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Shear Strength Models for Reinforced Concrete Slender Beams: Comparative Analysis and Parametric Evaluation

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Abstract: Shear failure of reinforced concrete (RC) members belongs to brittle failure and has always been concerned. In this paper, 16 existing shear strength models of RC slender beams have been selected and comprehensively compared based on a set of 781 experimental test results. These formulas from eight national codes and eight published papers are mainly the semi-empirical models or the analytical models. These experimental test results were collected from 66 published papers, and the range of key parameters is relatively wide. The accuracy of these shear strength models is evaluated from overall prediction level and the effect of key parameters. These key parameters mainly contain concrete compressive strength, shear-span-to-depth ratio, effective depth, and stirrup ratio. According to the results of overall prediction and evaluation of key parameters, the prediction results of Zsutty's, Gunawan's, and Bazant–Kim's models are more accurate than others for both beams with stirrups and without stirrups. The models of ACI and JSCE exhibit higher prediction accuracy and safety margin, and their average values are between 1.19 and 1.28. The results of this study may provide reference for the selection and/or improvement of the shear strength model for RC slender beams.

Keywords: reinforced concrete; beams; shear strength; prediction model

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1. Introduction

It is well acknowledged that brittle shear failure is one of the most dangerous failure modes for reinforced concrete (RC) slender beams. The RC slender beam is a kind of RC structural member with a relatively large shear span-to-depth ratio (a/d) or span-to-height ratio (a/h), and there are different values defined in various articles and codes, e.g., $a/d > 2.5$ in Kani [1] and ACI 318 [2], $a/d > 2$ in JSCE [3], and $a/h > 3$ in EC2 [4]. The shear behavior of RC slender beams is complicated and influenced by many factors, such as concrete compressive strength, stirrup ratio, shear span-to-depth ratio, and longitudinal reinforcement ratio [5–8]. Thus far, numerous research studies on shear performances of RC slender beams have been carried out, and a large number of shear models have been proposed. For practical engineering design, it is essential that the shear strength model has good prediction performance.

The existing shear strength formulas of RC beams can mainly be classified into five categories according to mechanisms and methodologies, including the empirical models, semi-empirical models, mechanics-based approaches, probabilistic models, and machine learning-based models. The empirical models are exclusively derived from the regression analysis of experimental data, which includes primary influencing factors in the formulas. The mechanics-based models are proposed based on a mechanics principle. The typical mechanical models contain a truss model [9–12], critical shear crack theory [13], modified compression field theory [14], compressive force path concept [15,16], strain-based shear strength model [17,18], splitting failure theory [19], and plasticity theory [20–22]. The

semi-empirical models are developed according to the mechanics-based models, and their coefficients are determined through the fitting of shear test results. The existing shear strength formulas in the literature and national codes are semi-empirical models, i.e., the shear models in ACI318 [2] and GB50010 [23] codes. Collins and Kuchma [24] modified the shear model of ACI 318 by considering the influence of the size effect. Khuntia et al. [25] considered beam size, type of loading and beam supports. Kuo et al. [26] modified the ACI 318 [2] model by combining the function of shear span-to-depth ratio from the UH method [27,28] and the web compression area from Frosch [29] for considering the arch action effect and the effect of longitudinal reinforcement ratio. Since the size effect is an important influencing factor for shear strength, it was also considered in CEB-FIP MC90 [30] and EC2 codes [4]. These models were established based on the variable angle truss model. The interaction between the bond action of longitudinal reinforcements and dowel action and the contribution of the dowel action, aggregate interlock, and the concrete compressive zone were considered in shear strength of RC slender beams of Kim [31]. On the basis of the fib model, Huber et al. [32,33] introduced a calibration factor to account for surface roughness. The models of CSA A23.3 [34] and level II approximation of the fib Model Code [35] were based on the modified compression field theory, in which the effect of the nominal maximum size of coarse aggregate was considered. In the semi-empirical model proposed by Zsutty [36], the concrete strength, shear span-to-depth ratio and longitudinal reinforcement ratio were determined by combining the dimensional analysis and regression analysis techniques.

A fact is the differences in forms, parameters and application scopes of these existing shear models. It is worth discussing how to select a model with good prediction accuracy. A series of related studies were carried out. Rao and Injaganeri [37] evaluated the models of codes [2,38] and Bazant's models [39,40] by adopting a database of 614 beam test data. The results showed that Bazant's models exhibit higher predictive accuracy. A comparative analysis of eight models based on a database containing 33 experimental studies was performed by Elsanadedy et al. [5]. It was found that the prediction results of the MCFT-based codes are more non-conservative than the other codes. Keskin [41] also conducted an evaluation study to compare the accuracy of nineteen models based on the test data of 1082 beams, including five design codes. The predictions from the models in design codes are poorly correlated with the experimental data, while the models proposed by researchers [19,42–45] present higher predictive accuracy. There is a contradiction that the evaluation results of the same model may be quite different in these studies. For example, Rahal [46] and Alqarni et al. [47] found that the prediction of the model of EC2 [4] performs better than CSA A23.3 [34]. EC2 [4] predictions are closer to the test data than ACI 318 [2]. However, Nawaz [48] obtained the opposite conclusion. Moreover, the result of EC2 [4] exhibits larger scatter than CSA A23.3 [34] and ACI 318 [2] when they were used in predicting RC beams with stirrups, as reported by Zhang et al. [49]. This phenomenon may be due to the insufficient number of test specimens in the experimental database or a different source for these test data.

In addition to the overall average prediction accuracy of these models, an ideal model should give high prediction accuracy in the variation range of parameters. Most of the existing models may be applicable to a certain range of parameters. The studies focused on the prediction accuracy of the models in different parameter ranges are not enough. These key parameters influencing the shear strength of RC slender beams mainly include the shear span-to-depth ratio, concrete strength, stirrup ratio, and effective depth. The effect of stirrup ratios on the prediction accuracy of shear strength of RC beams was discussed by Russo et al. [50] based on 474 experimental results of beams with stirrup. It was found that EC2 [4] and ACI 318 [2] models provided scatter and conservative predictions. Russo et al. [51] conducted a parametric comparison study concerning the effect of shear span-to-depth on the accuracy of seven models based on 917 experimental results of beams without stirrup. The results indicated that Nielsen's model [52] was more reasonable than other models in terms of accuracy and scatter of prediction. Arslan [43] studied the predic-

tion accuracy of the models on high-strength concrete beams without stirrup. This study concluded that the models of ACI 318 [2], CSA, EC2 [4], and Collins [24] are excessively conservative for most of the test members, whereas the NZS [53] and Bazant's model [40] produced unsafe predictions for RC slender beams. Gandomi et al. [54] found that the shear strength models in the above four codes were sensitive to shear span-to-depth ratio and concrete strength. Furthermore, in the report of Ahmad and Bhargava [55], the accuracy of eight models was discussed under various concrete compressive strengths, longitudinal reinforcement ratio, and size of the member. The discussions revealed that the models of Zararis et al. [19] showed higher prediction accuracy under an increased longitudinal ratio, and the model of Niwa et al. [56] exhibited worse accuracy for beams with an effective depth greater than 900 mm. In summary, the key parameters influencing shear strength of RC slender beams are not completely considered evaluated in these models, and the parameter classification interval is unclear, which should be further discussed for the existing models. Therefore, the evaluation study on the existing typical models may be conducted based on a representative database, and the influence of key parameters classification interval also needs further investigation.

In this paper, a detailed comparative analysis of 16 shear strength models for RC slender beams is carried out by using an experimental database with 781 slender beams, which are collected from 66 available studies. The evaluated models contain eight national codes and eight papers, which are mainly the semi-empirical models and the analytical models. The accuracy of these models is evaluated from the overall prediction level and the influence of key parameters. The purpose of this study is to provide a reference for the selection and/or improvement of the shear strength model for RC slender beams.

2. Shear Strength Models of RC Slender Beams and Experimental Database

2.1. Evaluated Shear Strength Models

Based on the experimental database of 781 RC slender beams, the accuracy of 16 shear strength models is evaluated. These models are shown in Appendix A, Tables A1 and A2, including eight national codes and eight models proposed by different researchers. For the models of national codes, it is recommended that the shear strength of RC beams is the sum of the shear contribution of the concrete term (V_c) and that of the stirrup (V_s). Furthermore, the coefficients of each parameter are determined by fitting the shear strength experimental results. The other eight models are selected from empirical models, mechanics-based models, and semi-empirical models. Among the national codes, the 45° truss model was adopted by GB50010 [23], TS500 [57], ACI 318 [2], and JSCE [3] to consider the shear contribution of the stirrup. TS500 [57] and ACI 318 [2] considered the shear contribution of concrete in the form of $\sqrt{f'_c}$, whereas it was considered by GB50010 [23] in the form of tensile strength. Furthermore, ACI 318 [2] considers the longitudinal reinforcement rate and shear span-to-depth ratio. In the JSCE [3] code, the aforementioned variables in addition to size effect (in the form of $d^{-0.25}$, d is the effective depth) are considered. In EC2 [4], BS8110 [38], and CEB-FIP MC90 [30], the calculations of vs. are recommended based on the variable angle truss model, and the size effect is considered in the three codes' calculations of V_c. A size effect factor is used by EC2 [4] to consider the effect of the effective depth. The form of $d^{-0.5}$ was adopted by CEB-FIP MC90 [30], while the form of $d^{-0.25}$ was adopted by BS8110 [38]. The shear design provisions given in CSA A23.3 [34] were based on the modified compression field theory, and the size effect and strain effect on tensile reinforcement and maximum coarse aggregate size were taken into consideration in the contribution of concrete shear strength in CSA A23.3 [34].

The eight models selected in the papers are based on different mechanisms and methods, Zsutty [36] established an empirical equation in the function of concrete strength, shear span-to-depth ratio, and longitudinal reinforcement ratio, and these parameters were determined by statistical analysis of available shear strength experimental data. Based on the mechanical model, Campione [58] proposed to evaluate the shear resistance contribution under beam and arch actions. The model considers the stirrup ratio, shear span-to-depth

ratio, crack spacing, bond resistance, and size effect. In the model of Tureyen et al. [59], a free-body diagram was used to describe the effect of principal tensile stress in the concrete above the neutral axis. The other five models are all semi-empirical models. By taking into account the effect of concrete strength, longitudinal reinforcement ratio, and shear span-to-depth ratio on the quasi-brittle nature of concrete, a model was obtained by Bazant et al. [40] based on fracture mechanics. Moreover, a modified model of the Bazant et al. model [40] was derived by Bazant et al. [39] by considering the effect of maximum aggregate on shear strength. In the model proposed by Kuo [26], Frosch's model [29] and the UH method were combined to take into account the shear contribution of arch action and the area of the uncracked compression zone. In the truss theory-based models of Gunawan [60], a reduction coefficient was applied to reflect the effect of the initiation force of the diagonal crack. Kotsovos's model [61] was based on the compressive force-path method, in which the shear resistance consists of three components, including the concrete strength, longitude reinforcement and stirrup.

It is generally known that the stirrup is a crucial component for the shear strength of RC slender beams. The primary functions of stirrups are to directly resist a portion of shear force, diminish the size effect, and restrict the development of shear cracks. The stirrup ratio is strongly related to the mechanism of stress transfer and failure mode of concrete beams. Furthermore, in various models, the form and contribution of stirrups are considered differently. In this study, the accuracy of these shear strength models is compared and assessed for two cases of RC slender beams with and without stirrup.

2.2. Experimental Database

In this paper, the 781 shear strength experimental results of RC slender beams from 66 published papers [8,33,46–48,60,62–119] were collected as a representative database (see Appendix A, Table A3). To ensure the consistency and reliability of the collected data, several criteria were used to select specimens. Firstly, RC slender beams failing in shear were selected. Secondly, specimens bear one or two concentrated loads, while beams bearing a uniformly distributed load or a continuous axial load were excluded. Thirdly, the database does not include beams made with waste materials other than recycled coarse aggregate (e.g., fly ash and bricks), fibers and reinforced with FRP. Additionally, the experimental data of specimens should be relatively complete so that the analysis presented in this paper can be carried out.

There are 428 groups of RC slender beams without stirrups and 353 groups with stirrups. The parameters of these beams in the database include the section width, shear span-to-depth ratio, effective depth, concrete strength, reinforcement diameter, reinforcement ratio, yield strengths of longitude reinforcement and stirrup, and stirrup spacing. The ranges of these parameters were as follows: the stirrup ratio ranges from 0.07% to 3.02%; the compressive strength of normal strength and high-strength concrete is in the range of 19.7 to 197 MPa; the effective depth ranges from 156 to 1925 mm; the shear span-to-depth ratio is in the range of 2.5 to 10.4.

3. Overall Evaluation Results and Discussion

In this section, the evaluation results of the 16 shear strength models based on the experimental database containing 781 RC slender beams are presented. The calculated results of the specimens without and with stirrup are given in Figures 1–4. The horizontal axis is the number of each model, and the vertical axis is normalized predictive shear strength (the ratio of the test shear strength to the calculated shear strength V_{test}/V_{cal}). The statistical indicators adopted to evaluate the models include the mean value, standard deviation, minimum value, maximum value, and quantiles. The whiskers represent the standard deviations (St.D), while the dots represent the means of the V_{test}/V_{cal}. The standard deviation reflects the trend of data dispersion, whereas the mean reflects the trend of data concentration. The lower and upper lines of a plot stand for the normalized strength values at 25% and 75% quartiles ($x_{25\%}$ and $x_{75\%}$), respectively. These values are shown

in the first row of the table in each figure ([25%, 75%]). The other two lines stand for the values of the data at 2.5% quartiles ($x_{2.5\%}$), and 97.5% quartiles ($x_{97.5\%}$), respectively. These values are shown in the second row of the table ([2.5%, 97.5%]). Furthermore, the frequency of the $V_{test}/V_{cal} \geq 1.0$ ($P(V_{test}/V_{cal} \geq 1.0)$) is shown in the third row. If the percentage of the ratio between the experimental value and the predicted value is greater than one, the model is assessed to be safe.

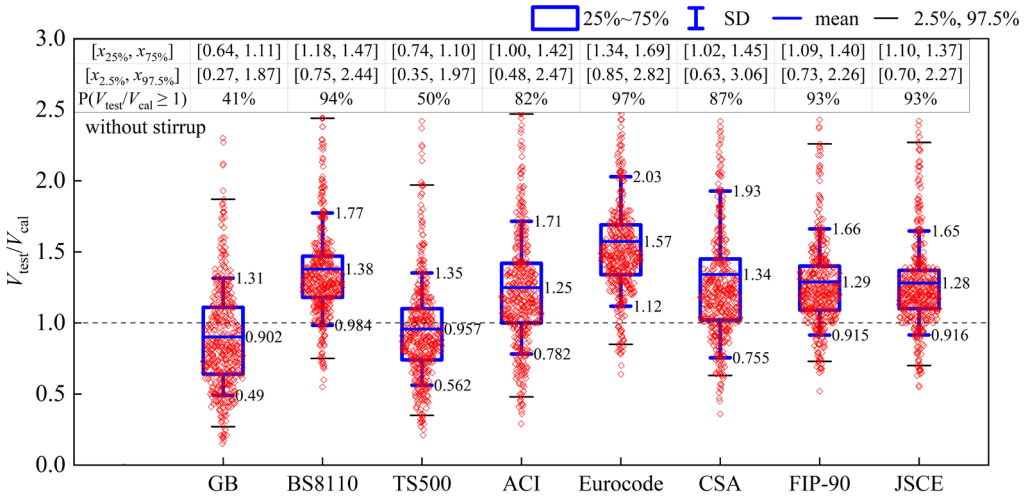


Figure 1. Codes: overall prediction results for beams without stirrup.

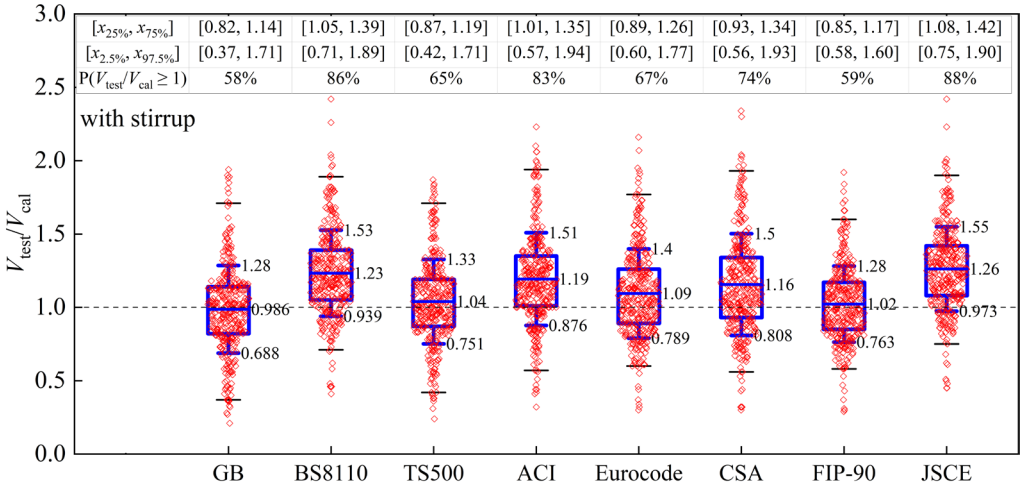


Figure 2. Codes: overall prediction results for beams with stirrups.

For models of eight codes, the results of the RC slender beams without and with stirrup are shown in Figures 1 and 2, respectively. It can be seen that the mean values of V_{test}/V_{cal} are greater than 1.0 for most of these models, except for that obtained by TS500 [57] and GB50010 [23] for RC slender beams without stirrup. The percentiles of safe prediction of the strength ratio from these two national codes are 41% and 50%, respectively. This indicates that the predictions from TS500 [57] and GB50010 [23] are mostly on the unsafe side. On the contrary, the mean value and the frequency of $V_{test}/V_{cal} \geq 1.0$ calculated by EC2 [4] are 1.57 and 97%, respectively, which exhibit highly conservative prediction results. For the RC slender beams with stirrup, most codes present reasonable estimations of the shear strength of RC slender beams compared to those without stirrup. The mean values of the codes GB50010 [23], CEB-FIP MC90 [30], and TS500 [57] are 0.99, 1.02, and 1.04, respectively, and the percentiles of safe prediction are 58%, 59%, and 65%, respectively. These results indicate that their predictions are closer to the experimental values, but their

safety percentiles are much lower than other codes. The presence of stirrup has little effect on the accuracy of BS8110 [38], ACI 318 [2], and JSCE [3] codes. The results of these three models exhibit similar safety percentiles and accuracy behaviors. For instance, the results of the beams without and with stirrup obtained by JSCE [3] are close to 1.3, the frequency values of $V_{test}/V_{cal} \geq 1.0$ are both larger than 85%, and that for beams with stirrup are the largest in all of the evaluated codes. Furthermore, the range of 2.5% and 97.5% quartiles provided by JSCE [3] are 0.27 and 2.2, respectively, which is smaller than BS8110 [38] and ACI 318 [2]. It is concluded that the results of JSCE [3] are relatively superior to other codes, which also remain as uniform safety margins.

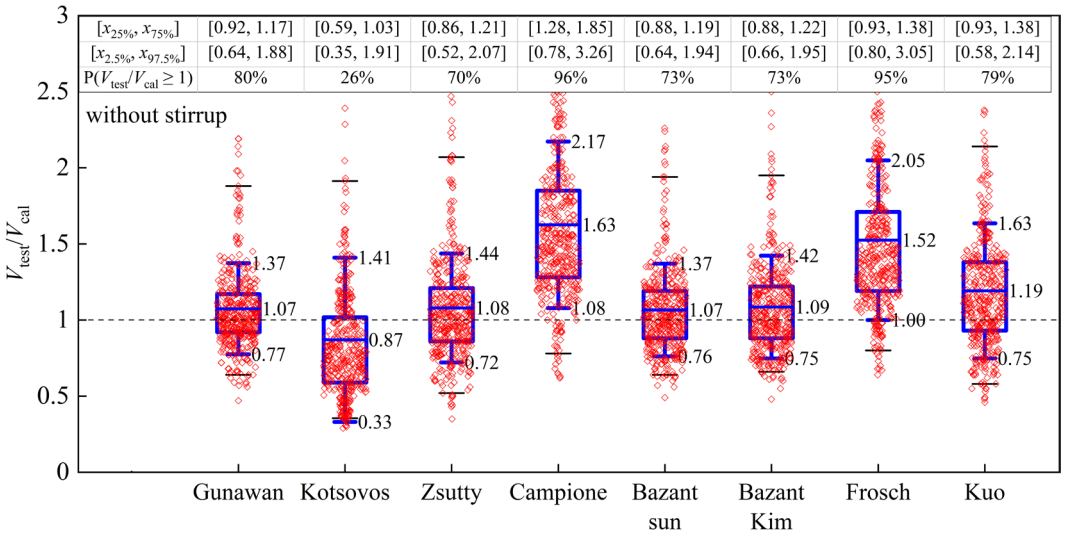


Figure 3. Researchers: overall prediction results for beams without stirrup.

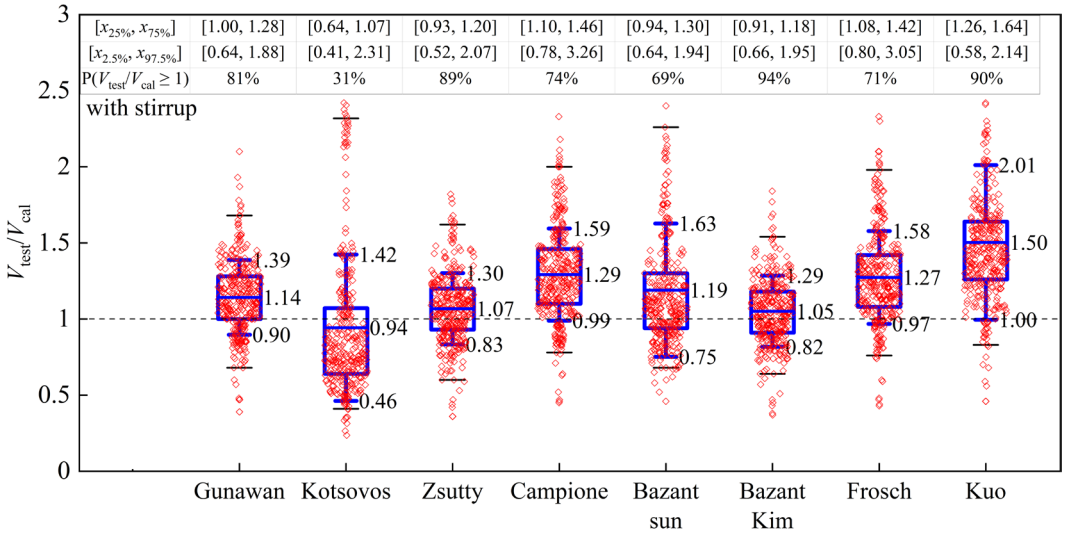


Figure 4. Researchers: overall prediction results for beams with stirrups.

It can be clearly seen from Figures 3 and 4 that the calculated mean values of the strength ratio by most models from published papers are greater than 1.0 for RC slender beams both without and with stirrup. For the RC slender beams without stirrup shown in Figure 3, the mean values of Gunawan [60], Zsutty [36], Bazant et al. [39], and Bazant et al. [40] are closer to 1 than other models. Among the above models, Gunawan [60] gives the lowest value of standard deviation and minimum range of quartile, which ranges from 0.30 to 0.25, respectively, and the distribution interval of values corresponding to

[$x_{2.5\%}$, $x_{97.5\%}$] is [0.64, 1.88]. It is thus believed that the model proposed by Gunawan [60] has higher prediction accuracy. For the RC slender beams with stirrups in Figure 4, the prediction values of models of Zsutty [36] and Bazant et al. [40] are close to the experimental values, in which the normalized mean values are 1.07 and 1.05, respectively.

Compared to Zsutty's model [36], the ranges of [$x_{25\%}$, $x_{75\%}$] and [$x_{2.5\%}$, $x_{97.5\%}$] calculated by both models suggest that Bazant et al. [40] presents a more uniform estimation of shear strength for RC slender beams with stirrup. Thus, Bazant et al.'s model [40] shows better performance than Zsutty's model [36] in predicting RC slender beams with stirrup. By comparing the estimated results in Figures 3 and 4, it is shown that the prediction accuracy of the studied models is affected to varying degrees by stirrup except for the models of Bazant et al. [40], Gunawan [60], and Zsutty [36], which remain uniform and close to 1.0 in both cases with and without stirrup. From the comparison of the above three models by standard deviation and the ranges of [$x_{25\%}$, $x_{75\%}$] and [$x_{2.5\%}$, $x_{97.5\%}$], the prediction results of Gunawan's model [60] are less scattered, as they have a tight distribution of strength ratios. Therefore, it is confirmed from the above evaluation that Gunawan's model [60] exhibits the most prediction performance. The reason for this result may be that in Gunawan's model [60], the amount of stirrups is considered by applying a decrement factor to the initiation force of the diagonal crack. However, other models are sensitive to stirrup, especially the models of Kotsovos [61], Campione [58], Frosch [59], and Kuo [26]. The prediction results obtained from Kotsovos's model [61] highly overestimate the test values for RC slender beams without stirrup and are scattered for RC slender beams with stirrup. Bazant et al.'s model [39] gives a reasonable prediction in the case of beams without stirrup, but conservative results are shown in the case of beams with stirrup. Furthermore, the models of Campione [58], Frosch [59], and Kuo [26] present conservative results for most of the test specimens in both cases. In addition, there are apparent differences in the dispersion of these models. The above discussions suggest that the influence of stirrup on shear strength is not well considered in these models.

4. Evaluation and Discussion of Key Parameters

4.1. Interval of the Key Parameter

The above discussions clearly show that the accuracy and conservativeness of each shear strength model are different. In order to better evaluate the selected shear strength models of RC slender beams under different ranges of each parameter, in this section, the influence of key parameters, which includes shear span-to-depth ratio (a/d), concrete compressive strength ($f_c{}'$), and the stirrup ratio ρ_{sw} on the V_{test}/V_{cal} in these models are discussed. The range of these parameters in the database is divided into several groups, and a single design parameter is varied for analysis at each time.

These key parameters are divided into different ranges according to the following constant convention.

The interval division of the shear span-to-depth ratio is mainly based on the analysis result of Subhan Ahmad's study [55], and the trend of nominal shear stress versus shear span-to-depth scatter distribution is shown in Figure 5a, which finally takes 3.5 as the distinction.

The interval division of the effective depth is mainly based on the experience data obtained by Collins and Kuchma [24], along with the trend of nominal shear stress versus effective depth scatter distribution shown in Figure 5b, which is limited to 225 and 450 and is divided into three intervals. According to the provisions in GB50010 [23], the minimum stirrup ratio limit is divided into two ranges. For the strength of concrete, the value of 50 MPa is used as a distinction between normal concrete and high-strength concrete.

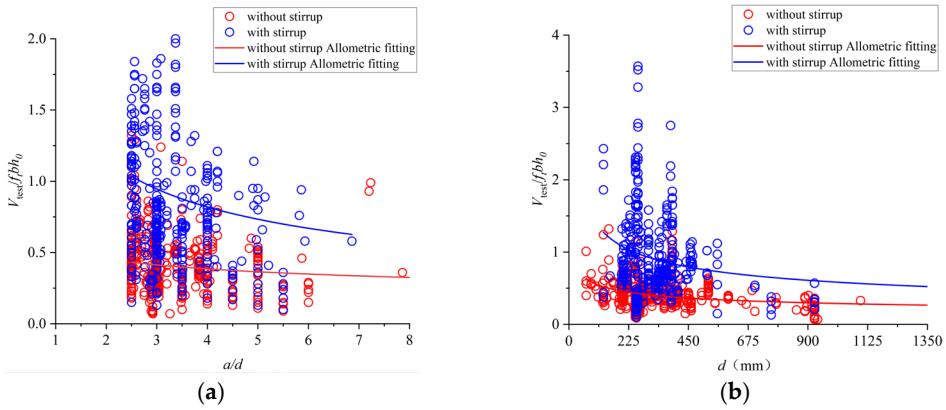


Figure 5. Normalized shear strength versus key parameters: (**a**) shear span-to-depth a/d and (**b**) effective depth d.

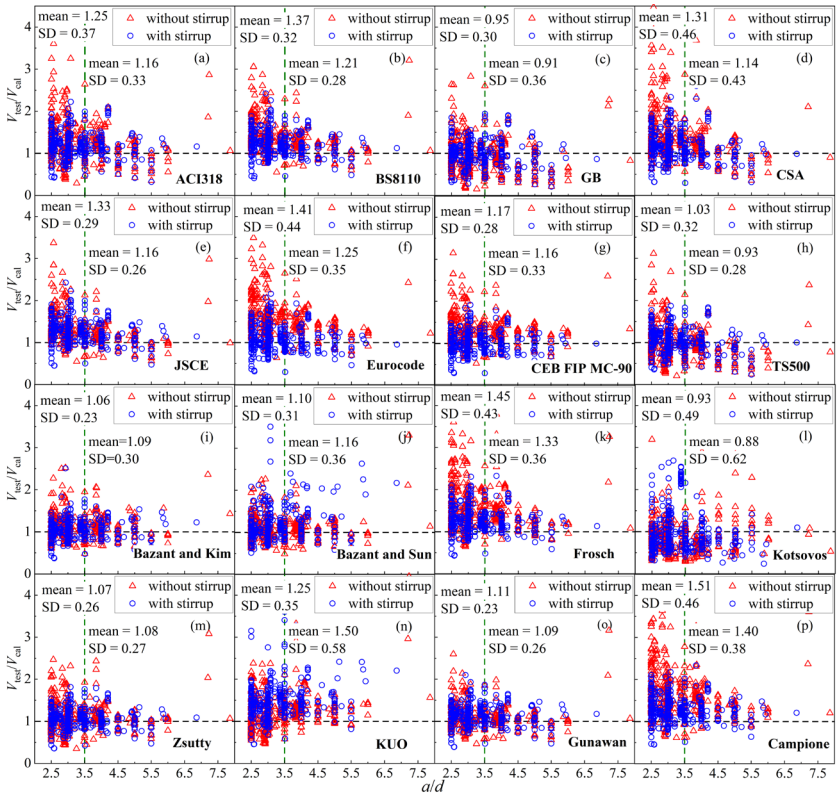


Figure 6. The relationship between the ratios ($V_{\text{test}}/V_{\mathbf{ca1}}$) and shear span-to-depth ratio of (**a**) ACI318; (**b**) BS8110; (**c**) GB; (**d**) CSA; (**e**) JSCE; (**f**) Eurocode; (**g**) CEB FIP MC-90; (**h**) TS500; (**i**) Bazant–Kim; (**j**) Bazant–Sun; (**k**) Frosch; (**l**) Kotsovos; (**m**) Zsutty; (**n**) Kuo; (**o**) Gunawan; (**p**) Campione.

4.2. Effect of Shear Span-to-Depth Ratio a/d

The shear span-to-depth ratio (a/d) is closely related to the shear transfer path, which further affects the failure mode of RC slender beams. In order to study the influence of a/d on the accuracy of the models in national codes and published papers, the ratio of experimental to predicted shear strength is plotted against a/d in Figure 6, and the comparison results are summarized in Figures 7 and 8.

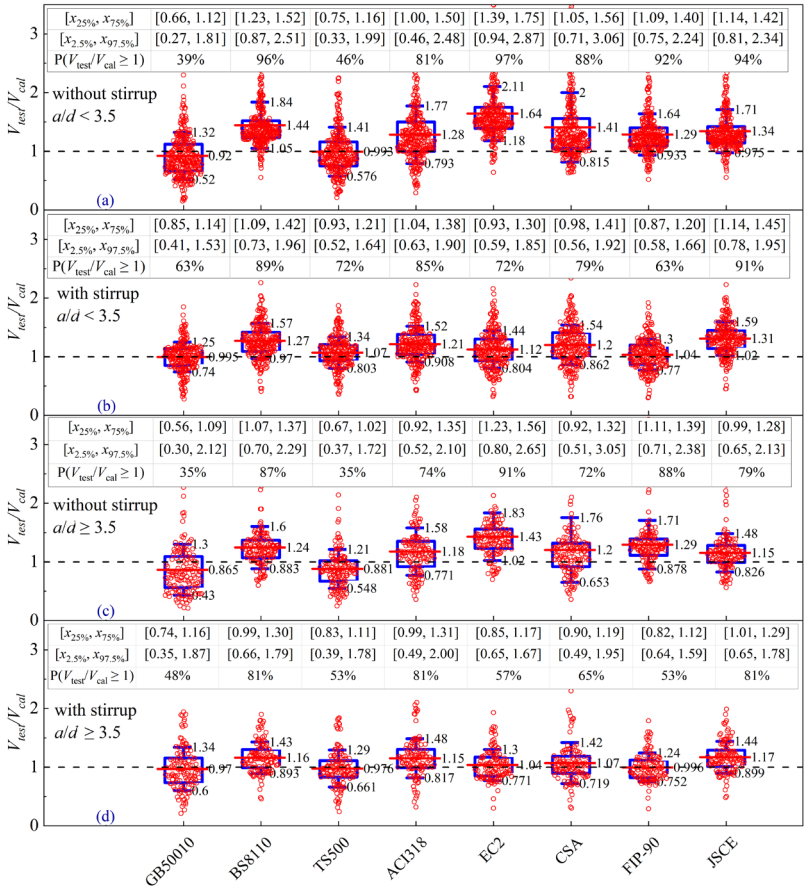


Figure 7. Codes: prediction results for beams without stirrup versus (**a**) $a/d < 3.5$ and (**c**) $a/d \geq 3.5$ and for beams with stirrup versus (**b**) $a/d < 3.5$ and (**d**) $a/d \geq 3.5$.

From Figure 6a–h, it can be clearly observed that most predicted values by the models of codes are distributed above the line of $V_{test}/V_{cal} = 1$ for $a/d < 3.5$, except for GB 50010 [23] and TS500 [57]. As can be seen from Figure 7, the safety percentiles predicted by EC2 [4] and CEB-FIP MC90 [30] are sensitive to stirrup, which drop from 97% to 92% for beams without stirrup and from 72% to 63% for beams with stirrup. Furthermore, the mean normalized values predicted by BS8110 [38] and CSA A23.3 [34] show significant variations in the cases of stirrups or not, although the predictive safety percentiles obtained from both models are high. The mean values obtained by BS8110 [38] and CSA A23.3 [34] are significantly influenced by the appearance of stirrup. The models of ACI 318 [2] and JSCE [3] present uniform mean values and relatively high safety estimation under $a/d < 3.5$.

When the a/d increases to above 3.5, the frequency values of $V_{test}/V_{cal} \geq 1.0$ calculated by national codes are affected to varying degrees, except for JSCE [3], ACI 318 [2], and BS8110 [38]. Compared to JSCE [3] and ACI 318 [2], the prediction from the model of BS8110 [38] is mostly in the safer range under $a/d \geq 3.5$. As mentioned above, for BS8110 [38], ACI 318 [2], and JSCE [3], their frequency values of $V_{test}/V_{cal} \geq 1.0$ are not sensitive to the a/d.

The models of Bazant et al. [40], Zsutty [36], and Gunawan [60], are independent of reinforcement types and are further accessed to investigate their sensitivity to a/d. For the beams under $a/d < 3.5$, it can be seen from Figure 6a,m,o that the predicted values of the three models are evenly distributed around the $V_{test}/V_{cal} = 1.0$. The calculated mean value and standard deviation obtained by Bazant et al.'s model [40] are 1.06 and 0.23, respectively, and those obtained by Zsutty's model [36] are 1.07 and 0.26, respectively, but those obtained by Gunawan [60] are 1.11 and 0.23, respectively. Moreover, the models of Bazant et al. [40] and Zsutty [36] provide the most reasonable prediction results at the given range of shear

span-to-depth ratio (i.e., $2.5 < a/d < 3$, and $3 < a/d < 3.5$). A conclusion can be drawn from Figure 8a,b that the models of Bazant et al. [40], Zsutty [36], and Gunawan [60] are more accurate with less discreteness by judging from the predictive values of [$x_{25\%}$, $x_{75\%}$] and [$x_{2.5\%}$, $x_{97.5\%}$] versus other models. From the comparison results of the above three models, it is shown that the models of Bazant et al. [40] and Zsutty [36] exhibit similar mean values, which are closer to 1 than Gunawan [60]. It is thus believed that the models of Bazant et al. [40] and Zsutty [36] are most accurate for RC slender beams under $a/d < 3.5$. With the increasing span-to-depth ratio ($a/d \geq 3.5$), the models of Gunawan [60] and Zsutty [36] provide more accurate prediction than Bazant et al.'s model [40], which can be seen from the fact that the mean values are close to 1.0, associated with smaller interquartile ranges and limits. It can also be seen from Figure 8 that the accuracy of Bazant et al.'s model [40] is more sensitive to stirrup than the other two models.

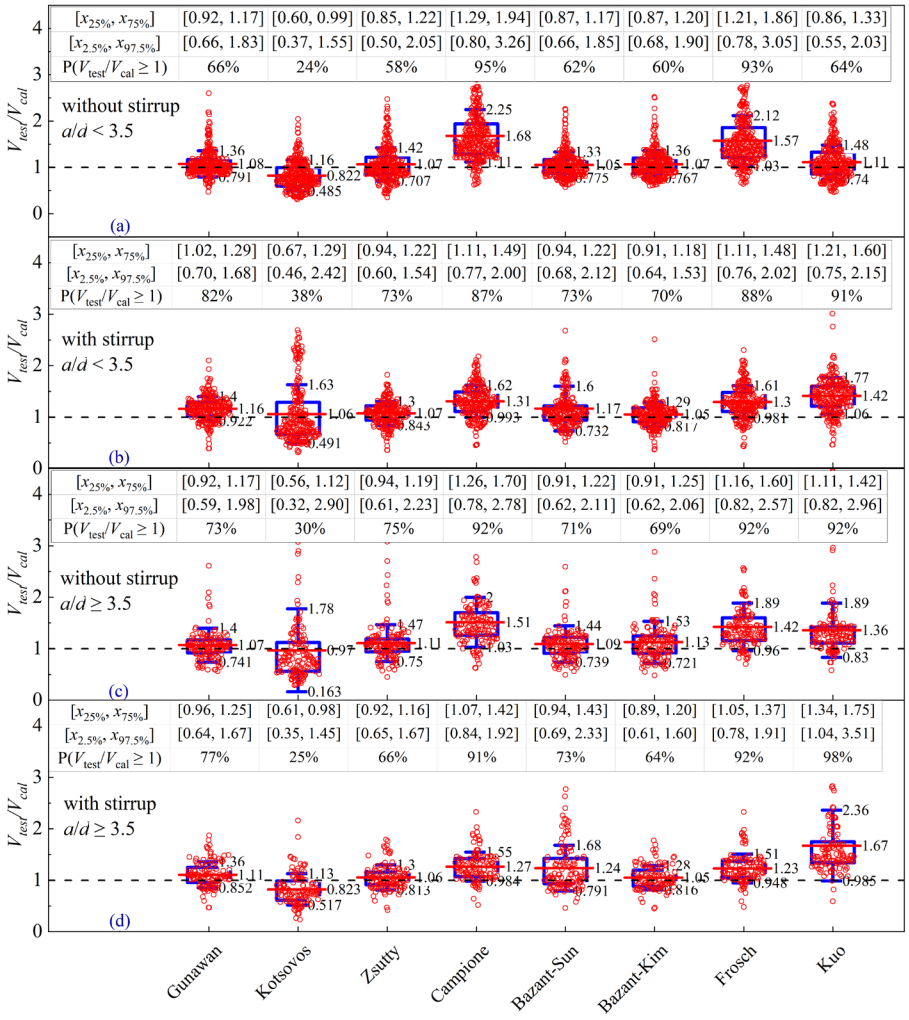


Figure 8. Researchers: prediction results prediction results for beams without stirrup versus (**a**) $a/d < 3.5$; (**c**) $a/d \geq 3.5$ and for beams with stirrup versus (**b**) $a/d < 3.5$; (**d**) $a/d \geq 3.5$.

The accuracy and dispersion of other models (i.e., Bazant–Sun [39], Frosch [59], Kotsovos [61], Kuo [26], and Campione's models [57]) can be evaluated by comparing Figure 6j–k,n,p. In the analyzed figures, these models are subjected to different shear span-to-depth ratios. As shown in Figure 6, the mean normalized values obtained by the models of Bazant et al. [39], Frosch [59], Kotsovos [61], Kuo [26], and Campione [58] are 1.10, 1.45, 0.93, 1.25, and 1.51, respectively, under $a/d < 3.5$. Therefore, the models of Frosch [59], Kuo [26], and Campione [58] are greatly conservative for most test data. When the value of a/d increases to above 3.5, the obtained mean values of these models change to

1.16, 1.33, 0.88, 1.50, and 1.40, respectively. From these values, it is apparent that the load carrying capacity is significantly underestimated by the models of Frosch [59], Kuo [26], and Campione [58], while it is overestimated by Kotsovos's model [61]. On the other hand, although the mean normalized value obtained by Bazant et al.'s model [39] is more even than the other mentioned models, it still shows a great dispersion. It is concluded that these mentioned models are all sensitive to a/d, which has not been well accounted for in these models.

In general, it can be summarized that Zsutty's model [36] presents the best accuracy for its good consideration of a/d. Similar to Zsutty's model [36], the accuracy of the models of Gunawan [60] and Bazant et al. [40] is not significantly sensitive to a/d. Therefore, the models of Bazant et al. [40] and Zsutty [36] are more suitable for predicting the shear behavior of RC slender beams with small shear span-to-depth ratio ($a/d < 3.5$). With an increase in shear span-to-depth ratio, the models of Gunawan [60] and Zsutty [36] show higher accuracy and reliability. It may be that Zsutty and Gunawan proposed models based on separate analyses of beam action and arch action to minimize extensive dispersion of shear stress values under arch action. As a result of regression analysis of the data, Gunawan and Zsutty considered the effects of shear span-to-depth ratio in the form of $(a/d)^{-1}$ and $(a/d)^{-1/3}$, respectively.

4.3. Effect of Effective Depth d

The increasing effective depth will induce an increase in the width of the inclined crack, thus reducing the transmission ability of shear stress along the crack interface. The accuracy of the model in national codes and researchers' works presents sensitivity to the size effect, as demonstrated in Figures 9–11. It is apparent from Figure 9 that the strength ratios obtained from models in national codes and most researchers' works tend to decrease with an increasing effective depth.

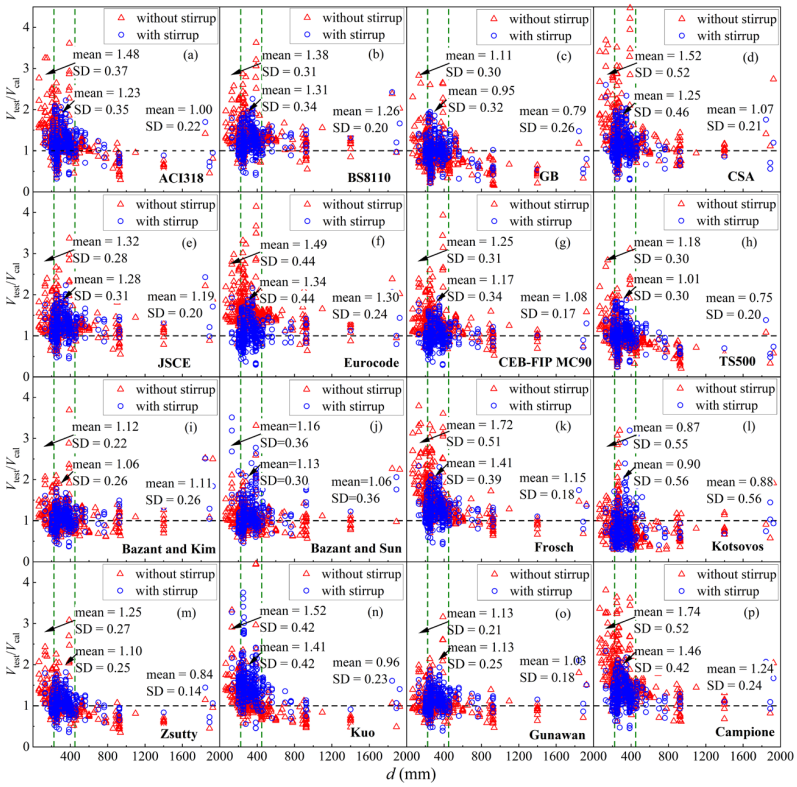


Figure 9. The relationship between the ratios (V_{test}/V_{cal}) and effective depth of (**a**) ACI318; (**b**) BS8110; (**c**) GB; (**d**) CSA; (**e**) JSCE; (**f**) Eurocode; (**g**) CEB FIP MC-90; (**h**) TS500; (**i**) Bazant–Kim; (**j**) Bazant–Sun; (**k**) Frosch; (**l**) Kotsovos; (**m**) Zsutty; (**n**) Kuo; (**o**) Gunawan; (**p**) Campione.

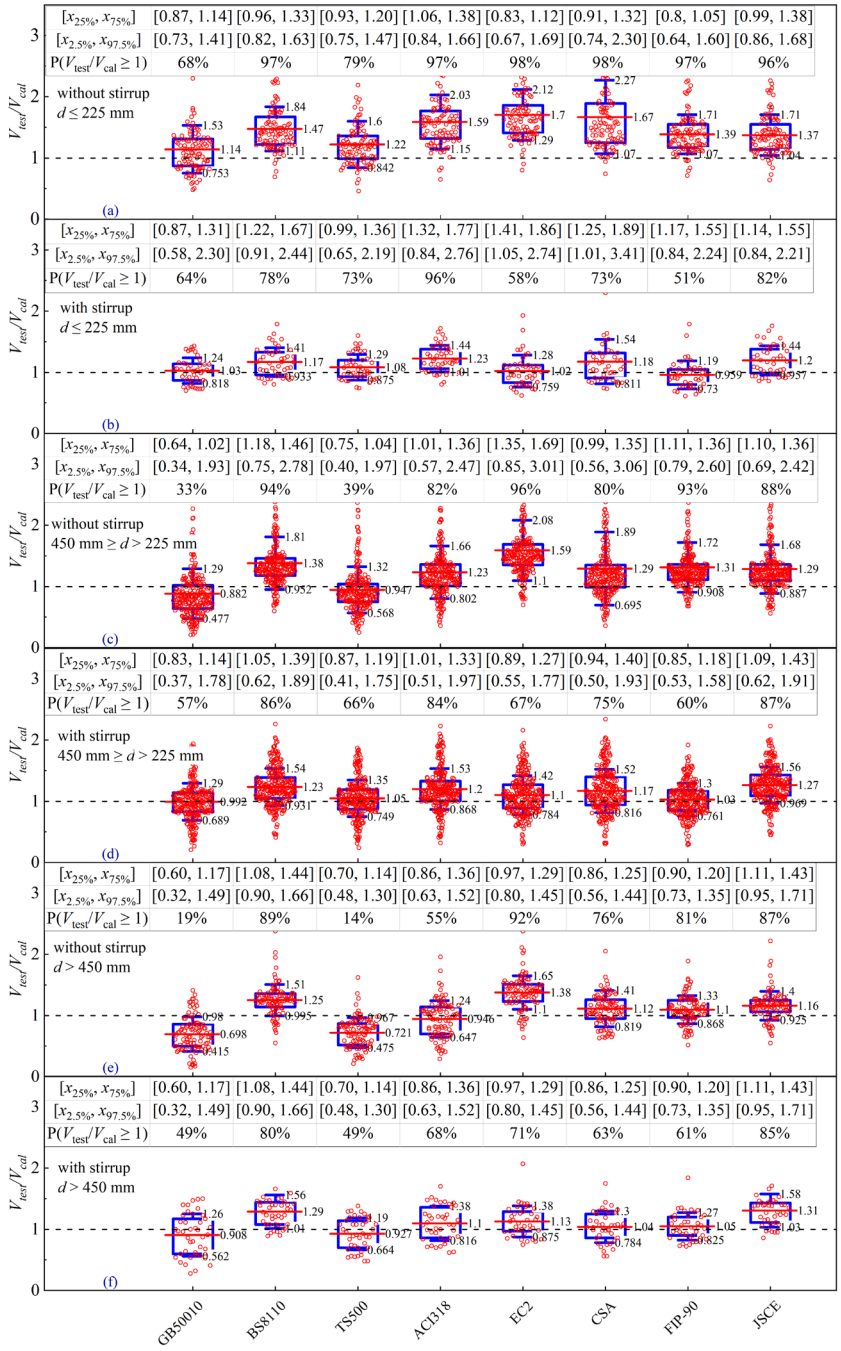


Figure 10. Codes: prediction results for beams without stirrup versus (**a**) $d \leq 225$ mm; (**c**) 225 mm $< d \leq$ 450 mm; (**e**) $d >$ 450 mm and for beams with stirrup versus (**b**) $d \leq 225$ mm; (**d**) 225 mm $< d \leq 450$ mm; (**f**) $d >$ 450 mm.

As presented in the third row of Figure 10, it can be seen that the stability margins of various national codes are also different from each other with the change of the effective depth. Among them, the frequency values of $V_{\text{test}}/V_{\text{cal}} \geq 1.0$ calculated by the models of GB50010 [23], TS500 [57], and ACI 318 [2] decrease significantly with the increasing effective depth, which change from 68% to 64% under $d \leq 225$ mm and from 19% and 49% under $d >$ 450 mm, from 79% and 73% to 14% and 49%, and from 97% and 96% to 55% and 68%, respectively. The results could be explained by the fact that the above three codes do not consider the size effect. In addition, it presents that the stability margins of EC2 [4] and CEB-FIP MC90 [30] are obviously affected by the appearance of stirrup. For example,

under $d \leq 225$ mm, the safe percentiles of specimen calculated by both codes decrease from 98% to 97% for beams without stirrup and from 58% to 51% for beams with stirrup. The prediction results of JSCE [3] remain at high safety levels for RC slender beams under $d \leq 225$ mm and $d > 450$ mm, and that of BS8110 [38] are adequately in favor of safety for RC slender beams under 225 mm $< d \leq 450$ mm.

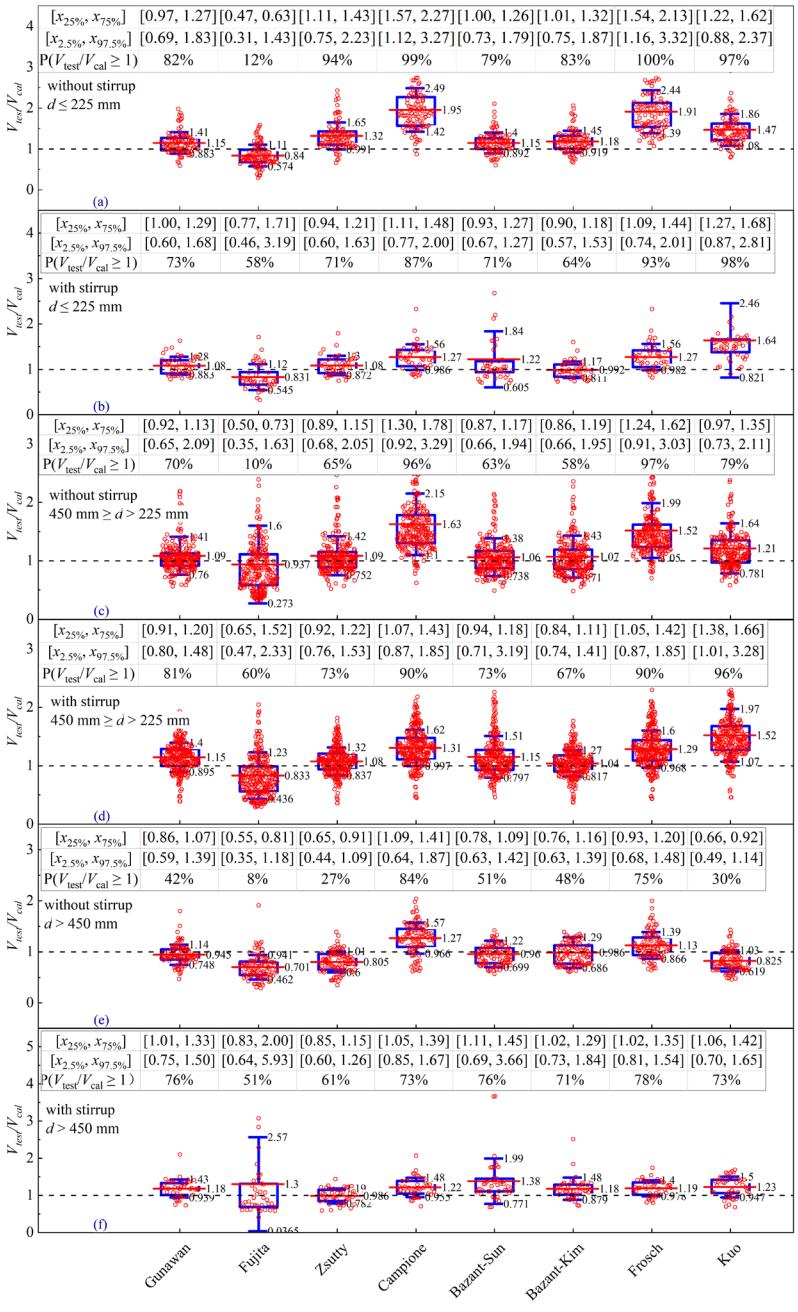


Figure 11. Researchers: prediction results for beams without stirrup versus (**a**) $d \leq 225$ mm; (**c**) 225 mm $< d \leq 450$ mm; (**e**) $d > 450$ mm and for beams with stirrup versus (**b**) $d \leq 225$ mm; (**d**) 225 mm $< d \leq 450$ mm; (**f**) $d > 450$ mm.

From the comparison of Figure 9i–p, the strength ratios calculated by the models of Bazant et al. [40], Bazant et al. [39], Zsutty [36], and Gunawan [60] are virtually unaffected by stirrup with tighter distribution and are closer to the line drawn at strength ratio = 1.0 than other models. The comparison results in Figure 11 clearly show that the predictions for beams without stirrup obtained by the models of Bazant et al. [39], Gunawan [60], and

Bazant et al. [40] are not sensitive to the effective depth, which are closer to 1.0 than Zsutty's model [36] over the entire range of the effective depth.

It can be thus believed that the models of Bazant et al. [39], Gunawan [60], and Bazant et al. [40] could precisely capture the size effect on shear strength of RC slender beams without stirrup. For beams with stirrup, Bazant et al.'s model [40] gives the most accurate and uniform estimations under $d \leq 225$ mm and 225 mm $< d \leq 450$ mm. It is evident that the predicted values given by Bazant et al.'s model [40] are approximately close to the experimental values with mean values of 0.99 and 1.04, respectively. The scatter of this model over both ranges is found below. Zsutty's model [36] shows the most precise prediction with low scatters for beams with stirrup under the effective depth > 450 mm. When Gunawan [60] is used to calculate the beams with stirrup, it is found that the prediction accuracy is slightly more conservative than Bazant et al.'s model [40] under $d \leq 225$ mm and 225 mm $< d \leq 450$ mm and Zsutty [36] in the case of $d > 450$ mm.

The distributions of strength ratio are found to be concentrated with the increasing effective depth. It is indicated that the accuracy of Gunawan [60] is not sensitive to the effective depth, and the size effect is reasonably considered.

Apart from the four models discussed above, the accuracy of the other four models shows large sensitivity to the size effect. As can be seen in Figure 11, the predicted values obtained by Kotsovos's model [61] greatly overestimate the experimental values for beams without stirrup, and it will exceedingly underestimate the experimental values with a large scatter. Furthermore, the models of Campione [58], Frosch [59], and Kuo [26] give conservative results with great dispersion. These four models are not reliable enough because the effect of the effective depth is not well considered.

4.4. Effect of Concrete Strength fc'

The influence of concrete strength on shear strength obtained by the models of national codes and researchers' works of RC slender beams is shown in Figure 12, and the evaluation results are summarized in Figures 13 and 14.

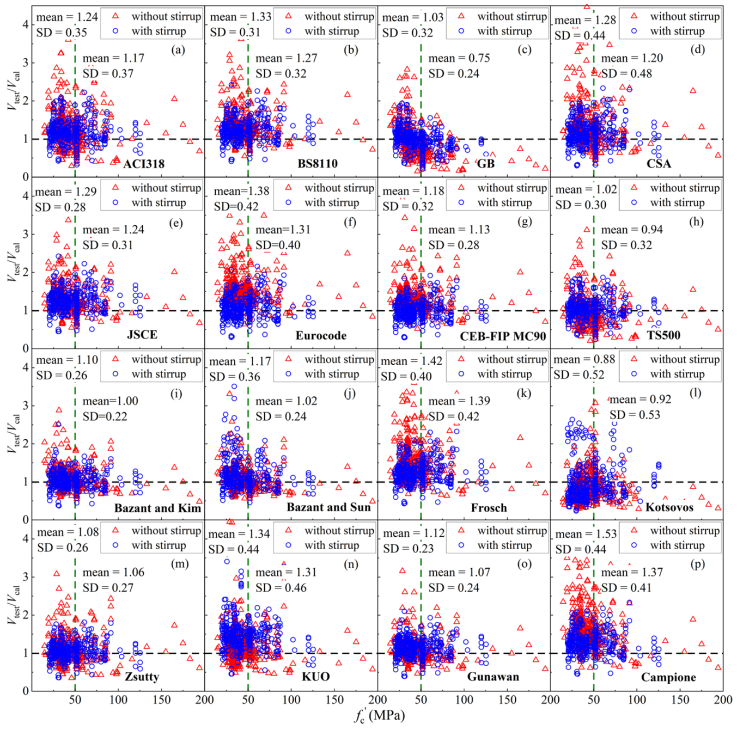


Figure 12. The relationship between the ratios (V_{test}/V_{cal}) and concrete strength of (**a**) ACI318; (**b**) BS8110; (**c**) GB; (**d**) CSA; (**e**) JSCE; (**f**) Eurocode; (**g**) CEB FIP MC-90; (**h**) TS500; (**i**) Bazant–Kim; (**j**) Bazant–Sun; (**k**) Frosch; (**l**) Kotsovos; (**m**) Zsutty; (**n**) Kuo; (**o**) Gunawan; (**p**) Campione.

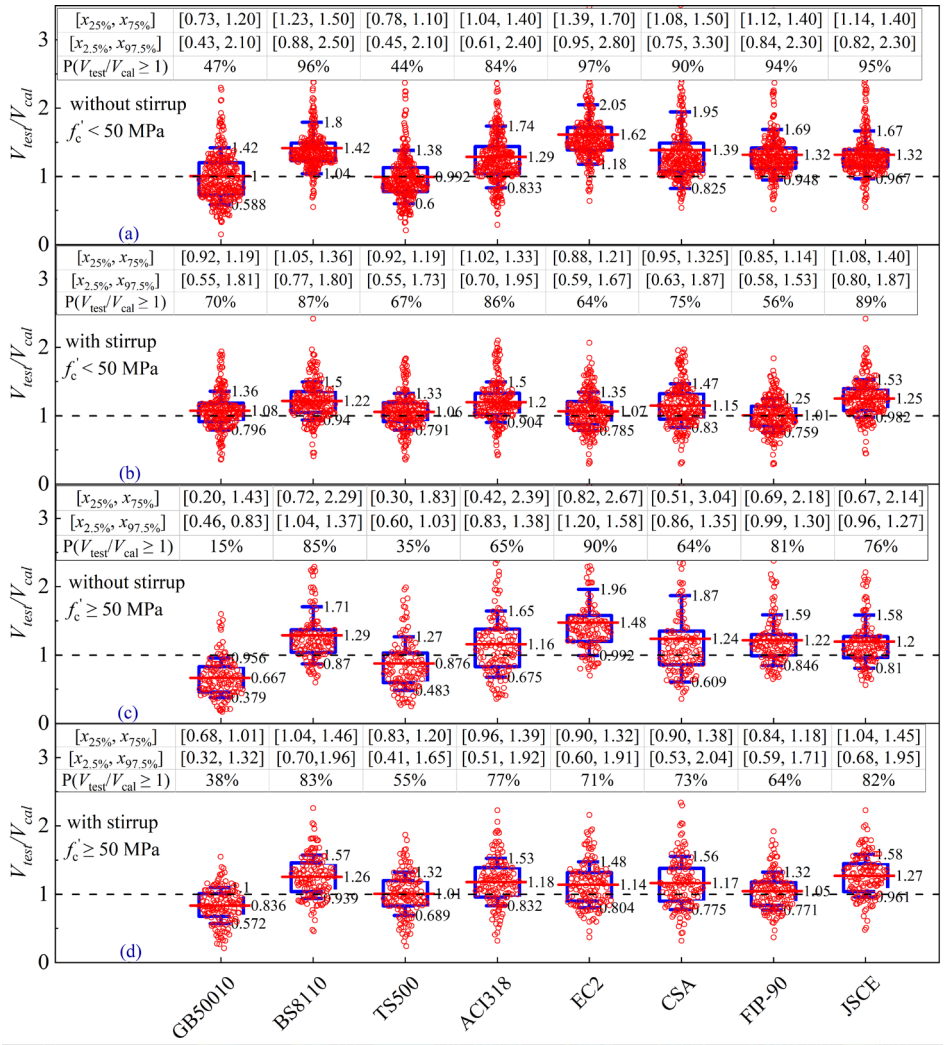


Figure 13. Codes: prediction results for beams without stirrup versus (**a**) normal concrete strength and (**c**) high concrete strength and for beams with stirrup versus (**b**) normal concrete strength and (**d**) high concrete strength.

Figure 12a–h show the distribution of strength ratios obtained by national codes against concrete strength. From Figure 13, it is visible that there is a large number of strength ratios yielded from codes except for GB50010 [23] and TS500 [57], which are distributed around the horizontal line at a strength ratio = 1.0. The safety percentile of specimens calculated by each code shows variations in concrete strength to varying degrees. Among them, the predictive results of GB50010 [23] and TS500 [57] constantly retain a low-security level, excluding predicting RC slender beams with stirrup and normal concrete strength, which are 70% and 67%, respectively, higher than CEB-FIP MC90 [30]. Furthermore, the amount of safety specimens obtained by EC2 [4] and CEB-FIP MC90 [30] are apparently influenced by the stirrup. For normal concrete strength, the frequency of $V_{\text{test}}/V_{\text{cal}} \geq 1.0$ of the models by EC2 [4] and CEB-FIP MC90 [30] range from 97% and 94% down to 64% and 56%, respectively, and those for high concrete strength range from 90% and 81% down to 71% and 64%. Compared to the above codes, the BS8110 [38], CSA A23.3 [34], and JSCE [3] models are not sensitive to concrete strength. In predicting the beams with normal and high concrete strength, the safety margins of the model of BS8110 [38] are comparatively high, which are consistently greater than 80%.

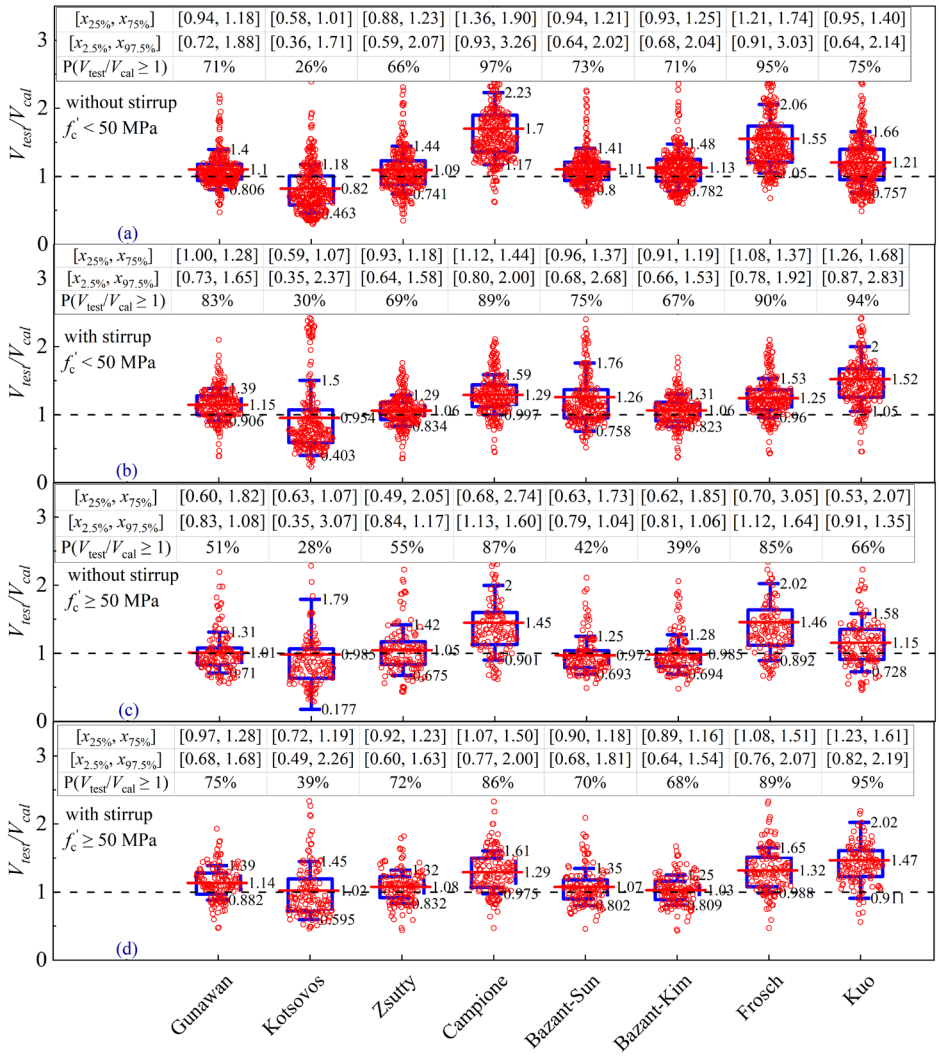


Figure 14. Researchers: prediction results for beams without stirrup versus (**a**) normal concrete strength and (**c**) high concrete strength and for beams with stirrup versus (**b**) normal concrete strength and (**d**) high concrete strength.

It can be observed from Figure 12i–p that the model of Zsutty [36], Gunawan [60], and Bazant et al. [40] are not significantly affected by the concrete strength and give a uniform prediction for beams with normal concrete strength. Gunawan's model [59] obtains approximately constant average ratios, with a uniform scatter across the range of less than 35 MPa while the models of Zsutty [36] and Bazant et al. [40] present more uniform scatters between 35 and 50 MPa. As summarized in Figure 14, these three models illustrate strength ratios that are closer to 1.0 than other models, and the standard deviation values and quartile range are smaller than other models. The same conclusion can be drawn in predicting beams with high concrete strength, and the models of Zsutty [36], Gunawan [60], and Bazant et al. [40] show the most accurate performance.

Kotsovos's model [61] gives unconservative and non-uniform estimations of shear strength since it yields the maximum number of unconservative results in predicting beams without stirrup and the highest scatter with a standard deviation of 60% in predicting beams with stirrup. Kotsovos's model [61] is the least accurate since it is heavily affected by stirrup. Figure 12 shows that the trend of average strength and the distribution of the scattering of models are not different as compressive strength increases, except for the model of Campione [58]. It can be concluded that the effect of compressive strength on shear strength is reasonable in existing models.

Among the above models, Gunawan's prediction results [60] are closer to the horizontal line at V_{test}/V_{cal} = 1. Thus, Gunawan [60] is more accurate. Gunawan [60] considers the effect of concrete strength on shear strength by introducing a parameter of $(f'_c)^{1/3}$ based on the model of JSCE [3]. When predicting the RC slender beams with stirrup, the model of Bazant et al. [40] is more accurate than the rest of the models because its average strength ratio is closer to 1.0, and the standard deviation and quartile range are smaller than other models. This may be due to the fact that Bazant et al. [40] applies fracture mechanics to the extension of mortar cracks in the contact area between large aggregate pieces, correlating the concrete strength with the maximum aggregate size.

4.5. Effect of Stirrup ρ_{sw}

Stirrups could directly provide a portion of shear force, which restricts the formation of diagonal cracks and thereby enhances the shear strength of RC slender beams. The distribution of V_{test}/V_{cal} versus the stirrup ratio is presented in Figure 15. The evaluation results for the models in codes and researchers' models are presented separately in Figures 16 and 17. From Figure 15a–h, it can be observed that for the beams with $\rho_{sw} < \rho_{sw,min}$, the models of national codes obtain larger strength ratios, which are distributed above the horizontal line at a strength ratio equal to 1.0, but they have a large scatter compared to those for the beams with $\rho_{sw} \geq \rho_{sw,min}$. As presented in Figure 16, among the considered national codes, the safety margins of EC2 [4], CSA A23.3 [34], and CEB-FIP MC90 [30] are significantly sensitive to stirrup ratio. The frequency of $V_{test}/V_{cal} \geq 1.0$ yields from the above three codes decreases to 50%, 69%, and 42% for $\rho_{sw} \geq \rho_{sw,min}$, which are 91%, 82% and 77% for $\rho_{sw} < \rho_{sw,min}$, respectively. Furthermore, the safety amount of prediction results by GB50010 [23] and TS500 [57] increases as the stirrup ratio increases. The models of BS8110 [38], ACI 318 [2], and JSCE [3] remain at a relatively high safety level as the stirrup ratios increase. The safe percentile of prediction results calculated by JSCE's model [3] (i.e., 96%) is second only to those obtained by BS8110 [38] (i.e., 97%) and the highest (i.e., 87%) compared to the other codes. It indicates that JSCE's model [3] is not sensitive to stirrup ratio and presents the safest predictions.

According to Figure 15i–p, the strength ratios obtained by the models of Zsutty [36] and Bazant et al. [40] are uniformly distributed along the horizontal line and become more concentrated with the increase in ρ_{sw}. As presented in Figure 17a, when predicting the beams with $\rho_{sw} < \rho_{sw,min}$, the mean values and standard deviations obtained from the models of Zsutty [36] and Bazant et al. [40] are found to be both equal to 1.15 and 0.22, respectively. However, the quartiles range from Zsutty's model [36] is 0.25, which is smaller than that from Bazant et al.'s model [40] (0.28), but when predicting the beams with $\rho_{sw} \geq \rho_{sw,min}$, the results are somewhat different. Bazant et al.'s model [40] yields an average strength ratio of 0.996, which is closer to 1.0 compared to the average strength ratio of 1.026 obtained by Zsutty's model [36]. In addition, the range of 2.5% and 97.5% quartiles obtained by the model of Bazant et al. [40] is 0.84, which is smaller than the one given by Zsutty [36] of 0.91.

It is shown that Zsutty's model [36] is more suitable for beams with $\rho_{sw} < \rho_{sw,min}$, and Bazant et al.'s model [40] is more suitable for predicting beams with $\rho_{sw} \geq \rho_{sw,min}$. The increase in stirrup ratios significantly affect the predicted value of the models of Campione [58] and Frosch [59]. The prediction results of Kotsovos's model [61] approximate to the experimental values in the case of $\rho_{sw} \geq \rho_{sw,min}$ but consistently exhibit high dispersion. On the contrary, the prediction results of the models of Campione [58] and Frosch [59] underestimate the experimental values in the case of $\rho_{sw} < \rho_{sw,min}$, but they become reasonable in the case of $\rho_{sw} \geq \rho_{sw,min}$. Furthermore, Kuo's model [26] yields conservative values of shear strength at each stirrup ratio. Although the stirrup rate has a slight effect on the predicted results for the models of Gunawan [60] and Bazant et al. [39], these are more conservative than the models of Bazant et al. [40] and Zsutty [36].

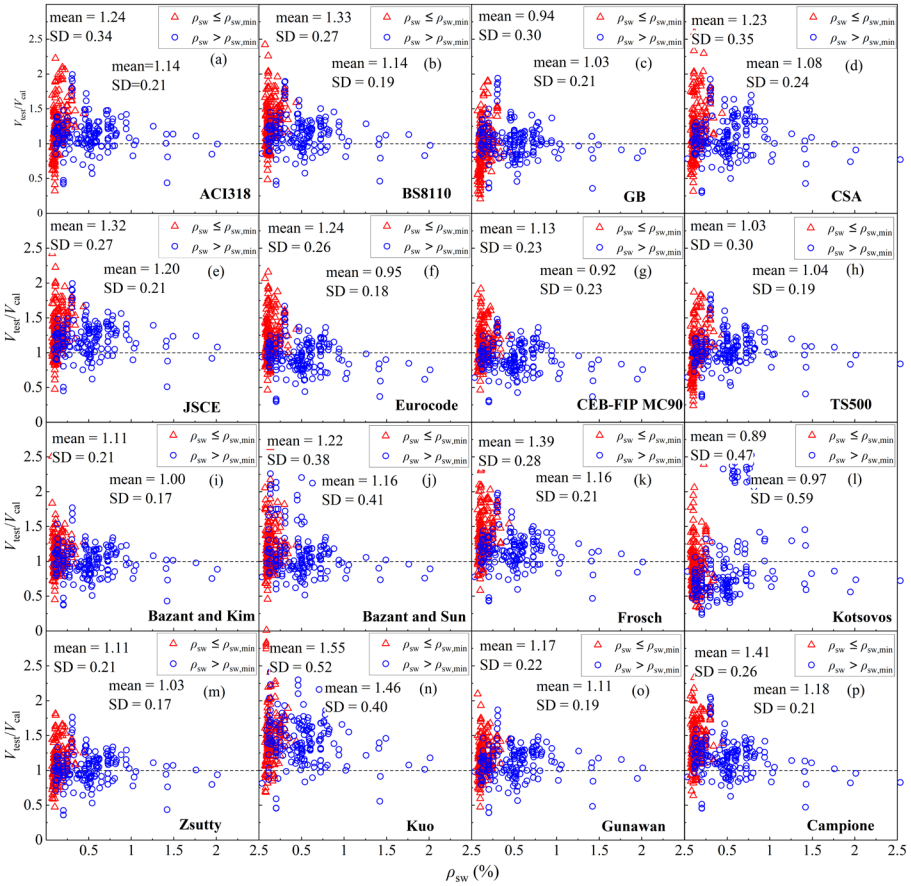


Figure 15. The relationship between the ratios (V_{test}/V_{cal}) and stirrup ratio of (**a**) ACI318; (**b**) BS8110; (**c**) GB; (**d**) CSA; (**e**) JSCE; (**f**) Eurocode; (**g**) CEB FIP MC-90; (**h**) TS500; (**i**) Bazant–Kim; (**j**) Bazant–Sun; (**k**) Frosch; (**l**) Kotsovos; (**m**) Zsutty; (**n**) Kuo; (**o**) Gunawan; (**p**) Campione.

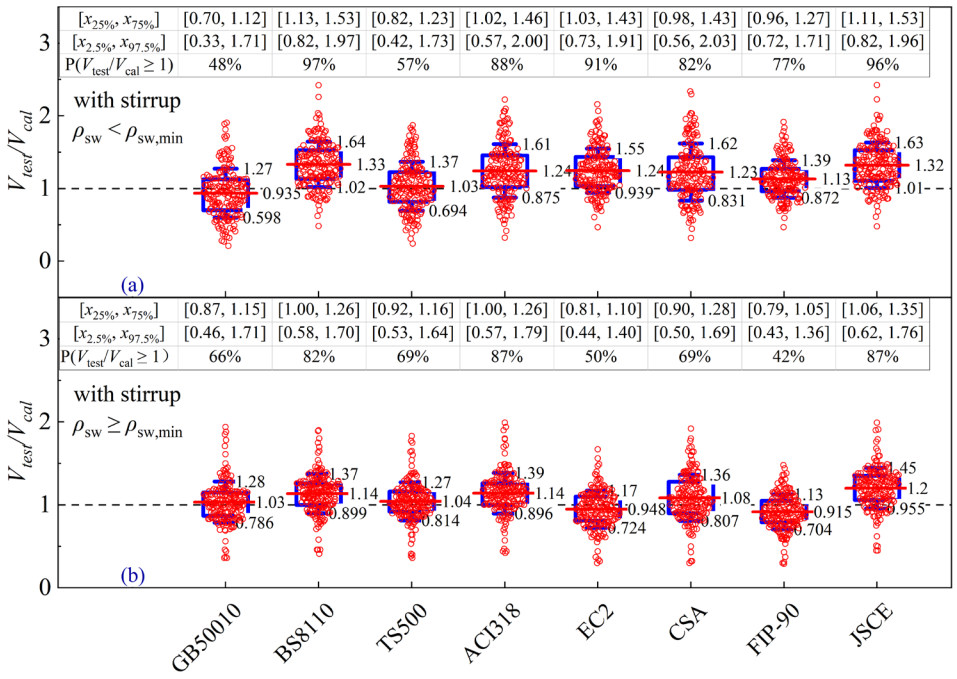


Figure 16. Codes: prediction results versus (**a**) $\rho_{sw} < \rho_{sw,min}$ and (**b**) $\rho_{sw} \ge \rho_{sw,min}$.

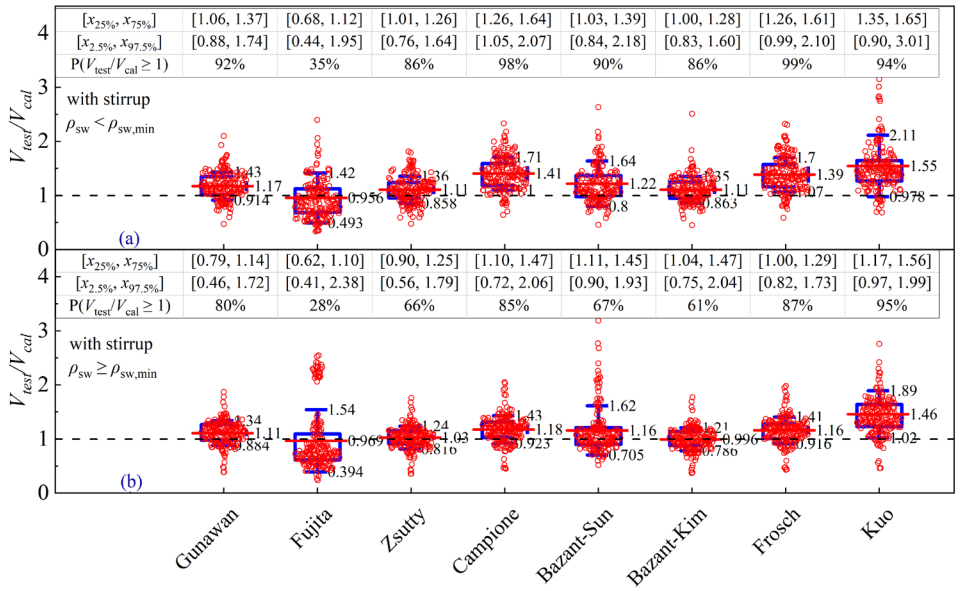


Figure 17. Researchers: prediction results versus (**a**) $\rho_{sw} < \rho_{sw,min}$ and (**b**) $\rho_{sw} \geq \rho_{sw,min}$.

5. Conclusions

In this paper, 16 shear strength formulas for RC slender beams, including eight national codes and eight published papers, were comparatively evaluated from overall level and effect level of key parameters by a large database consisting of 781 RC slender beams. The indexes for evaluation of these models mainly include the mean value, frequency of $V_{test}/V_{cal} \geq 1$, standard deviation, and quartiles range as well as 2.5th and 97.5th quartiles range. The following conclusions are drawn based on the detailed discussions:

- The main factors influencing the shear strength of slender beams mainly include the shear span-to-depth ratio, the effective depth, the concrete strength, and the stirrup ratio. Some factors considered by these sixteen models are different. According to overall prediction results, the prediction accuracy of the shear strength models from five researchers and seven codes is greatly affected by the presence or absence of stirrups. For researchers' models, the prediction results of Zsutty [36], Gunawan [60], and Bazant et al.'s models [40] are more reliable than others for both beams with stirrups and without stirrups, and their average values are between 1.05 and 1.14. For codes, except for GB50010 and TS500, the results of other code models for beams with stirrups are less than that for beams without stirrups. For both beams with stirrups and without stirrups, ACI and JSCE exhibit superior prediction accuracy and safety margins with mean values ranging between 1.19 and 1.28.
- According to evaluation and discussion of key parameters, the effects of shear span-to-depth ratio, concrete strength and stirrup ratio on shear strength are reasonably considered by Bazant et al. [40], Zsutty [36], and Gunawan's models [60]. Bazant et al. [40] and Zsutty [36] exhibit the most reasonable prediction for beams with $a/d < 3.5$, and Gunawan [60] and Zsutty's models [36] provide more accurate prediction than Bazant et al. [40] for beams with $a/d \geq 3.5$. For codes, the BS8110 code gives the most safety estimation for RC slender beams with $a/d \geq 3.5$. The prediction of JSCE exhibits superiority over those of others for beams with $a/d < 3.5$ and stirrup ratios ($\rho_{sw} \geq 0$).
- There exists a significant size effect in the models in national codes and some research, which has not been well considered. Bazant et al. [40] gave accurate results for the case of $d \leq 450$ mm, while Zsutty [36] accurately predicted the case of beams with stirrup under $d > 450$ mm. The prediction of Gunawan [60] is not sensitive to the effective depth, but slightly underestimates the experimental results. For codes, JSCE remained

at high safety levels for beams under $d \leq 225$ mm and $d > 450$ mm, and BS8110 gave a high safety margin for beams under 225 mm $< d \leq 450$ mm.

Author Contributions: C.M. mainly contributed to the design of this paper, methodology, establishment and analysis of database, formal analysis, writing—original draft, writing—review and editing, supervision, funding acquisition. Z.G. mainly contributed to conceptualization, methodology, formal analysis, writing—original draft, establishment of database, writing—review and editing. W.W. mainly contributed to the establishment of the database, formal analysis, resources, software, methodology. Y.Q. mainly contributed to the formal analysis, supervision, funding acquisition, investigation, project administration. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. National codes for the shear strength of beams.

Design Codes	Formulas
ACI 318 [2]	$V_u = (0.158\sqrt{f'_c} + 17.2\rho_v \frac{V_u d}{M_u}) b_w d$
GB 50010 [23]	$V_u = a_{cv} f_t b h_0 + f_{yv} \frac{A_{sv}}{s} h_0$
CSA A23.3 [34]	$V_u = V_c + V_s = \beta\sqrt{f'_c} b_w d + \frac{A_v f_y d \cot\theta}{s}$
EC2 [4]	$V_{Rd,c} = C_{Rd,c} k (100\rho_l f_c)^{\frac{1}{3}} b_w d + 0.9\frac{A_v f_y}{s} d \cot\theta$
BS8110 [38]	$V_u = \frac{0.79}{\gamma_m}\left(\frac{100A_s}{b_w d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\left(\frac{f_{cu}}{25}\right)^{1/3} b_w d + \frac{A_v f_y d}{s}$
CEB-FIP MC90 [30]	$V_u = 0.15(100\rho_l f'_c)^{1/3}\left(\frac{3}{a/d}\right)^{1/3}(1+\sqrt{\frac{200}{d}}) b_w d + \frac{A_v f_{yv}}{s} z(\cot\theta + \cot\alpha)\sin\alpha$
JSCE [3]	$V_u = \frac{\beta_d \beta_p f_{vcd} b_w d}{1.3} + \frac{A_v f_{yv}}{1.1s} z(\sin\alpha + \cos\alpha)$ $\beta_d = \sqrt[4]{1000/d}$; $\beta_p = \sqrt[3]{100\rho}$; $f_{vcd} = 0.2\sqrt[3]{f_{ck}}$
TS500 [57]	$V_u = (0.2275\sqrt{f'_c} + \rho_v f_{yv}) b_w d$

Notes: f'_c = cubic compressive strength; ρ_l = longitude reinforcement ratio; ρ_v = stirrup ratio; V_u = the ultimate shear force; M_u = the ultimate moment; a = shear span; d = the effective depth; b_w = the beam width; β = tensile stress factor; A_v = the cross section area of the stirrup; f_y = the yield strength of the stirrups; θ = compressive stress field inclination; s = the stirrups' spacing; $C_{Rd,c} = 0.18/\gamma_c$, in which γ_c (=1.5) is the material coefficient of concrete; $k = 1 + \sqrt{200/h_0}$; f_{ck} = the compressive strength of concrete with prism specimens of 150 × 150 × 300 mm; γ_m = partial factor for materials; f_{cu} = characteristic strength of concrete. β_d = coefficient to consider effect of effective depth on shear capacity; β_p = coefficient to consider effect of longitudinal reinforcement on shear capacity; z = distance between resultant compressive force and centroid of tension reinforcement; α = the inclination of the stirrups.

Table A2. Shear strength models proposed by researchers.

Researcher	Formulas
Gunawan [60]	$V_u = 0.2(f'_c 100\rho)^{1/3}\left(\frac{d}{1000}\right)^{-1/4}(0.75+\frac{1.4}{a/d})b_w d + A_w f_{yv}\frac{z}{s}$
Zsutty [36]	$V_u = 2.2(f'_c\rho\frac{d}{a})^{1/3} b_w d + \frac{A_v f_{yv}}{s} d$
Bazant and Kim [40]	$V_u = \frac{10\rho^{\frac{1}{3}}}{\sqrt{1+\frac{d}{25d_a}}}(0.083\sqrt{f'_c}+20.69\sqrt{\frac{\rho_l}{(\frac{a}{d})^5}})b_w d + \frac{A_v f_y d}{s}$
Bazant and Sun [39]	$V_u = 0.54\sqrt[3]{\rho_l}(\sqrt{f'_c}+249\sqrt{\frac{\rho_l}{(\frac{a}{d})^5}})(\frac{1+\sqrt{\frac{5.08}{d_a}}}{\sqrt{1+\frac{d}{25d_a}}})b_w\, d + \frac{A_v f_y d}{s}$
Campione [58]	$V_u = (1.07\sqrt{\rho}\sqrt{f'_c}+\rho_v f_{yv})b_w d$
Kotsovos [61]	$V_u = F_c[1-1/(1+5\,\lvert f_t\rvert/f_c)] + A_v f_y$
Kuo et al. [26]	$V_u = 1.17(\frac{a}{d})^{-0.7}\sqrt{f'_c} b_w c$
Tureyen and Frosch [59]	$V_u = \frac{2}{3} b_w c\sqrt{{f_t}^2+f_t\frac{\sigma_m}{2}} + \frac{A_v f_{yv}}{s} d$ $\sigma_m = 0.625\sqrt{f'_c}$

Notes: f'_c = cubic compressive strength; ρ_v = stirrup ratio; ρ_l = longitude reinforcement ratio; V_u = the ultimate shear force; a = shear span; d = the effective depth; b_w = the beam width; A_v = the cross section area of the stirrup; f_y = the yield strength of the stirrups; s = the stirrups' spacing; f_{ck} = the compressive strength of concrete with prism specimens of 150 × 150 × 300 mm; f_{cu} = characteristic strength of concrete. z = distance between resultant compressive force and centroid of tension reinforcement; c = cracked transformed section neutral axis depth; d_a = maximum aggregate size; F_c = compressive-stress resultant.

Table A3. Experimental database of RC slender beams.

Ref.	*a*/*d*	*d* (mm)	f'_c (MPa)	f_y (MPa)	ρ_l (%)	f_{yv} (MPa)	ρ_v (%)	V_u (kN)
Regan et al. [98]	3.7~3.8	200~350	34.6~41.0	475~600	2.39~4.19	0~350	0.25~0.5	50~207
Cucchiara et al. [62]	2.8	219~225	41.2	610	1.86~1.91	0~510	0~0.63	40~117
Angelakos et al. [64]	2.8~2.9	925	21~99	483~550	0.50~2.02	0~508	0~0.08	163~452
Lim et al. [76]	3.08	130	34	420	3.09	0~359	0~1.41	46~91
Rahal et al. [97]	2.7~3.0	300~330	60.9~66.4	440	2.16~3.96	0~305	0~0.30	125~270
Cao [102]	2.8~2.9	1845~1925	27.5~30.8	436	0.36~1.52	0~483	0~0.07	399~1225
Yoshida [110]	2.9~3.1	110~1890	35.4~37.7	437~550	0.74~0.91	0~470	0~0.07	40~550
Attar et al. [68]	4.12	170	30	390	0.89	0	0	26
López et al. [93]	2.6~10.4	384~389	22.3~36.4	218~544	1.63~2.29	0~549	0~0.21	81.8~281
Arowojolu et al. [66]	2.5	279.40	69.5	420	0.94	0~400	0~1.42	99~131
Rombach et al. [79]	3.0~5.0	302	49.2~54.8	550	1.56	0	0	75~84
Kim et al. [31]	2.5~4.0	300	34	476.5	1.43	0	0	62~107
Lee et al. [90]	3.05	442~459	23.1	396~408	1.66~1.82	504	0.11~0.22	187~273
Jeong et al. [86]	2.50	150~780	33.5	455~542	0.96~3.44	0	0	81.7~245
Choi et al. [73]	2.5~5.0	525	27	433	1.87	0	0	206~259
Alqarni et al. [47]	3.2	250	49.4~81.6	571	4.19	0~599	0~0.52	70.4~197
Jain et al. [85]	3.5	251	24.5~28.2	566	2.67	0~558	0~0.46	62~201
Huber et al. [33]	3.0	460	42.5~63.9	566	1.17	0	0	56~85
Perera et al. [113]	3.0~4.0	250	116~194	750	3.04	0	0	80~226
Wang et al. [114]	3.0	155	70.51	336.5	2.59	441.5	0.19	58
Nouri et al. [115]	3.0	135	44~85	439~447	1.2~2.3	0	0	23~30
Rahal et al. [46]	3.0	387	44.4~55.5	565	1.04	0	0	60~68
Deng et al. [75]	3.0	360	31.1~34.1	470.6	1.58	0	0	77~99
Hassan et al. [80]	2.8~3.8	100~667	47	480	1.09~3.14	0	0	78~340
Nawaz et al. [48]	3.4	260	19.8~50.3	530	0.73~1.08	530	0.56~0.83	228~271
Biolzi et al. [71]	2.5~4.5	260	42.1~75.0	516~589	0.91~1.03	0~589.6	0~0.10	34~109
Słowik [107]	2.5~4.1	220	35	453~545	1.80	0	0	40~50
Hu et al. [82]	2.60	270	32.0~40.5	551	2.36	0~442	0~0.62	117~258
Lee et al. [8]	2.5~4.0	300~520	25.0~50.3	530~707	2.7~4.6	187.1~758.7	0.3~0.5	187~758
Bukhari et al. [72]	2.5~6.0	267	44.8~52.7	420	0.58~1.07	0	0	24~70
Yi et al. [78]	2.6~3.4	335~338	29.9~53.6	552	1.52~1.84	414.3	0~0.19	80~215
Zu [112]	2.6~3.5	201~218	25.8	439~443	1.13~1.85	0	0	31~45
Yu et al. [111]	2.5~2.7	426~1098	27.3~33.8	500	0.67~1.19	0	0	150~630
Islam et al. [84]	3.00~3.9	203	50.8~83.3	532~554	2.02~3.22	0	0	45~117
Khaleel et al. [87]	2.5~3.5	220	56.1	401	1.52~3.05	313.6	0.24	62~87
Bentz et al. [69]	3.3~4.1	194~937	35.0~46.0	397~474	0.51~2.54	0	0	54~232

Table A3. *Cont.*

Ref.	*a*/*d*	*d* (mm)	f'_c (MPa)	f_y (MPa)	ρ_l (%)	f_{yv} (MPa)	ρ_v (%)	V_u (kN)
Thamrin et al. [116]	3.7	219	32.0	550	0.97~2.50	0	0	32~37
Lee et al. [117]	2.6	390	33.6~68.4	648	3.72	0~667	0~253.68	403~914
Lee et al. [91]	3.0	252~383	26.8~84.6	525~1068	1.44~6.92	508~510	0.20~3.02	188~836
Arslan et al. [67]	2.5	230	20.8	420	1.16~2.21	393	0.22~0.54	57~87
Gunawan et al. [60]	2.5~3.5	300	40.8~49.1	1152~1197	1.69	0~395	0~0.65	57~144
Tavio [77]	3.5	200~700	85.0~102.0	512	3.98	0~518	0~0.58	69~620
Lee et al. [6]	3.0~5.0	280~410	19.7~40.8	520~550	0.93~4.76	0~215	0~0.18	70~316
Kawamura et al. [87]	2.9	350	20.3~21.5	537	1.84	370~403	0.32~0.36	114~171
Vecchio et al. [109]	4.0~5.0	457~466	22.6~25.9	436~555	1.80~3.46	325~600	0.1~0.202	141~244
Rahal [96]	3.0	340	24.3~42.2	440	1.60~2.66	445	0.2	166~264
Mawlood et al. [92]	3.9	130	91.5	390	0.38~1.69	0~485	0~0.22	40~186
Shah et al. [101]	3.0~5.5	254	52.0	414	0.33~2.00	0~276	0~0.1	23~127
Bentz et al. [70]	2.9	84~333	34.3~36.1	400	1.55~1.63	0	0	14~43
Kim et al. [88]	2.5	300~600	31.8~34.9	529~651	1.90	0	0	60~261
Arezoumandi et al. [65]	3.0	407	30~37.2	414	1.24~2.49	0	0	111~173
Ignjatovic' et al. [83]	4.2	235	33.3~37.0	555~580	4.09	0~300	0~0.19	91~163
Pradhan et al. [95]	2.6	269	42.8	508~606	0.75~1.31	0~352	0~0.32	81~162
Knaack et al. [89]	3.8	200	31.2~46.4	572	1.34	0	0	31~44
Roller et al. [99]	2.5~3.0	559~762	72.0~125.4	436~489	1.61~6.89	406.7~457.8	0.07~1.76	301~226
Cladera et al. [74]	3.0~3.1	351~359	49.9~87.0	500	2.24~2.99	0~540	0~0.11	99~308
Sowik [106]	2.5~4.1	220	35.0	453~545	1.93	0	0	40~51
Ros et al. [100]	4.0	206	24.5~32.6	549~669	2.20	0~669.6	0~0.12	182~360
Sherwood et al. [103]	2.9	280~1400	28.1~77.3	452~494	0.83~1.34	494~496	0.10	36~298
Ghannoum [118]	2.5	65~889	34.2~58.6	385~477	1.15~2.00	0	0	42~386
Ahmad et al. [63]	3.0	198~216	38.3~84.8	421	0.96~4.51	0~421	0~0.78	23~120
Shin et al. [104]	2.5	215	52.0~73.0	414	3.77	0~388	0~0.94	56~183
Hong et al. [81]	3.0~6.0	142~915	53.7	477	1.01~4.68	0	0	39~332
Placas et al. [119]	3.4	272	24.8~30.3	621	0.98~1.46	0	0	45~54
Tan et al. [108]	2.8~3.1	400~448	54.7~74.7	498~538	2.00~5.8	353.2~385	0.48	135~265
Morrow et al. [94]	2.8~7.9	356~372	14.7~45.7	483	0.58~3.79	0	0	83~182

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