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Logic, Perception, and Beauty—An Outline of the Modern Proportion-Based Approach in Architecture

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Abstract: The present study contributes to the discourse on the environmental paradigm in architecture, recognizing beauty as essential alongside community and sustainability. It examines the modern proportion-based approach within the realm of formal aesthetics, which views proportional relationships as essential for architectural form and aims to develop methods to manipulate these relationships to achieve desired aesthetic outcomes. The main objective of this study is to characterize the modern approach as a current distinct from classical theories. The research seeks to explore the unique rationales behind a systematic approach to architectural proportions and to discern how these rationales distinguish the modern approach from its classical predecessor. The research encompasses theories from the mid-19th century to the present day. Through content and comparative analysis, this study explores theories by key figures such as A. Thiersch, H.P. Berlage, Le Corbusier, H. van der Laan, J. Żórawski, J. Hale, and P. Märkli. This study reveals the modern proportional approach as a unique current that adapts classical ideas to modern sensibilities, shifting from classical cosmological symbolism to a focus on logical consistency, visual perception, and artistic expression. It maintains a dual relationship with tradition, preserving past methods while introducing new interpretations of their aesthetic function. Despite its modest impact on contemporary practice, the proportional approach is significant, emphasizing the visual quality of the built environment and pursuing timeless design principles that transcend fleeting trends, focusing on enduring aesthetic values.

Keywords: proportion; proportional systems; regulating lines; architectural geometry

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1. Introduction

1.1. Scope of the Research

In mathematical terms, proportion signifies the equality of two numerical ratios. In architecture, however, the concept of proportion is more expansive, encompassing the entirety of metric and geometric relationships between the constituent parts of a form. A proportional system is a method of arranging these relationships, coordinated according to a specific geometric or numerical principle. Proportions and proportional systems have held a significant place in architectural theory and played an important role in medieval and Renaissance architecture [1–5]. Beginning in the 18th century, their influence waned but re-emerged in the mid-19th century with studies on historical architecture. Proportion-based theories have influenced the development of modernist architecture, only to lose prominence again in subsequent decades of the 20th century.

The issue of proportion is inextricably linked to the quest for beauty, which is variously understood and defined. The contemporary environmental paradigm recognizes beauty as an equivalent component of sustainability and community in a well-organized human living space [6]. There arises, therefore, a need for systematic studies on the concept of beauty in architecture, including a re-evaluation of existing concepts in this area, among which proportion-based theories hold a prominent position.

The subject of this study is the modern proportion-based approach to architecture.

Proportions are an aspect of every architectural form, but they are not always the primary focus or entirely within the designer's control. A proportion-based approach is a current in architectural theory that attributes particular importance to the issue of proportion and also seeks to develop methods for controlling it to achieve specific artistic or aesthetic effects. The term 'modern' in this context signifies 'non-classical' to distinguish it from ancient, medieval, and Renaissance theories. The modern proportion-based approach was born in the second half of the 19th century, reached its zenith in the mid-20th century, and persists in a limited form to this day.

1.2. Research Objectives and Inquiries

The aim of this study is an attempt to present the modern proportion-based approach as a distinct current in architectural theory that has not been sufficiently recognized and emphasized until now. This distinction is necessary to clarify the discourse on architectural proportions. Without it, the discussion becomes obscure, and proportions in architecture may be perceived in terms of esoteric and symbolic explanations that characterized previous epochs.

The primary research question concerns the theoretical foundations of this distinctiveness. It is therefore necessary to establish the basic assumptions regarding the key issue, namely, the application of proportional systems in architecture, as follows:

- What is the purpose of a systematic approach to the problem of proportion?
- What are its ideological sources?
- How do these assumptions differ from classical approaches, and in what ways are they similar?

An additional objective of this study is to provide a multifaceted characterization of the modern proportional approach, allowing for an assessment of its significance for contemporary architectural theory.

1.3. Thesis

According to the thesis of this study, the modern proportion-based approach is neither a simple continuation of classical proportion-based theories nor a collection of scattered concepts, but a theoretical and programmatic current that interprets classical ideas in the spirit of a modern worldview, while preserving certain threads of past theories. Traditional cosmological and anthropological symbolism is replaced or supplemented by new concepts, primarily based on the following views:

- The idea of logical consistency of form.
- The process of visual perception.
- The idea of a system of proportion as a tool for controlling artistic expression.

These concepts stand in contrast to the traditional rationales for proportional systems, which are articulated through technical imperatives and the symbolic embodiment of cosmological narratives.

The modern proportion-based approach incorporates two opposite explanations of the origin of proportional order. The traditional view holds that the need for proportion reflects the mathematical laws of nature, which humans, as part of the universe, aspire to mimic. Alternatively, a post-Kantian perspective sees mathematical laws as a unique construct of human reason, through which we shape and organize nature to suit our needs.

The content of the thesis is illustrated by Figure 1.

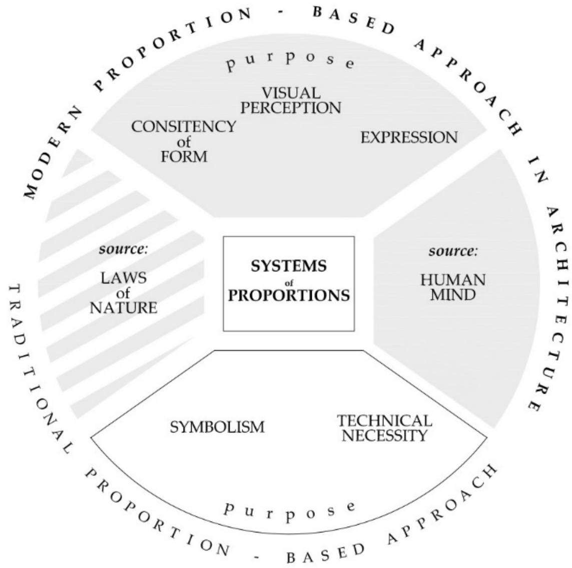

Figure 1. Graphical representation of the thesis. The drawing is carried out by the author.

1.4. State of Research

The subject of proportions in architecture has generated an extensive body of literature, which would require a separate bibliographic article for a comprehensive overview, such as the study by Ralph Weber and Sharon Larner [7]. A significant portion of this literature is associated with classical proportional theories, encompassing foundational texts like Vitruvius's *De Architectura* and Renaissance treatises, as well as historical studies on ancient systems of proportions and measures that have been systematically expanded since the 19th century. The early historical research in this field has particularly influenced the development of the modern proportional approach. Among the researchers whose theories have had a notable impact are E. Viollet-le-Duc, Georg Dehio, Auguste Choisy, and Marcel-Auguste Dieulafoy [8–11] Among later studies, Rudolf Wittkower's *Architectural Principles in the Age of Humanism* [2] and Colin Rowe's *Mathematics of the Ideal Villa* have achieved considerable renown [12].

Research into proportional systems remains a significant branch of architectural history, characterized by an increasingly critical methodology that emphasizes the historical context of construction, the precision of measurements, and the rigorous verification of sources. An example of this approach is the collective study *Proportional Systems in the History of Architecture: A Critical Reconsideration*, edited by Matthew A. Cohen and Maarten Delbeke in 2018 [13]. The past few decades have also yielded detailed monographic studies on mathematical principles in ancient and Gothic architecture [14,15]. It is important to note, however, that unlike the 19th-century theories mentioned, the outcomes of these studies do not typically influence architectural practice.

The primary material for this study consists of programmatic writings that form the core of the modern proportional approach. These include statements by theorizing architects such as August Thiersch, H.P. Berlage, Le Corbusier, Hans van der Laan, Juliusz Żórawski, Jonathan Hale, and Peter Märkli [16–24], who are discussed in greater detail in this study. This group also includes other authors who offer original perspectives on proportions, such as Frederik Macody Lund, Ernst Moessel, Jay Hambidge, and Milutin Borisavljević [25–28].

Supplementary literature includes review and synthesis studies. A classic of this genre is the Romanian philosopher and mathematician Matila Ghyka, author of popular publications such as *Le nombre d'or* (1931) [29] and *The Geometry of Art and Life* (1946) [30], which, in addition to expounding on various proportional theories, promoted the idea of 'sacred geometry' and the theory of common mathematical foundations underlying natural forms and works of art. Post-war art researchers such as Wittkower and Rudolf Arnheim have also addressed the issue of proportions in modern architecture [31,32].

In 1960, Scholfield published *The Theory of Proportion in Architecture*, in which he conducted a study of proportional systems, both classical and modern, presenting their evolution—according to the author—from numerical to geometric systems and from commensurable to incommensurable proportions [33]. Among more recent studies, one of the most comprehensive studies on the subject is Richard Padovan's *Proportion: Science, Philosophy, Architecture*, which explores the complex relationships between the theory of proportions, the scientific worldview, and art throughout history [34].

Christopher Height, in his study *Architectural Principles in the Age of Cybernetics*, undertook an analysis of post-World War II architectural discourses in relation to the Vitruvian paradigm of architecture. The study, whose title is a playful take on Wittkower's celebrated book, devotes extensive focus to the interpretation of the Vitruvian tradition as it is expressed by Le Corbusier and in postmodern approaches that encompass phenomenology and deconstruction [35].

One of the latest publications on the subject of proportions is the 2019 collective study *Proportions and Cognition in Architecture and Urban Design*, which includes a series of valuable studies on specific aspects of proportional theory as well as a collection of interviews with practicing architects who confront proportional systems with today's practice—including Peter Märkli [36]. Contemporary research into the connections between architectural design and mathematics, including studies on proportional systems, is also published in the *Nexus Network Journal* [37].

The aforementioned studies offer a wealth of insights and information that contribute to our understanding of proportion-based systems in architecture. However, they may not always provide a distinct delineation between classical and modern approaches to proportion, sometimes leaving this distinction less clear. The current study aims to offer a nuanced perspective that complements these existing studies by focusing specifically on the modern proportion-based approach and its unique characteristics within the broader discourse of architectural theory.

2. Materials and Methods

2.1. Objectives of this Study

The objective of this study is to investigate the theoretical foundations of the modern proportional approach in architecture, with a particular focus on explaining the purpose, significance, and justification for the use of proportional systems. This study also aims to understand the relationship between modern and classical proportional approaches. The preliminary theoretical framework suggests that the modern proportional approach differs from classical theories by proposing alternative justifications for proportional systems, such as the logical consistency of form, the shaping of visual perception, and artistic expression. This study aims to verify this thesis.

2.2. Research Materials

The primary research material is the subject literature, among which the most important position is occupied by studies presenting a theoretical approach to proportional systems. These studies have been categorized into several basic groups, depending on their significance and application in this study. The temporal scope of the main analysis covers the 20th century and the first decades of the 21st century (1904–2019). The core group contains selected theories that are most representative of the modern proportional approach. The theories of seven authors have been chosen: A. Thiersch, H.P. Berlage, Le Corbusier, H. Van der Laan, J. Żórawski, J. Hale, and P. Märkli. The selection of each theory is justified by its particular rank—primarily the range of influence of the given theory, or the originality and depth of the approach to the problem. The specific categories are presented in Table 1.

Table 1. Classification of theories and key references within the present study.

Categories	Characteristics	Application	References
Classic Theories	This group includes classic architectural treatises by authors such as Vitruvius, Alberti, Palladio, Serlio, and others. These studies are part of the paradigm of the Great Theory of aesthetics.	These studies are widely known and do not fall within the primary thematic scope of the study. They serve as a background and are not individually cited in the literature list.	-
Formative Theories	These are 19th-century studies that contributed to the formation of the modern proportional approach. They include historical studies and studies from the field of aesthetics (e.g., A. Zeising and the Beuron School).	These theories are mentioned in the chapter "The Formation of the Modern Proportion-Based Approach..." and serve as a background for the main analysis.	[8,9,11,38–43]
Core Theories A	This is the primary literature with a programmatic character. Representative positions include studies by A. Thiersch, H.P. Berlage, Le Corbusier, H. van der Laan, and Juliusz Żórawski.	A detailed content analysis of the individual positions is provided.	[16–24,44]
Core Theories B	Additional literature that fits within the modern proportional approach.	These studies are mentioned and serve as a background for the primary analysis.	[25–28,45–47]
Alternative Theories	Approaches that fall outside the modern proportional approach—historical-critical approaches and the so-called 'sacred geometry' current.	These theories are treated as polemical and are mentioned in the Discussion section.	[29,30,48–50]
Scholarly Studies	Non prescriptive studies, treating proportional systems as a field of research.	These positions have mainly informational value and, in terms of general conclusions, serve as a reference point for the theses of this study. Selected positions are mentioned in the Discussion section.	[31–36]

2.3. Analysis of Materials

The literature review will encompass a concise overview of traditional proportional concepts, serving as a reference point for the modern proportional approach. It will also include a brief historical sketch tracing the development of proportional theories from the 19th to the 20th century, situating the theories within a broader historical and theoretical framework. The main analysis covering the core group of literature is a content analysis with elements of comparative analysis. The basic research questions include the following:

- The context in which a given theory was formulated.
- The ideological or philosophical framework for the proportional system.
- The purpose or nature of the action is attributed to proportional systems.
- The practical and analytical methods used by the authors.

2.4. Expected Results

It is expected that the research will sufficiently justify the main thesis of this study and will also allow for a deeper characterization of the proportional approach, including an explanation of its relationship to historical theories and a general understanding of architecture within this approach. The results are presented in Section 5.

2.5. Discussion

In the section titled Discussion, an attempt is made to confront the results obtained with the current state of knowledge regarding proportional theories, highlighting areas where discrepancies occur. Here, the place of the proportional approach in contemporary architectural theory within the context of the environmental paradigm is also discussed. The limitations of the present research are defined, and directions for future studies are indicated.

2.6. Conclusions

The conclusion summarizes the research and draws general conclusions about the significance of the modern proportional approach in architecture.

3. The Formation of the Modern Proportion-Based Approach

3.1. Reference Point: Classical Proportional Approach

Proportional systems used in ancient architecture had, as some researchers suggest [4], a practical significance related to the methods of laying out and constructing buildings. Due to the imperfection of measuring tools, it was easier to define a building whose measurements were coordinated with each other, for example, through simple numerical ratios or geometric constructions obtained using a string. However, it would be a tremendous error to limit the significance of proportional systems in architecture to this pragmatic goal alone.

Ancient systems of proportion in architecture were shaped within the established European cultural tradition of aesthetics, which, due to its scope and influence on all artistic phenomena, is referred to by Władysław Tatarkiewicz as the Great Theory [51] (p. 125). The genesis of this tradition lies in ancient Greek philosophy, particularly in its Pythagorean-Platonic current, which was transmitted to the Christian Middle Ages by figures such as Boethius and St. Augustine, and revived in the 15th century at the Florentine Academy. According to this philosophy, the visible world is the result of a conscious act of creation that establishes a cosmic order out of formless chaos. The world is not self-sufficient; its existence is conditioned by transcendent ideas. These ideas have a moral character, such as Truth, Goodness, and Beauty, as well as a mathematical one. The world is harmonious, meaning that all its parts are arranged according to rational, numerical, and geometric relationships. This is expressed by the Pythagorean cosmological metaphor of the music of the planetary spheres, where simple numerical ratios represent the intervals of cosmic tones. These relationships are not subject to chance or change; they are eternal and are used by the Creator to impose order on the sensory world. They are a divine trace in the universe. They are beautiful because they lead to the very idea of Beauty, which exists in the Creator. Hence the centuries-old definition of beauty as proportion, which is the core of the Great Theory [51] (pp. 125–128). There is a profound kinship between the universe—the macrocosm—and man—the microcosm. The world, already presented in Plato’s Timaeus as a living corporeal-spiritual entity, corresponded in its structure to the soul and body of man, to which the principle of cosmic harmony applied. Hence the medieval concepts of *homo quadratus* and, above all, the famous statement by Vitruvius, which defines the harmonious human body as a model for architecture [52] (pp. 72–81).

Art was primarily the skill and knowledge of the laws governing a particular field of production. As Tatarkiewicz writes: “Doing anything without rules, only from inspiration or fantasy, was not considered art by the ancients or scholastics; it was its opposite” [51]. The perfection of art consisted of a deeper knowledge and understanding of its rules. “Ars sine scientia nihil est”—the well-known saying of the 14th-century architect Jean Mignot refers to the most abstract and general knowledge, which was considered the deepest—mathematics [5] (p. 19). In this spirit, both the ancient canons of Polyclitus and Vitruvius, the medieval schemes of Villard, and the Renaissance studies of proportion by Dürer, Piero della Francesca, Luca Pacioli, Francesco di Giorgio Martini, Alberti, and Palladio among others, were created [52] (pp. 61–96).

The early modern period brought a crisis to the Great Theory. Due to the study of Copernicus, Galileo, and Kepler, the ancient cosmology, with which classical aesthetics was organically linked, fell. The teachings of Descartes and Bacon undermined all judgments based on the authority of past ages and were not supported by strict deduction or experiment. In the 17th century, the Great Theory still shaped thinking about architecture. Influenced by Vitruvius and Italian treatises, Francois Blondel's (1618–1686) course for the Académie Royale d'Architecture set the direction for the academic tradition [53]. However, at the same time, Claude Perrault (1613–1688) openly questioned the authority of the ancients and called the proportions contained in mathematical formulas 'arbitrary' [54] (pp. 6–9).

The philosophical skepticism of the 18th century led to a widespread crisis in metaphysics and doubt about the idea of objective beauty. For David Hume (1711–1776), beauty was purely subjective and relative: "Beauty is not a property of things themselves. It exists in the mind that contemplates it, and each mind perceives a different beauty" [55]. Edmund Burke (1729–1797) openly questioned speculations about proportions as having no influence on the aesthetic perception of an object [52] (p. 97). The English aesthetes attacked the Palladian 'musical' proportions as derived from a false analogy between sight and hearing [33] (pp. 72–75). As Height notes, French Enlightenment theory, embodied by the proto-functionalism of N. J. Durand (1760–1834), seems to be the final end of the classical paradigm of proportion in architecture: "Two seemingly opposed developments eroded the Vitruvian tradition of bodily metaphor: on the one hand, the rise of rationalism and, on the other, the rise of taste determined by subjective fancy" [35] (p. 24).

3.2. The Development of the Modern Proportion-Based Approach in the 19th and 21st Centuries: An Outline

There was a renewed interest in proportional systems in the late first half of the 19th century, which was prompted by studies of Gothic architecture. Researchers such as Viollet le Duc (1814–1879) or Robert William Billings (1812–1874) perceived a geometric discipline in medieval buildings and endeavored to uncover its principles [8,39,40]. Over the subsequent decades, there was an increase in historical reconstructions of proportional systems. For example, the popular British *Encyclopaedia of Architecture* by J. Gwilt, published in 1849, in the section dedicated to architectural aesthetics, does not include any proportional systems, citing the views of English empiricists on the relativity and subjectivity of taste [56] (pp. 673–675). The same encyclopedia's edition from 1888, however, contains a full chapter on proportions, presenting several different geometric analyses of historical architecture [38] (pp. 935–963).

The efforts of 19th-century aestheticians to find scientific foundations for the experience of beauty were not without significance for proportional theories. The speculations of the psychologist Adolf Zeising (1810–1876), a proponent of the so-called golden ratio, were particularly important [43]. He promoted the golden ratio as a universal law governing the creation of forms, "striving for beauty and completeness in both nature and art". The beauty of the golden ratio was supposed to arise from the fact that it is, as Zeising argued, a principle of nature. Zeising sought the golden ratio throughout nature, for example, in the proportions of the human body as well as in outstanding works of architecture such as the Parthenon [43] (pp. 390–413) Contemporary research evaluates Zeising's theories rather critically. While the golden ratio was indeed known to the Greeks, there is a lack of evidence to suggest that it constituted a primary aesthetic principle—either in antiquity or, for example, during the Renaissance [57]. As Height observes, Zeising's theories stemmed from an ambition to explain aesthetic issues within the prevailing paradigm of naturalistic positivism that was triumphant at the time. It was intended to be a natural constant, akin to those discovered by modern science, such as the gravitational constant. [35] (pp. 143–147). The pioneer of experimental psychology, Gustav T. Fechner (1801–1887), also attempted to demonstrate the aesthetic value of the golden ratio [35] (pp. 148–152). The results of these studies did not resolve the issue, but Zeising's myth of the golden ratio proved to be remarkably resilient and became firmly established in design theory.

The late 19th century and the first decades of the 20th century in European culture were characterized by a general shift from positivism to spiritualism. One manifestation of this shift was the emergence of new forms of spirituality intended to be an alternative to scientific materialism. One of these was theosophy—a syncretic doctrine attempting to combine elements of European Hermeticism and Eastern religions with quasi-scientific rhetoric. Theosophy inherited the legacy of Pythagorean-Platonic cosmology, along with the symbolism of numbers and geometry. The problem of the geometric foundations of art was strongly present in theosophical speculations. It is significant that the direct inspiration for Helena Blavatsky to establish the Theosophical Society in 1875 was reportedly a lecture by George H. Felt titled *The Lost Canon of Proportion of the Egyptians* [58]). The significance of theosophy for the avant-garde of the first decades of the 20th century is well documented [59].

Less known is the influence of the 'Beuron School'—a movement in Catholic sacred art from the late decades of the 19th century. The leading figure of the movement was Peter Lenz (1832–1928), a German painter from the Nazarene circle, who took the monastic name Desiderius and joined the monastery in Beuron in 1872, where he founded an art school. The Beuron doctrine advocated for hieratic art inspired by models from ancient Egypt and Greek vase painting. Lenz criticized contemporary ecclesiastical art as capricious, sentimental, and superficial in its naturalism:

> "*For us, however, the ideas of the eternal and divine are abstract; they do not allow themselves to be brought down to this region without being weakened; they may not be translated into a purely human manner and style*". [41] (p. 27)

Therefore, sacred art should also be based on abstract, universal, and unchanging principles that can create a proper visual analogue of divinity—specifically, on mathematical canons. The compositional grid composed of lines and simple geometric figures, on which figurative scenes, devotional objects, liturgical furnishings, and architecture were constructed, became the hallmark of the Beuron School (Figure 2). The influence of the Beuron School extended beyond ecclesiastical circles. The mathematical mysticism and suggestive graphic schemes influenced French symbolists such as Les Nabis, the Vienna Secession, and some of the members of the avant-garde, including Le Corbusier himself, which was reluctantly confirmed in his *Modulor* [60] (pp. 218–219)). According to Ross Anderson, the Beuron geometric methods were also adopted by Peter Behrens [61].

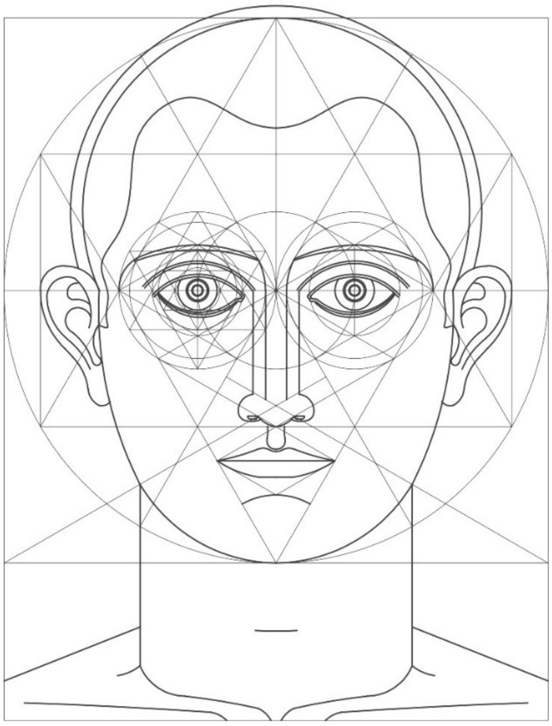

Figure 2. Proportional study of a man's head based on a hexagram, Beuron School, Desiderius Lenz, late 19th century. Adapted and CAD-Enhanced Illustration by the author based on [42].

Theosophical speculations and the doctrines of Beuron had a decisive influence on the emergence of the Dutch school of proportional systems in the first half of the 20th century. It was formed by architects such as Jan Hessel de Groot (1864–1916), J.L.M. Lauweriks (1864–1932), and K.P.C de Basel (1869—1923). H.P. Berlage significantly contributed to the dissemination of these theories [62].

In the interwar period, extensive theoretical publications appeared, containing comprehensive proportional systems. In 1926, Theodor Fischer's student Ernst Moessel (1881–1946) published his doctoral thesis, in which he presented a system of proportions based on the division of the circle, 'Kreuzteilung', which was to be—as the author argued—an archaic method of laying out building plans and facades used in antiquity and the Middle Ages. (Figure 3). The proportions resulted from the construction of regular polygons and multifaceted stars, which was both difficult to assimilate and graphically spectacular [26]. In 1921, the American artist Jay Hambidge (1867–1924) presented a system of dynamic symmetry—'discovered' as a result of studies of Greek ceramic artifacts and ancient architecture. Hambidge advocated for incommensurable proportions such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and φ, in contrast to the 'Renaissance' proportions of whole numbers (2/3, 3/5, etc.). Rectangles with these proportions (so-called dynamic rectangles) allowed for the creation of infinite sequences of divisions into similar figures. Hambidge's theory about the significance of incommensurability in ancient Greece was motivated by quotes from Plato's dialogue *Theaetetus* [27] (pp. 159–160)) (Figure 4). The Norwegian architect Frederik Macody Lund (1863–1943) presented his own reconstruction of a medieval system of proportions in the work *Ad Quadratum*. This system was used by the author himself in projects for the reconstruction of the Nidaros Cathedral in Trondheim [25]. Despite the vast material cited by the authors in support of their theses, none of the mentioned systems gained much popularity among practicing architects, which can be attributed to their considerable complexity.

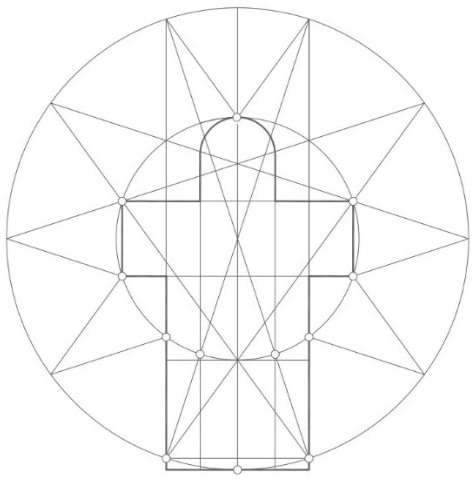

Figure 3. Scheme of proportions of a Gothic church according to Ernst Moessel's 'Kreuzteilung' system. Adapted and CAD-Enhanced Illustration by the author based on [26].

From the 1920s until his death in 1968, Le Corbusier was the main representative of the proportional approach in architecture. Through his publications *Vers une Architecture* and then Modulor, he dominated the debate on proportions. The authority of Le Corbusier, as well as the well-known publications of Wittkower, Rowe, or Ghyka, created a kind of fashion for proportions in the first post-war decades, which Height refers to as the 'mid-century renaissance'. The topic was of interest to leading architects such as Max Bill (1908–1994), Erno Goldfinger (1902–1987), and others [63,64]. In 1951, a conference called *La Divina Proportione* was organized at the Milan Triennale, aiming to explore the relationship between design and mathematics. Despite the announcements, subsequent editions of the conference did not take place [32]. The proportional theories flourishing in the esoteric atmosphere of the interwar avant-garde did not find fertile ground in the intellectual climate of the 1960s. The post-war advocates of the modern movement, including Gropius with his study *The New Architecture and the Bauhaus*, sought to make the principles of modernism accessible to a broad audience. They highlighted the movement's progressive and technically rational aspects rather than its more esoteric elements [65]. Proportional methods, which required not only geometric proficiency but also a suitable philosophical background, remained largely impenetrable to the general public. The architectural postmodernism initiated by Venturi, with its appreciation of contradictions, narrativity, and kitsch, was even less conducive to proportional theories. Among postmodernist architects, a few have declared their interest in employing proportional systems in their studies. One notable exception is the neorationalist Rob Krier (1938–2023), who dedicated an extensive chapter to the subject of proportions in his treatise *Architectural Composition* [47] and also applied proportional systems in his designs (Figure 5).

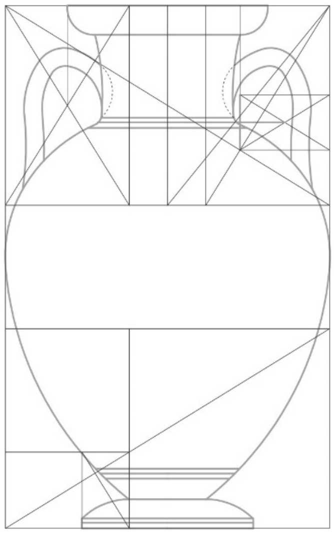

Figure 4. Proportional study of an ancient Greek amphora according to dynamic symmetry on a motif of golden ratio, Jay Hambidge. Adapted and CAD-Enhanced Illustration by the author based on: [27].

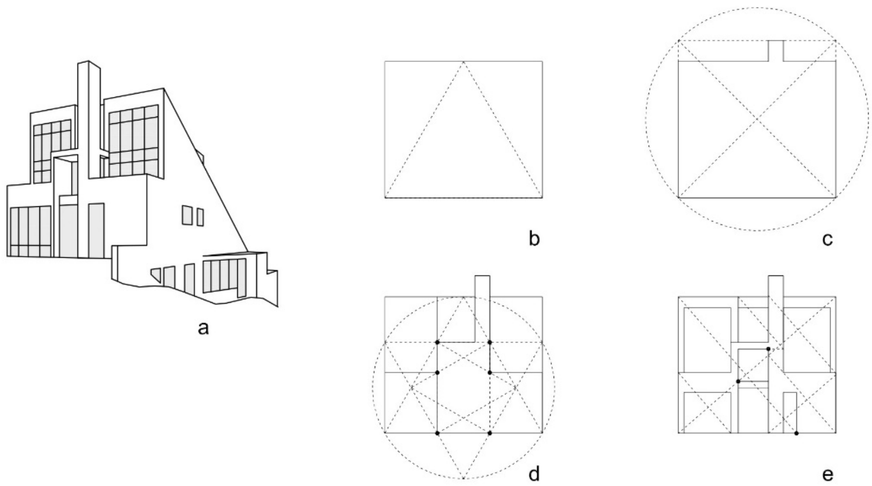

Figure 5. Geometric construction and proportional relationships of the façade of Siemer House, Warmbronn, Germany, 1968. Designed by Rob Krier. In his *Architectural Journal*, Krier presented several geometric interpretations of the façade of his well-known design, not always maintaining precision and geometric correctness. He utilized various principles such as the $\sqrt{3}$ motif (ad triangulum), the $\sqrt{2}$ motif (ad quadratum), or the golden section, which are not mutually consistent. The interpretation presented above is based on Krier's concepts, selecting one non-contradictory path to determine the building's proportions, proceeding from the general to the specific. It should be treated

as an illustration of the design process rather than a precise analysis of the proportions of the actual built structure: (**a**) an overall view of the building; (**b**) determining the proportions of the main façade volume based on an equilateral triangle (a height-to-width ratio of 1:2$\sqrt{3}/3$); (**c**) inscribing the façade, including the chimney, within a square form; (**d**) determining the main divisions of the façade based on a hexagram; and (**e**) establishing detailed points of the façade according to regulating lines. The author's CAD analysis is based on the ideas of Rob Krier.

In recent decades, interest in proportional systems has been gradually increasing. Several textbook-style publications have appeared, presenting the practical application of proportional systems in design [66,67]. Proportional systems are also taught outside the official academic circuit, for example, in traditionalist schools and institutes such as The King's Foundation School of Traditional Arts or the Institute of Classical Architecture & Art. Among architects operating within a non-traditionalist idiom, a notable proponent of the proportional approach is the Swiss architect Peter Märkli.

4. The Modern Proportion-Based Approach in Architecture: An Analysis of Selected Concepts

The upcoming section of this document will focus on the modern proportional approach to architecture, analyzing the theories of seven influential authors. These authors have each developed distinct systems and philosophies that contribute to the broader discourse on the role of proportions in architectural design. The analysis will provide an overview of their individual contributions, highlighting how their study collectively forms a significant current within modern architectural theory.

4.1. August Thiersch and the Principle of Similarity

At the beginning of the 20th century, August Thiersch (1843–1918), a German architect and professor at the Technical University of Munich, engaged with the problem of proportions. Thiersch dedicated a chapter to this issue in his *Handbook of Architecture*, published in 1904 [16] (pp. 37–39). Although Thiersch operated within the orthodox historicism movement, his theory of proportions was innovative. He questioned the value of previous research—such as that of Viollet-le-Duc or Zeising—insofar as it aimed to establish specific proportions like the Egyptian triangle or the golden ratio as an ideal for architecture. Thiersch sought to create a flexible theory that would account for the diverse characters of buildings: "The massive and bold is justified as well as the slender and graceful". [16] (p. 38) Simultaneously, he was one of the first to consider the realities of the perception process. Like many earlier authors, Thiersch challenged the value of musical analogies. These are unjustified because preferences for certain musical intervals result from the physical behavior of the eardrum, which is affected by sound waves of specific lengths. Humans do not possess an analogous visual apparatus that would react to the proportions of perceived objects in the same way. A slight difference in proportions goes unnoticed and is insignificant for aesthetic experiences. Thus, Thiersch joined the ranks of skeptics who, since the times of Perrault and Burke, have questioned the absolute aesthetic value of certain proportions. However, Thiersch also presented a positive theory: it is not the proportion itself that is the source of aesthetic satisfaction, but the repetition of the same proportional motif throughout the structure of the work. The harmony of form is nothing other than the appropriate participation of similar shapes in the architectural composition. This repetition should not be excessive, so as not to lead to monotony, but it should sufficiently bind the composition, creating 'similarity in diversity'. The aesthetic pleasure of applying the principle of similarity would result from a kind of economy of vision. As Thiersch explains:

> "*But another reason for a pleasing effect is the activity of the mind and consists in composing an image of the whole from views at different standpoints. The simpler the relation of the parts to each other, the more readily and willingly does the eye follow the lines, and the more easily is the internal intellectual image constructed*". [16] (p. 89)

Moreover, Thiersch emphasizes that, unlike a specific proportion, the similarity of forms is maintained in perspective foreshortening, in which buildings are usually viewed.

The strength of Thiersch's argumentation lay in his drawings, which illustrated the idea in a simple and suggestive manner. On plans and facades, Thiersch drew diagonals of rectangles representing various elements of the composition: main masses, intercolumniations, windows, portals, and also details such as entablatures, triglyphs, etc. These diagonals—usually parallel or perpendicular to each other—indicated the similarity of figures and reinforced the impression of a conscious design by the builders (Figure 6). Drawing diagonals was not entirely new in architectural theory—they were present in both medieval workshop drawings and treatises by Serlio, Blondel, and others. However, there the focus was on the geometric construction of form, whereas here it was on the visual relations.

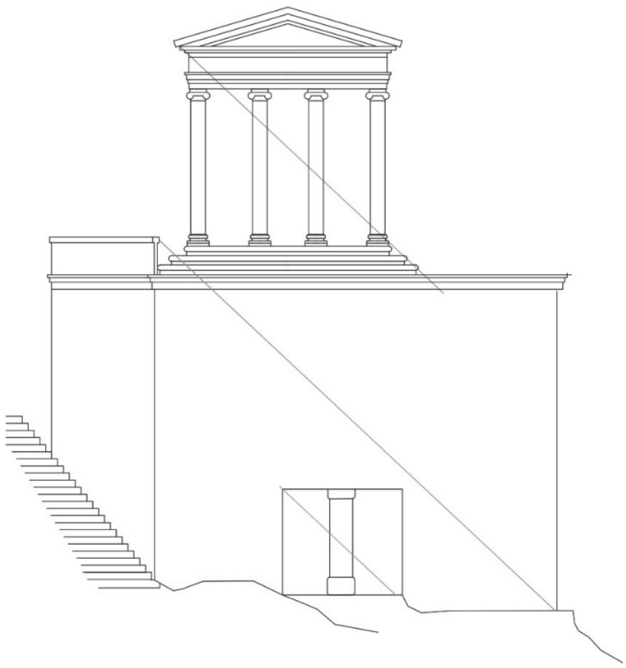

Figure 6. Scheme of proportions of the Temple of Nike on the Acropolis according to A. Thiersch. Adapted and CAD-Enhanced Illustration by the author based on [16].

The principle of similarity was meant to summarize and explain the history of architecture and link it with contemporary times. Thiersch illustrated it with a wide range of historical examples, trying to show that it was a secret of good composition passed down from ancient Egypt to the Renaissance. In a mysterious way, the principle also manifested itself in the studies of famous early 19th-century architects such as Schinkel or Klenze. Moreover, it was supposed to clarify the ambiguities of traditional theory. For example, the theory of similarity provided the correct interpretation of Vitruvius's concept of symmetry, and the method of diagonals was to be what Alberti had in mind when writing about 'lineamenta'. Thiersch also did not fail to refer to nature, whose forms obviously bear the characteristics of self-similarity:

> "*the entire form of the tree reappears in the branch (...) This repetition in plants results from growth, the first delicate twigs increasing to boughs and the germ becoming a complete organism. The completed building may be termed an organism. The whole grows out of a typical form and develops into numerous variations*". [16] (p. 89)

4.2. Berlage: Architecture as Crystallization

One of the most important advocates for proportional systems at the turn of the 19th and 20th centuries was Hendrik Petrus Berlage. Berlage, akin to his contemporaries Auguste Perret and Peter Behrens, held a view that valued geometry and construction logic in equal measure, echoing the ideals of medieval architecture. His views on proportions are to be found in his lecture *Foundations and Development of Architecture*, delivered in Zurich in 1908 [17]. The theme of architecture's mathematical foundations is embedded in a broad context, with a focus on exploring the deep essence of architecture and seeking the determinants of a new, non-eclectic style for the 20th century.

Berlage tried to overcome the baggage of subjectivism and 19th-century aesthetics, which were consumed with developing criteria of taste. For Berlage, the debate over whether something is beautiful or not is a barren one because only the principles that govern artistic creation can be the subject of debate, not individual impressions. The evaluation of a work is rather determined by the organization of the creative process:

> "*you should study how it has been done, in other words, how consistently the forms have been implemented. You should understand the logic with which the elevation has been developed out of the plan and the skill with which the respective building masses have been brought into accord*". [17] (p. 186)

This logic of formal relationships is the main focus of Berlage's interest, which he defines as the "ultimate condition of style".

Berlage adorns his reflections with a motto taken from the 18th-century furniture designer Thomas Sheraton: "Time changes fashion, but what is based on geometry and true knowledge remains unaltered". [17] (p. 186). It is precisely this immutability, timelessness, and stable theoretical foundations that seem to be what Berlage is looking for, which he does not find in the art of the 19th century, which was, in his opinion, superficial, capricious, and devoid of principles. However, true art cannot be arbitrary due to its transcendent goal, which Berlage defines, citing Hegel, as "the embodiment of the highest spiritual matters in our minds". Only such art—directed towards transcendence and subject to general principles—is noble and universal, which is synonymous with the creation of style.

The art of the 19th century was, as Berlage asserts, dominated by easel painting, which set standards for other fields, including architecture. In this way, romantic picturesqueness became a quality sought after in architecture as well, which led to the loss of its true essence: "From the moment architecture entered the sphere of 'fine arts', it was lost". [17] (p. 190).

The second malaise of 19th-century architecture was a misguided approach to the legacy of historic architecture. Berlage was convinced of the failure of 19th-century theory, which, although it had accumulated a vast amount of knowledge about styles, was unable to create a foundation for authentic creativity. The value of old buildings does not consist of ready-to-use forms but of universal laws of architecture, which were the deep foundation of these forms. Academic theory was unable to read these laws or transmit them selectively, for example, in the form of the teaching of columnar orders. Therefore, against the background of contemporary architecture, ancient buildings stand out with a certain essential quality:

> "*one major, might even say principal, characteristic becomes immediately apparent, and this is 'repose'. (...) In contrast, our present-day architecture gives a very restless impression". In another place, Berlage adds: "this repose is a consequence of style, and that style is in turn a consequence of 'order'—of a particular method of design*". [17] (p. 137)

Like many of his contemporaries, Berlage sought justification for his concept of art in nature. Unlike the romantics, he did not stop at the emotional reception of nature but referred to the mathematical laws that constitute its rational foundation. These laws, in combination with the variable specificity of conditions, determine the coherence and richness of natural forms—*unity in diversity*.

The naturalistic analogy allows us to understand the essence of architecture:

> "*Just as nature itself has laid claim to the simplest geometrical and stereotomic figures in creating crystals, so the architects in earlier epochs of style worked in a similar manner. And ultimately, because these figures have an immutable beauty, it is surely fitting to turn anew for instruction to our universal mother. Just as Hegel already asserted (...) architecture means crystallization*". [17] (p. 186)

According to Berlage, the practical methods of this 'crystallization' are abundantly provided by the history of architecture. Referring to researchers such as Dehio, Drah, Dieulafoy, and Viollet-le-Duc, Berlage presents a vision of architecture based on arithmetic or geometric systems of proportion. In his lecture, he describes in detail the geometric methods attributed to Gothic architecture. These are methods that organize form based on grids resulting from the multiplication and division of simple geometric shapes—the square and the triangle—known as quadrature and triangulation (Figure 7). Berlage himself used those methods in his study, following the so-called Egyptian triangle (an isosceles triangle, with a base-to-height ratio of 8:5) derived from Viollet-le-Duc, as illustrated by his project of the Amsterdam Stock Exchange (built in 1896–1903).

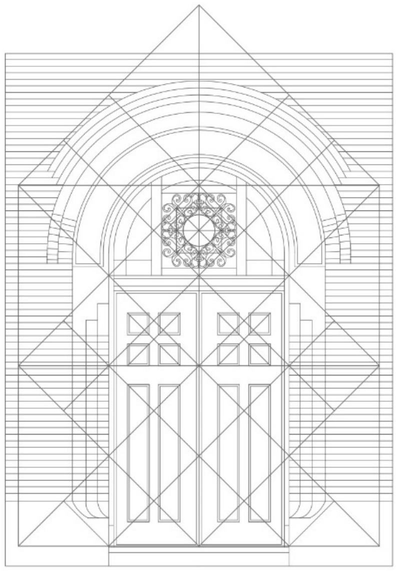

Figure 7. Design of a portal based on quadrature, H.P. Berlage. Adapted and CAD-Enhanced Illustration by the author based on [17].

These methods aim for a traditionally conceived 'symmetry'—linking parts and the whole through a network of related relationships, thereby contributing to order and coherence of form and consequently to the aforementioned 'repose'. Berlage emphasizes that the choice of a particular method is not essential; what is important is its consistent use throughout the project, which could be indicative of Thiersch's influence.

As Berlage emphasizes, geometric systems are not an end in themselves but a tool subordinate to the artistic idea. No rule or method will yield good results if the creativity does not stem from the artist's true love for the work. Here, Berlage seems to invoke the theses of Ruskin and the Arts and Crafts movement with their neo-medieval vision of inspired and engaged creativity. This vision is embodied in Berlage's dream of the reappearance in the 20th century of a 'master builder', engineer, and architect who would once again unite constructional logic and artistic expression based on the eternal laws of geometry [17] (p. 207).

4.3. Le Corbusier: The Sensible Mathematics

Le Corbusier was the most prominent advocate for proportional systems in 20th-century architecture. In his renowned manifesto *Vers une Architecture* from 1923, he popularized the concept of regulating lines and, after World War II, introduced his anthropometric system of measurement known as the Modulor.

The extensive experiences Charles-Édouard Jeanneret gathered before publishing *Vers une Architecture* undoubtedly influenced his interest in geometric proportional systems. The later Le Corbusier practiced under Perret and Behrens, both of whom used geometric methods in their projects, and was familiar with the theories of Berlage and the architecture of Lauweriks [68]. He also moved in the circles of avant-garde Parisian painters, such as those in the Section d'Or group, some of whom composed their paintings on geometric grids inspired by theories from Beuron. Le Corbusier was reluctant to acknowledge these influences, attributing his interests to his own reflections and discoveries. An exception was his reference in *Vers une Architecture* to the work of August Choisy, specifically his reconstruction of the proportions of the ancient Arsenal in Piraeus. Le Corbusier, as noted by Banham, committed an error in asserting that the drawing with regulating lines from Choisy's book is a work of the Greeks. In fact, the marble stele attributed to Philo did not contain a drawing but merely numerical descriptions of the building's proportions [69] (p. 226). The drawing itself is the work of Choisy and was intended to validate his thesis regarding the convergence of geometric and numerical methods among the ancients [10] (pp. 388–391), which for Corbusier was the ultimate confirmation of his earlier intuitions (Figure 8). The myth propagated by Le Corbusier that he was the rediscoverer of proportional systems in the 20th century provoked protests from the Academy and Berlage.

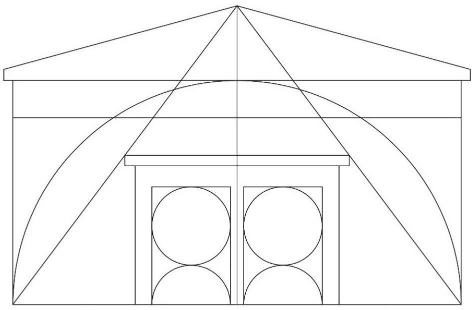

Figure 8. Reconstruction of the proportions of the Arsenal in Piraeus according to August Choisy. Adapted and CAD-Enhanced Illustration by the author based on [10].

Despite the considerable liberty in citing facts and sources, *Vers une Architecture* remains a key work in the modern proportional approach to architecture. The necessity for systematic reflection on proportions stems directly from a purist program for architecture.

This program, while attempting to incorporate standardization and scientific rationalism, is fundamentally formalistic. The technological civilization, to which Le Corbusier devotes much attention, inevitably conditions architecture but does not determine its deep content. If this were the case, architecture would remain a mere construction, an engineering and technical discipline, rather than an artistic field. Facts are evident and must be considered, but their organization takes place in the realm of abstract reasoning—in the world of ideas. Architecture is essentially 'pure'—transcendent to nature and technology—and also free from discursiveness—imitation, hidden meanings, literary descriptions, etc. Its means are plastic—volume, space, the play of light and shadow—and its tools are mathematical—number, and geometry. While engineering mathematics serves rational construction, mathematics in architecture controls proportions, which create 'plastic emotions':

> "*A thought which reveals itself without word or sound, but solely by means of shapes which stand in a certain relationship to one another. These shapes are such that they are clearly revealed in light. The relationships between them have not necessarily any reference to what is practical or descriptive. They are a mathematical creation of your mind. They are the language of Architecture. By the use of raw materials and starting from conditions more or less utilitarian, you have established certain relationships which have aroused my emotions. This is Architecture*". [18] (p. 153)

Mathematical rationality is, for Le Corbusier, a distinctive feature of humans among other organisms and the way humans act to transform the environment to their measure. In *Vers une Architecture*, this is illustrated by anecdotal stories about a *primitive man* building his dwelling in the wilderness or a tribe constructing a temple:

"But in deciding the form of the enclosure, the form of the hut, the situation of the altar and its accessories, he has had by instinct recourse to right angles—axes, the square, the circle. For he could not create anything otherwise which would give him the feeling that he was creating. For all these things—axes, circles, right angles—are geometrical truths, and give results that our eye can measure and recognize; whereas otherwise there would be only chance, irregularity and capriciousness. Geometry is the language of man". [18] (p. 72)

It would be a significant mistake to assume that Le Corbusier's creative approach was characterized by a detached rationalism similar to N.L. Durand's, simply because Le Corbusier valued mathematics. In *Vers Une Architecture*, we are constantly dealing with a 'sensual mathematics' ('mathématique sensible') that the architect intensely experiences during the creative process and which, as a 'plastic emotion', he strives to convey to the recipient of his work. The regulating line serves this purpose—acting as both a safeguard against arbitrariness and a foundation of 'spiritual order' ('d'ordre spirituel'), as well as a search for imaginative, harmonious relationships. The line is not a limitation but one of the possible paths of artistic expression: "The choice of the regulating line is one of the decisive moments of inspiration, it is one of the vital operations of architecture". [18] (p. 75).

Vers une Architecture contains three examples of the application of regulating lines in Jeanneret's own realizations (Figure 9). There is no doubt that Le Corbusier used the method of Thiersch's diagonals here, although the book lacks a reference to the theory of the German author.

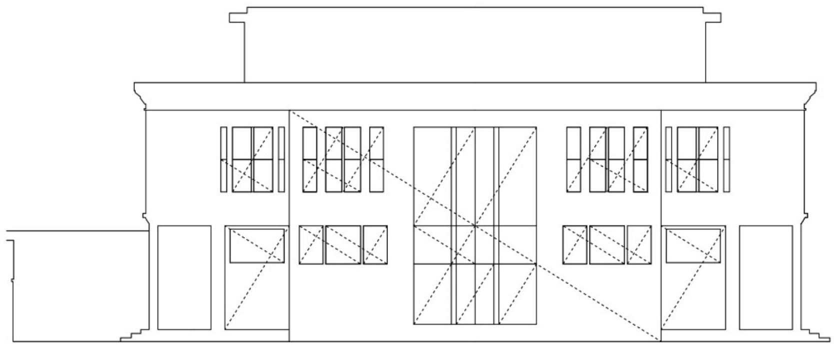

Figure 9. Façade of Villa Schwob in La Chaux-de-Fonds from 1912, with regulating lines applied. Author's CAD analysis of the original drawing by Le Corbusier. Adapted and CAD-Enhanced Illustration by the author based on [18].

The Modulor, developed during World War II, was already a completely original system of Le Corbusier, whose creation in a nearly heroic form is described in a book published in 1949 [60] (pp. 25–68). The Modulor was the realization of the ancient norm that "man is the measure of all things" and the Vitruvian analogy between the human body and architecture. At the same time, it had a practical goal—standardization. It was a kind of utopia, aiming to subject construction, industrial production, and art to one harmonic principle. Unlike the flexible theory of regulating lines, it privileges the golden ratio, thus returning to Zeising's ideas. The Modulor is based on geometric constructions of the golden ratio, that is, the construction of the golden rectangle based on the square and the golden section of the longer side of the double square (Figure 10).

The combination of these two constructions determines the basic matrix into which the human silhouette with its basic dimensions is inscribed: groin, navel, top of the head, and height of the raised hand. Specific absolute values were assigned to individual segments—in the final version of the system, the height of the top of the head—6 feet—about 1.83 cm—was intended to facilitate the use of the Modulor in imperial measurements. Further system measurements are obtained based on subsequent divisions or multiplications according to the golden ratio, resulting in two series of measurements that are expressions of the Fibonacci sequence. The system is both geometric and arithmetic. It is a system of proportions and, at the same time, a catalog of absolute dimensions allowed within the system.

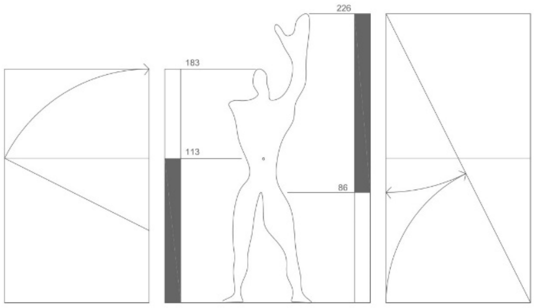

Figure 10. Construction of the basic dimensions of the Modulor. The author's drawing is based on ideas by Le Corbusier [60].

It seems that the significance of the Modulor for the modern proportional approach is ambiguous. Combining the old with the new, attractively illustrated, and passionately promoted, it undoubtedly contributed to the spread of the idea of mathematical order and the Vitruvian body-metaphor of architecture. As Height indicates, referring to the views of Alberto Pérez-Gómez: "The Modulor offered a glimpse into the possibility of recovering architecture's lost metaphysic adapted for modern conditions" [35] (p. 63).

On the other hand, the idea of proportions was long associated with Le Corbusier's prophetic rhetoric and his totalist vision of space—of which the Modulor, as an extremely universalist concept, was direct evidence. In this context, Height observes: "Far from reverting to a Renaissance humanism or recovering a lost tradition, then, the Modulor was to have operated as a distinctly modern and mediated anthropogenic machine". [35] (p. 180).

The dazzling popularity of the Modulor did not translate into its widespread use. Despite the reprinting of images of the Modulor in architectural textbooks, it remained more of a curiosity associated with the extraordinary personality and talent of its author than a widely used tool. For example, the neo modernist architect Richard Meier describes his youthful experiences with the Modulor in this way: "Early on, and for a very short time in fact, I used the Modulor as the dimensioning system, and then threw it away because it doesn't work in every condition. But what it does, it always relates you to human scale, which is important". [70].

4.4. Hans van der Laan—The Order of Size

Hans van der Laan's plastic number system occupies a unique position within the group of modern proportional systems. Its originality is defined by ideological premises that differ from the other theories mentioned here and by a specific design method.

Hans van der Laan (1900–1999) was a Dutch architect and Benedictine monk. He studied at the Delft School of Architecture, where he was a member of the discussion club led by Professor Marinus Jan Granpré Molière (1983–1972), the later long-standing dean of the faculty and a leader of the traditionalist Delft School. Without completing his studies, van der Laan entered a monastery in Oosterhout, where he engaged in activities such as the conservation of liturgical garments and the creation of architectural projects for the monastery's needs. In the years 1946–1973 in s-Hertogenbosch, he conducted a 3-year course on church architecture (Cursus Kerkelijke Architectuur), whose students formed the so-called Bossche School, a movement in post-war Dutch sacred architecture. He designed several sacred buildings in his characteristic ascetic style and published his architectural theories [71].

The uniqueness of van der Laan's approach stems directly from the specific environment in which it was developed and from the influence of scholastic philosophy, particularly the thought of St. Thomas Aquinas. Based on Aristotelian thought, Thomism relied on induction, deriving general concepts from observations of individual things and thus constructing a hierarchy of beings. This hierarchy is present in van der Laan's study. Man is 'thrown' into nature—naked and vulnerable, with nature not directly securing his existence. He needs to supplement nature with the works of his hands—clothing, everyday objects, and buildings. Living in the world has a hierarchical structure: the lowest level is the space associated with physical presence, above which is vision—related to the concept of form that constitutes the space—and finally, the intellectual understanding of the environment. Nature itself is incomprehensible to man, and its complexity is overwhelming. The ultimate goal of architecture is to create a space with a clear structure that allows for the reading of dimensional relations [20] (pp. 1–19).

According to Caroline Vogt, van der Laan's conviction about the importance of measure and rhythm in human life reflected ideas contained in the Rule of St. Benedict: the daily repeated pattern of the day and the strict division of time into periods of prayer and work. Van der Laan's ambition was to create a universal system or scale that would allow for the creation of such order in the visual structure of space. As Caroline Vogt points out, he wanted to achieve in architecture what the monks of Solesmes had achieved in music at the end of the 19th century with the renewed notation of Gregorian chant [72].

The starting point for constructing the system is the observation of how measures are perceived in the environment. This perception has its limits. When comparing two sizes, we will have difficulty distinguishing them if the difference is too small. If, however, it is too large, the two sizes cannot be related to each other and belong to different scale ranges. Hans van der Laan defined these limits as specific proportional relationships: 4/3 and 7:1 (respectively, the smallest and largest possible ratio of the larger measure to the smaller). These relationships, as van der Laan emphasized, are not abstract but empirical. To confirm them, van der Laan conducted experiments related to recognizing size differences in objects, constructing special sets of templates for this purpose. At the same time, he tried to give these observations a more precise formula. The rational number 4/3 (1.333. . .) is an approximation of the real number ρ, which is the solution to the equation $x + 1 = x^3$ (1.3247. . .). This number is the limit of the ratios of a sequence in which the sum of two consecutive elements is equal to the fourth element (1, 1, 1, 2, 2, 3, 4, 5, 7, 9. . .). Van der Laan was one of the first to study the properties of this number, which he called the plastic number.

'Plasticity' consists of specific properties: if we take a segment and divide it into two smaller segments so that the ratio of the larger to the smaller is ρ, and then divide the larger of these segments in the same way, we will obtain a sequence of 6 lengths increasing according to the progression ρ. If, for comparison, we perform a similar operation using the golden section, we will not obtain such a sequence because some lengths will be repeated. (Figure 11). In this property, Hans van der Laan saw the advantage of the plastic number over the golden section and established it as the basis of his system. These two discoveries—the limits of perception and the properties of the plastic number—form the basis for developing a scale of 8 measures based on the progression ρ (1, ρ, ρ^2, ρ^3, ρ^4, ρ^5, ρ^6, ρ^7), recorded for convenience in the form of rational numbers that are their approximations: 1, 4/3, 7/4, 7/3, 3, 4, 16/3, 7 [20] (pp. 70–98).

They constitute an 'order of size', the largest element of which is the smallest element of the next class, etc. (Figure 12). The greatest difference between the elements of a given scale, constituting a spatial unit (the so-called Cella), for example, the thickness of a wall and the size of the Cella, is 1:7. Larger wholes are created by making a leap to the next class—for example, a *courtyard* formed by grouping cells, where the same measurement principles apply [20] (pp. 20–31).

The information provided offers a foundational understanding of Van Der Laan's system, which is rich in detail. It includes the generation of derived dimensions, the superpositioning of various measures, and the creation and coordination of architectural rhythms. The exposition of these principles in Van der Laan's treatise *Architectonic Space* is relatively challenging to comprehend, which is why the studies by Caroline Voet [72], are very helpful in this context.

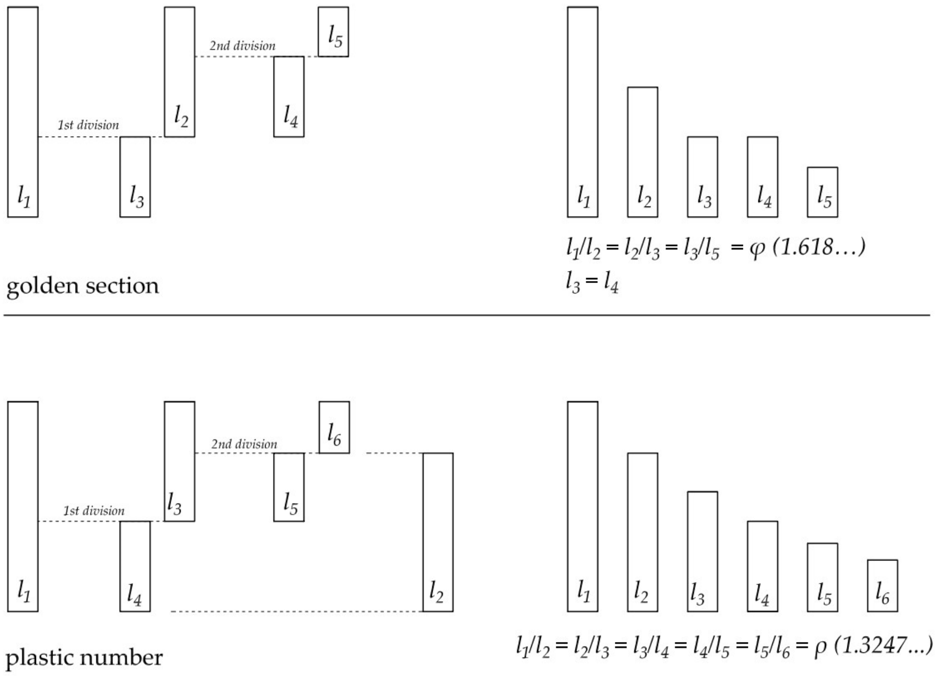

Figure 11. Comparison of the double division of a segment according to the golden ratio (at the top) and according to the plastic number (at the bottom). The drawing is carried out by the author.

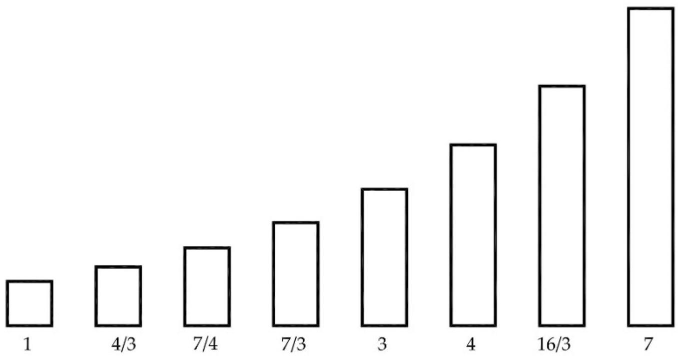

Figure 12. Order of size according to H. van der Laan. The drawing is carried out by the author.

Van der Laan's system was mainly applied in church projects realized in the Netherlands in the 1950s to 1990s by van der Laan's students and in several of his own realizations, characterized by a raw style and monumental dignity. Van der Laan also used it for furniture designs, liturgical garments, and church utensils, as well as to create his own typeface. Van der Laan's realizations, although spectacular, remain relatively unknown, and the influence of his system outside the Bossche School was small. Although it was developed before Le Corbusier's Modulor, translations from Dutch into other languages took place only in the 1980s, at a time when interest in proportional systems had waned.

4.5. Juliusz Żórawski—The Limited Complexity

Juliusz Żórawski (1898–1967), a Polish architect and architectural theorist, made a significant contribution to the discourse on architectural proportions. Active as a designer in the 1930s, he worked in the spirit of Le Corbusier. In 1943, during the German occupation, he defended his doctoral thesis at the clandestine Warsaw University of Technology under the supervision of the renowned philosopher and aesthetician Władysław Tatarkiewicz.

This thesis later became the foundation of his most important work, *O budowie formy architektonicznej* (On the Structure of Architectural Form), which was further developed in his posthumously published study, *Siatka prostych* (Network of Lines). Given the unavailability of English translations, the following section includes a larger number of direct quotations from Żórawski's works.

Żórawski was one of the first theorists to approach architectural theory from the perspective established by Gestalt theory. Although the issue of proportion was not the main thread of his theory of form, it is strongly present in his publications. Żórawski studied architecture from an intersubjective, or, as he called it, a supra-individual perspective. He understood architectural form as a psychological category—an image that takes shape in the human mind due to certain objective features of the object. He examined the factors that allow form to materialize, becoming a whole—a figure (Gestalt):

> "*Formation does not consist of adding, summing up, agglomerating, in short, gathering parts, but in shaping, that is, connecting, grouping, and arranging according to the discipline that constitutes the main value of the 'mother form', i.e., it takes place through categorical processes of wholeness proceeding according to the principles and laws governing the form as a phenomenon that illustrates, explains, and clarifies the content*". [21] (p. 40)

The form, for Żórawski, is a synthesis of parts that occurs in the mind. This synthesis can have varying degrees of intensity: the form can impose itself unequivocally (a strong form—square, circle, etc.), or it can be ambiguous—weak. It can permanently bind its constituent parts—a cohesive form—or it can be a loose grouping of them—a free form.

Żórawski's main thesis states that human perception naturally tends to move from a free form to a cohesive one. An example of this is the observation of the night sky:

> "*We observe triangles and irregular quadrilaterals formed by stars of similar brightness and it seems that they are connected by some ideally straight lines. We feel the urge to position the central star in Orion to obtain a geometric straight line. We compare the constellation of the Big Dipper to a wagon with a broken shaft... During all these observations, we are looking for form*". [21] (p. 26)

Among the many threads of Żórawski's theory of form, the geometrization, or more precisely, the motif of the straight line, plays a fundamental role. The straight line is one of the basic factors in building the cohesion of the form. In a form, there are lines visible as edges and outlines, but also implied lines that exist only as a correspondence between *formally important points*. These points "appear in the mother form where lines belonging to the parts of the form meet, intersect, or end". The clarity of the straight line fixes the position of the points and bonds the composition, while the lack of such bonding causes instability—the floating of elements. In this way, Żórawski interprets Alberti's theory of 'lineamenta' and Renaissance schemes of regulating lines, such as the famous drawing of Serlio's portal. (Figure 13). The function of these schemes is not to establish specific proportions but to situate formally important points on straight lines, thus increasing the cohesion of the form and its stability and legibility.

In his later studies, Żórawski gave even greater significance to straight lines. Visual perception is generalized here as the projection onto the plane of vision of a 'network of lines' drawn between corresponding points. The perception of a whole visually detached from the environment—a so-called pleiad of points—depends on the visual relationships of these points, that is, either the clarity of their overall arrangement or the common qualities of their individual parts. As Żórawski writes:

> "*Proportions, which play a decisive role in visual communication in architecture and cause the emergence of aesthetic feelings in the viewer, always play out between points that are in correspondence due to their position on a single straight line*". [22] (p. 31)

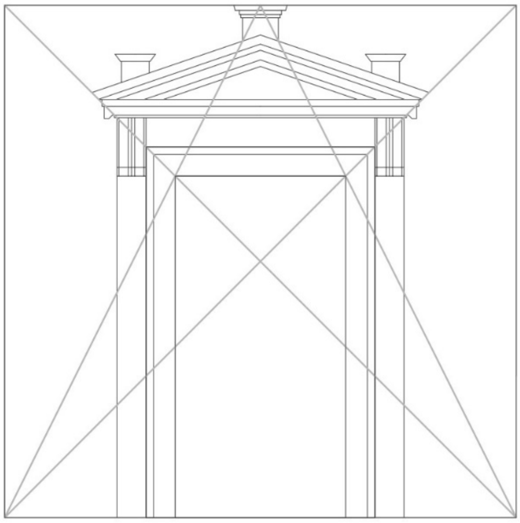

Figure 13. Drawing of a portal from Sebastian Serlio's treatise. Adapted and CAD-Enhanced Illustration by the author based on [73].

It should be emphasized that Żórawski firmly distanced himself from most earlier theories of proportion, especially those understood as recipes for beauty. He criticized, in particular, the belief in the absolute aesthetic value of specific proportions:

> "*If people who were engaged in the search for beauty in constant and unchanging distances between corresponding points in a network of lines did not manage to find these distances, then by their activity they proved that such universality does not exist at all. Today we can boldly state that the proportions, relationships, and dependencies between corresponding points that cause the appearance of aesthetic feelings cannot be constant*". [22] (p. 34)

Żórawski was also skeptical of Thiersch's concept, writing that

> "*if the application of such a rule guaranteed the beauty of a work, then the 'stone of beauty' would have been found. However, we know that this is not the case. Not all works in which the main compositional layout is repeated in the components are beautiful*". [21] (p. 148)

Żórawski understood beauty in line with the spirit of his time—subjectively and relatively. It was accessible only in individual feelings, dependent on the context of a given form, the perceiving subject, and their internal state field:

> "*Beauty is not synonymous with the strength of the form or its cohesion... Adhering to the natural inclination given to us by nature for straight lines... only results in legibility and facilitates orientation. However, it may hinder the path to beauty, imposing bonds and constraints, building a scaffolding that is rationally sound but emotionally neutral*". [22] (p. 92)

This does not mean arbitrariness in determining the principles of shaping architectural form. For Żórawski, the specificity of architecture, as opposed to other arts, is the "synthesis of a large number of impressions". When designing architecture, one cannot rely on a single gesture or ephemeral feeling: "Impressionist architecture is not among the successful ones". [21] (p. 144). Architecture has supra-individual features, that is, it defines form in a way that is equivalent for as large a group of recipients as possible. It is therefore inherently a social art, and its aim is to increase the visual communicativeness of the environment. Architectural thinking is the creation of a limited complexity of pleiads within the infinite complexity of the environment. Therefore, cohesion—built also through regulating lines—is a default guideline for architecture:

> "*No detail should 'float' unconnected, unconditioned formally, if such its 'suspension in uncertainty' is not the result of compositional actions*". [21] (p. 90)

4.6. Jonathan Hale—The Way of Seeing

Jonathan Hale's book, *The Old Way of Seeing*, addresses the issue of proportions in architecture in a fundamentally innovative manner. Written in 1994, it is significantly later than the studies discussed so far. The backdrop for the issues raised in the book is the popular postmodern architecture of the United States—houses, office buildings, suburban shopping centers, etc.—mass-produced buildings of low aesthetic value that superficially referenced tradition or modernity, creating bland and impersonal spaces. Hale contrasts these with older American architecture from before 1830 and, further in his considerations, also with the studies of modernist masters such as Frank Lloyd Wright, Le Corbusier, Mies van der Rohe, and Walter Gropius.

Hale focuses on the emotional reception of buildings and, in particular, on the problem of the 'presence' of architecture in environmental perception, which can be described as the intensity or integrity of form impacting the observer: "The old buildings smiled, while our new buildings are faceless" [23]. According to Hale, 'presence' is not synonymous with originality or distinctiveness from the surroundings but is related to the strength of the visual pattern defined by relationships such as contrasts, tensions, balance, and proportions between elements. The perception of architecture, especially facades, as pattern of light and shadow is, according to Hale, a characteristic of a properly developed creative intuition that he calls 'the old way of seeing'. This is a direct vision in which style, the meanings of forms, symbolism, connotations, and denotations are pushed into the background—what remains is a pure, two-dimensional pattern and its effect. Hale asserted that Enlightenment rationalism and industrial pragmatism in the early 19th century irreversibly deprived culture of this intuition, leading to a situation where the formal aspect of architecture became a game of discourses:

> "*The incomplete, unrecognizable patterns, up and down every street, become a kind of noise and the bubble of symbolic messages*". [23] (p. 22)

One of Hale's key theses is crucial in the context of the discussed issues: thinking and creating in terms of patterns implies a proportional system, a geometric principle that orders the arrangement of elements. This principle may be unconscious, but it is discernible through analysis. Hale's book includes examples of analyses of traditional New England house facades, where the form is organized according to regulating lines and figures (Figure 14). The question of whether this geometry was the result of conscious construction is secondary:

> "*One wants to know, were these hidden lines ever meant to be seen? Is it a deliberate geometric message? What is beyond doubt is the pattern itself, the pleasure of organization, color, texture—it is very much like a piece of music*". [23] (p. 15)

The examples presented by Hale are difficult to consider as convincing scientific evidence because Hale's geometric methodology is notably liberal, making his thesis hard to falsify. Similar to many other theories discussed in this article, it should be recognized rather as a manifestation of a particular artistic sensitivity and a way of perceiving space in architectural terms. Hale's prioritization of visual structure over meaning structure leads him to interesting conclusions about contemporary architecture, which contrast with the most established schemas, such as the distinction between modernist and traditional architecture. According to Hale, the merits of visual form have nothing to do with which idiom a given building belongs to. For example, they are present in the studies of modernist masters but are rarely found in the studies of their numerous imitators:

> "*In the hands of masters, the International Style did achieve the old spirit, but these designers were specialists in seeing buildings as compositions in light and shade... Modernism was an elite style. Only an artist could make it work... The simplicity of the International Style made it much easier to express elemental form. But the designer who used it walked a knife edge. The style offered true greatness to those who could stay on that edge, but on each side was an ice-cold abyss*". [23] (p. 130)

Similarly, Hale refers to the achievements of architectural traditionalism, particularly the New Urbanism popular in the United States. Urbanists like A. Duany and E. Plater-Zyberk have attempted to recreate the spirit of traditional American towns in new developments through detailed codes specifying the character of the planned buildings. However, in Hale's view, such ensembles—like the well-known Kentlands community (begun ca. 1988)—still have a hint of falsification.

Hale suggests that a design can come alive within the constraints of strict rules, but only if the designer has a larger goal, which is to imbue the design with grace:

> "*There were restrictions of many kinds in the days when buildings came alive. But no code required the old way of seeing. If we could devise such a code, it might say something like this: The process of design shall be a play. The designer shall experience great pleasure in the work, or the design shall be deemed to have failed. A rich geometric pattern shall underlie the design. The designer shall not be aware of how this pattern was arrived at*". [23] (pp. 116–117)

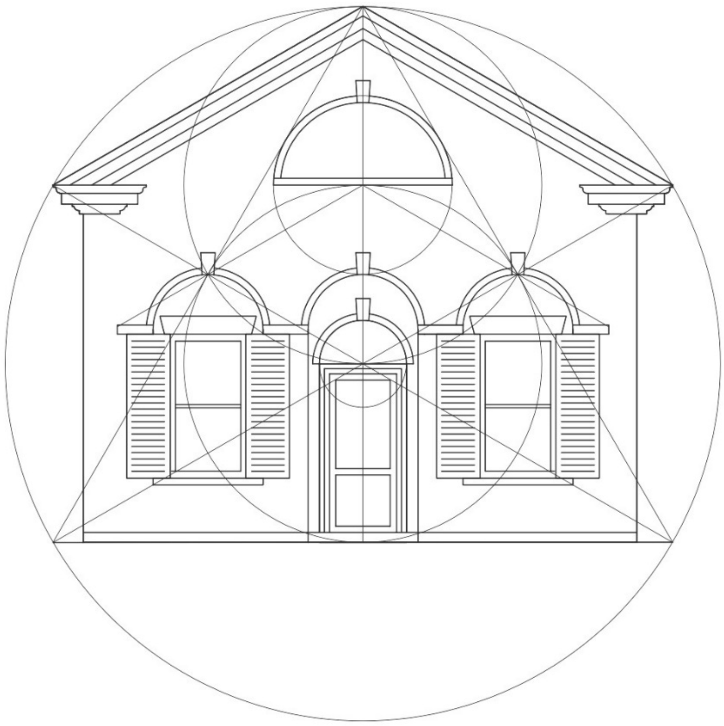

Figure 14. Façade of the little office building in Chelsea, Vermont, ca. 1825. The author's CAD geometric analysis is based on the photo by Jonathan Hale [23].

Hale's seemingly sentimental or poetic approach places him among other American authors such as Robert Pirsig and Christopher Alexander, who challenge the traditional epistemological division between objective facts and subjective feelings. In this anti-Cartesian worldview, 'quality' is a pivotal concept that transcends individual evaluation or taste, constituting a real category accessible through direct experience, facilitated by properly oriented visual perception and rational analysis.

4.7. Peter Märkli—Indesctructible Solidity

Peter Märkli, a Swiss architect born in 1953, is one of the few contemporary designers recognized for using proportional systems in his study. Märkli studied at ETH Zurich and has run his own practice since 1978, completing projects of various scales. From 2002 to 2015, he was a professor at ETH Zurich, and in 2017, he was awarded the Swiss Grand Award for Art.

Märkli's example, though isolated, is significant due to his specific design philosophy and work ethic, which provide a context for the idea of proportional systems. His creative stance contrasts with the dominant practices of the contemporary profession. While studying architecture in the 1970s, Märkli encountered an educational environment "*dominated by strong sociological tendencies, coupled with an almost institutionalized modernism that was heavily influenced by Le Corbusier but lacked the life of his buildings*" [44] (p. 8). Seeking more inspiring models, Märkli came into contact with the renowned Swiss architect Rudolf Olgiati (1910–1995), whose study presented an original synthesis of modernism, Swiss vernacular, and classical architecture. The architecture of the houses Olgiati designed in the mountainous landscape was stylistically distinct from conventional post-war modernism and avoided the narrativity of emerging postmodernism. It operated with the textures of raw materials and visual tensions, especially the contrast between the mass of the wall and its openings. Olgiati believed that form affects observers "independent of the vagaries of fashion or time" [44] (p. 11), and he described his approach as 'optical objectivism'. The second mentor and later collaborator of Märkli was the Swiss sculptor Hans Josephsohn (1920–2012), whose art—like Olgiati's—explored timeless sculptural themes and emphasized the materiality of the study. All of this influenced the formation of the basic framework of Peter Märkli's creative worldview. He perceives architecture in terms of temporal unity. His private studies included Greek buildings from the archaic period, Romanesque architecture, Sicilian Baroque, and the architecture of Le Corbusier and Mies van der Rohe:

> "*Contrary to the school's teaching, I saw classical modernism as the most recent chapter of history. . . If I had to describe the architecture that inspires me, it would be a combination of the legacy of classical modernism and the traditional principles of architecture*" [44] (p. 48)

As the author of the monograph on Märkli's work, M. Mostafavi, writes: "Märkli's interest in painting and sculpture, coupled with his long associations with Hans Josephsohn and Rudolf Olgiati, have instilled in him an acute visual and optical exactitude, perceptive of small differences" [44] (p. 18). This sharpening of attention to the visual aspects of form—reminiscent of Hale's old way of seeing or Le Corbusier's plastic emotion—is practiced by Märkli throughout his career through drawings. These are often detached from actual design tasks, gestalts—miniature sketches balancing between pure abstraction and quasi-architectural arrangements, whose main theme is the tension between verticals and horizontals, masses and voids, and color contrasts [44] (pp. 20–48).

Commenting on contemporary paradigms in architecture, Märkli distinguishes two basic models: the first emphasizing the universalism of perceptual processes and the second, discursive, which assumes that a particular way of perceiving is the result of social convention: "*The first point assumes there are laws governing human behaviour that correspond with the laws of perception. These laws are open to interpretation, it is true; but they stand above the individual or the collective will. Among other things this assumption is supported by the typological similarities between products of different cultures and times (. . .) But opposing this system there is another conceptual model of the collective definition of architecture: the 'discursive' model, in which the collective rules are determined by conventions, as in language*". [44] (p. 49). Märkli advocates for the first model: "*The second argument, that language influences perception and vice versa, seems to be beyond question, but is a secondary, less important influence*". [44] (p. 49).

The emphasis on formal relationships, combined with an ahistorical view of architecture, leads directly to proportional systems. Märkli became interested in proportions during his studies when, while working on course projects, he encountered the problem of how to determine the dimensions of a plan based on a sketch: "*My story began during my first semester at the ETH, when we had to create a sketch and then turn this into a plan. I didn't know how to draw a plan, because I didn't know where to put the line on the paper in relation to the scale on the ruler. We were not provided with the answer to that question at the time. Or to me, at least, it was not self-explanatory*" [24] (p. 118). A hand-drawn, preliminary sketch is expressive and contains—using Le Corbusier's term—a 'plastic emotion' that the artist's hand transfers to paper. It is a search for beauty that is expressed in the relationships and tensions between elements. However, the sketch is inaccurate and unstable in its proportions, while a detailed plan and the final building require precise dimensions. There is a lack of an intermediary element—a tool that allows for controlling the tensions between individual metric dimensions to preserve the original idea's values:

> "*In order for a sketch to have stability, the dimensions must be clear. Since the architect does not construct a building by hand, this is the only way to achieve the intended impression. Orthogonal lines must recreate the tension of the sketch in drawings. That is our task. Without proportions, I would be completely lost... So to me, dimensions and their proportions are at the top of the hierarchy for conceiving a design, just after the idea itself. Not the material... Proportional relations give a building an indestructible solidity*". [24] (p. 121)

Märkli is the author of his own original proportional system, which is, however, a synthesis of many older proportional principles that Märkli intensively studied in his youth. The starting point is a modification of Leonardo da Vinci's famous drawing of Vitruvian Man—a circle placed on a square so that the circle touches the center of the square's lower base and intersects its two upper vertices. The center of such a circle is determined by the intersection of a vertical line passing through the center of the square and a line perpendicular to the diagonal of half the square passing through its center. This diagonal line divides the sides of the square in a 3:5 ratio and a 1:7 ratio, which corresponds approximately to the golden section and the proportions of an equilateral triangle (the ratio of the base to the height is about 8:7) (Figure 15). The result is a kind of geometric matrix in which specific characteristic proportions can be easily determined and individual sizes can be assigned fractional values in relation to the base dimension. This is usually the side of the square, which is typically taken as one of the largest dimensions of the building. The basic division is into eighths, and then sixteenths, thirty-seconds, etc., can be introduced. This is an arithmetic and geometric system resulting from the construction of figures and oblique regulating lines. The system also has the characteristics of a modular grid (Figure 16).

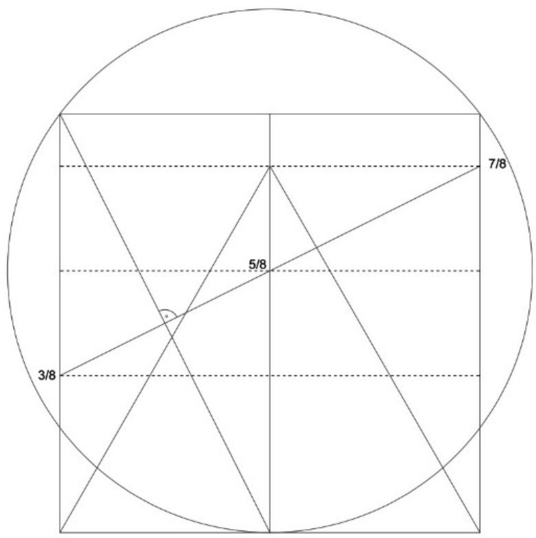

Figure 15. Scheme of Peter Märkli's proportional system. The drawing is carried out by the author.

The calculations of the building's proportions are a kind of predefined structure based on which a detailed building design is developed. Märkli's sketches and proportional calculations are performed by him in an individual studio, often in a long and laborious process, only later passing the material on to collaborators [44] (p. 8).

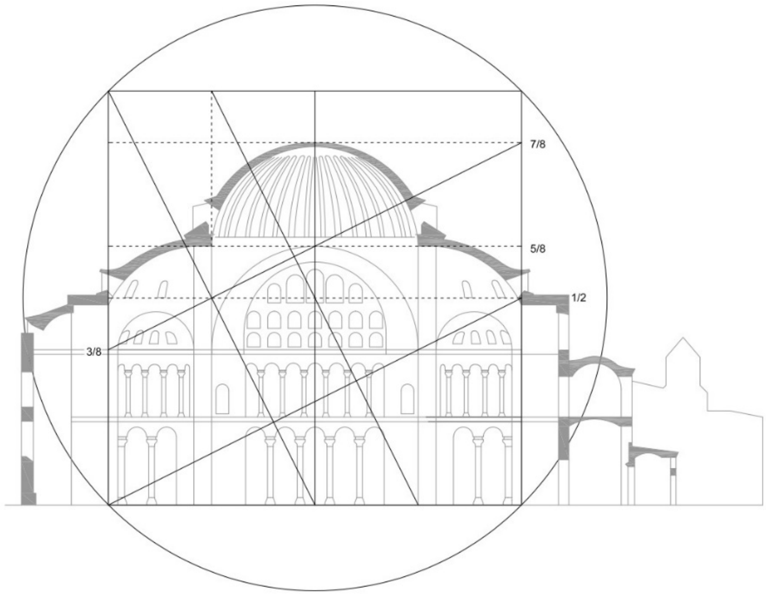

Figure 16. Proportional analysis of Hagia Sophia. CAD geometrical analysis is performed by the author based on the hand-drawn sketch by Peter Märkli [24].

Märkli's example is symptomatic of today's proportional approach. His views and methods were shaped as a result of a combination of personal experiences and searches outside the system of architectural education and detached from the broader professional environment. It raises the question of how compatible the realities of today's dominant design practice are with a proportional approach or, more broadly, with any form of 'optical objectivism'. The latter seems to imply a 'meditative' method of work and deep reflection on form. The architect works in a way similar to that of a plastic artist, and the individual experience of beauty, which requires time, silence, and engagement, is an indispensable part of the process. These are all characteristics of an authorial design practice. The model of a corporate design office, focused on efficiency and collectivity, seems to eliminate these elements from the profession.

5. Results: Modern Proportion-Based Approach—An Attempt at Characterization

As a result of the conducted research, we obtain a clear picture of the modern proportional approach in architecture. The analysis confirms the validity of the fundamental hypotheses and allows for additional observations regarding the characteristics of this trend. The key findings are as follows:

5.1. General Remarks

The modern proportional approach is a current within architectural theory that has been developing since the late 19th century, reaching its peak in the mid-20th century, and persisting in a limited form to this day. Its defining characteristic is the focus on the significance of proportional relationships within architectural works and the endeavor to establish methods for their regulation. The dissemination of proportional theories coincided with the advent of the modern movement in architecture and influenced its development. However, the significance of this trend extends historically and thematically beyond modernism.

5.2. Distinctiveness from Classical Theories

The modern proportional approach is clearly related to ancient proportional theories but is a distinct phenomenon in itself. This stems from the discontinuity in the development of proportional concepts in the 18th century. The emerging trend in the second half of the 19th century was no longer a simple continuation of ancient theories but was based on different assumptions. The architectural analogy to music, central to Renaissance architecture, was abandoned in favor of primarily geometric methods, which were contemporary reconstructions of ancient systems.

5.3. Versatile Understanding of Beauty

The theorists of this approach depart from the traditional conviction that the application of a simple arithmetic ratio or a regular geometric figure inherently constitutes beauty. They also eschew the cosmological symbolism, where such relationships were thought to mirror the structure of the universe. Beauty, it appears, remains the intention of the authors, but its contemporary definitions, marked by relativity and subjectivity, are considered. Consequently, more objective categories are sought, offering a rationale for spatial order and preventing the arbitrariness of forms. Beauty can thus be understood as an abstract mathematical structure of form, as a personal visual impression, or as an individual experience of the artist, who aims to communicate it to others through the judicious choice of proportions.

5.4. Idea of Logical Coherence of Form

The logical coherence, which Berlage termed crystallization, is considered a value in itself. It is independent of individual taste and fleeting aesthetic impressions. It is related to the discipline of the creative process and the creation of form in the artist's mind. As Le Corbusier's 'learned game of forms' or Peter Märkli's 'indestructible solidity', it is a testament to artistic skill and clarity of idea.

5.5. Consideration of Visual Perception

Perceptual categories concern the form as an image experienced in the process of vision. They define the legibility of this image, the ease of its distinction from the environment, its stability, and its intensity. These categories are not absolute, but due to the relatively stable characteristics of vision, they apply to the majority of observers and remain closely related to the geometric structure of the form. These intersubjective principles include Thiersch's similarity in diversity, Żórawski's cohesion and limited complexity, Van der Laan's concept of measure, Berlage's repose, and Hale's concept of aliveness.

5.6. Proportions as Tools of Expression

In the theories discussed, proportions serve as instrumental elements, integral to the broader artistic vision, as Berlage underscored. For Le Corbusier and Märkli, they are vehicles for expressing plastic emotion, while for Van der Laan, they are mechanisms for managing visual tensions. Proportional systems, therefore, are not abstract impositions but rather a means for architects to articulate their aesthetic experiences within the architectural form, enabling the communication of individually perceived beauty to others.

5.7. Nature and Reason—Two Sources of Proportions

For Berlage and Thiersch—shaped by 19th-century positivism—geometric order in architecture is a continuation of the laws shaping the forms of living and non-living nature. This is the legacy of Zeising, later supported by authors such as Matila Ghyka. Le Corbusier is less decisive on this issue. In *Vers une Architecture*, he outlines a broad analogy between the sensation of architecture, rationally shaped form, and cosmic order, thereby inscribing himself into the Pythagorean-Platonic worldview. At the same time, he presents geometry as a specifically human way of thinking and organizing the environment, through which man marks his distinctiveness in nature. Van der Laan is even more radical in this respect: nature is incomprehensible to man, its laws are illegible. Man must introduce his own mathematical-architectural order to understand space and inhabit it.

5.8. Ahistoricity of the Approach

Although the modern proportional approach revises classical aesthetic views, it seeks continuity with ancient architecture and aims to establish timeless principles. It wants to bridge the gap created by the 18th-century aesthetic shift. Architecture, as a discipline, constitutes a spatiotemporal unity. In the proportional approach we are experiencing, in Banham's terms, a 'praesens historicum'—a moment where history feels immediate and tangible, as if it were unfolding in the present. For example, 19th-century studies on proportions were ambiguous in their purpose—they served to understand historical truth but also discovered systems that could be applied in design. Rejecting historicism, Berlage sought support for the same principles that he saw in Gothic cathedrals. Le Corbusier, negating all formal characteristics of ancient architecture with equal force, advocated for historical—in his concept—systems of proportions. Even completely innovative systems had to obtain some form of confirmation in the distant past: the dimensions of the Modulor mysteriously found themselves in ruined medieval abbeys [60] (p. 192), and the measures of the plastic number in Stonehenge [20] (p. 185–204).

5.9. Formal Universalism

The proportional approach does not limit itself to the postulate of applying systems but carries a deeper concept, touching on the essence of architecture as a creative discipline. This concept, most succinctly expressed by Le Corbusier, presents architecture as a mental construct organizing empirical facts into a form—a plastic idea. Form does not actualize without matter; function, technology, economy, and culture are condicio sine qua non for architecture, yet architecture transcends these categories and establishes a certain autonomous order in the sphere of 'pure' plastic relationships. These relationships, which are mathematical in nature, can be studied and controlled using specific geometric and numerical methods, which, however, remain in harmony with the artistic sensitivity and creative intuition of the architect.

5.10. Diversity of Methods

The proportional approach abounds in various practical methods. These include 19th-century reconstructions of medieval constructive geometry or original systems with varying degrees of formalization (e.g., Thiersch's free method and Hambidge's formalized system). Some are geometric (e.g., Berlage, Thiersch, and Le Corbusier's regulating lines), others purely arithmetic (Van der Laan), and still others combine both features (e.g., the Modulor and Peter Märkli's system). It can be generally stated that the modern proportional approach does not prefer any specific proportions or systems, treating them as tools serving the general idea of formal order.

6. Discussion

The present analysis, though limited in scope, allows for a reflection on some previous attempts to summarize the topic of proportions in architecture. The modern proportional approach is a complex and multifaceted phenomenon, making it susceptible to overly simplified interpretations.

It is critical to evaluate any attempts to identify the ideal or most 'advanced' theory or system of proportion. It is difficult today to point to a specific proportional method that has gained exceptionally wide recognition at the expense of others. This mistake was made in the mid-20th century by Matila Ghyka, who criticized the theories of Viollet-le-Duc or Dehio as 'out-of-date' and praised the systems of Hambidge or Moessel as scientifically credible [30] (p. 124). Similarly, Scholfield believed that the characteristic of the modern approach to the issue of proportions was the rejection of arithmetic systems and static geometric systems based on simple figures to be replaced by Hambidge's system based on incommensurable ratios. [33] (pp. 82–125). Despite its merits, Hambidge's system did not dominate the debate on proportions. It is also difficult to find convincing evidence of its application in 20th-century architectural design. The modern proportional approach is pluralistic by nature, favoring the general idea of proportional order over specific mathematical methods and ratios.

One should also treat with caution approaches under the banner of 'sacred geometry' that attribute inherent symbolic or even metaphysical meanings to systems of proportion. A classic of these theories was Matila Ghyka, with contemporary continuators such as Keith Critchlow and Steve Bass [49,50]. These theories are incompatible with the modern proportion-based approach due to their deliberate anachronism. They seek to resurrect the pre-scientific Pythagorean worldview, where cosmological symbolism is seen as the core function of proportional systems. In contrast, modern proportion theory, while acknowledging historical and philosophical influences as seen in Le Corbusier's work, eschews esoteric interpretations. It emphasizes logical coherence and perceptual processes, distancing itself from literal or mystical numerology as propagated by proponents of 'sacred geometry'.

At the opposite pole from these approaches is the skeptical stance, represented today by, among others, M.A. Cohen. In this view, systems of proportion are a historical phenomenon with symbolic significance important in the context of pre-scientific civilization, now completely anachronistic. Their objective visual impact is impossible to empirically verify. As Cohen writes:

> "*You cannot distinguish between a root 2 rectangle and a slightly stretched one (. . .) and in any case, why should a geometrical figure for which we have a name be more visually pleasant than figures for which we have no names?*" [48]

These views are nothing new and fit into a long tradition of modern skepticism. It should be emphasized that the modern proportional approach is in a post-skeptical direction. It does not ignore modern doubts; on the contrary, as already stated, it tries to respond to them and their consequences—the arbitrariness and inconsistency of architectural forms. It must be noted, of course, that the views of the authors regarding the visual impact of systems do not have the status of scientifically proven truths and remain hypotheses. Architecture is not experimental psychology but a creative discipline in which intellectual speculations are inextricably linked to artistic intuition and mutually verified. The proportional approach should therefore be considered more in terms of a creative paradigm than a scientific theory of form perception.

The statements of proportion theorists shed light primarily on the philosophical foundations of systems of proportion. It is difficult to fully agree with Rayner Banham's thesis that these systems—and the turn towards geometrization in the 20th century in general—were an attempt to find a language of forms appropriate for 'the first machine age'. According to Banham, this would result from the conviction that only the most universal, i.e., Platonic forms, correspond to the spirit of industrial mass production [69] (p. 328). These themes certainly appear in the doctrine of purism or Werkbund, and the 'instrumental' theory of proportion might have seemed valid when Banham wrote his book. However, the continued use of the proportional approach beyond the modern movement seems to contradict this. The significance of geometry and proportion is far more universal, representing the fundamental language of architecture. Likewise, the need for their application appears universal, serving as an antidote to randomness and arbitrariness and providing 'indestructible solidity'.

Richard Padovan's much later concept seems more accurate, which used the scheme proposed by W. Worringer in 1907, according to which human artistic intentions can be explained using the polar categories of 'empathy' and 'abstraction':

> "*From the viewpoint of empathy, to know something is to belong to it: we can know and understand nature because we are of it; and mathematics is the key to this understanding because nature is essentially mathematical. But from the viewpoint of abstraction, to know something is to have made it oneself: we cannot know nature because we have not made it; but we can interpret it through mathematics because mathematics is our own creation*". [34] (pp. 18–19)

The statements presented in the study seem to confirm this duality noticed by Padovan. Systems of proportion are, on the one hand, read as a continuation of the laws of nature, and on the other, as a tool for taming the element of space. While the first way of seeing can be called traditional—built on the foundation of the cosmological certainty of the pre-scientific worldview—the second is already a modern feature and seems to respond to the experience of the infinity and endless complexity of the universe. However, the opposition made by Padovan between Le Corbusier—the alleged representative of empathy—and Van der Laan—the advocate of abstraction—is unconvincing. Le Corbusier, although he refers to the laws and harmony of the universe, also strongly emphasizes that architecture is something purely human, a creation of the mind, and a mathematical abstraction. For Van der Laan, the measure given to space by man is a scholastic analogy to the act of creation by God—with the difference that the Creator uses an infinite number of measures, while man uses seven. The empathetic and abstract positions seem to interpenetrate and complement each other. Ultimately, the relationship between architecture and nature remains a philosophical enigma.

Are the assumptions of the proportional approach—partially characterized here—in any way relevant to the contemporary debate on architecture? Contemporary discourses oscillating around the issue of sustainability consider a wide range of aspects, among which the aesthetics of the built environment play a significant role, affecting the sense of place, the well-being of space users, and thus the greater stability and resilience of urban ensembles. In 2012, Peter Buchanan, in a well-known series of articles titled *The Big Rethink*, outlined a program for integral architectural design, in which functional, technical, economic, energy efficiency, and other measurable aspects accompany psychological, cultural, and aesthetic factors on an equal footing. Among many postulates, Buchanan called for a new 'facadism'—renewed concern for the external appearance of buildings, seeking principles that create a sense of aliveness of space. Referring to the theories of Hale and Christopher Alexander, Buchanan called for abandoning the postmodern understanding of architecture as a narrative game in favor of returning to formal aesthetics, including geometric relationships [74]. If such a postulate is considered valid, a contemporary interpretation of the assumptions of the proportional approach would be necessary.

However, it should be emphasized that the proportional approach has always been more of a margin of modern architectural theory, and even at the peak of its popularity in the mid-20th century, it was neither universally recognized nor established in the system of architectural education. Usually, systems of proportion, if they function at all in individual practice, are more of a personal work philosophy that a designer does not feel obliged to justify or promote more broadly. They remain—as Rudolf Arnheim put it—"a venerable game" that architects sometimes play [75] (p. 259), using—within a series of complex professional conditions—the creative freedom of the artist.

Limitations and Future Research

The exploration of proportional theories in architecture, while historically rich and theoretically engaging, offers numerous avenues for further research that could enhance our understanding of their practical and perceptual implications:

- First, there is potential for a more profound exploration of the modern proportional approach. This study has focused on theoretical aspects, but there is a need for a more substantial engagement with practical methodologies and a deeper exploration of their interconnections. It would be particularly illuminating to investigate the degree to which 19th–20th century proportional theories were grounded in historical systems of proportion or if they were largely speculative constructs serving the agendas of their authors.
- Second, the presented theories do not offer a proper scientific explanation of perceptual mechanisms in the context of proportions. The intricate relationship between visual perception and geometric principles is yet to be fully elucidated. Future studies could aim to unravel these mechanisms, providing empirical evidence to either validate or

refute the visual efficacy of proportional systems. Given the interdisciplinary nature of architectural theory, it would be beneficial to explore the intersection of proportional systems with other fields, such as psychology, neuroscience, and cognitive science. This could provide a deeper understanding of how proportional relationships influence human emotions, cognition, and behavior within built environments, thereby enriching the discourse on architectural beauty and human experience.

- Third, this study focuses on traditional tools without delving into the integration of contemporary design technologies like CAD and parametric modeling with classical systems. However, the emergence of advanced computational technologies, such as artificial intelligence and parametric design, presents new possibilities for this study and application of proportional systems. Investigating the potential of AI to detect and optimize proportional regularities, as well as the intersection between proportional systems and parametric architecture, could lead to innovative insights and practices that bridge traditional and contemporary design methodologies.

7. Conclusions

This study presents an outline of a contemporary approach within architectural theory that places particular emphasis on the concept of proportion and utilizes arithmetic or geometric methods to regulate proportional relationships for specific aesthetic effects. Within this framework, a series of practical techniques has been developed to aid the design process. The analysis has been conducted from a theoretical standpoint, focusing on the objectives, concepts, and justifications that underlie this approach. This modern perspective, while rooted in traditional theories, stands as a distinct entity, favoring three fundamental principles: the rational consistency of form, the quality of visual stimuli, and artistic expression. Both nature and reason serve as sources for proportional order within these theories, reflecting a dual approach to comprehending beauty and harmony in architecture.

Despite the shift in theoretical basis, this approach strives to maintain the continuity between contemporary and historical architectural practices, seeking enduring principles that transcend temporal constraints. By emphasizing the significance of architectural form, this contemporary approach provides a significant counterpoint to perspectives that view architecture either as a mere expression of practical necessities or as a purely semiotic construct. It suggests that, despite the relative and particular nature of architectural creation, there exists a universal foundation for this process. It would be a mistake to treat the proportional approach as a set of anachronistic prejudices from a bygone era with no application in the contemporary world. While the justifications and explanations offered by the presented theories are not scientific evidence, they can be intellectually satisfying in our time. Proportional systems—these or others—do not guarantee beauty, yet they focus the designer's attention on formal relationships and composition, providing tools for their management. The mere reemphasis of the importance of these issues in today's architecture could, hopefully, have a beneficial impact on architectural education and practice. Being aware of the significant role of the parts in the perception of the whole, we can anticipate that this could influence the shaping of beauty in the built environment.

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