

Article

Nonlinear Deformation Model for Analysis of Temperature Effects on Reinforced Concrete Beam Elements

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Abstract: The results of the study are aimed at developing a nonlinear strain model for reinforced concrete beam elements in the general case of force actions and temperature effects for different durations. As a rule, temperature effects on structures involve temperature drops, causing heterogeneity of mechanical and rheological properties of concrete and reinforcement. The article has approximating expressions needed to take into account the effects of temperature and heating time on the values of temperature-induced strain and mechanical and rheological properties of heavy concretes with C30–C60 strength classes. The properties of concrete and rebars are heterogeneous from top to bottom and across the width of the cross section. A physically nonlinear problem is solved using the method of elastic solutions combined with the method of stepwise increases in temperature and force loading. The cooling of a heated reinforced concrete element to normal temperature is considered a short-term effect. Strength criteria of portions of a concrete cross section, crack closure conditions, and the ability of cracked sections to take loads in compression amid a change in the sign of stresses are determined. The stress–strain state (SSS) analysis of reinforced concrete beams, made according to the proposed method, is compared with the experimental studies using (i) values of thermal bending moments in statically indeterminate structures, (ii) cracking forces, and (iii) values of deformations (elongations and curvatures) of the elements in the longitudinal axis. Good agreement between the calculated and experimental values of controllable criteria confirms the reliability of physical relationships (i) developed for heterogeneous reinforced concrete beam elements and (ii) applied to the complex cases of temperature and force effects.

Keywords: reinforced concrete; beam; force and temperature effects; heterogeneity; deformation model; stresses; strains; forces; relaxation



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1. Introduction

Many structures of industrial buildings and engineering structures (chimneys, cooling towers, tanks, etc.) are characterized by complex loading and heating modes corresponding to the stages of construction and subsequent operation. Temperature effects, produced on structures, are thermal gradients. Nonuniform heating of reinforced concrete structures (the most frequent case) is the reason for heterogeneous strength and deformation properties of concrete and rebars, and so-called “temperature” forces are triggered in statically indeterminate structures. Temperature forces in structures change (relax) over time due to the cracking of reinforced concrete, differences in dimensions and rates of shrinkage, and creep deformation of nonuniformly heated concrete [1–3].

The accuracy of the calculated results of reinforced concrete structures depends, to a large extent, on the reliability of physical relations for concrete and reinforcement, taking into account significant influencing factors [4–6]. Sufficiently numerous numerical studies of the behaviors of reinforced concrete beam-type elements under fire conditions have been performed under conditions of symmetric heating of a section with a limited time

of temperature actions [7–14]. In this case, creep deformations of concrete, as a rule, are not taken into account. For structures that are subjected to systematic effects of elevated process temperatures, different modes of heating, loading, and cooling after prolonged heating are possible [2,3,15–17]. The modeling of such modes requires taking into account in physical relations the possibility of previous loading.

Characteristics of mechanical and rheological properties of concrete, subjected to high temperatures, depend on a number of factors, such as temperature and time of heating, age of concrete before heating and loading, heating and loading modes, and the scale factor [18–23].

The complexity of the tasks lies in the fact that all of the above-mentioned factors are manifested simultaneously in the case of nonuniform heating of reinforced concrete structures. The independent evaluation of the effect, produced by each factor, is too difficult, because it requires the implementation of special heating, loading, and strain measurement programs within the framework of experimental studies. The differences between testing methods determine discrepancies between the results of experiments reported by a number of researchers [1–3,24–26].

Experimental studies [27–29] were conducted using the same testing methods and concretes, with similar compositions. These studies allowed for a quantitative evaluation of the effect produced by the main factors on the physical–mechanical and rheological properties of heavy concretes with C30–C60 strength classes. However, analytical expressions, based on the experimental data obtained using prism-shaped specimens under isothermal heating conditions, need to be verified as part of analytical models of reinforced concrete structures subjected to characteristic force effects and heterogeneous temperature effects.

The objective of the study is to develop physical relationships for inhomogeneous reinforced concrete beam elements and to verify them by comparing the computed values with the data of experimental studies conducted under the effect of (i) temperature gradients, with different time frames, and (ii) loadings in differing planes.

The subject of the research encompasses (i) physical correlations for heterogeneous reinforced concrete elements of a structure subjected to combined force and temperature effects, over different durations, and (ii) characteristics of the stress–strain state (SSS) of reinforced concrete elements subjected to characteristic force and temperature effects.

Objectives of the study:

1. To develop analytical expressions to evaluate the effect of temperature, time, and modes of loading and heating on the values of temperature-induced strain, characteristics of mechanical and rheological properties of concrete.
2. To develop a system of equations to describe the equilibrium between external and internal forces for normal cross sections of reinforced concrete elements within the framework of a nonlinear strain model. The system of equations is to take into account principal factors triggered by temperature and loading effects, or heterogeneous characteristics of mechanical and rheological properties, temperature shrinkage and creep deformations of concrete, physical nonlinearity of concrete deformation and cracking.
3. To verify the calculation model by comparing it with the data of experimental studies.
4. To identify the effects of major factors on the SSS of reinforced concrete structures, given that these factors include the temperature difference, values of longitudinal forces, and bending moments in the general case of misalignment with the main axes of the cross section of elements.

2. Method

2.1. Mechanical Properties of Concrete under Exposure Conditions to High Temperatures

In the case of the axial compression force $R_{b,tem}$, the axial tension force $R_{bt,t}$, the initial modulus of elasticity $E_{b,t}$, high temperature values t^0 during time T , and values of concrete

strength are found using the parameters $\gamma_{b,i}$, $\gamma_{t,i}$, and $\beta_{b,i}$ ($i = t^0, T, \eta_{sl}$), taking into account long-term compressive preloading η_{sl} :

$$\begin{aligned} R_{b,tem}(t^0, T, \eta_{sl}) &= R_b \cdot \gamma_{bt} \cdot \gamma_{b,\eta} \\ R_{bt,t}(t^0, T, \eta_{sl}) &= R_{bt} \cdot \gamma_{tt} \cdot \gamma_{t,\eta} \\ E_{b,t}(t^0, T, \eta_{sl}) &= E_b \cdot \beta_b \cdot \beta_\eta \end{aligned} \quad (1)$$

Expressions for the parameters γ_{bt} , γ_{tt} , and β_b are taken from [3] as

$$\begin{aligned} \gamma_{bt} &= [1 - F_1 + F_4 \cdot F_b(t^0, T)] \\ \gamma_{tt} &= [1 - F_2 + F_5 \cdot F_{bt}(t^0, T)] \\ \beta_b &= [1 - F_3 + F_6 \cdot F_b(t^0, T)] \end{aligned} \quad (2)$$

In Equation (2), F_1 , F_2 , and F_3 are the parameters that take into account the effect of destructive processes in the structure of concrete during first short-term heating, which reduce the strength and the modulus of elasticity of concrete; F_4 , F_5 , and F_6 are the parameters that take into account structural processes in concrete during long-term heating, if these processes contribute to the partial recovery of property characteristics; $F_b(t^0, T)$ and $F_{bt}(t^0, T)$ are the parameters that take into account the rate of increment of concrete strength and the modulus of elasticity in case of long-term heating, determined using the following expressions [3]:

$$F_i = a_i (t^0 - 20^\circ)^3 + b_i (t^0 - 20^\circ)^2 + c_i (t^0 - 20^\circ) \quad (i = 1, 2, \dots, 6). \quad (3)$$

where a_i , b_i and c_i ($i = 1, 2, \dots, 6$) are the numerical coefficients, given as follows:

$$\begin{aligned} a_1 &= 2.7 \times 10^{-7}; b_1 = -9.5 \times 10^{-5}; c_1 = 9.8 \times 10^{-3}; \\ a_2 &= 3.25 \times 10^{-7}; b_2 = -1.16 \times 10^{-4}; c_2 = 1.2 \times 10^{-2}; \\ a_3 &= 1.3 \times 10^{-7}; b_3 = -5.8 \times 10^{-5}; c_3 = 8 \times 10^{-3}; \\ a_4 &= 1.7 \times 10^{-7}; b_4 = -5.5 \times 10^{-5}; c_4 = 5.55 \times 10^{-3}; \\ a_5 &= 1.95 \times 10^{-7}; b_5 = -7.7 \times 10^{-5}; c_5 = 7.7 \times 10^{-3}; \\ a_6 &= 1.05 \times 10^{-7}; b_6 = -3.3 \times 10^{-5}; c_6 = 2.2 \times 10^{-3}. \end{aligned} \quad (4)$$

The parameters $\gamma_{b,\eta}$ and β_η are used to take into account the effect of the modes of preliminary long-term loading on the strength and strain of concrete at high temperatures in case of further supplementary compressive loading [3] as

$$\gamma_{b,\eta} = A_1 \cdot (1 - \eta_{lt})^k + B_1 \cdot (1 - \eta_{lt}) \quad (5)$$

$$\beta_\eta = A_2 \cdot (1 - \eta_{lt})^n + B_2 \cdot (1 - \eta_{lt}) \quad (6)$$

$$\eta_{lt} = \sigma_l / R_{b,tem}(t^0, T) \quad (7)$$

where $B_i = 1 - A_i$, ($i = 1, 2$), η_{lt} is the long-term confinement of concrete.

The parameters A_i , k , and n are approximated as the functions of temperature t^0 :

$$A_1 = 2.7 \times (t^0 - 20^\circ)^3 \times 10^{-7} - 1.12 \times (t^0 - 20^\circ)^2 \times 10^{-4} + 1.66 \times (t^0 - 20^\circ) \times 10^{-2} + 2 \quad (8)$$

$$A_2 = -1.22 \times (t^0 - 20^\circ)^3 + 3.17 \times (t^0 - 20^\circ)^2 \times 10^{-4} - 0.022 \times (t^0 - 20^\circ) + 3.5 \quad (9)$$

$$k = 1.94 \times (t^0 - 20^\circ) \times 10^{-3} + 0.25 \text{ and } n = 0.5 \quad (10)$$

The ultimate strain $\varepsilon_{u,tem}$ values of preconfined concrete can be determined with sufficient accuracy in case of subsequent axial compression, accompanied by the heating

and axial tension strain $\varepsilon_{u,tt}$, depending on the characteristics of ε_u and ε_{ut} under normal temperature conditions using the following relationships:

$$\varepsilon_{u,tem} = \varepsilon_u \cdot \frac{\gamma_{bt} \cdot \gamma_{b,\eta}}{\beta_b \cdot \beta_\eta}; \varepsilon_{u,tt} = \varepsilon_{ut} \cdot \frac{\gamma_{tt}}{\beta_b} \quad (11)$$

2.2. Thermal Shrinkage of Concrete

The thermal shrinkage of concrete during isothermal heating at temperature t° during time T is identified using the formula [3] as

$$\varepsilon_{bt}(t^\circ, T) = [\alpha_{bt}(t^\circ) + \Delta\alpha_{bt}(t^\circ, T)] \cdot \Delta t^\circ - \varepsilon_{cs}(t^\circ, T) \text{ and } \Delta t^\circ = t^\circ - t_0^\circ \quad (12)$$

where t_0° is the values of the initial temperature of concrete before heating, $\alpha_{bt}(t^\circ)$ is the coefficient of the reversible thermal strain of concrete after long-term heating ($T \rightarrow \infty$) at temperature t° and drying to moisture equilibrium with the medium, and $\Delta\alpha_{bt}(t^\circ, T)$ is the increment value of the linear temperature-induced strain coefficient of concrete in the initial stages of heating if the humidity value exceeds the equilibrium humidity with the medium.

$$\alpha_{bt}(t^\circ) = [10 - 0.00001 (\Delta t^\circ)^2] \cdot 10^{-6} \quad (13)$$

$$\Delta\alpha_{bt}(t^\circ, T) = [(4.9 - 0.02 \cdot \Delta t^\circ) \cdot e^{-0.16 \cdot \Delta t^\circ \cdot T \cdot 10^{-2}}] \cdot 10^{-6} \quad (14)$$

$\varepsilon_{cs}(t^\circ, T)$ is the concrete shrinkage strain values by time T at heating temperature t° :

$$\varepsilon_{cs}(t^\circ, T) = 1.6 \cdot (\Delta t^\circ)^{0.2} \cdot (1 - e^{-0.29 \cdot \Delta t^\circ \cdot T \cdot 10^{-2}}) \cdot 10^{-4} \quad (15)$$

2.3. Time-Dependent Strains of Concrete at Elevated Temperatures

Under conditions of long-term exposure to compressive stress σ and high temperature t° by time T , the values of creep strain of concrete, ε_c , are found using methods described in [2,3] and the formula as

$$\varepsilon_c = (t^\circ, T_{red}, \tau_{red}, \eta_\sigma) = C(t^\circ, T_{red}, \tau_{red}, \eta_\sigma) \cdot \sigma \quad (16)$$

The total unit creep strain of concrete takes into account the nonlinear component at high temperatures:

$$C(\sigma, t^\circ, T_{red}, \tau_{red}, M_0) = (1 + \Delta f(\eta_\sigma, t^\circ)) \cdot C_l(t^\circ, T_{red}, \tau_{red}, M_0) \quad (17)$$

where $\Delta f(\eta_\sigma, t^\circ)$ is the function that takes into account the nonlinearity of concrete creep strain, $\eta_\sigma = \frac{\sigma}{R_b}$ is the long-term concrete confinement, and T_{red} is the reduced time of exposure to high temperatures from the beginning of heating and is computed according to the following method [2]:

$$T_{red} = \int_{\tau_i}^T F(t^\circ) \cdot dT \quad (18)$$

τ_i is the time value corresponding to the beginning of heating; τ_{red} is the reduced time of exposure to high temperatures from the beginning of heating to the beginning of loading and is calculated as

$$\tau_{red} = \int_{\tau_i}^{\tau_\sigma} F(t^\circ) \cdot d\tau \quad (19)$$

C_l is the linear unit creep strain of old drying concrete subjected to isothermal heating; it is found using the following method [2]:

$$C_l(t^\circ, T_{red}, \tau_{red}, M_0) = \theta(t^\circ, \tau_{red}, M_0) \cdot \{1 - \beta(t^\circ) \cdot \exp[-0.05 \cdot (T_{red} - \tau_{red})]\}, \quad (20)$$

where $\beta(t^0)$ is the coefficient that takes into account the fast component of the concrete creep strain [2]:

$$\beta(t^0) = 0.85 \cdot [1 - 0.0027 \cdot (t^0 - 20)] \quad (21)$$

$$\theta(t^0, \tau_{red}, M_0) = \theta(t^0, \tau_{red} = 0, M_0) \cdot \left\{ (0.07 + t^0 \cdot 10^{-3}) + (0.93 - t^0 \cdot 10^{-3}) \cdot \exp \left[- (0.08 + t^0 \cdot 6 \cdot 10^{-4}) \cdot \tau_{red} \right] \right\} \quad (22)$$

The analysis of the experimental data [3,7] serves as the basis for the following recommended expressions, describing the functions included in Formulas (17) and (22):

$$\Delta f(\eta_\sigma, t^0) = 0.5 \eta_\sigma \cdot (1 + \eta_\sigma) \cdot [1 + 1.5 (t^0 - 20) \cdot 10^{-3}] \quad (23)$$

$$\theta(t^0, \tau_{red}=0, M_0) = 0.77 \{12.7 - 0.018 t^0 - 9.3 \exp[-0.01 (t^0 - 20)]\} \cdot 10^{-5} \cdot \gamma_{c, M_0} \quad (24)$$

where γ_{c, M_0} is the function that takes into account the effect of massiveness of the structure (the scale factor) on the ultimate values of unit creep strain of concrete [3,7]:

$$\gamma_{c, M_0} = 0.6 + 0.4 \cdot \left(\frac{M_0}{30} \right)^{\frac{1}{2}} \quad (25)$$

where M_0 is the open surface modulus, defined as the ratio of the area of the lateral surface of the structure that is open for drying to its volume (m^{-1}).

Analytical Expressions (1)–(25) are applicable to heavy old drying concrete with C30–C60 strength classes in the temperature range of +20 °C to +200 °C and at heating rates of 10–20 °C/h.

2.4. Nonlinear Deformation Model

A nonlinear deformation model of an inhomogeneous reinforced concrete element is based on the following general design assumptions:

1. A reinforced concrete beam element, featuring inhomogeneous physical–mechanical and rheological properties of concrete and rebars across the width and throughout the height of its section, loaded by a longitudinal compressive (tensile) force and bending moments in vertical and horizontal planes, is considered.
2. A method of independent analysis of (a) long-term processes that are underway in concrete and (b) the phenomenon of physical nonlinearity of deformation is used.
3. Bernoulli's hypothesis is assumed to be true.
4. Regularities that govern relations between stresses and strains in concrete and rebars are identified using analytical expressions for preset diagrams of deformation of materials, taking into account the effects of temperature, heating time, and long-term preloading.
5. Temperature, shrinkage, and creep deformations of concrete are identified by using applicable methods and added to the deformations triggered by stresses in concrete and rebars (application of the superposition principle).
6. The behavior of reinforced concrete with cracks in the tensile zone of an element is modeled using the averaged stress for sections with and without cracks. This is achieved by averaging the values of longitudinal stiffness of reinforcement bars.

The inhomogeneity of the design cross section of a reinforced concrete element is modeled using the following supplementary assumptions:

1. The nonuniform heating of the cross section of reinforced concrete beam elements is considered in case of the arbitrary orientation of the temperature plane relative to the main axes of the element cross section. It is assumed that the element is free to elongate along axes X and Y, or $\sigma_x = 0$ and $\sigma_y = 0$.
2. The temperature distributions over the height and width of the section is assumed to be preset, with temperature being a function of the coordinates x_{ji} and y_{ji} . Initial

mechanical characteristics of concrete and rebars are assumed to be functions of the temperature effect and its duration.

3. The cross section of an inhomogeneous reinforced concrete element is represented as a system of hypothetically homogeneous small portions of concrete, having the dimensions $dF = dx \cdot dy$ and rebars with the cross-sectional area $f_{s,k}$ (Figure 1).
4. Stresses arising within the boundaries of each dF section are assumed to be uniformly distributed according to the values of force-triggered deformations at the centers of gravity of the design sections. It is assumed that within the boundaries of each small element and at each stage of loading, mechanical and rheological properties, as well as stresses in concrete, $\sigma_{b,ji}$, and in rebars, $\sigma_{s,k}$, are constant and correspond to the temperature at the center of gravity of each design section.
5. The inability of part of the concrete section to take load as a result of its failure in compression or cracking in tension is modeled by assigning the value of the modulus of deformation equal to $E'_{b,ji} = 0$.
6. The criteria of concrete strength inside layers are determined by the values of ultimate deformations.

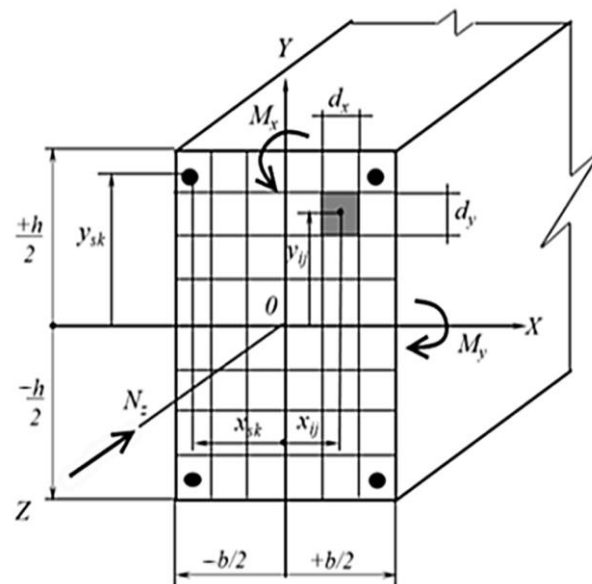


Figure 1. Model of an inhomogeneous reinforced concrete element.

Physical relationships linking internal forces and deformations at the level of the median axis convey the independent analysis of the physical nonlinearity of deformation and long-term processes in concrete.

Total deformations of the small concrete portion, $\varepsilon_{b,ji}$, and the k -th rebar in the direction of the Z axis are assumed as the sum of deformations triggered by the stresses $\sigma_{z,ji}$ and $\sigma_{s,k}$, thermal expansion shrinkage $\varepsilon_{cs,ji}$, and creep $\varepsilon_{c,ji}$ of concrete as

$$\varepsilon_{b,ji} = \frac{\sigma_{z,ji}}{E'_{b,ji}} + \alpha_{bt,ji} \cdot \Delta t_{ji} + \varepsilon_{cs,ji} + \varepsilon_{c,ji} \quad (26)$$

$$\varepsilon_{cs,ij} = \frac{\sigma_{s,k}}{E'_{s,k}} + \alpha_{st,k} \Delta t_{s,k} \quad (27)$$

$$E'_{s,k} = E_s \frac{\beta_{s,k}}{K_{s,k}}. \quad (28)$$

where $E'_{b,ji}$ and $E'_{s,k}$ are the moduli of the concrete portion with the cross-sectional coordinates x_{ji} and y_{ji} and k -th rebar with the cross-sectional area $f_{s,k}$ and coordinates $x_{s,k}$ and $y_{s,k}$:

$$E_{b,ji} = E_b \cdot \beta_{b,ji}, E_{s,k} = E_s \cdot \beta_{s,k}. \quad (29)$$

The values used in Expressions (26)–(29) are given as follows: $\alpha_{bt,ji}$ and $\alpha_{st,k}$ are the coefficients of temperature deformation of the ji -th portion of concrete and the k -th rebar, respectively; $\beta_{b,ji}$ and $\beta_{s,k}$ are the coefficients showing the temperature effect on the moduli of elasticity of concrete and rebars; $K_{s,k}$ is the coefficient showing the cracking effect on a specific rebar, assumed to be equal to 1 in the absence of cracks and $K_{s,k} = \psi_s$ in case of cracking; and ψ_s is the coefficient showing deformations in elongated concrete between the cracks.

The relationship between the deformations in small portions of concrete and rebars and total deformations (elongations, curvatures) of the longitudinal axis of a structural element based on the law of plane sections is described as follows:

$$\begin{aligned} \varepsilon_{b,ji} &= \varepsilon_{0z} + \chi_x \cdot y_{ji} + \chi_y \cdot x_{ji}, \\ \varepsilon_{s,k} &= \varepsilon_{0z} + \chi_x \cdot y_{s,k} + \chi_y \cdot x_{s,k}. \end{aligned} \quad (30)$$

3. Results and Discussion

Conditions of the static equivalence of stresses and internal forces serve to obtain the following expressions for the bending moments M_x and M_y and longitudinal force N_z in the normal section of an element:

$$\begin{aligned} M_x &= \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \sigma_{z,ji} dx dy y_{ji} + \sum_{k=1}^{ns} \sigma_{s,k} f_{s,k} y_{s,k} \\ M_y &= \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \sigma_{z,ji} dx dy x_{ji} + \sum_{k=1}^{ns} \sigma_{s,k} f_{s,k} x_{s,k} \\ N_z &= \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \sigma_{z,ji} dx dy + \sum_{k=1}^{ns} \sigma_{s,k} f_{s,k} \end{aligned} \quad (31)$$

The resulting physical relations can be obtained by extracting the stresses $\sigma_{b,ji}$ and $\sigma_{s,k}$ from Equations (26) and (27), replacing the deformations $\varepsilon_{b,ji}$ and $\varepsilon_{s,k}$ by Equation (30), and substituting them into Equation (31). The system of equations can be represented in a more compact matrix form after grouping the terms containing the longitudinal relative deformations of the element, $\varepsilon_{0,z}$, at the level of the median axis and the curvatures χ_x and χ_y with respect to the X and Y axes:

$$\begin{vmatrix} M_x \\ M_y \\ N_z \end{vmatrix} = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} * \begin{vmatrix} \chi_x \\ \chi_y \\ \varepsilon_{0z} \end{vmatrix} + \begin{vmatrix} M_{0x} \\ M_{0y} \\ N_{0z} \end{vmatrix} \quad (32)$$

where the coefficients of the stiffness matrix, $A_1, \dots, C_3$, and the elements of the free vector-column are represented by the following expressions:

$$A_1 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_b dx dy y_{ji}^2 + \sum_{k=1}^{ns} y_{s,k}^2 f_{s,k} E'_{s,k} \quad (33)$$

$$B_1 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_b dx dy x_i y_i + \sum_{k=1}^{ns} x_{s,k} y_{s,k} f_{s,k} E'_{s,k} \quad (34)$$

$$C_1 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_b dx dy y_i + \sum_{k=1}^{ns} y_{s,k} f_{s,k} E'_{s,k} \quad (35)$$

$$B_2 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_b dx dy x_i^2 + \sum_{k=1}^{ns} x_{s,k}^2 f_{s,k} E'_{s,k} \quad (36)$$

$$C_2 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_b dx dy x_i + \sum_{k=1}^{ns} x_{s,k} f_{s,k} E'_{s,k} \quad (37)$$

$$C_3 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_b dx dy + \sum_{k=1}^{ns} f_{s,k} E'_{s,k} \quad (38)$$

$$A_2 = B_1; A_3 = C_1; B_3 = C_2. \quad (39)$$

$$M_{ox} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_{b,ji} y_i (\alpha_{bt,ji} \Delta t_{ji} + \varepsilon_{cs,ji} + \varepsilon_{c,ji}) dx dy - \sum_{k=1}^{ns} y_{s,a} f_{s,a} \alpha_{s,a} \Delta t_{s,a} E'_{s,k} \quad (40)$$

$$M_{oy} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_{b,ji} x_i (\alpha_{bt,ji} \Delta t_{ji} + \varepsilon_{cs,ji} + \varepsilon_{c,ji}) dx dy - \sum_{k=1}^{ns} x_{s,a} f_{s,a} \alpha_{s,a} \Delta t_{s,a} E'_{s,k}; \quad (41)$$

$$N_{oz} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} E_{b,ji} (\alpha_{bt,ji} \Delta t_{ji} + \varepsilon_{cs,ji} + \varepsilon_{c,ji}) dx dy - \sum_{k=1}^{ns} f_{s,a} \alpha_{s,a} \Delta t_{s,a} E'_{s,k}; \quad (42)$$

Due to a stepwise change in the material properties and stresses in the design section, namely, in the zone of transition from one portion of concrete to another, integral expressions in Equations (33)–(42) are replaced by quadrature expressions.

3.1. Solving the Nonlinear Problems for Reinforced Concrete Elements That Have Inhomogeneous Properties

Inhomogeneous reinforced concrete elements subjected, in the general case, to effects of longitudinal forces featuring various durations, bending moments, and temperature differences are considered here. Solutions, obtained for specific types of structures, are in agreement due to the implementation of respective boundary conditions in elements. The stress–strain state of reinforced concrete elements is identified for some averaged hypothetical sections, characterizing the behavior of an element as a whole whose sections, with and without cracks, have averaged properties. If long-term processes, which are underway

in concrete, and the physical nonlinearity of deformation are analyzed separately, the SSS of structural elements is reduced to the consecutive solution of two types of problems:

- Identification of the SSS under the effect of regular temperature and load and taking into account the time factor;
- Identification of the SSS under supplementary short-term loading, if the load value is increased to trigger the failure using methods of the theory of deformation of reinforced concrete with cracks.

The SSS of elements, identified as a result of the first stage of analysis, serves as the initial value for the second stage.

Under the long-term effects of (i) temperature, (ii) a longitudinal force, and (iii) bending moments, the rheological problem is solved using the theory of concrete characteristic deteriorations over time, developed in [2,30–33] for conditions of elevated temperatures. The principle of superposition of incremental creep deformations at a preset temperature is assumed to be applicable.

When an element is exposed to elevated temperature, the time of exposure is divided into separate intervals of variable duration, changing in proportions close to the logarithmic law. It is assumed that the changes in temperature and stresses in layers of concrete and rebars occur instantaneously at the onset of each stage. Temperature shrinkage and creep deformations of concrete under heating conditions are identified using Equations (12)–(25).

The cooling of reinforced concrete elements to normal temperature and their one-sided freezing within the framework of the simulation of the winter period of operation are considered as short-term effects, assuming that shrinkage and creep deformations do not develop at these stages, and inelastic deformations are taken into account by the deformation model.

In the process of making calculations at each stage of loading of each small portion of concrete and rebars, temperature, stresses, and physical and mechanical properties of materials are assumed to be constant and equal to their values in the center of gravity of the section element. The presence or absence of cracks is determined separately for each small concrete portion under consideration. Following (1) cracking or (2) the failure of small portions of concrete, the values of their moduli are assumed to be equal to zero.

3.2. Solution to the Problem under the Effect of Temperature and Short-Term Incremental Loading

The process of loading of structural elements is divided into a number of stages, including elevating temperature, longitudinal forces N_z , and/or bending moments M_x and M_y . The secant moduli of concrete deformation are assumed for each small portion of concrete in each approximation to be constant and equal to the values determined at the previous iteration.

The method of elastic solutions is employed in combination with the method of stepwise elevation of temperature and loading to solve the physically nonlinear problem. At each step of loading, the values of the moduli are identified iteratively for each design portion of concrete and span of rebars.

Strength conditions are verified separately for each small portion of concrete using the deformation criterion, which allows for taking into account the behavior of concrete within the descending branch of the load–deformation curve. When the sign of the force changes, cracks close and the earlier cracked concrete element takes compression load when compressive stresses in the layer adjacent to the cracked one reach the value of, at least, 0.5 MPa. The agreement of the computational process is considered to be achieved if the moduli in each layer of concrete and rebars at the beginning and at the end of the iteration differ by no more than 5%.

A transformable concrete load–deformation diagram, for example, a well-known Sargin diagram, is preferable. The effects of temperature, heating time, and loading history can be taken into account by adjusting the position of the diagram top by the following Equation (11).

The mechanical properties of concrete under temperature effects, e.g., compressive and tensile strengths, and initial modulus of elasticity, taking into account the prehistory of loading, can be calculated by applying analytical expressions [3]. Temperature shrinkage deformations and creep deformations of medium-strength concretes are calculated for conditions of elevated temperatures using methods addressed in [2,3], and the same values are obtained for modified high-strength concretes using methods described in [20,21].

For the range of negative temperatures down to $-50\text{ }^{\circ}\text{C}$, the characteristics of temperature deformations and compressive and tensile strengths can be described by approximating Expressions (1) and (11).

The agreement between solutions, developed for beam elements according to the described method and solutions for real beam structures is achieved by implementing appropriate boundary conditions in the calculation. The types of problems to be solved are listed below.

Problem 1. The beam is fixed to prevent the rotation around the axis OX ($\chi_x = 0$), and it is loaded with the bending moment $M_x = var$ and the longitudinal force $N_z = var$. The values to be identified are M_y , χ_y and ε_{oz} . If $N_z = 0$, it is the case of diagonal bending.

Problem 2. The beam is fixed to prevent the rotation around the axis OY ($\chi_y = 0$), and it is loaded with the bending moment $M_y = var$ and the longitudinal force $N_z = var$. The values to be identified are M_x , χ_x , and ε_{oz} .

Problem 3. A nonbending element ($\chi_y = 0$; $\chi_x = 0$) is loaded with the longitudinal force $N_z = var$. The values to be identified are M_x , M_y , and ε_{oz} . $N_z = 0$ is the case of diagonal bending.

Problem 4. An inhomogeneous element is loaded by the forces $M_x = var$, $M_y = var$, and $N_z = var$. The values to be identified are χ_x , χ_y , and ε_{oz} for the case of diagonal eccentric compression/tension.

Under the preset boundary conditions, static equilibrium equations are satisfied, and the values are found for each type of problems from the solution to the system of Equation (32).

The SSS characteristics of reinforced concrete beams, calculated according to the proposed method, are compared with the results of experimental studies ([3] on Figures 2–5; [6] on Figures 6 and 7) using the values of thermal bending moments in statically indeterminate structures, as well as the values of cracking forces and deformations of the longitudinal axis of elements.

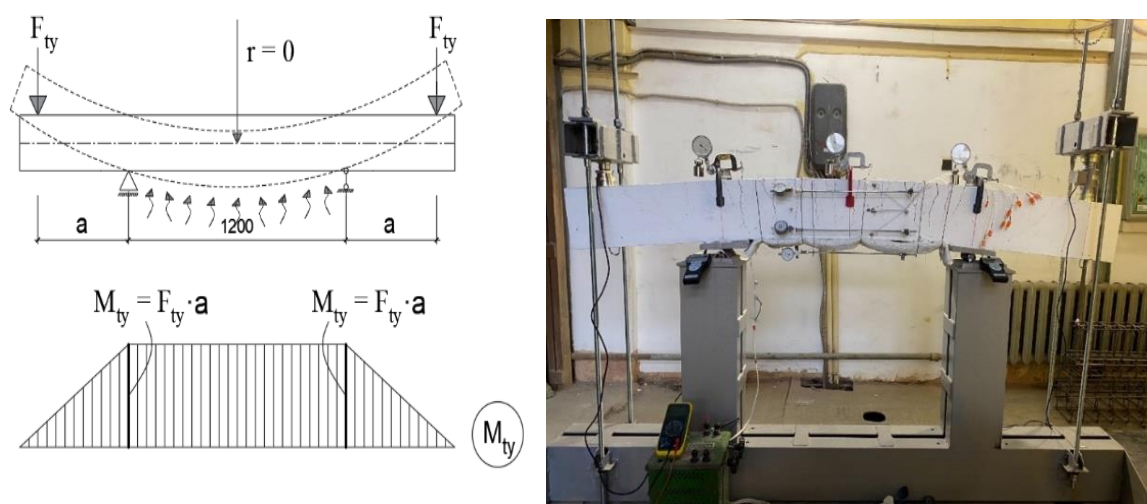


Figure 2. Scheme for determining temperature moments in a beam when eliminating axis curvature from uneven heating.

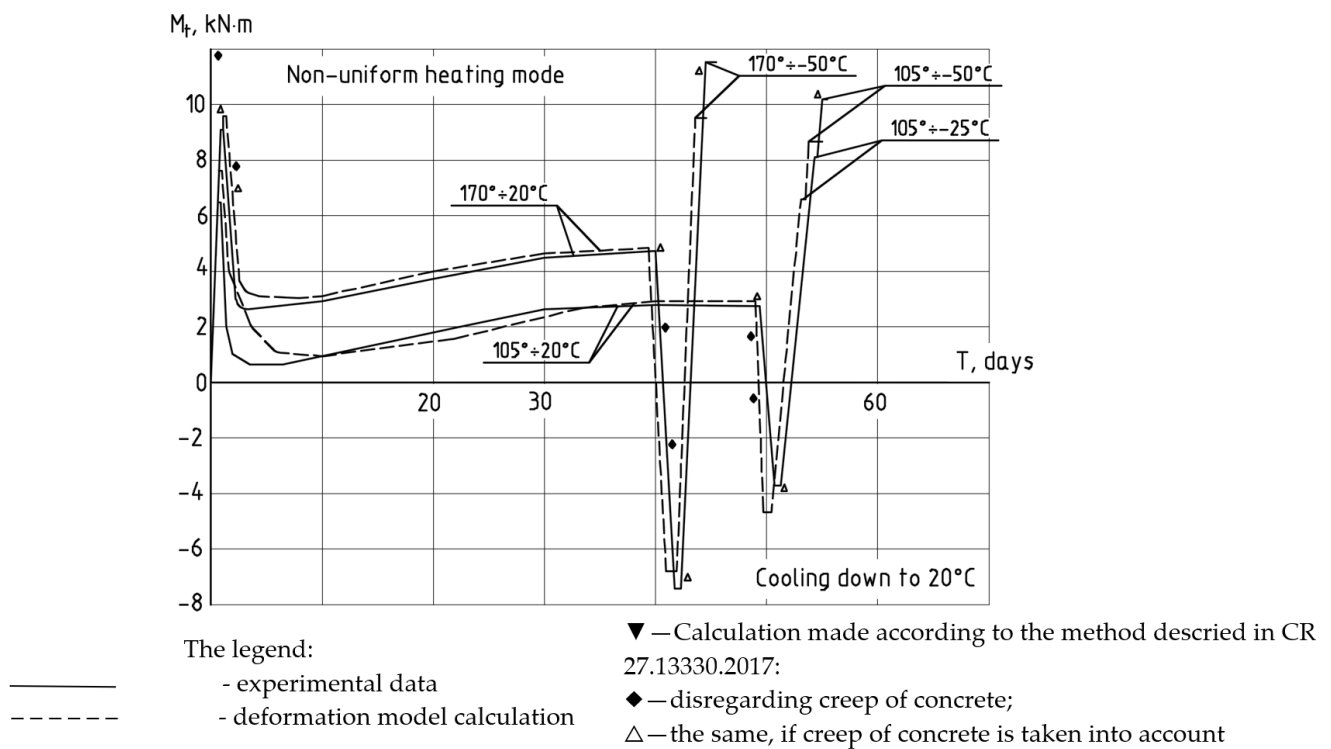


Figure 3. Temperature moments in reinforced concrete beams ($b \times h = 120 \text{ mm} \times 250 \text{ mm}$), loaded by longitudinal compressive force $N = -200 \text{ kN}$, accompanied by one-sided heating below.

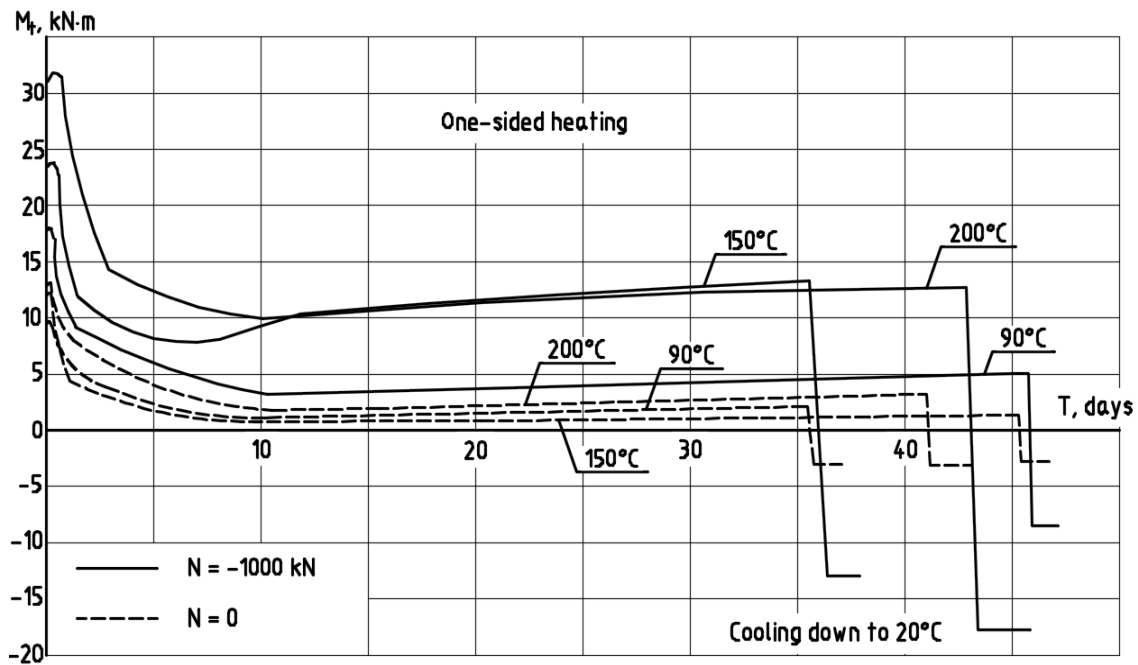


Figure 4. The effect of temperature of one-sided long-term heating and longitudinal compressive force N on values of thermal moments in reinforced concrete beams ($b \times h = 1000 \text{ mm} \times 150 \text{ mm}$).

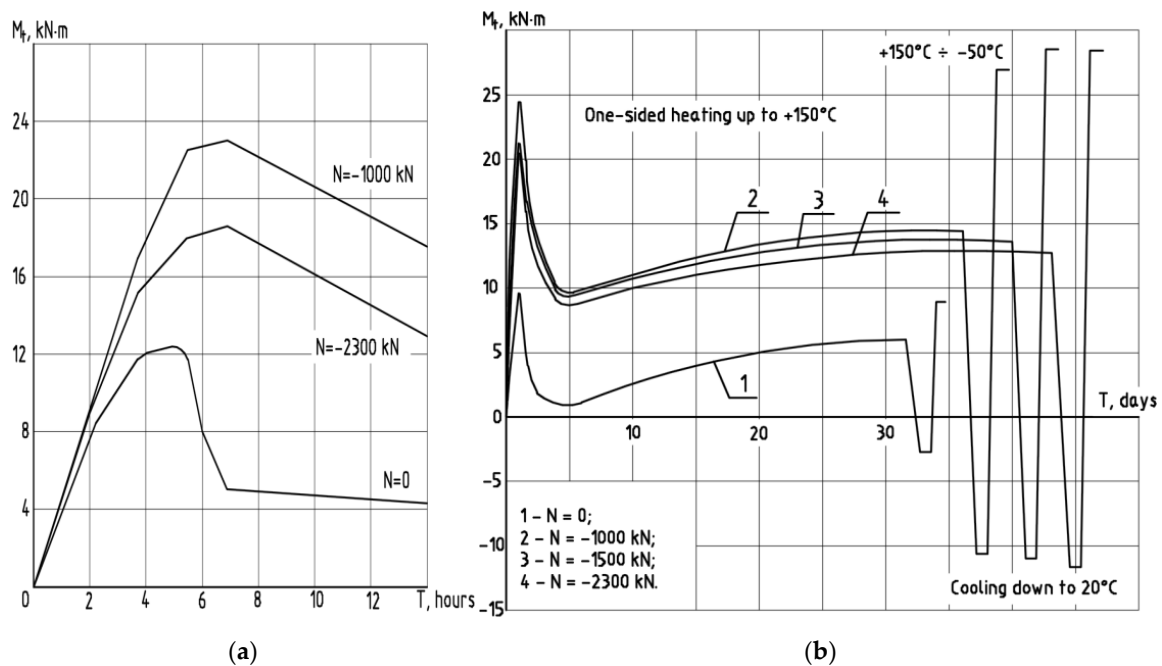


Figure 5. The effect of the value of longitudinal compressive force N on values of thermal moments in reinforced concrete beams ($b \times h = 1000 \text{ mm} \times 150 \text{ mm}$): (a) in case of short-term heating up to 150°C from below; (b) in case of long-term heating from below and short-term cooling down to -50°C from above.

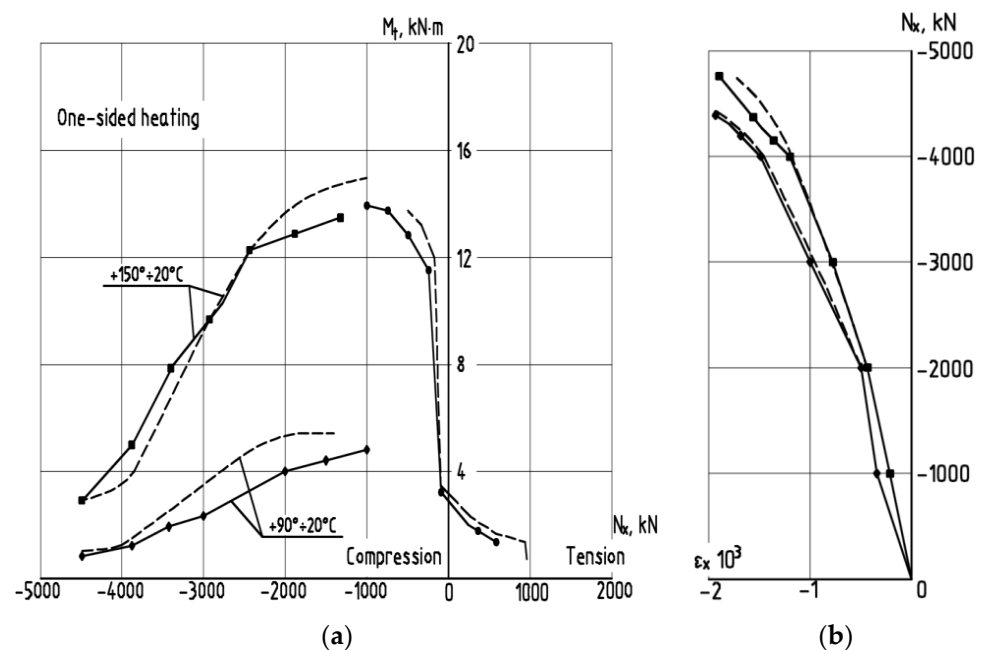


Figure 6. Changes in (a) temperature moments and (b) elongation values of the longitudinal axis of elements when supplementary longitudinal compressive (tensile) force N_z is applied to beam specimens ($b \times h = 1000 \text{ mm} \times 150 \text{ mm}$). The legend: ———— experimental data; - - - - calculation made using the deformation model.

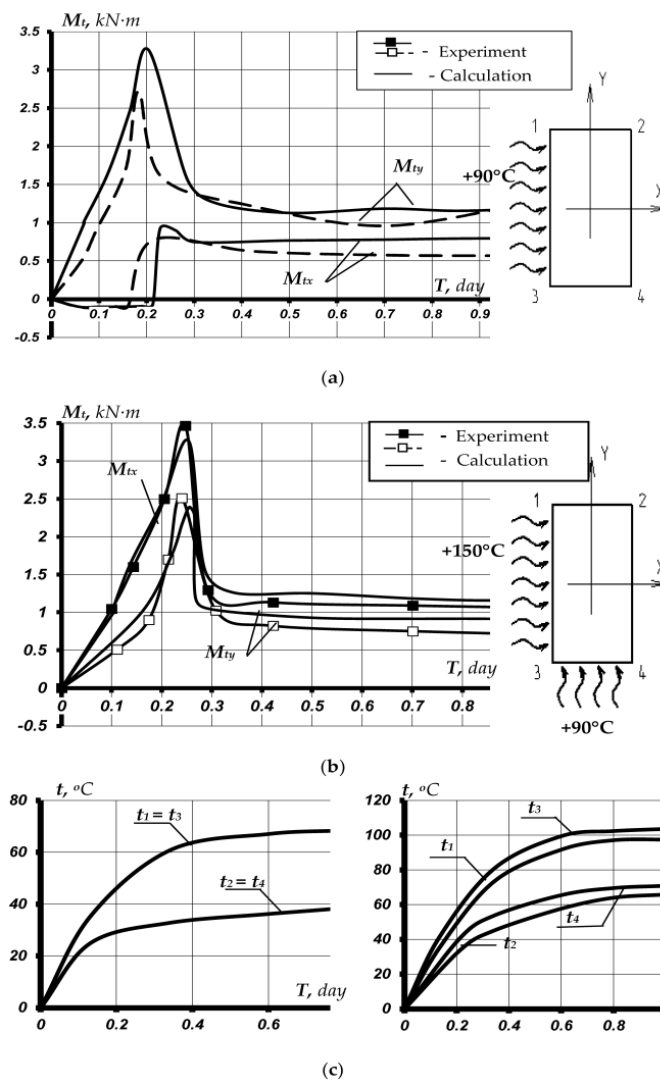


Figure 7. Changes in (a,b) temperature moments M_{tx} and M_{ty} in the case of nonuniform lateral and bottom heating of beams of $b \times h = 120 \text{ mm} \times 250 \text{ mm}$, and (c) temperature values in angular points of sections.

The convergence of the computational process for the inhomogeneous beam element was achieved in 3–5 iterations at each stage of temperature or force loading. For a close correspondence to the experimental data, it was sufficient to divide the cross section into 6 and 10 layers in the width and height.

For cases of diagonal bending and diagonal eccentric compression (tension), the value of the coefficient ψ_s in the design model of an inhomogeneous reinforced concrete element is calculated for each rebar separately. The calculation procedure is similar to the one applied to a hypothetical reinforced concrete element whose cross section is (i) equal to the total area of portions, crossed by a crack, and (ii) under the state of uniaxial stress reduced to the hypothetically homogeneous state of its cross section:

$$\Psi_s = 1 - b \frac{N_{b,crc,z}}{N_{z,k}} \leq 1 \quad (43)$$

$$N_{b,crc,z} = \sum_{i=1}^m \bar{\sigma}_{b,i,z} dx_i dy_i \quad (44)$$

$$N_{s,k} = \sigma_{s,k} f_{s,k} \quad (45)$$

where b is the coefficient describing the short-term effect of temperature ($T \leq 1$ day) and loading equal to 0.7, while for the long-term temperature and load effect, it equals 0.35; m is the number of portions of the concrete cross section crossed by a crack and surrounding the rebar under consideration; $\sigma_{b,i,z}$ is the limit value of tensile stress in the i -th portion of concrete at the moment of cracking, extracted from calculations made in the course of earlier iterations; and $\sigma_{s,k}$ is the value of stress in rebar, S_k , calculated using the value of ψ_s identified in the course of earlier iterations.

This method of identifying the coefficient ψ_s allowed for obtaining values of the temperature moments M_{tx} and M_{ty} in beam specimens, having a nonbending longitudinal axis. These values are close to the experimental ones (Figures 6 and 7).

4. Conclusions

From the research results in this study, the following conclusions can be drawn:

1. The nonlinear deformation model is improved by a system of equations applied to the inhomogeneous reinforced concrete beam elements in the general case of temperature and force effects, with different durations.
2. The proposed system of equations allows for obtaining solutions in terms of deformations (elongations, curvatures), forces in design sections, stresses and deformations in small portions of the concrete cross section, and rebars for any moment during the temperature and load action. The general case of diagonal eccentric compression (tension) is considered in respect of an inhomogeneous beam element. This general case serves as the basis for special cases of diagonal and plane bending, plane eccentric compression, and tension.
3. Independent analysis of the physical nonlinearity of deformation and long-term processes, which are underway in concrete, allows for modeling long-term operational temperature effects and a short-term increase in loading within the framework of analysis of structures.
4. The results of calculations made using the model are in good agreement with the experimental data on the characteristic modes of loading and nonuniform heating.
5. The calculation model allows for identifying stresses, deformations, cracking in design elements of the inhomogeneous cross section of a structure, predicting changes in deformations, and temperature forces in statically indeterminate reinforced concrete structures as a result of time-dependent cracking and inhomogeneous development of long-term and short-term inelastic deformations of concrete along the height and width of the cross section.
6. The most important factors affecting the stress–strain states of reinforced concrete structures under the combined effects of temperature gradients and loading are dependent, with respect to the deformation and strength properties of concrete, on the temperature and duration of its action, inhomogeneity of temperature shrinkage deformations, and creep of concrete, as well as cracking.

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