buildings

MDPI

Article

A Three-Dimensional Seismic Damage Assessment Method for RC Structures Based on Multi-Mode Damage Model

Xiang Guo [1,2,3], Shuo Liu [1], Xiang Wang [4], Fujian Yang [1] and Yantai Zhang [4,*]

[1] School of Urban Construction, Changzhou University, Changzhou 213164, China; guoxiang2131@cczu.edu.cn (X.G.); 20485108@smail.cczu.edu.cn (S.L.); fjyang@cczu.edu.cn (F.Y.)
[2] School of Civil Engineering, Tianjin University, Tianjin 300072, China
[3] Jujiang Construction Group, Jiaxing 314599, China
[4] College of Civil Engineering, Nanjing Forestry University, Nanjing 210037, China; wangx@njfu.edu.cn
* Correspondence: atai1991@njfu.edu.cn

Abstract: To rationally evaluate the seismic damage of RC structures comprehensively and multi-dimensionally, a damage index calculation method is proposed. This is a macroscopic global seismic damage model that considers torsional damage, damage in two perpendicular horizontal directions, as well as the overall damage, based on the modal characteristics of the three-dimensional structure and the multi-mode damage model. Formulas are derived, and the steps for damage evaluation are summarized. To better illustrate the results of the proposed method, an example of an asymmetric 6-story frame-shear wall structure is built using the OpenSees program. Thirteen ground motions are selected for incremental dynamic analysis. The structure's damage indexes are evaluated according to the proposed method and compared with the corresponding structural responses, He et al.'s index, and the Final Softening index. The results demonstrate that the proposed method can fully reflect the macroscopic damage state of the structure from different perspectives. Additionally, the results show that, despite the ground motion only acting in the *y*-direction, the structure exhibits responses and damage in both the *x*-direction and the torsional direction. The overall damage to the structure is primarily controlled by the torsional damage, attributed to the asymmetric arrangement of shear walls. The torsional effect is the key factor leading to the failure of asymmetric structures during earthquakes. Therefore, ensuring the torsional strength of the structure is crucial during the structural design process.

Keywords: three-dimensional structure; multi-mode; earthquake; damage model; torsional damage

check for updates

Citation: Guo, X.; Liu, S.; Wang, X.; Yang, F.; Zhang, Y. A Three-Dimensional Seismic Damage Assessment Method for RC Structures Based on Multi-Mode Damage Model. *Buildings* **2024**, *14*, 739. https://doi.org/10.3390/buildings14030739

Academic Editor: Harry Far

Received: 21 January 2024
Revised: 26 February 2024
Accepted: 7 March 2024
Published: 9 March 2024

Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

To quantitatively describe the damage state of components or structures, numerous damage indexes or damage models are put forward. However, among these seismic damage models, seldom do they considered three-dimensional seismic damage, especially for torsional damage. The assessment of reinforced concrete buildings for three-dimensional seismic damage is a challenging structural problem, and it is also a key issue for disaster mitigation and resilience [1,2]. Previous studies have recognized the importance of considering torsional damage in three-dimensional structures, but few have established quantitative evaluation models specifically for torsional damage.

The development of three-dimensional seismic damage assessment of building structures is difficult. The true sense of a three-dimensional damage model that considers torsional effects was first proposed by Jeong and Elnashai [3]. In Jeong and Elnashai's [4] early research, the three-dimensional damage was considered by incorporating the influence of torsion damage as a reduction factor in their comprehensive damage index. However, they did not provide a direct quantitative assessment of the structural torsional damage. Later, they [3] first proposed a three-dimensional damage model at the structural level that truly

considers the torsional effect. Similarly, Prakash and Belarbi [5] acknowledged this limitation in their study and proposed a component-level torsional damage model based on the Park-Ang damage model. Other researchers, such as Wang et al. [6], proposed a structural story damage index for a three-dimensional asymmetric multi-story structure based on mode shapes and frequencies. This index can describe the seismic story damage state of asymmetric structures, but it does not provide a measure of the overall global damage to the structure. Guo et al. [7] extended the Park-Ang damage model to three-dimensional structures but did not address the overall torsion problem of the structure, as they relied on the moment–curvature relationship of the structural components. Mirshafiei et al. [8], for structures with slight damage, established a three-dimensional seismic damage assessment method based on on-site environmental vibration measurements of acceleration and velocity sensors. This method considers the torsion effect and has been extended to more severely damaged structures [9]. However, they actually established a structure response and vibration data output system for damage assessment of three-dimensional structures. They did not develop a quantitative evaluation model for structural damage, especially for torsional damage. Bernagozzi et al. [10] proposed a damage diagnosis method based on modal flexibility. This method can be used for the damage diagnosis of a symmetrical box-type three-dimensional building structure with a rectangular plane where asymmetric damage occurs. It is a kind of three-dimensional seismic damage model. The strategy adopted by their method is to find the inspection load that can decouple the deformation based on modal flexibility, without in-depth quantification of structural torsional damage research in their study.

Comparing three-dimensional and torsional damage, the investigation of torsional responses are fruitful. Through the efforts of scholars, fruitful results have been achieved in reversing the problem. Torsional resistance has gradually been included in national codes. At present, the torsional control indexes adopted by national codes mainly include the torsional displacement ratio and relative eccentricity, and the torsional cycle ratio is also specified in the "Technical specification for concrete structures of tall buildings" [11] in China. However, the provisions in the specification are basically based on the research results obtained by the elastic analysis method, which may not be applicable to the torsional control in the inelastic stage of the structure. The study by Li Yingmin et al. [12] showed that the torsional period ratio did not play an effective role in controlling the elastic-plastic torsional response of the structure. In view of the problem that stiffness eccentricity is not a good indicator of the torsion degree of the structure after entering the nonlinear state, Sadek and Tso [13] studied the influence rule of strength eccentricity on the torsion effect of single-layer eccentric structures and believed that strength eccentricity plays a controlling role after the structure enters the nonlinear state. In addition to eccentricity, scholars also studied other parameters to index the torsional irregularity of the structure, such as modal eccentricity [14], effective modal mass ratio correlation parameters [15], rigid floor angle correlation parameters [10], etc. Efforts have also been made in building-idealized three-dimensional reduced-order models that can account for plan-irregular structures with few degrees of freedom [16].

The aforementioned studies offer methodologies and insight for the quantitative evaluation of seismic damage in three-dimensional structures. Of course, the development of finite element software such as OpenSees 3.5.0 [17] and numerical analysis methods such as incremental dynamic analysis of RC buildings [18,19] also provide a basis for the study. Offering open-source flexibility and a broad range of applications, OpenSees plays a critical role in the precise modeling of complex structural behaviors under seismic loads [20]. The visualization method makes modeling and analysis much easier [21,22]. The development of the incremental dynamic analysis method in three-dimensional structures makes it possible to analyze the seismic damage of complex structures [18,23]. It should also be noted that prior research primarily concentrates on three-dimensional structures that exhibit torsional responses, yet it often neglects a critical detail: even in the absence of the torsional response, the torsional resistance of the structure may decrease, resulting in

torsional damage. Nonetheless, research on the quantitative assessment of this particular aspect is still lacking. Another related phenomenon often mentioned in previous studies is that damage in two orthogonal horizontal directions is correlated. For example, Marusic and Fajfar [24] mentioned in their study on the response of asymmetric structures under bi-directional seismic excitation that stiffness degradation caused by plastic deformation in one direction may greatly affect the performance in the orthogonal direction. Shibata et al. [25] also mentioned in their research that yielding in one direction affects the response in the orthogonal direction. Similarly, Marante and Flórez-López [26] observed the same phenomenon. For the overall structure, the damage in the two translational directions is closely related to the torsional damage to the structure, and there is a coupling relationship between the two translational damages, making the problem of three-dimensional seismic damage extremely complex.

Since the modal parameters contain the three-dimensional information of the structure, the modal damage model using modal parameters is likely to be a solution to this problem. The original concept of the modal damage index was first adopted by Koyluoglu et al. [27], which was actually the Final Softening (*FS*) index proposed by DiPasquale et al. [28]. The formula of the *FS* index is defined as:

$$FS = 1 - \frac{T_{0,\text{initial}}^2}{T_{0,\text{final}}^2} \tag{1}$$

where $T_{0,\text{initial}}$ and $T_{0,\text{final}}$ are the fundamental period before and after the earthquake, respectively. As we all know, the *FS* index only considers the fundamental mode of the structure. Later, scholars Zhu et al. [29], Wang et al. [30], etc. also adopted the concept of modal damage in their papers. Zhu et al. [29] extended modal damage from the fundamental mode to the form of multi-modal damage combination, and then through the improvement and expansion of He et al. [31–33], a modal damage model that can be used to describe the structural damage state was formed. On this basis, a new multi-mode damage model was proposed to evaluate the seismic damage of single-layer steel latticed shells [34]. This model is similar to the damage model proposed by Ghobarah et al. [35] (called G model hereafter). The only difference is that the two models apply different forces. In the model proposed by Zhang et al. [34], a specially constructed equivalent static earthquake load is applied that can consider multi-modal contribution, whereas the G model employed an inverted triangular load, which only considers the effect of the first mode. This kind of method is different from the modal damage model proposed by He et al. [33], in which the modal analysis method is adopted twice before and after damage to the structure. He et al.'s approach may be simpler with regard to analytical techniques and computational processes. The calculation formula for He et al.'s damage model is as follows:

$$D_n = 1 - \lambda_n \frac{T_{n,\text{initial}}^2}{T_{n,\text{final}}^2} \tag{2}$$

$$\lambda_n = \frac{M_{n,\text{final}} / \psi_{n,\text{final}}^2}{M_{n,\text{initial}} / \psi_{n,\text{initial}}^2} \tag{3}$$

$$D = \sqrt{1 - \prod_{n=1}^{m} (1 - D_n^2)} \tag{4}$$

where D_n is the nth order modal damage index of the structure; $T_{n,\text{initial}}$ is the nth order period of the initial structure; $T_{n,\text{final}}$ is the nth order period of the damaged structure; λ_n is the mass contribution factor of the nth order mode, which can be taken as 1.0 in simplified calculation; $M_{n,\text{initial}}$ and $M_{n,\text{final}}$ are the modal mass before and after the earthquake, respectively; $\psi_{n,initial}$ and $\psi_{n,final}$ are the maximum elements (absolute values) of the vibration modes of the structure before and after an earthquake, respectively; m is the number of modes involved in the combination considered, and the value of m can be taken from

the number of modes required for the cumulative equivalent mode mass participation coefficient to reach 90% in the Code for Seismic Design of Buildings of China [36]; and D is the overall damage index of the structure.

For modal damage, the structural damage can be decomposed into different modes of damage, each mode has its own contribution to the structural damage, and the combination of different modes of damage in a certain way can obtain the overall damage to the structure. The modal damage calculation is convenient and efficient, and it does not need to involve inertial force, damping, or the response time history of the structure, nor does it need to know the maximum displacement of the structure during earthquake action or the deformation under critical collapse. The overall damage index of the structure can be obtained without weighting, which has a lot of advantages. And this is also the reason why it is selected as the basis of three-dimensional seismic damage assessment of structures in this study.

The purpose of this paper is to propose a new three-dimensional seismic damage model on the basis of previous modal damage models, which can be used to comprehensively and multi-dimensionally evaluate the seismic damage of structures.

2. Derivation of Three-Dimensional Seismic Damage Model

2.1. Discussion of the Referenced Multi-Mode Damage Model

Based on the modal decomposition method and the characteristic that a damaged structure can undergo elastic micro-vibration at its equilibrium position after an earthquake, the multi-mode damage model has been derived [33]. Although it is not specified that the multi-mode damage model is only applicable to plane structures according to the process of derivation, its derivation background and application are not suitable for the three-dimensional structure. Therefore, when considering the characteristics of a three-dimensional structure, especially with regard to the torsional damage of the entire structure, further improvement is necessary for the multi-mode damage model. And this is also the purpose of this study.

When it comes to three-dimensional structures, there are several aspects that need to be taken into consideration. The first thing to consider is the characteristics of the three-dimensional structural modes. Three-dimensional structures have additional degrees of freedom compared to plane structures. The modes of vibration in three-dimensional structures can involve not only translational motion in the x- and y-directions (vertical direction is not considered in general) but also rotational motion around the vertical axial. And the mode shapes of three-dimensional structures can exhibit more complex patterns compared to planar structures. In planar structures, the mode shapes are typically characterized by simple patterns such as bending and shearing. However, in three-dimensional structures, the mode shapes can involve a combination of bending, shearing, twisting, and other complex deformations. Finally, the natural frequencies associated with the modes of vibration in three-dimensional structures can vary significantly depending on the structural geometry and material properties. This variation in natural frequencies can have implications for the dynamic behavior and response of the structure to external loads or seismic events. Considering these characteristics, it is assumed that each mode of a three-dimensional structure contains three components, namely two translational components and one torsional component. Based on this assumption, the proposed method is derived.

Considering what has been discussed, the next part focuses on the three-dimensional modal characteristics and considers the combination method of three-dimensional damage to derive the three-dimensional seismic damage model for the structure.

2.2. Three-Dimensional Seismic Damage Model

For an n-story structure, considering three degrees of freedom per story, the total degrees of freedom is $N = 3n$. The instantaneous displacement vector $\mathbf{u}$ and the ith-order modal shape vector $\boldsymbol{\phi}_{\mathrm{i}}$ can be written as:

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_x \\ \mathbf{u}_y \\ \mathbf{u}_\theta \end{bmatrix}_{N\times 1} \tag{5a}$$

$$\boldsymbol{\phi}_i = \begin{bmatrix} \boldsymbol{\phi}_{xi} \\ \boldsymbol{\phi}_{yi} \\ \boldsymbol{\phi}_{\theta i} \end{bmatrix}_{N\times 1} \tag{5b}$$

where $\boldsymbol{u}_{\mathrm{x}}$, $\boldsymbol{u}_{\mathrm{y}}$, $\boldsymbol{u}_{\theta}$, and $\boldsymbol{\phi}_{x\mathrm{i}}$, $\boldsymbol{\phi}_{y\mathrm{i}}$, $\boldsymbol{\phi}_{\theta\mathrm{i}}$ are the two translational and one torsional component of the displacement vector and the modal shape vector, respectively. The components of the modal shape vector can be further expressed as:

$$\boldsymbol{\phi}_{xi} = \begin{bmatrix} \mathrm{u}_{i1} \\ \vdots \\ \mathrm{u}_{ij} \\ \vdots \end{bmatrix}_{n\times 1,} \quad \boldsymbol{\phi}_{yi} = \begin{bmatrix} \mathrm{v}_{i1} \\ \vdots \\ \mathrm{v}_{ij} \\ \vdots \end{bmatrix}_{n\times 1,} \quad \boldsymbol{\phi}_{\theta i} = \begin{bmatrix} \theta_{i1} \\ \vdots \\ \theta_{ij} \\ \vdots \end{bmatrix}_{n\times 1} \tag{6}$$

where u_{ij}, v_{ij}, and θ_{ij} are the x-direction, y-direction, and torsional direction components of mode $\boldsymbol{\phi}_i$, respectively.

From the perspective of energy, the kinetic energy of the structure can be expressed as:

$$T = \frac{1}{2}\dot{\mathbf{u}}^T\mathbf{K}\dot{\mathbf{u}} = \frac{1}{2}\sum_i^n \boldsymbol{\phi}_i^T\mathbf{M}\boldsymbol{\phi}_i Y_i \tag{7}$$

where $\dot{\boldsymbol{u}}$ is the instantaneous velocity vector of the structure; $\mathbf{K}$ is the stiffness matrix of the structure; Y_i is the generalized coordinates; $\boldsymbol{\phi}_i$ is the ith order modal vector of the structure, for which the expression is shown in Equation (5); and $\mathbf{M}$ is the mass matrix of the structure, which can also be written as:

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}_x & & \\ & \mathbf{m}_y & \\ & & \mathbf{m}_\theta \end{bmatrix}_{3n\times 3n} \tag{8}$$

where $\mathbf{m}_x$, $\mathbf{m}_y$, and $\mathbf{m}_\theta$ are the mass matrix or moment of inertia matrix in the three directions, $\mathbf{m}_x = \mathrm{diag}(m_{x1}, \ldots, m_{xj}, \ldots, m_{xn})_{\mathrm{n}\times\mathrm{n}}$, $\mathbf{m}_y = \mathrm{diag}(m_{y1}, \ldots, m_{yj}, \ldots, m_{yn})_{\mathrm{n}\times\mathrm{n}}$, and $\mathbf{m}_\theta = \mathrm{diag}(J_1, \ldots, J_j, \ldots, J_n)_{\mathrm{n}\times\mathrm{n}}$, in which m_{xj}, m_{yj}, and J_j are the mass and moment of inertia in the x-direction, y-direction, and torsional direction of jth story, respectively.

Consider $\boldsymbol{\phi}_i^T\mathbf{M}\boldsymbol{\phi}_i$ for each mode in Equation (7), set $E_i = \boldsymbol{\phi}_i^T\mathbf{M}\boldsymbol{\phi}_i$, and submit Equation (5b), Equation (6), and Equation (8) into it, then the following equation can be obtained:

$$E_i = \boldsymbol{\phi}_i^T\mathbf{M}\boldsymbol{\phi}_i = \sum_j^n (m_{xj}u_{ij}^2 + m_{yj}\mathrm{v}_{ij}^2 + J_j\theta_{ij}^2) \tag{9}$$

Thus, in mode i, the direction coefficients of the structure can be obtained from the perspective of energy, respectively. And it can be denoted as:

$$C_{xi} = \frac{\sum_j^n m_{xj}\mathrm{u}_{ij}^2}{E_i} \tag{10a}$$

$$C_{yi} = \frac{\sum_{j}^{n} m_{yj} v_{ij}^2}{E_i} \tag{10b}$$

$$C_{\theta i} = \frac{\sum_{j}^{n} J_j \theta_{ij}^2}{E_i} \tag{10c}$$

According to the magnitude of directional coefficients C_{xi}, C_{yi}, and $C_{\theta i}$, the direction of the modes can be defined accordingly. Thus, the modal shape vectors $\boldsymbol{\Phi}_i$ and $\boldsymbol{\Phi}'_i$ of the initial and the damaged structure, respectively, can be sorted and re-expressed as shown in Table 1 according to each principal vibration direction. The corresponding vibration periods are taken as T_{xi}, T_{yi}, $T_{\theta i}$ and T'_{xi}, T'_{yi}, $T'_{\theta i}$ (i = 1, 2...n) for the initial and the damaged structure, respectively.

Table 1. The mode parameters sorted according to the direction of the principal vibration.

Principal Vibration Direction	The First Mode		The Second Mode		...	The nth Mode	
	Initial	Damaged	Initial	Damaged		Initial	Damaged
x	Φ_{x1}	Φ'_{x1}	Φ_{x2}	Φ'_{x2}	...	Φ_{xn}	Φ'_{xn},
y	Φ_{y1}	Φ'_{y1}	Φ_{y2}	Φ'_{y2}	...	Φ_{yn}	Φ'_{yn},
torsion	$\Phi_{\theta 1}$	$\Phi'_{\theta 1}$	$\Phi_{\theta 2}$	$\Phi'_{\theta 2}$	...	$\Phi_{\theta n}$	$\Phi'_{\theta n}$

If the modes of each principal vibration direction (x, y, and torsion) are considered separately, the case is no different from that of a plane structure. That allows for the use of existing methods and formulas for calculating modal damage in plane structures. By decomposing the three-dimensional structure into three "plane" structures, it simplifies the analysis process and the modal damage in the three principal directions (x, y, and torsion) can be obtained. Here, the simplified calculation method is considered, that is, the mass contribution factor λ_n is taken as 1.0, and the result is as follows:

$$d_{ri} = 1 - \frac{T_{ri}^2}{T'^2_{ri}}, \ (r = x, y, \theta) \tag{11}$$

$$D_r = \sqrt{1 - \prod_{i}^{m} (1 - d_{ri}^2)}, \ (r = x, y, \theta) \tag{12}$$

where d_{ri} is the ith mode damage index of the x-, y-, and torsional directions after reordering according to the principal vibration direction; D_r is the total damage to the structure in the x-, y-, and torsional directions, and $0 \leq D_r \leq 1$; and m is the number of modes involved in the combination considered, and each principal direction can have different m values according to the needs.

It should be noted that the vibration in each principal direction is assumed to be separate in Equations (11) and (12), and then the modal damage index in each direction is calculated using the plane method. However, in reality, the vibration and damage are coupled. If the plane method is still used to perform the combination to obtain the overall damage index of the whole structure, it is obviously inappropriate. This will be discussed exclusively next.

2.3. The Combination of Overall Damage for the Whole Three-Dimensional Structure

The coupled effect between the damage in different directions can be clearly seen from the components contained in the mode vector of each principal vibration direction and the correlation between the stiffness of the structure in each direction. When there is damage in one direction, it can lead to changes in the stiffness properties of the structure, which can then affect the response and damage in other directions. This phenomenon is similar to the relationship between structural shear damage and bending damage. This

coupling effect arises from the interconnected nature of the structure and the redistribution of forces and stresses due to the damaged components. For example, in a beam, if there is bending damage, it can lead to local deformation and changes in the stiffness properties. This altered stiffness can then affect the response and damage from shear. Similarly, shear damage can also affect the bending behavior of the beam. It is important to consider these coupled effects when evaluating the structural performance and determining failure criteria. When damage reaches a critical level in one direction, it can significantly affect the behavior and capacity of the structure in other directions. And when one of these kinds of damage reaches 1.0, the structure fails. Therefore, failure criteria should take into account the interaction and coupled effects between different types of damage to ensure an accurate assessment of the structural integrity.

By referring to the method of Mergos and Kappos [37], which was proposed for the combination of structural bending damage and shear damage, the overall damage of the three-dimensional structure can be obtained by combining the damage in the x-direction, y-direction, and torsional direction. The calculation formula is as follows:

$$D = 1 - (1 - D_x)^{\alpha} \cdot (1 - D_y)^{\beta} \cdot (1 - D_\theta)^{\gamma} \tag{13}$$

where α, β, and γ are the importance coefficients of structural damage in the x-, y-, and torsional directions, respectively.

The calculation of global damage index D is greatly affected by the values of α, β, and γ. The range of these parameters can be determined on the basis of the physical interpretation of the directional and overall structural damage. The overall damage to the structure is obtained by the combination of x-direction damage, y-direction damage, and torsional-direction damage. Thus, the overall damage should obtain greater values than the respective directional damage. Nevertheless, according to the definition of damage, the overall damage index D of the structure should not be greater than 1.0. Therefore, the above restriction can only be satisfied if the values of α, β, and γ are greater or equal to 1. In addition, the overall damage to the structure can be controlled by the damage in a single direction (such as the x-direction), i.e., $D = 1 - (1 - D_x)^{\alpha} \approx D_x$, when it is assumed that damage in the other two directions of the structure (such as the y-direction and the torsional direction) can be ignored. It is physically meaningful. To satisfy this condition, α needs to be approximately equal to 1. Therefore, it is suggested to take $\alpha = \beta = \gamma = 1$ in actual calculation. Then, Equation (13) becomes:

$$D = 1 - \prod_{r=x,y,\theta} (1 - D_r) \tag{14}$$

As can be seen from Equation (14), as long as the damage in any x-, y-, or torsional direction reaches 1.0, the overall damage to the whole structure will reach 1.0 and the three-dimensional structure will reach its ultimate state.

3. Three-Dimensional Seismic Damage Evaluation Process

In summary of the derivation shown above, the process of the three-dimensional seismic damage assessment method for RC structures can be expressed as shown in Figure 1. A key step of the process is to determine the modal correlations between the modes for each principal vibration direction before and after the act of earthquake. Due to the complexity of the three-dimensional structure, it may not be easy to establish a simple one-to-one modal correlation between the modes before and after seismic excitation. This problem is more prominent in large-span lattice shell structures [32,34], which is usually determined by using the linear modal assurance criteria (LMAC) approach [38]. This method can also be referred to for complex high-rise building structures. For the example of the low-rise building structure used in this study, the direction coefficient sorting method can be directly used to determine the modal correlations. Details will be shown in the following example part.

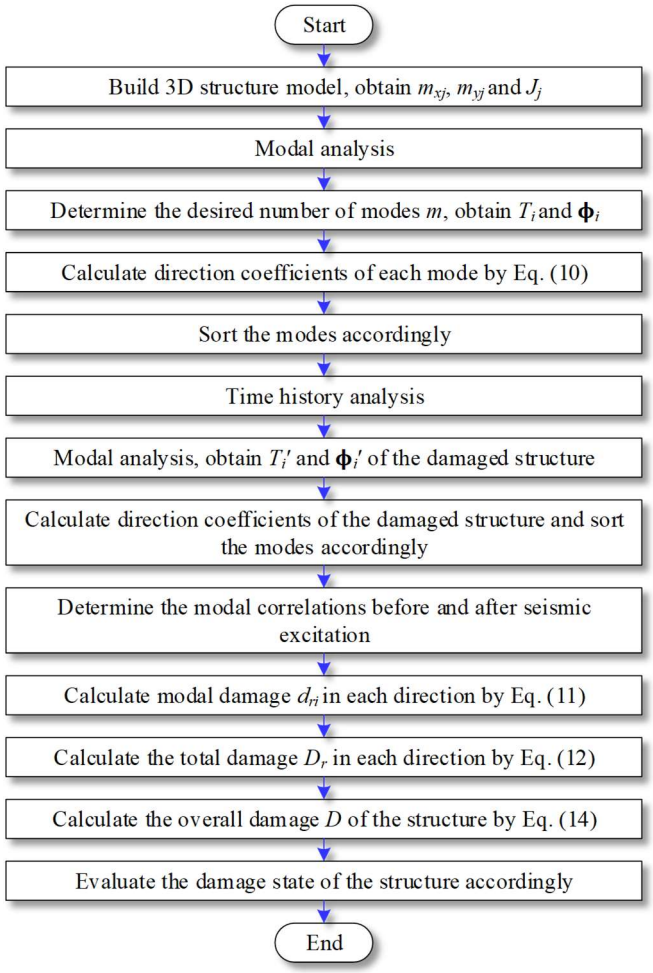


Figure 1. The outline flow chart of three-dimensional seismic damage assessment.

The methodology presented in this study relies on the three-dimensional modal characteristics of structures. Whether the buildings are rigid, flexible, or coupled torsional, there appear to be no discrepancies in terms of the underlying formulas or mathematical principles. However, the specific application may necessitate variations in the combination method or parameter values. Based on the current research example, there are no apparent issues in implementing this approach for coupled torsional buildings. And according to the characteristics of the modal damage model, the higher-order modal damage to irregular structures may be more prominent than that to regular structures. Nonetheless, further investigation is required to ascertain the applicability of this method to other building types or buildings with different construction materials [39], which is also an avenue for our future research endeavors.

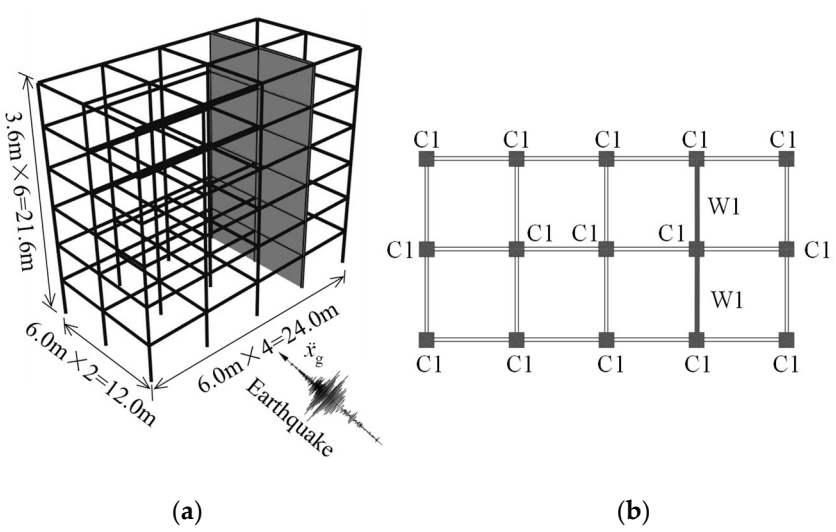
4. Example

4.1. Model Introduction and Ground Motion Selection

The example model is a six-story frame-shear wall structure with eccentric arrangement. The purpose of choosing an asymmetric structure specifically as the example is to better demonstrate the characteristics of the three-dimensional seismic damage model. In asymmetric structures, these torsional forces can have a dominant influence on the structural response during seismic events, which means that the torsional damage may play a dominant role. This feature allows for a thorough examination of the proposed three-dimensional seismic damage model's validity.

The structural model and component arrangement are shown in Figure 2. The section size and reinforcement of each member of the structure are shown in Table 2. The concrete

strength grade is C35, the longitudinal reinforcement is HRB335, and the stirrup is HRB235. Because the structure is not high, and to make the problem as simple as possible, the section of the shear wall does not change along the height. Incremental dynamic analysis (IDA) [40] is performed using the OpenSees program [20]. The version used is the latest OpenSees 3.5.0 (See "https://opensees.berkeley.edu/index.php", accessed on 10 May 2023). The modeling is manually coded, and the diagram of model in Figure 2 is aided by the graphical user interface OSLite-0.7.3 [22]. The shear wall is simulated by the ShellMITC4 shell element combined with the LayeredShell section, Planestsusermaterial material constitutive model, and PlateFromPlaneStress material constitutive model. The dispBeamColumn element combination of Concrete02 material and Steel4 material is applied to model the nonlinear beams and columns with fiber-discretized cross sections. Thirteen ground motions (shown in Table 3) selected according to Chinese design code are used for IDA, and the response spectra are shown in Figure 3.



(**a**) (**b**)

Figure 2. The three-dimensional asymmetric structure model and layout of components. (**a**) Three-dimensional asymmetric structure model; (**b**) Structural plane layout.

Table 2. Cross section and reinforcement details of 6-story wall-frame structure components.

Component	Cross Section (mm × mm)	Longitudinal Reinforcement			Transverse Reinforcement	
		Yield Strength (MPa)	Amount		Yield Strength (MPa)	Amount
			Top	Bottom		
Beam	300 × 600	335	6d25	6d25	235	d12@100
Column (C1)	500 × 500	335	16d25		235	d12@100
Shear Wall (W1)	200 × 6000	335	d10@150		335	d8@100

Table 3. The selected ground motions.

No.	Name	Station	M
1	Hector Mine	Heart Bar State Park	7.13
2	Imperial Valley	El Centro Array #13	6.53
3	Kobe, Japan	Tadoka	6.9
4	Loma Prieta	Saratoga—Aloha Ave	6.93
5	Kobe, Japan	Abeno	6.9
6	Manjil, Iran	Abbar	7.37
7	Superstition Hills	Brawley Airport	6.54
8	Cape Mendocino	Loleta Fire Station	7.01
9	San Fernando	Whittier Narrows Dam	6.61

Table 3. *Cont.*

No.	Name	Station	M
10	Chi-Chi, China	San Onofre—So Cal Edison	7.62
11	Borrego Mtn	Bishop—LADWP South St	6.63
12	Chalfant Valley	R109 (temp)	6.19
13	Denali Alaska	TAPS Pump Station #08	7.9

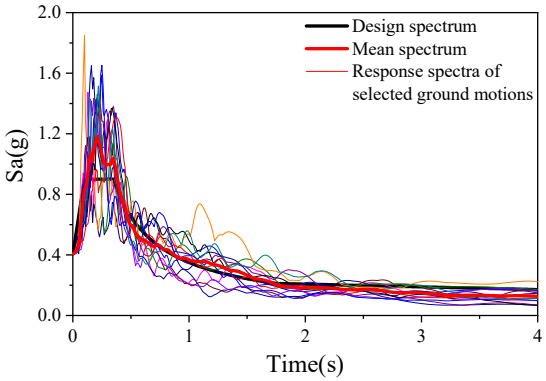


Figure 3. Response spectra of the selected ground motions.

4.2. Modal Analysis

Modal analysis is applied to the initial structure, and the modal characteristics of each mode of the undamaged three-dimensional asymmetric structure are shown in Table 4. For this three-dimensional 6-story structure, three degrees of freedom are taken from each layer. By using Equation (10), the direction coefficients of the 18-order mode shape vectors are calculated and sorted. The first to sixth modes in the *x*-, *y*-, and torsional directions are obtained, respectively. The results are shown in Table 4.

Table 4. Modal characteristics of the initial structure.

Mode	Period (s)	Direction Coefficients			Principal Vibration Direction
		x-Direction	*y*-Direction	Torsional-Direction	
1	0.600	1.00	0.00	0.00	*x*
2	0.596	0.00	0.35	0.65	Torsion
3	0.191	0.00	0.38	0.62	Torsion
4	0.190	1.00	0.00	0.00	*x*
5	0.179	0.00	0.61	0.39	*y*
6	0.106	0.00	0.34	0.66	Torsion
7	0.104	1.00	0.00	0.00	*x*
8	0.071	0.00	0.34	0.66	Torsion
9	0.066	1.00	0.00	0.00	*x*
10	0.051	0.00	0.35	0.65	Torsion
11	0.047	0.00	0.66	0.34	*y*
12	0.043	1.00	0.00	0.00	*x*
13	0.039	0.00	0.36	0.64	Torsion
14	0.024	0.00	0.66	0.34	*y*
15	0.024	1.00	0.00	0.00	*x*
16	0.017	0.00	0.66	0.34	*y*
17	0.013	0.00	0.65	0.35	*y*
18	0.012	0.00	0.64	0.36	*y*

It can be seen from Table 4 that the first to sixth modes in the *x*-direction are the 1st, 4th, 7th, 9th, 12th, and 15th modes of the original structure, the first to sixth modes in the

y-direction are the 5th, 11th, 14th, 16th, 17th, and 18th modes of the original structure, and the first to sixth modes in the torsional direction are the 2nd, 3rd, 6th, 8th, 10th, and 13th modes of the original structure, respectively. Due to the eccentric arrangement of the shear wall, the eccentricity of the structure is more serious, resulting in a higher degree of torsion. And the torsion mode is more forward than the usual structure. This means that the structure is more susceptible to torsional forces during seismic events.

Similarly, after the earthquake, modal analysis is carried out on the damaged structure at its equilibrium position. The corresponding modal parameters can be obtained, and the direction coefficients are calculated and sorted. It should be noted that, due to the phenomenon of the modal jump effect that may occur after the act of earthquake, the modes of the damaged structure do not necessarily correspond to the original structure one by one, that is, the 1st to 6th mode in the *x*-direction of the damaged structure do not necessarily correspond to the original 1st, 4th, 7th, 9th, 12th, and 15th modes of the structure. For example, when the intensity of the Hector Mine ground motion is 1.0 g, the 1st to 6th modes of the damaged structure in the *x*-direction are the 2nd, 5th, 7th, 9th, 12th, and 15th modes of the structure, respectively. The mode sequence numbers in the *y*-direction and the torsional direction also change, as shown in Table 5. It is essential to consider this modal jump effect when analyzing and interpreting the modal parameters of a damaged structure to accurately assess its seismic damage.

Table 5. Modal characteristics of the damaged structure (1.0 g).

Mode	Period (s)	Direction Coefficients			Principal Vibration Direction
		x-Direction	*y*-Direction	Torsional-Direction	
1	0.958	0.00	0.36	0.64	Torsion
2	0.779	1.00	0.00	0.00	*x*
3	0.319	0.00	0.65	0.35	*y*
4	0.285	0.00	0.33	0.67	Torsion
5	0.240	1.00	0.00	0.00	*x*
6	0.154	0.00	0.34	0.66	Torsion
7	0.130	1.00	0.00	0.00	*x*
8	0.100	0.00	0.34	0.66	Torsion
9	0.082	1.00	0.00	0.00	*x*
10	0.070	0.00	0.34	0.66	Torsion
11	0.057	0.00	0.66	0.34	*y*
12	0.051	1.00	0.00	0.00	*x*
13	0.050	0.00	0.36	0.64	Torsion
14	0.027	0.00	0.66	0.34	*y*
15	0.026	1.00	0.00	0.00	*x*
16	0.019	0.00	0.66	0.34	*y*
17	0.015	0.00	0.65	0.35	*y*
18	0.012	0.00	0.65	0.35	*y*

4.3. Result Analysis and Discussion

Figure 4 illustrates the response of the structure at both the maximum response point and the geometric center in all directions, accompanied by the peak ground acceleration (PGA) during IDA for the 13 selected ground motions. As the ground motion primarily acts in the *y*-direction, the structure's response is notably larger and develops more rapidly in this direction, demonstrated by the Drift-(*y*) curve in Figure 4b,e. However, due to the eccentric layout of shear walls, there is also significant eccentricity in the *x*-direction, which results in deformation not only in directions perpendicular to the ground motion but also in torsional directions. The variations in response along these two directions relative to the PGA can be seen in Figure 4a,d for Drift-(*x*) and in Figure 4e,f for torsion (θ). Due to the existence of the shear wall and its numerical simulation method [41], the IDA curves obtained do not have obvious displacement divergence points like general RC frames.

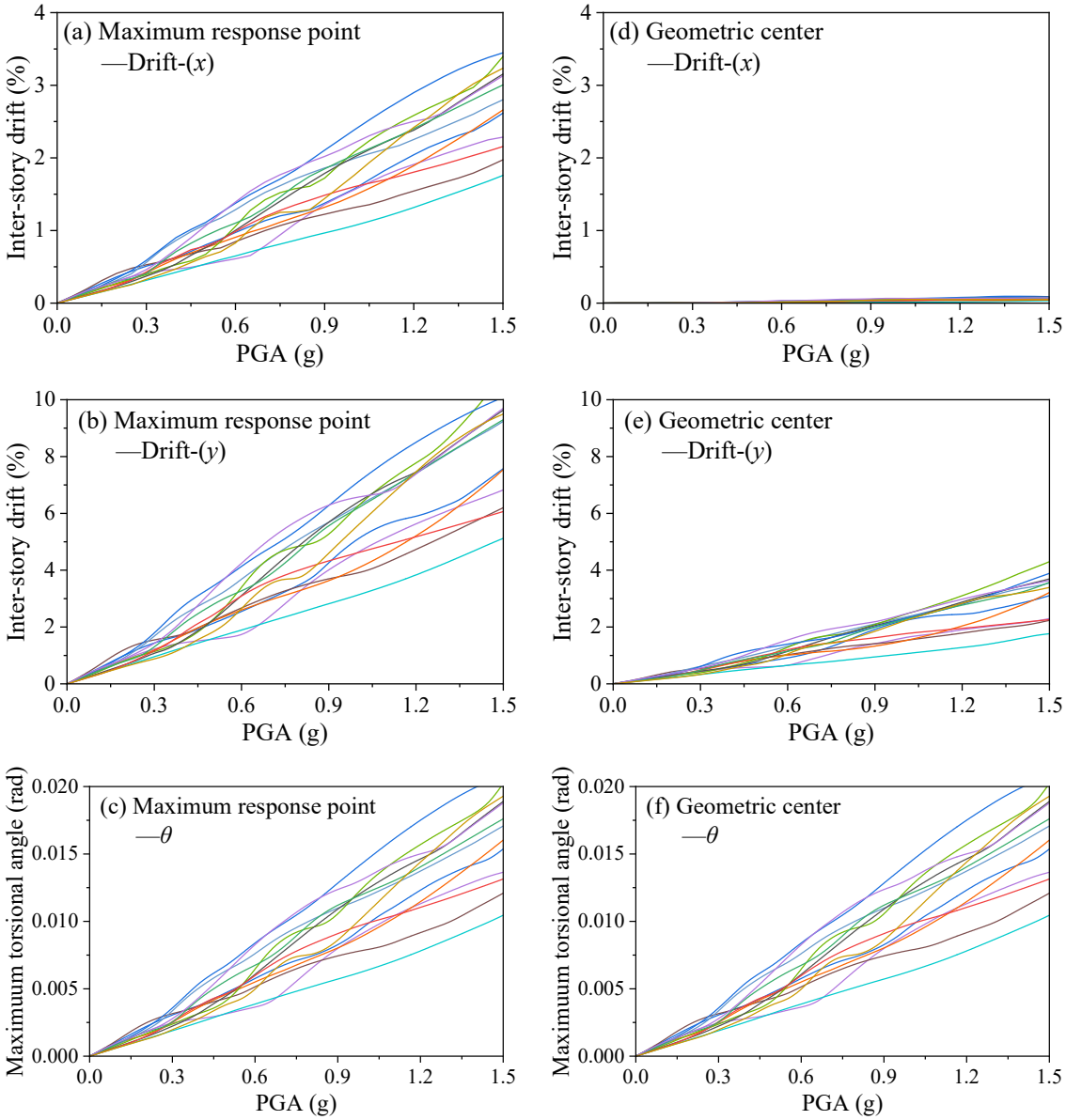


Figure 4. The maximum response of the structure under the 13 selected ground motions at the maximum response point and geometric center.

Figure 4 also reveals that the structural response in the *x*- and *y*-directions is not evenly distributed in the plane. The response at the geometric center is significantly smaller than that at the maximum response point. This discrepancy is attributable to the geometric center's proximity to the center of torsion, whereas the maximum response manifests at the point furthest from this center. Because of this uneven distribution of deformation, structural members distant from the center of torsion are more susceptible to failure, which is evident in the hysteresis curves of these members. For instance, at a ground motion intensity of 1.4 g for the Hector Mine event, the bending moment–curvature curves for the first-story corner column—located at the side away from the shear wall—and the first-story column at the geometric center are depicted in Figure 5. In Figure 5, the corner column is observed to undergo significant nonlinear deformation, while the central column remains within the elastic range. Moreover, due to the assumption of a rigid floor, the rotational angle is uniform at all points on the floor plan, leading to a consistent torsional response throughout the structure. This uniformity is confirmed in Figure 4c,f, which show identical torsional responses at both the maximum response point and geometric center.

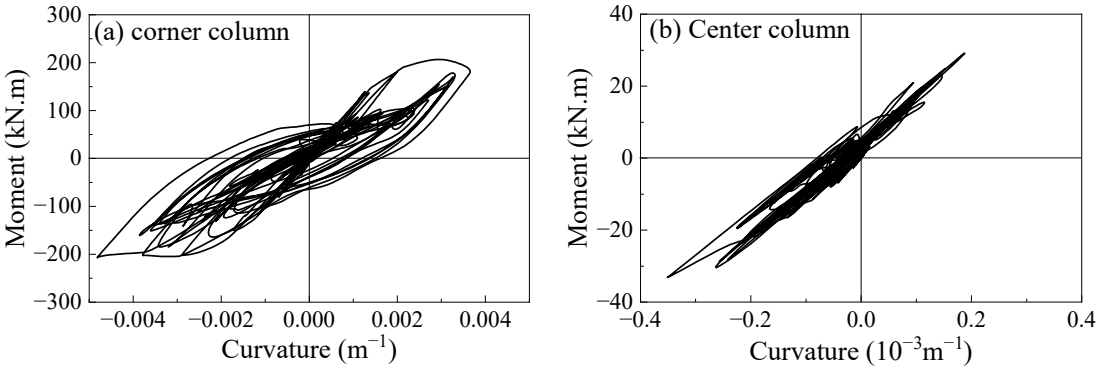


Figure 5. Hysteresis curves of the corner column and center column of the first story at 1.4 g of Hector Mine (No. 1) ground motion.

The overall seismic damage indexes under the 13 ground motions obtained by the proposed method are shown in Figure 6. As can be seen from the figure, the damage to the structure basically follows the typical S-type damage evolution rule, and the development of structural damage basically corresponds to the response of the structure.

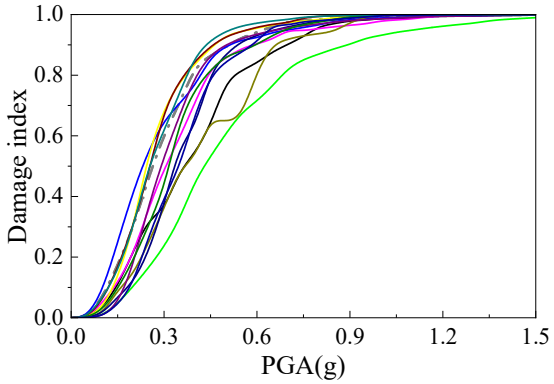


Figure 6. The overall seismic damage under the 13 ground motions obtained by the proposed method.

Take the No. 1 to No. 4 ground motions as an example, the total damage in the three principle directions, the overall seismic damage of the three-dimensional structure calculated using the proposed method, the overall damage indexes obtained using He et al.'s [33] method, as well as the *FS* index proposed by DiPasquale et al. [28] change along with the PGA levels, as shown in Figure 7. The damage model proposed by He et al. [33] does not account for translational and torsional damage separately; thus, only the overall damage indexes are presented in Figure 7d. The damage method proposed in this study offers advantages in representing damage in all principle directions of the structure. It is observable from the damage curves that the progression of He et al.'s index is somewhat gradual, which correlates with an underestimation of the structural torsional damage. By using their combination method [33], the modal damage with significant contributions is emphasized in the overall damage index. Consequently, if the translational modal damage is big enough, it could overshadow the torsional damage, potentially leading to an underestimation of the structure's torsional damage. However, in the damage model proposed in this study, the translational damage and the torsional damage are calculated separately, and the effects of the three principle directions are considered equally in the final combination (as shown in Equation (14)), which is more appropriate. Compared to the *FS* index, if only the fundamental mode is considered, the global damage indexes are relatively small, with a maximum value of around 0.6, far below the 1.0 limit. This indicates that the structural seismic performance would likely be overestimated when using the *FS* index. The damage curves of the proposed model suggest that structural damage is not expected to occur

when the PGA level is below 0.05 g, while the calculated damage indexes are about 0, and that structural failure initiates when the PGA level surpasses approximately 1.2 g, while the calculated damage indexes are greater than 0.99, and the corresponding maximum inter-story drift is about 6%. As indicated by the curves, different damage stages can be identified. As the PGA level increases, torsional damage occurs first within the structure, followed by damage in the *x*-direction. Damage in the *y*-direction—the direction in which the shear wall is located—occurs last, yet it develops swiftly upon initiation and ultimately surpasses the damage in the *x*-direction. This phenomenon is related to the fact that the stiffness of the shear wall is much larger than that of the frame, and the failure form of the shear wall is closer to brittle failure. And throughout the entire IDA process, the torsional damage maintains a dominant position. This confirms that the torsional effect is a critical factor in structural failure during earthquakes. Therefore, the torsional resistance of the structure must be ensured during the design process. Moreover, it is noteworthy that no additional incremental damage is anticipated even with the PGA intensity exceeds 1.2 g. Thus, from a numerical standpoint, the structure is deemed susceptible to collapse. This can also be seen from the hysteresis curves of the structural members, as shown in Figure 5, in which the skeleton curve is already in the descending section at a PGA level of 1.4 g.

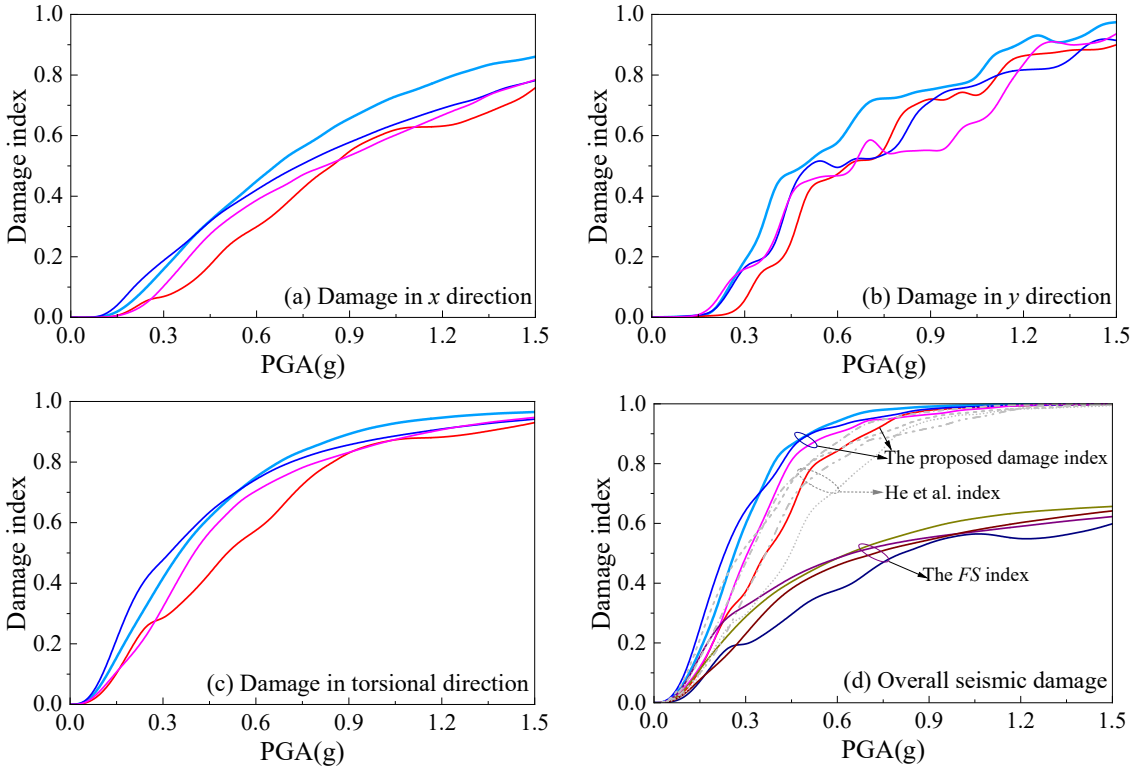


Figure 7. Damage in the three principle directions and comparison of different overall damage indexes under the No. 1 to No. 4 ground motions.

5. Conclusions

Through derivation and example analysis, the following conclusions can be drawn:

A method is proposed to calculate modal damage, total damage in each principal direction, and overall damage of the entire structure. Each mode of a three-dimensional structure may consist of two translational components and one torsional component. By prioritizing the principal vibration direction of each mode based on energy and integrating it with a multi-mode damage model, a three-dimensional seismic damage assessment method with a new modal damage combination method is derived. The proposed damage index can effectively reflect the damage state in each considered direction and the overall structure. In addition, the calculation process of the damage index is simple, and compared

to the Final Softening index, the effect of higher modes on structural damage can be considered, while compared to He et al.'s modal damage model, the torsional damage can be considered separately and equally. The basic mathematical formula and calculation method of the damage model are the same for different three-dimensional structure types, and the combination method or parameter value may be different for different types of structures or for structures with different irregularities in practical application.

The proposed method in this study can account for the correlation of damage across all principal directions. Ground motion acting in one direction can induce damage in the other horizontal direction and torsional direction, and it can be reasonably reflected by the proposed damage model. By comparing the directional damage indexes, engineers can identify the most effective design measures to enhance the seismic performance of the structure, especially when the structure has potential torsional performance deficiencies. Example results show that torsional damage may have a dominant role in the damage development process of asymmetric structures, highlighting the importance of considering torsional effects in structural damage during earthquakes. For this kind of structure, the structural design should aim to minimize the possibility of a torsion failure mode.

Future research can focus on further validating and refining the proposed method through experimental testing and field measurements. Additionally, practical application of the method can be extended to consider other types of structures, such as high-rise buildings, buildings with kind of irregularities, etc., and to incorporate probabilistic approaches for assessing the seismic risk of structures.

Author Contributions: Conceptualization, X.G. and Y.Z.; methodology, X.G.; software, X.G.; validation, X.G. and F.Y.; formal analysis, X.G., S.L. and X.W.; investigation, X.G. and S.L.; resources, X.G., F.Y. and Y.Z.; data curation, X.G., S.L. and X.W.; writing—original draft preparation, X.G., S.L. and X.W.; writing—review and editing, X.G., F.Y. and Y.Z.; visualization, X.G., S.L. and X.W.; supervision, Y.Z.; project administration, X.G., F.Y. and Y.Z.; funding acquisition, X.G., F.Y. and Y.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was financially supported by the National Natural Science Foundation of China (Grant No. 52108457, 52208460) and the Students' innovation and entrepreneurship training program of Jiangsu, China (Grant No. 202310292072Y).

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the first and the corresponding authors.

Conflicts of Interest: Author Xiang Guo was employed by the company Jujiang Construction Group. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

1. Makhloof, D.A.; Ibrahim, A.R.; Ren, X. Damage Assessment of Reinforced Concrete Structures through Damage Indices: A State-of-the-Art Review. *CMES—Comput. Model. Eng. Sci.* **2021**, *128*, 849–874. [CrossRef]
2. Bhatta, S.; Dang, J. Seismic damage prediction of RC buildings using machine learning. *Earthq. Eng. Struct. Dyn.* **2023**, *52*, 3504–3527. [CrossRef]
3. Jeong, S.H.; Elnashai, A. New three-dimensional damage index for RC buildings with planar irregularities. *J. Struct. Eng.* **2006**, *132*, 1482–1490. [CrossRef]
4. Jeong, S.H.; Elnashai, A. Analytical and experimental seismic assessment of irregular RC buildings. In Proceedings of the 13th World Conference on Earthquake Engineering, Vancouver, BC, Canada, 1–6 August 2004.
5. Prakash, S.S.; Belarbi, A. Towards damage–based design approach for RC bridge columns under combined loadings using damage index models. *J. Earthq. Eng.* **2010**, *14*, 363–389. [CrossRef]
6. Wang, J.F.; Lin, C.C.; Lin, G.L.; Yang, C.H. Story damage identification of irregular buildings based on earthquake records. *Earthq. Spectra* **2013**, *29*, 963–985. [CrossRef]
7. Guo, J.; Wang, J.J.; Li, Y.; Zhao, W.G.; Du, Y.L. Three dimensional extension for Park and Ang damage model. *Structures* **2016**, *7*, 184–194. [CrossRef]
8. Mirshafiei, F.; Mirshafiei, M.; McClure, G. A new three–dimensional seismic assessment method (3D–SAM) for buildings based on experimental modal analysis. *Comput. Struct.* **2017**, *180*, 125–137. [CrossRef]

9. Mirshafiei, F.; McClure, G. Modified three–dimensional seismic assessment method for buildings based on ambient vibration tests: Extrapolation to higher shaking levels and measuring the dynamic amplification portion of natural torsion. *Earthq. Eng. Struct. Dyn.* **2016**, *45*, 2011–2026. [CrossRef]
10. Bernagozzi, G.; Ventura, C.E.; Allahdadian, S.; Kaya, Y.; Landi, L.; Diotallevi, P.P. Output–only damage diagnosis for plan–symmetric buildings with asymmetric damage using modal flexibility–based deflections. *Eng. Struct.* **2020**, *207*, 110015. [CrossRef]
11. *JGJ 3-2010*; Technical Specification for Concrete Structures of Tall Building. China Architecture and Building Press: Beijing, China, 2010. (In Chinese)
12. Li, Y.; Han, J.; Liu, J. Study on torsional period ratio control index for seismic design of building structures. *J. Build. Struct.* **2009**, *30*, 77–85. (In Chinese)
13. Sadek, A.W.; Tso, W.K. Strength eccentricity concept for inelastic analysis of asymmetrical structures. *Eng. Struct.* **1989**, *11*, 189–194. [CrossRef]
14. Georgoussis, G.K. Modal eccentricities of asymmetric structures. *Struct. Des. Tall Spec. Build.* **2006**, *15*, 339–361. [CrossRef]
15. Özmen, G.; Girgin, K.; Durgun, Y. Torsional irregularity in multi-story structures. *Int. J. Adv. Struct. Eng.* **2014**, *6*, 121–131. [CrossRef]
16. Ruggieri, S.; Chatzidaki, A.; Vamvatsikos, D.; Uva, G. Reduced-order models for the seismic assessment of plan-irregular low-rise frame buildings. *Earthq. Eng. Struct. Dyn.* **2022**, *51*, 3327–3346. [CrossRef]
17. Shabani, A. A review of graphical user interfaces of OpenSees software framework. *Front. Built Environ.* **2023**, *9*, 1233116. [CrossRef]
18. Kostinakis, K.; Athanatopoulou, A. Incremental dynamic analysis applied to assessment of structure-specific earthquake IMs in 3D R/C buildings. *Eng. Struct.* **2016**, *125*, 300–312. [CrossRef]
19. Zaherdannak, A.; Shabani, A.; Erfani, S. Seismic performance evaluation of special RC frames with gravity steel columns under the base level. *Shock Vib.* **2020**, *2020*, 8825258. [CrossRef]
20. McKenna, F. OpenSees: A framework for earthquake engineering simulation. *Comput. Sci. Eng.* **2011**, *13*, 58–66. [CrossRef]
21. Yang, T.; Schellenberg, A.; Kohama, E. OpenSees Navigator. Berkeley, CA, USA: Pacific Earthquake Engineering Research Center. 2017. Available online: https://openseesnavigator.berkeley.edu/ (accessed on 9 December 2023).
22. Chen, J. OSLite-0.7.3 [Software]. 2022. Available online: http://www.oslite.tech/ (accessed on 30 September 2022).
23. Zhou, Y.; Ge, P.; Han, J.; Lu, Z. Vector-valued intensity measures for incremental dynamic analysis. *Soil Dyn. Earthq. Eng.* **2017**, *100*, 380–388. [CrossRef]
24. Marusic, D.; Fajfar, P. On the inelastic seismic response of asymmetric buildings under bi–axial excitation. *Earthq. Eng. Struct. Dyn.* **2005**, *34*, 943–963.
25. Shibata, A.; Onose, J.; Shiga, T. Torsional response of buildings to strong earthquake motions. In Proceedings of the 7th World Conference on Earthquake Engineering, Istanbul, Turkey, 8–13 September 1980; pp. 123–138.
26. Marante, M.; Flórez–López, J. Model of damage for RC elements subjected to biaxial bending. *Eng. Struct.* **2002**, *24*, 1141–1152. [CrossRef]
27. Köylüoğlu, H.U.; Nielsen, S.R.; Abbott, J.; Cahmak, A.S. Local and modal damage indicators for RC frames subject to earthquakes. *J. Eng. Mech.* **1998**, *124*, 1371–1379. [CrossRef]
28. DiPasquale, E.; Ju, J.W.; Askar, A.; Çakmak, A.S. Relation between global damage indices and local stiffness degradation. *J. Struct. Eng.* **1990**, *116*, 1440–1456. [CrossRef]
29. Zhu, H.W.; Wang, K.P.; Tang, S.G. Modal damage index and its application in structure damage assessment. *J. Tongji Univ. Nat. Sci. Ed.* **2004**, *32*, 1589–1592. (In Chinese)
30. Wang, M.F. Seismic damage model of tall building structure based on modal damage indices. *Earthq. Resist. Eng. Retrofit.* **2008**, *30*, 24–32. (In Chinese)
31. He, Z.; Ou, X.; Ou, J. A macro-level global seismic damage model considering higher modes. *Earthq. Eng. Eng. Vib.* **2014**, *13*, 425–436. [CrossRef]
32. He, Z.; Zhu, Z.; Guo, X.; Liu, T.; Hu, Y. Development of a macroscopic global seismic damage model for lattice shell structures. *J. Struct. Eng.* **2017**, *143*, 04016174. [CrossRef]
33. He, Z.; Guo, X.; Zhang, Y.; Ou, X. A global seismic damage model of RC structures based on structural modal properties. *J. Struct. Eng.* **2018**, *144*, 04018171. [CrossRef]
34. Zhang, Y.; Luo, Y.; Guo, X.; Li, Y.; Huang, Q. A new seismic damage assessment method for single-layer steel latticed shells considering multi-modal contribution. *Structures* **2021**, *33*, 4670–4689. [CrossRef]
35. Ghobarah, A.; Abou-Elfath, H.; Biddah, A. Response-based damage assessment of structure. *Earthq. Eng. Struct. Dyn.* **1999**, *28*, 79–104. [CrossRef]
36. *GB 50011-2010*; Code for Seismic Design of Buildings (2016 Edition). China Architecture and Building Press: Beijing, China, 2016. (In Chinese)
37. Mergos, P.E.; Kappos, A.J. Seismic damage analysis including inelastic shear–flexure interaction. *Bull. Earthq. Eng.* **2010**, *8*, 27–46. [CrossRef]
38. Morales, C.A. Comments on the MAC and the NCO, and a linear modal correlation coefficient. *J. Sound Vib.* **2005**, *282*, 529–537. [CrossRef]

39. Zhang, Q.; Wei, Z.Y.; Gu, X.L.; Yang, Q.C.; Li, S.Y.; Zhao, Y.S. Confinement behavior and stress–strain response of square concrete columns strengthened with carbon textile reinforced concrete (CTRC) composites. *Eng. Struct.* **2022**, *266*, 114592. [CrossRef]
40. Vamvatsikos, D.; Cornell, C.A. Incremental dynamic analysis. *Earthq. Eng. Struct. Dyn.* **2002**, *31*, 491–514. [CrossRef]
41. He, Z.; Sun, C. Seismic response analysis of super high-rise frame-core tube structure using improved nonlinear layered shell model. *Eng. Mech.* **2018**, *35*, 173–181. (In Chinese)

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.