

Article

Plastic Zone Radius Criteria for Crack Propagation Angle Evaluated with Experimentally Obtained Displacement Fields

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Abstract: The monitoring and maintenance of cracked structures are generally carried out using structural integrity assessments. The plastic zone (PZ) crack path (CP) criteria state that a crack grows in a specific direction when the radius of the plastic zone ahead of the crack tip reaches a minimum value. The PZ can be evaluated using stress intensity factors (SIFs). The SIFs under mixed-mode loading were extracted from the literature from three samples: two single edge notch tension (SENT) samples ($E = 2.5$ GPa, $\nu = 0.38$) made from polycarbonate and one modified compact test (C(T)) sample made from low-carbon steel ($E = 200$ GPa, $\nu = 0.3$). In addition, the CP angle was evaluated for the W and R criteria with experimental data, which included non-linear effects such as fatigue-induced plasticity, crack roughness, and debris. It was found that both can predict the CP for lateral cracks in both tested materials and monotonic and cyclic load when the mode mixity does not change considerably from one crack length to the next or goes beyond 0.2. Moreover, the R criterion exhibited an error as high as 1.7%, whereas the W criterion showed a 6% error on the last crack length for the low-carbon steel sample under cyclic load, which had a 100% increase in mode mixity. Finally, the applicability of LEFM was checked, while the CP was sought by finding the size of the PZ.

Keywords: LEFM; crack kinking; crack path; plastic zone



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1. Introduction

In the maintenance of cracked components and structures, the crack front advance rate needs to be established and monitored [1–3]. However, in the case of mixed-mode loading, factors such as plasticity [4], crack interlocking, crack rugosity [5], trapped debris [6], and crack tip vicinity to defects [7], among others, may affect the crack growth rate and crack path (CP). Thus, CP prediction is not entirely understood [1,2,4,5,8–10]. According to [11], the more kinked the crack is, the more crack growth might be retarded. Nevertheless, for crack kinking and equivalent SIFs, there are already several postulated criteria [1,12], mainly based on linear elastic fracture mechanics (LEFM), with the maximum tangential stress criteria being the most used for mixed-mode loading [10]. However, the applicability of LEFM must be checked [13,14] before a suitable model can be applied. Using a model that checks the applicability and predicts CP simplifies those two tasks.

Overall, it can be said that all CP prediction models state a hypothesis for a crack to change direction by a small $\Delta\theta$ from its original path. Such an increment is generally created when an equivalent SIF (K_{eq}) exceeds material toughness (K_{IC}) [10,13] in the LEFM approach. Wasiluk and Golos [8] proposed a model based on a dimensionless plastic zone radius, P_{zr} . Ren et al. [15] proposed a model based on the plastic zone radius and evaluated

it for plane stress and plane strain conditions. Both models were devised for a central crack. Comprehensive reviews of CP models can be found in [1,12].

Although the CP models described here use SIFs (monotonic load), in the literature [12,16,17], a swap between monotonic SIF (K) and fatigue SIF range (ΔK) for proportional load was found to be acceptable. This means that a crack grows under fatigue loading if ΔK_{eq} exceeds the fatigue threshold (ΔK_{th}), or the crack grows under static loading when the equivalent SIF (K_{eq}) exceeds the fracture toughness (K_c), as has been postulated before [16]. Finally, we can say that SENT samples have gained popularity in testing pipelines [18] and welds or offer less restriction [6,19], and C(T) samples offer a larger space to observe CP evolution [20–22]. Lately, some researchers have devoted efforts to evaluating CP in complex problems using artificial intelligence, such as heterogeneous materials [7], metamaterials [23], or composites [24]. However, the computational cost might not be worth it for isotropic materials where parametric and less computationally costly models are sufficient for prediction.

Although both evaluated models were proposed for monotonic central cracks in infinite plates, this paper evaluates them on lateral cracks and, at the same time, assesses the applicability of LEFM in fatigue and monotonic loading. Furthermore, the models are evaluated with experimental data that include non-linear effects such as fatigue-induced plasticity (which induces hardening ahead of the crack tip, which may produce crack retardation), crack roughness, crack interlocking (which might hinder the full development of mode II), and trapped debris (which keep the crack open, thus accelerating mode I propagation). Our literature review revealed that the combination of such non-linearities cannot yet be numerically reproduced. Therefore, the first set of experimental data is for proportional fatigue load [25], whereas the second set is for proportional monotonic load [26], both under different mode mixity ratios (K_{II}/K_I). The first data set was obtained using the digital image correlation (DIC) technique, whereas the second set was obtained through finite element modeling (FEM) and validated with photoelasticity. The performance of both CP models is shown along the crack length.

The rest of the paper is organized as follows: Section 2 describes the basis for the CP used; Section 3 explains the experimental details of where the data were obtained from; Section 4 groups the CP prediction and discusses the results attributable to the appropriate phenomena; and Section 5 draws the conclusions and presents the findings.

2. Criteria for the Crack Path

The stress field in a cracked body under an applied force F is schematically represented in Figure 1, where σ and τ are the normal and shear stress, respectively, r and θ are the radial and tangential directions, respectively, and a is the crack length.

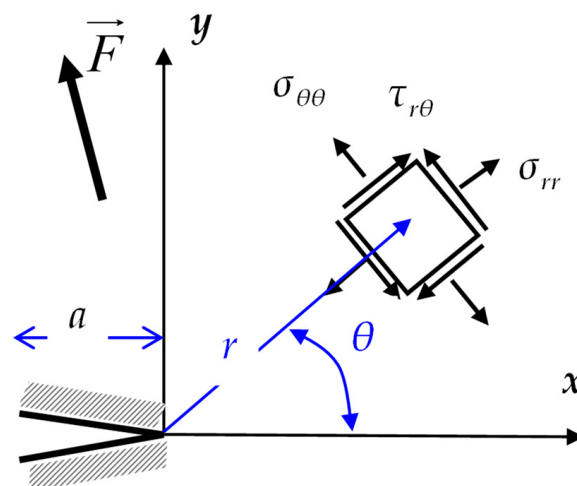


Figure 1. Schematics and notation of acting stress in a cracked body.

In 2000, Wasiluk and Golos proposed the W criterion [8]. It states that a central crack under mixed-mode loading grows in the direction where the radius from the crack tip to the boundary of the plastic zone is the shortest [8]. They started from the radius of the plastic zone, r_{pz} , proposed by Pook [27] for mode I and II loading and compared the stress field with the von Misses yield criteria for a plate with a centrally inclined crack, as shown in Equation (1).

$$r_{pz} = \frac{1}{2\pi\sigma_y^2} \left[K_I^2 \cos^2\left(\frac{\theta}{2}\right) \left(1 + 3 \sin^2\left(\frac{\theta}{2}\right)\right) + K_I K_{II} \sin \theta (3 \cos \theta - 1) + K_{II}^2 \left(3 + \sin^2\left(\frac{\theta}{2}\right) \left(1 - 9 \sin^2\left(\frac{\theta}{2}\right)\right)\right) \right] \quad (1)$$

where σ_y is the yield stress and K_I and K_{II} are the SIFs in modes I and II, respectively. Then, W is the factor of dividing r_{pz} over half the crack length, and using $K_I = \sigma \sin 2\theta \sqrt{\pi a}$ and $K_{II} = \sigma \sin \theta \cos \theta \sqrt{\pi a}$, we obtained Equation (2).

$$W = \frac{\sigma^2}{2\sigma_y^2} \left[\sin^4(\theta_W) \cos^2\left(\frac{\theta}{2}\right) \left(1 + 3 \sin^2\left(\frac{\theta}{2}\right)\right) + \sin^4(\theta_W) \sin(\theta) (3 \cos \theta - 1) + \sin^2(\theta_W) \cos^2(\theta_W) \left(3 + \sin^2\left(\frac{\theta}{2}\right) \left(1 - 9 \sin^2\left(\frac{\theta}{2}\right)\right)\right) \right] \quad (2)$$

Thus, taking the first and second derivative of W , Equation (2) with respect to θ , as seen in Equation (3), gives a $W_{\min}$ at which the direction of crack growth, θ_W , will happen.

$$\left. \frac{\partial W_{(\theta, \theta_*, \sigma)}}{\partial \theta} \right|_{\theta=\theta_*} = 0 \text{ and } \left. \frac{\partial^2 W_{(\theta, \theta_*, \sigma)}}{\partial \theta^2} \right|_{\theta=\theta_*} > 0 \quad (3)$$

Then, if the applied load is cyclic, SIF, K , becomes the SIF range, ΔK , and the fracture toughness, K_c , becomes the fatigue threshold, ΔK_{th} , as proposed and tested in [17], so Equation (1) can be rewritten as Equation (4).

$$r_{pz-W} = \frac{1}{2\pi\sigma_y^2} \left[\Delta K_I^2 \cos^2\left(\frac{\theta}{2}\right) \left(1 + 3 \sin^2\left(\frac{\theta}{2}\right)\right) + \Delta K_I \Delta K_{II} \sin \theta (3 \cos \theta - 1) + \Delta K_{II}^2 \left(3 + \sin^2\left(\frac{\theta}{2}\right) \left(1 - 9 \sin^2\left(\frac{\theta}{2}\right)\right)\right) \right] \quad (4)$$

On the other hand, in 2014, Ren [15] proposed a model based on the PZ radius and evaluated it for plane stress and plane strain conditions for a centrally inclined crack. The radius of the plastic zone is shown in Equation (5).

$$r_{pz} = \frac{1}{2\pi\sigma_y^2} \left[K_I^2 \frac{2(1-2\nu)^2}{3} [(1 + \cos \theta) + \sin^2 \theta] + 2K_I K_{II} \left(\sin 2\theta - \frac{2(1-2\nu)^2}{3} \right) + K_{II}^2 \left(1 + \frac{2(1-2\nu)^2}{3} \right) (1 - \cos \theta) (3 + \cos^2 \theta) \right] \quad (5)$$

where ν is Poisson's modulus. Moreover, assuming the interchangeability of K and ΔK and K_c with ΔK_{th} , Equation (5) for monotonic loading can be rewritten as Equation (6) for fatigue loading.

$$r_{pz} = \frac{1}{2\pi\sigma_y^2} \left[\Delta K_I^2 \frac{2(1-2\nu)^2}{3} [(1 + \cos \theta) + \sin^2 \theta] + 2\Delta K_I \Delta K_{II} \left(\sin 2\theta - \frac{2(1-2\nu)^2}{3} \right) + \Delta K_{II}^2 \left(1 + \frac{2(1-2\nu)^2}{3} \right) (1 - \cos \theta) (3 + \cos^2 \theta) \right] \quad (6)$$

where ΔK is the SIF range, and ΔK_{th} is the fatigue threshold. It is noted how the PZ can give an insight into crack initiation at the shortest radius. Furthermore, an analysis of Equations (4) and (6) gives the angles at which the minimum radius occurs. This is 0° for mode I and 82.5° for pure mode II in the R criterion and 77° in the W criterion. Furthermore, Figure 2 shows how the PZ changes for different mode mixity (K_{II}/K_I) ratios for both the W and R criteria, with the latter plotted for plain stress. The PZ is plotted for different mode mixity ratios, from pure mode I, $K_{II}/K_I = 0$, until pure mode II, $K_{II}/K_I = \infty$, but keeping K_{eq} constant, $K_{eq} = \sqrt{(K_I^2 + K_{II}^2)} = 1$. Both K_I and K_{II} were always kept positive. A negative K_I does not make physical sense as it requires the crack lips to overlap, whereas a negative K_{II} implies the relative displacements of two opposite-to-crack points. It can be seen how the PZ changes the applied load's direction.

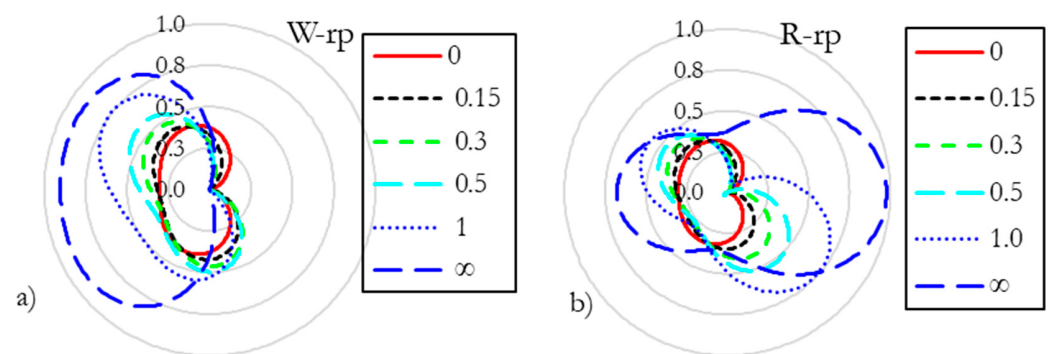


Figure 2. Dimensionless PZ for different mode mixity ratios (K_{II}/K_I). (a) W criterion; (b) R criterion.

One can see in Figure 2 that the crack orientation of the PZ changes from perpendicular to the tensile stress, mode I, to parallel to the shear stress, mode II. Therefore, a parameter, M [9,28], is used to tell whether the crack growth is dominated by the tensile mode or the shear mode, as shown in Equation (7).

$$M = \frac{2}{\pi} \tan^{-1} \left(\frac{K_{II}}{K_I} \right) \quad (7)$$

3. Materials and Methods

The data used to evaluate the two CP models were obtained from two samples subjected to proportional loading tests. The first was an 8.7 mm thick modified compact test C(T) specimen made out of low-carbon steel ($E = 200$ GPa, $\nu = 0.3$) with a drilled hole in front of the CP to modify the stress field [25], as seen in Figure 3a, with the testing setup shown in Figure 3b. Miranda tested a similar sample [29], with the results computationally verified later on [22]. The experimentally observed CP for this sample and the six measured points are shown in Figure 3a. The hole modified the stress field, which curved the crack and induced opening mode II over the applied mode I load. Moreover, because the load inversion ratio was 0.1, the SIF range (ΔK) was calculated as $K_{max} - K_{min}$ from DIC displacement measurements and verified through FEM [30]. Table 1 shows this sample's retrieved crack length, CP angle, SIF ranges, and mode mixity for the six measured points. Finally, the fatigue loading rate for the modified C(T) sample was applied at 10 Hz, and for about every 1 mm of crack growth, the load frequency was lowered to 0.1 Hz to allow for image recording for DIC analysis.

Because the applied load on this sample was cyclic, Equations (4) and (6) were used to establish the sample's PZ radius and shape. Extensive details about this test and specifics on how the SIFs were calculated can be found in [25].

As stated before, the displacement fields for the modified C(T) were measured using the DIC technique. It provides independent displacement fields using non-contact optical measurements. Once displacements are obtained, one can compute strains, and by

using a constitutive law, stresses can be calculated. In-depth details for DIC, including its capabilities, limitations, and implementation, can be found in [30].

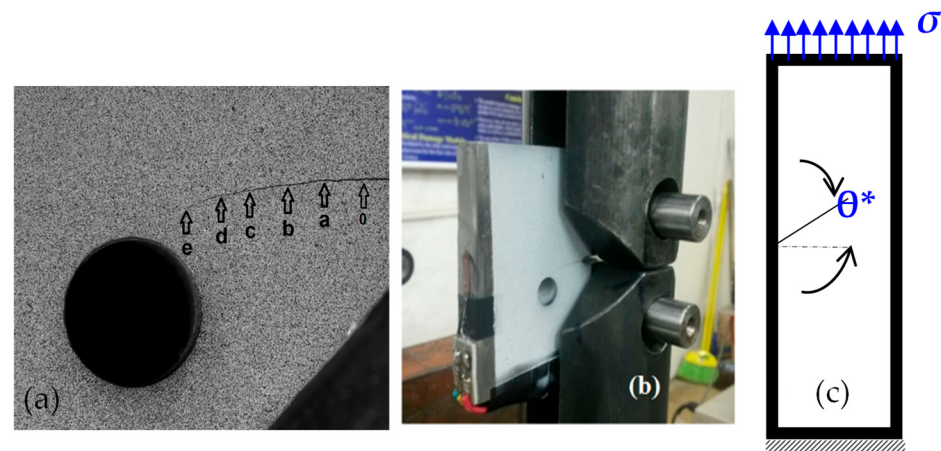


Figure 3. Schematics of the tests. (a) Measured points for the holed CT specimen; (b) testing setup for the holed CT sample; (c) SENT with the inclined crack. (a,b) from [25].

Table 1. Experimental parameters for the modified C(T) at different crack lengths, from [25].

Point	a [mm]	θ^*	ΔK_I , MPa $\sqrt{m}$	ΔK_{II} , MPa $\sqrt{m}$	$\Delta K_{II}/\Delta K_I$
0	2.1	0	13.12	0.46	0.04
a	4.1	0	17.78	0.47	0.03
b	6.3	−5	18.14	0.59	0.03
c	8.2	−5	19.67	1.17	0.06
d	10.2	−7	22.00	1.55	0.07
e	11.9	−24	26.85	3.55	0.13

The second type of samples was two 3 mm thick polycarbonate ($E = 2.5$ GPa, $\nu = 0.38$) SENT samples [26], see the schematics in Figure 3c, subjected to simple remote tension with a 22.5° crack and another with a 45° inclined pre-existing crack producing mixed-mode (I and II) loading conditions. The retrieved data included SIF modes I and II, obtained through numerical simulation and validated with photoelasticity, shown in Table 2, where a/W is the dimensionless residual ligament. Because the applied loading on these samples was monotonic, Equations (1) and (5) were used to establish the PZ radius and shape.

Table 2. Experimental parameters for the thin strips from [26].

a/W	$K_I/(\sigma\sqrt{\pi a})$	$K_{II}/(\sigma\sqrt{\pi a})$	K_{II}/K_I
45°			
0.3	0.83	0.48	0.58
0.4	0.96	0.50	0.52
0.5	1.22	0.57	0.47
0.6	1.42	0.65	0.46
22.5°			
0.3	1.48	0.35	0.24
0.4	1.80	0.41	0.23
0.5	2.28	0.54	0.24
0.6	2.99	0.68	0.23

Finally, the performance of the CP models was evaluated with the expression proposed in Equation (8), as recently proposed [31].

$$err = \frac{\theta_{exp} - \theta^*}{\pi} \quad (8)$$

where θ_{exp} is the experimental CP angle and θ^* is the angle predicted by either model.

4. Results and Discussion

4.1. CP Prediction for Modified C(T)

Figure 4 compares the experimentally observed CP for the W and R criteria predicted angle for the six measured points in the modified C(T) sample. One can see that the prediction is very close for short cracks as the mode mixity ratio, $\Delta K_{II}/\Delta K_I$, is low. However, at the last measurement before unstable crack growth, the $\Delta K_{II}/\Delta K_I$ rises to 0.13, and the predicted angle gives a 6% error for the W criterion. In contrast, the R criterion shows a 1.7% error. The incremental change in mode mixity is attributed to the machined hole, which caused an asymmetry to the stress field, producing larger stresses toward the hole that ultimately caused the crack to kink towards it. As the crack grew, the stress field became even more asymmetric, continuously increasing K_{II} , kinking the crack more pronouncedly every loading cycle until it finally met the hole.

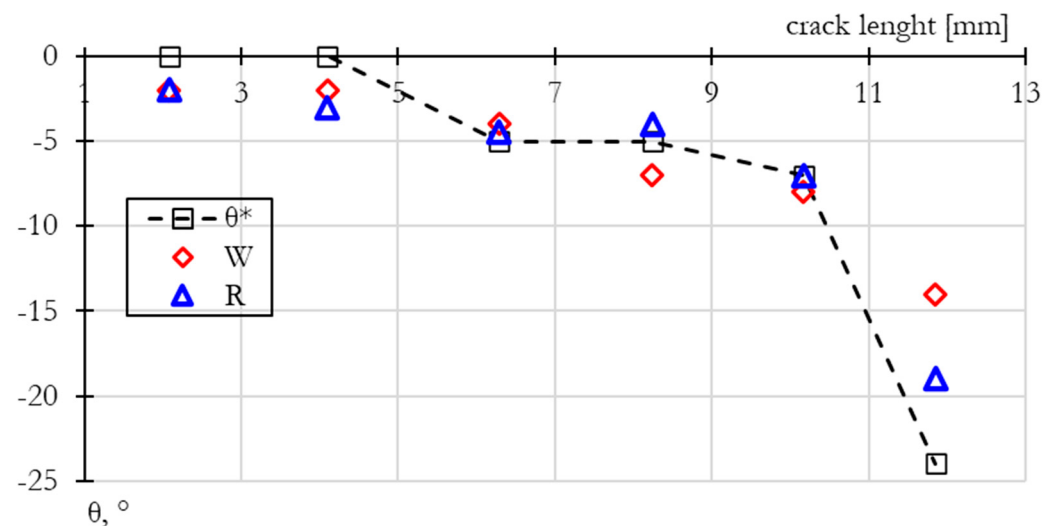


Figure 4. Evaluation of crack kinking models for the modified C(T) sample.

The W criteria made a close prediction for most of the crack lengths. Figure 5 shows the comparative size and shape of the PZ, Equation (4) W criterion, and the Equation (5) R criterion, for six different crack lengths in this specimen. It can be seen how the PZ orientation turns as the crack length increases and the mode mixity changes. This turning is attributed to the increase in ΔK_{II} , which makes the crack deviate from the applied mode. I load Moreover, it can be observed that the size of the largest PZ, about 2.54 mm for the W criterion and about 2.9 mm for the R criterion, in their largest axis, which are below the thickness of the specimen, 8.7 mm. Thus, LEFM adequately describes the stress field. For this sample, the W criterion showed larger PZ for the first five crack sizes, whereas the R criterion retrieved a more extensive zone for the last one, where K_{II} reaches $3.55 \text{ MPa}\sqrt{\text{m}}$ and the mode mixity ratio, K_{II}/K_I , is 0.13.

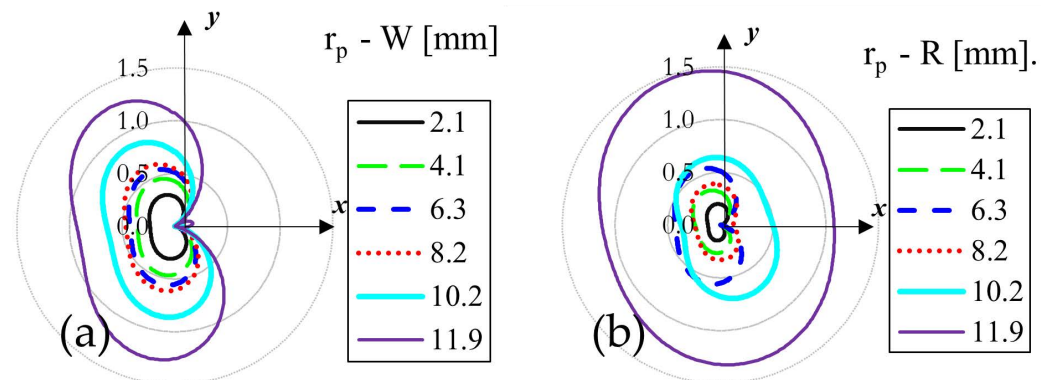


Figure 5. Size and shape of plastic zone for the modified C(T) specimen at different crack lengths: (a) from the W criterion and (b) from the R criterion.

4.2. CP Prediction for SENT Samples

On the other hand, the performance of the CP models for both SENT specimens for dimensionless crack length is shown in Figure 6. The models were evaluated with the retrieved K_I and K_{II} . Unlike the C(T) sample, the applied load is monotonic, so there is no need to assume that K is ΔK and K_c is ΔK_{th} . The SENT with a 45° crack showed a K_{II}/K_I ratio from 0.46 at the longest to 0.58 for the shortest a/W . Polycarbonate is a material prone to shear-dominated failure, and the 45° crack induces a high K_{II}/K_I ratio. Therefore, the W criterion is not expected to perform well under these conditions, so it predicts a quasi-constant 29° , whereas the R criterion predicts a closer CP between 43 and 47° .

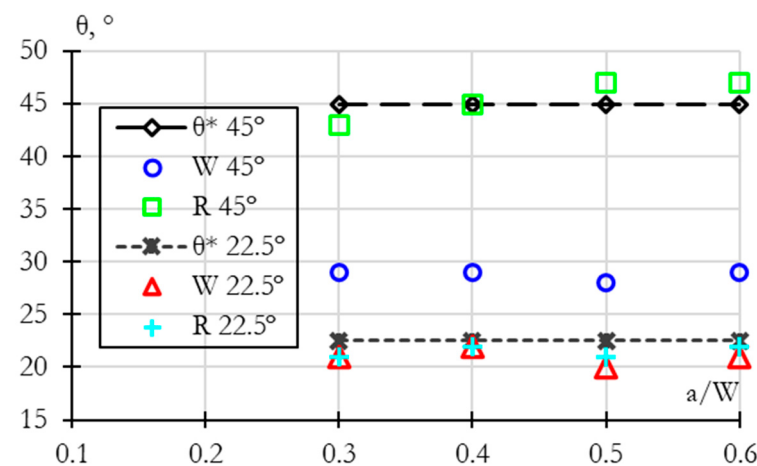


Figure 6. Evaluation of crack kinking models for SENT samples.

Conversely, the SENT with a 22.5° crack showed a quasi-constant K_{II}/K_I ratio of 0.23. Because of this constant K_{II}/K_I ratio, the predicted CP angle is also quasi-constant, about 21° degrees for the W criterion, very close to the experimentally observed angle of 22.5° . Conversely, the R criterion predicts a quasi-constant angle of 17° .

The shape and size of the plastic zone are shown in Figure 7 for the SENT sample with an initial crack of 45° for the four different reported crack lengths. It can be seen how the orientation of the plastic zone stays pretty much constant as the crack length increases for the W and R criteria. This situation is attributed to the steady K_{II}/K_I ratio, as seen in Table 2, making the crack grow straight from its original path, even though only mode I is applied. Moreover, it can be observed that the size of the largest plastic zone on its largest axis is about 0.7 mm for the W criterion and 0.58 mm for the R criterion. In both cases, this size is below the thickness of the specimen, 3.0 mm, so LEFM can be used to describe the stress field satisfactorily. Finally, it is noted that the W criterion predicts a slightly larger PZ than the R criterion.

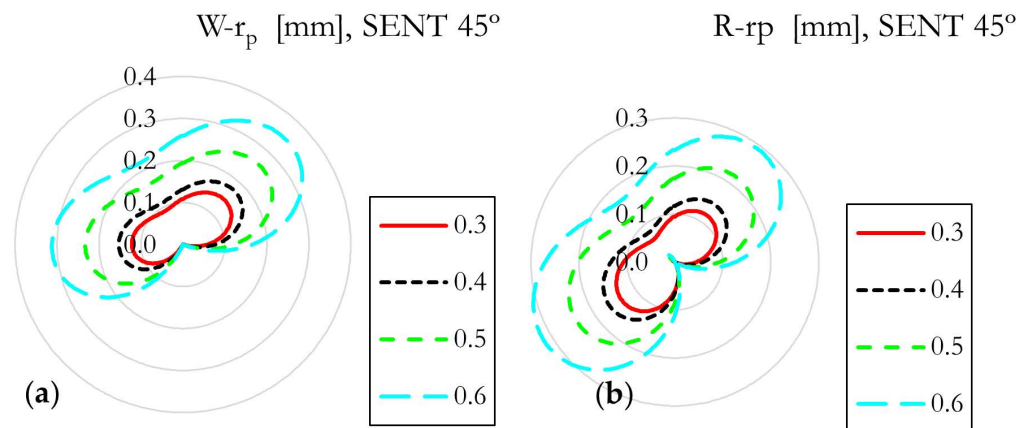


Figure 7. Size and shape of the plastic zone for the SENT sample 45°: (a) from the W criterion, (b) from the R criterion.

Figure 8 shows the PZ size for the SENT sample with an initial crack of 22.5° for the four different reported crack lengths. It can be seen how the orientation of the plastic zone stays pretty much constant as the crack length increases. This situation is attributed to the steady K_{II}/K_I ratio, making the crack grow straight from its original inclined path, even though there is an applied load in mode I. Moreover, it was observed that the size of the largest PZ, about 1.8 mm for the W criterion and about 1.54 mm for the R criterion in its largest axis, is below the thickness of the specimen, 3.0 mm, so LEFM can be used to properly describe the stress field. In this sample, it was also observed that the W criterion predicts a slightly larger PZ than the R criterion.

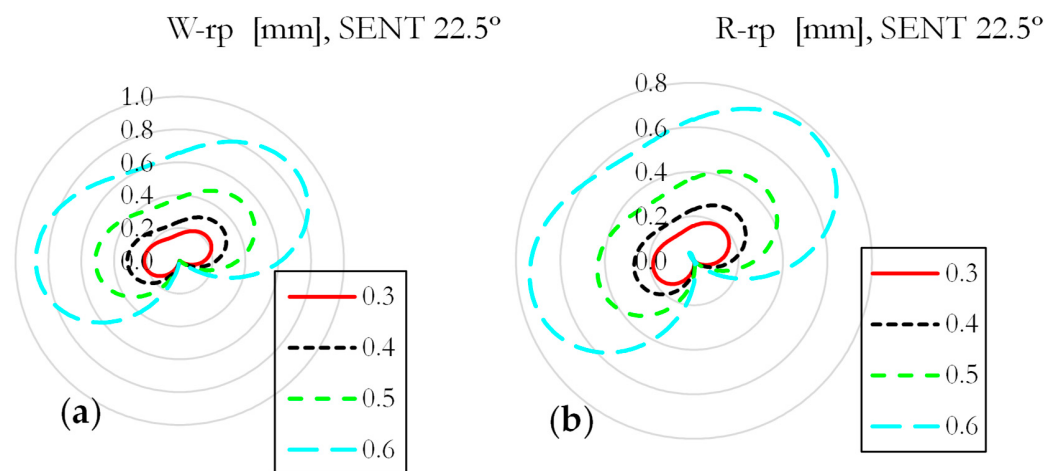


Figure 8. Size and shape of the plastic zone for the SENT sample 22.5°: (a) from the W criterion and (b) from the R criterion.

4.3. Discussion

Highsmith [10] stated that a crack under mixed mode will grow when the equivalent SIF exceeds the material toughness, K_c , or the equivalent SIF range exceeds the material fatigue threshold, K_{th} . In both cases, the crack grew under either monotonic or fatigue loading. Therefore, the K_{eq} was larger than the K_c for the monotonic load, and the ΔK_{eq} was larger than the K_{th} for the cycling load. However, because more than one loading mode was present, the CP must be established to describe crack growth completely [31].

On the other hand, Vormwald et al. [9] argued that the mode mixity ratio [28] might be the parameter that can be used to tell when a crack growth scenario can change from tensile to shear-dominating load. It could make a crack turn when it reaches 0.5. Figure 9 shows the variation in the CP angle with mode mixity for both samples. For the SENT

samples, neither the load nor the mode mixity changed much with crack length. For the 22.5° SENT sample, the K_{II}/K_I ratio kept a 0.24 constant value, whereas for the 45° sample, the K_{II}/K_I ratio went from 0.56 to 0.46, as seen in Figure 6. Vormwald et al. [9] argued that when transitioning from tensile to entirely dominated shear crack growth, such mode mixity parameters might not be able to tell the CP. Therefore, the CP for the SENT samples was not expected to deviate from its initial angle. This can be seen in Figure 9a for the three samples and Figure 9b for a close-up of the SENT 22.5° sample.

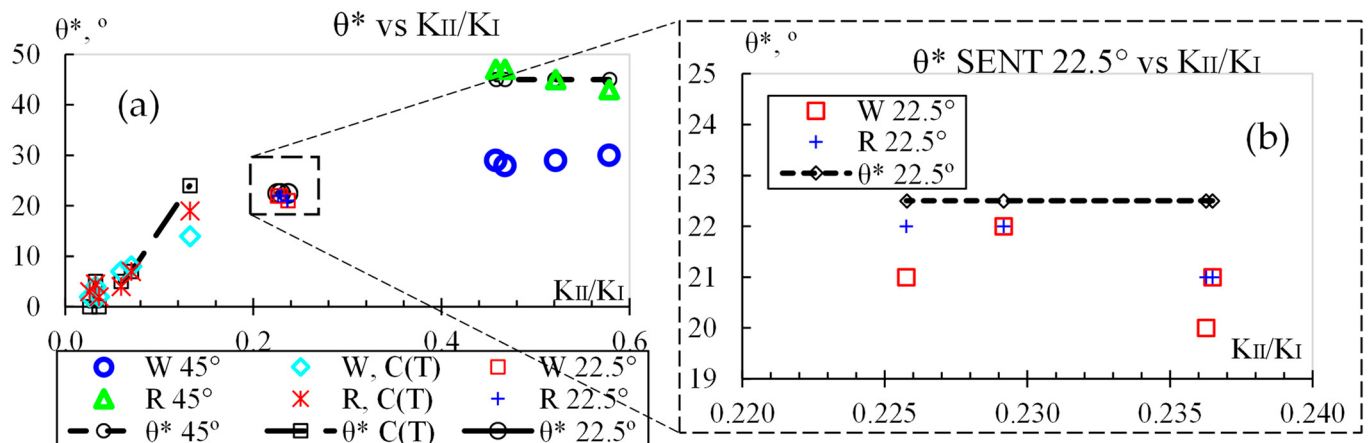


Figure 9. CP angle variation with mode mixity: (a) modified C(T) and SENT samples and (b) close-up of the SENT 22.5° sample.

On the other hand, for the modified C(T) sample, the mode mixity changes constantly, as explained in Section 4.1, and is attributed to the constant change in the stress field, starting at 0.04 and reaching 0.13. As a result, the CP for the modified C(T) sample changes as well, as seen in Figure 9 in the black squares. Thus, the mode mixity indeed influences the CP angle. This analysis shows that a change in the K_{II}/K_I ratio involves changing the CP angle.

Furthermore, in Figure 8, one can see that the size of the largest plastic zone is about 1.5 mm, so it does not exceed the holed C(T) sample thickness, 8.7 mm. On the other hand, the largest plastic zone size for the SENT sample is seen in Figure 7 for the 22.5° sample and Figure 8 for the 45° sample, where the plastic zone does not exceed the 3 mm sample thickness. Therefore, the applicability of LEFM is checked at once.

In [25], one can see how the traditional butterfly-like plastic zone in the modified C(T) evolved to be asymmetric, showing larger areas and higher stress values towards one side, the side of the stress concentrator. That stress asymmetry is assumed to cause the crack to kink. One can see in Figure 5 how the PZ turns. Although [26] did not show the evolution of the fringes for the SENT samples, they show how the fringes also exhibit asymmetry, piling up on one side. This fringe concentration reflects the stress gradient that most likely produces the crack to deviate from the direction of the applied load. The orientation change of the PZ on the C(T) or the quasi-constant orientation for the SENT is interpreted as the angle for crack kinking. Therefore, the PZ stays the same, even for the relatively large error shown by the W model. In addition, it is important to note that the C(T) sample was produced via fatigue loading, whereas the SENT sample was initially cut with a sharp razor blade and later propagated under monotonic loading. This is important because a flaw with a blunt radius does not fully comply with LEFM.

On the other hand, the CP prediction error is shown in Table 3. Both criteria are close in prediction, with errors as low as 0.28% and as high as 1.67% for the modified C(T) sample. For the largest mode mixity on the modified C(T) sample, the W criterion gives 5.56, whereas the R criterion shows a 2.78% error. The same case for the SENT 45° sample is 8.33 versus 1.11% error. In both cases, the mode mixity ($\Delta K_{II}/\Delta K_I$) had a wide range of

about 0.1. For the SENT 22.5 sample, both are close to each other; the largest difference is 0.56%. In this case, this could be explained by the quasi-constant $\Delta K_{II}/\Delta K_I$.

Table 3. Error percentage in crack kinking angle prediction.

C(T)			SENT 45°			SENT 22.5°		
$\Delta K_{II}/\Delta K_I$	W, C(T)	R, C(T)	$\Delta K_{II}/\Delta K_I$	W 45°	R 45°	$\Delta K_{II}/\Delta K_I$	R 22.5°	R 22.5°
0.04	−1.11	−1.11	0.58	8.33	1.11	0.24	0.83	0.83
0.03	−1.11	−1.67	0.52	8.89	0.00	0.23	0.28	0.28
0.03	0.56	0.28	0.47	9.44	−1.11	0.24	1.39	0.83
0.06	−1.11	0.56	0.46	8.89	−1.11	0.23	0.83	0.28
0.07	−0.56	0.00						
0.13	5.56	2.78						

As a final note, it has to be said that the computational cost of these two criteria is high compared to other ones, as recently evaluated [31]. For this reason, the computational cost, in terms of floating-point operations (FLOPS), using a benchmark from [32], is compared in Table 4. The R criterion is slightly less costly than the W criterion.

Table 4. Computational cost for the evaluated criteria; baseline values from [32].

Operator	+, −, ×, /	√	Sin, Cos, Atan	Acoss	Tan	ABS, SGN	^	Total Cost
Operator Cost	2	2	5	4	6	2	8	
Cost in W model	23	0	10	0	0	0	10	176
Cost in R model	33	0	5	0	0	0	8	155

5. Conclusions

The crack path was calculated for specimens under fatigue and monotonic load using the LEFM-based plastic zone radius criteria. The LEFM parameters for the fatigued sample were characterized experimentally with DIC, whereas the monotonic samples' SIFs were obtained numerically through FEM and validated with photoelasticity; for both samples, SIFs were obtained from the literature. An advantage of the plastic zone-based criteria is that the applicability of LEFM is checked in the same equation. Furthermore, using prediction models rather than FEM modeling or re-meshing makes a faster prediction by avoiding time-consuming modeling and simulation. Finally, the selected models worked for numerical and experimentally obtained SIFs. The literature has discussed how the experimentally acquired SIFs include non-linear effects such as rugosity, plasticity-induced closure, trapped debris, and crack flank interlocking.

The W and R criteria were devised for monotonic load, and here, they were tested for monotonic and fatigue load. The swap of K with ΔK was checked, which was applicable in these cases. The error analysis showed that the W and R criteria predicted angles very close for short cracks as the mode mixity ratio, $\Delta K_{II}/\Delta K_I$, is below 0.25, with the R criterion exhibiting lower error values. However, at the last measurement for the modified C(T) low carbon steel sample before unstable crack growth, the $\Delta K_{II}/\Delta K_I$ doubles its value and makes an unstable W criterion prediction. The R criterion has about 88% of the W criterion's computational cost, which could impact performance for recurrent calculations. Furthermore, the models were devised for central cracks, and here, they were tested in lateral cracks. The predicted versus experimental CP comparison showed that the models could be used for fatigue and monotonic loads.

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Acronyms and Nomenclature

Plastic zone	PZ
Crack path	CP
Single edge notch tension	SENT
Compact test	C(T)
Linear elastic fracture mechanics	LEFM
Floating-point operations	FLOPS
Increment in crack kinking angle	$\Delta\theta$
Stress intensity factor	SIF, K
Equivalent stress intensity factor	K_{eq}
Stress intensity factor range factor	ΔK
Plastic zone radius	Pzr.
Fatigue threshold	ΔK_{th}
Digital image correlation	DIC
Finite element modeling	FEM
Applied force	F
Normal stress	σ
Shear stress	τ
Radial direction	r
Tangential direction	θ
Yield stress	σ_y
Poisson's modulus	ν
Residual ligament size	(a/W)
Measure crack kinking angle	θ_{exp}
Predicted crack kinking angle	θ^*

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