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Study on Structural Reliability Analysis Method Based on Chance Theory

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Abstract: Many factors influence structural reliability in practice engineering. Some factors can be measured to obtain lots of data, but others are difficult to acquire statistical data. In view of this situation, a new structural reliability analysis method is proposed using chance theory, which is composed of probability theory and uncertainty theory and can reflect random uncertainty and cognitive uncertainty simultaneously. The performance function of a structural mechanical element is defined, and when it is a random uncertain variable, the chance distribution is established. Then the calculated method of failure measures and reliability measures for the structural mechanical element is put forward. Furthermore, considering the series system and parallel system, the performance function of the structural system is proposed, and the calculated method of failure measure is determined by theoretical proof. The results can provide a new approach to analyzing structural reliability under the uncertain circumstance of lack of statistical data.

Keywords: structural reliability; chance theory; performance function; failure measure; uncertainty theory

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1. Introduction

Structural reliability is the ability of the structure to complete the predetermined function in the specified time and under specified conditions. It includes safety, adaptability and durability. In order to make the structure achieve its intended use, structural reliability should be required. Analyzing structural reliability is a key tool for estimating the safety levels and the most probable failure state during the service life of the structure. The analysis method of structural reliability is usually discussed under uncertain conditions.

In the early 20th century, probability theory began to be used to analyze structural reliability, marking the emergence of structural reliability theory [1]. Freudenthal [2] proposed fundamental structural safety problems under random load using the total probability method, which became the foundation of structural reliability theory. Longo et al. [3,4] proposed a probabilistic approach based on the combination of probabilistic seismic hazard analysis, probabilistic seismic demand analysis and probabilistic seismic capacity analysis to analyze the influence of design criteria on the seismic reliability of centrally X-braced frames and investigate the seismic reliability of the designed concentrically "V" braced steel frames. Zhao et al. [5] proposed that the structural safety coefficient could be analyzed using the first-order second-moment method and then proposed the numerical simulation method of structural reliability, such as the Monte Carlo simulation, response surface method, and hybrid simulation method. Lv et al. [6] proposed the uniform design response surface method to analyze structural reliability. Jafari-Asl et al. [7,8] proposed a novel framework that integrates the line sampling method with the slime mold algorithm to solve complex structural reliability problems and a new framework for accurate reliability analysis based on improving the directional simulation by using metaheuristic algorithms.

Random analysis needs large quantity statistical data, but the data is sometimes difficult to obtain from practical engineering. Therefore, the fuzzy theory was adopted to analyze the structural reliability. Fabio et al. [9] carried out a reliability analysis on the concrete structure under a fuzzy environment. Adduri et al. [10] analyzed the structural reliability under the environment with fuzzy variables and random variables. Furthermore, Wang et al. [11] established the analysis method of single-mode and multi-mode fuzzy random reliability for seismic structures. Li and Nie [12] proposed a structural reliability calculation method with fuzzy random variables using the error principle.

Fuzzy theory was used to deal with cognitive uncertainty, but it is based on possibility theory, and it is challenging to be self-consistent in the practice of reliability. In order to better analyze cognitive uncertainty and deal with circumstances that lack statistical data, Liu [13] established uncertainty theory based on an axiomatic system, which was a new mathematic theory. Uncertainty theory has been used to analyze system reliability. Liu [14] proposed an uncertain reliability analysis method according to uncertainty theory. Wang [15] regarded structural resistance and load as uncertain variables and proposed the definition and theorem of the structural reliability index. Furthermore, Miao [16] took a space truss and a continuous beam as examples and analyzed the structural reliability using uncertainty theory.

System reliability is very complex in practice engineering. There are many influence parameters, of which some can be measured to obtain lots of data, and others are difficult to acquire statistical data [17]. Therefore, it is unreasonable to analyze the above system reliability by using probability theory or uncertainty theory. In order to solve the problem, Liu [18] proposed chance theory, which is the combination of probability theory and uncertainty theory, and it can be used to study random uncertainty and cognitive uncertainty simultaneously. Based on chance theory, Wen and Kang [19] established an uncertain random reliability analysis method with the concept of reliability index. For structural reliability, it is also easy for some influence parameters and hard for others to obtain lots of statistical data. Therefore, chance theory is an excellent choice to analyze structural reliability. At present, no research in this field has been found.

In this paper, the structural reliability analysis with a new approach is further studied. The structural performance function, in which the influence parameters can be random variables or uncertain variables, is given, and its chance distribution is established. The calculated methods of the failure measure and reliability measure of structural mechanical elements are proposed. Based on the above, the determination method of failure measure for the structural system is given when it is a series system or a parallel system. Based on the proposed new approach, the structural reliability analysis can be carried out without the limitation of large statistical data and can be applied in a situation where larger statistical data can be obtained for some influence factors and not for some other influence factors.

2. Preliminary

In order to better describe the structural reliability analysis method established in this paper, the following basic concepts and operational laws of uncertainty theory (Liu, 2007) and chance theory (Liu, 2014) will be introduced first.

(1) Let Γ be a nonempty set, and $\mathcal{L}$ be a σ -algebra over Γ. If the function $\mathcal{M}$ satisfies:

(i) $\mathcal{M}\{\Gamma\} = 1$, that is normality;
(ii) For any $\Lambda \in \mathcal{L}$, $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$, which is duality;
(iii) For every sequence of $\Lambda_i \in \mathcal{L}$, it holds that

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \leq \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$$

is subadditivity. $\mathcal{M}$ and $(\Gamma, \mathcal{L}, \mathcal{M})$ are known as the uncertain measure and the uncertainty space, respectively.

(2) An uncertain variable ξ is a measurable function from $(\Gamma, \mathcal{L}, \mathcal{M})$ to the real numbers set.

(3) The uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_i \in B_i)\right\} = \bigwedge_{i=1}^{n}\mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \cdots, B_n$ of real numbers.

(4) For any real number x, the uncertainty distribution Φ of ξ is defined as

$$\Phi(x) = \mathcal{M}\{\xi \leq x\},$$

and the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.

(5) Let $(\Gamma, \mathcal{L}, \mathcal{M})$ be an uncertainty space, and let $(\Omega, \mathcal{F}, P)$ be a probability space. Then the product $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{F}, P)$ is called a chance space.

(6) Let $\Theta \in \mathcal{L} \times \mathcal{F}$ be an event in $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{F}, P)$. Then the chance measure of Θ is defined as

$$Ch\{\Theta\} = \int_0^1 P\{\omega \in \Omega | \mathcal{M}\{\gamma \in \Gamma | (\gamma, \omega) \in \Theta\} \geq x\} \mathrm{d}x.$$

(7) An uncertain random variable is a function ξ from a chance space $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{F}, P)$ to the set of real numbers such that $\{\xi \in B\}$ is an event in $\mathcal{L} \times \mathcal{F}$ for any Borel set B of real numbers.

$$\{\xi \in B\} = \{(\gamma, \omega) | \xi(\gamma, \omega) \in B\}.$$

(8) Let ξ be an uncertain random variable. The chance distribution of ξ is defined as

$$\Phi(x) = Ch\{\xi \leq x\}$$

for any $x \in$ R.

(9) Let $\eta_1, \eta_2, \cdots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_m$, and let $\tau_1, \tau_2, \cdots, \tau_n$ be independent uncertain variables with uncertainty distributions $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_n$, respectively. If f is a measurable function, then the uncertain random variable

$$\xi = f(\eta_1, \eta_2, \cdots, \eta_m, \tau_1, \tau_2, \cdots, \tau_n)$$

has a chance distribution

$$\Phi(x) = \int_{\mathrm{R}^m} F(x; y_1, y_2, \cdots, y_m) \mathrm{d}\Psi_1(y_1), \mathrm{d}\Psi_2(y_2), \cdots, \mathrm{d}\Psi_m(y_m),$$

where

$$F(x; y_1, y_2, \cdots, y_m) = \mathcal{M}\{f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n) \leq x\}$$

is the uncertainty distribution of $f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)$ for any real numbers $y_1, y_2, \cdots, y_m$, and is determined by $\Upsilon_1, \Upsilon_2, \cdots, \Upsilon_n$.

If $f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)$ is a monotonically increasing function for $\tau_1, \tau_2, \cdots, \tau_n$, the inverse function of $F(x; y_1, y_2, \cdots, y_m)$ is

$$F^{-1}(\alpha; y_1, y_2, \cdots, y_m) = f\left(y_1, y_2, \cdots, y_m, \Upsilon_1^{-1}(\alpha), \Upsilon_2^{-1}(\alpha), \cdots, \Upsilon_n^{-1}(\alpha)\right).$$

If $f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)$ is a monotonically decreasing function for $\tau_1, \tau_2, \cdots, \tau_n$, then the inverse function of $F(x; y_1, y_2, \cdots, y_m)$ is

$$F^{-1}(\alpha; y_1, y_2, \cdots, y_m) = f\left(y_1, y_2, \cdots, y_m, \Upsilon_1^{-1}(1-\alpha), \Upsilon_2^{-1}(1-\alpha), \cdots, \Upsilon_n^{-1}(1-\alpha)\right).$$

3. The Performance Function of Structure

In the process of structural reliability analysis, the structure is usually regarded as a system and contains different types of mechanical elements. For the structural mechanical element, some influence parameters can be measured to obtain a large number of sample data, which can be regarded as random variables and analyzed by using probability. Other influence parameters, of which the statistical data is difficult to obtain through the actual measurement, can be seen as uncertain variables and conducted by using uncertainty theory. Therefore, the structural mechanical element can be called an uncertain random structural mechanical element; namely, the influence parameters of the structural mechanical element include random variables and uncertain variables simultaneously. When all the parameters of a structural mechanical element are random variables, it degenerates into a random structural mechanical element. When all the parameters of a structural mechanical element are uncertain variables, it degenerates into an uncertain structural mechanical element. Obviously, the random and uncertain structural mechanical elements are unique forms of uncertain random structural mechanical elements.

Definition 1. *The basic variable X is influenced by factors $\xi_1, \xi_2, \cdots, \xi_m$, that is*

$$X = g(\xi_1, \xi_2, \cdots, \xi_m)$$

where g is the factor function of X. If $\xi_1, \xi_2, \cdots, \xi_m$ include random and uncertain variables simultaneously, X is an uncertain random variable; if $\xi_1, \xi_2, \cdots, \xi_m$ are all random variables, X is a random variable; if $\xi_1, \xi_2, \cdots, \xi_m$ are all uncertain variables, X is an uncertain variable. The latter two are exceptional cases of the former.

Definition 2. *If the function of structural mechanical elements is influenced by basic variables $X_1, X_2, \cdots, X_n$,*

$$Z = f(X_1, X_2, \cdots, X_n)$$

is called the performance function of the mechanical element.

If $Z > 0$, the structural mechanical element is in the reliable state; if $Z < 0$, the structural mechanical element is in the failure state; if $Z = 0$, the structural mechanical element is in the limit state.

When $X_1, X_2, \cdots, X_n$ contain random variables and uncertain variables simultaneously, or one or more uncertain random variables, Z is an uncertain random variable; when $X_1, X_2, \cdots, X_n$ are all random variables, Z is a random variable; when $X_1, X_2, \cdots, X_n$ are all uncertain variables, Z is an uncertain variable.

For the above three cases, chance theory, probability theory and mathematical statistics, and uncertainty theory can be used to analyze the reliability, respectively.

In the performance function of structural mechanical elements, two basic variables were usually considered. One basic variable is structural resistance R, which is the ability of the structure to withstand the action effect. Another is the action effect S, which is the internal force and deformation of the structure resulting from the loads. In this case, the performance function of a structural mechanical element can be written as Equation (1).

$$Z_0 = g(R, S) = R - S \tag{1}$$

where R is the function of material property indexes and geometric parameters, of which the statistical data can be obtained by measuring. Hence, the material property indexes and geometric parameters are random variables, resulting in R as a random variable. S is the function of loads borne by the mechanical element and geometric parameters. Because the loads are difficult to be measured, they can be regarded as uncertain variables, of which the empirical data can be acquired through expert investigation. Therefore, S can be determined as an uncertain random variable, so Z_0 is also an uncertain random variable.

In order to facilitate analysis, Q and G are chosen to denote the loads borne by the mechanical element and the geometric parameters, respectively. Hence, S can be expressed as the product of Q and G, and according to Equation (1), Z_0 can be represented as the function of variables R, Q and G, as shown in Equation (2).

$$Z_0 = g(R, Q, G) = R - QG \tag{2}$$

4. Failure Measure and Reliability Measure of Structural Mechanical Element

Definition 3. *If the performance function of structural mechanical element Z is an uncertain random variable, its chance distribution* $\Phi_Z(x)$ *is*

$$\Phi_Z(x) = Ch\{Z \le x\}$$

for any real number x, *where* $Z = f(X_1, X_2, \cdots, X_n)$, *and* $X_1, X_2, \cdots, X_n$ *are basic variables.*

Theorem 1. *If the basic variable* $X_i(i = 1, 2, \cdots, n)$ *in the performance function of the structural mechanical element* $Z = f(X_1, X_2, \cdots, X_n)$ *is a random variable or an uncertain variable, let* $X_1, X_2, \cdots, X_p$ *be random variables with a probability distribution* $\Psi_{X_1}, \Psi_{X_2}, \cdots, \Psi_{X_p}$, *and let* $X_{p+1}, X_{p+2}, \cdots, X_n$ *be uncertain variables with an uncertain distribution* $\Upsilon_{X_{p+1}}, \Upsilon_{X_{p+2}}, \cdots, \Upsilon_{X_n}$. *Z is a monotonically increasing function with respect to* $X_{p+1}, X_{p+2}, \cdots, X_q$ $(p + 1 \le q \le n)$, *and a monotonically decreasing function with respect to* $X_{q+1}, X_{q+2}, \cdots, X_n$. *The chance distribution of Z can be expressed as Equation (3).*

$$\Phi_Z(x) = \int_{\mathrm{R}^p} F(x; a_1, a_2, \cdots, a_p) \mathrm{d}\Psi_{X_1}(a_1) \mathrm{d}\Psi_{X_2}(a_2) \cdots \mathrm{d}\Psi_{X_p}(a_p) \tag{3}$$

where $\Phi_Z(x)$ *is the chance distribution of Z;* $a_1, a_2, \cdots, a_p$ *are any representative values of* $X_1, X_2, \cdots, X_p$; $F(x; a_1, a_2, \cdots, a_p)$ *is the uncertain distribution of the uncertain variable* $f(a_1, a_2, \cdots, a_p, X_{p+1}, X_{p+2}, \cdots, X_q, X_{q+1}, X_{q+2}, \cdots, X_n)$, *which can be determined according to its inverse uncertainty distribution as Equation (4).*

$$\begin{aligned} &F^{-1}(\alpha; a_1, a_2, \cdots, a_p) \\ &= f\left(a_1, a_2, \cdots, a_p, \Upsilon^{-1}_{X_{p+1}}(\alpha), \cdots, \Upsilon^{-1}_{X_q}(\alpha), \Upsilon^{-1}_{X_{q+1}}(1-\alpha), \cdots, \Upsilon^{-1}_{X_n}(1-\alpha)\right) \end{aligned} \tag{4}$$

Set x = 0 in Equation (3), the failure measure of the structural mechanical element can be obtained, as shown in Equation (5).

$$\begin{aligned} M_\mathrm{f} &= \Phi_Z(0) = Ch\{Z < 0\} \\ &= \int_{\mathrm{R}^p} F(0; a_1, a_2, \cdots, a_p) \mathrm{d}\Psi_{X_1}(a_1) \mathrm{d}\Psi_{X_2}(a_2) \cdots \mathrm{d}\Psi_{X_p}(a_p) \end{aligned} \tag{5}$$

According to the self-duality of chance measure, the reliability measure of the structural mechanical element M_r can be obtained as Equation (6).

$$M_\mathrm{r} = Ch\{Z \ge 0\} = 1 - M_\mathrm{f} \tag{6}$$

The performance function of structural mechanical element Z_0 shown in Equation (1) conforms to the condition of Theorem 1. Assuming that the probability distributions of variables R and G are Ψ_R and Ψ_G respectively, and the uncertain distribution of the variable Q is Υ_Q. Because Z_0 is a monotonically decreasing function for Q, the chance distribution of Z_0 can be obtained as Equation (7) according to Equations (3) and (4).

$$\Phi_{Z_0}(x) = \int_{\mathrm{R}^2} \left[1 - \Upsilon_Q\left(\frac{y_1 - x}{y_2}\right)\right] \mathrm{d}\Psi_R(y_1) \mathrm{d}\Psi_G(y_2) \tag{7}$$

where $\Phi_{Z_0}(x)$ is the chance theory of Z_0; y_1, y_2 are any representative values of R and G, respectively, and $1 - \Upsilon_Q\left(\frac{y_1 - x}{y_2}\right)$ is the uncertain distribution of uncertain variable $y_1 - Qy_2$, which can be determined by Equation (8).

$$F^{-1}(\alpha; y_1, y_2) = y_1 - \Upsilon^{-1}_Q(1 - \alpha) y_2 \tag{8}$$

According to Equations (5) and (6), the failure measure M_{f0} and reliability measure M_{r0} of the structural mechanical element can be expressed as Equations (9) and (10), respectively.

$$M_{\mathrm{f0}} = \Phi_{Z_0}(0) = Ch\{Z_0 < 0\} = \int_{\mathrm{R}^2} \left[1 - \Upsilon_Q\left(\frac{y_1}{y_2}\right)\right] \mathrm{d}\Psi_R(y_1)\mathrm{d}\Psi_G(y_2) \tag{9}$$

$$M_{\mathrm{r0}} = Ch\{Z_0 \geq 0\} \; = 1 - M_{\mathrm{f0}} \tag{10}$$

5. Reliability Analysis of Structural System

The structural system is composed of mechanical elements. If the correlations of these mechanical elements are simple, the structural system may be regarded or simplified as a series system or a parallel system.

In the series system (Figure 1), all structural mechanical elements are connected in series. The characteristic of a series system is that the failure of any structural mechanical element will make the whole structure system fail; that is, if and only if all structural mechanical elements work, the whole system works.

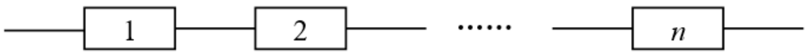

Figure 1. Series system.

In the parallel system (Figure 2), all structural mechanical elements are connected in parallel. The parallel system is characterized by the fact that the whole system works whenever at least one mechanical element works.

There are many basic variables in the performance function of structural mechanical elements, including random variables, uncertain variables, and uncertain random variables, and therefore, it is difficult to analyze the reliability of a structural system. Consequently, the performance function of a structural system will be established based on the performance function Z_0 of the structural mechanical element shown in Equation (2), which is influenced by only three variables.

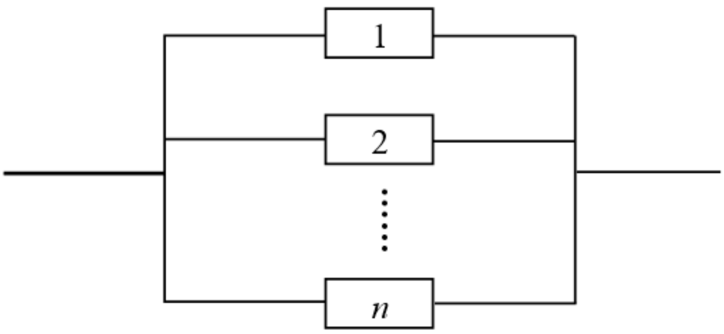

Figure 2. Parallel system.

Definition 4. *A structural system has n mechanical elements, and the performance functions of each mechanical element are uncertain random variables $Z_{01}, Z_{02}, \cdots, Z_{0n}$. When the whole system is a series system, its performance function $Z_{0\mathrm{ss}}$ can be defined as*

$$Z_{0\mathrm{ss}} = Z_{01} \wedge Z_{02} \wedge \cdots \wedge Z_{0n},$$

where $\wedge$ represents a minimum.

Theorem 2. *A structural system is a series system, and it includes n mechanical elements, of which the performance functions $Z_{01}, Z_{02}, \cdots, Z_{0n}$ are random uncertain variables and independent of each other. $R_1, R_2, \cdots, R_n$ and $G_1, G_2, \cdots, G_n$ are random variables, which are independent and have probability distributions $\Psi_{R_1}, \Psi_{R_2}, \cdots, \Psi_{R_n}$ and $\Psi_{G_1}, \Psi_{G_2}, \cdots, \Psi_{G_n}$, respectively.*

$Q_1, Q_2, \cdots, Q_n$ are relatively independent uncertain variables with uncertainty distributions $\Upsilon_{Q_1}, \Upsilon_{Q_2}, \cdots, \Upsilon_{Q_n}$. The failure measure of the series structural system is shown in Equation (11).

$$M_{\mathrm{f0,ss}} = \int_{\mathrm{R}^{2n}} \alpha \mathrm{d}\Psi_{\mathrm{R}_1}(y_{\mathrm{R}_1})\mathrm{d}\Psi_{\mathrm{R}_2}(y_{\mathrm{R}_2})\cdots\mathrm{d}\Psi_{\mathrm{R}_n}(y_{\mathrm{R}_n})\mathrm{d}\Psi_{\mathrm{G}_1}(y_{\mathrm{G}_1})\mathrm{d}\Psi_{\mathrm{G}_2}(y_{\mathrm{G}_2})\cdots\mathrm{d}\Psi_{\mathrm{G}_n}(y_{\mathrm{G}_n}) \tag{11}$$

where

$$\alpha = \left[1 - \Upsilon_{Q_1}\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \vee \left[1 - \Upsilon_{Q_2}\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right] \vee \cdots \vee \left[1 - \Upsilon_{Q_n}\left(\frac{y_{\mathrm{R}_n}}{y_{\mathrm{G}_n}}\right)\right].$$

Proof. According to the definition and operational law of chance measure, the failure measure of the series structural system can be expressed as

$$\begin{aligned} M_{\mathrm{f0,ss}} &= Ch\{Z_{0\mathrm{ss}} < 0\} \\ &= Ch\{Z_1 \wedge Z_2 \wedge \cdots \wedge Z_n < 0\} \\ &= Ch\{(R_1 - Q_1G_1) \wedge (R_2 - Q_2G_2) \wedge \cdots \wedge (R_n - Q_nG_n) < 0\} \\ &= \textstyle\int_{\mathrm{R}^{2n}} \alpha \mathrm{d}\Psi_{\mathrm{R}_1}(y_{\mathrm{R}_1})\mathrm{d}\Psi_{\mathrm{R}_2}(y_{\mathrm{R}_2})\cdots\mathrm{d}\Psi_{\mathrm{R}_n}(y_{\mathrm{R}_n})\ \mathrm{d}\Psi_{\mathrm{G}_1}(y_{\mathrm{G}_1})\mathrm{d}\Psi_{\mathrm{G}_2}(y_{\mathrm{G}_2})\cdots\mathrm{d}\Psi_{\mathrm{G}_n}(y_{\mathrm{G}_n}) \end{aligned}$$

Obviously, $Z = Z_1 \wedge Z_2 \wedge \cdots \wedge Z_n$ is a strictly decreasing function of $Q_1, Q_2, \cdots, Q_n$. Therefore,

$$\begin{aligned} M_{\mathrm{f0,ss}} = &\textstyle\int_{\mathrm{R}^{2n}} G(y_{\mathrm{R}_1}, y_{\mathrm{R}_2}, \cdots, y_{\mathrm{R}_n}, y_{\mathrm{G}_1}, y_{\mathrm{G}_2}, \cdots, y_{\mathrm{G}_n}) \\ &\mathrm{d}\Psi_{\mathrm{R}_1}(y_{\mathrm{R}_1})\mathrm{d}\Psi_{\mathrm{R}_2}(y_{\mathrm{R}_2})\cdots\mathrm{d}\Psi_{\mathrm{R}_n}(y_{\mathrm{R}_n})\mathrm{d}\Psi_{\mathrm{G}_1}(y_{\mathrm{G}_1})\mathrm{d}\Psi_{\mathrm{G}_2}(y_{\mathrm{G}_2})\cdots\mathrm{d}\Psi_{\mathrm{G}_n}(y_{\mathrm{G}_n}) \end{aligned}$$

where $G(y_{\mathrm{R}_1}, y_{\mathrm{R}_2}, \cdots, y_{\mathrm{R}_n}, y_{\mathrm{G}_1}, y_{\mathrm{G}_2}, \cdots, y_{\mathrm{G}_n})$ is α, which is the root of the following equation

$$\left(y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1}\right) \wedge \left(y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}\right) \wedge \cdots \wedge \left(y_{\mathrm{R}_n} - \Upsilon_n^{-1}(1-\alpha)y_{\mathrm{G}_n}\right) = 0.$$

For simplicity, the case $n = 2$ will be proved. In this case,

$$\left(y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1}\right) \wedge \left(y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}\right) = 0$$

There are two subcases.

Subcase one: Assume $y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} \le y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}$, thus

$$y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} = 0,$$

then

$$\alpha = 1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)$$

can be obtained.

Meanwhile $y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2} \ge 0$, thus

$$\alpha \ge 1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right).$$

Hence,

$$\alpha = \left[1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \vee \left[1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right].$$

Subcase two: Assume $y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} > y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}$, thus

$$y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2} = 0,$$

then

$$\alpha = 1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)$$

can be obtained.

At the same time, $y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} \geq 0$, so

$$\alpha \geq 1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right).$$

Hence

$$\alpha = \left[1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \vee \left[1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right].$$

By combining subcase 1 and subcase 2, it can be proved that:

$$\alpha = \left[1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \vee \left[1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right].$$

Therefore,

$$\alpha = \left[1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \vee \left[1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right] \vee \cdots \vee \left[1 - \Upsilon_n\left(\frac{y_{\mathrm{R}_n}}{y_{\mathrm{G}_n}}\right)\right].$$

Thus, the theorem is proved. □

Definition 5. *A structural system has n mechanical elements, and the performance functions of each mechanical element are uncertain random variables $Z_{01}, Z_{02}, \cdots, Z_{0n}$. When the whole system is a parallel system, its performance function $Z_{0\mathrm{ps}}$ can be defined as*

$$Z_{0\mathrm{ps}} = Z_{01} \vee Z_{02} \vee \cdots \vee Z_{0n},$$

where $\vee$ represents a maximum.

Theorem 3. *A structural system is a parallel system, and it includes n mechanical elements, of which the corresponding performance functions $Z_{01}, Z_{02}, \cdots, Z_{0n}$ are independent random uncertain variables. $R_1, R_2, \cdots, R_n$ and $G_1, G_2, \cdots, G_n$ are random variables, which are independent and have probability distributions $\Psi_{R_1}, \Psi_{R_2}, \cdots, \Psi_{R_n}$ and $\Psi_{G_1}, \Psi_{G_2}, \cdots, \Psi_{G_n}$, respectively. $Q_1, Q_2, \cdots, Q_n$ are uncertain variables, which are relatively independent and have uncertainty distributions $\Upsilon_{Q_1}, \Upsilon_{Q_2}, \cdots, \Upsilon_{Q_n}$. The failure measure of the parallel structural system is shown in Equation (12).*

$$M_{\mathrm{f0,ps}} = \int_{\mathrm{R}^{2n}} \alpha \mathrm{d}\Psi_{\mathrm{R}_1}(y_{\mathrm{R}_1})\mathrm{d}\Psi_{\mathrm{R}_2}(y_{\mathrm{R}_2})\cdots\mathrm{d}\Psi_{\mathrm{R}_n}(y_{\mathrm{R}_n})\mathrm{d}\Psi_{\mathrm{G}_1}(y_{\mathrm{G}_1})\mathrm{d}\Psi_{\mathrm{G}_2}(y_{\mathrm{G}_2})\cdots\mathrm{d}\Psi_{\mathrm{G}_n}(y_{\mathrm{G}_n}) \tag{12}$$

where

$$\alpha = \left[1 - \Upsilon_{Q_1}\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \wedge \left[1 - \Upsilon_{Q_2}\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right] \wedge \cdots \wedge \left[1 - \Upsilon_{Q_n}\left(\frac{y_{\mathrm{R}_n}}{y_{\mathrm{G}_n}}\right)\right].$$

Proof. According to the definition and operational law of chance measure, the failure measure of a parallel structural system can be expressed as

$$\begin{aligned} M_{\mathrm{f0,ps}} &= Ch\{Z_{0\mathrm{ps}} < 0\} \\ &= Ch\{Z_{01} \vee Z_{02} \vee \cdots \vee Z_{0n} < 0\} \\ &= Ch\{(R_1 - Q_1G_1) \vee (R_2 - Q_2G_2) \vee \cdots \vee (R_n - Q_nG_n) < 0\} \\ &= \int_{\mathrm{R}^{2n}} \alpha \mathrm{d}\Psi_{\mathrm{R}_1}(y_{\mathrm{R}_1})\mathrm{d}\Psi_{\mathrm{R}_2}(y_{\mathrm{R}_2})\cdots\mathrm{d}\Psi_{\mathrm{R}_n}(y_{\mathrm{R}_n})\mathrm{d}\Psi_{\mathrm{G}_1}(y_{\mathrm{G}_1})\mathrm{d}\Psi_{\mathrm{G}_2}(y_{\mathrm{G}_2})\cdots\mathrm{d}\Psi_{\mathrm{G}_n}(y_{\mathrm{G}_n}) \end{aligned}$$

Obviously, $Z = Z_1 \vee Z_2 \vee \cdots \vee Z_n$ is a strictly decreasing function of $Q_1, Q_2, \cdots, Q_n$, and therefore

$$M_{\mathrm{f0,ps}} = \int_{\mathrm{R}^{2n}} G(y_{\mathrm{R}_1}, y_{\mathrm{R}_2}, \cdots, y_{\mathrm{R}_n}, y_{\mathrm{G}_1}, y_{\mathrm{G}_2}, \cdots, y_{\mathrm{G}_n}) \\ \mathrm{d}\Psi_{\mathrm{R}_1}(y_{\mathrm{R}_1})\mathrm{d}\Psi_{\mathrm{R}_2}(y_{\mathrm{R}_2})\cdots\mathrm{d}\Psi_{\mathrm{R}_n}(y_{\mathrm{R}_n})\mathrm{d}\Psi_{\mathrm{G}_1}(y_{\mathrm{G}_1})\mathrm{d}\Psi_{\mathrm{G}_2}(y_{\mathrm{G}_2})\cdots\mathrm{d}\Psi_{\mathrm{G}_n}(y_{\mathrm{G}_n})$$

where $G(y_{\mathrm{R}_1}, y_{\mathrm{R}_2}, \cdots, y_{\mathrm{R}_n}, y_{\mathrm{G}_1}, y_{\mathrm{G}_2}, \cdots, y_{\mathrm{G}_n})$ is α, which is the root of the following equation.

$$\left(y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1}\right) \vee \left(y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}\right) \vee \cdots \vee \left(y_{\mathrm{R}_n} - \Upsilon_n^{-1}(1-\alpha)y_{\mathrm{G}_n}\right) = 0.$$

For simplicity, case $n = 2$ will be proved, and in this case:

$$\left(y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1}\right) \vee \left(y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}\right) = 0.$$

There are two subcases.

Subcase one: Assume $y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} \le y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}$, thus

$$\left(y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}\right) = 0,$$

then

$$\alpha = 1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)$$

can be obtained.

Meanwhile $y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} \le 0$, thus

$$\alpha \le 1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right).$$

Hence

$$\alpha = \left[1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \wedge \left[1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right].$$

Subcase two: Assume $y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} > y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2}$, thus

$$y_{\mathrm{R}_1} - \Upsilon_1^{-1}(1-\alpha)y_{\mathrm{G}_1} = 0,$$

then

$$\alpha = 1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)$$

can be obtained.

At the same time $y_{\mathrm{R}_2} - \Upsilon_2^{-1}(1-\alpha)y_{\mathrm{G}_2} \le 0$, so

$$\alpha \le 1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right).$$

Combining subcase 1 and subcase 2,

$$\alpha = \left[1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \wedge \left[1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right]$$

can be obtained. Therefore,

$$\alpha = \left[1 - \Upsilon_1\left(\frac{y_{\mathrm{R}_1}}{y_{\mathrm{G}_1}}\right)\right] \wedge \left[1 - \Upsilon_2\left(\frac{y_{\mathrm{R}_2}}{y_{\mathrm{G}_2}}\right)\right] \wedge \cdots \wedge \left[1 - \Upsilon_n\left(\frac{y_{\mathrm{R}_n}}{y_{\mathrm{G}_n}}\right)\right]$$

Thus, the theorem is proved. □

6. Numerical Examples

According to the above, the reliability of the structural system can be conducted as shown in Figure 3. In order to show the application of the new method, two examples, including a series system and a parallel system, were given as follows.

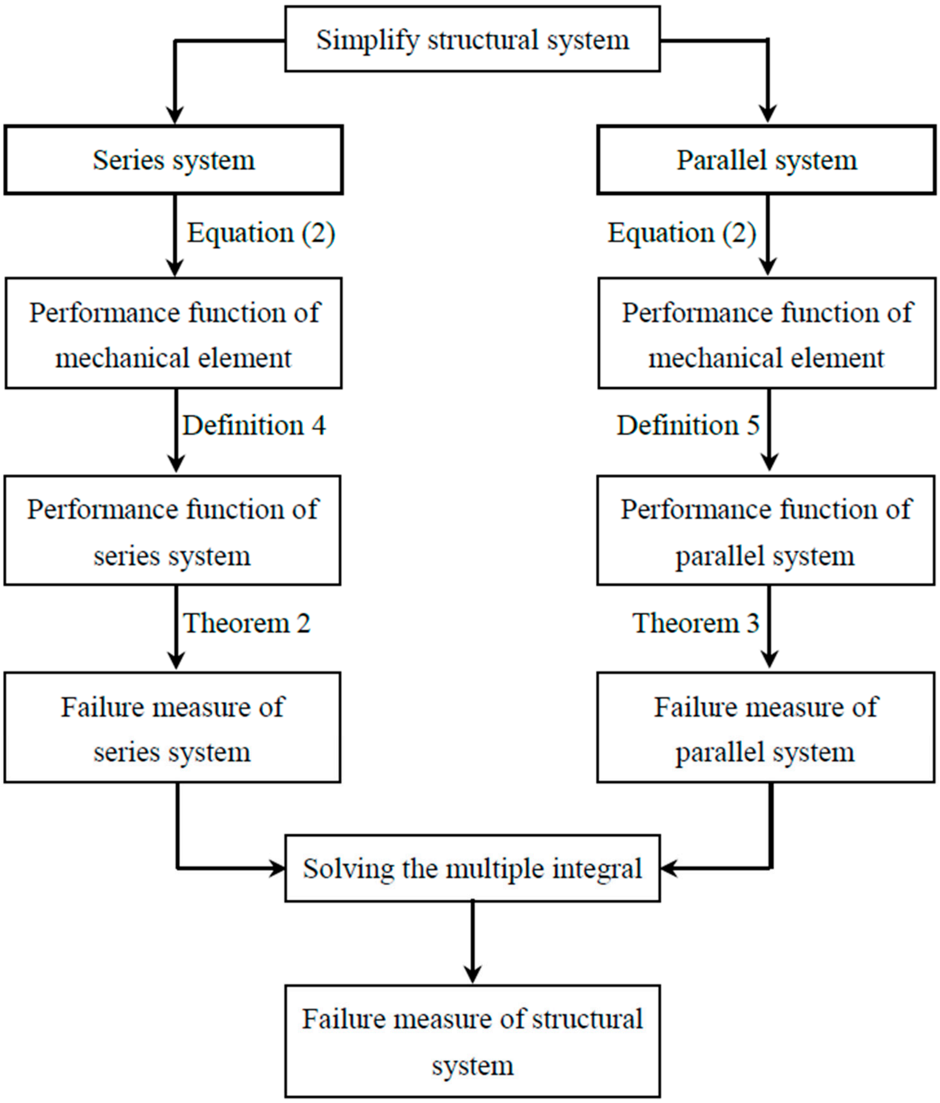

Figure 3. Flowchart of the structural reliability analysis.

Example 1. *In a system shown in Figure 4, there are two rods connected by a unidirectional hinge, and the same uniform load is applied at both free ends. The two rods, which are produced by two different factories, respectively, are both randomly selected from a large number of rods. Therefore, the cross-sectional areas (unit: mm^2) recorded as A_1 and A_2 and the tensile capacities (unit: N) recorded as F_1 and F_2 are independent random variables, respectively. According to probability statistics, the probability distributions of A_1 and A_2 are Ψ_{A_1} and Ψ_{A_2}, respectively, and the probability distributions of F_1 and F_2 are Ψ_{F_1} and Ψ_{F_2} respectively. The uniform load (unit: N/mm^2) recorded as q is an uncertain variable, and the uncertainty distribution is Υ_q according to the uncertain statistics. Try to analyze the reliability of the system based on the tensile performance of rods.*

Assume that A_1, A_2, F_1 and F_2 are all normal variables and are N (2800, 126), N (2900, 153), N (630,000, 9850) and N (650,000, 11,200), respectively. q is linear uncertain variable L (180, 220).

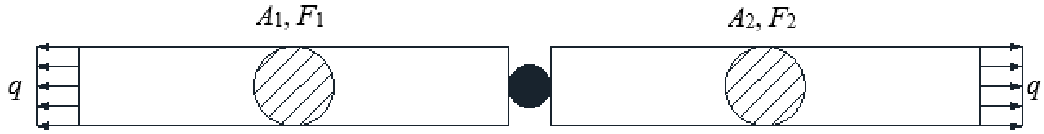

Figure 4. An example of a series system.

This system is a series system and includes two mechanical elements, namely two rods. Based on the tensile performance, the performance function of the rod can be expressed as

$$Z_{0i} = F_i - qA_i$$

in which i = 1 or 2.

According to Definition 4, the performance function of the series system is

$$Z_{0\text{ss}} = Z_{01} \wedge Z_{02} = (F_1 - qA_1) \wedge (F_2 - qA_2)$$

According to Theorem 2, the failure measure of the series system can be presented as

$$M_{\text{f0,ss}} = \int_{\text{R}^4} \alpha \text{d}\Psi_{\text{F}_1}(y_{\text{F}_1})\text{d}\Psi_{\text{F}_2}(y_{\text{F}_2})\text{d}\Psi_{\text{A}_1}(y_{\text{A}_1})\text{d}\Psi_{\text{A}_2}(y_{\text{A}_2})$$

$$\alpha = \left[1 - \Upsilon_{\text{q}}\left(\frac{y_{\text{F}_1}}{y_{\text{A}_1}}\right)\right] \vee \left[1 - \Upsilon_{\text{q}}\left(\frac{y_{\text{F}_2}}{y_{\text{A}_2}}\right)\right]$$

where $y_{\text{F}_1}, y_{\text{F}_2}, y_{\text{A}_1}$ and y_{A_1} are any representative values of F_1, F_2, A_1 and A_2 respectively.

Hence,

$$M_{\text{f0,ss}} = \int_{\text{R}^4} \left\{\left[1 - \Upsilon_{\text{q}}\left(\frac{y_{\text{F}_1}}{y_{\text{A}_1}}\right)\right] \vee \left[1 - \Upsilon_{\text{q}}\left(\frac{y_{\text{F}_2}}{y_{\text{A}_2}}\right)\right]\right\} \text{d}\Psi_{\text{F}_1}(y_{\text{F}_1})\text{d}\Psi_{\text{F}_2}(y_{\text{F}_2})\text{d}\Psi_{\text{A}_1}(y_{\text{A}_1})\text{d}\Psi_{\text{A}_2}(y_{\text{A}_2})$$

The solution of multiple integrals is usually difficult, but every integral can be regarded as the mathematical expectation of a random variable [20]. Therefore, $M_{\text{f0,ss}}$ can be solved by using the following method.

$$\begin{aligned}
M_{\text{f0,ss}} &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{\left[1 - \Upsilon_{\text{q}}\left(\tfrac{y_{\text{F}_1}}{y_{\text{A}_1}}\right)\right] \vee \left[1 - \Upsilon_{\text{q}}\left(\tfrac{y_{\text{F}_2}}{y_{\text{A}_2}}\right)\right]\right\} \text{d}\Psi_{\text{F}_1}(y_{\text{F}_1})\text{d}\Psi_{\text{F}_2}(y_{\text{F}_2})\text{d}\Psi_{\text{A}_1}(y_{\text{A}_1})\text{d}\Psi_{\text{A}_2}(y_{\text{A}_2}) \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{\left[1 - \Upsilon_{\text{q}}\left(\tfrac{y_{\text{F}_1}}{y_{\text{A}_1}}\right)\right] \vee \left[1 - \Upsilon_{\text{q}}\left(\tfrac{y_{\text{F}_2}}{y_{\text{A}_2}}\right)\right]\right\} f_{\text{F}_1}(y_{\text{F}_1})f_{\text{F}_2}(y_{\text{F}_2})f_{\text{A}_1}(y_{\text{A}_1})f_{\text{A}_2}(y_{\text{A}_2})\text{d}y_{\text{F}_1}\text{d}y_{\text{F}_2}\text{d}y_{\text{A}_1}\text{d}y_{\text{A}_2} \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{\left[1 - \Upsilon_{\text{q}}\left(\tfrac{y_{\text{F}_1}}{y_{\text{A}_1}}\right)\right] \vee \left[1 - \Upsilon_{\text{q}}\left(\tfrac{y_{\text{F}_2}}{y_{\text{A}_2}}\right)\right]\right\} f(y_{\text{F}_1}, y_{\text{F}_2}, y_{\text{A}_1}, y_{\text{A}_2})\text{d}y_{\text{F}_1}\text{d}y_{\text{F}_2}\text{d}y_{\text{A}_1}\text{d}y_{\text{A}_2} \\
&= E\left\{\left[1 - \Upsilon_{\text{q}}\left(\tfrac{F_1}{A_1}\right)\right] \vee \left[1 - \Upsilon_{\text{q}}\left(\tfrac{F_2}{A_2}\right)\right]\right\}
\end{aligned}$$

where $f_{\text{F}_1}, f_{\text{F}_2}, f_{\text{A}_1}, f_{\text{A}_2}$ are the probability density functions of F_1, F_2, A_1, A_2, respectively; $f(y_{\text{F}_1}, y_{\text{F}_2}, y_{\text{A}_1}, y_{\text{A}_2})$ is the probability density function of four dimensional random variables F_1, F_2, A_1 and A_2.

It can be seen that $M_{\text{f0,ss}}$ is the mathematical expectation of a random variable function $\left[1 - \Upsilon_{\text{q}}\left(\frac{F_1}{A_1}\right)\right] \vee \left[1 - \Upsilon_{\text{q}}\left(\frac{F_2}{A_2}\right)\right]$. It is difficult to obtain the exact solution, but the approximate solution can be acquired by using the Monte Carlo method.

According to the numerical characteristics of the normal distributions for A_1, A_2, F_1 and F_2 and the linear uncertainty distribution of q, the failure measure of the series system can be calculated as $M_{\text{f0,ss}}$ = 0.1092.

Example 2. *In a system shown in Figure 5, there are two simply supported beams for transporting goods from C to D. The two beams, which are produced by two different factories, respectively, are both randomly selected from a large number of beams. Therefore, the lengths (unit: m) recorded as l_1 and l_2 and the flexural capacities (unit: kN·m) recorded as M_1 and M_2 are independent random variables, respectively. According to probability statistics, the probability distributions of l_1 and*

l_2 are Ψ_1 and Ψ_2 respectively, and the probability distributions of M_1 and M_2 are Υ_1 and Υ_2 respectively. The weight of the goods and transport vehicle (unit: kN) recorded as P is regarded as a concentrated load and is an uncertain variable. According to uncertain statistics, the uncertain distribution of P is Φ. Without considering the dynamic effect of the transport vehicle, try to analyze the reliability of the system based on the flexural performance of the beam.

Assume that l_1, l_2, M_1 and M_2 are all normal variables and are N (9.5, 0.7), N (9.7, 0.9), N (550, 37) and N (560, 41), respectively. q is linear uncertain variable L (190, 210).

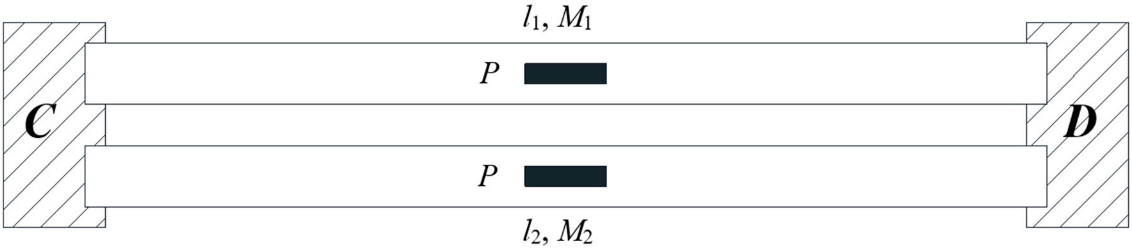

Figure 5. An example of a parallel system.

This system is a parallel system and includes two mechanical elements, namely two simply supported beams. According to the mechanical analysis, the performance function of the simply supported beam based on the flexural performance can be expressed as

$$Z_{0i} = M_i - \frac{1}{4}Pl_i$$

where i = 1 or 2.

According to Definition 5, the performance function of the parallel system is

$$Z_{0\mathrm{ps}} = Z_{01} \vee Z_{02} = \left(M_1 - \frac{1}{4}Pl_1\right) \vee \left(M_2 - \frac{1}{4}Pl_2\right)$$

According to Theorem 3, the failure measure of the parallel system can be presented as

$$M_{\mathrm{f0,ps}} = \int_{\mathrm{R}^4} \alpha \mathrm{d}\Psi_{\mathrm{M}_1}(y_{\mathrm{M}_1})\mathrm{d}\Psi_{\mathrm{M}_2}(y_{\mathrm{M}_2})\mathrm{d}\Psi_{\mathrm{l}_1}(y_{\mathrm{l}_1})\mathrm{d}\Psi_{\mathrm{l}_2}(y_{\mathrm{l}_2})$$

$$\alpha = \left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_1}}{y_{\mathrm{l}_1}}\right)\right] \wedge \left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_2}}{y_{\mathrm{l}_2}}\right)\right]$$

where y_{M_1}, y_{M_2}, y_{l_1} and y_{l_2} are any representative values of M_1, M_2, l_1 and l_2 respectively.

Hence,

$$M_{\mathrm{f0,ps}} = \int_{\mathrm{R}^4} \left\{\left[1 - \Upsilon_{\mathrm{P}}\left(\frac{4y_{\mathrm{M}_1}}{y_{\mathrm{l}_1}}\right)\right] \wedge \left[1 - \Upsilon_{\mathrm{P}}\left(\frac{4y_{\mathrm{M}_2}}{y_{\mathrm{l}_2}}\right)\right]\right\}\mathrm{d}\Psi_{\mathrm{M}_1}(y_{\mathrm{M}_1})\mathrm{d}\Psi_{\mathrm{M}_2}(y_{\mathrm{M}_2})\mathrm{d}\Psi_{\mathrm{l}_1}(y_{\mathrm{l}_1})\mathrm{d}\Psi_{\mathrm{l}_2}(y_{\mathrm{l}_2})$$

The simplifying calculation process of $M_{\mathrm{f0,ps}}$ is shown as follows.

$$\begin{aligned}
M_{\mathrm{f0,ps}} &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{\left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_1}}{y_{\mathrm{l}_1}}\right)\right] \wedge \left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_2}}{y_{\mathrm{l}_2}}\right)\right]\right\}\mathrm{d}\Psi_{\mathrm{M}_1}(y_{\mathrm{M}_1})\mathrm{d}\Psi_{\mathrm{M}_2}(y_{\mathrm{M}_2})\mathrm{d}\Psi_{\mathrm{l}_1}(y_{\mathrm{l}_1})\mathrm{d}\Psi_{\mathrm{l}_2}(y_{\mathrm{l}_2}) \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{\left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_1}}{y_{\mathrm{l}_1}}\right)\right] \wedge \left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_2}}{y_{\mathrm{l}_2}}\right)\right]\right\} f_{\mathrm{M}_1}(y_{\mathrm{M}_1}) f_{\mathrm{M}_2}(y_{\mathrm{M}_2}) f_{\mathrm{l}_1}(y_{\mathrm{l}_1}) f_{\mathrm{lA}_2}(y_{\mathrm{l}_2})\mathrm{d}y_{\mathrm{M}_1}\mathrm{d}y_{\mathrm{M}_2}\mathrm{d}y_{\mathrm{l}_1}\mathrm{d}y_{\mathrm{l}_2} \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{\left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_1}}{y_{\mathrm{l}_1}}\right)\right] \wedge \left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4y_{\mathrm{M}_2}}{y_{\mathrm{l}_2}}\right)\right]\right\} f(y_{\mathrm{M}_1}, y_{\mathrm{M}_2}, y_{\mathrm{l}_1}, y_{\mathrm{l}_2})\mathrm{d}y_{\mathrm{M}_1}\mathrm{d}y_{\mathrm{M}_2}\mathrm{d}y_{\mathrm{l}_1}\mathrm{d}y_{\mathrm{l}_2} \\
&= E\left\{\left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4M_1}{l_1}\right)\right] \wedge \left[1 - \Upsilon_{\mathrm{P}}\left(\tfrac{4M_2}{l_2}\right)\right]\right\}
\end{aligned}$$

where f_{M_1}, f_{M_2}, f_{l_1}, f_{l_2} is the probability density functions of M_1, M_2, l_1, l_2, respectively; $f(y_{\mathrm{M}_1}, y_{\mathrm{M}_2}, y_{\mathrm{l}_1}, y_{\mathrm{l}_2})$ is the probability density function of four dimensional random variables M_1, M_2, l_1 and l_2.

According to the Monte Carlo method, the numerical characteristics of the normal distributions for l_1, l_2, M_1 and M_2 and the linear uncertainty of P, the failure measure of the system can be calculated as $M_{\mathrm{f0,ps}} = 0.0103$

7. Conclusions

Based on the chance theory, a new analysis method for structural reliability under uncertain conditions was proposed, and the following conclusions were drawn.

(1) According to the different types of basic variables, the performance function of structural mechanical elements can be regarded as an uncertain random variable, a random variable or an uncertain variable;

(2) If the performance function of a structural mechanical element only includes the structural resistance and the action effect, it is determined as an uncertain random variable, which can reflect the influence of random uncertainty and cognitive uncertainty simultaneously, and its equivalent expression is proposed to facilitate analysis;

(3) For the performance function of a structural mechanical element, which is an uncertain random variable, the chance distribution is established, and the calculated methods of failure measure and reliability measure are put forward. It can be used for the structural mechanical element, for which the large statistic data can be obtained for some influence factors and not for some other influence factors;

(4) For the structural system, which is a series system or parallel system, the performance function is defined, and the calculated method of failure measure is proposed by theoretical proof. It can be used for the structural reliability of practical structures, which can be simplified to series systems or parallel systems.

This paper presented the preliminary study for structural reliability based on the chance theory. The results can be used for the series system and parallel system only. However, If the practical structures are more complex and can not be simplified to series systems or parallel systems, the new method can not be used to analyze the structural reliability. Therefore, future studies need to solve the problem of how to improve the proposed structural reliability method to better apply it to practical structures, especially complex practical structures.

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Nomenclature

Ch	Chance measure
f	Measurable function
G	Geometric parameters
M_{f}	Failure measure
M_{r}	Reliability measure
Q	Load
R	Structural resistance
S	Action effect
X	Basic variable
Z	Performance function

ξ	Random variable, Uncertain variable or Uncertain random variable
Υ	Uncertainty distribution
Υ^{-1}	Inverse uncertainty distribution
Φ	Uncertainty distribution or Chance distribution
Φ^{-1}	Inverse uncertainty distribution
Ψ	Probability distribution
$\mathcal{M}$	Uncertain measure

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