

Article

Identification and Calibration of Advanced Hysteresis Models for Recycled Rubber–Fiber-Reinforced Bearings

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Abstract: Several studies have investigated the feasibility of reducing the implementation cost of base isolation. In this optic, recycled rubber–fiber-reinforced bearings (RR–FRBs) represent a suitable solution for structures in developing countries. Such devices can be produced using simple manufacturing procedures at a limited cost with respect to conventional isolators. Full-scale tests on RR–FRBs featured energy dissipation values similar to those associated with high-damping natural rubber bearings (HDRBs). Equivalent viscous damping, ranging from 10 to 15%, resulted from testing of RR–FRBs, with poor degradation after cyclic loading. On the other hand, a sensible softening response, associated with the axial–shear interaction, which is much more significant compared to that exhibited by HDRBs, was observed. As a result, the numerical description of the cyclic behavior of the RR–FRBs appears to be more challenging than that of HDRBs. In past studies, simple bilinear hysteresis models were adopted to describe the cyclic behavior of low-cost rubber bearings, thus completely neglecting the P-delta effects which significantly influence the dynamic behavior of such bearings. In this paper, advanced hysteresis numerical models, able to capture the nonlinear response of RR–FRBs, were examined and properly calibrated using a powerful optimization technique, the differential evolution algorithm. Preliminary results of the numerical analyses, performed in OpenSees, were described and compared with those of experimental tests on low-cost rubber bearings. The findings of this study represent the first step of a characterization procedure aimed to provide an accurate representation of the dynamic behavior of these particular bearings. Obviously, additional studies are needed to compare results of response history analyses with those of experimental tests for real structures on RR–FRBs. In this optic, the present paper, along with further studies, could provide a new impulse for the application of low-cost rubber-based devices in current practice.

Keywords: recycled rubber—fiber reinforced bearings; nonlinear system parametric identification base isolation; differential evolution algorithm



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1. Introduction

Base isolation [1] is an appealing strategy for the seismic protection of new and existing buildings. It is based on the reduction of seismic accelerations, hence the seismic forces, acting on the structure. Generally speaking, the most used typologies of isolation bearings are represented by friction pendulum bearings (FPBs) and laminated rubber bearings (LRBs). FPBs consist of a (rigid or articulate) slider moving on a sliding surface, generally composed of steel and PTFE. The initial version of such bearings features one sliding surface maintaining constant lateral stiffness and dynamic period for any displacement (hazard) level. To improve the seismic performances of the FPBs, a double or triple sliding surface can be introduced. Moreover, the displacement capacity being equal, double or triple surface bearings are characterized by lower dimensions with respect to the single pendulum.

LRBs are composed of a sequence of rubber layers and steel shims. The rubber layers provide lateral flexibility and energy dissipation, while the steel shims are used to prevent bulging and provide axial and “tilting” stiffness to the bearing. The rubber compound adopted for the manufacturing of the bearing strongly affects its inherent behavior. In this optic, LRBs can be classified as low-damping and high-damping rubber bearings.

Low-damping natural rubber bearings feature a quasi-elastic behavior quite linear until shear strain equal to 100%. Equivalent viscous damping of the order of 5–10% (at 100% shear strain) is generally obtained. High-damping rubber bearings exhibit damping of approximately 20% at 100% shear strain. This is related to the use of special additives in the rubber compound. On the other hand, such bearings are characterized by a rate, temperature, and pressure-dependent behavior and by significant aging effects.

One of the main disadvantages of LRBs is their high production cost, directly related to the complex manufacturing process [2]. To reduce the effective cost of such bearings, fiber fabric layers can be adopted in lieu of the traditional steel shims. The main advantages of fiber-reinforced bearings (FRBs) over traditional rubber-based bearings are: (i) the lightweight of the individual device; and (ii) the possibility of cutting an individual bearing from a pad of bigger dimensions. To further reduce the construction costs of FRB, Calabrese et al. [2] verified the feasibility of replacing the natural rubber of conventional FRBs using a recycled rubber material derived from the reuse of scrap tires and industrial unused. The resulting device was named recycled rubber–fiber-reinforced bearing (RR–FRBs). In RR–FRBs, the bonding between the rubber and the reinforcement layers is achieved by gluing, with a polyurethane binder, the layers of reinforcement and the layers of elastomer, with no need for vulcanization of the entire device.

An important research effort has been dedicated to the study of RR–FRBs [3] in the attempt to extend the range of application of base isolation to low-rise residential buildings in developing countries, where the typical costs associated with the standard technology are unaffordable.

While many hysteresis models have been developed to describe the nonlinear response of conventional LRBs [4–7] and FPBs [8–10], the applicability of these models to describe the cyclic response of RR–FRBs has to be verified. An attempt in this direction was made by assessing the effectiveness of simple bilinear or trilinear models of hysteresis in capturing the response of structures on RR–FRBs [3,11]. However, such models completely neglect the P-delta effects which significantly influence the dynamic behavior of the low-cost rubber bearings.

Recently, advanced hysteresis models have been proposed and included in the OpenSees framework library [12]. In particular, three main “element objects”, featuring increasing complexity, are examined in the present study: (i) the elastomeric bearing (Bouc–Wen) element, (ii) the HDR element [13] and (iii) the Kikuchi bearing element [14]. The above-mentioned advanced numerical models were calibrated matching the experimental dynamic behavior of RR–FRBs, obtained from the tests carried out by Calabrese et al. [3]. To do that, a suitable evolutionary computation strategy, the differential evolution algorithm [15], was adopted to identify the key parameters of the above-mentioned numerical models.

In Section 2, the set-up and the main results of experimental tests on RR–FRBs carried out at the Department of Industrial Engineering of the University of Naples “Federico II” are described. In Section 3, the three advanced models examined in this study are presented. In Sections 4 and 5 the differential evolution method is described and then applied to each of the aforementioned model for the parametric identification and calibration. The main results of the model calibration procedure are discussed in Section 6.

It is worth noting that the present study is just a preliminary step in a larger characterization procedure to accurately represent the dynamic behavior of RR–FRBs. Additional studies, comparing the results of response history analyses with those of experimental tests for real structures on RR–FRBs, are in progress.

2. Experimental Tests on RR-FRBs

Experimental tests were run at the Department of Industrial Engineering of the University of Naples “Federico II” to identify the nonlinear shear response of full-scale RR-FRBs (base dimensions 210×210 mm and total height 210 mm) [3]. Figure 1 shows a RR-FRB sample and the setup of the experimental testing apparatus. The latter is a bidirectional machine featuring a maximum vertical static load equal to 190 kN. A base plate, sliding on roller devices, is used to apply the horizontal load. More in detail, a manual hydraulic jack together with an air accumulator were used to apply the vertical static load. On the other hand, a dynamic actuator (50 kN maximum capacity, ± 200 mm maximum stroke and 2.2 m/s maximum speed) imposed the horizontal dynamic load to the sliding plate. The testing apparatus was also equipped with a set of transducers to capture the horizontal displacement and the horizontal/vertical forces.

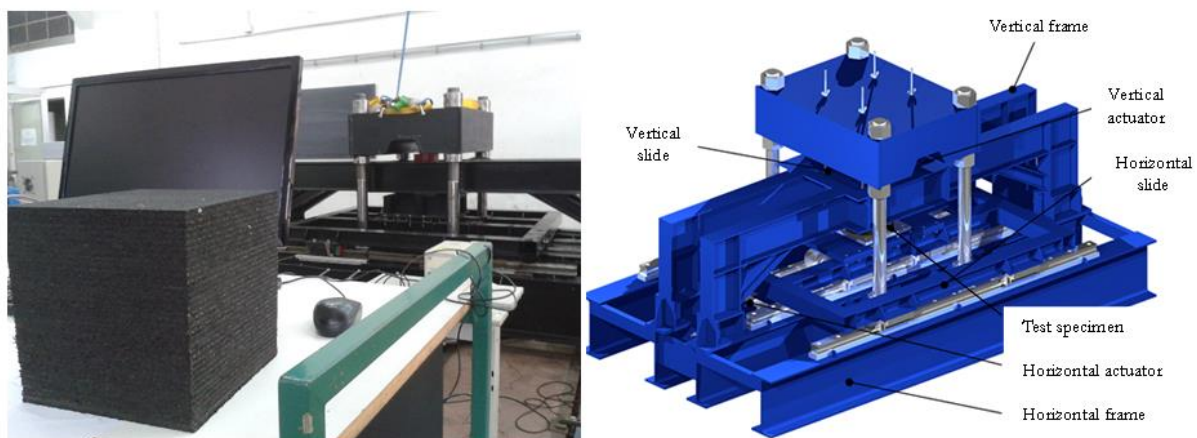


Figure 1. Setup of the experimental tests.

The bearings were tested under displacement control by applying axial and lateral loads, simultaneously. In particular, cyclic shear tests were performed applying a constant vertical pressure (3.85 MPa) and imposing increasing horizontal deformations, ranging from 15% to 60%, at 0.5 Hz frequency. The imposed displacement history is shown in Figure 2a.

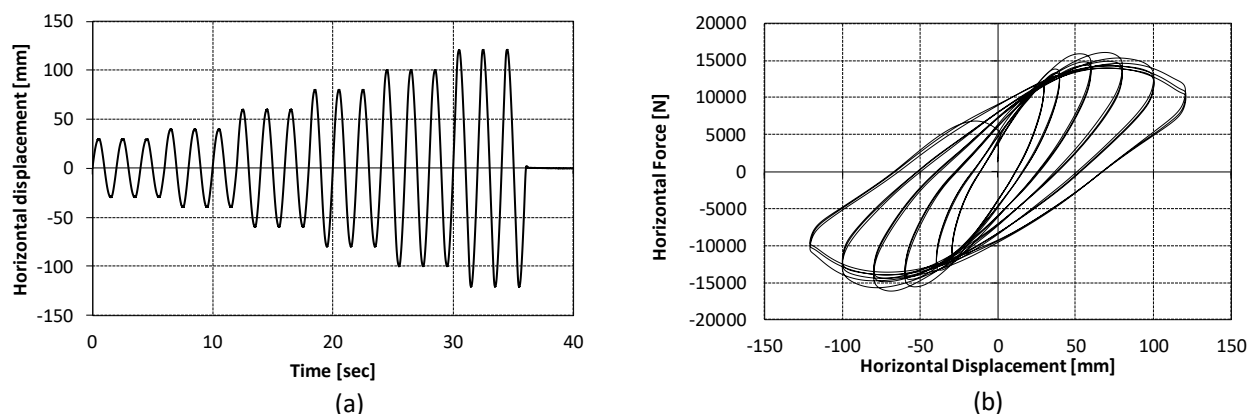


Figure 2. Setup of the experimental tests: (a) loading protocol—displacement time history during the shear testing; and (b) force vs displacement response of the full-scale RR-FRB.

As is clearly pointed out by the experimental hysteresis loops of Figure 2b: (i) the bearings show little degradation after many cycles of deformation, (ii) for small lateral deformations, the response of the devices is close to linear, and (iii) the device exhibits a negative tangent stiffness for shear deformations larger than 40% (softening effect).

3. Numerical Modeling

In the present study, the experimental data presented in Section 2 were used for the parametric identification and calibration of three advanced numerical models (already included in the OpenSees library) to describe the cyclic behavior of a low-cost rubber bearing.

The elastomeric bearing (Bouc–Wen) Element is a two-node element based on a 3D formulation characterized by coupled plasticity properties for the shear deformations in the two horizontal directions. More in details, the bilinear shear behavior, implicitly defined within the mathematical formulation of the numerical model, is an elastic–plastic behavior with hardening completely defined by a set of 8 parameters. The first two, i.e., K_{init} and F_y , represent the initial elastic stiffness and the characteristic strength (i.e., yield force) of the bearing. In particular, the initial stiffness can be expressed as:

$$K_{init} = G_r A / T_r \quad (1)$$

where T_r is the total rubber height, A is the area of the bearing while G_r is the shear modulus of rubber.

The parameters α_1 and α_2 , showed in Figure 3, represent the post yield stiffness ratio of linear and non-linear hardening components, respectively. The latter component is also characterized by a hardening exponent (μ). The parameters β and γ govern the size and shape of the hysteresis loops. Finally, the sharpness of the transition from initial to post elastic slope is controlled by the η parameter (see Table 1).

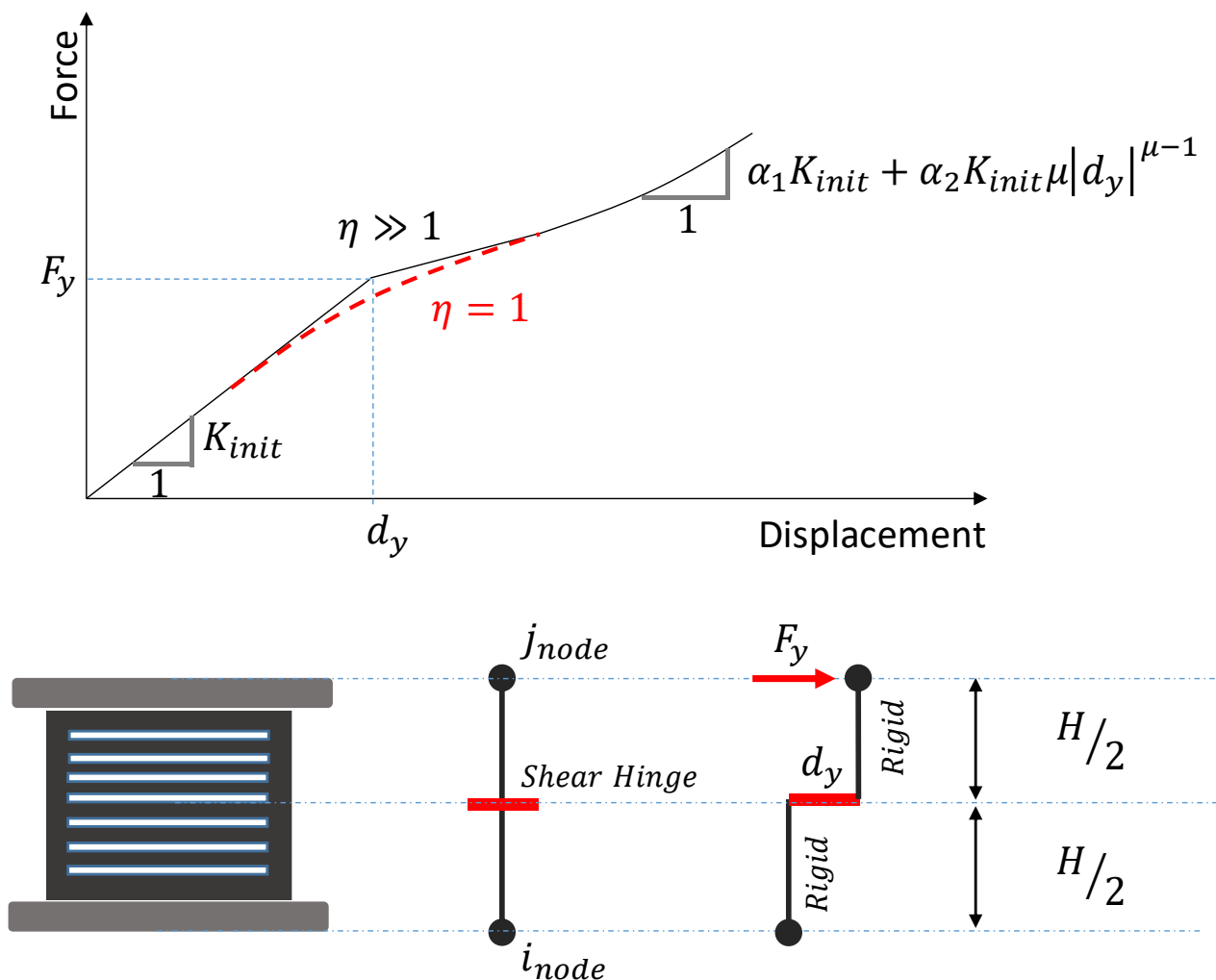


Figure 3. Spring model representation of the Elastomeric Bearing (Bouc–wen) Element.

Table 1. Defining parameters of the Elastomeric Bearing (BoucWen) Element.

Parameter	Definition	Description	Source
K_{init}	Initial elastic stiffness	Lateral stiffness	From Equation (1)
E_y	Characteristic strength	Shear strength	From Experimental acceptance tests
α_1	Post yield (linear) stiffness ratio	Linear hardening	A-dimensional parameters derived in this study based on experimental results using evolutionary algorithms (see Section 5)
α_2	Post yield (non-linear) stiffness ratio	Non-linear hardening	
μ	Hardening exponent		
β, γ, η	Wen's parameters [2]	Size, shape and sharpness of the hysteresis loops	

The force–deformation behaviors associated with the remaining four directions in the 3D domain are defined by adopting uniaxial materials. In the present study, the Axial SP material, available in OpenSees, was associated with the vertical behavior in tension and compression. The material is completely defined by the following parameters: (i) initial elastic stiffness along vertical direction E_{init} , (ii) post-elastic stiffness in compression ($1/2 E_{init}$) and in tension ($1/100 E_{init}$), (iii) cavitation stress ($f_{ty} = 2 G_r$ [6]) and (iv) yielding stress in compression ($f_{cy} = 100$ MPa [16]).

According to [16], the following relationship can be adopted to calculate the elastic E_{init} :

$$E_{init} = K_{bulk} \left\{ 1 - \frac{2 I_1(\bar{\lambda})}{\bar{\lambda} I_0(\bar{\lambda})} \right\} \quad (2)$$

where I_1 and I_0 represent the modified Bessel functions of the first kind of order 1 and 0 respectively, K_{bulk} is the bulk modulus and $\bar{\lambda}$ is the dimensionless parameter

$$\bar{\lambda} = \frac{D}{t_r} \sqrt{\frac{3G_r}{K_{bulk}}} \quad (3)$$

in which D and t_r represent the size and thickness of single rubber layer, respectively.

Uniaxial material elastic featuring very large values of elastic stiffness (two order of magnitude larger than lateral stiffness) are adopted in the remaining directions.

It is worth noting that the aforesaid model can capture the P- Δ effect. However, the effect is taken into account using a simplified approach. More in detail, in the default configuration, P- Δ moments are equally distributed to the two end-nodes of the element.

The HDR element is a two-node element with 12 degrees-of freedom. It is characterized by computational efficiency and easy implementation (see Figure 4). The element is composed by six springs representing global mechanical behavior: (i) a non-linear vertical spring governing the axial behaviour, (ii) two coupled horizontal springs for the shear behavior and (iii) three linear uncoupled springs governing torsion and the two rotations, respectively.

The mentioned element extends the formulation of the elastomeric bearing (Bouc–Wen) element. In this case, instead of providing specific material models as input, only material and geometric properties of the real elastomeric bearing are requested as input. In other words, the material models in the mentioned six directions are formulated within the element from (geometric and material) input parameters. The specific material parameters for RR–FRB derived from experimental tests and the literature review are shown in Table 2.

The mathematical model developed by Kumar et al. [13] was adopted to represent the behaviour in the axial direction. The model captures: (i) the cavitation phenomenon associated to a tensile force value producing a tensile stress of $3G$ and post-cavitation behaviour in tension; and (ii) the variation of the critical buckling load and the vertical axial stiffness with horizontal displacement in compression. A linear elastic model is assumed to represent the rotational behaviour about the two horizontal directions. The bidirectional model proposed by Grant et al. [17] is assumed to describe the behaviour in the coupled shear directions. The aforementioned model is also able to take into account

the degradation of bearing stiffness and damping due to the scragging effect in shear, particularly important for high damping bearings [5].

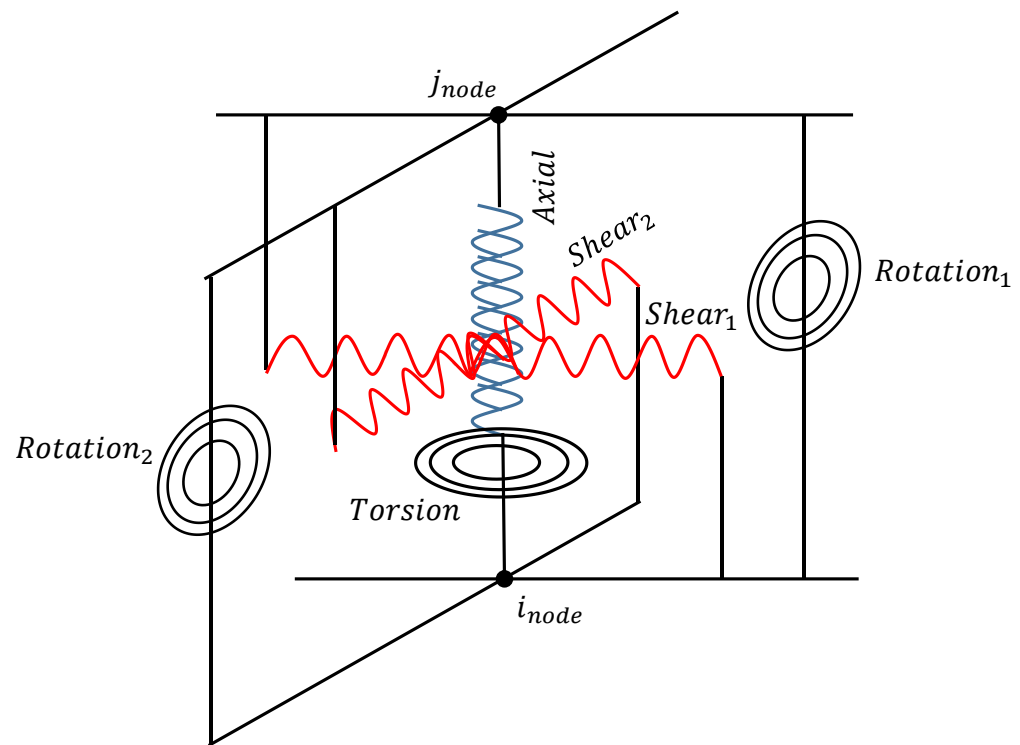


Figure 4. Model representation of the of the HDR Element.

Table 2. Defining parameters of the HDR Element.

Parameter	Definition	Description	Source
G_r	Shear modulus of rubber	Lateral stiffness	From experimental acceptance tests or Manufacturer's catalogue
K_{bulk}	Bulk modulus of rubber	Axial stiffness	2000 MPa [14]
D t_s t_r n	Bearing size Steel (fiber) shim thickness Rubber layer thickness Number of rubber layers	Device's geometry	From manufacturer's catalogue
a_1 a_2 a_3 b_1 b_2 b_3 c_1 c_2 c_3 c_4	See Equations (4)–(6)	Elastic component of shear behaviour Hysteretic inelastic component of shear behaviour Stiffness and damping degradation associated to Scragging and Mullin's effects	A-dimensional parameters ($\alpha_i, \beta_i, \chi_i$) derived in this study based on experimental results using evolutionary algorithms (see Section 5)

Beyond the geometric variables and the shear modulus (G_r), the Grant model is based on a set of ten “bearing” parameters. In particular, the first three parameters a_1, a_2, a_3 control the elastic component while b_1, b_2, b_3 are directly related to the hysteretic inelastic component. The stiffness and damping degradation associated to the scragging and Mullin's effect is controlled by the last sub-set of parameters (c_1, c_2, c_3 and c_4). Simple relationships relating the mentioned bearing parameters ($a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3, c_4$) to a-dimensional rubber parameters ($\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \chi_1, \chi_2, \chi_3$ and χ_4) were developed by Cardone et al. [18] as a function of the rubber area (A) and total rubber thickness (T_r). For completeness, they are summarized in the following equations:

$$a_1 = \frac{\alpha_1 A}{T_r}; a_2 = \frac{\alpha_2 A}{T_r^3}; a_3 = \frac{\alpha_3 A}{T_r^5}; \quad (4)$$

$$b_1 = \beta_1 A; b_2 = \frac{\beta_2 A}{T_r^2}; b_3 = \frac{\beta_2}{T_r}; \quad (5)$$

$$c_1 = \frac{\chi_1}{T_r^3}; c_2 = \frac{\chi_2}{T_r^3}; c_3 = \chi_3; c_4 = \frac{\chi_4}{T_r^3}. \quad (6)$$

As mentioned before, the HDR element is able to capture the coupling between vertical and horizontal directions only partially, by considering the variation of the critical buckling load capacity due to lateral displacement. On the other hand, the response in the horizontal directions does not depend on the response in the vertical direction and the P-Δ effects, due to post buckling behaviour, are substantially neglected.

Finally, the Kikuchi bearing element is a two-nodes element featuring a multiple spring mechanical model including a double set of multiple axial springs, at the top and the bottom of the bearing (multiple normal spring element, MNS) and a set of mid-height multiple radial shear springs (multiple shear spring element, MSS), all bound together by rigid links, see Figure 5. The geometric parameters defining the Kikuchi bearing element are summarized in Table 3.

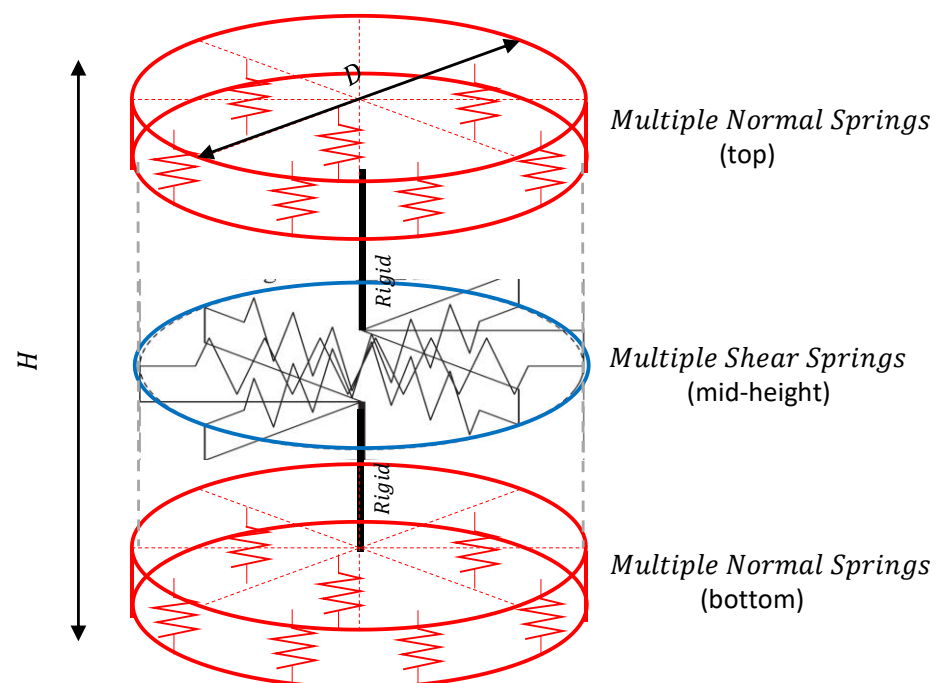


Figure 5. Model representation of the Kikuchi bearing element.

Table 3. Geometric Parameters of the Kikuchi Bearing Element.

Parameter	Definition	Description	Source
D	Bearing size	Lateral stiffness	From manufacturer's catalogue
A	Area of rubber	Shear strength	
H	Bearing height	Linear hardening	
T_r	Rubber Thickness	Non-linear hardening	
$nMSS$	n. of springs in MSS	Size, shape and sharpness of the hysteresis loops	$nMSS = 8$, $nMNS = 30$ [14]
$nMNS$	n. of springs in MNS		
c_g	Modification factors	Correction coefficient for the equivalent shear modulus	A-dimensional parameters derived in this study based on experimental results using evolutionary algorithms (see Section 5)
c_h		Correction coefficient for the equivalent viscous damping	
c_u		Correction coefficient for the shear force at zero displacement	

For the axial springs of the MNS, the above-mentioned AxialSP material has been adopted. On the other hand, the KikuchiAikenHDR Uniaxial Material [19] has been selected to describe the inherent behaviour of the radial springs. It is worth noting that the original formulation of the selected material model (i.e., KikuchiAikenHDR) is suitable for a limited set of pre-calibrated rubber compounds. However, to better fit specific rubber compounds, the latter can be slightly adjusted using a set of correction coefficients. More in details, the mentioned coefficients (namely, c_g , c_h and c_u) control the equivalent shear modulus, the equivalent viscous damping ratio, and the ratio of shear force at zero displacement, respectively. The parameters defining the KikuchiAikenHDR material are resumed in Table 3. In the present paper, reference to the X0.6 rubber compound has been made [13]. The main advantage of the Kikuchi bearing element is represented by the inherent coupled behavior between vertical and horizontal direction, taken into account through its large displacement formulation. In particular, the Kikuchi element is able to capture the P- Δ effect, therefore describing the pre- and post- buckling behavior of the bearing [6]. The aforementioned model is also able to correctly predict the post-buckled behavior even for square bearings and variable vertical loads. This is an attractive peculiarity, to represent the real behavior of isolation bearings subjected to strong earthquakes producing the rocking motion of the superstructure. The main peculiarities of each of the selected models, to properly simulate the bearing response, are reported in Table 4.

Table 4. Main peculiarities in capturing the bearing response of the examined numerical models.

Physical Phenomenon	Bouc–Wen	Element HDR	Kikichi
Coupled bi-directional motion in horizontal direction	✓	✓	✓
Coupling between vertical and horizontal motion	×	Partially	✓
Cavitation and post-cavitation behaviour	✓	✓	✓
Strength degradation due to cavitation	×	✓	×
Mullin’s effect	×	✓	×
Variation in critical buckling load capacity	×	✓	✓
Post buckling behaviour (including P- Δ effects)	Simplified way	×	✓

✓ the numerical model captures the physical phenomenon; × the numerical model does not capture the physical phenomenon.

As can be observed, none of the selected models were is able to capture all the physical phenomena at the same time.

4. General Overview on the Differential Evolution Algorithm

The parametric identification and model calibration of each examined model can be formulated as an optimization problem. Several traditional analytical methods are available for the parametric identification of linear and non-linear systems [20–22]. However, the traditional methods are generally affected by certain peculiarities limiting their applicability to simple systems. Most of the methods based on numerical algorithm require an initial guess to start the process. As a consequence, such methods can be very sensitive to the initial choice and, moreover, by searching the optimal solution near the initial guess, they are prone to get trapped into “local optima”. In some cases, the optimal values of the parameters cannot be found, with a consequent deterioration of the model performances. Promising results in solving complex optimization problems have been obtained using heuristic methods. Among those methods, the evolutionary algorithms (EAs) represent an interesting category of optimization procedures. Such procedures are based on imitating the biological evolution process, relying on biologically inspired operators such as mutation, crossover, and selection [23].

One of the main advantages of the EAs is that, unlike the traditional methods, they do not require prior knowledge about the optimization problem. As a matter of fact, in the EAs, the population of possible solutions is randomly initialized at the first start and then

evolved to better solutions by applying the following three operators: mutation, crossover and selecting.

Within an EA, a solution is considered ‘good’ only in comparison to other, previously determined solutions. In other words, the algorithm basically misses the idea of an optimal solution, or any way to test whether a given solution is optimal. As a consequence, EAs never really know when to stop. Generally speaking, heuristic rules can be adopted to stop the procedure when the best solution found has not improved in a long time. In practice, such algorithms are stopped manually by the operator, or by considering a pre-determined limit on the number of iterations or time.

In this paper, a novel evolutionary approach, the differential evolution algorithm (DEA) approach has been adopted in the parametric identification of the examined numerical models. The DEA represents a hybrid heuristic approach characterized by attractive potentialities: robustness, simplicity to use, and numerical efficiency. In particular, DEA features the basic structure of a classical evolutionary algorithm differing in the generation of new candidate solutions, adopting the concept of larger population from the genetic algorithms, and the use of a greedy solution approach [23] (i.e., greedy selection scheme).

The algorithm was proposed by Storn and Price [15] and is basically employed for the minimization of a specific objective function. The above-mentioned objective function can be expressed as follows:

$$f(\theta) = \frac{1}{S \cdot \text{var}(F_{exp})} (F_{num} - F_{exp})^T (F_{num} - F_{exp}) \quad (7)$$

where θ represents the vector collecting the n parametric variables of the examined model, F_{num} and F_{exp} are the predicted and experimental force values, while S and $\text{var}(F_{exp})$ represent the number of samples and the variance of F_{exp} , respectively. The best estimation θ_{best} is assumed as the global minimum of the objective function f . The DEA approach framework is summarized in Figure 6.

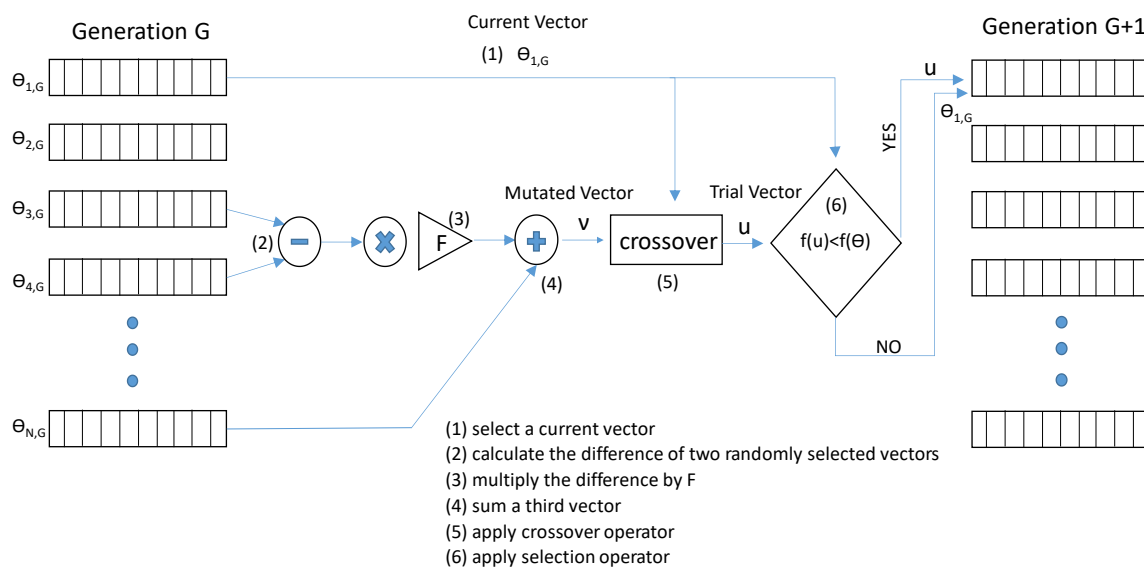


Figure 6. Differential Evolution Algorithm.

The first step of the DEA approach consists in randomly initializing a population of N solution vectors, with N representing the selected population size. In other words, if the optimization procedure consists of n -parameters, a set of N n -dimensional vectors is randomly generated. The first algorithm generation (G) is thus obtained:

$$\theta_i^G = (\theta_{i1}, \theta_{i2}, \dots, \theta_{in}) \in D, \quad i = 1, 2, \dots, N \quad (8)$$

where D is the selected search space.

Subsequently, the mutation operator is used to introduce diversity in the parameters space. In particular, for each n -dimensional vector θ_i^G , a mutation vector (also named as target vector) is created using the following relationship:

$$v_i^{G+1} = \theta_{r1}^G + F(\theta_{r2}^G - \theta_{r3}^G) \quad (9)$$

where $r1$, $r2$ and $r3$ represent three integer numbers, mutually different and randomly selected, while $F > 0$ is a mutation constant.

It is worth noting that the mathematical structure of the mutation operator can be selected among many different alternatives [17].

At this point, for each mutant vector, the offspring $u_i^{G+1} = u_{i1}^{G+1}, u_{i2}^{G+1} \dots u_{in}^{G+1}$ is generated, adopting the following crossover rule:

$$u_{ij}^{G+1} = \begin{cases} v_{ij}^{G+1} & \text{if } rj \leq k \\ x_{ij}^G & \text{if } rj > k \end{cases} \quad (10)$$

where rj is a (randomly selected) index ($j = 1, 2, \dots, n$) and k is the crossover constant, controlling the population diversity.

Finally, to decide if the generic vector $u_i^{(G+1)}$ is suitable to be included in the next generation population ($G + 1$), the selection operator is adopted, comparing the mentioned $u_i^{(G+1)}$ vector with the corresponding θ_i^G vector.

In particular, the following selection rule, aimed at preserving the best individuals in the population, is adopted (one-to-one competition scheme):

$$\theta_i^{G+1} = \begin{cases} u_i^{G+1} & \text{if } f(u_i^{G+1}) < f(\theta_i^G) \\ x_i^G & \text{if } f(u_i^{G+1}) \geq f(\theta_i^G) \end{cases} \quad (11)$$

The output of the selection is a set of N n -dimensional vectors representing the next algorithm generation. The algorithm search is stopped when a predetermined number of iterations (P) is reached.

5. Parametric Identification and Model Calibration

The following parameters vectors have been identified for the three examined models: (i) elastomeric bearing (Bouc–Wen) element, (ii) HDR element, and (iii) Kikuchi bearing element, considering the remaining parameters as invariant (see Table 5):

Table 5. Constant parameters assumed in the DEA approach.

Parameter	Value
G_r (MPa)	1.1
K_{bulk} (MPa)	2000
D (mm)	210
H (mm)	210
t_s (mm)	0.1
t_r (mm)	5
n (–)	40
T_r (mm)	200

for the elastomeric bearing (Bouc–Wen) element:

$$\theta_{BW} = \{F_y, \alpha_1, \alpha_2, \mu, \beta, \gamma, \eta\} \quad (12)$$

for the HDR Element:

$$\theta_{HDR} = \{\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \chi_1, \chi_2, \chi_3, \chi_4\} \quad (13)$$

for the Kikuchi bearing element:

$$\theta_{KIK} = \{c_g, c_h, c_u\} \quad (14)$$

Generally speaking, in the parametric identification of structural systems and elements, a significant number of parameters sets could provide unrealistic solutions. A restriction of the search space to a feasible range is thus desirable. In this optic, proper search ranges for the pre-defined model parameters have been defined, based on available information (i.e., personal experience, literature review, etc.).

Once the lower and upper bounds of each search range have been selected, DEA is performed adopting a population size (N) and a maximum number of iterations (P) equal to 50 and 150, respectively. Moreover, a mutation constant (F) equal to 0.8 and a crossover constant (k) equal to 0.5, have been assumed based on literature review [24].

The final values obtained solving the optimization problem, for each of the examined numerical models, are summarized in Tables 6–8. The considered range of variations are also included.

Table 6. Identified model parameters and search space for the elastomeric bearing (Bouc–Wen) element.

Parameter	f_y	α_1 [-]	α_2 [-]	μ [-]	β [-]	γ [-]	η [-]
Identified	11,000	0.28	−0.0003	2.40	0.75	−0.25	3.0
Lower Bound	9000	0.24	−0.0002	2.25	0.55	−0.45	1.5
Upper Bound	12,500	0.32	−0.0004	2.60	0.90	0.10	4.0

Table 7. Identified model parameters and search space for the HDR Element.

Parameter	α_1 [-]	α_2 [-]	α_3 [-]	β_1 [-]	β_2 [-]	β_3 [-]	χ_1 [-]	χ_2 [-]	χ_3 [-]	χ_4 [-]
Identified	0.32	−0.01	0.003	0.188	0.187	5.77	0.012	0.086	1.836	0.00025
Lower Bound	0.12	−0.03	0.001	0.10	0.10	2.50	0.005	0.025	0.500	0.00010
Upper Bound	0.50	−0.005	0.01	0.25	0.25	10.0	0.06	0.100	1.950	0.00045

Table 8. Identified model parameters and search space for the Kikuchi Element.

Parameter	c_g [-]	c_h [-]	c_u [-]
Identified	2.0	0.80	1.12
Lower Bound	1.85	0.70	1.00
Upper Bound	2.3	1.10	1.30

6. Results

The DEA was performed 10 times for each examined model. Table 9 summarizes the best and worst values of the objective functions (f) obtained applying the aforementioned algorithm.

Table 9. Estimated best and worst objective function values obtained applying the DE algorithm.

Element	Objective Function f	
	Best Value	Worst Value
Elastomeric (BW)	1.541	1.690
HDR	>5	>5
Kikuchi Bearing	1.418	1.552

On the other hand, objective values larger than five were obtained in the application of the DEA algorithm to the HDR element, pointing out a poor optimization of the parametric identification.

The comparison between the experimental and identified responses is presented in Figure 7, for each of the examined numerical models. For the sake of brevity, only the results associated with the best fit (best value of the objective function) are shown.

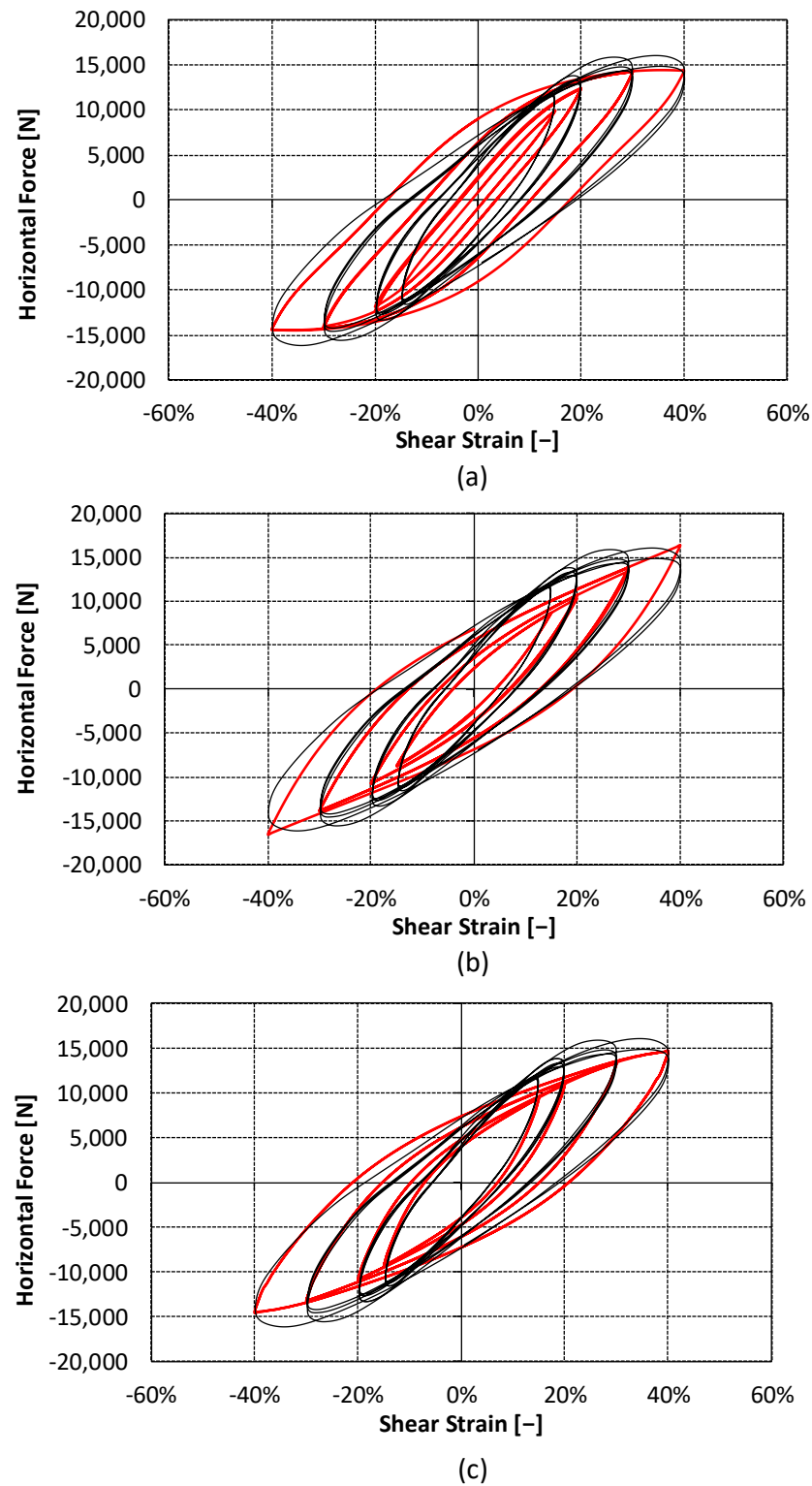


Figure 7. Differential Evolution Algorithm. Numerical Analysis vs Test Data, (a) BOUC–WEN Element, (b) HDR Element and (c) Kikuchi Bearing Element.

As can be observed, objective function values always lower than five were obtained for the elastomeric bearing (Bouc–Wen) element and the Kikuchi bearing element. According to [24,25], this result is generally associated to a good parameter estimation. Moreover, a significant stability of the algorithm emerged, since the best and worst values of the objective function are very close.

The very good parameter estimation obtained for the elastomeric bearing (Bouc–Wen) element and the Kikuchi bearing element is confirmed. In particular, for the elastomeric bearing (Bouc–Wen) element (see Figure 7a), a proper simulation of the experimental response is observed, with the exception of the first cycles. As a matter of fact, a significant underestimation of both lateral stiffness and dissipated energy capacity can be noticed at very low deformations (see Figure 8). Better results are obtained using the Kikuchi bearing element (see Figure 7c and Table 10). In this case, the hysteresis loops are well identified at each shear strain amplitude. The dissipated energy is properly captured at any cycle, while a little underestimation of the lateral stiffness still remains at very low deformations (Figure 8).

Table 10. Numerical Analysis versus Experimental Test results: Errors in terms of Effective Lateral Stiffness, Dissipated Energy and Equivalent Viscous Damping.

Shear Deformation [-]	15%	20%	30%	40%	50%	60%
Error [%] Lateral Secant Stiffness						
Elastomeric (BW)	14%	9%	0%	24%	8%	6%
HDR	24%	22%	3%	10%	56%	88%
Kikuchi Bearing	17%	20%	6%	22%	3%	5%
Error [%] Dissipated Energy						
Elastomeric (BW)	78%	53%	13%	2%	4%	8%
HDR	50%	38%	20%	15%	7%	2%
Kikuchi Bearing	5%	13%	9%	7%	1%	5%
Error [%] Equivalent Viscous Damping						
Elastomeric (BW)	75%	49%	14%	9%	3%	5%
HDR	35%	22%	18%	20%	40%	52%
Kikuchi Bearing	12%	7%	4%	2%	4%	1%

The poor optimization obtained for the HDR element is clearly represented in Figure 7b. As can be observed, although the experimental response of the bearing is well captured until shear strain equal to 40%, it is completely missed beyond this limit (Figure 8). This is probably due to the inherent mathematical formulation of the HDR element, that is not able to completely account the axial-shear load interaction, as well as P- Δ effects (see Section 3). As a consequence, the reduction of the horizontal stiffness in the post-buckling phase is not considered, thus potentially biasing, to some extent, the results of the simulation analyses.

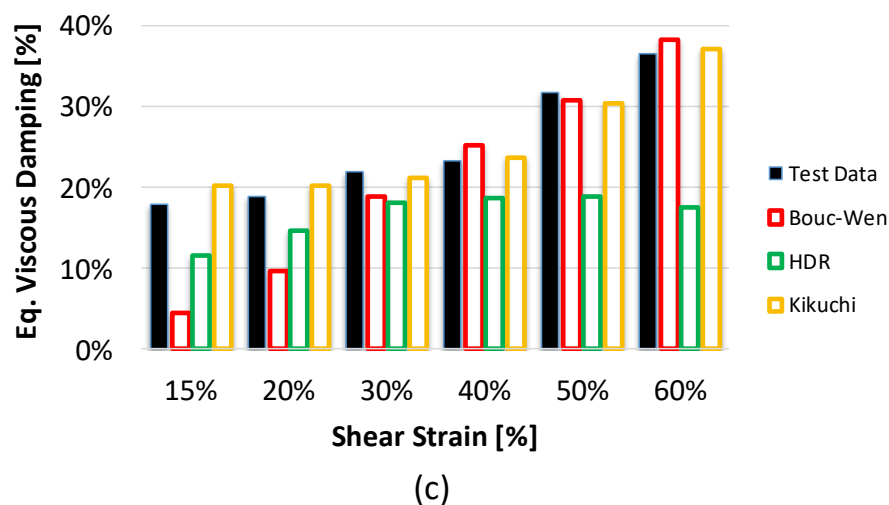
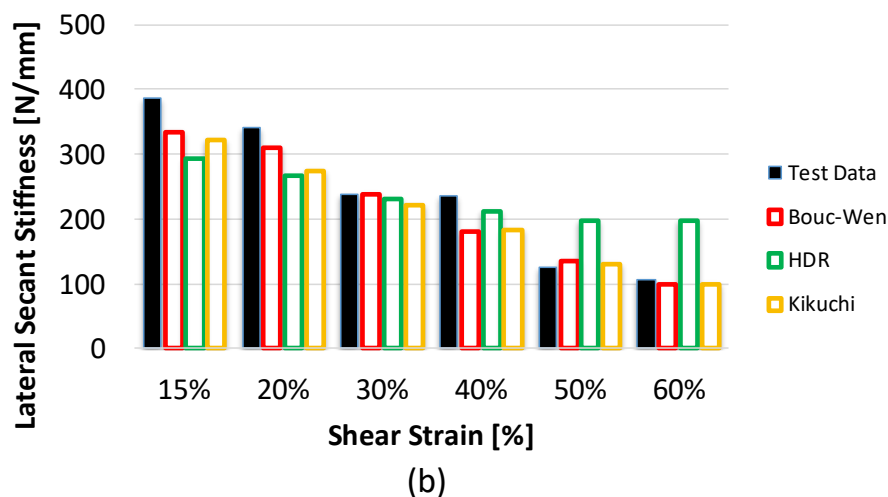
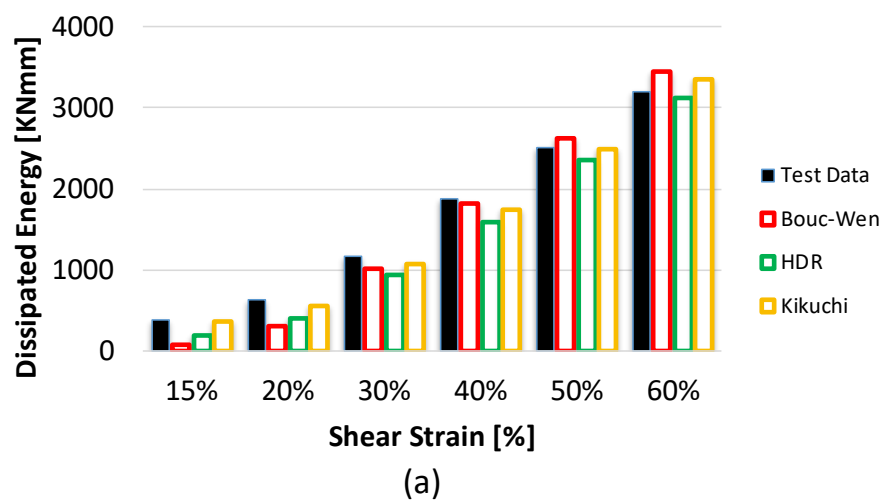


Figure 8. Numerical Analysis vs Test findings in terms of: (a) dissipated energy; (b) lateral secant stiffness; and (c) equivalent viscous damping.

7. Conclusions

Recycled rubber–fiber-reinforced bearings represent a promising and attractive solution for the base isolation of ordinary low-rise buildings. Such bearings can be adopted to reduce and mitigate the consequences of seismic events on structures. The dynamic behavior of the RR–FRBs is significantly influenced by P-delta effects, which cannot be described using simple bilinear or trilinear models of hysteresis (as done for conventional elastomeric bearings). In this paper, the applicability and calibration of advanced 3D hysteresis models, already included in the OpenSees framework library, to capture the cyclic response of RR–FRBs were discussed. In particular, the elastomeric bearing (Bouc–Wen) element, the HDR element and the Kikuchi bearing element were examined. The key parameters of such models have been identified in this work, based on experimental results using suitable evolutionary algorithms.

Based on the main results obtained within the present study, the following observations can be derived:

- a very good fitting has been obtained for the Kikuchi bearing element and the elastomeric bearing (Bouc–Wen) element, in particular for shear strain values larger than 20%;
- on the contrary, the HDR element seems not suitable to represent the real shear behavior of such bearings due to the specific peculiarities of the inherent mathematical formulation, accounting for the effective axial-shear interaction only in a partial and simplified way.

The findings of this study could provide a new impulse to the application of low-cost rubber-based devices in current practice. Additional studies are needed to compare the results of response history analyses and the results of experimental tests for real structures on RR–FRBs.

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