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Experimental Investigation of Shear Behavior in High-Strength Concrete Beams Reinforced with Hooked-End Steel Fibers and High-Strength Steel Rebars

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Abstract: Shear failure is an unfavorable phenomenon as it is a brittle type of failure; however, adding rebars and fibers to a concrete beam can minimize its detrimental effects. The objective of this study was to experimentally investigate the shear behavior of high-strength concrete (HC) beams reinforced with hooked-end (H) steel fibers and high-strength steel (HS) rebars under three-point bending tests. For this purpose, nine HC beams (300 × 250 × 1150 mm in dimension) were cast with 0%, 1%, and 2% H fibers by volume in three longitudinal rebar ratios (i.e., 1.5%, 2.0%, and 3.1%) and compared with beams without fibers. Furthermore, numerical analyses were performed to validate the experimental results and compare them with design codes. The results showed that, irrespective of the fiber content or longitudinal rebar ratio, the beams failed in shear. Increasing the rebar ratio and fiber content increased the shear capacity to as high as 100% (for the specimen with 3% rebar and 2% fiber compared to its counterpart with 1% rebar and 2% fiber). In addition, the research-based equations proposed in the literature either overestimated or underestimated the shear capacity of fibrous HC beams significantly. The level of overestimation or underestimation was closely related to the sensitivity of the proposed model to the shear span ratio and the fiber content. Rebars proved to be more beneficial in contributing to the shear capacity, but the rate of this positive contribution decreased as the fiber ratio increased. Finally, the inverse analysis approach adopted herein proved to be an efficient tool in estimating the shear response of fiber-reinforced beams failing in shear (margin of error: less than 10%).

Keywords: fiber-reinforced concrete (FRC); high-strength concrete (HC); high-strength steel (HS); hooked-end steel fiber



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1. Introduction

The expansion and progress of the construction sector and infrastructure amenities worldwide necessitate the utilization of concrete. To mitigate the environmental impact, it is imperative to curtail concrete ingredient consumption by enhancing its material strength. Apart from decreasing concrete volume, employing high-strength concrete augments its mechanical attributes and resilience against environmental and chemical factors. Recent technological and industrial advancements have significantly increased the compressive strength of cementitious materials. Conventional concrete has given way to specimens with compressive strengths exceeding 100 MPa (notably, ACI 363R [1] defines high-strength concrete (HC) as having a compressive strength surpassing 55 MPa). Despite this improved strength, concrete still exhibits weaknesses in tension and under conditions of fire or elevated temperatures [2]. High-strength concretes bolster the compressibility of the mix, elevating the strength of cement paste and its interface with coarse aggregates. In

these cases, cracks form more gradually with fractured surfaces, often extending through the aggregates. This type of failure diminishes the shear capacity relative to the compressive capacity, raising concerns about the shear strength [3]. Consequently, diverse fibers—synthetic, mineral, steel, and natural—have been introduced to concrete. Experimental studies have demonstrated that sufficient steel fiber incorporation enhances the shear capacity of reinforced concrete. Steel fibers bolster concrete's tensile strength, limiting crack formation and growth, promoting crack containment, and improving the tensile crack distribution [4,5]. Research on fiber-reinforced concrete (FRC) underscores the advantageous characteristics fibers confer, depending on the fiber type and loading conditions. This encompasses topics such as the flexural behavior of FRC beams using single and hybrid steel fibers [6–10], shear behavior [11–13], performance under cryogenic temperatures [14,15], seismic response [16], effects of age [17], and more. The addition of fibers, especially steel ones, enhances not only tensile strength but also other crucial parameters in concrete behavior, such as compressive, shear, and flexural strength, corrosion resistance, and freeze–thaw resilience.

Studies indicate that the performance of fiber-reinforced concrete hinges on the fiber type and arrangement. Influential factors regarding steel fibers, including the type, shape, aspect ratio, volume fraction, creep, and tension, which affect the bending and shear strength in various reinforced concrete scenarios, will be discussed further:

Laxmi et al. [18] scrutinized the impact of hooked-end steel fibers on the potency and endurance of geopolymer concrete cured under ambient conditions. They formulated geopolymer concrete using fly ash, ground granulated blast furnace slag, manufactured sand, natural coarse aggregates, and hooked-end steel fibers with an aspect ratio of 67. Varying volume fractions of 0%, 0.5%, 1%, 1.5%, and 2% were tested. Findings revealed that incorporating steel fibers at an optimal 1% dosage enhanced the compressive, split tensile, and flexural strength, contributing to environmentally friendly construction materials.

An inventive design approach (with a relatively high steel fiber content) was introduced to develop high-performance fiber-reinforced concrete (HPFRC) with appropriate mechanical attributes for manufacturing precast prestressed concrete components [5]. The mix included cement, fly ash, limestone filler, superplasticizer, water, three aggregate types, and 35 mm long hooked-end steel fibers. These fibers had an aspect ratio of 64 and a yield stress of 1100 MPa. This work yielded a design flowchart for determining the optimal material dosage.

Hoang and Nguyen [19] conducted a comparison of the flexural and tensile behaviors of ultra-high performance fiber-reinforced concrete (UHPFRC) using different steel fiber types. They employed six high-strength steel fiber types, consisting of four micro steel fibers (two smooth straight fibers and two crimped fibers) and two indented macro steel fibers (hooked-end and straight). Pull-out tests were performed to ascertain the average fiber-matrix bond strength, while the flexural and tensile performances of UHPFRC with these steel fibers were investigated. Their results demonstrated that employing 1.5 vol.% of the most effective fiber (with $L_f/D_f = 100$) guarantees the attainment of a strain-hardening UHPFRC with an ultimate tensile strength exceeding 10 MPa.

Furthermore, Mineiro et al. [20] introduced an approach for converting the experimental findings of the pullout tests of hooked-end steel fibers from a cementitious matrix into a multiscale model. This model utilized a discrete and clear representation of the steel fibers. The authors aimed to precisely elucidate the interaction between the fibers and the matrix, specifically focusing on the anchoring mechanism of hooked-end steel fibers. By comparing the results of experimental pullout tests conducted on hooked-end steel fibers and straight fibers, they derived input parameters to explain the anchorage mechanism. The outcomes highlight the potential utility of combining experimental and multiscale methods to establish connections between the physical and numerical behaviors of SFRCC (steel fiber-reinforced cementitious composite) containing hooked-end steel fibers. Consequently, the resulting numerical tool offers valuable insights into comprehending the failure processes in this particular type of composite material.

Venugopal et al. [21] conducted a statistical analysis on hooked-end steel fibers in concrete. They employed machine learning techniques like non-linear regression on a dataset of 146 samples gathered from the literature. Their aim was to create predictive models for the compressive and flexural strengths of hooked-end concrete. The study focused on four main parameters: fiber factor, water-to-binder ratio, coarse aggregate content, and silica fume content (referred to as the fiber factor). Notably, when adjusting the water-to-binder ratio, there was a significant enhancement in the compressive strength from 55 MPa to 85 MPa at an aspect ratio of 60. Moreover, the flexural strength experienced a substantial improvement of around 200% to 300%, with an increase in the fiber factor from 0 to 150.

Khakseifi et al. [22] investigated the behavior of bond-slip in HSS rebars embedded within ultra-high-performance concrete (UHPC) and normal strength concrete (NSC) specimens. The study involved 60 cubic concrete specimens reinforced with a central rebar and different bond lengths (2, 4, and 6 times the diameter) using high-strength and normal-strength rebars. Their findings indicated that AIV rebars outperformed conventional AIII rebars in load-bearing capacity. The bond strength ratio between UHPC and NSC was found to be 5, eliminating the need for extensive embedment lengths. Unlike NSC specimens, UHPC specimens demonstrated reduced bond stress as the bond length increased.

Iftekhair et al. [23] delved into the long-term creep effects of steel fiber reinforcement in ultra-high-performance fiber-reinforced concrete (UHPC). Their study explored the impact of fiber embedment length, angle, and sustained load on pull-out creep. They concluded that varying the embedment length had a minimal influence on the pull-out creep in hooked-end fibers. However, the angle between loading and the fiber-embedded axis played a significant role in concrete breakage at the loading end during sustained loading.

Alternative methods to boost the shear capacity of concrete beams include the utilization of carbon fiber-reinforced polymer (CFRP) and externally bonded reinforcement (EBR) techniques. Nikoloutsopoulos et al. [24] compared different forms of shear strengthening, such as side-bonded FRP strips, U-FRP wraps, and FRP wraps, to minimize drilling and costs. They highlighted the advantages of CFRP, including its high tensile strength, lightweight nature, and easy installation.

Quada and Mashrei [25] performed an experimental study on the shear strength of steel fiber concrete beams strengthened with CFRP through various techniques. Their findings showcased that higher fiber volume fractions led to increased shear strength and ductility. SFRC beams reinforced with CFRP exhibited a 33% to 68% higher load-carrying capacity compared to those without steel fibers. Strengthening beams with CFRP sheets enhanced both strength and stiffness.

Regarding reinforcing rebars, it is important to note that steel reinforcing bars are termed HSS bars when their yield strength exceeds 400 MPa (ACI 439.6 [26]). These bars go by different names based on regional standards, like ASTM A706 Grade 550 [27] and ASTM A1035 Grade 690 and Grade 830 [28] in the United States and Canada and grade 500E and 500N in Australia and New Zealand [29].

An overall examination of the literature reveals a scarcity of studies on HC beams reinforced with hooked-end (H) steel fibers and high-strength steel (HSS) rebars. This study aims to contribute to this limited body of knowledge by casting nine HC beams with varying ratios of H fibers (volume fractions of 0%, 1%, and 2%) and HSS rebars (reinforcement longitudinal ratios of 1.5%, 2%, and 3.1%). The beams then underwent three-point bending tests, and the subsequent sections discuss the results, including the modulus of elasticity, energy absorption, cracking patterns, shear strength, and other pertinent parameters aligned with the study's objectives.

2. Experimental Program

2.1. Materials and Mix Design

The high-strength concrete utilized for creating our nine samples consisted of the subsequent elements: Type II Portland cement, silica fume that had a maximum size of 229 μm , a liquid form of a high-range water-reducing admixture (HRWRA) which was formulated with polycarboxylates (known as AURAMIX), finely sifted sand passed through a No. 16 sieve (less than 1.1 mm), gravel with the largest dimension reaching 10 mm, fibers with hooked ends, and water. The measurement of fibers incorporated in the concrete mixture was determined by the portion of volume they occupied. The L_f/d_f ratio of the fibers, referred to as the aspect ratio, is an important parameter taken into consideration in fiber technology. Table 1 shows the constituents and materials of the mix design used in making the concrete beam specimens of 0%, 1%, and 2% fiber by volume fraction. Similarly, Table 2 shows the properties of H steel fibers and Table 3 shows the properties of AIV reinforcing rebars.

Table 1. Mix design for specimens.

Specimen ID	Cement	Micro Silica (kg/m ³)	Fine Sand (kg/m ³)	Gravel (kg/m ³)	Water (L)	SP (L/m ³)	Fiber (kg/m ³)	W/C	W/B
HC-F0	650	70	700	1030	130	20	0	0.2	0.18
HC-F1	650	70	700	985	130	20	78	0.2	0.18
HC-F2	650	70	700	950	130	20	156	0.2	0.18

SP: superplasticizer; W: water; C: cement; and B: binder.

Table 2. Properties of hooked-end steel fibers.

d_f (mm)	L_f (mm)	(L_f/d_f)	Density (g/cm ³)	f_t (MPa)	E_f (GPa)
0.8	30	37.5	7.5	1900	200

d_f : diameter; L_f : length; f_t : tensile strength; and E_f : elastic modulus.

Table 3. Properties of high-strength steel rebars.

Reinforcement	E (GPa)	f_y (MPa)	f_u (MPa)
AIV	200	500	650

E : modulus of elasticity; f_y : yield strength; and f_u : ultimate tensile strength.

Figure 1 shows the steel fiber shape and high-strength concrete with fibers during mixing. Steel fibers were added to the concrete during mixing to have the best orientation.



Figure 1. (a) steel fibers (b) high-strength concrete with fibers during mixing.

2.2. Casting and Curing of Specimens

After the concrete was placed in formworks, the specimens were subsequently held in the curing tank for 28 days. Nine concrete beams were made with dimensions of $300 \times 250 \times 1150$ mm according to Table 4 below. In Table 4, HSS rebars in three ratios, namely, 1.50%, 2.50%, and 3.10%, were incorporated into HC specimens with 0%, 1%, and 2% H fiber by volume. The notation used for the beams was as follows: HC: high-strength concrete; HS: high-strength rebar; and F: hooked-end fiber. The digits following “HS” denote the longitudinal reinforcement ratio, i.e., “1” denotes 1.50%, “2” denotes 2.00%, and “3” denotes 3.10% rebar ratios. Similarly, the digits following the letter “F” denote the fiber content by volume. The formworks, curing specimens, and the final specimens ready for the test are shown in Figure 2a–c, respectively. For each beam, three 100 mm cubic and three 150×300 mm cylindrical specimens were tested for the compression and modulus of elasticity tests. The findings presented in Table 4 suggest that the inclusion of fibers has a minimal impact on the compressive strength of the HC samples.

Table 4. Details of the specimens and their compressive strength.

ID	No. of Rebars	ρ (%)	Fiber Ratio (% by Volume)	f'_c (No Fiber) (MPa)	f'_c (1% Fiber) (Mpa)	f'_c (2% Fiber) (Mpa)
HCHS1F0	3 Φ 20	1.50	0	84.2	-	-
HCHS2F0	2 Φ 22 + 1 Φ 25	2.00	0	84.2	-	-
HCHS3F0	4 Φ 25	3.10	0	84.2	-	-
HCHS1F1	3 Φ 20	1.50	1	-	86.1	-
HCHS2F1	2 Φ 22 + 1 Φ 25	2.00	1	-	86.1	-
HCHS3F1	4 Φ 25	3.10	1	-	86.1	-
HCHS1F2	3 Φ 20	1.50	2	-	-	91.2
HCHS2F2	2 Φ 22 + 1 Φ 25	2.00	2	-	-	91.2
HCHS3F2	4 Φ 25	3.10	2	-	-	91.2

Note: ρ : longitudinal reinforcements ratio; f'_c : average compressive strength (Mpa).



(a)



(b)

Figure 2. Cont.



(c)

Figure 2. Preparation of the HC beams: (a) formwork and rebars, (b) curing specimens, and (c) final specimens.

2.3. Uniaxial Compression Tests

A sequence of compression examinations was conducted on cylindrical specimens measuring $100 \times 100 \times 100$ mm and 150×300 mm. The modulus of elasticity evaluations were executed in accordance with ASTM C469/C469M [30] and using cylindrical samples sized at 150×300 mm. The outcomes were derived from the average of three specimens, as given in Equation (1):

$$E_c = \frac{0.4f'_c - f_{c1}}{\varepsilon_2 - 0.00005} \quad (1)$$

where E_c represents the modulus of elasticity for the mixture; $0.4f'_c$ signifies the compressive stress linked to strain ε_2 ; and f_{c1} is the compressive stress corresponding to an axial strain of 0.00005. The axial strains across the cylinder's height were monitored via LVDTs affixed to a set of parallel rings, as illustrated in Figure 3. The outcomes of the tests are laid out in Table 5.



Figure 3. Modulus of elasticity test setup.

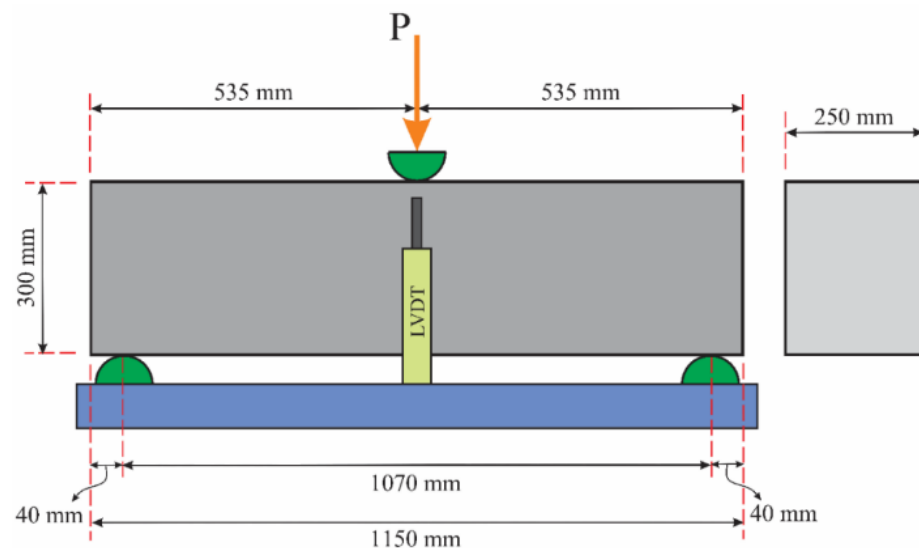
Table 5. Compression and moduli of elasticity properties of the specimens.

Specimen ID	f'_c (MPa)	ϵ_2 (mm/mm)	f'_{c1} (Mpa)	E (GPa)
HC-F0	84.2	0.00109	1.81	30.54
HC-F1	86.1	0.00104	2.74	31.76
HC-F2	91.2	0.00101	3.87	33.94

ϵ_2 : strain corresponding to $0.4f'_c$; f'_{c1} : compressive stress corresponding to an axial strain of 0.00005.

2.4. Three-Point Bending Tests

Three-point bending tests were carried out on beams with a clear span of 1070 mm and a shear span to a depth of 2.14 ($a/d = 2.14$ where a is the shear span and d is the effective depth of the beam) according to ASTM C1609/C1609M [31]. The beams were loaded with a three-point flexural load until failure and were carried out at a rate of 0.5 mm/min in a displacement-controlled manner with LVDTs recording mid-span deflections. The load was measured via a load cell attached to the bottom of the crosshead. Tests were performed on a three-point bending machine in a rigid Lab at Islamic Azad University, Maragheh Branch. A schematic configuration of the three-point bending test setup is shown in Figure 4.

**Figure 4.** Three-point bending test details.

Furthermore, to determine the shear capacity of the beams based on the recommendations of the FIB Model Code [32], notched beams were cast with a 1% and 2% fiber content by volume with an initial notch 5 mm in width and 25 mm in depth. The so-called beams were tested under three-point bending tests to plot the load-crack mouth-opening displacement (CMOD) curves and determine the residual flexural strength values; $CMOD_1 = 0.5$ mm and $CMOD_3 = 2.5$ mm according to Equation (2) given below and presented in Figure 5a,b:

$$f_{Rj} = \frac{3f_j L}{2bh_{sp}^2} \quad (2)$$

where f_{Rj} is the residual flexural tensile strength corresponding to $CMOD_j$; f_j is the load corresponding to f_j ; L is the span length; b is the specimen width, and h_{sp} is the distance between the notch tip and the top of the specimen.

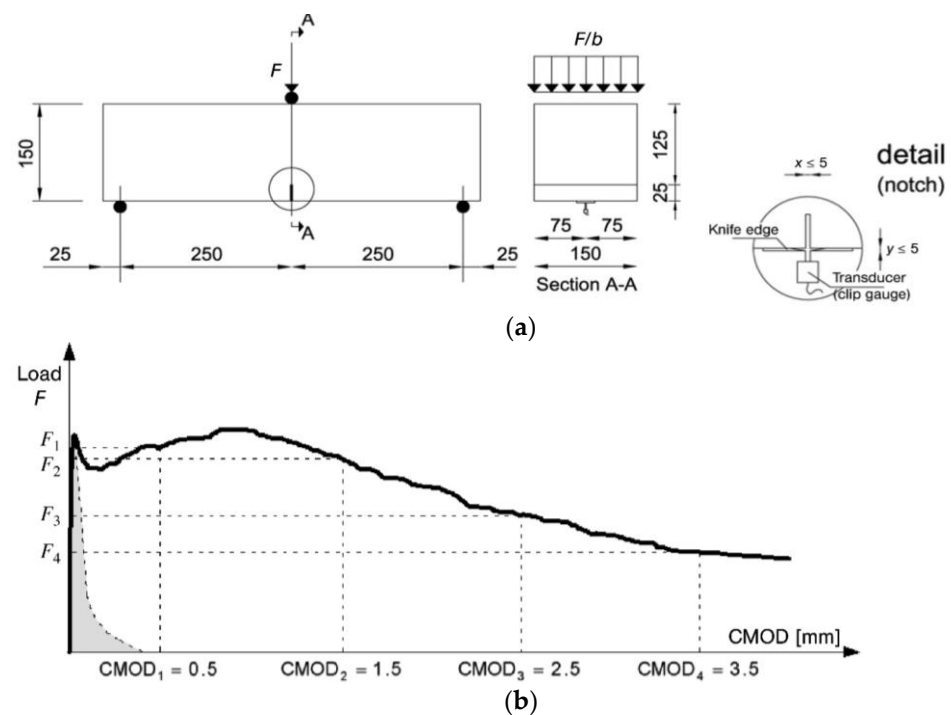


Figure 5. Three-point bending test of the FIB Model Code [32]: (a) notched beams and (b) definition of $CMOD_{j(j=1-4)}$.

3. Results and Discussions

3.1. Modulus of Elasticity

A stress-strain curve for the cylindrical specimens is depicted in Figure 6, highlighting the section used to calculate the modulus of elasticity. The obtained average value of 32 GPa is compared with the values predicted by the existing equations for the HC beam elasticity modulus in the literature [33–38], as displayed in Table 6. The table demonstrates that these equations generally provide reasonable estimations for the modulus of elasticity; except for Kollmorgen’s equation [33], which notably overestimates it by 61%. This overestimation can be attributed to the fact that Kollmorgen’s equation is designed for a wide array of concrete specimens varying greatly in compressive strengths. Suksawang et al.’s equation [38] only considers fiber ratios regardless of their types. In contrast, Lee et al.’s equation [35], unique to this study, directly incorporates fiber geometry into the modulus of elasticity value, while other equations indirectly involve fiber by relying on the compressive strength. All equations tend to overestimate the response, with Lee et al.’s [35] equation providing the closest approximation to reality with a 9% error. Despite extensive research on concrete’s modulus of elasticity, various factors influencing different concrete types have made accurate parameter estimation challenging. Evidently, practical codes have set thresholds for different cementitious material classes, beyond which material characteristics diverge. Anticipating a single equation to effectively represent the mechanical properties of diverse concrete classes is implausible.

3.2. Cracking Pattern and Failure of Beams

Figure 7 illustrates the pattern of cracks in the test beams during the ultimate stage. Initially, flexural cracks emerged, corresponding to an average load range of 20–50 kN, exhibiting consistent spacing along the mid-section of the beams. As loading progressed to 60–70% of the ultimate load capacity, diagonal cracks developed within the shear span, aligning with the supports and the loading point. These cracks propagated towards the center compressive sections of the beam, leading to an abrupt shear failure caused by the crushing of concrete. This outcome was anticipated due to the choice of a beam with a geometry ($a/d < 3$, where a represents the shear span and d indicates the effective depth of

the beam) conducive to shear behavior. In fiber-reinforced concrete (FRC) beams featuring a sufficient steel fiber content, inclined cracks appeared at relatively higher load levels compared to plain concrete beams. Notably, these cracks were significantly constrained in FRC specimens. Observing Figure 7a–f, it becomes evident that shear failure occurred regardless of the longitudinal rebar ratio or fiber content. The steel fibers effectively bridged minor cracks before the concrete reached its tensile strength. Following this stage, their efficacy contributed to the gradual and smooth decline in post-peak behavior.

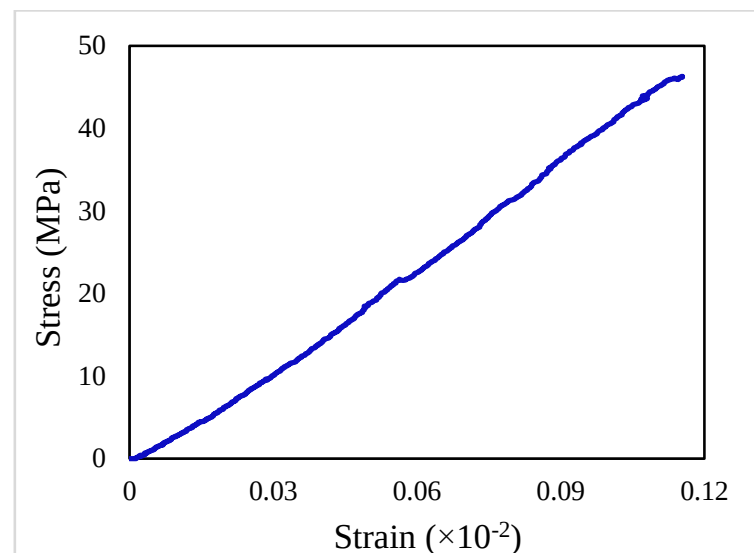


Figure 6. Average stress-strain curve of cylindrical specimens for the determination of the modulus of elasticity.

Table 6. Equations concerning the modulus of elasticity of HC found in various literature sources.

Researcher (s)	Equations (GPa)	Note	Ratios
Kollmorgen [33]	$E_c = 11,800(f'_c)^{\frac{1}{3.14}}$	$34 \leq f'_c \leq 207$ MPa	1.61
KCI [34]	$E_c = 8500\sqrt[3]{f'_c + 8}$	---	1.27
Lee et al. [35]	$E_c = (-367V_f \frac{L_f}{d_f} + 5520)f_c'^{0.41}$	---	1.09
Als Salman et al. [36]	$E_c = 8010(f'_c)^{0.36}$	$31 \leq f'_c \leq 235$ MPa	1.32
Haber et al. [37]	$E_c = 3755\sqrt{f'_c}$	$64.8 \leq f'_c \leq 153$ MPa	1.19
Suksawang et al. [38]	$E_c = 4700\lambda\sqrt{f'_c}$	$\lambda = (1 + 0.7V_f)/2$	1.11
Average of the current study	---	$f'_c = 87.5$ MPa, $E_c = 32$ GPa	---

E_c : modulus of elasticity of concrete; f'_c : compressive strength of concrete; d_f : diameter of hooked-end steel fibers; L_f : length of hooked-end steel fibers; V_f : fiber by volume; and λ : reduction factor in the elastic modulus as the fiber volume increases.

3.3. Load-Deflection Curves

Figure 8 depicts the load-deflection curves derived from the three-point bending tests on the specimens. These curves exhibit three distinct regions: (1) a phase of linear elastic behavior up to the point of initial cracking, (2) a subsequent stage with a quasi-linear region where the curve's slope, influenced by the initial crack, slightly decreases, and substantial deformations occur alongside increasing load, and (3) a third stage where the non-linear deflection becomes more significant, leading to failure while the load remains nearly constant. This phase signifies the plastic behavior of the concrete beam, inducing significant plastic deformations before failure. Following the development and propagation of cracks, specimens lacking steel fibers (F0) experience a rapid stiffness loss and fail without undergoing substantial deflection compared to beams with steel fibers.

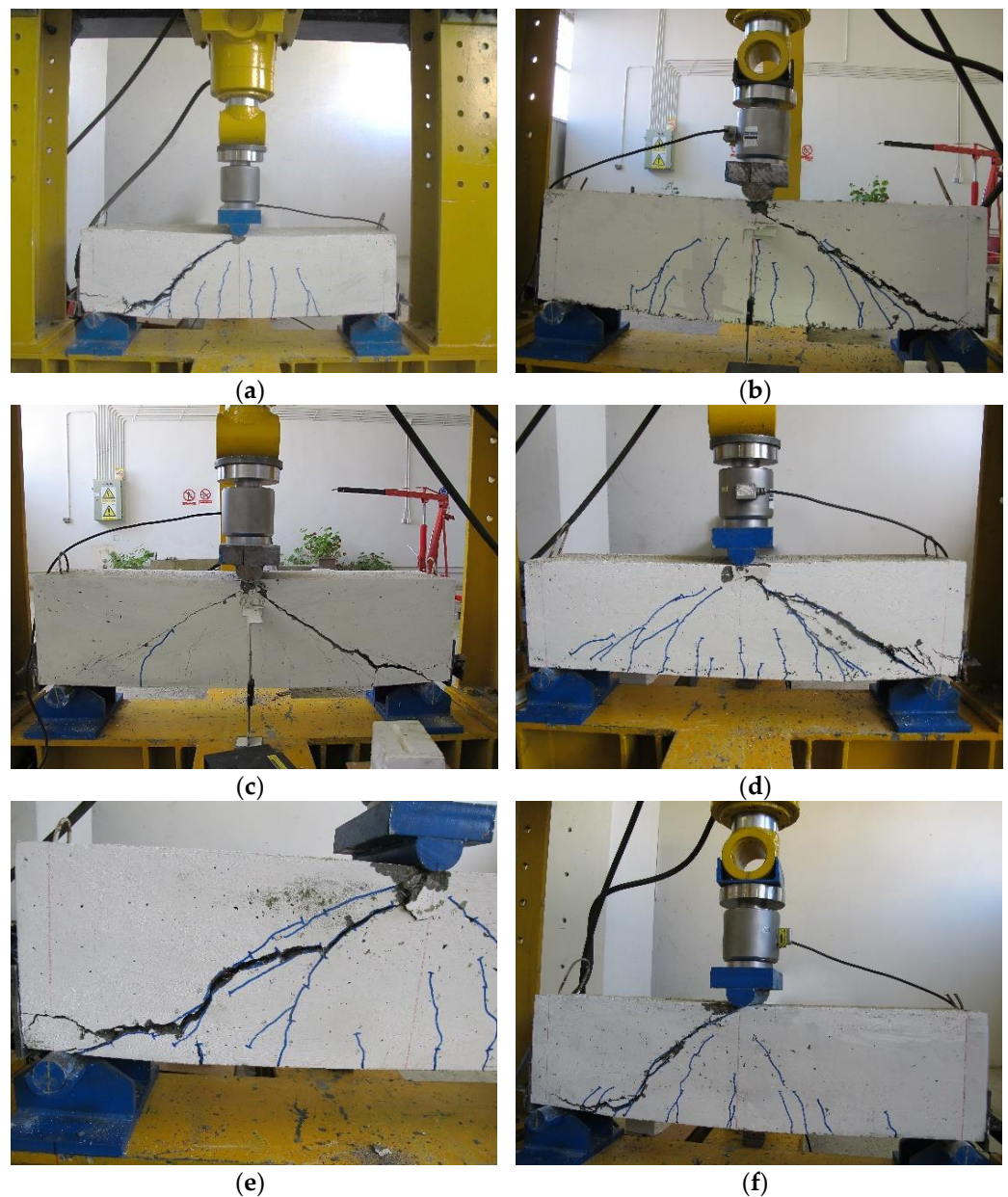


Figure 7. Crack pattern of fiber-reinforced HC specimens (a) HCHS1F1, (b) HCHS1F2, (c) HCH2F1, (d) HCHS2F2, (e) HCHS3F1, and (f) HCHS3F2.

Moreover, Figure 8 distinctly illustrates that the inclusion of steel fibers shifts the deflection corresponding to the peak load towards greater deflection values. These fibers act as transverse reinforcements, impeding crack growth, thereby resulting in narrower crack widths within these beams. Importantly, unlike the two initial stages observed in specimens without steel fibers (F0), FRC specimens exhibit a third stage characterized by nonlinear deformation and plastic behavior before failure. This shift, which signifies an increased peak deflection value, is more prominent in cases of low rebar ratios where the rebar demonstrates a higher level of ductility.

Furthermore, insights can be drawn from Figure 8 regarding specimens lacking fibers. An increase in the rebar ratio corresponds to an increase in the load-bearing capacity and deflection aligned with peak load values. However, with increasing reinforcing rebar ratios, the disparity in shear capacity between the specimens without fibers and those containing 2% fiber by volume diminishes (i.e., increases of 79%, 59%, and 53%, respectively). This trend also holds true when comparing 1% fiber content to 2% fiber content. This indicates

that the impact of the reinforcing rebar ratio is more prominent than that of steel fibers, as indicated in Figure 8.

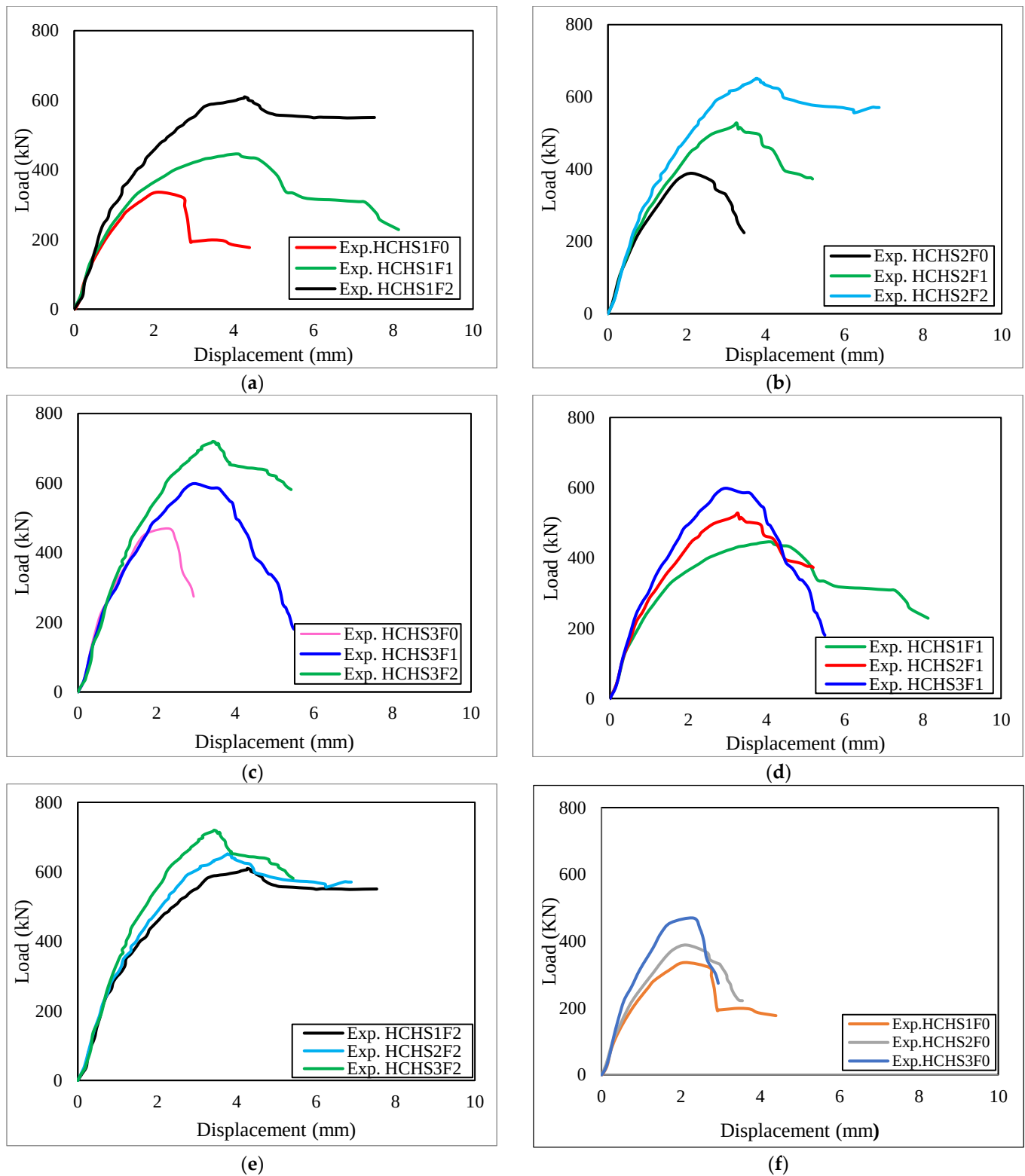


Figure 8. Load-deflection curves for various test specimens. (a) HCHS1F0, HCHS1F1, and HCHS1F2; (b) HCHS2F0, HCHS2F1, and HCHS2F2; (c) HCHS3F0, HCHS3F1, and HCHS3F2; (d) HCHS1F1, HCHS2F1, and HCHS3F1; (e) HCHS1F2, HCHS2F2, and HCHS3F2; and (f) HCHS1F0, HCHS2F0, and HCHS3F0.

A summary of the above-mentioned discussions is given in Tables 7 and 8 where the role of rebars in their contribution to the shear capacity is presented in Table 7 and that of fibers is presented in Table 8. According to Table 8, in beams without fibers, the role of rebars in contributing to the shear capacity in comparison to their plain counterparts is more noticeable than in cases where fibers are present (21%, and 44% for HCHS2F0 and HCHS3F0). Also, for a given fiber volume, the role of rebars in contributing to the shear capacity diminishes as the fiber percentage increases (44%, 33%, and 19% for specimens HCHS3F0, HCHS3F1, and HCH3F2, respectively). It is worth noting that a higher shear capacity should not be the sole indicator when assessing the performance of beams but rather a simultaneous consideration of the shear capacity and satisfactory deflection of the beam should be taken into account as well.

Table 7. Ultimate loads of the test beams.

Sample ID	V_u (kN)	Role of Rebars in Increasing V_u HC-HSk-Fm/HC-HS1-Fm
HCHS1F0	326.15	-
HCHS2F0	395.23	+21%
HCHS3F0	469.15	+44%
HCHS1F1	448.34	-
HCHS2F1	521.45	+16%
HCHS3F1	598.25	+33%
HCHS1F2	611.55	-
HCHS2F2	651.82	+7%
HCHS3F2	725.35	+19%

Note: V_u : ultimate load (kN); $k = 2, 3$, and $m = 0, 1, 2$.

Table 8. Ultimate loads of the test beams.

Sample ID	V_u (kN)	Role of Fibers in Increasing V_u HC-HSi-Fj/HC-HSi-F0
HCHS1F0	326.15	-
HCHS1F1	448.34	+37%
HCHS1F2	611.55	+88%
HCHS2F0	395.23	-
HCHS2F1	521.45	+32%
HCHS2F2	651.82	+65%
HCHS3F0	469.15	-
HCHS3F1	598.25	+28%
HCHS3F2	725.35	+55%

Note: V_u : ultimate load (kN); $i = 1, 2, 3$; and $j = 1, 2$.

3.4. Energy Absorption

The area under the load-deflection curves denotes the capability of the specimens to absorb energy. To account for this parameter, the energy values per cross-sectional area were calculated at $L/400$ and $L/250$ for without fiber and fibrous concrete, where L : clear span. The choice of the foregoing the clear span ratios corresponded roughly to the post-peak value of the curves. Results for the role of rebars and fibers in contributing to the absorbed energy are given in Tables 9 and 10. It is clear from Table 8 that the inclusion of fibers contributes to an energy absorption improvement by at least 100%, the increase rate of which decreases with the increase in the fiber and rebar ratio (28%, 19%, and 16% for specimens HCHS1F2, HCHS2F2, and HCHS3F2, respectively). This corresponds to the occurrence of the peak load at larger displacement values with more or less the same trend.

Table 9. Energy absorption and peak deflection values for test specimens.

Sample ID	Energy Absorption E (N.mm)	Peak Deflection D_p (mm)	Role of Rebars in Increasing E HC-HSk-Fm/HC-HS1-Fm
HCHS1F0	8.61	2.08	-
HCHS2F0	9.78	2.15	+14%
HCHS3F0	11.78	2.32	+37%
HCHS1F1	18.83	4.11	-
HCHS2F1	21.67	3.27	+15%
HCHS3F1	24.53	2.95	+30%
HCHS1F2	24.01	4.27	-
HCHS2F2	25.75	3.78	+7%
HCHS3F2	28.33	3.43	+18%

Note: $k = 2, 3$; $m = 0, 1, 2$; and E : energy absorption (N.mm).

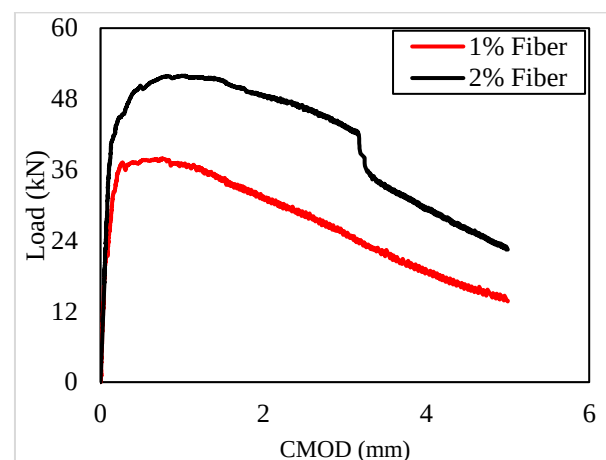
Table 10. Energy absorption and peak deflection values for test specimens.

Sample ID	Energy Absorption E (N.mm)	Peak Deflection D_p (mm)	Role of Fibers in Increasing E HC-HSi-Fj/HC-HSi-F0
HCHS1F0	8.61	2.08	-
HCHS1F1	18.83	4.11	+119%
HCHS1F2	24.01	4.27	+179%
HCHS2F0	9.78	2.15	-
HCHS2F1	21.67	3.27	+122%
HCHS2F2	25.75	3.78	+163%
HCHS3F0	11.78	2.32	-
HCHS3F1	24.53	2.95	+108%
HCHS3F2	28.33	3.43	+140%

Note: $i = 1, 2, 3$; $j = 1, 2$; and E : energy absorption (N.mm).

3.5. Load-CMOD Curves

The load-crack mouth opening displacement (CMOD) curve given in Figure 9 was used to calculate the residual flexural strengths f_{R1} and f_{R3} as the required parameters to determine the shear capacity of FRC beams according to the FIB Model Code [32]. The values obtained for the aforementioned parameters were equal to 6 and 4.48 MPa for specimens with 1% fiber content and 8 and 7.45 MPa for their 2% fiber counterparts. These values will be used in calculating the shear capacity of Section 3.7 based on the FIB Model Code [32].

**Figure 9.** Loading vs. crack mouth opening displacement (CMOD).

3.6. Numerical Simulation and Results

The simulation of fiber-reinforced concrete (FRC) beam behavior was carried out using ATENA [39] in conjunction with the GID [40] pre-processor software. This software has been previously used in numerous research to simulate the behavior of normal concrete [41–45] and FRCs [7–10]. As there is currently no standardized method for modeling FRC, an inverse analysis based on the software developers' guidelines [39] was employed to derive the post-cracking tensile stress versus fracture strain relationship for FRCs. The NonlinearCementitious2User material model, utilizing a fracture-plastic approach, was used to simulate concrete behavior. Linear elastic steel plates were employed for loading plates and supports. In cases where specimens lacked fibers, the NonlinearCementitious2 model was used, automatically considering compressive, tensile, shear, and other relevant criteria, requiring only user-defined parameters. Input parameters included the moduli of elasticity (32 GPa), direct tensile strength (4 and 3.5 MPa), and compressive strength (87.15 MPa, with the cubic compressive strength set at 105 MPa). Notably, due to the close proximity of the compressive strength values for plain and fibrous concrete, the average value of 87.15 MPa was used in calculations.

To model concrete, steel supports, and loading plates, eight-node hexahedral elements were utilized. The mesh size chosen was 30 mm. Additionally, finer and coarser mesh sizes of 10, 20, and 40 mm were tested, resulting in negligible variations in the results compared to the initial mesh size, as shown in Figure 10.

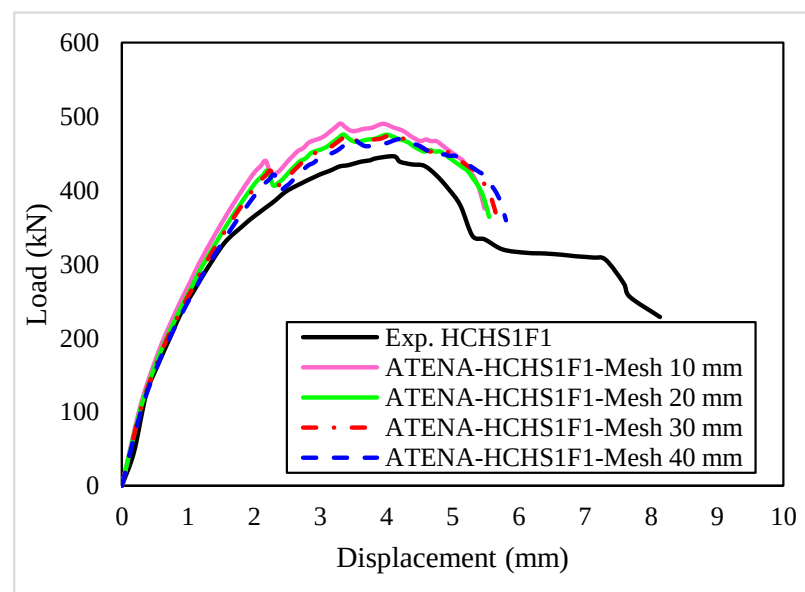


Figure 10. Mesh sensitivity analysis.

The load was applied using a displacement-controlled approach at a rate of 0.1 mm/step until failure took place. Monitoring points were strategically placed to record the reaction values at both the load plates and the mid-span of the beam. The nonlinear set of equations was solved utilizing the Newton–Raphson method in conjunction with the line-search method. The fracture strain used to define the tensile function can be expressed as follows (refer to Equation (3)):

$$\varepsilon = \frac{w}{L_t} \quad (3)$$

where ε is the fracture strain; w is the crack width; and L_t , as illustrated in Figure 11a, which is accompanied by a 3D view of the beam in Figure 11b, distinct mesh sizes in Figure 11c, and the tensile function in Figure 11d. The procedure for simulating the behavior of fiber-reinforced composites is outlined below:

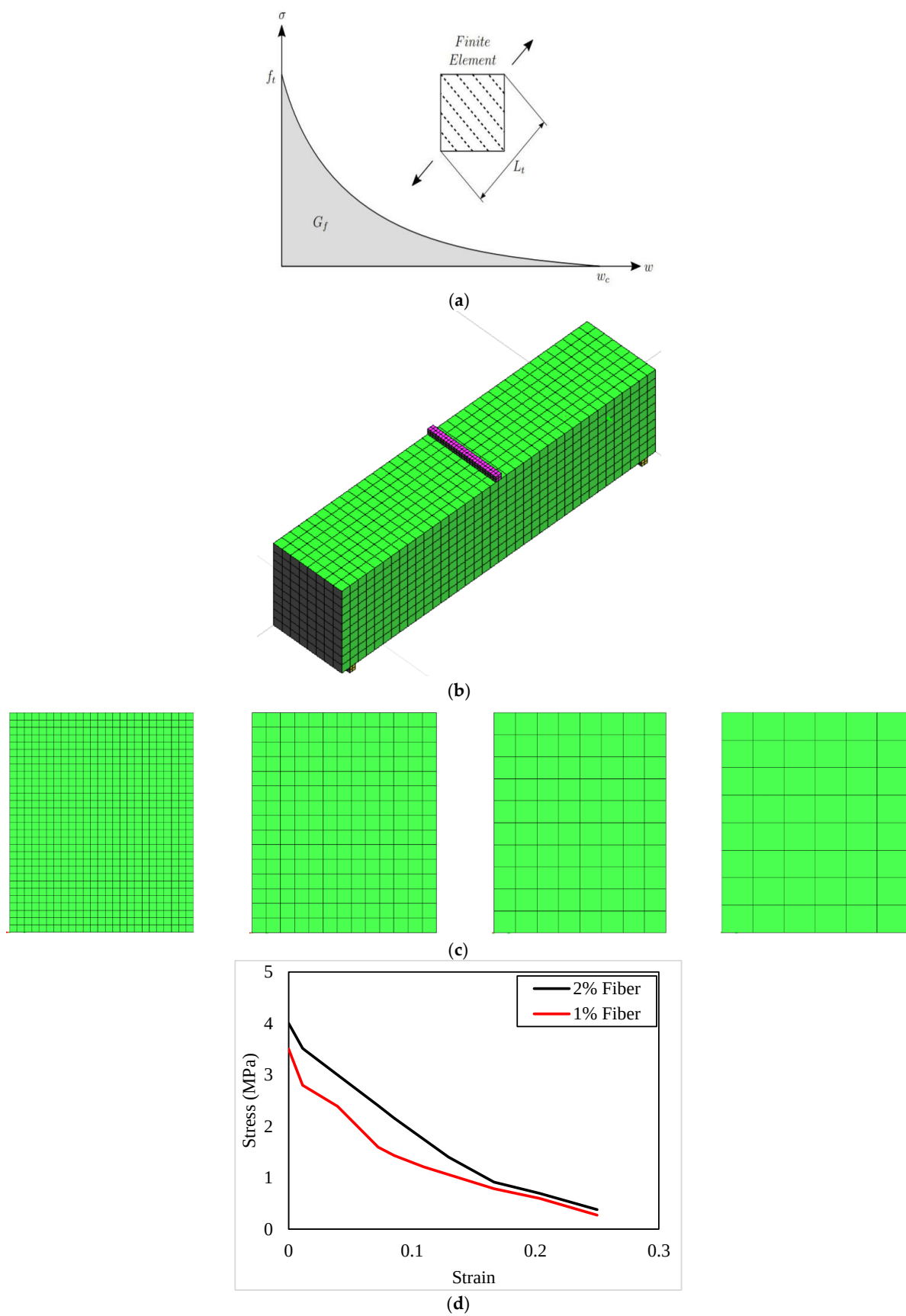


Figure 11. (a) Definition of the characteristic length and (b) 3D view of the beam, (c) from left to right, mesh sizes of 10, 20, 30, and 40 mm, and (d) tensile function.

The first crack's fracture strain and tensile stress are established as the x and y axes of the tensile function, respectively. This is based either on experimental findings or the user's expertise.

An analysis is then conducted by drawing parallels between the experimental and numerical outcomes in the subsequent steps.

If the results exhibit negligible discrepancies, the analysis is considered complete. However, if significant disparities are present, the fracture strain must be recalculated using the crack widths obtained from analyses at distinct deflection values, as per Equation (2). The revised fracture strain then leads to adjustments in the initial tensile function, and stress values are multiplied by the ratio of experimental-to-numerical outcomes.

It is worth highlighting that the accuracy of the tensile function and its associated results depends on the number of trial-and-error attempts and the desired precision set by the user. In the present study, a substantial number of trial-and-error analyses were conducted to achieve results that closely match experimental observations (Figure 12). Figure 13a presents the primary cracks along the failure path, while Figure 13d indicates that the concrete's performance failed before the longitudinal rebars yielded, signifying a brittle failure (for the sake of brevity and similarity, results for only one specimen were presented).

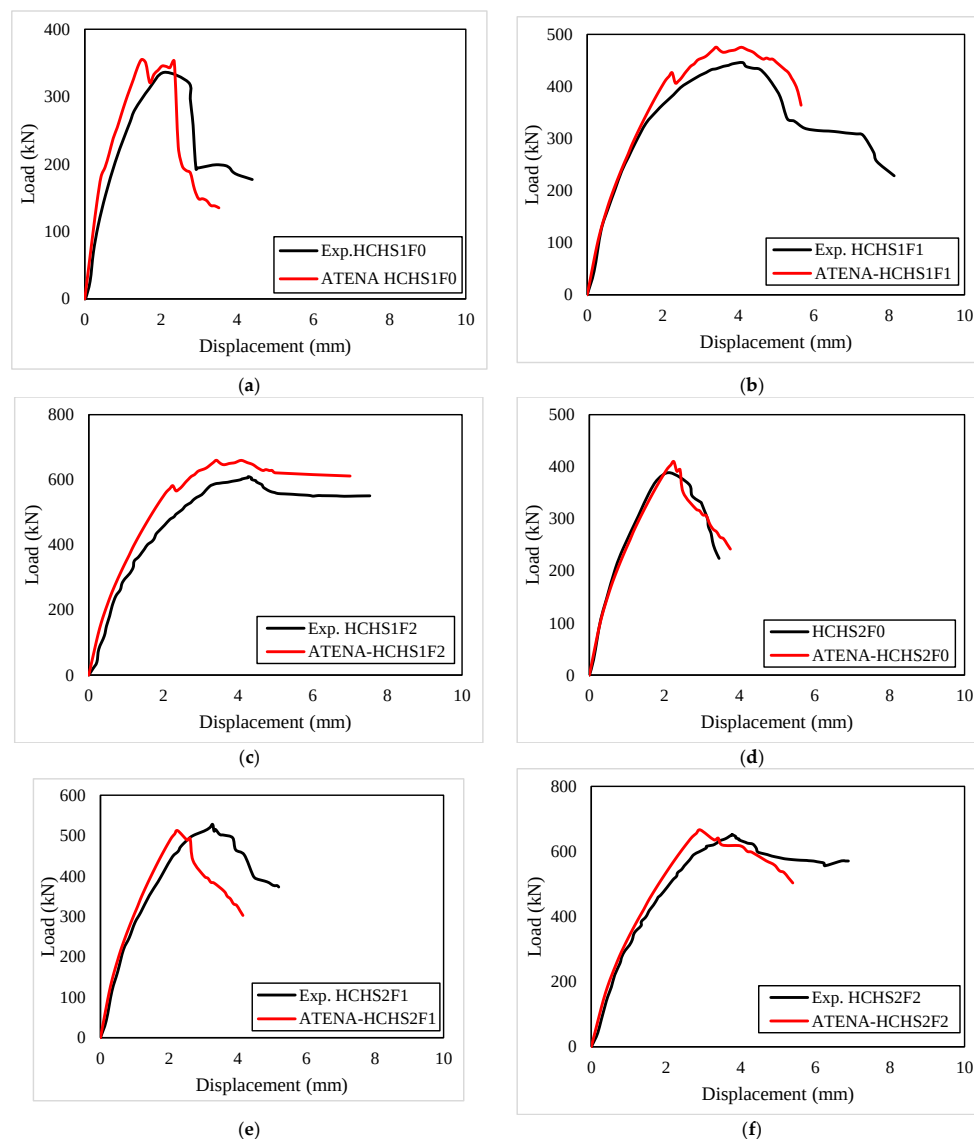


Figure 12. Cont.

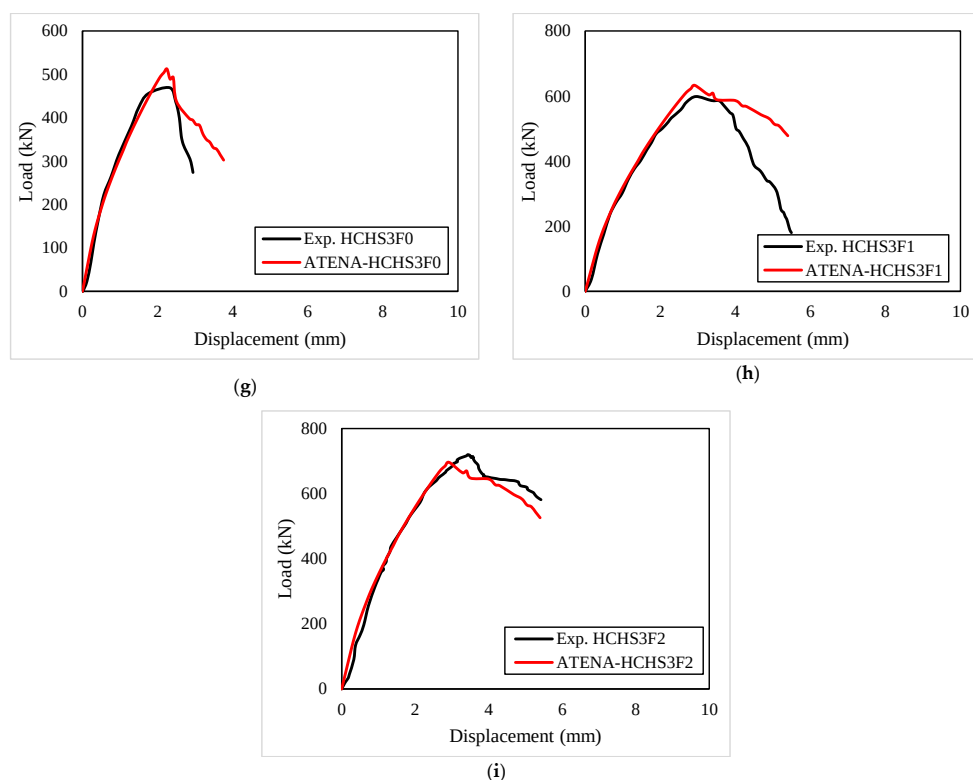


Figure 12. Comparison of the experimental and numerical load-deflection curves. Comparison of the experimental and numerical load-deflection curves: (a) HCHS1F0, (b) HCH1F1, (c) HCH1F2, (d) HCHS2F0, (e) HCHS2F1, (f) HCHS2F2, (g) HCHS3F0, (h) HCHS3F1, and (i) HCHS3F2.

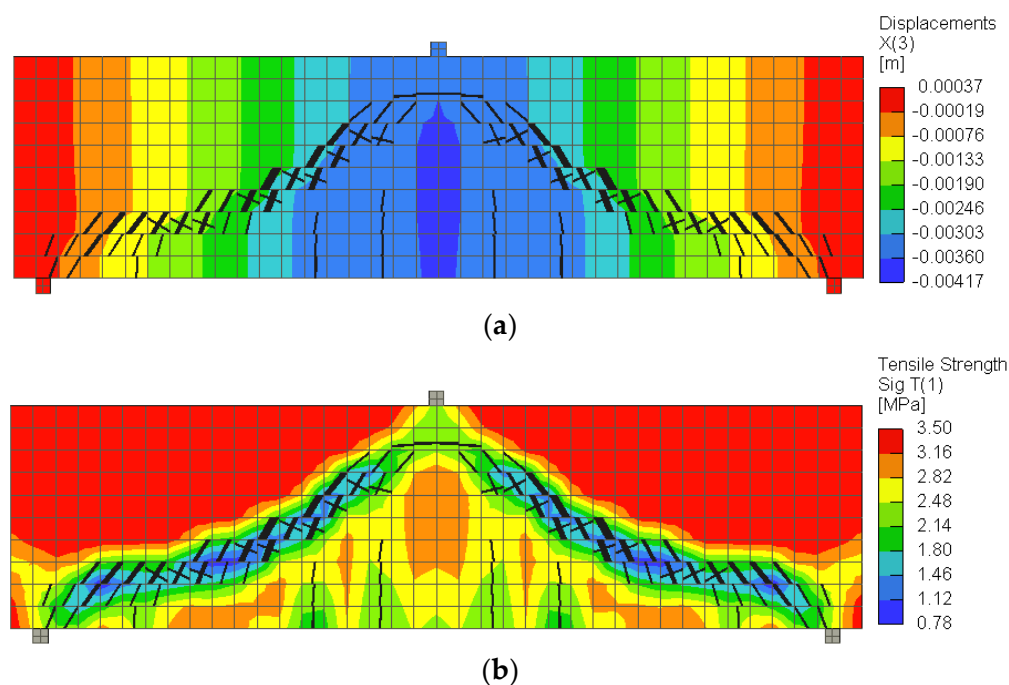


Figure 13. Cont.

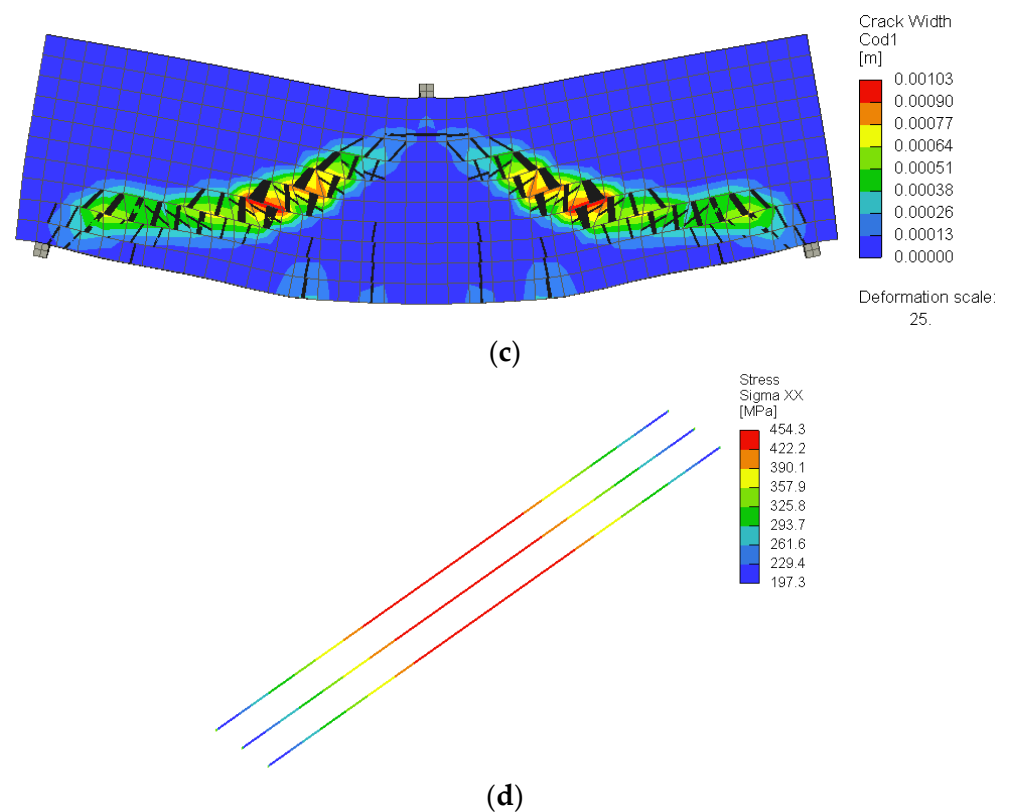


Figure 13. Numerical results: (a) Displacement contour, (b) tensile strength contour, (c) crack width contour, and (d) stress in rebars for specimens HCHS1F1.

A comparison of the experimental and numerical load-deflection curves in Figure 12 shows that ATENA [39] is capable of capturing the shear behavior of fiber-reinforced HC beams successfully with a good margin of error. This finding is further validated by the cracking pattern presented in Figure 13, which obviously shows the shear failure of beams. It can also be seen in Figure 12b that the tensile strength is lost along the cracking path, which proves the shear failure induced by diagonal tensile stress. In addition, Figure 13c shows the width of the main cracks along the failure path and Figure 13d reveals that prior to yielding longitudinal rebars, the concrete has failed to perform, which signifies brittle failure (for brevity and similarity, only the results for one specimen were presented).

3.7. Evaluation of Shear Strength

To evaluate the shear response of the samples in this article, some existing models published in the articles were considered, a list of which is presented in Table 11.

The shear capacities of the tested specimens were computed based on the data provided in Table 11. The outcomes of these calculations are presented in Tables 12 and 13. It becomes evident that the equation proposed by Narayanan and Darwish [46] offers the most accurate estimation of the experimental results when compared to other models.

Table 11. Available equations in the literature for the shear capacity of concrete beams.

Authors	Formula	Comments
Narayanan and Darwish [46]	$v_u = e \left[0.24 f_{spfc} + 80 \rho \frac{d}{a} \right] + v_b$	e = arch action factor (1.0 for $a/d > 2.8$ and $2.8d/a$ for $a/d \leq 2.8$); f_{spfc} = computed value of split tensile strength of concrete $(= \frac{f_{cu}}{20 - \sqrt{F}} + 0.7 + 1.0 \sqrt{F})$ f_{cu} = cube strength of FRC; F = fiber factor $(= l_f / d_f v_f D_f)$; v_f = fiber volume fraction D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber); l_f = fiber length, d_f = fiber diameter a = shear span, d = effective depth ρ = reinforcement ratio v_b = shear strength attributed to fibers $(= 0.41 \tau \cdot F)$; τ = average bond stress $(= 4.15 \text{ MPa})$
Faisal et al. [47]	$v_u = \left(0.7 \sqrt{f'_c} + 7F \right) \frac{d}{a} + 17.2 \rho \frac{d}{a}$	a = shear span, d = effective depth f'_c = concrete cylinder strength; F = fiber factor $(= l_f / d_f v_f D_f)$; v_f = fiber volume fraction D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber); l_f = fiber length, d_f = fiber diameter ρ = reinforcement ratio;
Ashour et al. [48]	For $a/d \geq 2.5$; $v_u = \left(2.11 \sqrt[3]{f'_c} + 7F \right) \cdot \left(\rho \frac{d}{a} \right)^{0.333}$ For $a/d < 2.5$ $v_u = \left(2.11 \sqrt[3]{f'_c} + 7F \right) \cdot \left(\rho \frac{d}{a} \right)^{0.333} \times 2.5 \frac{d}{a} + \left(2.5 - \frac{a}{d} \right) \cdot v_b$	f'_c = concrete cylinder strength; F = fiber factor $(= l_f / d_f v_f D_f)$; v_f = fiber volume fraction D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber); l_f = fiber length, d_f = fiber diameter a = shear span, d = effective depth ρ = reinforcement ratio; $v_b = 0.41 \tau \cdot F$; τ = average bond stress
Kwak et al. [49]	$v_u = 3.7 e_a f_{spfc}^{\frac{2}{3}} \cdot \left(\rho \frac{d}{a} \right)^{\frac{1}{3}} + 0.8 v_b$	e_a = arch action factor (1.0 for $a/d > 3.4$ and $3.4d/a$ for $a/d \leq 3.4$); f_{spfc} = computed value of split tensile strength of concrete F = fiber factor $(= l_f / d_f v_f D_f)$; l_f = fiber length, d_f = fiber diameter D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber); a = shear span, d = effective depth ρ = reinforcement ratio; $v_b = 0.41 \tau \cdot F$; τ = average bond stress
Imam et al. [50]	$v_u = 0.6 \Psi \sqrt[3]{\omega} \left[(f'_c)^{0.44} + 275 \sqrt{\frac{\omega}{(\frac{a}{d})^5}} \right]$	Ψ = size effect factor $(= \frac{1 + \sqrt{(5.08/d_a)}}{\sqrt{1 + d/(25d_a)}})$; f'_c = concrete cylinder strength; d_a = maximum aggregate size; ω = reinforcement factor $(= \rho(1 + 4F))$; F = fiber factor $(= l_f / d_f v_f D_f)$; l_f = fiber length, d_f = fiber diameter D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber); a = shear span, d = effective depth ρ = reinforcement ratio;

Table 11. Cont.

Authors	Formula	Comments
Padmarajaiah and Ramaswamy [51]	$v_u = \zeta \left[e_a \left\{ 0.32 \left(\frac{\sqrt{f_{cu}}}{3} + 1.918 \frac{l_f}{d_f} \cdot \rho \right) + 75 \rho \frac{d}{a} \right\} + g \cdot v_b \right]$	ζ = size effect factor ($= 1 / \sqrt{1 + d / (25d_a)}$); f'_{cu} = concrete cubic strength e_a = arch action factor (1.0 for $a/d > 3.4$ and $3.4d/a$ for $a/d \leq 3.4$); d_a = maximum aggregate size; a = shear span, d = effective depth g = an empirical coefficient (1.0 for $a/d > 2.8$ and 1.3 for $a/d \leq 2.8$); $v_b = 0.645 \tau \frac{l_f}{d_f} \eta_f v_f$; τ = average bond stress and η_f is a fiber shape coefficient, which for hooked-end steel fibers is equal in this model to 0.75 l_f = fiber length, d_f = fiber diameter
Shin et al. [52]	For $a/d \geq 3$; $v_u = 0.19 f_{sp} + 93 \rho \cdot \left(\frac{d}{a} \right) + 0.834 v_b$ For $a/d < 3$; $v_u = 0.22 f_{sp} + 217 \rho \cdot \left(\frac{d}{a} \right) + 0.834 v_b$	f_{sp} = splitting tensile strength of cylinder is assumed to be equal to f_{spfc} F = fiber factor ($= l_f / d_f v_f D_f$); l_f = fiber length, d_f = fiber diameter a = shear span, d = effective depth D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber); ρ = reinforcement ratio; $v_b = 0.41 \tau \cdot F$; τ = average bond stress
Ding et al. [53]	$v_u = \zeta \left[0.97 \rho^{0.46} f_c^{0.5} + 0.2 \rho^{0.91} f_c^{0.38} f_{yl}^{0.96} (a/d)^{-2.37} \right] + v_b$	ζ = size effect factor ($= 1 / \sqrt{1 + d / (25d_a)}$); f'_c = concrete cylinder strength; a = shear span, d = effective depth ρ = reinforcement ratio; f_{yl} = yield strength of the longitudinal reinforcement $v_b = 0.5 \tau \cdot F \cdot \cot \alpha$; α = is the inclination between the longitudinal reinforcement and the shear crack, and is assumed to be equal to 45°
Khuntia et al. [54]	$v_u = (0.167 \alpha + 0.25 F) \sqrt{f'_c}$	α = arch action factor (1.0 for $a/d \geq 2.5$ and $2.5d/a \leq 3$ for $a/d < 2.5$); F = fiber factor ($= l_f / d_f v_f D_f$); l_f = fiber length, d_f = fiber diameter D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber); f'_c = concrete cylindrical strength
Sarveghadi et al. [55]	$v_u = \rho + \frac{\rho}{v_b} + \frac{1}{a/d} \left(\frac{\rho f'_t (\rho + 2) \left(f'_t \frac{a}{d} - \frac{3}{v_b} \right)}{a/d} + f'_t \right) + v_b$	f'_t = specified tensile strength of concrete ($= 0.79 \sqrt{f'_c}$) f'_c = concrete cylindrical strength $v_b = 0.41 \tau \cdot F$; τ = average bond stress F = fiber factor ($= l_f / d_f v_f D_f$); l_f = fiber length, d_f = fiber diameter D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber a = shear span, d = effective depth ρ = reinforcement ratio
Al-Táan-Al-Feel [56]	$v_u = \left(\frac{1}{9} \right) \cdot \left[1.6 \sqrt{f'_c} + 960 \rho \left(\frac{d}{a} \right) \cdot e + 8.5 K v_f \frac{l_f}{d_f} \right]$	e = arch action factor (1.0 for $a/d > 2.5$ and $2.5d/a$ for $a/d \leq 2.5$); K = introduced to reflect the fiber shape, for hooked fibers = 1.2; l_f = fiber length, d_f = fiber diameter D_f = bond factor (0.5 for round fiber, 0.75 for crimped fiber, and 1.0 for hooked fiber; f'_c = concrete cylindrical strength a = shear span, d = effective depth ζ = size effect factor ($= 1 / \sqrt{1 + d / (25d_a)}$) ρ = reinforcement ratio; f_{yl} = yield strength of the longitudinal reinforcement; f'_c = concrete cylindrical strength; a/d = shear span-to-depth ratio; ρ_v = transverse reinforcement ratio; f_{yv} = yield strength of transverse reinforcement a = shear span, d = effective depth
Russo et al. [57]	$v_u = 0.72 \zeta \left[\rho^{0.4} f_c^{0.39} + 0.25 f_c^{0.89} f_{yl}^{0.96} (a/d)^{-1.2-0.45 \frac{a}{d}} \right] + 0.075 f_c^{0.5} (\rho_v f_{yv})^{0.7}$	ζ = size effect factor ($= 1 / \sqrt{1 + d / (25d_a)}$) ρ = reinforcement ratio; f_{yl} = yield strength of the longitudinal reinforcement; f'_c = concrete cylindrical strength; a/d = shear span-to-depth ratio; ρ_v = transverse reinforcement ratio; f_{yv} = yield strength of transverse reinforcement a = shear span, d = effective depth

Table 11. Cont.

Authors	Formula	Comments
FIB Model Code [32]	$V_{Rd,F} = \left\{ \frac{0.18}{\gamma_c} k \left[100 \rho \left(1 + 7.5 \frac{f_{t,uk}}{f_{ctk}} \right) f_{ck} \right]^{1/3} + 0.15 \sigma_{cp} \right\} b_w d$	γ_c = the partial safety factor for the concrete without fibers = 1.5; k = size effect factor $\left(= 1 + \sqrt{\frac{200}{d}} \leq 2.0 \right)$ a = shear span, d = effective depth ρ = reinforcement ratio $f_{t,uk}$ = the characteristic value of the ultimate residual tensile strength for FRC, by considering $w_u = 1.5$ mm f_{ctk} = the characteristic value of the tensile strength for the concrete without fibers f_{ck} = the characteristic value of cylindrical compressive strength $\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{cd}$ is the average stress acting on the concrete cross-section A_c for an axial force N_{Ed} due to loading or prestressing actions ($N_{Ed} < 0$ for compression)

Table 12. Experimental and predicted value of the load capacity of concrete beams.

Sample ID	Experiment (kN)	Narayanan and Darwish [46] (kN)	Faisal et al. [47] (kN)	Ashour et al. [48] (kN)	Kwak et al. [49] (kN)	Imam et al. [50] (kN)	Padmarajaiah and Ramaswamy [51] (kN)
HCHS1F0	326.15	325.75	396.86	262.33	462.57	273.35	187.52
HCHS1F1	448.34	436.05	550.18	364.66	565.88	460.21	300.65
HCHS1F2	611.55	528.59	703.51	466.98	645.75	613.48	413.77
HCHS2F0	395.23	355.94	401.82	288.28	508.38	319.34	207.54
HCHS2F1	521.45	466.25	555.15	397.89	615.60	546.39	320.66
HCHS2F2	651.82	558.78	708.48	507.50	697.06	734.30	433.78
HCHS3F0	469.15	425.63	413.27	334.96	590.79	413.17	253.74
HCHS3F1	598.25	535.93	566.60	457.67	705.05	725.23	366.86
HCHS3F2	725.35	628.47	719.93	580.37	789.37	986.89	479.98
Sample ID	Shin et al. [52] (kN)	Ding et al. [53] (kN)	Khuntia et al. [54] (kN)	Sarveghadi et al. [55] (kN)	Al-Ta'an and Al-Feel [56] (kN)	Russo et al. [57] (kN)	FIB Model Code [32] (kN)
HCHS1F0	421.28	256.91	227.66	---	317.21	206.67	---
HCHS1F1	442.68	354.18	337.06	583.16	370.34	---	133.95
HCHS1F2	451.63	451.45	446.46	675.82	423.46	---	152.88
HCHS2F0	483.88	314.46	227.66	---	353.16	243.31	---
HCHS2F1	505.28	411.73	337.06	607.16	406.28	---	133.95
HCHS2F2	514.23	508.99	446.46	704.09	459.41	---	152.88
HCHS3F0	628.35	437.26	227.66	---	436.12	317.82	---
HCHS3F1	649.75	534.53	337.06	662.97	489.24	---	133.95
HCHS3F2	658.70	631.79	446.46	769.85	542.37	---	152.88

Table 13. Comparison of the experimental shear capacity with the equations available in the literature.

Sample ID	$\frac{V_{Experiment}}{V_{Prediction}}$ [46]	$\frac{V_{Experiment}}{V_{Prediction}}$ [47]	$\frac{V_{Experiment}}{V_{Prediction}}$ [48]	$\frac{V_{Experiment}}{V_{Prediction}}$ [49]	$\frac{V_{Experiment}}{V_{Prediction}}$ [50]	$\frac{V_{Experiment}}{V_{Prediction}}$ [51]	$\frac{V_{Experiment}}{V_{Prediction}}$ [52]
HCHS1F0	1.01	0.82	1.24	0.71	1.19	1.74	0.77
HCHS1F1	1.03	0.81	1.23	0.79	0.97	1.49	1.01
HCHS1F2	1.16	0.87	1.31	0.95	1.00	1.48	1.35
HCHS2F0	1.11	0.98	1.37	0.78	1.24	1.90	0.82
HCHS2F1	1.12	0.94	1.31	0.85	0.95	1.63	1.03
HCHS2F2	1.17	0.92	1.28	0.94	0.89	1.50	1.27
HCHS3F0	1.10	1.14	1.40	0.79	1.14	1.85	0.75
HCHS3F1	1.12	1.06	1.31	0.85	0.82	1.63	0.92
HCHS3F2	1.15	1.01	1.25	0.92	0.73	1.51	1.10
Average	1.11	0.95	1.30	0.84	0.99	1.64	1

Sample ID	$\frac{V_{Experiment}}{V_{Prediction}}$ [53]	$\frac{V_{Experiment}}{V_{Prediction}}$ [54]	$\frac{V_{Experiment}}{V_{Prediction}}$ [55]	$\frac{V_{Experiment}}{V_{Prediction}}$ [56]	$\frac{V_{Experiment}}{V_{Prediction}}$ [57]	$\frac{V_{Experiment}}{V_{Prediction}}$ [32]
HCHS1F0	1.27	1.43	--	1.03	1.58	---
HCHS1F1	1.27	1.33	0.77	1.21	---	3.35
HCHS1F2	1.35	1.37	0.90	1.44	---	4.00
HCHS2F0	1.26	1.74	--	1.12	1.62	---
HCHS2F1	1.27	1.55	0.86	1.28	---	3.89
HCHS2F2	1.28	1.46	0.93	1.42	---	4.26
HCHS3F0	1.07	2.06	--	1.08	1.48	---
HCHS3F1	1.12	1.77	0.90	1.22	---	4.47
HCHS3F2	1.15	1.62	0.94	1.34	---	4.74
Average	1.23	1.59	0.88	1.24	1.56	4.12

The equation introduced by Kwak et al. [49] exhibits poor performance, particularly when dealing with low fiber contents and rebar ratios, leading to an underestimation of over 20%. However, this underestimation can be mitigated by increasing the amount of rebar and fibers in the mix.

The equation presented by Imam et al. [50] results in an overestimation of around 20% for specimens lacking fibers, while underestimating specimens containing fibers by approximately 12%. This discrepancy in estimation becomes more pronounced with higher fiber content ratios.

On the other hand, the equation provided by Al-Ta'an-Al-Feel [56] appropriately predicts the shear capacity of concrete beams without fibers. However, it tends to overestimate by approximately 30% when applied to beams with added fibers.

One of the most noteworthy instances of overestimation in the realm of research is found in the work of Padmarajaiah and Ramaswamy [51]. Their estimates excessively inflate the response of both plain and fibrous concrete. This overestimation ranges from a minimum of 48% for the HCHS2F2 specimen to a maximum of 90% for the HCHS2F0 specimen [53–55]. Similar trends are observed in the studies by Khuntia et al. [54] and Russo et al. [57].

It is noteworthy that the equation proposed by Ding et al. [53] is the only equation that considers the role of reinforcing rebars' yield stress in the calculation. Models proposed by Ding et al. [53] and Imam et al. [50] are sensitive to the variations of the a/d ratio, the power of which for the former is -2.37 , and for the latter, -2.5 .

Moreover, design codes such as the FIB Model Code [32] have given parameters such as the rebar ratio and residual flexural strength of fibrous concrete the power of $1/3$, which decreases their contribution to the overall shear capacity and is surprising because the role of the rebar ratio is by no means negligible as experimental results assert; furthermore, giving a negligible share to the role of fibers results in a conservative estimation of the shear

capacity of beams with a higher fiber content. This has resulted in conservative estimations as high as four times.

Additionally, a comparison of numerical and experimental results (Table 14) reveals that the numerical estimations of shear capacity are within an acceptable range (margin or error less than 10%) from an engineering perspective and, hence, they have successfully captured the behavior of test specimens [58–60].

Table 14. Ratios of the experimental shear capacities to that of the numerical values.

Sample ID	$\frac{V_{Exp.}}{V_{ATENA}}$
HCHS1F0	0.95
HCHS1F1	0.94
HCHS1F2	0.93
HCHS2F0	0.94
HCHS2F1	1.03
HCHS2F2	0.98
HCHS3F0	0.92
HCHS3F1	0.95
HCHS3F2	1.04

4. Conclusions

An experimental investigation was conducted in this paper to examine the shear behavior of high-strength concrete (HC) reinforced with high-strength steel (HSS) rebars and hooked-end fibers in varying ratios. Nine beams were cast and subjected to three-point bending tests. The beam dimensions were selected such that the shear behavior dictated the failure mode. Our conclusions include the following.

- Fibers had a minimal influence on the compressive strength of the concrete. Specifically, incorporating 2% fibers by volume only contributed to a 7% increase to the compressive strength of the concrete.
- Upon comparing the experimental modulus of elasticity with values from the literature, it was observed that all equations overestimated the value. The overestimation ranged from 9% for the equation proposed by Lee et al. [35], which accounted for fiber geometry, to a significant 61% from Kollmorgen's equation [33].
- For specimens lacking fibers, an increase in the rebar ratio resulted in a higher load-bearing capacity (up to a 32% increase for HCHS3F0 compared to HCHS1F0) and deflection at the peak load. However, the rate of increase in these parameters decreased as the rebar ratio increased. Introducing fibers enhanced the energy absorption by at least 100%, with the improvement increasing alongside the rebar ratio and fiber content. This characteristic is particularly important in earthquake-prone areas.
- Although the initial stiffness of the load-deflection values was minimally impacted by H fibers, their inclusion led to stiffer yet ductile post-peak responses, in contrast to beams without fibers that exhibited sudden failure.
- Incorporating H fibers shifted the mid-span deflection corresponding to the peak load towards higher deflection values compared to specimens without fibers. The rate of this deflection increase reduced with higher rebar ratios.
- Comparing the effect of increasing fiber content from 0% to 1% and from 1% to 2% revealed that the latter had a more pronounced effect on increasing the load-bearing capacity, with the difference narrowing as the rebar ratio increased (34%, 29%, and 28% increase for 1% fiber specimens relative to plain specimens, in comparison to a 28%, 23%, and 20% increase for 2% fiber specimens).
- The inverse analysis method employed in this study effectively captured the shear response of fiber-reinforced concrete (FRC) beams within a 10% margin of error.
- The equation proposed by Narayanan and Darwish [46] yielded the best results, with a maximum overestimation of 17% for specimen HCHS2F2. This suggests a need for further research to better understand the shear response of fibrous concrete.

- Comparing research-based shear equations with the experimental results highlighted notable discrepancies, either underestimating or overestimating (up to double). Similarly, design codes like the FIB Model Code 2010 [32] provided conservative results by assigning rebar ratios and fiber content a power of $1/3$, leading to potential overestimations as high as 3.92 times.
- Ding et al.'s [53] equation stands out as the sole equation considering the role of rebar yield stress in calculations. Both the Ding et al. [53] and Imam et al. [50] models are sensitive to variations in the a/d ratio, with respective powers of -2.37 and -2.5 .
- In general, equations that fail to logically incorporate various parameters into their shear capacity predictions yield either high or low estimates.

For future research, it is recommended to consider various types of fibers and re-bars [61,62].

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