\phi 68 \times 3.5$	7.09	18	$\phi 108 \times 5$	16.18	33	$\phi 152 \times 8$	36.19
4	$\phi 68 \times 4$	8.04	19	$\phi 114 \times 5$	17.12	34	$\phi 159 \times 8$	37.95
5	$\phi 70 \times 4$	8.29	20	$\phi 108 \times 5.5$	17.71	35	$\phi 146 \times 9$	38.74
6	$\phi 73 \times 4$	8.67	21	$\phi 121 \times 5$	18.22	36	$\phi 168 \times 8$	40.21
7	$\phi 70 \times 4.5$	9.26	22	$\phi 114 \times 5.5$	18.75	37	$\phi 152 \times 9$	40.43
8	$\phi 83 \times 4$	9.93	23	$\phi 127 \times 5.5$	20.99	38	$\phi 159 \times 9$	42.41
9	$\phi 70 \times 5$	10.21	24	$\phi 121 \times 6$	21.68	39	$\phi 180 \times 8$	43.23
10	$\phi 73 \times 5$	10.68	25	$\phi 127 \times 6$	22.81	40	$\phi 168 \times 9$	44.96
11	$\phi 83 \times 4.5$	11.10	26	$\phi 133 \times 6$	23.94	41	$\phi 194 \times 8$	46.75
12	$\phi 89 \times 4.5$	11.95	27	$\phi 133 \times 6.5$	25.83	42	$\phi 180 \times 9$	48.35
13	$\phi 95 \times 4.5$	12.79	28	$\phi 133 \times 7$	27.71	43	$\phi 203 \times 8$	49.01
14	$\phi 89 \times 5$	13.19	29	$\phi 140 \times 7$	29.25	44	$\phi 194 \times 9$	52.31
15	$\phi 102 \times 4.5$	13.78	30	$\phi 146 \times 7$	30.57	45	$\phi 203 \times 9$	54.85

Table 8. Allowable discrete set of cable sections [42].

No.	Sectional Area (mm ²)	No.	Sectional Area (mm ²)
1	49.48	14	153.73
2	56.30	15	182.80
3	59.69	16	196.44
4	67.35	17	216.62
5	72.23	18	238.76
6	78.94	19	244.39
7	87.96	20	297.57
8	94.15	21	302.18
9	96.27	22	355.98
10	100.88	23	382.92
11	116.24	24	385.10
12	125.50	25	464.96
13	152.81		

4.3. Wind Resistance Optimization Process

The wind resistance optimization design process in this study can be divided into two parts: one part is the finite element analysis of the cable-stiffened SLSRS, and the other part is the iterative updating of the optimization variables. Therefore, this study adopted a method combining ANSYS and MATLAB to develop the wind resistance optimization design program. The parametric modeling and finite element analysis of the cable-stiffened SLSRS were carried out in ANSYS, while the iterative updating of optimization variables using the mixed-variable QPSO algorithm was implemented in MATLAB. The shell members and struts were simulated using the BEAM188 element, both of which were regarded as ideal elastic–plastic elements, and the bilinear isotropic hardening model was used. The prestressed cables were simulated using the LINK10 element and were considered ideal elastic materials. Since the cables stopped working under compression in the actual working state, the LINK10 element was set to be tension only, and the initial strain method was used to introduce the cable prestress. Compared to the limitations of using Euler’s method in large deformation and nonlinear deformation analyses, employing nonlinear geometrical analysis for cable-stiffened SLSRSs under dynamic loads, which exhibit significant deformations, was considered feasible. This approach facilitates obtaining highly accurate and reliable analytical results through numerical simulations [20]. It is noteworthy that the block Lanczos method was employed in this study for the analysis of the shell’s natural frequencies. Figure 9 displays the displacement cloud diagram of the cable-stiffened SLSRS, which is mentioned in Section 4.1, under the condition of only bearing the cable prestress. It can be observed that the maximum displacement of the shell is only 12.6 mm, which is relatively very small compared to the dimensions of the shell. It can be concluded that the application of the initial prestress of the cables would not significantly influence the structural elements before the impact of wind loads.

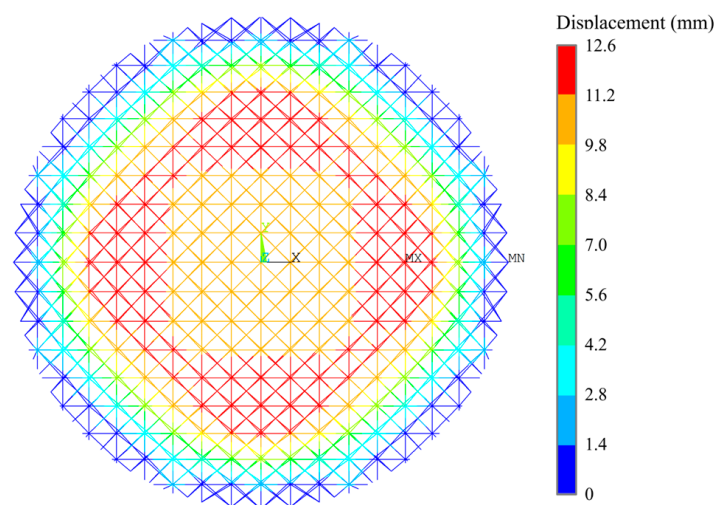


Figure 9. Displacement contour of the cable-stiffened SLSRS without wind load applied.

The specific steps for the wind resistance optimization design of the cable-stiffened single-layer spherical shell based on the mixed-variable QPSO algorithm are described as follows:

Step 1: Determine parameters such as the population size, maximum number of iterations, particle dimension, particle feasible domain, number of constraints, and allowable values of constraints, and randomly initialize the positions of the population particles using Equation (21).

Step 2: Use MATLAB’s parallel computing toolbox to assign each particle in the population to different CPUs for computation.

Step 3: Match the section sizes of the members, struts, and prestressed cables with the sizes in the steel pipe database and cable database, respectively, to achieve discretization, and then output the values of each design variable.

Step 4: Use MATLAB's system function to call ANSYS, and then read the values of each design variable on the ANSYS platform to complete the parametric modeling of the cable-stiffened SLSRS.

Step 5: Based on the newly established shell model, update the first-order natural frequency of the shell and the equivalent static wind load on the shell according to Equations (33)–(36), apply the updated load to the shell, and perform finite element analysis.

Step 6: After the finite element computation, obtain the calculated values of the constraint variables such as the maximum strength stress, maximum node displacement, maximum slenderness ratio of members, maximum stability stress, and the volume of the shell through ANSYS post-processing, and output the results.

Step 7: Read the ANSYS output results on the MATLAB platform, calculate the redundancy of each constraint variable, and then calculate the fitness value of the particle according to Equation (22).

Step 8: Summarize the computation results from each CPU to the current MATLAB workspace, and update the global best solution and individual best solution according to Equations (15)–(17) based on the fitness values of the particles.

Step 9: Update the positions of the particles in the next generation according to the particle position evolution formula, Equation (13), of the QPSO algorithm, and if a particle flies out of the feasible domain, pull it back into the feasible domain.

Step 10: If the function reaches the maximum number of iterations, select the current global extremum as the optimal solution; otherwise, return to Step 2.

5. Wind Resistance Optimization of Cable-Stiffened SLSRS

5.1. Section Optimization

The objective of the section optimization was to minimize the total volume of the shell. The design variables included the member section dimension, cable section dimension, strut section dimension, and cable prestress. The population size of the algorithm was set to 20, and the maximum number of iterations was set to 400. The optimization iterative process curves for the shell under six different wind load conditions were obtained, as displayed in Figure 10. It can be observed that the optimization iterations of the shell show a good trend of gradually decreasing to convergence, with the optimization of the shell under each condition stably converging to the optimal solution before 300 iterations.

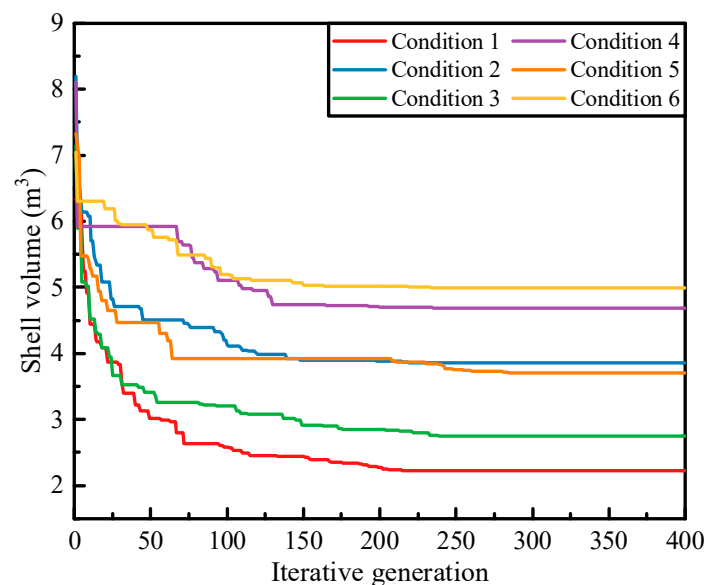


Figure 10. The iterative process of SLSRS section optimization under various load conditions.

The section optimization results of the cable-stiffened SLSRSs under various load conditions are listed in Table 9. It was found that the optimized volume of the shell under condition 6 was the largest, at 4.986 m³, indicating that condition 6 was the controlling load condition for the shell. Therefore, the optimization result under condition 6 was selected as the final result for the section optimization of the cable-stiffened SLSRSs. Compared to the initial design volume of 10.996 m³, the optimized volume under condition 6 decreased by 54.7%, significantly reducing the steel consumption of the shell. The calculation results of the constraints after the optimization of the cable-stiffened SLSRS under each condition are listed in Table 10. As shown in the table, the calculated values of the constraints under each condition were less than their respective limits, indicating that the wind resistance section optimization results of this section were effective.

Table 9. Wind-resistant section optimization results under various load conditions.

Variable		Condition					
		1	2	3	4	5	6
Member (mm)	1	$\phi 83 \times 4$	$\phi 121 \times 5$	$\phi 121 \times 5$	$\phi 133 \times 6$	$\phi 121 \times 5$	$\phi 133 \times 7.0$
	2	$\phi 83 \times 4$	$\phi 127 \times 5.5$	$\phi 73 \times 4$	$\phi 133 \times 6$	$\phi 121 \times 5$	$\phi 133 \times 6$
	3	$\phi 83 \times 4$	$\phi 121 \times 5$	$\phi 73 \times 4$	$\phi 127 \times 6$	$\phi 121 \times 5$	$\phi 133 \times 6$
	4	$\phi 83 \times 4$	$\phi 127 \times 5.5$	$\phi 73 \times 4$	$\phi 133 \times 7$	$\phi 127 \times 5.5$	$\phi 133 \times 6$
Strut (mm)		$\phi 60 \times 3.5$	$\phi 60 \times 3.5$	$\phi 60 \times 3.5$	$\phi 60 \times 3.5$	$\phi 60 \times 3.5$	$\phi 60 \times 3.5$
Cable area (mm ²)		49.48	49.48	49.48	49.48	49.48	49.48
Cable stress (MPa)		128.76	194.79	310.43	108.60	178.58	113.53
Shell volume (m ³)		2.223	3.863	2.753	4.686	3.707	4.986

Table 10. The constraint condition results after section optimization under various conditions.

Constraint	Limiting Value	Condition					
		1	2	3	4	5	6
Strength stress	0.7	0.699	0.322	0.574	0.304	0.325	0.301
Stability stress	0.7	0.241	0.698	0.357	0.699	0.658	0.699
Displacement	1.0	0.237	0.089	0.105	0.136	0.071	0.146
Slenderness ratio	1.0	0.876	0.995	0.995	0.919	0.995	0.916

5.2. Shape Optimization

The shape optimization was also performed for the cable-stiffened SLSRSs. In comparison to the four variables used in wind resistance section optimization, shape optimization additionally considered two variables related to the shape of the shell: shell rise and strut length, making a total of six variables. The introduction of these two variables allowed for the shape of the shell to change during the optimization process. The optimization objective remained the total volume of the shell. The population size of the algorithm was set to 20, and the maximum number of iterations was set to 400. The wind resistance shape optimization of the shell yielded optimization iteration process curves under six different wind load conditions, as displayed in Figure 11. It can be seen that the optimization iterations of the shell show a good trend of gradually decreasing to convergence, with the optimization of the shell under each condition stably converging to the optimal solution before the 300th iteration.

The shape optimization results for the cable-stiffened SLSRSs under different load conditions are listed in Table 11. It was observed that the optimized volume under load condition 6 was the largest, at 4.803 m³, indicating that condition 6 was the governing load case for the shell. Consequently, the optimization result for condition 6 was selected as the final shape optimization outcome. Compared to the initial design volume of 10.996 m³, the optimized volume under condition 6 was reduced by 56.3%, significantly decreasing both the steel usage and construction cost. The constraint values after shape optimization

for each load condition are listed in Table 12. The data demonstrate that all constraint values for the various load conditions are within their respective limits, confirming the effectiveness and validity of the wind resistance shape optimization results.

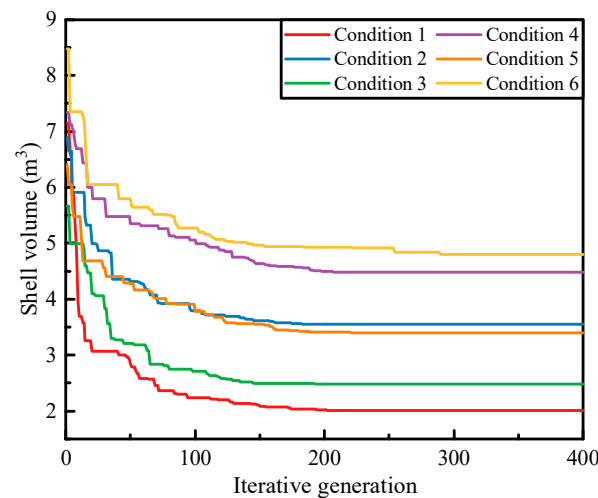


Figure 11. The iterative process of model II shape optimization under various load conditions.

Table 11. Wind-resistant shape optimization results under various load conditions.

Variable		Condition					
		1	2	3	4	5	6
Member (mm)	1	$\phi 70 \times 4.5$	$\phi 121 \times 5$	$\phi 114 \times 5$	$\phi 133 \times 6$	$\phi 114 \times 5$	$\phi 133 \times 6.5$
	2	$\phi 70 \times 4$	$\phi 114 \times 5$	$\phi 68 \times 3.5$	$\phi 127 \times 6$	$\phi 114 \times 5$	$\phi 133 \times 6$
	3	$\phi 73 \times 4$	$\phi 114 \times 5$	$\phi 68 \times 3.5$	$\phi 127 \times 5.5$	$\phi 114 \times 5$	$\phi 133 \times 6$
	4	$\phi 70 \times 4.5$	$\phi 114 \times 5$	$\phi 68 \times 3.5$	$\phi 127 \times 5.5$	$\phi 114 \times 5$	$\phi 133 \times 6$
Strut (mm)		$\phi 60 \times 3.5$	$\phi 60 \times 3.5$	$\phi 60 \times 3.5$	$\phi 68 \times 4$	$\phi 60 \times 3.5$	$\phi 60 \times 3.5$
Cable area (mm ²)		49.48	56.30	49.48	49.48	49.48	49.48
Cable stress (MPa)		128.76	194.79	310.43	108.60	178.58	113.53
Shell volume (m ³)		2.223	3.863	2.753	4.686	3.707	4.986

Table 12. The constraint condition results after shape optimization under various conditions.

Constraint	Limiting Value	Condition					
		1	2	3	4	5	6
Strength stress	0.7	0.700	0.290	0.698	0.324	0.318	0.308
Stability stress	0.7	0.198	0.699	0.325	0.700	0.654	0.699
Displacement	1.0	0.278	0.143	0.314	0.138	0.149	0.146
Slenderness ratio	1.0	0.995	0.976	0.976	0.899	0.965	0.882

5.3. Comparison of the Results

The wind resistance section optimization results were compared with the wind resistance shape optimization results for the cable-stiffened SLSRSs, as displayed in Figure 12. It can be seen that, based on the section optimization, the shell volume can be further reduced through shape optimization. The reduction in steel usage for each load condition is listed in Table 13. It was found that, compared to section optimization, the reduction in steel usage for the shell after shape optimization ranged from 3.7% to 10.2%. This indicates that the shape of the shell significantly impacts the amount of steel used and the overall cost, highlighting the importance of considering shape optimization in the design process.

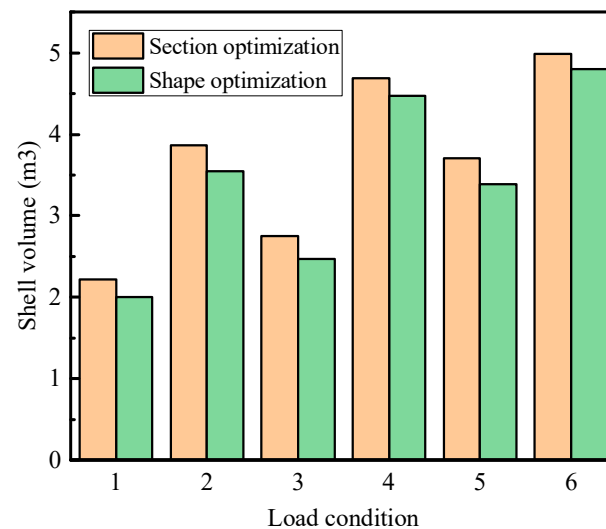


Figure 12. Comparison of section optimization and shape optimization results.

Table 13. Comparison of steel consumption between section optimization and shape optimization.

Optimization Type	Condition					
	1	2	3	4	5	6
Section optimization	2.223 t	3.863 t	2.753 t	4.686 t	3.707 t	4.986 t
Shape optimization	2.006 t	3.549 t	2.473 t	4.477 t	3.392 t	4.803 t
Reduction	9.8%	8.1%	10.2%	4.5%	8.5%	3.7%

6. Conclusions

This study proposed an improved mixed-variable QPSO algorithm based on the traditional continuous QPSO algorithm. This algorithm was then applied to the wind resistance optimization of cable-stiffened SLSRSs, including both section optimization and shape optimization. The main conclusions of this study are drawn as follows:

- (1) Through the case study analysis of the K6-type SLSRS, the optimization results obtained using the proposed mixed-variable QPSO algorithm were compared with the optimization results obtained using other algorithms in the existing literature. This comparison verified the high accuracy and reliability of the improved QPSO algorithm proposed in this study.
- (2) The proposed mixed-variable QPSO algorithm was used to study the wind resistance optimization method for cable-stiffened SLSRSs. A mathematical model for wind resistance optimization was established, and the wind resistance optimization process based on the mixed-variable QPSO algorithm was presented. An optimization design program was developed using a combination of MATLAB and ANSYS. Finally, wind resistance section optimization and shape optimization were performed on the cable-stiffened SLSRSs. The optimization results indicated that the proposed mixed-variable QPSO algorithm has good stability and a strong global search capability, providing an effective approach to solving the wind resistance optimization problem of cable-stiffened SLSRSs.
- (3) By comparing the results of wind resistance section optimization and shape optimization, it was found that the steel usage of SLSRSs can be further reduced after shape optimization based on section optimization. After shape optimization, the steel usage of the shell could be reduced by up to 10.2% compared to section optimization, indicating that the shape of the shell significantly impacted the steel usage and overall cost. Therefore, shape optimization should be given considerable attention in the design process.

Author Contributions: Conceptualization, Y.Z. and P.L.; Methodology, Y.Z. and G.C.; Software, G.C., S.S., M.H. and T.Z.; Formal analysis, M.H.; Investigation, G.C., M.H. and T.Z.; Resources, P.L.; Data curation, T.Z.; Writing—original draft, Y.Z., G.C. and S.S.; Writing—review & editing, M.H. and T.Z.; Visualization, G.C. and G.X.; Supervision, S.S. and G.X.; Project administration, Y.Z., S.S. and G.X.; Funding acquisition, Y.Z. and P.L. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Chongqing Construction Science and Technology Plan (Grant No. 2023-3-19).

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Conflicts of Interest: The authors declare no conflict of interest.

References

- Li, P.; Wu, M.; Xing, P. Novel cable-stiffened single-layer latticed shells and their stabilities. *J. Constr. Steel Res.* **2014**, *92*, 114–121. [\[CrossRef\]](#)
- Bulenda, T.; Knippers, J. Stability of grid shells. *Comput. Struct.* **2001**, *79*, 1161–1174. [\[CrossRef\]](#)
- Cai, J.; Xu, Y.; Feng, J.; Zhang, J. Nonlinear stability of a single-layer hybrid grid shell. *J. Civ. Eng. Manag.* **2012**, *18*, 752–760. [\[CrossRef\]](#)
- Li, P.; Wu, M. Parametric study of cable-stiffened single-layer cylindrical latticed shells with different supporting conditions. *J. Constr. Steel Res.* **2016**, *121*, 457–467. [\[CrossRef\]](#)
- Li, P.; Wu, M. Stabilities of cable-stiffened cylindrical single-layer latticed shells. *Steel Compos. Struct.* **2017**, *24*, 591–602.
- Cui, X.Z.; Li, Y.G.; Hong, H.G. Reliability of stability of single-layer latticed shells with spatially correlated initial geometric imperfection modeled using conditional autoregressive model. *Eng. Struct.* **2019**, *201*, 109787. [\[CrossRef\]](#)
- Manuello, A. Multi-body rope approach for grid shells: Form-finding and imperfection sensitivity. *Eng. Struct.* **2020**, *221*, 111029. [\[CrossRef\]](#)
- Hossein, M.; Karim, A.; Behzad, S. Collapse behavior of cable-stiffened single-layer barrel vaults with novel forms of out-of-plane cables. *Int. J. Struct. Stab. Dyn.* **2021**, *21*, 2150135.
- Yang, D.; Guo, B.; Chong, C.; Liu, C. Modeling and structural behavior of cable-stiffened single-layer latticed domes of hexagonal meshes. *J. Constr. Steel Res.* **2015**, *114*, 237–246. [\[CrossRef\]](#)
- Zhao, L.; Cao, Z.; Wang, Z.; Fan, F. Comparative study and optimal selection for different cable-stiffened latticed shells. *Thin-Walled Struct.* **2022**, *181*, 110083. [\[CrossRef\]](#)
- Zhang, L.; Chen, W.; Dong, S. Natural vibration and wind-induced response analysis of the non-fully symmetric Geiger cable dome. *J. Vibroeng.* **2014**, *16*, 31–41.
- Ma, T.; Zhao, L.; Ji, T.; Tang, T. Case study of wind-induced performance and equivalent static wind loads of large-span openable truss structures. *Thin-Walled Struct.* **2022**, *175*, 109206. [\[CrossRef\]](#)
- Sun, W.; Zhang, Q. Geometrically nonlinear wind-induced responses of prestressed cable-suspended roof. *Structures* **2021**, *29*, 883–898. [\[CrossRef\]](#)
- He, Y.; Zhu, M.; Zhao, Y.; Li, X. Influence of different cable–membrane connection models on wind-induced responses of an air supported membrane structure with orthogonal cable net. *Thin-Walled Struct.* **2022**, *180*, 109840. [\[CrossRef\]](#)
- Zhou, Z.; Li, Z.; Meng, S.; Wu, J. Wind-induced vibration responses of prestressed double-layered spherical latticed shells. *Int. J. Steel Struct.* **2011**, *11*, 191–202. [\[CrossRef\]](#)
- Wang, F.; Wang, X. Wind vibration response analysis for plate-cone reticulated shell. *Adv. Mater. Res.* **2011**, *163*, 4222–4226. [\[CrossRef\]](#)
- Lu, W.; Wang, J.; Guo, H.; Li, H.; Sun, J. Wind-induced dynamic collapse analysis of single-layer cylindrical reticulated shells considering roof slabs and support columns. *Int. J. Heat Technol.* **2020**, *38*, 180–186. [\[CrossRef\]](#)
- Alizadeh, A.; Shekastehband, B. Wind-induced instability analysis of large-span single-layer lattice domes. *Thin-Walled Struct.* **2022**, *181*, 110109. [\[CrossRef\]](#)
- Li, P.; Wang, L. Nonlinear stability behavior of cable-stiffened single-layer latticed shells under earthquakes. *Int. J. Struct. Stab. Dyn.* **2018**, *18*, 1850117. [\[CrossRef\]](#)
- Li, P.; Chen, G.; Luo, Y.; Zhang, T.; Ren, S.; Liu, C.; Jian, B.; Xiong, G. Seismic behavior analysis of cable-stiffened cylindrical latticed shells. *J. Constr. Steel Res.* **2024**, *219*, 108786. [\[CrossRef\]](#)
- Zhang, Z.; Li, P.; Zhang, T.; Hu, H.; Wang, J.; Xiong, G. Dynamic behavior of cable-stiffened cylindrical single-layer latticed shell based on shaking table tests. *Structures* **2024**, *66*, 106857. [\[CrossRef\]](#)
- Fu, J.; Wu, B.; Wu, J.; Deng, T.; Pi, Y.; Xie, Z. Wind resistant size optimization of geometrically nonlinear lattice structures using a modified optimality criterion method. *Eng. Struct.* **2018**, *173*, 573–588. [\[CrossRef\]](#)
- Xu, A.; Lin, H.; Fu, J.; Sun, W. Wind-resistant structural optimization of supertall buildings based on high-frequency force balance wind tunnel experiment. *Eng. Struct.* **2021**, *248*, 113247. [\[CrossRef\]](#)

24. Phan, D.; Lim, J.; Sha, W.; Siew, C.Y.; Tanyimboh, T.T.; Issa, H.K.; Mohammad, F.A. Design optimization of cold-formed steel portal frames taking into account the effect of building topology. *Eng. Optim.* **2013**, *45*, 415–433. [\[CrossRef\]](#)
25. Qiu, Y.; Yu, R.; San, B.; Li, J. Aerodynamic shape optimization of large-span coal sheds for wind-induced effect mitigation using surrogate models. *Eng. Struct.* **2022**, *253*, 113818. [\[CrossRef\]](#)
26. San, B.; Xu, C.; Qiu, Y. Three-dimensional aerodynamic optimization of single-layer reticulated cylindrical roofs subjected to mean wind loads. *Adv. Civ. Eng.* **2019**, *2019*, 4156319. [\[CrossRef\]](#)
27. Li, P.; Chen, G.; Qiu, J.; Qian, J.; Ding, D.; Jian, B.; Zhang, X.; Xiong, G. Experimental and numerical investigation into the load-carrying capacity of aluminium alloy H-sectional stocky columns under axial compression. *J. Build. Eng.* **2024**, *87*, 108777. [\[CrossRef\]](#)
28. Fu, J.; Wu, J.; Dong, C. Optimization design of large span portal-rigid steel frame with tapered sections under wind-induced drift constraint. *Eng. Struct.* **2019**, *194*, 396–405. [\[CrossRef\]](#)
29. Kociecki, M.; Adeli, H. Two-phase genetic algorithm for size optimization of free-form steel space-frame roof structures. *J. Constr. Steel Res.* **2013**, *90*, 283–296. [\[CrossRef\]](#)
30. Kociecki, M.; Adeli, H. Two-phase genetic algorithm for topology optimization of free-form steel space-frame roof structures with complex curvatures. *Eng. Appl. Artif. Intell.* **2014**, *32*, 218–227. [\[CrossRef\]](#)
31. Hsaine, N. Structural Optimization of Self-Supported Dome Roof Frames under Gust Wind Loads. Ph.D. Thesis, Newcastle University, Newcastle, UK, 2020.
32. Bergh, F. An Analysis of Particle Swarm Optimizers. Ph.D. Thesis, University of Pretoria, Pretoria, South Africa, 2001.
33. Bergh, F. A new locally convergent particle swarm optimizer. In Proceedings of the IEEE International Conference on systems, Man and Cybernetics, Yasmine Hammamet, Tunisia, 6–9 October 2002.
34. Sun, J.; Fang, W.; Wu, X.; Wu, X.J.; Xu, W.B. *Quantum-Behaved Particle Swarm Optimization: Principles and Applications*; Tsinghua University Press: Beijing, China, 2011.
35. Cai, B. Optimization of the Frame Based on Particle Swarm Algorithm. Ph.D. Thesis, Dalian University of Technology, Dalian, China, 2014.
36. Shi, K.; Pan, W.; Jiang, Z.; Lin, J.; Wei, D. Optimization method of spatial structures based on growth space limited & parallel search-based plant growth simulation algorithm. *J. Build. Struct.* **2021**, *42*, 85–94.
37. GB 50017-2017; Standard for Design of Steel Structures. National Standards of the People's Republic of China: Beijing, China, 2018.
38. JGJ 7-2010; Technical Specification for Space Frame Structures. Industry Standards of People's Republic of China: Beijing, China, 2010.
39. Ruan, Z. Research on Optimization Design of Prestressed Steel Structure Based on Improved PGSA and Cable Force Identification Method. Ph.D. Thesis, South China University of Technology, Guangzhou, China, 2015.
40. Wang, J.; Ye, J. Application of genetic algorithm in single-layer spherical shell mass optimization. *J. Vib. Shock* **2009**, *28*, 1–7.
41. GB/T 17395-2008; Dimensions, Shapes, Masses and Tolerances of Seamless Steel Tubes. National Standards of the People's Republic of China: Beijing, China, 2008.
42. GB/T 20492-2019; Zinc-5% Aluminum-Mixed Mischmetal Alloy-Coated Steel Wire. National Standards of the People's Republic of China: Beijing, China, 2019.
43. JGJ/T 481-2019; Standard for Wind Loads on Roof Structures. Industry Standards of People's Republic of China: Beijing, China, 2019.
44. Yang, Q.; Tian, Y.; Chen, B. Main contents of the standard for wind loads on roof structures. *Eng. Mech.* **2018**, *35*, 1–6.
45. GB 50009-2012; Load Code for the Design of Building Structures. National Standards of the People's Republic of China: Beijing, China, 2012.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.