SUPPLEMENTAL MATERIAL

This supplemental material provides more detailed information about fuzzy BWM-TOPSIS methodology, triangular fuzzy numbers (TFNs), basic fuzzy arithmetic operations, detailed calculations for identifying the key products in GP, and fuzzy DEMATEL method.

S1. Fuzzy BWM-TOPSIS methodology

Fuzzy BWM-TOPSI methodology steps are as follows [1]:

S1. 1.Determining the weighs of criteria/ factors by Fuzzy BWM

S1.1.1 Determine the most and the least important criteria

The decision makers are asked to determine the most and the least important criteria/factors among the finalized criteria/factors.

S1.1.2. Execute the fuzzy reference comparisons for the most important criterion/factor and the least important criterion/factor

By considering the linguistic terms presented in table S1, the fuzzy preference of the most important criterion / factor over all the selected criteria/factors are determined by the decision makers in a group meeting. Similarly, the fuzzy preference comparison of all criteria/factors over the least important criterion/factor are determined. The obtained fuzzy important-to-others vector and others-to-least important vectors can be obtained as Eqs. (S1) and (S2) respectively.

$$\tilde{A}_B = (\tilde{a}_{B1}, \tilde{a}_{B2}, \dots, \tilde{a}_{Bn}) \quad \text{(S1)}$$

$$\tilde{A}_W = (\tilde{a}_{1W}, \tilde{a}_{2W}, \dots, \tilde{a}_{nW})^T \quad \text{(S2)}$$

where $\tilde{a}_{Bj}$ denotes the preference of the most important criterion/factor over criterion/factor j and $\tilde{a}_{jW}$ indicates the preference of criterion/factor j over the least important criterion/ factor. In addition, it is known that $\tilde{a}_{BB} = (1,1,1)$ and $\tilde{a}_{WW} = (1,1,1)$.

Table S1. Linguistic Terms and Corresponding Membership Functions

Linguistic Term	Membership Function
Equally Important	(1,1,1)
Weakly important	(2/3,1,3/2)
Fairly important	(3/2,2,5/2)

Linguistic Term	Membership Function
Very important	(5/2,3,7/2)
Absolutely important	(7/2,4,9/2)

S1.1.3. Calculate the optimal weighs of criteria/factors

Let $\widetilde{w}_j = ({w_j}^l, {w_j}^m, {w_j}^u)$, then the graded mean integration representation of a triangular fuzzy number (TFN), $GMRI(\widetilde{w}_j)$ can be calculated by the following equation.

$$GMRI(\widetilde{w}_j) = \frac{{w_j}^l + 4{w_j}^m + {w_j}^u}{6} \tag{S3}$$

Finally, the optimal fuzzy weights of criteria/factors can be determined by solving the following nonlinear optimization problem:

$$\begin{aligned} &\min \tilde{k} \\ &|\frac{\widetilde{w}_B}{\widetilde{w}_j} - \tilde{a}_{Bj}| \leq \tilde{k} \\ &|\frac{\widetilde{w}_j}{\widetilde{w}_w} - \tilde{a}_{jw}| \leq \tilde{k} \\ &\sum_{j=1}^{n} GMRI(\widetilde{w}_j) = 1 \\ &{w_j}^l \leq {w_j}^m \leq {w_j}^u \\ &{w_j}^l \geq 0, \text{ foa all } j \end{aligned} \tag{S4}$$

where $\tilde{k}$ shows the consistency of the comparisons. It should be noted that the crisp values of the weights can be obtained by Eq. (S3).

S1.2. Ranking the products using fuzzy TOPSIS

S1.2.1. Construct a comparison matrix

Based on the linguistic terms presented in table S2, a comparison matrix of products with different criteria is constructed which is called a decision matrix. In this matrix, each value is shown as a TFN ($\tilde{s}_{ij} = (a_{ij}, b_{ij}, c_{ij})$).

Table S2. Linguistic Terms and Corresponding Membership Functions in TOPSIS

Linguistic Term	Membership Function
None	(0,0,1)
Low	(1,1,3)
Moderate	(1,3,5)
Considerable	(3,5,7)
High	(5,7,9)
Extreme	(7,9,9)

S1.2.2. Create the weighted normalized decision matrix

The normalized decision matrix can be obtained as follows:

$$\tilde{s}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right) \quad ; \quad c_j^* = max_i\, c_{ij} \text{ ; } \textit{Positive ideal solution} \tag{S5}$$

$$\tilde{s}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) \quad ; \quad a_j^- = min_i\, a_{ij} \text{ ; } \textit{Negative ideal solution} \tag{S6}$$

Considering the fuzzy weights derived from the fuzzy BWM, the weighted normalized matrix can be calculated as follows [1]:

$$\tilde{v}_{ij} = \tilde{s}_{ij}.\widetilde{w}_{ij} \tag{S7}$$

S1.2.3. Determine the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS)

FPIS and FNIS can be calculated according to Eqs. (S8) and (S9).

$$A^* = \{\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+\} = \{(\max v_{ij} \,|j \in J), (\min v_{ij} \,|j \in J')\} \tag{S8}$$

$$A^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-\} = \{(\min v_{ij} \,|j \in J), (\max v_{ij} \,|j \in J')\} \tag{S9}$$

where J and J' represent the positive and negative ideal solutions, respectively.

The distance of each alternative from FPIS and FNIS can be calculated as below:

$$Dis_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+) \tag{S10}$$

$$Dis_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-) \tag{S11}$$

where d is the distance function between two fuzzy numbers. Let $\tilde{P}_1 = (a_1, b_1, c_1)$ and $\tilde{P}_2 = (a_2, b_2, c_2)$ be two TFNs. The distance between these TFNs can be obtained by the following equation:

$$d(\tilde{P}_1, \tilde{P}_2) = \sqrt{\frac{1}{3}[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2]} \quad \text{(S12)}$$

S1.2.4.Calculate the closeness coefficient (CC_i) of each product as follows:

$$CC_i = \frac{Dis_i^-}{Dis_i^+ + Dis_i^-} \quad \text{(S13)}$$

The products are finally ranked based on CC_i values in their descending order.

S2. TFNs' basic arithmetic operations

Given two TFNs $\tilde{x}_1 = ({x_1}^l, {x_1}^m, {x_1}^u)$and $\tilde{x}_2 = ({x_2}^l, {x_2}^m, {x_2}^u)$, the fuzzy sum, the fuzzy subtraction, the multiplication between two fuzzy numbers, and the fuzzy inverse can be respectively determined as follows:

$$\tilde{x}_1 \oplus \tilde{x}_2 = ({x_1}^l + {x_2}^l, {x_1}^m + {x_2}^m, {x_1}^u + {x_2}^u) \quad \text{(S14)}$$

$$\tilde{x}_1 \ominus \tilde{x}_2 = ({x_1}^l - {x_2}^l, {x_1}^m - {x_2}^m, {x_1}^u - {x_2}^u) \quad \text{(S15)}$$

$$\tilde{x}_1 \otimes \tilde{x}_2 = ({x_1}^l {x_2}^l, {x_1}^m {x_2}^m, {x_1}^u {x_2}^u) \quad \text{(S16)}$$

$$\tilde{x}_1^{-1} = (1/{x_1}^u, 1/{x_1}^m, 1/{x_1}^l) \quad \text{(S17)}$$

Moreover, the crisp value of a TFN can be obtained by the center of area approach as follows:

$$x_1 = \frac{{x_1}^l + {x_1}^m + {x_1}^u}{3} \quad \text{(S18)}$$

S3. Key Products Determination in GP

S3.1. Determining the weighs of criteria by Fuzzy BWM

At this stage, the weights of criteria including loss of revenue (Ca), degree of damage on company's reputation (Cb), importance of product for the country according to the country policies

(Cc), defection of customers (Cd), and loss of interested parties supports (Ce) are determined by using fuzzy BWM. Four top managers of the company (i.e., GP manager, sales manager, marketing manager, and financial manager) were asked to fill out the BWM questionnaire indicating the most and the least important criteria and comparing other criteria with them. The results are shown in tables S3 and S4. By solving Eq (S4) for the derived preference matrices, the fuzzy weights of the criteria are obtained as table S5.

Table S3. The TFNs for fuzzy preferences of the most important criterion over all the criteria

Criteria		Ca	Cb	Cc	Cd	Ce
The Most Important criteria	Ca	(1,1,1)	(3/2,2,5/2)	(2/3,1,3/2)	(2/3,1,3/2)	(7/2,4,9/2)

Table S4. The TFNs for fuzzy preferences of criteria over the least important criterion

Probability Factors	The Least Important Probability Factor
	Ce
Ca	(7/2,4,9/2)
Cb	(3/2,2,5/2)
Cc	(5/2,3,7/2)
Cd	(3/2,2,5/2)
Ce	(1,1,1)

Table S2. Fuzzy weighs of criteria

Fuzzy Weighs of Probability Factors
$\widetilde{w}_{ca} = (0.264,0.29,0.326)$
$\widetilde{w}_{cb} = (0.13,0.165,0.211)$
$\widetilde{w}_{cc} = (0.214,0.260,0.301)$
$\widetilde{w}_{cd} = (0.168,0.2,0.236)$
$\widetilde{w}_{ce} = (0.08,0.082,0.086)$

S3.2. Rank the products by fuzzy TOPSIS

Table S6 is the decision matrix whose values represent the arithmetic mean of the top managers. Tables S7, S8, and S9 show the weighted normalized decision matrix, distance from positive and negative ideal solutions, and closeness coefficient of each product, respectively.

Table S6. Decision Matrix

	Ca	**Cb**	**Cc**	**Cd**	**Ce**
condensate	(6.5,8.5,9.)	(4,6,8)	(4.5,6.5,8.5)	(3,5,7)	(3,5,6.5)
elemental sulfur	(3,5,7)	(2,4,6)	(1,3,5)	(1.5,3.5,5.5)	(2,4,6)
propane	(5,7,9)	(1.5,3.5,5.5)	(2.5,4.5,6.5)	(2.5,4,6)	(3.5,5.5,7.5)
butane	(4.5,6.5,8.5)	(1,3,5)	(2.5,4.5,6.5)	(2.5,4,6)	(3.5,5.5,7.5)

Table S7. The weighted normalized decision matrix

	Ca	**Cb**	**Cc**	**Cd**	**Ce**
condensate	(0.088,0.102,0.15)	(0.016,0.028,0.053)	(0.113,0.199,0.301)	(0.036,0.060,0.118)	(0.025,0.033,0.057)
elemental sulfur	(0.113,0.174,0.326)	(0.022,0.041,0.106)	(0.025,0.092,0.177)	(0.046,0.086,0.236)	(0.027,0.041,0.086)
propane	(0.088,0.124,0.196)	(0.024,0.047,0.141)	(0.063,0.138,0.230)	(0.042,0.075,0.142)	(0.021,0.030,0.049)
butane	(0.093,0.134,0.217)	(0.026,0.055,0.211)	(0.063,0.138,0.230)	(0.042,0.075,0.142)	(0.021,0.030,0.049)

Table S8. Distance from positive and negative ideal solutions

	Distance from positive ideal	**Distance from negative ideal**
condensate	0.005	0.398
elemental sulfur	0.342	0.061
propane	0.159	0.246
butane	0.214	0.191

Table S9. Closeness coefficient

	Ci	rank
condensate	0.987	1
elemental sulfur	0.152	4
propane	0.607	2
butane	0.472	3

Based on the closeness coefficients and top managers’ preferences, the ethylene is considered as the key product of GP.

S4. Fuzzy DEMATEL Approach

Fuzzy DEMATEL steps are as follows [2]:

S4.1 Calculate the Average Matrix

Each member of experts' committee is asked to indicate in what extent he/she believes a risk j affects risk j' in terms of an uncertain linguistic term expressed as a triangular fuzzy number (TFN) ($\tilde{E}_{jj'k} = (e^1_{jj'k}, e^2_{jj'k}, e^3_{jj'k})$) according to Table S1. The average matrix $\widetilde{N}$ (whose (j, j') element is expressed as $\tilde{n}_{jj'k} = (n^1_{jj'k}, n^2_{jj'k}, n^3_{jj'k})$), can be calculated based on the different experts' opinions about relations between risks by the following equation:

$$n^t{}_{Jj'} = \frac{\sum_k e^t_{jj'k}}{g} \qquad \forall j, j' \qquad \text{(S.1)}$$

where k is index of asset management committee's experts and g denotes the number of experts, and $n^t{}_{Jj'}$ denotes the t[th] element of triangular fuzzy number $\tilde{n}_{jj'k}$.

1- *Calculate the Normalized Direct- Relation Matrix*

The normalized direct relation matrix $\widetilde{D}$ whose (j, j') element is expressed as $\tilde{d}_{jj'} = \left(d^1_{jj'k}, d^2_{jj'k}, d^3_{jj'k}\right)$, is calculated by the following equation:

$$d^t{}_{Jj'} = \frac{n^t{}_{Jj'}}{\max\limits_{j} \sum_{j'} n^t{}_{Jj'}} \qquad \forall t \qquad \text{(S.2)}$$

where $d^t{}_{Jj'}$ denotes the t[th] element of triangular fuzzy number $\tilde{d}_{jj'}$

S4.2. Acquire the Total Relation Matrix

1. The total relation matrix $\widetilde{H}$ whose (j, j') element is expressed as $\tilde{h}_{jj'} = \left(h^1_{jj'}, h^2_{jj'}, h^3_{jj'}\right)$, is calculated as follows:

$$\widetilde{H} = \lim_{n\to\infty} (\widetilde{D} + \widetilde{D}^2 + \cdots + \widetilde{D}^n) \qquad \text{(S.3)}$$

2. Defuzzify the Total Relation Matrix and Define Threshold Value to Build Initial Interdependency Graph

The total relation matrix is defuzzified as follows:

$$h_{jj'} = \frac{h^1_{jj'} + h^2_{jj'} + h^3_{jj'}}{3} \qquad \forall j, j' \qquad \text{(S.4)}$$

Table S30. Linguistic Terms and Corresponding Membership Functions

Linguistic Term	Membership Function
None	(0,0,1)
Low	(1,1,3)
Moderate	(1,3,5)
Considerable	(3,5,7)
High	(5,7,9)
Extreme	(7,9,9)

References

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2. Choua, Y.-C.; Sun, C.-C.; Yen, H.-Y. Evaluating the criteria for human resource for science and technology (HRST) based on an integrated fuzzy AHP and fuzzy DEMATEL approach. Appl. Soft Comput. 2012, 12, 64–71..