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Shear Capacity Stochasticity of Simply Supported and Symmetrically Loaded Reinforced Concrete Beams

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Abstract: For shear tests of reinforced concrete (RC) beams, a simply supported and symmetrical loading system is usually applied. In deterministic analysis, shear capacities of the paired shear spans of such beams are the same. However, considering the randomness of concrete strength, geometric dimension, and other factors, shear failure often occurs in the weaker one of the paired shear spans of a beam rather than occurring in the two shear spans simultaneously. Therefore, from the perspective of probability theory, the shear capacities of the paired shear spans of such simply supported and symmetrically loaded beams can be regarded as two random variables with the same distribution. The beam shear capacity, which is the minimum of the two random variables, is also a random variable. Hence, probabilistic differences exist between the shear capacities of shear spans and beams. In this paper, the transformation relationship between the stochasticities of shear span shear capacity and beam shear capacity is theoretically derived. By taking the RC beams without web reinforcement as an example, the shear capacity stochasticities of shear spans and beams, which are valuable for reliability-based design codes, are quantitatively analyzed based on three shear strength models in design codes and a reliable experimental database. Their probabilistic differences are identified and verified to have an impact on the model calibration in the reliability analysis. The results also show that there are obvious differences in the shear capacity stochasticities obtained by different models. It indicates that to obtain the real stochasticity of the shear capacity, it is not enough to consider the model uncertainty merely but to minimize it. Therefore, models based on a solid understanding of the shear mechanisms are urgently needed for practical design.

Keywords: shear capacity; simple beam; stochasticity; model uncertainty; model calibration; database

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1. Introduction

In shear tests of RC beams, a symmetrical three- or four-point loading system is widely used, as shown in Figure 1. It is impossible to predict in advance that shear failure will occur in which shear span. As the load increases, the flexural-shear diagonal cracks appear gradually in the shear spans. When the beam reaches its ultimate shear capacity, shear failure occurs with one of the two shear spans separated along the critical shear crack. At this point, generally, less damage can be observed in the other shear span. If the failed shear span is reinforced (such as by external stirrups) and then re-loads to shear failure of the other shear span, the ultimate capacity is often higher than that in the first load. This phenomenon can be observed in the experiments performed by Feldman and Siess [1], Leonhardt and Walther [2], Chana [3], Collins and Kuchma [4], Lubell et al. [5,6], and Sherwood et al. [7,8].

In deterministic analysis, for a symmetrically loaded and simply supported beam, the capacities (all of the following "capacity" refers to "shear capacity") of the two spans (all of the following "span" refers to "shear span") V_s are the same due to their identical values of geometry parameters and material strength. In this case, there is no difference between span capacity V_s and beam capacity V_b. On the other hand, considering the randomness of concrete strength, geometric dimension, and other factors, the shear failure occurs in one of the paired spans of the beam, which has a lower shear capacity than the other one. In this case, the beam capacity equals the lower span capacity.

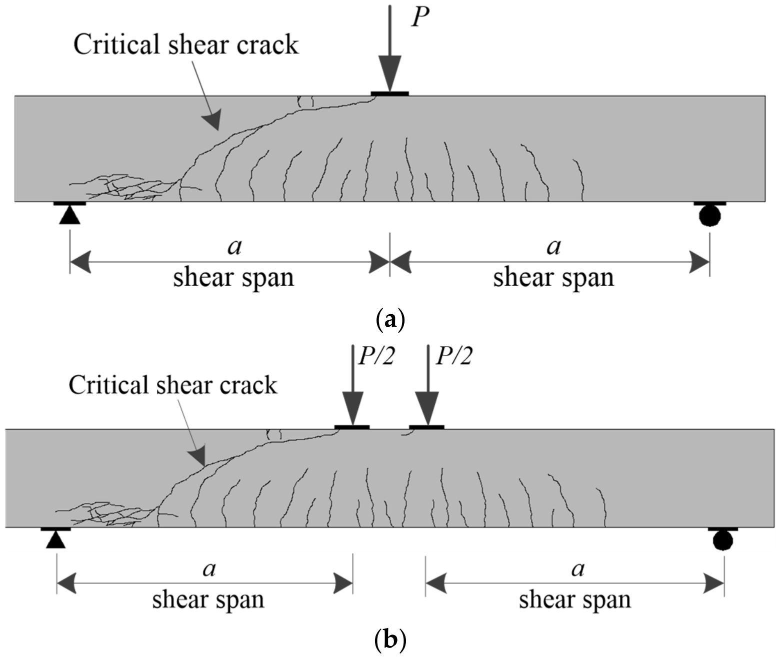

Figure 1. Typical shear test for RC beams. (**a**) three-point symmetrical loading. (**b**) Four-point symmetrical loading.

Suppose the capacities of the two spans of a symmetrically loaded and simply supported beam are regarded as two random variables with an identical distribution. In that case, the capacity of the beam is a function of the two variables and also a random variable. For a real beam, each span's capacity can be considered a sample of the corresponding random variable, and the smaller one of the span capacities determines the capacity of the beam.

Yi and Chen [9] assumed that shear capacities V_{s1} and V_{s2} of the two spans of a symmetrically loaded and simply supported beam obey the same normal distribution (the mean value was 300 kN, and the standard deviation was 50 kN). By Monte Carlo sampling, 500,000 pairs of data were generated as 500,000 virtual beams. The smaller value of each pair of data was selected as the shear capacity V_b of the virtual beam. The probability density function (PDF) curves of V_{s1}, V_{s2}, and V_b indicate significant differences between the stochasticities of the span capacity and the beam capacity. However, as the PDF of the span capacity was entirely hypothetical, and the PDF of the beam capacity was obtained by numerical simulation, they cannot truly reflect the differences and relationships between the stochasticities of the span capacity and the beam capacity.

According to the probability theory [10], the transformation relation between the stochasticities of span capacity V_s and beam capacity V_b of symmetrically loaded simple beams was established in this paper. By taking the RC beams without web reinforcement as an example, the stochasticity of V_b was obtained based on a reliable shear test database. On this basis, V_s was theoretically derived, and the probabilistic differences between the stochasticities of V_s and V_b were identified.

The shear capacity stochasticity is important in the reliability analysis. In practical design, shear capacity models of design codes are used to calculate the shear capacity of a shear span or the critical (diagonal) section in a shear span. However, most test results used to calibrate the models are beam capacities of symmetrically loaded simple beams. The discrepancy between the prediction and calibration of the models and the influence on reliability were discussed. In addition, this study also explored the influences of different shear models (i.e., different model uncertainties) on the shear capacity stochasticity.

2. Methodology: Formulation of Shear Capacity Stochasticity

When the shear capacity is regarded as a random variable, the span capacity V_s and beam capacity V_b, respectively, are

$$V_s = V_P K_{Ps} \tag{1}$$

$$V_b = V_P K_{Pb} \tag{2}$$

where V_P is the shear capacity predicted by shear models, and K_{Ps} and K_{Pb} are the model uncertainties corresponding to V_s and V_b, respectively.

2.1. Stochasticity of Beam Capacity

As most shear tests of simple beams are symmetrically loaded, the tested beam capacity is the smaller value of the capacities of the paired spans. According to the Equation (2), there is

$$K_{pb} = \frac{V_b}{V_P} \tag{3}$$

The samples of the model uncertainty K_{pb} can be obtained by Equation (3) with the samples of the beam capacity V_b. When the V_P in Equation (3) is calculated, the measured values of material properties and geometrical dimensions should be used to exclude material uncertainties and geometric uncertainties.

After the samples of K_{pb} are obtained, the PDF of K_{pb} can be obtained by fitting, and then the PDF of V_b can be obtained according to Equation (2). However, the number of samples of the span capacity V_s is very limited. Thus, the PDF of V_s cannot be determined by this method.

2.2. Stochasticity of Span Capacity

Assuming that the shear capacities of the paired spans of a simple beam are random variables V_{s1} and V_{s2} respectively, the beam capacity V_b is

$$V_b = \min(V_{s1}, V_{s2}) \tag{4}$$

For a symmetrically loaded simple beam with identical structural characteristics in the paired spans, the span capacities V_{s1} and V_{s2} are assumed to be statistically independent and identically distributed. According to probability theory [10], the cumulative distribution function (CDF) $F_Y(y)$ of the minimum Y of the sample random variables X_1, X_2, $\cdots$, X_n, which are statistically independent and identically distributed, is

$$F_Y(y) = 1 - [1 - F_X(y)]^n \tag{5}$$

The corresponding PDF $f_Y(y)$ of Y is

$$f_Y(y) = n[1 - F_X(y)]^{n-1} f_X(y) \tag{6}$$

The above general conclusion can be used for the establishment of the transformation relationship between the stochasticity of span capacity V_s and beam capacity V_b.

$$\begin{cases} F_{Vb}(y) = 1 - [1 - F_{Vs}(y)]^2 \\ f_{Vb}(y) = 2[1 - F_{Vs}(y)] f_{Vs}(y) \end{cases} \tag{7}$$

$$\begin{cases} F_{Vs}(y) = 1 - \sqrt{1 - F_{Vb}(y)} \\ f_{Vs}(y) = \frac{f_{Vb}(y)}{2\sqrt{1 - F_{Vb}(y)}} \end{cases} \tag{8}$$

where $F_{Vb}(y)$ and $f_{Vb}(y)$ are the CDF and PDF of V_b respectively, and $F_{Vs}(y)$ and $f_{Vs}(y)$ are the CDF and PDF of V_s respectively.

Thus, once the stochasticity of the beam capacity V_b is known, the stochasticity of the span capacity V_s can be further determined by Equation (8).

It should be noted that although there is a certain correlation between the span capacities of a beam, this correlation is difficult to be quantified and verified. Moreover, considering the correlation will make the theoretical transformation relationship much more complicated [11]. Therefore, for the sake of simplicity, this study adopted the assumption that the paired span capacities in a symmetrically loaded simple beam are independent. Similarly, the independent assumption was also applied to adjacent strips (macroelements) for numerical analysis of the statistical size effect of span in four-point bending beams [12,13].

3. Example: Shear Capacity Stochasticity of Simple RC Beams without Stirrups

3.1. Shear Tests Database

In this paper, the ACI-DAfStb database established by Reineck et al. [14] is considered. The shear failure of slender beams, characterized by diagonal tension, differs from the shear-compression failure of deep beams [15–19]. The transition of slender and deep beams occurs at a shear span-to-depth ratio a/d of 2.0 to 2.5 [20]. Therefore, in order to keep a consistent shear failure mode (i.e., diagonal tension failure), 605 point-loaded rectangular RC beams with shear span-to-depth ratio a/d greater than 2.5 from the database are used to obtain the statistical samples required for this study.

Of the 605 beams, 573 simple beams with symmetrical structural characteristics were symmetrically loaded. The test results of these beams can be regarded as the samples of beam capacity V_b. For the removed 32 beams [1–8,21–23], the shear failures of 4 beams (specimens H50/5 and H100/5 in [23], SB2012/0, and SB2003/0 in [22]) were fixed in the selected spans by reinforcing the other spans with stirrups, which can be regarded as the samples of the span capacity V_s.

3.2. Shear Capacity Models

In this study, the shear capacity models of RC beams without stirrups in the European code EC2 [24], the American code ACI 318-14 (ACI) [25], and the Chinese code GB 50010-10 (GB) [26] are selected and listed in Table 1. Since the bending moment weakens shear capacity in the ACI model, it is necessary to determine the critical cross-section. As the shear failure surface involves a length along the beam axis approximately equal to effective depth d, sections closer than d to the face of the support or the face of the load will not be critical [27,28], as shown in Figure 2. Therefore, the cross-section with a distance d from the loading point is selected as the critical section in the ACI model.

Table 1. Shear capacity models for RC beams without stirrups in the codes.

Code	Shear Capacity Model	Note
EC2	$V_{P,EC2} = 0.18\left(1+\sqrt{\frac{200}{d}}\right)(100\rho f_c)^{1/3}bd$	where b is the width of the beam; d is the effective depth of the beam; ρ is the ratio of longitudinal reinforcement; a is the shear span measured center-to-center from load to support; f_c and f_t are the compressive and tensile strength of concrete; bending moment $M_{p,ACI}$ occurs simultaneously with $V_{p,ACI}$ at the section considered; and β_d in GB is the factor considering the influence of d on shear capacity.
ACI	$V_{P,ACI} = \left(0.16\sqrt{f_c}+17\rho\frac{V_{P,ACI}d}{M_{P,ACI}}\right)bd = \left(0.16\sqrt{f_c}+17\rho\frac{d}{a-d}\right)bd$	
GB	$V_{P,GB} = \beta_d\frac{1.75}{a/d+1}f_tbd,\ 1.5 \le a/d \le 3.0$ where $\beta_d = \left(\frac{800}{d}\right)^{1/4},\ 800 \le d \le 2000$	

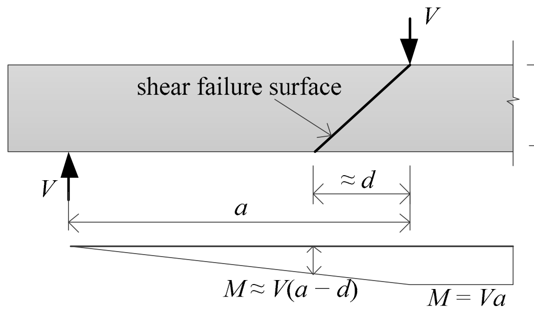

Figure 2. Critical section in ACI model.

3.3. Model Uncertainty K_{pb} of RC Beams without Stirrups

By filtering the ACI-DAfStb database, 573 samples of beam capacity V_b are obtained, while there are only four samples of span capacity V_s. Therefore, the samples of V_b are used to calculate the samples of model uncertainty K_{pb} according to Equation (3). In order to exclude the impact of material uncertainty and geometrical uncertainty, the measured values of material properties and geometrical dimensions should be used for V_p. Then, the distribution function of K_{pb} can be obtained by fitting.

The shear capacities of the 573 beams are calculated by the models in Table 1, and the comparison of the model predictions and the test results are shown in Figure 3. The correlation coefficient R between the predictions by the EC2 model and the test results is the highest, reaching 0.876. Figure 3a shows the prediction points of the EC2 model are closest to the red line, which indicates that the predicted value is equal to the test value. In comparison, the R of the GB model is the lowest, only 0.566. Figure 3c shows the prediction points by the GB model are most significantly scattered on both sides of the red line. The comparison shows that the EC2 model best predicts the shear capacity, followed by the ACI model, while the GB model performs worst.

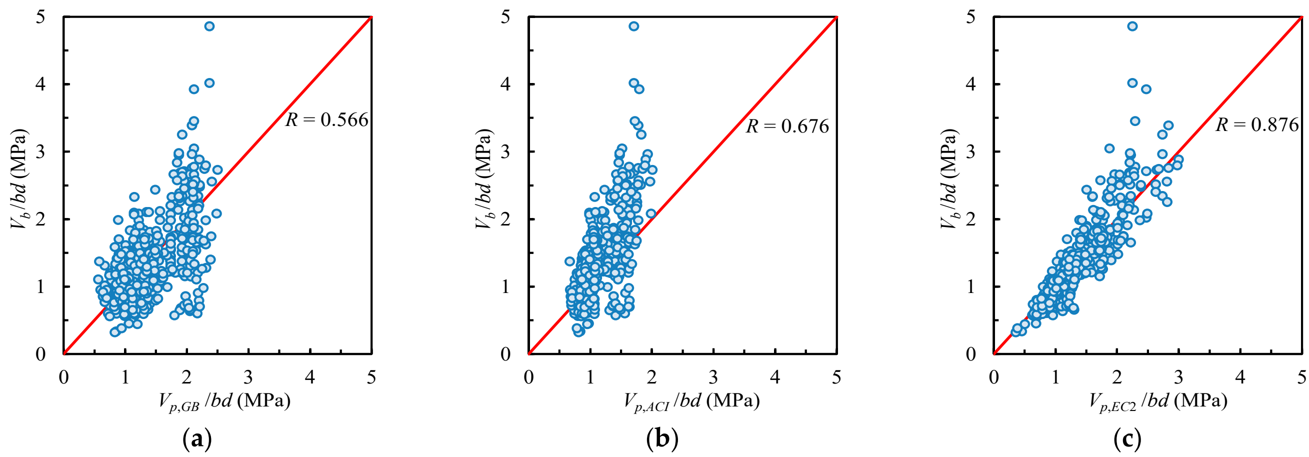

Figure 3. Comparison of model predictions with test results. (**a**) EC2. (**b**) ACI. (**c**) GB.

After the samples of the model uncertainty K_{pb} are obtained, they are fitted by the normal distribution, lognormal distribution, generalized extreme value (GEV) distribution, logistic distribution, and log-logistic distribution, respectively. The Kolmogorov-Smirnov (KS) test is carried out on whether K_{pb} obeys the distributions at the 0.05 significance level, and the results are shown in Table 2. For the distributions accepted by the KS test, their fitting results are shown in Figure 4, and the fitting degree is quantified in the log-likelihood value shown in Table 2. The results indicate that the logistic distribution is accepted by the KS test for all the shear capacity models, and its fitting degree is relatively high. Therefore, the logistic distribution is selected for K_{pb}, and its estimated parameters (i.e., mean μ_{Kpb} and standard deviation σ_{Kpb}) are shown in Table 3.

Table 2. Fitting results of the model uncertainty K_{pb}.

Code	Fitting Result	Distribution Type				
		Normal	**Lognormal**	**Generalized Extreme Value (GEV)**	**Logistic**	**Log-Logistic**
EC2	KS test	Rejected	Accepted	Accepted	Accepted	Accepted
	log-likelihood value	-	229.678	222.272	237.324	247.25
ACI	KS test	Rejected	Rejected	Rejected	Accepted	Rejected
	log-likelihood value	-	-	-	−175.865	-
GB	KS test	Accepted	Rejected	Accepted	Accepted	Accepted
	log-likelihood value	−179.592	-	−178.583	−181.497	−197.03

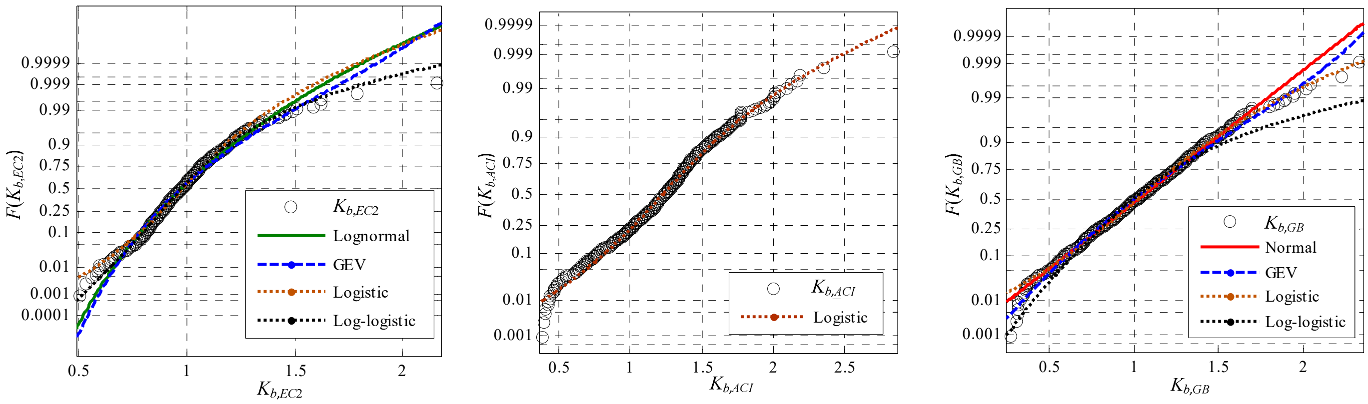

Figure 4. Fitting of the model uncertainty K_{pb} by the distributions accepted by KS test.

Table 3. Parameter estimation for the logistic distribution of model uncertainty K_{pb}.

Code	Parameter of Logistic Distribution	
	μKpb	σKpb
EC2	0.978	0.161
ACI	1.225	0.332
GB	1.022	0.339

3.4. Beam Capacity V_b of RC Beams without Stirrups

The stochasticity of the model shear capacity V_p can be determined by the random variables considered. According to JCSS Probabilistic Model Code [29], the distribution types and probabilistic properties of the geometric and material variables (including b, d, a, ρ, f_c and f_t) in the shear capacity models are defined [30], as shown in Table 4.

The concrete compressive strength is defined as [29]

$$f_c = \alpha(t,\tau)(f_{co})^{\lambda} Y_1 \tag{9}$$

where f_{co} is the basic concrete compression strength; $\alpha(t,\tau)$ is a deterministic function which takes into account the concrete age at the loading time t and the duration of loading τ; λ is a lognormal variable with mean 0.96 and coefficient of variation 0.005, and generally it suffices to take λ deterministically; Y_1 is a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete.

The concrete tensile strength is defined as [29]

$$f_t = 0.3(f_c)^{2/3} Y_2 \tag{10}$$

where the variable Y_2 mainly reflects variations due to factors not well accounted for by concrete compressive strength (e.g., gravel type and size, chemical composition of cement and other ingredients, climatical conditions).

Table 4. Probabilistic properties of the variables considered by the models.

Variable			Distribution Type	Unit	Parameters of the Distribution			Note
					μ	σ	COV	
Geometry	b		Normal	mm	b_m	$4 + 0.006\, b_m \leq 10$	-	where COV is the coefficient of variation, and equals σ/μ; b_m, d_m, a_m, $A_{s,m}$, and $f_{co,m}$ are the mean values of the corresponding variables.
	d		Normal	mm	d_m	10	-	
	a		Normal	mm	a_m	$4 + 0.006\, a_m \leq 10$	-	
	A_s		Normal	mm	$A_{s,m}$	-	0.02	
Material	f_c		-	MPa	-	-	-	
		$\alpha\ (t,\tau)$	Deterministic	-	1.0	-	-	
		f_{co}	Lognormal	MPa	$\mu\ (f_{co,m})$	$\sigma\ (f_{co,m})$	-	
		λ	Deterministic	-	0.96	-	-	
		Y_1	Lognormal	-	1.0	-	0.06	
	f_t		-	MPa	-	-	-	
		Y_2	Lognormal	-	1.0	-	0.3	

By referring to the specimen OA1 tested by Bresler and Scordelis [31], the values of the distribution parameters are assumed as follows: b_m = 310 mm, d_m = 556 mm, a_m = 1830 mm, $A_{s,m}$ = 2579 mm^2, $\mu\ (f_{co,m})$ = 22.6 MPa, and $\sigma\ (f_{co,m})$ = 2.5 MPa. According to Equation (2), the Monte Carlo method is used to simulate 100,000 samples of V_p and K_{pb} each to obtain the samples of V_b, which are then fitted by appropriate distribution types. The fitting results are shown in Figure 5, and the estimation values of the distribution parameters are shown in Table 5.

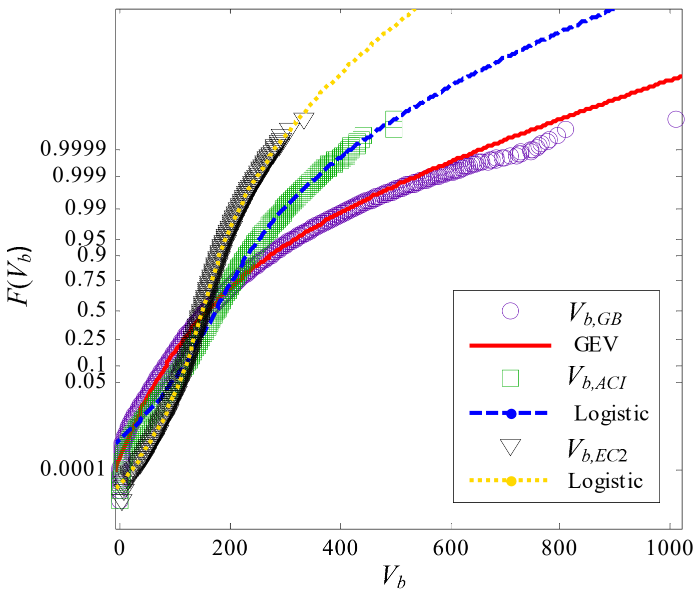

Figure 5. Distribution fitting of the beam capacity V_b.

Table 5. Parameter estimation for the distributions of beam capacity V_b.

Beam Capacity V_b	Distribution Type	Parameter of Distributions		
		μ_{Vb}	σ_{Vb}	k_{Vb}
$V_{b,EC2}$	Logistic	149.918	25.748	-
$V_{b,ACI}$	Logistic	173.894	48.140	-
$V_{b,GB}$	GEV	148.545	68.506	−0.034

Note: k_{Vb} is the scale parameter of GEV distribution.

3.5. Span Capacity V_s of RC Beams without Stirrups

The stochasticity of the span capacity V_s is determined by Equation (8) after obtaining the stochasticity of the beam capacity V_b, and the PDFs of V_s are shown in Table 6. From the comparison of the PDFs of V_b and V_s in Figure 6, it can be seen that the mean and standard deviation of the span capacity V_s are larger than the beam capacity V_b, which is consistent with the conclusion by Nowak et al. [32,33] that both the mean value and the variance decrease with an increasing sample random variable number (i.e., n in Equations (5) and (6)). Therefore, the difference between the stochasticities of V_b and V_s is theoretically verified, and its influence on the reliability analysis is discussed in Section 3.6.

Table 6. Probability density functions for the shear span's shear strength V_u.

Span Capacity V_s	**Probability Density Function**
$V_{s,EC2}$ $V_{s,ACI}$	$f_{Vu}(y) = \frac{\left(\exp\left(\frac{y-\mu_{Vb}}{s_{Vb}}\right)\right)^{1/2}}{2s_{Vb}\left(1+\exp\left(\frac{y-\mu_{Vb}}{s_{Vb}}\right)\right)^{3/2}}$, where $s_{Vb} = \frac{\sqrt{3}\sigma_{Vb}}{\pi}$
$V_{s,GB}$	$f_{Vu}(y) = \frac{\frac{1}{\sigma_{Vb}}\exp\left(-\left(1+k\frac{(y-u_{Vb})}{\sigma_{Vb}}\right)^{-1/k}\right)\left(1+k\frac{(y-u_{Vb})}{\sigma_{Vb}}\right)^{-1-1/k}}{2\sqrt{1-\exp\left(-\left(1+k\frac{(y-u_{Vb})}{\sigma_{Vb}}\right)^{-1/k}\right)}}$

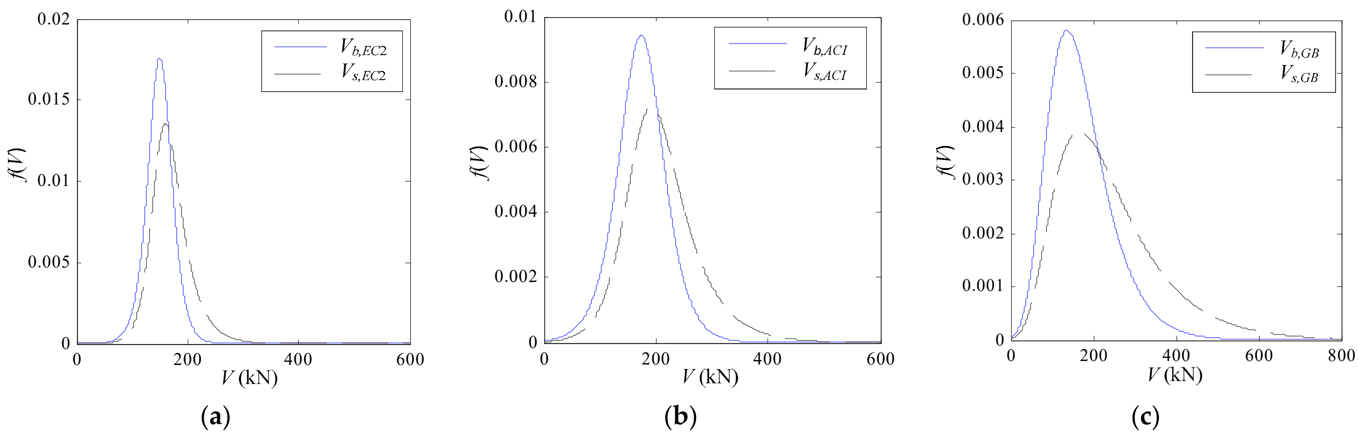

Figure 6. Comparison of the PDFs of V_b and V_s. (**a**) EC2. (**b**) ACI. (**c**) GB.

The stochasticities of beam and span capacities of RC simple beams are inherent characters and should be independent of the design models. However, by comparing the calculated PDFs of V_b and V_s obtained based on different models, as shown in Figure 7, it can be seen that there are great differences among them. It can be inferred that the differences are transferred from the various model uncertainties K_{pb}, which quantify the deficiencies of the empirical models. To obtain the real stochasticity of the shear capacity, it is not enough to consider the model uncertainty but also to make the model as far as possible to reflect the mechanism of shear failure, i.e., to minimize the model uncertainty. Therefore, models based on a solid understanding of the shear mechanisms are urgently needed for practical design.

3.6. Reliability Analysis of Span and Beam Capacities

In order to achieve the predetermined target reliability of designed structures, design models in codes need to be calibrated using test results [32,33]. The shear capacity models in the design codes are used to calculate the shear capacity of a shear span or the critical (diagonal) section in a shear span. However, most test results used to calibrate the models are beam capacities of symmetrically loaded simple beams, which are the lower span capacities of the paired spans. The discrepancy between the prediction and calibration of the models and the influence on reliability need to be evaluated.

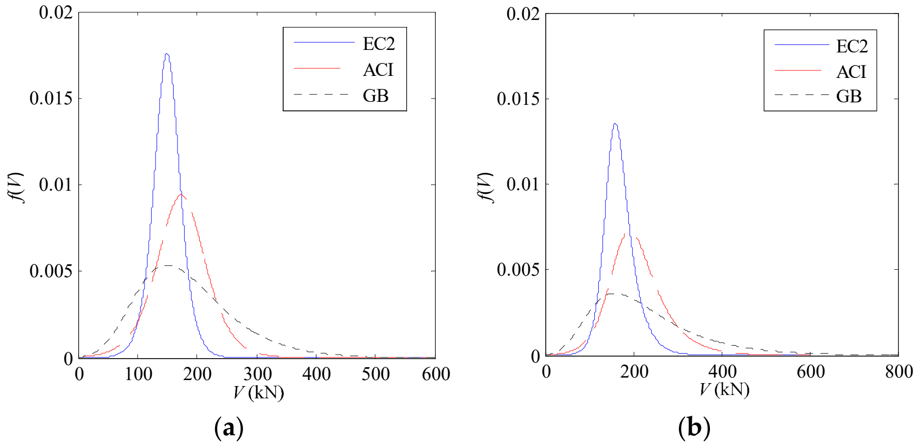

Figure 7. Comparison of the PDFs of shear capacity by different models. (**a**) PDFs of V_b. (**b**) PDFs of V_s.

In this study, the reliability analysis is carried out by using the ACI model and specimen OA1 [31] as an example. The dead load D and the live load L are determined according to Equation (11) and Table 7.

$$1.2D_n + 1.6L_n \leq \phi V_{P,ACI} \tag{11}$$

where D_n and L_n are the nominal values of D and L, respectively, and their statistical parameters are shown in Table 7 [32,33]. Resistance factors ϕ is 0.75 for shear failure according to ACI [25].

Table 7. Probabilistic properties for the dead load D and live load L.

Load Type	Distribution Type	Statistical Parameters	
		$\mu(D)/D_n$	COV
D	Normal	1.05	0.10
L	Extreme type I	1.00	0.18

The limit state functions Z_{Vs} and Z_{Vb} for the shear failure of span and beam, respectively, are formulated as Equations (12) and (13).

$$Z_{Vs} = V_P K_{Ps} - D - L = V_s - D - L \tag{12}$$

$$Z_{Vb} = V_P K_{Pb} - D - L = V_b - D - L \tag{13}$$

Using Monte Carlo simulations, the reliability indexes for V_s and V_b are shown in Figure 8, showing that the reliability index of V_s is about 0.25 higher than that of V_b, which means the failure probability of V_s is about half of that of V_b under the same load combination.

It should be noted that the reliability indexes obtained in this study are lower than those provided by Szerszen and Nowak [33]. The main reason is that the COV (about 0.28) of the shear capacity obtained in this study is much larger than the COV (about 0.11) used by Szerszen and Nowak [33]. If the COV = 0.11 is used in the reliability analysis of this study, the reliability indexes will be close to those provided by Szerszen and Nowak [33].

As previously mentioned, in practical design, the shear capacity models are used to calculate the shear capacity of shear spans. However, most test results available to calculate the model uncertainty are beam capacities of symmetrically loaded simple beams, so the shear models are actually calibrated only at the beam-level, which causes the reliability of shear spans designed by the beam-level calibrated shear models to be underestimated.

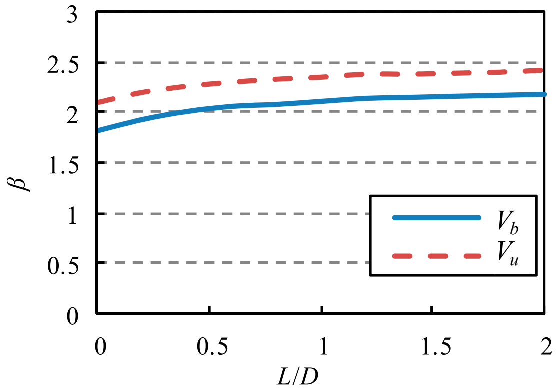

Figure 8. Reliability indexes β of V_b and V_u.

To more reasonably calibrate the reliability of the beam shear capacity, attentions should be paid to (1) the selection criteria of test results in the database, (2) the inconsistency of the shear capacity stochasticities between the shear span and the beam for symmetrically loaded simple beams, and (3) the minimizing of the model uncertainty. On the other hand, the independence assumption of the paired span capacities of symmetrically loaded simple beams is adopted in this study, which still needs to be further discussed.

4. Summary and Conclusions

1. The transformation relationship between the stochasticities of span capacity and beam capacity was theoretically derived. It is applicable to shear controlled members with symmetrical boundary conditions and structural parameters, including symmetrically loaded simple and continuous beams with and without stirrups.
2. By taking the RC beams without web reinforcement as an example, the stochasticities of the span and beam capacities, which are valuable for reliability-based design code, were quantitatively analyzed on the basis of three shear strength models in design codes and a reliable experimental database. The results theoretically verified the probabilistic difference between the stochasticities of V_b and V_s.
3. Differences in the shear capacity stochasticities obtained by different models were also identified, which indicated that to obtain the real stochasticity of the shear capacity, it is not enough to merely consider the model uncertainty, but to minimize it.
4. The reliability analysis showed that the reliability index of V_s is higher than that of V_b, and the failure probability of V_s is about half of V_b under the same load combination. In addition, the discrepancy between the prediction and calibration of the models and the influence on reliability were evaluated, indicating the reliability of shear spans designed by the beam-level calibrated shear models is underestimated.

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Conflicts of Interest: The authors declare no conflict of interest.

Notations

a	shear span measured center-to-center from load to support
A_s	area of longitudinal reinforcement
b	width of the beam
D	dead load
f_c	compressive strength of concrete
f_{co}	basic concrete compression strength
f_t	tensile strength of concrete
k	scale parameter of generalized extreme value distribution
K_{Ps}, K_{Pb}	model uncertainties corresponding to V_s and V_b, respectively
L	live load
M_p	bending moment occurs simultaneously with V_P at the section considered
s	scale parameter of logistic and log-logistic distribution
V_b	beam shear capacity
V_P	shear capacity predicted by model
V_s	shear capacity of shear span
Y_1	log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete
Y_2	variable reflects variations due to factors not well accounted for by concrete compressive strength
Z	limit state functions
$\alpha(t,\tau)$	deterministic function which takes into account the concrete age at the loading time t and the duration of loading τ
β_d	factor considering the influence of d on shear capacity
μ	mean value of random valuable
ρ	ratio of longitudinal reinforcement
σ	standard deviation of random valuable

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