



# Article Data-Driven Rock Strength Parameter Identification Using Artificial Bee Colony Algorithm

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Abstract: Rock strength parameters are essential to understanding the rock failure mechanism and safely constructing rock excavation. It is a challenging problem for determining the rock failure criterion and its parameters due to the complexity of rock media. This study adopts an artificial bee colony (ABC) algorithm to determine the Hoek-Brown failure criterion, widely used in rock engineering practice, based on experimental data. The ABC-based approach is presented in detail and applied to a collection of experimental data collected from the literature. The ABC-based approach successfully determines the Hoek-Brown failure criterion, and the determined failure envelope is in excellent agreement with the measured curve. The maximum relative error obtained by ABC is only 2.15% and is far less than the 12.24% obtained by the traditional method. Then, the developed approach is applied to the Goupitan Hydropower Station, China, and determines the rheological parameters of soft rock based on the Burgers model. The deformation of an experiment located in the Goupitan Hydropower Station is evaluated based on obtained parameters by the developed approach. The predicted deformation matches the monitored displacement in the field. The obtained parameters of the failure criterion characterize the mechanical behavior of rock mass well. Thus, the method used provides a reliable and robust approach to determining the mechanical parameters of the failure criterion.

Keywords: rock material; strength; test data; artificial bee colony; intelligent optimization

## 1. Introduction

Stability analysis is one of the most challenging issues in rock excavation activities, such as tunnelling, sloping, coal mining, etc. The rock failure criterion is critical to estimating the stability of rock structures. Various failure criteria have been developed for rock engineering in the last decades [1–6]. The Hoek-Brown failure criterion, derived for brittle and jointed rock masses by Hoek and Brown, has been widely used in practical rock engineering for the last several decades [7–10]. In order to estimate and predict the accuracy of the failure behavior for a rock mass, it is essential to determine the Hoek-Brown failure criterion, which is difficult to measure experimentally in laboratory and field tests. The rational determination of parameters for the model have drawn increased attention from researchers and engineers in rock engineering.

In order to understand and characterize the failure mechanism and deformation behavior of a rock mass, the strength failure criterion is necessary for stability analysis and construction in rock engineering. The Hoek-Brown failure criterion is a relatively simple function with a small number of parameters; it defines rock strength as a nonlinear function of confining pressure. The Hoek-Brown failure criterion provides a standardized method for determining rock mass strength parameters based on intact laboratory test data [11]. Mathematical and statistical methods are essential to approximate the failure criterion and



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**Copyright:** © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). its parameters [12,13]. Simple linear regression was used to determine the rock failure criterion, illustrated by three examples [14]. The simplex reflection method was applied to fit the rock failure criterion based on the laboratory test data [15]. A bonded block numerical model was developed to numerically quantify the strength of fractured rock mass [16]. A GSI-softening model was proposed to capture the strength behavior of thermally damaged rock based on the Hoek-Brown failure criterion [17]. With the development of computational technology, nonlinear regression was also used to determine the rock failure criterion [18,19]. The multiple regression method was developed to estimate the rock failure criterion [20]. Using a single empirical model is not enough to describe the failure envelope in tension and high compression zones. A bi-segmental Hoek-Brown failure criterion was developed to overcome the above problem by considering the failure mechanism in different zones [21]. A mathematical formula was developed to characterize the fatigue and strength of concrete using symbolic regression [22]. Recently, researchers applied Bayes theory to determine the rock failure criterion [23,24]. Bayesian inference was used to determine the rock strength anisotropy and evaluate the uncertainty of the Hoek-Brown failure criterion [25]. With the development of artificial intelligence, the soft computing method provides an optimal method for determining the parameters of the model [26,27]. The genetic algorithm was used to determine the Mohr-Coulomb constitutive model [28]. Symbolic regression-based data were developed to determine the fatigue equation [22]. Zhao et al. adopted particle swarm optimization (PSO) to determine the geomechanical parameters and perform reliability analysis for geotechnical engineering [29,30]. A fuzzy inference system, artificial neural network, and adaptive neuro-fuzzy inference system were utilized to predict the Brazilian tensile strength of rock samples [17]. The artificial Bee Colony (ABC) algorithm, which simulates the intelligent foraging behavior of honey bee swarms, was proposed by Karaboga for optimizing numerical problems [31]. Sonmez and Kang applied it to structural engineering [32,33]. Zhao et al. applied ABC to the back analysis in geotechnical engineering [34]. Villegas et al. applied ABC to optimize the values for the variables of a proportional integral controller for observing the behavior of the controller [35]. Caraveo et al. developed a modification of a bio-inspired algorithm based on ABC for optimizing fuzzy controllers [36]. An improved artificial bee colony algorithm was proposed to address the optimization of the injection scheme of steam flooding [37]. A hybrid model was proposed for predicting the cemented paste backfill strength based on an adaptive neuro-fuzzy inference system and artificial bee colony [38]. Yavuz et al. developed the artificial bee colony algorithm with distant savants for solving constrained optimization problems [39]. Uncertainty quantification was utilized to capture the strength of the reinforcement rock mass [40]. The maximum entropy probability density function was adopted to obtain the uncertainty of the shear strength of the intact rock in the Jinping II project [41]. In this study, ABC is adopted to determine rock strength parameters based on testing data for rock engineering.

It is not easy to determine the strength parameters due to the complexity and unclear failure mechanism of the rock mass. ABC is a straightforward, robust, and populationbased stochastic optimization algorithm that does not depend on the specific problem and is suitable for the black-box problem. The significant advantage of ABC is it is easy to implement without any gradient information. Moreover, ABC offers a good balance between fast convergence and exploratory search, allowing for escaping from local minima and aiming for global instead. Meanwhile, ABC could provide a data-driven model capable of correctly determining the strength parameters of the rock mass based on laboratory tests. The remainder of this paper is organized as follows. Firstly, the Hoek-Brown failure criterion and the Burgers model of rock mass are introduced in detail. Secondly, the algorithm of ABC is described, and the procedure of determining the rock failure criterion and its parameters is presented briefly. Then, the proposed method is verified and illustrated using the test data of the uniaxial compression test, triaxial compression test, and is applied to the soft rock mass in a practical experimental tunnel. Finally, some conclusions are presented.

## 2. Rock Failure Criteria

# 2.1. Hoek-Brown Failure Criterion

Rock failure originates from micro-cracks or flaws in the rock mass. Griffith developed the tensile failure and deepened it to obtain a nonlinear compressive failure envelope for brittle materials [42]. On the basis of the above nonlinear Griffith failure criterion, Hoek and Brown proposed the generalized Hoek-Brown criterion for rock mass strength [7]. The generalized Hoek-Brown failure criterion could be expressed as follows:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left( m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a \tag{1}$$

where  $\sigma_1$  and  $\sigma_3$  denote the major and minor principal stresses, respectively;  $\sigma_{ci}$  denotes the unconfined compressive strength;  $m_b$ , s, and a denote the rock mass material constants, determined as follows:

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$$m_b = m_i e^{\frac{GSI - 100}{28 - 14D}}$$
(2)

$$s = e^{\frac{GST - 100}{9 - 3D}} \tag{3}$$

$$a = \frac{1}{2} + \frac{1}{6} \left( e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}} \right)$$
(4)

where *GSI* denotes the geological strength index, which presents the failure criterion based on the engineering geology observations in the field; *D* denotes a factor that considers the degree of the rock mass disturbance by blast damage and stress relaxation. For the intact rock, the material constants are adopted by  $m_i$ , s = 1, and a = 0.5. The Hoek-Brown failure criterion was employed to capture the results of a wide range of triaxial tests on intact rock samples.

$$\sigma_1 = \sigma_3 + \sigma_{ci} \sqrt{m_i \frac{\sigma_3}{\sigma_{ci}} + 1} \tag{5}$$

In order to characterize the Hoek-Brown failure criterion for intact rock mass,  $m_i$  and  $\sigma_{ci}$  are determined based on the experimental data. This study utilizes ABC to characterize the Hoek-Brown failure criterion and determine its material constants based on laboratory tests.

#### 2.2. Burgers Model

The rheological properties of rock mass characterize the deformation behavior of the surrounding rock mass over time during rock excavation. The rock failure criterion of the combination model is commonly adopted in rheological numerical calculation and analysis. Its advantage is that the appropriate combination of idealized basic elements can be utilized to capture the complex rheological properties of the actual rock mass, and the mechanical concept and the physical meaning are also simple and clear. The Burgers model is generally adopted to capture the mechanical and deformation behavior of soft rock [43]. It consists of the Kelvin model and the Maxwell model (Figure 1). The Burgers model of rock failure can be expressed as follows [44–46]:

$$\ddot{\sigma} + \left(\frac{E_1}{\eta_2} + \frac{E_1}{\eta_1} + \frac{E_2}{\eta_1}\right)\dot{\sigma} + \frac{E_1E_2}{\eta_1\eta_2}\sigma = E_1\ddot{\varepsilon} + \frac{E_1E_2}{\eta_2}\dot{\varepsilon}$$
(6)

where  $\sigma$  and  $\varepsilon$  denote the stress and strain of the Burgers model,  $E_1$  and  $\eta_1$  denote the elastic modulus and viscosity for the Kelvin model, and  $E_2$  and  $\eta_2$  denote the elastic modulus

and viscosity for the Maxwell model. Under constant load  $\sigma_0$ , the Burgers equation can be changed into the following equation:

$$\varepsilon = \sigma_0 \left\{ \frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2} \left[ 1 - \exp(-\frac{E_2 t}{\eta_2}) \right] \right\}$$
(7)

The Burgers model is utilized in a soft rock mass with instantaneous deformation, deceleration creep, constant velocity creep, relaxation, and elastic after-effect.

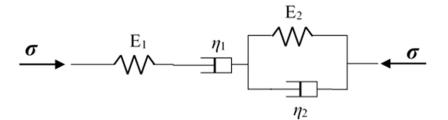


Figure 1. Burgers model.

## 3. The Artificial Bee Colony Algorithm

The artificial bee colony (ABC) algorithm was first developed for unconstrained optimization problems, where it showed good performance [47]. ABC was inspired by the intelligent foraging behavior of bee swarms. There are various tasks done by specialized bee individuals in the actual bee colony. In the ABC algorithm, the colony of artificial bees includes three groups of bees, i.e., employed bees, onlooker bees, and scout bees. Employed bees are used to exploit the nectar sources previously explored and then transfer information to the other waiting bees in the hive about the quality of the food source they seek. Onlooker bees wait in the hive and build a food source to exploit based on the information gained from the employed bees. Scout bees seek an environment in order to obtain a new food source. Once the ABC algorithm is initialized, it needs a cycle of three phases: employed bee phase, onlooker bee phase, and scout bee phase.

## 3.1. Initialization Phase

Firstly, the ABC algorithm generates the initial population of *SN* solutions/bees randomly and calculates the fitness function of each solution/bee.

$$x(i,j) = x_{\min}^{j} + rand(0,1) \left( x_{\max}^{j} - x_{\min}^{j} \right)$$
(8)

where *SN* is an integer; x(i,j) denotes the candidate solution of problem; i = 1, 2, ..., SN/2 and *SN*/2 denotes the size of population; j = 1, 2, ..., Dim and *Dim* denotes the dimension number of each solution; *rand*(0,1) denotes a random number between [0, 1];  $x^{j}_{min}$  and  $x^{j}_{max}$  denote the upper and lower bound of each solution.

#### 3.2. Employed Bee Phase

While initialization is finished, employed bees generate the new solution based on the ABC algorithm (Figure 2). A candidate solution can be obtained using the memory of bees, as follows:

$$v(i,j) = x(i,j) + \varphi_{ij}(x(i,j) - x(k,j))$$
(9)

where *k* is different from *i* and denotes the indexes randomly chosen from {1, 2,..., SN/2}, *j* also denotes randomly chosen indexes from {1, 2,..., Dim},  $\varphi_{ij}$  denotes a random number in [-1, 1], characterizes the generation of neighbor food sources around x(i,j), and denotes the comparison of two food positions seen by a bee.

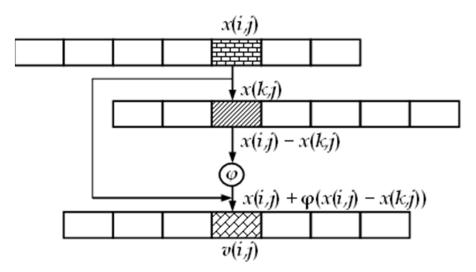


Figure 2. A simple position update scheme.

## 3.3. Onlooker Bee Phase

Onlooker bees determine a solution based on the fitness function gained from employed bees, generate the solution to be abandoned, and allocate its employed bees as scout bees. The probability of being selected for each fitness function  $p_i$  can be computed as follows:

$$p_{i} = \frac{fitness_{i}}{\sum\limits_{n=1}^{SN/n} fitness_{n}}$$
(10)

where *fitness<sub>i</sub>* denotes the fitness value of the solution.

#### 3.4. Scout Bee Phase

In the ABC algorithm, scout bees randomly seek a new solution in the predetermined searching ranges. No further improved solution is assumed to be abandoned by onlookers in a predetermined number of cycles. The abandoned solution x(i,j) is updated by a new solution x'(i,j), generated by the scout bees using Equation (9). Each candidate solution v(i,j) generated by x(i,j) can be determined using the comparison between x(i,j) and its old solution. The old solution will be updated by the new solution when it equals or is better than the old solution. Otherwise, the old solution is unchanged in the memory.

## 3.5. Procedure of ABC

ABC mainly depends on three controlling parameters: the number of food sources equals the number of employed or onlooker bees (*SN*), the value of limit, and the maximum cycle number (*MCN*). The fitness function is essential to ABC algorithm, which depends on the specific problem. In this study, mean absolute error and mean relative error are utilized to determine the fitness function. The detailed formation is presented in Section 4.1. The procedure of ABC can be described in detail as follows:

Step1: Set the value of control parameters SN, MCN, and "limit" of the ABC algorithm.

Step2: Initialize the population *x*(*i*,*j*) using Equation (8) and calculate the fitness value of each solution.

- Step3: For each employed bee, generate a new solution v(i,j) using Equation (9) and calculate its fitness.
- Step4: Determine the probability  $p_i$  for the solution x(i,j) using Equation (10).
- Step5: For each onlooker bee, determine a solution x(i,j) based on  $p_i$ , generate a new solution v(i,j), and calculate the fitness.
- Step6: If there is an abandoned solution for the scout, it is in place of a new solution that will be randomly generated using Equation (8).

Step7: Record the best solution. Step8: Repeat Step3 to Step7 until reaching the maximum cycle.

## 4. Determination of Hoek-Brown Failure Criterion Based on Artificial Bee Colony

Based on the laboratory test, the ABC-based approach is developed to characterize the Hoek-Brown failure criterion. The ABC is employed to seek the material constant  $m_i$  and rock compressive strength  $\sigma_{ci}$  based on the error function, which is the difference between the predicted and actual values.

## 4.1. Fitness Function

In order to use the ABC algorithm to characterize the Hoek-Brown failure criterion, as in any conventional optimal method, it is necessary to construct the error function/fitness function for ABC. In this study, the absolute error and relative error that Douglas recommended are defined as follows [20]:

$$fitness = \begin{cases} \frac{\sum \sigma_1 - \sigma'_1}{n} & \sigma_1 > -3\sigma_3\\ \frac{\sum \sigma_3 - \sigma'_3}{n} & \sigma_1 \le -3\sigma_3 \end{cases}$$
(11)

$$fitness = \begin{cases} \frac{\sum \frac{\sigma_{1} - \sigma_{1}'}{\sigma_{1}'}}{n} & \sigma_{1} > -3\sigma_{3} \\ \frac{\sum \frac{\sigma_{3} - \sigma_{3}'}{\sigma_{3}'}}{n} & \sigma_{1} \le -3\sigma_{3} \end{cases}$$
(12)

where  $\sigma_{1'}$  and  $\sigma_{3'}$  denote the predicted stresses using the Hoek-Brown failure criterion,  $\sigma_1$  and  $\sigma_3$  denote the measured stress values from the experiment, and *n* denotes the number of experimental data. In this study, the root mean squared error is also utilized to illustrate the performance of ABC.

$$fitness = \begin{cases} \sqrt{\frac{\sum (\sigma_1 - \sigma_1')^2}{n}} & \sigma_1 > -3\sigma_3\\ \sqrt{\frac{\sum (\sigma_3 - \sigma_3')^2}{n}} & \sigma_1 \le -3\sigma_3 \end{cases}$$
(13)

$$fitness = \begin{cases} \sqrt{\frac{\sum \left(\frac{\sigma_1 - \sigma_1'}{\sigma_1'}\right)^2}{n}} & \sigma_1 > -3\sigma_3\\ \sqrt{\frac{\sum \left(\frac{\sigma_3 - \sigma_3'}{\sigma_3'}\right)^2}{n}} & \sigma_1 \le -3\sigma_3 \end{cases}$$
(14)

## 4.2. Procedure of Determination of Hoek-Brown Failure Criterion

Once the rock laboratory experiments are finished, the fitness value can be determined based on testing data and the error function in the above section. ABC is utilized to seek the material parameters of the Hoek-Brown failure criterion based on the error function. The analysis proceeds through these steps as follows (Figure 3):

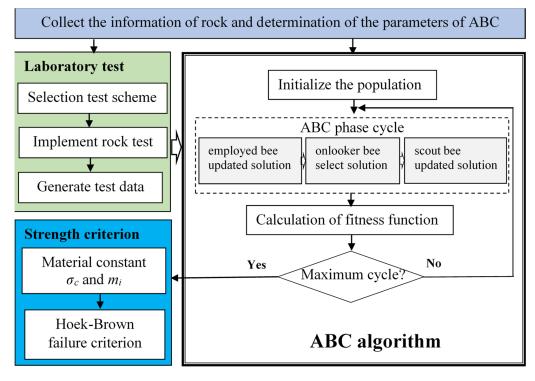


Figure 3. The ABC-based approach for determination of Hoek-Brown failure criterion.

- Step 1: Collect the information on the rock and determine the parameters of ABC.
- Step 2: Determine the test scheme based on rock mass property.
- Step 3: Implement the rock test according to the test standard.
- Step 4: Generate the test data.
- Step 5: Calculate the fitness value based on the fitness function using the testing data above.
- Step 6: Use the ABC algorithm to seek material constants of the Hoek-Brown failure criterion.
- Step 7: Characterize the Hoek-Brown failure criterion.

## 4.3. Verification

The Hoek-Brown failure criterion is utilized to capture the rock properties based on experimental data. For a good estimation, the more experimental data, the more accurate the estimation of the rock properties will be. Sari collected 266 experimental data points, including the uniaxial compression test, Brazilian tensile test, and triaxial compression test [48]. The data points are very widely scattered (Figure 4). Sari adopted the linear regression method and non-linear Levenberg-Marquardt fitting algorithm to determine the Hoek-Brown failure criterion based on the above experimental data. This study utilizes ABC to characterize the Hoek-Brown failure criterion (Figure 5). Based on the Hoek-Brown criterion (Equation (1)) and fitness function (Equations (11)–(14)), material constant  $\sigma_{ci}$  and  $m_i$  were determined using ABC. The major principal stress  $\sigma_1$  and minor principal stress  $\sigma_3$  are the input parameters from 266 experimental data for the ABC model based on the fitness function. The results are compared with the different methods. The performance of ABC and estimation of rock properties are illustrated using the different parameters of ABC, searching range, and fitness/error function.

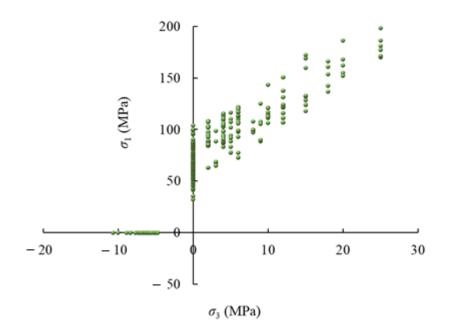


Figure 4. The experimental data.

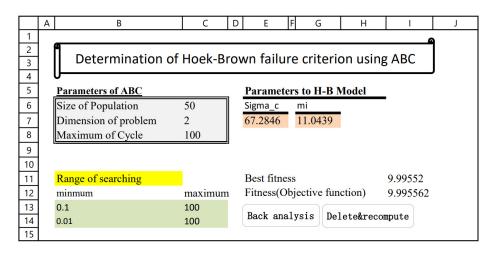
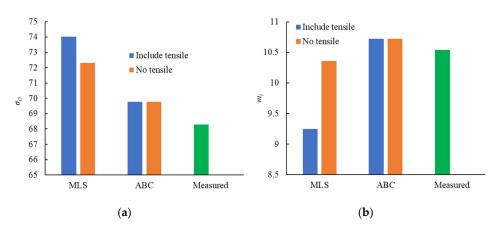


Figure 5. The determination of the Hoek-Brown failure criterion using ABC.

Hoek-Brown failure criterion  $\sigma_c$  and  $m_i$  parameters are listed in Table 1 based on the 266 experimental data points using the different methods. The obtained strength parameters are closer to the measured parameters than the modified least square method (MLS). Based on the MLS, the maximum relative error is 12.24%, but the maximum relative error is only 2.15% based on ABC. Meanwhile, the obtained parameters are almost identical to the data, including and excluding tensile experiments using ABC. This shows that ABC can capture the strength property of rock mass better than MLS. Figure 6 shows the comparison of the obtained parameters using the different methods. The above investigation shows that the proposed method has an optimal global performance. The envelopes measured and obtained by ABC and MLS is shown in Figure 7. The envelope obtained using ABC is closer to the measured envelope than that obtained using MLS when using data included in the tensile experiment. The proposed method is obviously superior to MLS for the envelope excluding tensile experiment data. With the increase of the confining pressure, the difference between the measured parameters and those predicted using ABC will increase. However, the difference is very insignificant. This further proves that the proposed method reasonably determines the Hoek-Brown failure criterion using the experimental data points.



**Figure 6.** The comparison between the predicted and measured coefficients: (a)  $\sigma_{ci}$ ; (b)  $m_i$ .

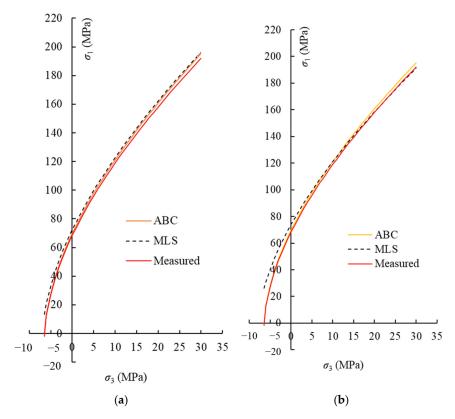
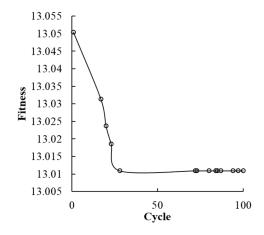


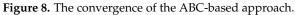
Figure 7. The Hoek-Brown failure envelope determined using different methods: (a) tensile; (b) no tensile.

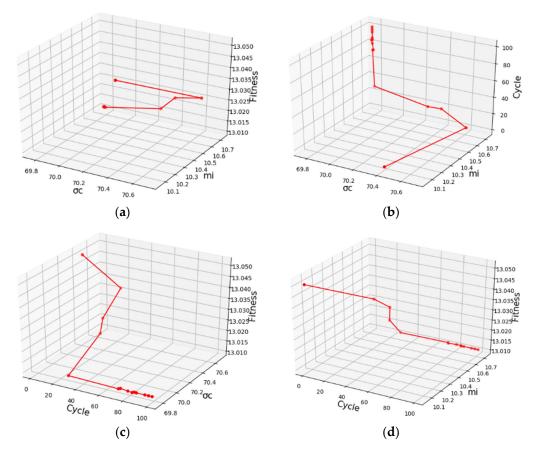
<b>Table 1.</b> The obtained strengths using different methods.
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Material	Include Tensile		No Te	No Tensile	
Constants	MLS	ABC	MLS	ABC	— Measured
$\sigma_{ci}$	74.02	69.77	72.30	69.76	68.30
Relative error (%)	8.37	2.15	5.86	2.14	-
m <sub>i</sub>	9.25	10.72	10.36	10.72	10.54
Relative error (%)	12.24	1.71	1.71	1.73	-

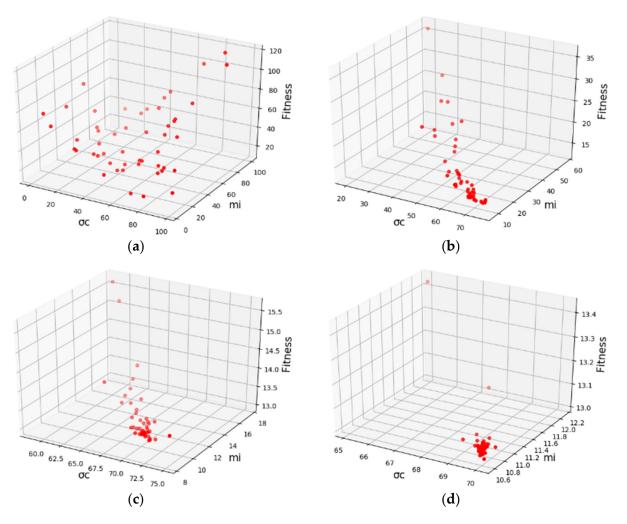
The convergence performance of the proposed method is shown in Figure 8. Figure 9 shows the variation process of parameters of the failure criterion. It shows that the proposed method has good convergence and can quickly determine the Hoek-Brown failure criterion based on the experimental data points. The fitness of the population is shown in Figure 10. The results are similar to the above results. This shows that ABC has a fast convergence performance. Based on the test data, the ABC-based approach can accurately determine the Hoek-Brown failure criterion.







**Figure 9.** The searching process of the parameters of the failure criterion: (**a**),  $\sigma_c$ ,  $m_i$ , and fitness; (**b**)  $\sigma_c$ ,  $m_i$ , and cycle; (**c**)  $\sigma_c$ , cycle, and fitness; (**d**) cycle,  $m_i$ , and fitness.



**Figure 10.** The fitness value of population with the cycle: (**a**) initial cycle, (**b**) cycle 20, (**c**) cycle 40, and (**d**) cycle 60.

The optimal global performance is an essential index to the optimal method. In order to verify the ABC algorithm, a different searching range is used to determine the parameters of the failure criterion. Table 2 lists the results in different searching ranges. The ABC-based approach can determine well the parameters of the failure criterion and find the minimum fitness. The proposed method has a good global optimal performance. The ABC-based approach can avoid the optimal local solution and determine the best solution for the Hoek-Brown failure criterion. Figure 11 shows the convergence in different ranges. The ABC-based approach can determine the solution at about Cycle 10 in a smaller range, and it can determine the solution at about Cycle 20 in a more extensive range. This shows that the proposed method is suitable and can quickly determine the solution in a smaller range.

Table 2. The parameters of the failure criterion and fitness value in different ranges.

Range	$\sigma_c$ (MPa)	$m_i$	Fitness
[0, 100]	67.2842	11.0439	9.99552
[0, 200]	67.2848	11.0437	9.99552
[0, 400]	67.2827	11.0442	9.99552
[0, 1000]	67.2832	11.0449	9.99552

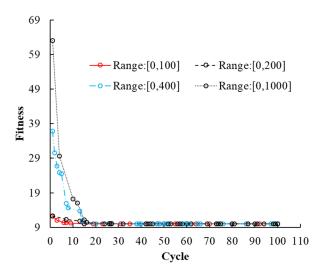


Figure 11. The convergence in different searching ranges.

The objective function drives the optimal method to search for the solution and is essential to the optimal method. Sari investigated the fitness function in multiple regression and found that the fitness function is essential for determining the Hoek-Brown failure criterion [48]. In this study, various fitness functions were used to verify the performance of the ABC-based approach. Figure 12 shows the Hoek-Brown failure envelope using a different fitness function. The fitness function has little influence on the Hoek-Brown failure criterion because of the difficulty of determining the fitness function. This shows that the ABC-based approach is a reliable and robust method for determining the Hoek-Brown failure criterion.

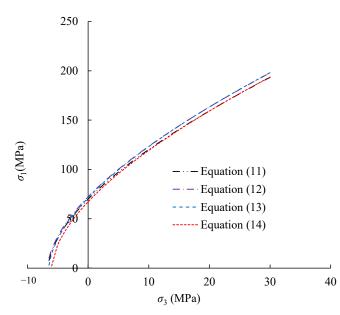
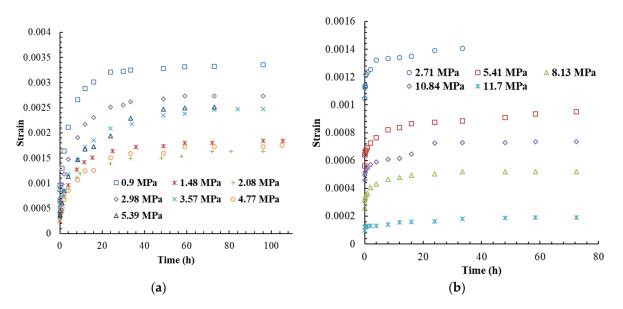


Figure 12. The Hoek-Brown failure envelope using different fitness functions.

## 5. Application

The Goupitan Hydropower Station is located in Wujiang, Guizhou Province, China. It is a landmark electricity transmission project ranging from the west to the east of the province [49]. In order to characterize the rheological property of the rock mass, rock samples of gray-green claystone and mauve claystone were analyzed using the uniaxial compression rheological test in the laboratory. The samples were taken as blocks on site and processed into  $\phi$ 50 mm × 100 mm. The test was implemented on an LYJ rheometer.

The loading method was step-by-step loading, and the load direction was perpendicular to the rock layer. After loading the first stage load, the instantaneous displacement was measured immediately; then, the creep displacement was measured at 5 min, 10 min, 15 min, 30 min, 1 h, 2 h, 4 h, 8 h, 12 h, 16 h, and 24 h (cumulative time). After 24 h, the creep displacement was measured twice a day until the creep displacement was stable (24 h displacement was no more than 0.001 mm). Then, the next shear load was applied, and the above measurements were repeated until the specimen was rheologically damaged. The strain increment was determined based on the relationship between the displacement and time at the various stress levels to the step-by-step loading. The curve between time and the increment of strain is shown in Figure 13 for different stress levels.



**Figure 13.** The curve of the uniaxial compression rheological test: (**a**) grey-green claystone, (**b**) mauve claystone.

Once the testing data were obtained, ABC could be used to determine the rheological parameters of the Burgers model. The size of the population and the maximum cycle of ABC were 50 and 100, respectively. The range of rheological parameters was determined based on the field measurements and laboratory tests (Table 3). The rheological parameters of grey-green claystone are shown in Figure 14 for the different stress levels. We can see from Figure 14 that the rock rheological parameters depend on the stress level (loading) and are not constant under different stress conditions. It is difficult to reveal the complex and uncertain relationship between rheological parameters and stress levels (loading). Generally, the average value of different stress levels is regarded as the identified rheological parameters in rock engineering. In this study, the rheological parameters were determined based on the stress conditions of the Goupitan experiment tunnel. The width, height, and buried depth of the tunnel were 2 m, 2 m, and 70 m, respectively. The experimental tunnel was excavated through rock layer  $S_{2h}^{1-1}$ ; the  $S_{2h}^{1-2}$  stratum lies about 3 m below the tunnel, and layer  $S_{1a}^1$  lies 30 m above it (Figure 15). According to the buried depth, the testing data at 1.48 MPa and 2.71 MPa was adopted to determine the rheological parameters of grey-green claystone and mauve claystone, respectively. The values of the rheological parameters are listed in Table 4. Figure 16 shows the comparison between testing data and predicted data based on the obtained rheological parameters using the Burgers model. The rheological parameters are determined based on the testing data of stress level 1.48 MPa. The predicted strain is in excellent agreement with the testing data (Figure 16b). This shows that ABC can better characterize the mechanical properties. For the other stress level, there is a higher error. The predicted curve is better closer to the 1.48 MPa stress level (Figure 16a,c,d) than further away (Figure 16e-g). There is a larger discrepancy

between the test data and predicted values in Figure 16f because the stress level is 4.77 MPa, far removed from 1.48 MPa. This shows that the rock rheological parameters should be determined based on the practical in situ stress conditions. A comprehensive comparison of the predicted curve based on the full test data is shown in Figure 16h. It shows that the rheological parameters should be determined based on the field stress conditions. Figure 17 shows the results of mauve claystone at a 2.71 MPa stress level. This proves that the parameters obtained by ABC characterize well the deformation and mechanical properties of rock.

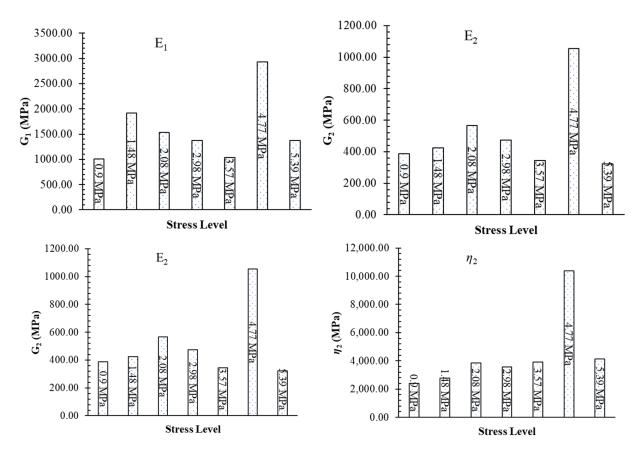


Figure 14. The rheological parameters based on different stress levels using ABC.

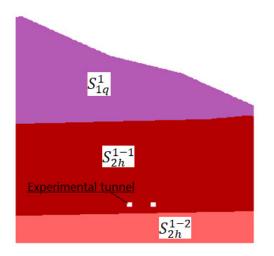
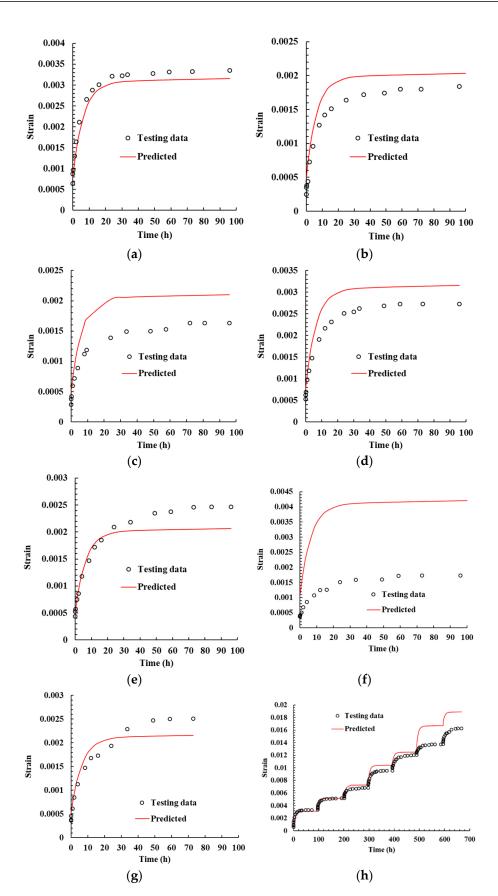


Figure 15. The geology map of the Goupitan experimental tunnel.



**Figure 16.** The curve comparison between test data and values predicted by the obtained rheological parameters using the Burgers model: (**a**) 0.9 MPa, (**b**) 1.48 MPa, (**c**) 2.08 MPa, (**d**) 2.98 MPa, (**e**) 3.57 MPa, (**f**) 4.77 MPa, (**g**) 5.39 MPa, and (**h**) predicted curve based on the full test data.

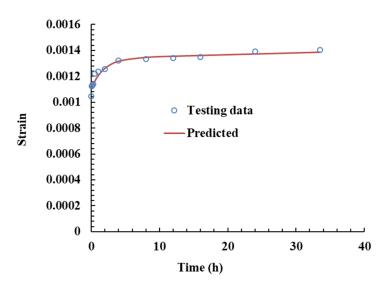


Figure 17. The predicted and testing data at 2.71 MPa stress level for mauve claystone.

Table 3. The ranges of rheological parameters.

Rock Type	<i>E</i> <sub>1</sub> (MPa)	<i>E</i> <sub>2</sub> (MPa)	$\eta_1$ (MPa·h)	η <sub>2</sub> (MPa·h)
Grey-green claystone Mauve claystone	$\begin{array}{c} 5.0\times10^{2}  4.5\times10^{3} \\ 1.0\times10^{3}  1.5\times10^{4} \end{array}$	$\begin{array}{c} 1.0 \times 10^2  3.5 \times 10^3 \\ 5.0 \times 10^3  2.0 \times 10^4 \end{array}$	$\begin{array}{c} 3.6 \times 10^4  8.4 \times 10^6 \\ 3.6 \times 10^5  1.08 \times 10^8 \end{array}$	$\begin{array}{c} \textbf{2.4}\times 10^2  \textbf{8.4}\times 10^4 \\ \textbf{2.4}\times 10^3  \textbf{3.6}\times 10^6 \end{array}$

Table 4. The value of rheological parameters.

Rock Type	<i>E</i> <sub>1</sub> (MPa)	<i>E</i> <sub>2</sub> (MPa)	$\eta_1$ (MPa·h)	$\eta_2$ (MPa·h)
Grey-green claystone	1169.0985	390.9448	1,041,173.4899	2500.9780
Mauve claystone	2432.5317	10,800.7721	36,000,000.0000	24,000.1511

Figure 18 shows the changing process of population fitness with the increase of the cycle. The fitness of the population is converged to the constant with the cycle. Figure 19 shows the convergence of the ABC algorithm. This shows that the developed method has a good global optimal and convergence performance. This proves again that the developed method has a good performance and can quickly determine the solution in a more extensive range.

In the field, an experimental tunnel was excavated to observe the rheological behavior of the claystone. Some monitoring sites were set up to investigate the deformation of the surrounding rock during excavation. The experimental tunnel was analyzed based on the determined rheological parameters in this study. The width, height, and buried depth of the tunnel were 2 m, 2 m, and 70 m, respectively. Fast Lagrangian analysis of continua (FLAC) software was utilized to compute the deformations of the surrounding rock mass. The deformation of the surrounding rock mass was evaluated based on the rheological parameters of the Burgers model. The displacements of the monitored borehole were then evaluated by a numerical model that used the determined rheological parameters. These are compared in Figure 20, in which it is evident that the predicted displacement meets the laws of monitored displacement with increasing time. This proves that the testing data are helpful for characterizing the mechanical behavior of the rock mass. The geotechnical parameters obtained in this way can be helpful for design, stability analysis, construction, and safety management in rock excavation.

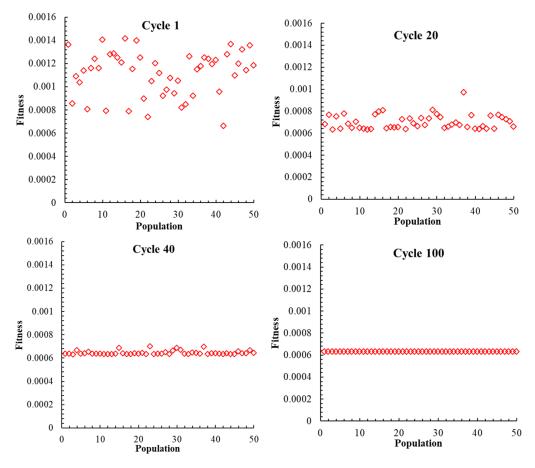


Figure 18. The varied process of fitness with the cycle.

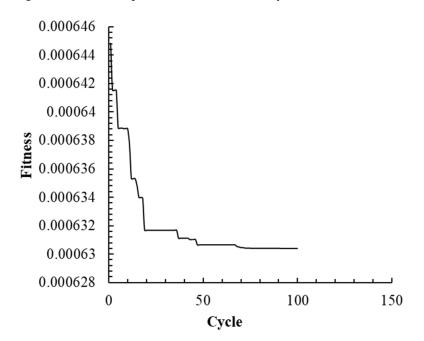
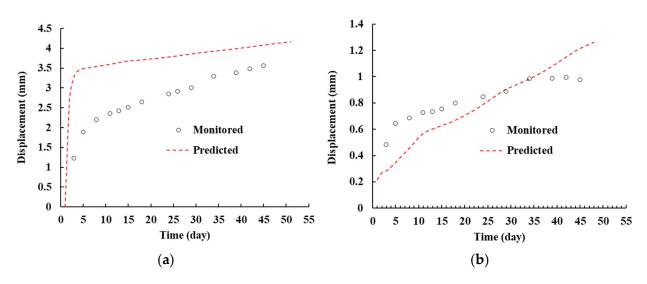


Figure 19. The convergence of ABC algorithm.



**Figure 20.** The displacement comparison between monitored data and values predicted by the numerical model based on obtained rheological parameters: (**a**) the monitored point at #4 borehole and (**b**) the monitored point at #6 borehole.

# 6. Conclusions

The rock failure criterion is critical to rock excavation. However, the determination of rock strength is a challenging issue because of the unclear mechanism of rock failure and the complexity of geology conditions. This study develops an ABC-based approach to determine the rock strength parameters by combing them with the experimental data.

- (1) In this study, ABC was utilized to determine the strength parameters, which characterize the rock failure mechanism and deformation behavior. Once the strength parameters were determined, the corresponding failure criterion could be used to evaluate the stability of the rock mass and determine the supporting pattern of the surrounding rock mass in practical engineering.
- (2) Laboratory testing is a common way to determine rock mass strength. This study developed an ABC-based approach to characterize the rock strength properties based on the Hoek-Brown and Burges models. Test data, hiding the rock failure mechanism, were utilized to capture the rock failure criterion based on the mathematical tool. The determined Hoek-Brown failure envelope was in excellent agreement with the experimental curve. The ABC-based approach provides a promising and scientific way to determine the rock failure criterion.
- (3) The fitness function is an essential component of the developed approach. The ABCbased approach can determine the optimal material constants using different fitness functions based on experimental data and avoids the limitations and disadvantages of the traditional methods. Meanwhile, the strength parameters obtained by the developed approach characterize well the deformation and strength properties of the rock mass.
- (4) It is challenging to understand and characterize the failure mechanism and deformation behavior of a rock mass. Thus, it is not easy to determine the rock strength criterion and its coefficient. The ABC-based approach has a good performance for global optimization; it can avoid the optimal local solution and provides an alternative tool to address it, which is illustrated by its performance. The developed approach focuses on the Hoek-Brown failure criterion and the Burger model for rock mass. It is worth noting that the proposed ABC approach can be employed to determine the strength parameters of other failure criteria in rock mechanics and engineering. Further study is necessary for various strength criteria in the future.
- (5) In this study, ABC was adopted to determine the strength parameters of rock mass based on laboratory data. ABC has been proven to have a strong global searching

capability, which can significantly increase the efficiency of the strength parameter determination process. However, the efficiency depends on the number of laboratory data. With the increase in laboratory data, the computation time will increase. The developed method will be further studied by combining it with a new seeking strategy in the future.

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