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Architectural Design Optimisation in Reticulated Free-Form Canopies

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Abstract: The search for the structural form of reticulated roofs is significant in interdisciplinary Architectural Design optimisation. Combining parametric design with structural logic influences the visual perception of the shape by choosing the most suitable technical solutions. Therefore, the divisions of reticulated structures should be determined to pursue structural, material and fabrication advancement. Structural divisions of free-formed canopies should simultaneously be solved in architectural and structural design at an early stage. Choosing a proper design becomes a complicated process, requiring the ability to select a type of production and rationalise technical solutions mainly due to the computer-aided design supported by algorithmic tools. Based on searching for optimal geometrical divisions, the case study investigates the differences between planar quadrilateral and triangular mesh panelisation. The study concludes the assets and flaws of both geometry shaping methods of reticulated structures based on minimal weight and fabrication aspects. The study concludes that implementing the manufacturing method of the chosen type of gridshells divisions into the architectural design optimisation enhances the resulting free-form structures at the early design stage.

Keywords: canopies; creative designing; generative designing; gridstructures; structural optimisation; topology optimisation

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1. Introduction

In the 21st century, a change in designing activity in the Architectural, Engineering, and Construction sectors is noticeable, mainly due to digital tools. Progressive digitisation "permeates" all processes related to creating architecture, and design-based digital tools are increasingly used in Architectural Design Optimisation (ADO). Sustainable Development adopted by the United Nations Assembly [1,2], influences construction development mainly in natural resource usage without destroying natural ecosystems. It results in optimisation methods and tools in designing, constructing, and exploiting buildings' life. Particularly notable is the tendency to act more comprehensively in the execution of the manufacturing process, which amounts, among other things, to greater control of fabrication or shaping the form with unprecedented precision in the manner of a medieval craftsman [3]. The rapidly progressing digitisation in ADO is now "filtering" the design techniques and methods associated with structure optimisation. [4]. Parametric design and guided simulations make it possible to optimise materials usage and production costs and reduce construction time and the carbon footprint [5,6]. It is visible in supervising the entire production process, from BIM environment designing, by controlling the construction process and optimising the material and shape with unprecedented precision. In design practice; however, architects and structural designers still encounter significant difficulties in using algorithmic designing: "insufficient knowledge of optimisation based on multivariant simulations, use of inefficient optimisation methods based on genetic algorithms, lack of modern, easy-to-use optimisation tools and lack of integration of optimisation into architectural design." [7].

Computer-based digital modelling methods in curvilinear structure designing improve the aesthetic quality and the efficient use of materials [8]. Understanding parametric form designing becomes a significant design aspect in contemporary construction practice. It provides new possibilities in a creative search for “eco-efficient” architectural forms and technical solutions. Structural optimisation achievements have remarkable outputs thanks to the availability of advanced, generative techniques and engineering methods [9,10]. The design based on forms inspired by nature [9] provides numerous innovative insights with interdisciplinary scientific analysis [11]. The biomimetic trend in architecture can be used on different designing levels, from imitating natural patterns to understanding natural ecosystem behaviour and implementing them into the building scheme. The most effective way of using building materials is to apply their mechanical properties to calculation algorithms. It is essential for the material optimisation and the fabrication process in the concept phase. In Modernism, Architect F. L. Wright noticed the need to implement material properties into the project’s concept phase: “Every new material means a new form, a new application if used according to its nature.” [12].

The development of curvilinear geometries has intensified the search for various technologies. A complicated process requires selecting main factors such as production and technical solutions based on computer-aided design, supported by algorithmic tools. Fundamental issues in the problems undertaken by designers are:

- The use of digitalized mathematical algorithms and tools in the search for optimal shapes, with the help of biomimetics, in search of optimal forms and behavioural systems;
- Finding the ways of fabrication components concerning shape formation and preparing members for efficient assembly;
- Minimising the energy required for the manufacture and assembly of components,
- The use of environmentally neutral materials or minimisation of environmental degradation (including carbon footprint).

Different approaches to designing freeform architectural installations are discussed in the field [13,14]. The unique and large-scale structures demand more in-depth investigations, which now take place in the interdisciplinary application of new design methods. The former physical process evaluates towards a numerical algorithm design based on the same equilibrium state of free-form structures [15–17]. Based on the new complex approaches to the freeform structure design as [18,19] present, this method can be divided into geometric [20], biomimetic, and form-finding approaches.

Analysing the differences between triangular and quadrilateral gridshell divisions becomes an essential technological issue in architectural design. Triangular mesh divisions were the most used solutions for free-form canopies. It lies under the facility of creating flat glass panels. However, quadrilateral panels surpass triangular panels in many aspects of the fabrication process. However, designing flat quadrilateral glassing surfaces still appears problematic. Choosing a structure division method becomes an architectural concern and influences the material and fabrication [21]. The case study was conducted on an open pavilion canopy design with curvilinear surfaces due to generative optimisation. The study is conducted in form-finding software called Grasshopper/Rhinoceros and in structural analysing Robot Structural Analysis program due to the optimal weight of the structure.

The comparative analyses presented in this paper describe the differences in structures based on triangular or quadrilateral gridshells patterns. Under certain selected assumptions, pavilion roofs of free-form geometries were analysed, aiming at weight minimisation. Reducing weight is a vital part of the optimisation process, but as stated in various research, not the most important [22,23]. The need to balance the structure’s weight must be compromised with the production and assembling processes through the components (beams, joints, coverage material), manufacturing technologies and materials. The paper will compare the total weight of the structure with the total length of structural elements and the number of joints.

1.1. Background

Striving for rapid element fabrication should not exclude unique element design in contemporary manufacturing. Mathematical algorithms based on proportionality requirements analyse various variant solutions to find the optimal curvilinear geometries. It is thought-provoking, particularly for prefabrication, which is increasingly characterised by postfordism [24].

Learning the computational capacity of computers has become the basis for formal optimisation of designing experiments [25–27]. Current research on curvilinear structures results from inspiration from nature-based systems [28,29]. The search for biomimetic algorithms that describe living organisms' growth principles are a matter of survival. It is visible in the architectural realisations of the XXI century worldwide.

Designing lightweight canopy structures with transparent coverings raises architectural aesthetic, structural complexity and technological aspects of fabrication. Form-finding algorithms improve shape-oriented design, but technical optimisation and gridshell pattern adjustments remain a vast question to address. This paper raises two main directions for designing the free-formed canopy panels: triangular panels (Figure 1) and quadrilateral panels (Figure 2). The significant differences between those two methods of surface discretisation in architectural and structural concerns were raised.

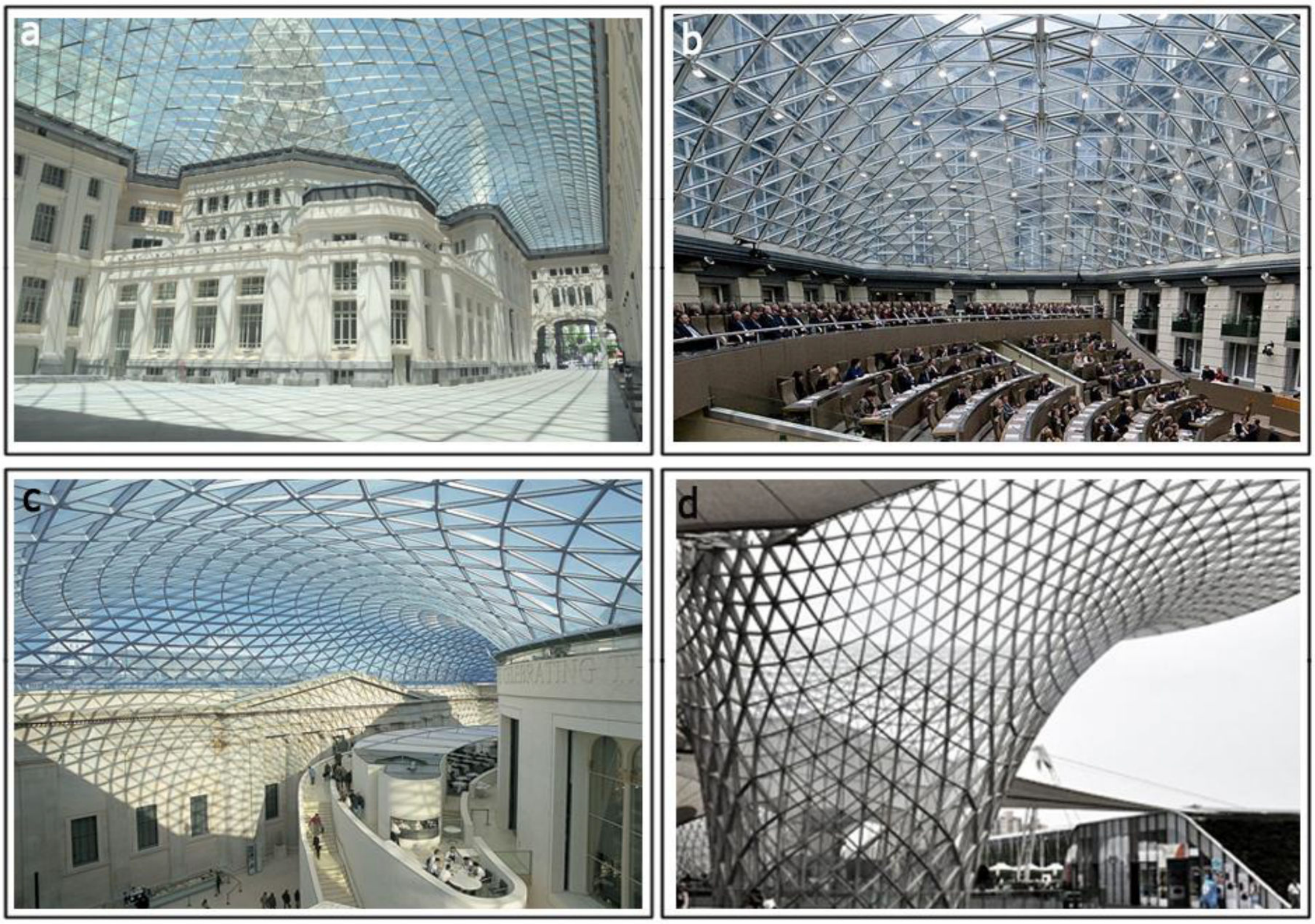


Figure 1. Curvilinear gridshell canopy with mesh triangular panelisation: (**a**) Madrid City Hall; (**b**) Meeting Hall Flemish Council in Belgium; (**c**) British Museum of Art; (**d**) Shanghai EXPO pavilion.

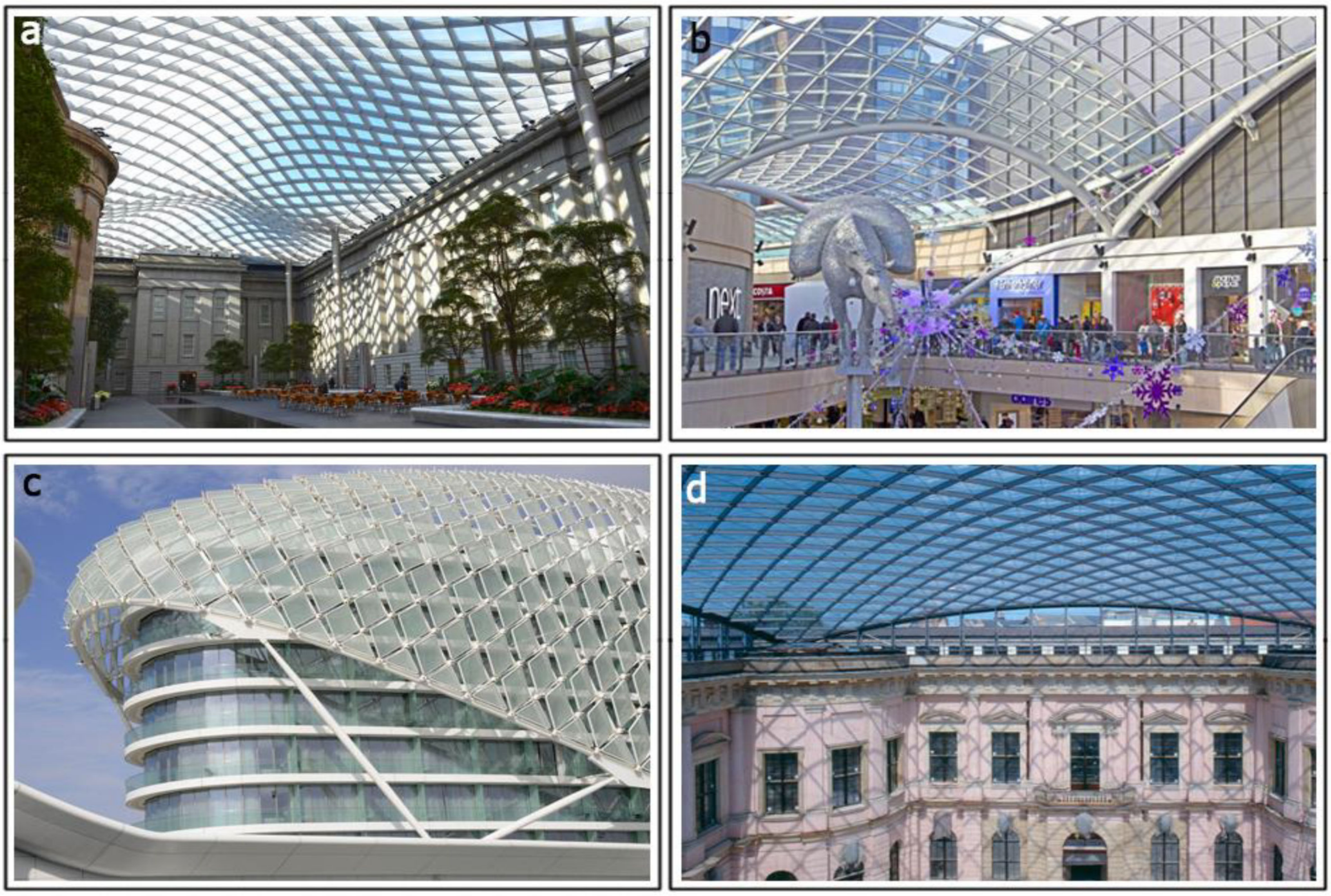


Figure 2. Curvilinear grid-shell canopy with rectangular mesh panelisation: (**a**) Kogod Courtyard w Smithsonian; (**b**) Trinity w Leed mall centre; (**c**) Yas Island Hotel in Abu Dhabi; (**d**) German Historical Museum in Berlin.

1.2. Terminology

Curvilinear canopy design arises from interdisciplinary boundary conditions at the early design stage. While determining a free-form geometry of curvilinear gridshells, designers should implement interdisciplinary knowledge from structural designing, material engineering, the available technologies and assembling possibilities with labour knowledge.

An experimental search for lightweight curvilinear canopies requires an interdisciplinary approach, possibly already at the preliminary architectural design stage. The search for curvilinear structures with computer-aided architectural design tools makes it possible to achieve curvilinear forms whose primary goal is to achieve a state of equilibrium of forces in the structure using model-shaping algorithms. On the other hand, the search for optical divisions (gridshell pattern), as a technological issue considered with the fabrication specification, makes it possible to shape efficient roofs not only in terms of the use of material but also from the viewpoint of technological aspects of production and subsequent operation of objects. The fundamental question of Architectural Sustainability is the cost, use of renewable materials, and the minimisation of carbon footprint and energy required for the construction joints.

In interdisciplinary designing, combining architectural design with technological aspects of designing, i.e., surface discretization to manufacturable panels, providing support systems, load-bearing structure and minimizing cost, is based on geometric modelling and processing; thus, it is widely called Architectural Geometry [30]. In architectural design, it is mainly connected to the form-finding approach, the form evolves from "figures of equilibrium" or "forms of equilibrium" [18]. Form-finding approach is then not based on the geometry or on mimicking the actual bio-forms, but what is the most important follows the equilibrium of forces in given boundary conditions [31]. It results from numerical methods of surface discretization with dynamic relaxation [32,33], alongside the force density method [34] and particle spring system [35,36]. This theoretical case study designed a reticulated structure with simplified computational models of curvilinear forms.

As an integral part of Architectural Geometry, the technology and materials of the free-form structures are a vital point of the optimisation processes, they have a significant impact on the design and assembling processes. Douthe et al. [37], among others, indicate that finding the new materials for form-finding of free-forms, such as polymers [19,38] of non-standard composites [37] helps improve the optimisation processes.

Developments in manufacturing technology led to the steady growth of curvilinear geometries, but are not altering the technological systems used in the construction sector [39]. According to the new approaches to designing gridshell divisions [40–43], it is essential to search for the shape of the curvilinear geometry and search for the structural divisions that allow an equilibrium state to be reached. For this reason, the study attempts to investigate different types of structural meshes in free curvilinear canopy structures and to recognise other boundary conditions of their usage.

The interdisciplinary design raises the integration between aesthetic effects and sustainable solutions. In the presented theoretical case study, the aesthetic factors were considered boundary factors, such as the maximum height and the gridshell division patterns (Delaunay triangulation, regular quadrilateral and triangular division). The algorithm created the curvature (catenary model). Structural solutions and optimisation processes were bound to the initial architectural conditions. Hence, the interdisciplinary optimisation process discusses only a chosen boundary factor of the theoretical case study.

2. State of the Art

The literature analysis gave rise to the need to research reticulated structures due to the material optimisation and the ease with which components can be prepared for fabrication. The principle of form-finding can be illustrated as a process of shaping structures with an assumed load, as physical models made by A. Gaudi or F. Otto in the previous century. It is: "finding an (optimal) shape of a (form-active structure) that is in (or approximates) a state of static equilibrium" [44]. Designing curvilinear canopy structures must compromise aesthetics and technology [22,45,46]. Planar glass panels filling the structural grid to achieve material savings have become a technological requirement [47]. However, the aesthetic needs to obtain a smooth surface division (possibly without sharp edges) [48]. The application of structural optimisation of curvilinear structures of planar triangular and quadrilateral meshes both bring limitations on the technological level. While triangular panels are more accessible in designing flat panels [23], quadrilateral panels are more accessible in production and create less waste.

While analysing the research literature, some material minimisation and affordability factors were found [49]. While triangular and quadrilateral panels are the most common to be designed, they provide practical difficulties in the three most important phases of the design method, execution technology and architectural quality. Examples of curvilinear gridshell structures with triangular and quadrilateral panels are presented in Table 1. The proposed conclusions were made based on the analysis of research from recent years [8,45,50,51] (Table 1). The presented methods of optimising curvilinear gridshells compromise the differences in various patterning systems and highlight that the interdisciplinary optimisation in design requires numerous boundary conditions at the early stages of the projects.

Table 1. A comparison of characteristics of free-form canopies in triangular and quadrilateral panels.

	Triangular Panels	Rectangular Panels
Design method	+ accessible subdivision of curvilinear geometries into any flat triangular panels	− difficulty in developing flat-panel geometries on a quadrangle: when creating quadrilateral double-sided panels, further technical problems arise, such as thermal deformability; in the case of developing flat panels, designers often had to compensate for difficulties in achieving the correct curvature in the outer part of the cover (e.g., Courtyard Coord at Smithsonian)
the technology of execution	− difficulty in making joints in which six bars merge at different angles − more waste glass than in quadrilateral panels	+ ease of fabrication of joints where only 4 rods merge + minimising the amount of waste of covering material of rectangular panels (compared to triangular ones)
architectural quality	− a larger ratio of the surface area of the rod structural elements in comparison to the quadrilateral grids, which is equivalent to less translucency	− higher translucency and interior illumination thanks to a smaller ratio of structural elements is a covering material.

Legend: advantages are marked as "+", disadvantages are marked as "−".

3. Theoretical Study Methodology on Curvilinear Gridshell Patterns Optimisation

The theoretical case study was based on selected canopy structure models and was conducted on the preliminary case study made by the authors [52]. The following research extends and clarifies the theme of multivariate optimization, in which the architect should participate. They indicate that the search related to the algorithmic determination of the forming curve in parametric design programs is essential, and the search for the appropriate way to divide and scale the divisions of the structural grid. Similar case studies are examined, but only on geometrical transformations of the structure's shape [20,53].

The research compares different structural meshes of the Delaunay triangulation, regular triangle, and quadrilateral divisions. The Delaunay diagram was used because of the possibility of using random node and bar length selections without designer interference. The resultant geometry with the minimal total weight of the structure was the basis for subsequent experiments. The second phase of the research was based on finding key factors of Delaunay divisions such as the total length of all bars, the exact size of single bars, and the number of joints in regular triangular and quadrilateral structural meshes.

Grasshopper/Rhinoceros was used to generate the form by imitating physical forces. Adjusting the structure's curvature was based on generative shape optimisation in the Kangaroo 2 plug-in, which provides the physical forces to create the form of the structure. Structural optimisation was performed in Robot Structural Analysis. The initial differences in irregularity shaping in the variants were not discussed in the research. However, the random displacement of the initial nodes might impact the topological differences in meshes and affect the optimisation process. This aspect of architectural design optimisation was omitted in the study.

Adopting generative tools helped to distribute loads in the structures more efficiently. The differences between the catenary and parabola models are almost non-noticeable, but improve material optimisation (Figure 3a), similar to the chain models designed by A. Gaudi (Figure 3b). This numerical method aims to find a geometry in which all forces are in equilibrium, allowing the structure's minimum area (length) to be located. Dynamic relaxation significantly minimizes materials in the systems visible in earlier studies of curvilinear designs [54]. The generative design tool used in the research was chosen based on the availability and accessible of using this tool by architects to generate the catenary model based on selected boundary conditions.

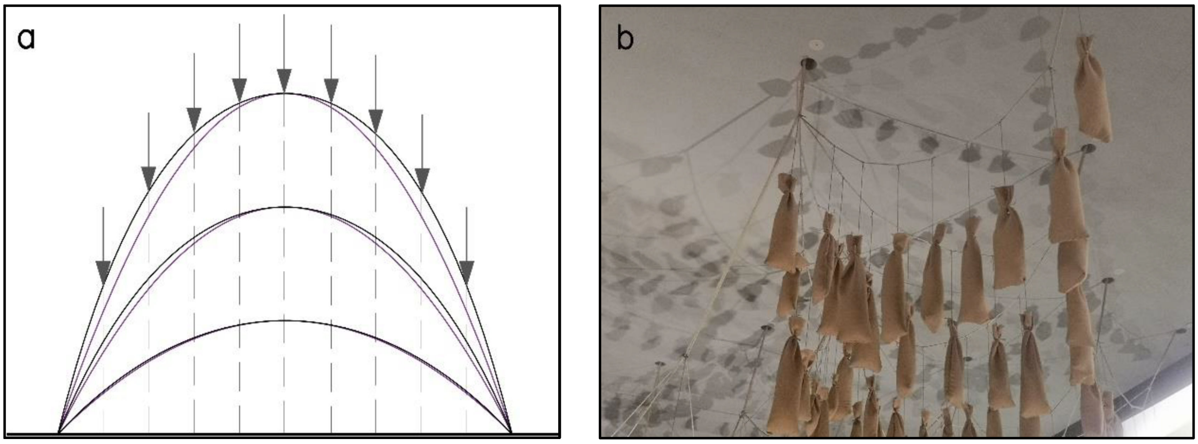


Figure 3. Catenary curves: (**a**) comparison of the curvature of a parabola (purple curves) and catenary (black curves); (**b**) catenary curves in the catenary model in Sagrada Familia, Barcelona (authors compilation).

According to the flowchart (see Figure 4), all analysed structures' design boundary conditions were the same. All the structures cover the circular plan; each structure's total area is 900 m^2. All the systems are based on three supports with the ratio P/W = 2.0 (distance between supports (P) to the cantilever length (W) (see Figure 5a). The support location was also based on a prior theoretical case study made by the authors.

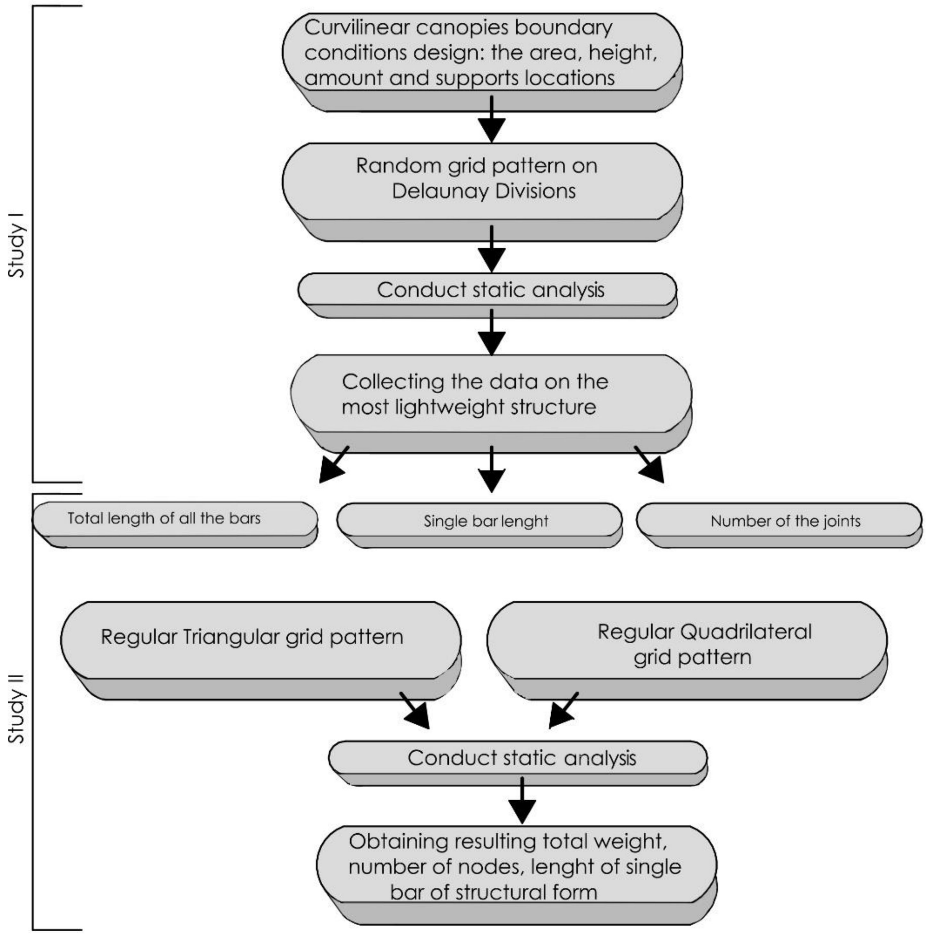


Figure 4. The Methodology flowchart of conducted theoretical studies (authors compilation).

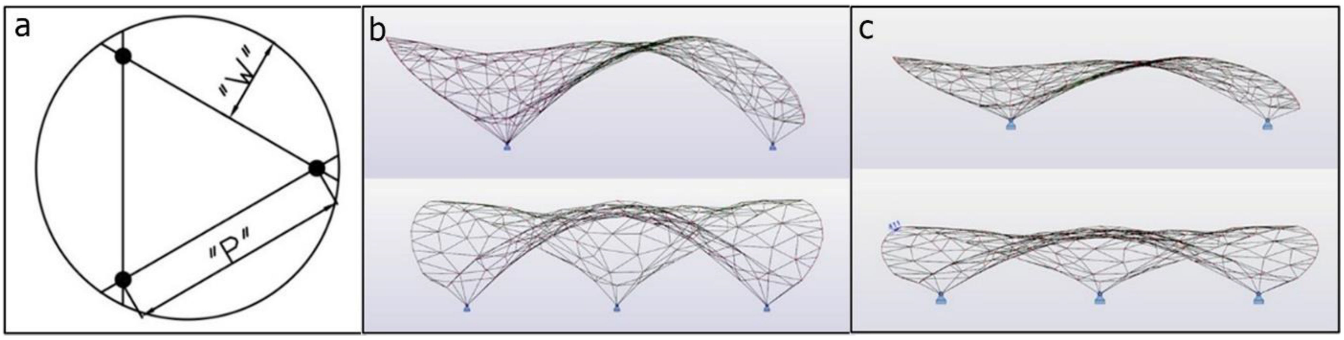


Figure 5. (**a**) Analysed canopy plan, with supports location (dots) and representation of the ratio between cantilever and span between supports; (**b**) side views of the canopy with the highest curvature of the geometry (The curvature height to the support span ratio: 1/3); (**c**) side views of the canopy with the slightest curvature of the geometry (The curvature height to the support span ratio: 1/5) (authors compilation).

Curvilinear variants (on three supports) were shaped using a generative algorithm, catenary model, and dynamic relaxation (with the Kangaroo2 plug-in). Due to the algorithmic design, the structure's curvilinear shape, the heights vary, and the height to supports span ratio used in the study was 1/3, 1/4, 1/5.

Two-stage analyses of various grid canopy geometries were carried out (according to the Figure 4):

Study 1—analysis of the structural mesh based on Delaunay triangulation (the analysed geometries were previously developed within the author's research [52]).
Study 2—a comparative study on triangular and quadrilateral regular mesh division, based on the Study 1 guidelines.

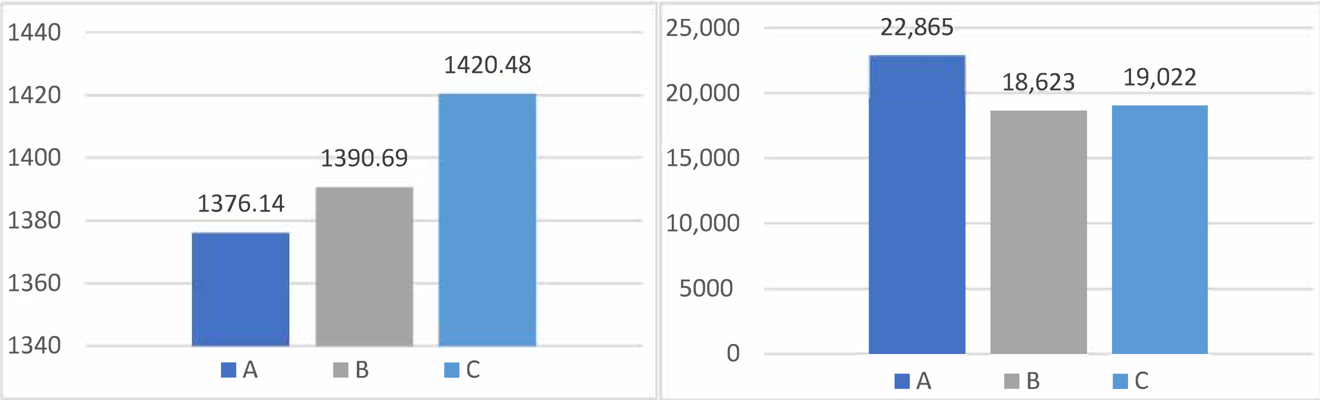
Model tests were carried out due to the dead load (according to the PN-EN 1991-1:2004 standard), snow and wind loads (according to the PN-EN 1991-1-3/4:2005/2008 standard) the load of the covering material. Load combinations, according to PN-EN 1990:2004. The analysed structures were designed from S355 steel round hollowed profiles (TRON). Due to polycarbonate as a covering material and variable geometry, acceptable global deformations on cantilevers were set as L/125, which was developed in detail in individual variants. The optimisation criteria were the minimum total weight [kg] in the conducted analyses.

4. Results

4.1. Study 1–Reticulated Structure Optimisation Based on Delaunay Triangular Divisions

Study 1 aimed to identify the most beneficial divisions in the random Delaunay triangulation pattern of a gridshell. Geometries were generated based on the previous authors' research experiment [54,55].

The case study of this experiment phase consisted of 36 covering structures, but only the three best ones were represented in the paper (Scheme 1). The selection of best geometries was chosen based on the parameter of the minimum total weight of the structure. The variants were selected based on the max lengths of individual bars throughout the structure. Geometries with max bar lengths up to 3.0, 3.5, and 4.0 m were found too dense and too heavy, compared to geometries with max bar lengths up to 4.5 m and 5.0 m. Structures with bars longer than 5.0 m were rejected due to excessive deflections and stresses in the structure.



Scheme 1. Left, the total length of the variants A–C, right, the total weight of the variants A–C (authors compilation).

The chosen structures with form-finding algorithm were based on catenary models with various maximum lengths of a single bar (4.5 m and 5.0 m) and structure height to the support span ratio:

- 1/5, the full height of the structure is equal to 3.97 m;
- 1/4, the full height of the structure is equal to 4.98 m;
- 1/3, the full height of the structure is equal to 6.58 m.

The support location was also based on the authors' prior theoretical case study, constituting the optimum span between supports and the longest cantilever ratio as 2.0.

A comparison of freeform structure systems showed that the average total weight decreased as the structure's full height increased. The average total weight of the canopy was 20,170 kg. The most optimal variant (with the lowest total weight) was *Variant B*. The total weight of this arrangement is 18,623 kg (with a covering area of 963.41 m^2), and the average total length of all the bars is 1390.69 kg.

As mentioned in the introduction, reducing the total weight of the structure is not the most important aspect of optimisation processes, among others, it should also reduce the production components (i.e., total length of the structural elements) and simplify assembling processes (i.e., the number of joints). Hence to compare the Delaunay mesh of the gridshell from Study 1 with the regular triangular and quadrilateral meshes in Study 2—the following parameters were observed and became the guideline for the following study:

- **Finding 1:** The average total length of the bars was 1390.69 m;
- **Finding 2:** the length of each bar did not exceed 5.0 m;
- **Finding 3:** the total number of nodes was 203.

4.2. Study 2–Reticulated Structure Optimisation Based on Regular Triangular and Quadrilateral Divisions

Study 2 aimed to identify the most beneficial divisions based on quadrilateral and regular triangular meshes. Geometries were generated based on the conclusions of Study I. In this study, the ratio of the structure's height to the supports span remained the same as in Study 1: 1/3; 1/4; 1/5.

The theoretical case study of the second phase covers the regular triangular and quadrilateral meshes consists of 18 covering variants, divided into three groups according to: the total length of all bars (variants 1.1.–1.6.), a single bar length of about 5.0 m (variants 2.1–2.6.), and 203 nodes (variants 3.1.–3.6.), in each group three different heights of the structure were used (see Table 2).

Table 2. The variants of Study 2.

Height to Support Span Ratio	Variants					
	The Total Length of the Bars		Fixed Length of All the Bars to 5.0 m		The Total Number of Joints Equals 203	
	Triangular	Quadrilateral	Triangular	Quadrilateral	Triangular	Quadrilateral
1/5	1.1.	1.4.	2.1.	2.4.	3.1.	3.4.
1/4	1.2.	1.5.	2.2.	2.5.	3.2.	3.5.
1/3	1.3.	1.6.	2.3.	2.6.	3.3.	3.6.

The significant difference in structural smoothness and bar density is visible; the difference between the most lightweight geometries in each group is shown in Figures 6–8. The analysis results showed in the Scheme 2 present the total weight of regular triangular and quadrilateral meshes.

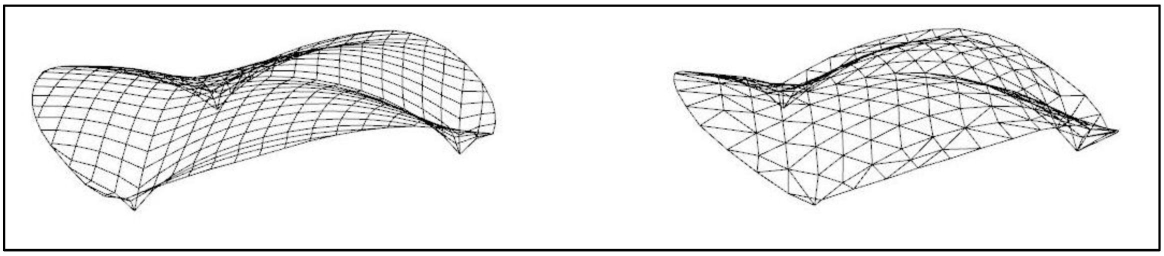


Figure 6. Geometries generated according to Finding 1–fixed total length in the grid-shell~1390 m (left–rectangular, right–triangular). Both meshes present a smooth surface but with dense mesh (authors compilation).

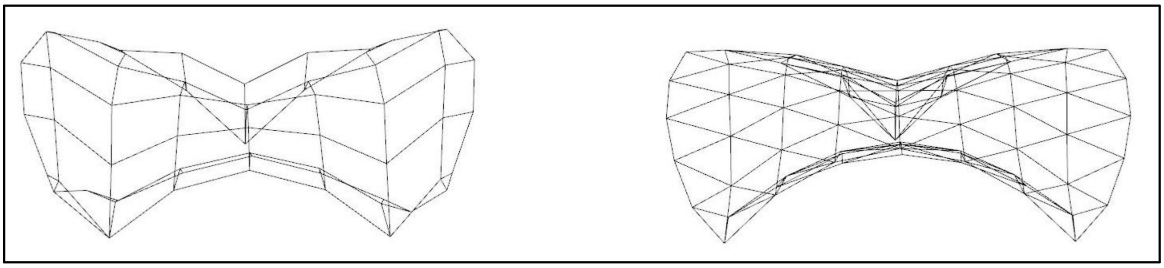


Figure 7. Geometries generated according to Finding 2–fixed length of each bar~5 m (left–rectangular, right–triangular). The quadrilateral mesh lacks surface smoothness (authors compilation).

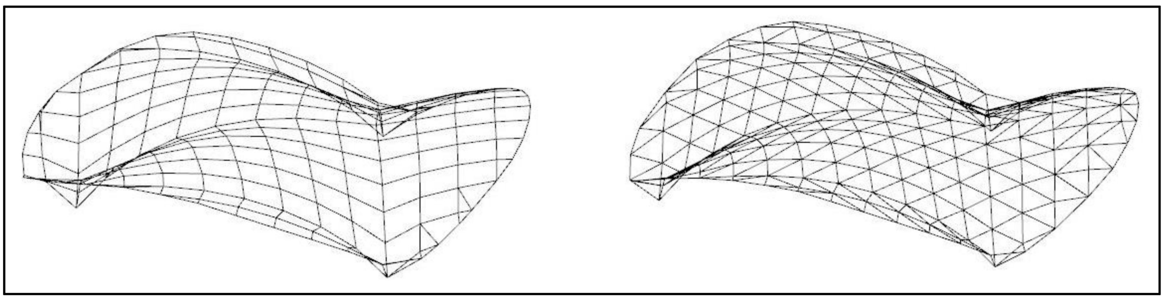
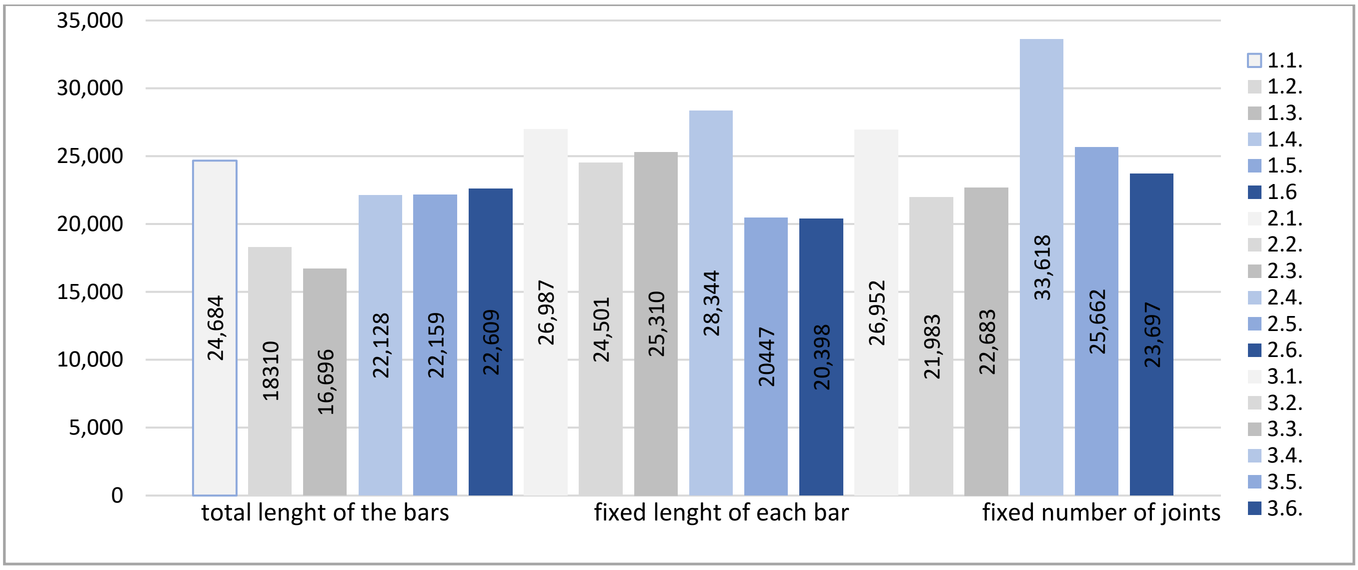


Figure 8. Geometries are generated according to Finding a 3-fixed number of joints (left–rectangular, right triangular). Both meshes present a smooth surface with less dense mesh (authors compilation).



Scheme 2. Results of Study 2–the total weight of the structures [kg]. Grey colour represents regular rectangular patterns; blue represents the regular triangular pattern.

5. Findings

The lowest total weight of the structure was achieved in quadrilateral panels *Variant 1.3* (with a full bar length of 1390 m) with a total weight of 16,696 kg. Variant 2.6, with a total weight of 20398 kg, was the most lightweight among the variants with triangular divisions. The difference between the best quadrilateral and triangular variants was almost 4 tons (~29%). However, triangular panelled structures are still more frequently chosen structures. It is due to technical difficulties from the design to the execution stage. Assembling the quadrilateral meshes increases the need to prepare the planar panels, due to the deflection and stresses produced and the cost reduction. The triangular panels are now easier to design (as the triangle is always planar), whereas planar quadrilateral meshes are still challenging for architects and structural engineers.

As highlighted in literature analyses, it was noticed that the construction of nodes connects only 4 bars in the quadrilateral mesh increases, whereas in the triangular mesh, the joint connects 6 bars. This results in the more complex joint systems in triangular meshes and reduces the amount of sunshine inside triangular meshes when the amount of joints is equal.

Triangular divisions also showed a significant reduction in node usage (while comparing the variants with the smallest total weight of the structure), which may ultimately affect the choice of this particular geometry rather than merely optimising the structure's weight in a quadrilateral one. Table 3 presents the main differences in observed structural parameters between the best geometries in each type of mesh division in canopies (according to the total weight, length, and number of joints).

Table 3. A comparison of chosen fabrication factors of Delaunay, regular triangular, and quadrilateral grid divisions (authors compilation).

Factor	Delaunay Triangulation	Regular Rectangular	Regular Triangular
Weight [kg]	18,623	**16,696**	20,398
Total length [m]	1390	1390	**858.57**
Length of individual rods [m]	1–5	2.3	5
Number of joints	203	452	**75**

By comparing the best structures, the lowest needed total length and number of joints present a *regular triangular* structure, indicating better access to the sun in the interior (due to the significantly smaller number of joints). However, a significant weight reduction is visible, favouring the Planar Quadrilateral (PQ) mesh. There is a vast difference in the number of nodes in both systems (where the nodes give the most shadows). The most lightweight quadrilateral Variant 1.3. has 452 nodes, while the regular triangular Variant 2.6. has only 75. The quantitative comparison of the weight of the whole structure is crucial. Nevertheless, many aspects, such as the number and type of node, become fundamental to designers' work.

Manufacturing technology and structural design are the main points in interdisciplinary generative design. ADO implements the production methods as crucial as aesthetic appearance in the initial design phase. The choice of the fabrication methods, traditional welding or 3D printing, becomes a fundamental aspect of price and carbon footprint emission optimisation in the AEC sector. In digital fabrication, the most crucial question is to minimise the material and printing time usage, not the number of elements that merge in one node.

A comparison of free-form structures only in terms of minimum weight presents that quadrilateral structures require less material than triangular ones. Furthermore, based on their structural behaviour, they can use more in-depth material optimisation than triangular structures. In curvilinear forms with panels generated by Delaunay triangulation and PQ, the maximum deformations were the main limiting parameter for bar optimisation. In systems with regular triangular panels, it is visible that maximum stresses were reached, which limited the crosssection material optimisation. It indicates optimal solutions could be sought in triangular structures due to better material utilisation, using different crosssections based on bar section ratios.

6. Conclusions

The presented theoretical studies indicate that optimising the division patterns of structural gridshells influences the architectural aesthetics, structural rigidity, and manufacturing aspects. Obtaining efficient structural divisions of grids becomes an interdisciplinary task. In Table 4, the authors' result highlights that choosing the pattern division method depends on multiple factors. Such factors have proper software and knowledge on designing particular divisions, the availability of advanced manufacturing tools, and obtaining architectural quality in shading and material optimisation. From the structural point of view, as J.W. Lewis indicates, understanding how patterns and divisions influence the structural rigidity from receiving structural stresses in the whole structure at a high ratio influences further optimisation possibilities [56].

Table 4. A summarize authors' opinions on free-form canopies in triangular and quadrilateral panels.

	Triangular Panels	**Rectangular Panels**
Design method	The simplified method of planar triangular panel divisions	The complex method of designing planar quadrilateral panels in curvilinear gridshell, especially in significant curvature surface
the technology of execution	Besides the disadvantage of joint complexity (6 bars in one node), the total amount of all the joints is significantly smaller than in the corresponding quadrilateral-more waste glass than in quadrilateral panels.	Depending on chosen fabrication method, constructing a greater amount of joints with greater facility lowers the cost of manufacturing, with the use of less professional labour
architectural quality	Thanks to the reduced amount of joints and almost 40% reduced total length of all the bars, the regular triangular pattern of the mesh improves the aesthetic lightness of the whole structure and visual users' comfort.	Despite the total length and number of joints outnumbering the triangular panels, the total weight of the whole structure remains ~29% lower than other variants, which improves the sustainable usage of the material.

The selection of the manufacturing techniques influences the process of design. Choosing the gridshell pattern becomes an interdisciplinary question to address structural material optimisation, fabrication technologies, and architectural aesthetics. The study indicates that designing free-form structures should always be followed by in-depth multidisciplinary research, as those structures present the non-linear trends in structural optimisation [57]. Implementing the production and cost optimisation type is crucial and often limits choosing specific technical solutions in the construction phase [58]. The research limitations presented in the paper consider choosing the specific geometry boundary conditions, which cannot be applied broadly to all free-form geometries. Searching for aesthetic effects and waste reduction can be attained due to the algorithmisation of design tools, which needs to be improved and applied in the early designing phase. Due to multiple boundary conditions, which vary in each design process, the algorithmic design allows for correcting assumptions: "this also includes the algorithm's usefulness as a tool to verify design concepts and the talent of their creators." [59]. Visible in XXI Century trend in research-based designing improves the more profound understanding of the biomimicry and structural logic in the interdisciplinary designing process. The future scope of research aims to review the possibilities of implementing new fabrication methods such as additive manufacturing and formative technologies to generate free-form geometries [60].

7. Discussion

The generative design represents new possibilities in searching for multivariant solutions, depending on boundary conditions. In the design of relatively simple utility functions, it is possible to identify selected design aspects under certain assumptions and constraints. The choice of materials and technologies becomes the key to broadening the knowledge and perception skills boundaries—the new possibilities of computational design change the multidisciplinary environment [61]. In the case of designing, the usage of relatively simple tools benefits the in-depth optimisation process.

Many architectural activities have made attempts to test modern materials and construction methods. The emergence of new materials and technologies is becoming an essential stimulus in extending the boundaries of knowledge and broadening the perceptual skills of designers. Contemporary trends in the design of canopies also include the parameterisation of tectonic and structural solutions, which may be inspired by bionics. The digitalisation of the procedure depends on architects' perceptual and cognitive abilities.

The authors highlight the need to investigate further the algorithmic design of canopies' shape and structural divisions. A more in-depth interdisciplinary optimisation according to sustainable development regulations should be addressed. Therefore, obtaining practical structural divisions of spatial gridshells in architectural-structural shaping becomes interdisciplinary. Determining the optimal structural form is challenging to look for plastic effects conditioned by the rationalisation of technical solutions, especially in applying algorithmic design tools. A new parametric and generative approach with bionic algorithms and outstanding computation opportunities still need to be addressed, such as redefining the designing factors such as new material and fabrication technologies. This paper and other cited research articles [32–36] provide creative inspiration for developing more optimal and sustainable solutions in interdisciplinary design. Comparing the three types of gridshell patterns presents the multidisciplinary approach's broader aspect of architectural design. Using parametric software might improve the cost of structural forms without losing the aesthetic architectural approach to contemporary "free-form" design.

This paper presents the differences in various patterns of structural divisions of curvilinear canopies. Showing the weight and number of nodes results serves as a starting point for a deeper exploration of architectural and structural optimisation with minimum mass, lowering the number of nodes and aesthetics. Further research should be conducted to explore the architectural aspects of algorithmic form and pattern division findings and the engineering aspects of material and topology optimisation.

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