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Software for the Multi-Criteria Design of the External Walls Based on User Priority

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Abstract: The external walls of buildings account for a substantial part of the financial costs of the entire construction, and there can be a loss of up to 35% of thermal energy through them. By properly optimizing the price for the construction of m^2 of the external wall structure and its thermal technical parameters, interesting savings can be achieved. At present, there is no multi-criteria analysis for designing external wall structure compositions involving broader input options according to the user's technical parameters and priorities. There is a large selection of special software in the Czech and European markets, but the software is focused only on the narrower area of design and ignores the issue of building material prices. The aim of this work is to create an algorithm that reliably finds the composition that best meets the user's requirements using a wide database of materials and selected mathematical methods. This article presents an algorithm that would design the ideal composition of an external wall. This algorithm has two options for searching. The first is based on eight technical criteria and the prices of materials used in combination with user priorities. The second option is to find the best composition based only on the specified interval of the selected technical parameters. Materials databases and the use of existing computational methods, such as the Saaty method and the WSM—weighted sum method, applied to the algorithm are essential to find the composition. According to the assignment, the structures will be clearly quantified in values from 1 (best) to 0 (worst). The algorithm, which is based on the analysis of data, sources, and theories of multi-criteria decision-making, should, therefore, facilitate the design of the external wall. At the end of this article, there is a verification of the functionality of the algorithm on a case study. We believe that software that uses the proposed algorithm could be very useful for practice.

Keywords: multi-criteria analysis; external walls; design; priority

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1. Introduction

The external walls of buildings account for a substantial part of the financial costs of the entire construction. The complete structure of the external walls of the average house in the Czech Republic accounts for approximately 11% of the total cost of the house. [1] In addition, there is a loss of about 35% of thermal energy through the external wall [2]. The combination of these two factors clearly shows that by properly optimizing the price for the construction of m^2 of the external wall structure and its thermal technical parameters, we can achieve interesting savings. A common problem in construction practice is that the client (investor) does not know the price levels of the individual parts of the house construction. Often, the client wants to build a house but has an unrealistic idea of the costs. In the final phase of thinking about the client's house, the total price is always the determinative factor, as it is when we buy a car, shoes, a vacation, and other things for our lives. The proposed algorithm can quickly point to the price gap between houses in a high thermal technical standard and a standard on the edge of standard requirements. Today, the construction market offers a large number of materials and their potential combinations. Even for the professional public, it is often difficult to decide which is the most suitable option under the solved conditions. As model examples, we can mention the plot in the gap site of the city, with a view to the construction of an apartment building. Therefore, we have a clearly defined plot. The client's priority is to maximize the usable area. Therefore, we are looking for a structure of the narrowest possible dimensions while meeting the standard requirements. For interest, the price of m^2 of usable area for new apartments in Prague is 107,100 CZK/m^2 [3]. Each m^2 of the extra usable area has great value for the investor. Another model situation can be a house in the passive standard, where we are looking for a structure with high heat capacity and, at the same time, a low value of the heat transfer coefficient U (W/(m^2·K)). We can meet a house that requires higher masonry compressive strength f_k (MPa) and, at the same time, increased requirements for the acoustic attenuation R_w (dB). An algorithm, with the help of multi-criteria analysis of user priorities, can significantly facilitate the design of the structure in these and many other situations of construction practice.

1.1. Multi-Criteria Analyses in Construction

Multi-criteria decision-making plays an increasingly important role in decision-making situations that happen every day. In most cases, we do not even realize that this is a type of task. An essential factor is that these decision-making situations do not have only one criterion on the basis of which to decide. Consequently, multi-criteria decision-making methods are becoming increasingly important [4]. Today, there is an increasing and broader choice of materials, each with its specific properties, advantages, disadvantages, or various limitations. Material selection is a typical task for multi-criteria decision making. Materials are often specified by clearly given physical or quantitative values so that we can specify limit values and priorities. If we can only express a material qualitative value, the decision-making process is not as accurate [5]. In practice, there is often a need for a compromise between conflicting objectives. The goal of material selection is to optimize many performance metrics in the environment for which it is used. Standard criteria such as price, thermal resistance, heat capacity, strength, acoustic attenuation, etc. are often conflicting because choosing a material that optimizes one metric will generally not do the same for others. Then, the best choice is a compromise that does not optimize any property to perfection but pushes everything as close to their optimism as their interdependence allows [6].

An optimized approach to building design is currently popularly spread around the world. This approach is an effective measure that reduces construction costs. The optimized approach aims, for example, to increase the thermal comfort of users, improve their comfort in the interior of buildings, and, especially, reduce the energy consumption of buildings in order to achieve sustainability. All of these considerations lead to environmental friendliness, which we also find necessary, and, therefore, our work is aimed towards a similar direction of optimization.

This article focuses on the issue of the quality of the building envelope, specifically the quality of the structure of the external walls made from bricks, using a new computational algorithm that verifies this quality. The article partly follows previous work by one of the authors [7], in which the self-sufficiency of the house was evaluated using a simple point method according to the technologies used, the heat transfer coefficient, the output of the photovoltaic power plant, the disposal of rainwater and sewage, and the storage of surplus energy. The computational algorithm is created to help designers, architects, construction companies, and investors design the structure of the external wall. The calculation tool designed in this way facilitates the correct design of the structure of the external wall for a specific building and model situation. This proposal is made based on defined user criteria while meeting the relevant standards and laws, namely, ČSN 73 0540-2 [8], ČSN 73 0810 [9], ČSN 73 0802 [10], ČSN 73 0532 [11], ČSN EN 1996-1-1+A1 [12], ČSN EN 1996-1-2 [13], and ČSN 73 0540-4 [14]. The proposed algorithm will be especially useful in pre-project preparation for optimizing the design of external walls. In this case, it may mean that the user directly targets his required parameters to improve the thermal parameters of the composition, which is associated with a consequent reduction in the cost of heating the building. The user does not directly target this, but the resulting composition is designed to exactly meet his requirements and meet all the essentials, which secondarily leads to a reasonable design, which does not waste building materials and requires no additional interventions.

1.2. Actual Situation in the World

Several research groups have implemented an optimized building design in their research. Machairas et al. [15] summarized in their work proposals for optimizing the energy consumption processes of buildings. This article highlights the ever-increasing importance of optimization methods used in building design and the ever-increasing number of studies dealing with this topic. Nguyen et al. [16] provide an overview of the applications of simulation-based optimization methods in the building sector, aiming at outlining the major obstacles in building design optimization, which are the complex natures of building simulation outputs, the expensive computational cost, the scale of the problems, multi-objective design problems, and the uncertainty of many factors during the optimization, including design variables, environmental variables, model and constraint uncertainty, etc.

In their article from 2018, Lupíšek, Růžička, Tywoniak, Hájek, and Volf [17] evaluated the use of the evaluation of the quality of housing in buildings based on multi-criteria analyses. Saviz Moghtadernejad dealt with multi-criteria optimization for the preliminary design of a sustainable façade composition [18]. He further developed his work by creating multi-criteria decision-making tools [19]. One of the articles that partly deals with the economic side of construction and the economic return of the selected composition of the walls is discussed in work [20]. Researchers from China applied a multi-objective genetic algorithm to optimize energy efficiency and thermal comfort in building design and presented a case study with the aid of the multi-objective approach [21]. This work, like ours, deals with the solution to the collision of conflicting requirements in the design of a building using the algorithm. However, their research focuses on energy consumption and the indoor thermal comfort status of residential buildings. Wright et al. [22] presented the results obtained by applying a multi-objective genetic algorithm search method in the identification of the optimum pay-off characteristic between the energy cost of a building and the occupant thermal discomfort for three design days and three building weights. The next research work [23] is focused on the comparison analysis of simplification methods of building performance optimization for passive building design. This study of building performance optimization firstly develops a simplification matrix based on two criteria of complexity and flexibility to categorize existing passive design options. Secondly, four simplified configurations are compared to evaluate their impact on the quality and correlation of optimized solutions. A prefabricated building was selected as a case study, and three performance metrics were defined as energy use intensity, thermal discomfort ratio, and dissatisfaction with daylight illuminance. Bambrook et al. [24] optimized a model for a detached house in Sydney using a building energy simulation program to reduce the annual heating and cooling requirements to the point where a heating and cooling system is no longer necessary. It was shown that by using building energy simulation and optimization, the space heating and cooling energy requirements of a new house in Sydney can be cost-effectively reduced by up to 94% compared to Australia's legislated requirements. Zhuang et al. [25] presented a performance integrated BIM (P-BIM) framework for building life cycle energy efficiency and environment optimization. The optimized design was applied to a case study of a school building using a BIM tool implemented directly into the Autodesk Revit platform. The focus on the building envelope is, for example, in work [26]. This study presents the application of an analytic network process (ANP) model indicating the order of priority for high performance criteria that must be considered in the assessment of the performance of adaptive facade systems for complex commercial buildings. Sambou et al. [27] used genetic algorithms for the optimization of a building wall. The aim of optimization was to maximize thermal insulation and maximize thermal inertia. Research work [28] deals with the optimized design of building materials and states that functional, structural, design, economic, and defect factors must be comprehensively considered in tandem with objective evaluation criteria when deciding on exterior building materials. If end-users or engineers are not knowledgeable about the durability and defect factors of exterior building materials, there is a high probability of making an inappropriate selection; they should clarify the various requirements for cladding and then implement the proper performance evaluation. Of course, the optimized approach can also be applied to the design of sustainable energy retrofit in buildings [29], where the Analytic Hierarchy Process (AHP) is used to determine the relative weight of each criterion. Research work [30] is also interesting; the purpose was to optimize building envelope parameters and the ventilation systems by considering passive cooling strategies to improve the indoor thermal comfort. The algorithm was applied in this optimization study with three objectives: (a) the minimization of indoor thermal discomfort, (b) the minimization of construction costs, and (c) the minimization of the building weight. Furthermore, we should not forget a review on the optimization methods applied in energy-efficient building geometry and envelope design [31], which states that building envelope parameters and geometric configurations can considerably influence the building energy performance.

The authors of the article know of only one algorithm dealing with the design of external walls. The design of the external walls by linear programming was discussed in an article by Saladin and Soler from 2018 [32]. An ideal structure of the external wall is found when entering specific materials for up to six layers of external walling. The main idea of the software is to find the structure of the external wall in an ideal ratio of heat transfer coefficient U and price. Other criteria, such as total masonry thickness, construction time, and maintenance costs in the 10-year horizon, are included in the program. This application works only with a clearly defined six-layer wall structure. Combining different materials in each layer, the highest thermal benefit at the lowest possible cost is sought. However, combinations do not consider omitting a layer. It is, therefore, a clearly defined wall structure that cannot be changed. Furthermore, it does not consider in any way the importance of the individual criteria. Price with a heat transfer coefficient may not always be the dominant priority. The program is set to assess only five zones of the heat transfer coefficient according to the Spanish standard. The range of zones cannot be changed. Therefore, it is not possible to set any interval and individualize for other national standards. The program works only with five criteria. There is no option to enter a material discount. Manufacturers of materials have various special offers that can significantly advantage their products over the competition.

The algorithm in this article is a much more universal tool. It can solve and design external wall structures for a wide range of practical problems. These problems do not have to be defined only by price. It also allows us to design structures with a changing number of layers. The range of criteria is more extensive in total; it works with nine criteria, including criteria with an ecological focus. By entering a discount on materials, the current market situation can be clearly defined, allowing us to set criteria at any interval. Thanks to this, it can also be set for the standards of other countries. A combination of AHP (Analytic Hierarchy Process) and WSM (Weighted Sum Method) was used to create the proposed algorithm. AHP is one of the main mathematical models currently available to support the decision theory. AHP was developed in the 1970s by Thomas L. Saaty and is therefore often referred to as the Saaty method [33]. The WSM constructs a rating for each variant. So, it can be used both to search for one of the most advantageous variants and to organize the variants from best to worst.

This article first describes in detail the method of the proposed algorithm based on multi-criteria analysis. Another section provides an overview of all input criteria and the procedure of selecting the most suitable structure composition according to defined criteria using flowcharts. This is followed by a case study, where an example of four structures is used to show how the proposed algorithm selects the most suitable one. Finally, the results obtained from the case study are discussed.

2. Materials and Methods

The methodology is based on a multi-criteria analysis which deals with the design of the best compromise solution of the structure of the external wall with specific priorities of the user. The proposed algorithm works with nine criteria. The criteria include price levels P (CZK/m^2), heat transfer coefficients U (W/(m^2·K)), labor consumption walling per hour W (h/m), fire resistance F (min), specific heat capacity C (J/(kg·K)), acoustic attenuation A (dB), compressive wall strength S (Pa), and an environmental criterion or life cycle assessment. There are two subcriteria, namely, the amount of waste in production T (kg/FU) and the global warming potential G (kg CO_2/FU). The designation of the criteria corresponds to the designation in the flowcharts. The algorithm works with two options to search for the structure of the external wall. The first deals with the selection of the value of the heat transfer coefficient (W/(m^2·K)) according to ČSN 73 0540-2 [8] and finding the compositions with the lowest price. The second option is to select at least two criteria. Priority to other criteria must be given to each criterion. Using multi-criteria analysis, the importance value of each criterion is then obtained. The result is to find the structure with the greatest benefit according to the given priorities.

2.1. Multi-Criteria Analyses

The tasks of multi-criteria analysis can be solved from two primary points of view. The first is according to the goal of solving the problem. The second is according to the information with which the task works. According to the goal of the solution, we divide the tasks into three essential areas of the tasks. The first is to select one variant marked as a compromise. It is about finding the solution that is the best solution according to the given criteria. Second, the tasks aim at the complete arrangement of a set of variants. Usually, we rank the variants from best to worst. Thirdly, the tasks are to divide the set of variants into good and bad variants. In these tasks, one finds not so much about the order of variants as about deciding whether the considered variant is acceptable or not. The proposed algorithm searches for the structures with the best compromise for the user.

The tasks are also divided according to the type of information. It is the information about the preferences between the criteria and variants. If we do not have any information, the user cannot or does not want to decide which criterion is essential. In this case, it is possible to assign the same weight to each criterion. In the case of nominal information, its value is expressed using the aspiration level. The aspiration level divides our values into acceptable and unacceptable. Ordinal information expresses the arrangement of criteria according to importance. Therefore, the user is able and willing to express the order of importance of the criteria. And the last option is according to cardinal information. The proposed algorithm is based on this type of information. The user is able and willing to express not only the order of importance but also their ratio. This type of information ensures that the proposed algorithm can unambiguously and comprehensively describe the priority of individual criteria.

2.2. Evalution of the Importance of Individual Criteria

To determine the individual weights of the criteria, the most appropriate method was chosen, requiring cardinal information about the preferences of the criterion. The Saaty method (AHP) [4] was chosen to determine the total weights of the individual criteria. The Saaty method is ideal when only one person evaluates the criteria. The Saaty method was also chosen because it can quantify the problem sought in detail and, therefore, is informative and global. This is a method that is mathematically easy to solve and use. Therefore, this method is very suitable for this kind of algorithmic task. The application of the Saaty method begins with a problem being decomposed into a hierarchy of criteria so as to be more easily analyzed and compared in an independent manner (Figure 1).

After this logical hierarchy is constructed, the decision makers can systematically assess the alternatives by making pair-wise comparisons for each of the chosen criteria [33]. The last method used by the algorithm is WSM. Among other things, the method has a relatively simple procedure; for this reason, it was chosen for the algorithm. A more detailed description of these methods will be given in Sections 2.6.3 and 2.6.4.

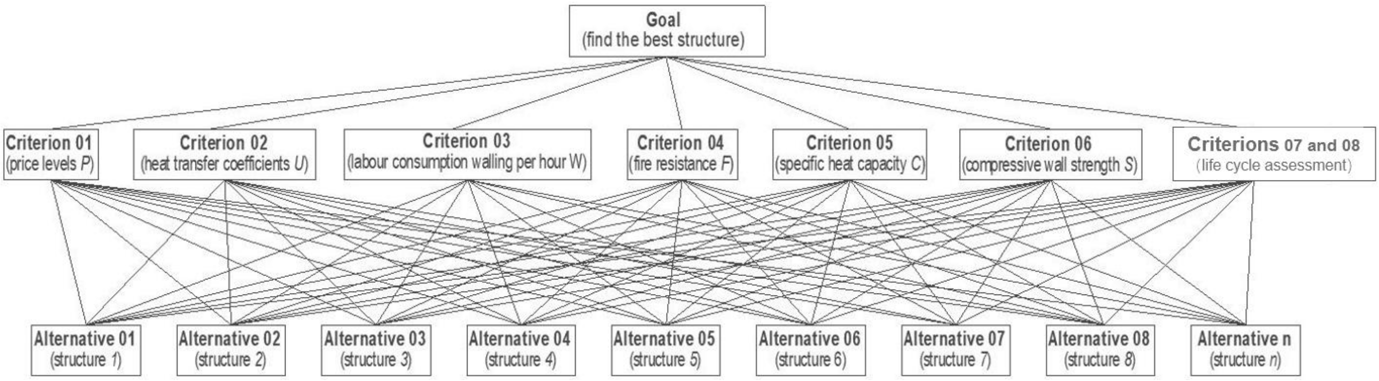

Figure 1. Example of a hierarchy of criteria.

2.3. Creating a Database

The compilation of individual structures is possible thanks to extensive databases of materials, plasters, masonry, and insulation. These databases were created using the technical documents of the manufacturers. All necessary technical data sheets are available on the manufacturers' websites [34–43]. In the case of missing parameters, individual manufacturers were asked to provide data. The required values of materials were then tabulated into the basic database. The data sets will then be evaluated by a technical-economic method that uses multi-criteria decision-making and leads to multi-criteria optimization.

2.4. Formulas Using the Algorithm for Compiling Individual Structures

2.4.1. Price Level (CZK/m^2)

The price will be formed from the price of the material (P_m) and the price of the execution of the work (P_w). The calculation of the final price of individual compositions will be made according to Formulas (1)–(3).

$$P = P_m + P_w \tag{1}$$

$$P_m = price_{\ exterior\ plaster\ m^2} + price_{\ interior\ plaster\ m^2} + price_{\ bricks\ m^2} + price_{\ insulation\ m^2} \tag{2}$$

$$P_w = price_{\ work\ on\ exterior\ plaster\ m^2} + price_{\ work\ on\ interior\ plaster\ m^2} + price_{\ work\ on\ walling\ m^2} + price_{\ work\ on\ insulation\ m^2} \tag{3}$$

2.4.2. Heat Transfer Coefficient (W/(m^2·K))

This will be calculated according to ČSN 73 0540-4 (Thermal protection of buildings part 4: Calculation methods) [14]. The calculation will be performed according to Formulas (4)–(6).

$$R_j = \frac{d_j}{\lambda_j} \tag{4}$$

$$R = R_{exterior\ plaster} + R_{Bricks} + R_{insulation} + R_{interior\ plaster} \tag{5}$$

$$U = \frac{1}{R_{si} + R + R_{se}} \tag{6}$$

where R_{si} is 0.04 (m^2·K/W) and R_{se} is 0.13 (m^2·K/W).

2.4.3. Labor Consumption Walling per Hour (h/m^2)

This will be calculated from the values of the plaster execution, the masonry of the brick system itself, and the execution of the thermal insulation. The value will be calculated per 1 m^2 of masonry performed per hour of work.

$$W = W_{interior\ plaster} + W_{bricks\ walling} + W_{insulation} + W_{exterior\ plaster} \quad (7)$$

2.4.4. Acoustic Attenuation (dB)

This will be considered with the values of the masonry material only without the influence of thermal insulation. To calculate the effect of thermal insulation, it is necessary to know the values of the dynamic stiffness of the insulation material. These values are only declared by the manufacturer for acoustic insulation. The use of acoustic insulation is not considered on external load-bearing structures due to the price and small thicknesses.

2.4.5. Fire Resistance (min)

This will be stated according to the values given by the manufacturers of brick elements. Insulation has no effect on internal fire. The user will be able to filter polystyrene or mineral insulation. Therefore, in the case of a house higher than 22.5 m, according to ČSN 73 0810 (Fire safety of buildings—Common provisions) [9], the condition of using only non-flammable thermal insulation is met.

2.4.6. Heat Capacity (J/(kg·K))

Only the values of the heat capacity of the brick elements will be considered. The heat capacity of thermal insulation materials is small. The thermal capacity of the plasters is neglected due to its thickness.

2.4.7. Compressive Wall Strength (Pa)

Only the masonry strength values will be considered. Added thermal insulation and plaster do not affect this value. The calculation will be performed according to Formula (8) based on the standard [13].

$$f_k = K \times f_b^{0.7} \times f_m^{0.3} \quad (8)$$

where K is a constant depending on the masonry elements, f_b is the standard strength of masonry elements, and f_m is the strength of the mortar.

2.5. The First Method of Searching for Structures by ČSN 73 0540-2

The first main possibility of using the proposed algorithm is to find the ideal structure of the external walls by the three limit categories according to ČSN 73 0540-2 (Thermal protection of buildings—Part 2: Requirements) [8]. By this Czech energy standard, there are three categories of the heat transfer coefficient. Each house built in the Czech Republic must meet one of these categories. The procedure of the algorithm will be described in the following paragraphs.

2.5.1. Selection of Category by ČSN 73 0540-2

In the standard ČSN 73 0540-2 (Thermal protection of buildings—Part 2: Requirements) [8], the categories for external structures are listed in Table 1.

Table 1. Categories for external structures according to standard ČSN 73 0540-2.

	ČSN 73 0540-2 (W/(m^2·K))		
	$U_{rec,20}$ Required	$U_{pas,20}$ Recommended	$U_{pas,20}$ Recommended
External wall	0.30	0.25–0.20	0.18–0.12

In the first step, the user must select the category according to ČSN 73 0540-2 [8]. Thereafter, the program will select structures that are in a selected category. In the flowchart, these categories are referred to as U_h and U_d.

2.5.2. Definition of Insulation Selection

In this step, the user can choose whether he wants to compare the structure with the mineral and polystyrene insulation or exclude any of them. This function can be useful; for example, when we want to build buildings higher than 22.5 m, we must use only mineral insulation according to [9].

2.5.3. Entering Discount from Retail Price

The user is asked if he has a discount on a particular material or if a specific manufacturer does not have a special offer. This feature will be especially useful for construction companies that often have specific prices in the stores. Entering a discount on a specific product brand will significantly affect the results. Without this feature, the program would only compare the external wall structures based on retail prices, which would be very distorting.

2.5.4. Other Evaluation Criteria

In the end, the user can enter any other criterion and set limit values. Possible criteria were specified in Section 2. The user can choose how many of the following criteria to add. For example, one criterion may be selected: an increased compressive wall strength. This may be desirable if we want a passive house, but we also know that the masonry must have a strength of at least 15 MPa. It is up to the user to select or skip this step.

2.5.5. Getting a Final Result

After the completion of the discount part, the program will sort the resulting structures of the external walls according to the entered criteria and by price. The user gets the best possible external wall structures according to his criteria for the best current price on the market.

2.5.6. The Flowchart

Figure 2 is a flowchart showing, in a simplified manner, the steps that are shown in detail in Figure 3. Figure 3 shows the whole algorithm process. Part 1 of Figure 3 shows what the user must do in the input step.

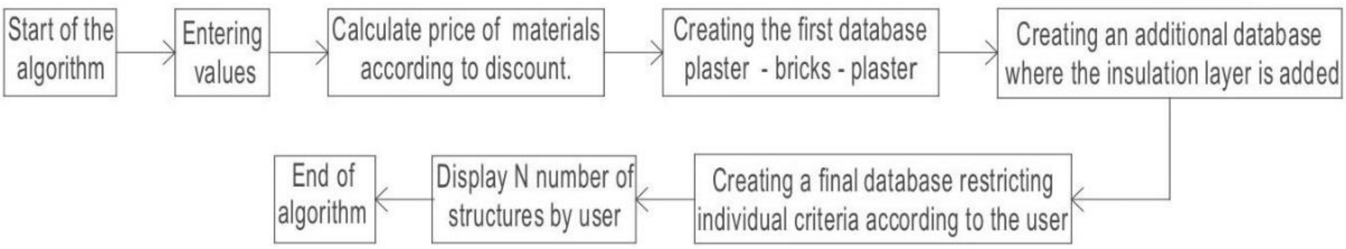

Figure 2. Flowchart showing a brief algorithm progress.

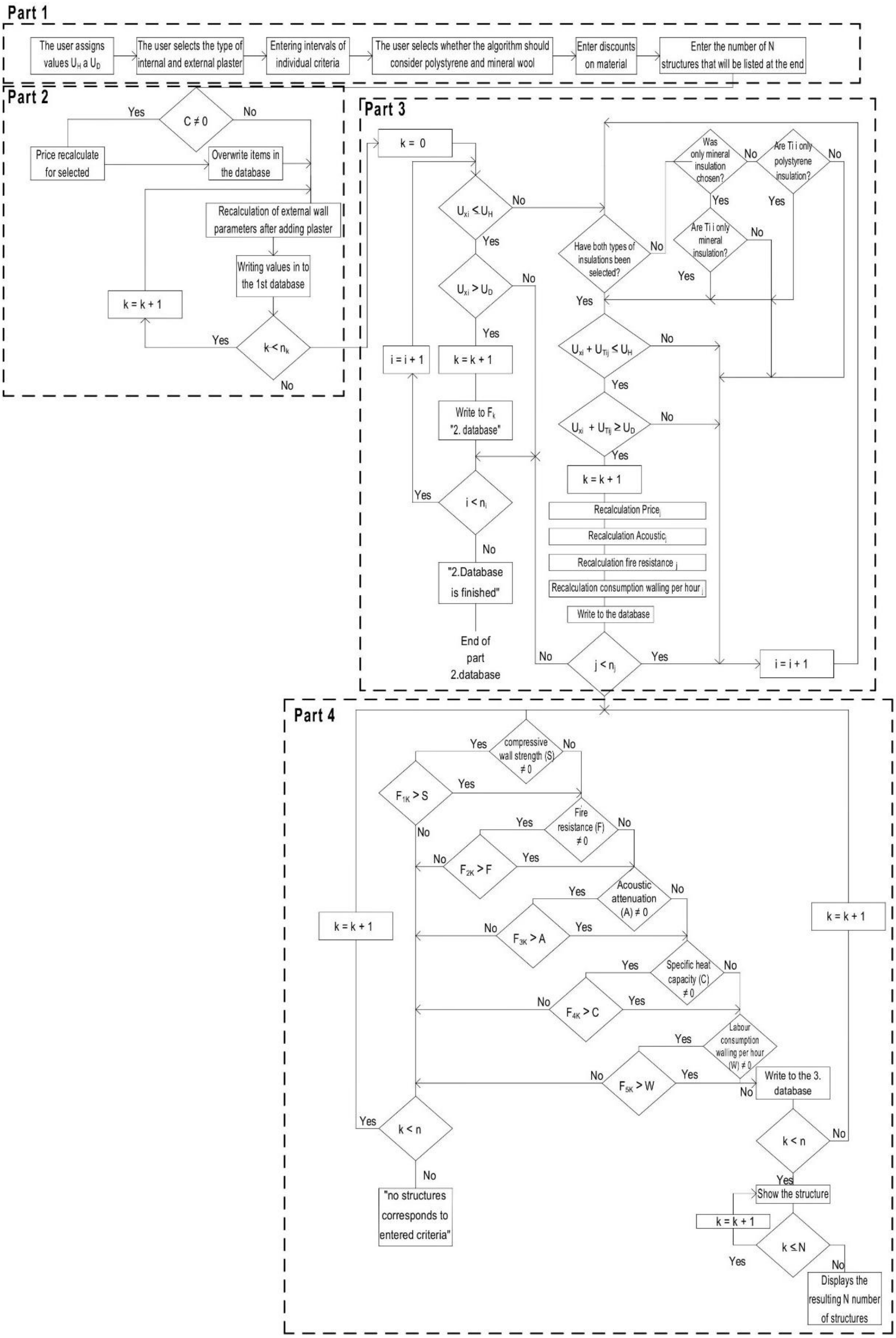

Figure 3. The whole process of the first method of searching structures by ČSN 730540-2.

The input prices of the materials are recalculated after the assignment is complete. This step takes place only if the discount was specified in the assignment. Otherwise, this step is skipped. The part of the flowchart depicting this section is shown in Part 2. Part 2 also shows the creation of the first database containing only plaster–masonry–plaster structures.

Part 3 shows a section of the flowchart in which individual pieces of bricks are evaluated in the first section to see if they fall into the given category according to Section 2.5.1. If not, the insulation is added step by step according to the type of insulation until it reaches the required values of U_h and U_d, and then it is written in the database. The insulation has a maximum thickness of 300 mm. Part 4 shows a section of the flowchart where, according to Section 2.5.4., individual structures of the external wall are sorted according to other criteria selected by the user. This part also shows where the program lists the number of structures of external walls according to the user input.

2.6. The Second Method of Searching for Structures by the Priority of Each Criterion

The second main option to find the ideal structure of an external wall using an algorithm is to specify the priorities of each criterion. The number of criteria must be between two and nine. The criteria that can be chosen are described in Section 2 of this article. The algorithm, using the Saaty method, determines the weight of each criterion, and the weighted sum method finds the best structure of an external wall according to the user's priorities. The procedure of this part of the algorithm will be described in the following paragraphs.

2.6.1. Setting Criteria Limit Values

In the first step, the user must enter the limit values U (heat transfer coefficients). The second step is the same as in Section 2.5.2. The third step is the possibility to set the interval of values of individual criteria in which the program will search for the ideal structure. This can also significantly reduce the computational complexity of the algorithm. This function can be handy, for example, if the investor is working with a specific budget. Entering this constraint will get the best possible structure of the external wall up to its budget possibilities.

2.6.2. Selection of Criteria and Set up Their Importance

In the fourth step, the algorithm asks the user how many and which criteria will be evaluated. The user must rate each criterion with a number between 1 and 9.

2.6.3. Saaty Method

Now, we have to evaluate the selected criteria using the Saaty method. A 9-point scale is used to evaluate the pairing criteria, and it is also possible to use intermediate steps (values 2, 4, 6, 8):

1—equivalent criteria i and j, 3—weakly preferred criterion i before j, 5—strongly preferred criterion i before j, 7—very strongly preferred criterion i before j, 9—absolutely preferred criterion i before j.

The user compares each pair of criteria and the sizes of the preferences of the i-th criterion in relation to the j-th criterion; then, the algorithm writes into the Saaty matrix $S = (s_{ij})$:

$$S = \begin{pmatrix} 1 & s_{12} & \dots & s_{1n} \\ 1/s_{12} & 1 & \dots & s_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/s_{1k} & 1/s_{12} & \dots & 1 \end{pmatrix} \quad (9)$$

If the i-th and j-th criteria are equivalent, $s_{ij} = 1$. If the user weakly prefers the i-th criterion over the j-th, $s_{ij} = 3$. If he strongly prefers the i-th criterion over the j-th, $s_{ij} = 5$. With very strong i-th criterion preference $s_{ij} = 7$, and with absolute preference, $s_{ij} = 9$. If the j-th criterion is preferred to the i-th criterion, the inverse of the Saaty matrix is written

(s_{ij} = 1/3 for weak preference, s_{ij} = 1/5 for strong preference, etc.). The size of the matrix depends on the number of selected criteria k, so it has a size $k \times k$. When five criteria are selected, there will be a matrix size of 5 × 5. The matrix is reciprocal, meaning that $s_{ij} = 1/s_{ji}$, and expresses an estimate of the weights of the i-th and j-th criteria. On the diagonal of the Saaty matrix, the values are always equal to one (each criterion is equivalent to itself).

The elements of the Saaty matrix are usually not entirely consistent, which means that $s_{hj} = s_{hi} \cdot s_{ij}$ does not apply to all $h, i, j = 4, 5, \ldots, n$. If we assemble a matrix $V = (v_{ij})$ whose elements would be real proportional weights ($v_{ij} = v_i/v_j$), for the elements of this matrix, the above condition will apply. Consistency is measured, for example, by the consistency index that Saaty defined as:

$$I_S = \frac{l_{max} - n}{n - 1} \quad (10)$$

where l_{max} is the largest eigenvalue of the Saaty matrix and n is the number of criteria. The Saaty matrix is considered sufficiently consistent if $I_S < 0.1$. The weights v_j could be estimated from the condition that matrix S should differ as little as possible from matrix V. Saaty has proposed several very simple numerical methods by which weights v_j can be estimated. The most commonly used method is to calculate the weights as a normalized geometric diameter of the rows of the Saaty matrix (the logarithmic least squares method). We compute the b_i values as the geometric mean of the rows of the Saaty matrix:

$$b_i = \sqrt[n]{\prod_{j=1}^{n} S_{ij}} \quad (11)$$

The individual weights are then calculated by normalizing the b_i values:

$$v_i = \frac{b_i}{\sum_{i=1}^{n} b_i} \quad (12)$$

Cases where the Saaty matrix is inconsistent are widespread, especially for larger tasks. Inconsistency may be caused by an error when entering weighting estimates when the user does not check their estimates. In this case, it is necessary to re-quantify the Saaty matrix based on the weight estimates to meet the consistency requirement and then make a new weight estimate. In this interactive way, excellent results can be achieved.

2.6.4. Weighted Sum Method

After obtaining the individual weights of each criterion, it is necessary to recalculate the individual technical values of the external wall structures in an interval from 0 to 1 by the weighted sum method [44], where 0 is the worst variation and 1 is the ideal variation. We obtain these values by creating the criterion matrix R and the criterion weights vector $\underline{v}$ (using the Saaty method). This method constructs an overall rating for each variation, so it can be used both to search for one of the most advantageous variations and to organize variations from best to worst. This method is based on the principle of maximizing utility. If variant a_i according to criterion j reaches a specific value of y_{ij}, it brings the user a benefit, which can be expressed using a linear utility function. The total benefit of the variant is expressed by the weighted sum of the values of the partial utility functions:

$$u(a_i) = \sum_{i=1}^{m} v_j u_j(y_{ij}) \quad (13)$$

where u_j are the subfunctions of the benefit of individual criteria and v_j are the weights of the criteria.

The steps for this weighted sum method are as follows:

The ideal variant h_j and the worst variant d_j are determined. A standardized criterion matrix R, whose elements are obtained by Formula (14), is created.

$$r_{ij} = \frac{y_{ij} - d_j}{h_j - d_j} \quad (14)$$

Matrix R already represents a matrix of values of the function of benefit from the i-th variant by the j-th criterion, since the elements of this matrix are the linearly transformed criteria values such that r_{ij} belongs to [0; 1]. Then, the basal variant is 0, and the ideal variant is 1. For each variant, we calculate the aggregate utility function:

$$u(a_i) = \sum_{j=1}^{n} v_j r_{ij} \quad (15)$$

2.6.5. The Final Result

The algorithm sorts the variants in descending order according to the values u (a_i). We consider the highest values to be the best solution.

2.6.6. The Flowchart

Figure 4 shows a summary of the algorithm progress for the second method of searching for structures by the priority of each criterion. Figure 5—Part 1 shows what the user has to enter before creating the structure databases.

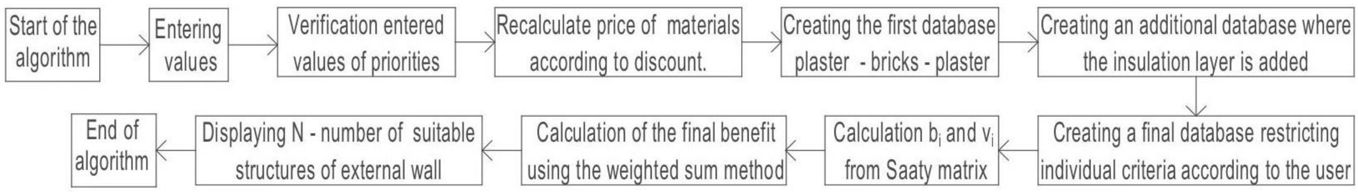

Figure 4. Simplified summary procedure of the algorithm for the second possibility to search for the structures of external walls.

Figure 5—Part 2 shows the verification of the Saaty matrix in order to know if the consistency condition of (10) is met; if the consistency measure corresponds to the limit value, the algorithm proceeds by creating the Saaty matrix according to (9).

After checking the consistency of the matrix, the algorithm recalculates the basic material prices according to the discount, as shown in Figure 3—Part 2. The creation of the first two databases is then the same as shown in Figure 3—Parts 3 and 4. The final utility of the external wall structure from the Saaty matrix and the weighted sum method is shown in Figure 5—Part 6, and the final result is also shown there.

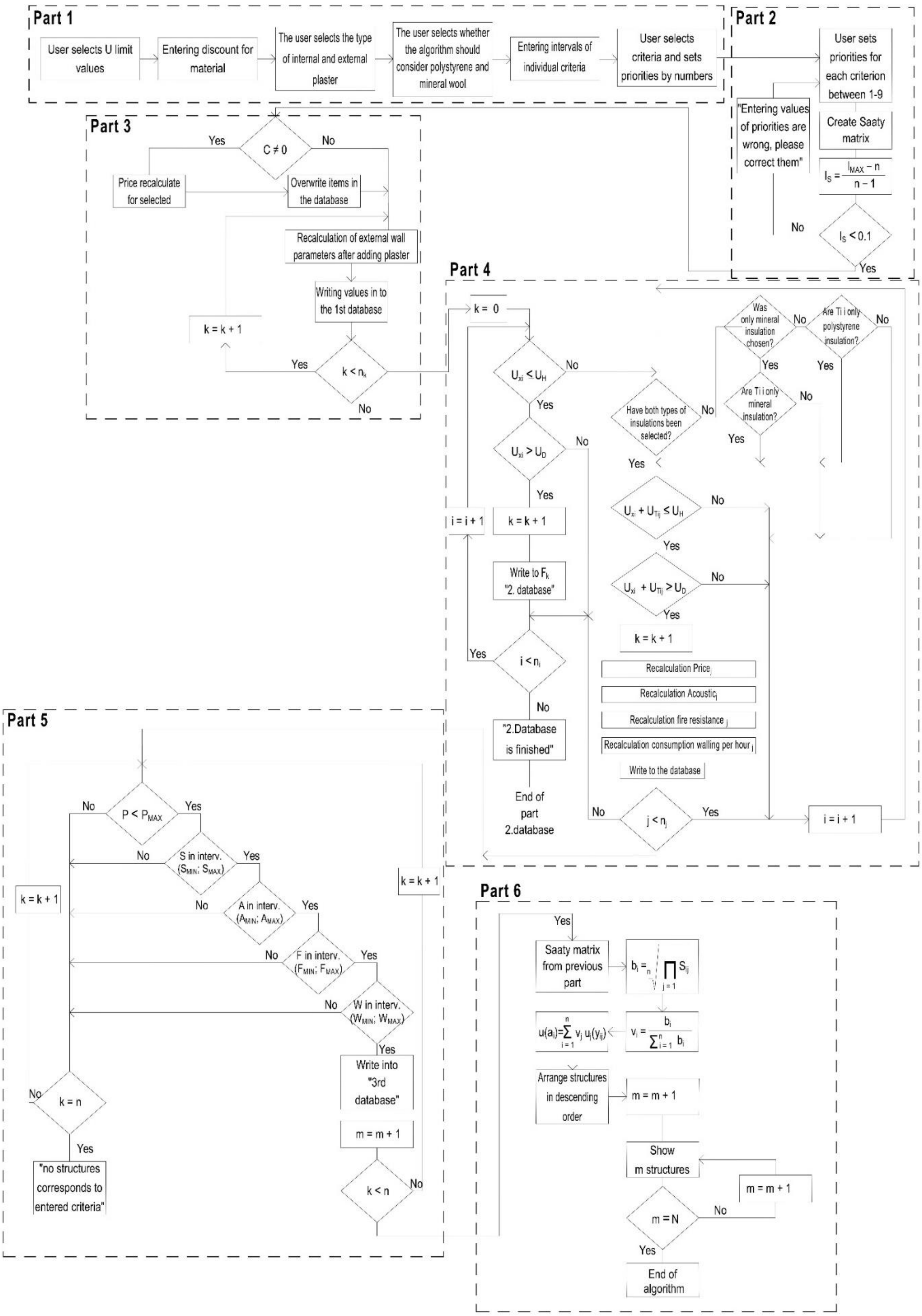

Figure 5. The whole process of the second method of searching for structures by the priority of each criterion.

3. Case Study and Discussion

It is not possible to obtain a clear conclusion or a single general result with this algorithm. It always depends on the specific situation and the interests of the user. For the sake of clarity, this article presents a case study that briefly describes the practical procedure of the whole design process of the structure. The user is looking for an external wall structure for his low-energy house. The plot is located near a busy road. Therefore, the user has the following requirements: building sound insulation of at least 48 dB, maximum heat transfer coefficient of 0.20 W/m^2·K, and masonry compressive strength of at least 8 MPa. For clarity, only four structures will be listed (Table 2, Figure 6). Table 3 shows the technical parameters of these external walls.

Table 2. Structures of four random external walls.

Structure	Exterior Plaster	Insulation	Brick	Interior Plaster
1	Silicon Baumit Nanopor Top	EPS Grey Wall 180 mm	Porotherm 30 Profi, 300 mm	Gypsum Baumit Ratio Glatt L
2	Silikate Baumit Star EPS	EPS 70-F 160 mm	Ytong P6-650, 375 mm	lime-cement Baumit MPI
3	Silicone-Silicate Ceresit CT 174	-	Porotherm 44 Eko + 440 mm	Gypsum Baumit Ratio Glatt L
4	Silicon Baumit Nanopor Top	EPS 70-F 200 mm	Sendwix 14DF-LDE 200 mm	gypsum Cemix 136

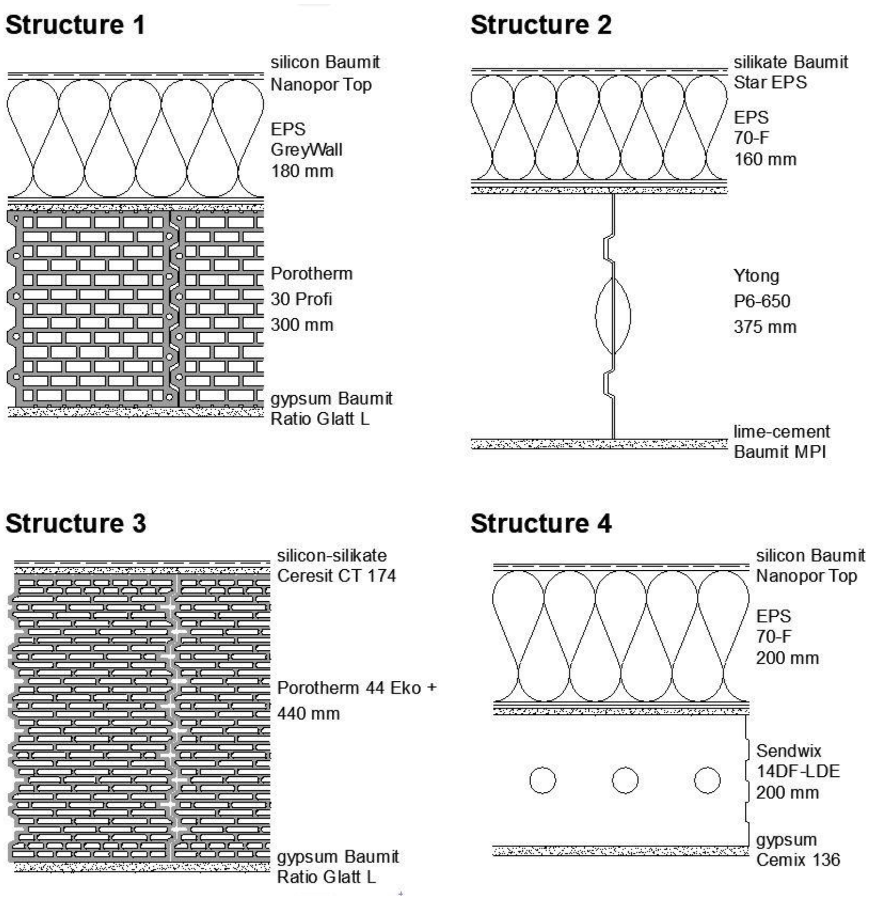

Figure 6. Structures according to Table 2.

Table 3. Technical parameters of external walls.

Structure	Heat Transfer Coefficients U (W/(m^2·K))	Compressive Wall Strength (MPa)	Price Levels (CZK/m^2)	Acoustic Attenuation (dB)
1	0.137	8	4 271	47
2	0.162	6.5	4 058	49
3	0.190	8	3 998	48
4	0.170	25	3 797	49

The combinations of compositions are created in a simple way. From the databases of individual materials, the algorithm gradually combines only the plaster–masonry–plaster layers. If the user's required technical parameters are met, the combination is entered into the final list of compositions. Then, the algorithm starts to combine the composition: plaster–insulation–masonry–plaster. Again, if the user's required technical parameters are met, the combination is entered into the final list of compositions. The compositions found in this way have physical properties and prices; this creates a list of many compositions, which are then recalculated using WSM. Finally, the composition with the greatest benefit to the user is obtained. The algorithm creates thousands of combinations, which will then be evaluated.

Composition number 2 will be eliminated from the beginning because the minimum strength condition of at least 8 MPa has not been met. However, it will be retained in Tables 4 and 5 for illustration and explanation of the case study. Now, in this step, the user has to enter the priorities of the individual criteria according to Section 2.6.3. The user sets the priorities as follows:

Table 4. The Saaty matrix and values of b_i and v_i.

	Heat Transfer Coefficients	Compressive Wall Strength	Price Levels	Acoustic Attenuation	b_i	v_i
Heat Transfer Coefficients	1	5	1/2	5	1.88	0.32
Compressive Wall Strength	1/5	1	1/7	1	0.41	0.07
Price Levels	2	7	1	7	3.15	0.54
Acoustic Attenuation	1/5	1	1/7	1	0.43	0.07

Table 5. Values after recalculating Table 3 by Section 2.6.4.

Structure	Heat Transfer Coefficients U (W/(m^2·K))	Compressive Wall Strength (MPa)	Price Levels (CZK/m^2)	Acoustic Attenuation (dB)
1	1	0.08	0	0
2	0.53	0	0.45	1
3	0	0.08	0.58	0.50
4	0.38	1	1	1

Now, the algorithm checks the consistency of the matrix according to Formula (10). If the I_S consistency condition < 0.1 is met, the algorithm can proceed to the calculation of the geometric diameter of the matrix row and the calculation of the weights of the individual criteria. Formulas (11) and (12) will be used for these calculations. Now, the algorithm uses the weighted sum method and recalculates the values in Table 3 to the values at the interval

<0;1>, where 0 is the worst option and 1 is the best option. The algorithm uses Formula (13) to calculate these values. The resulting values of the criteria matrix are listed in Table 5.

The resulting utility value of the external wall structures according to the entered user priorities is shown in Figure 7. The utility values were calculated using Formula (14).

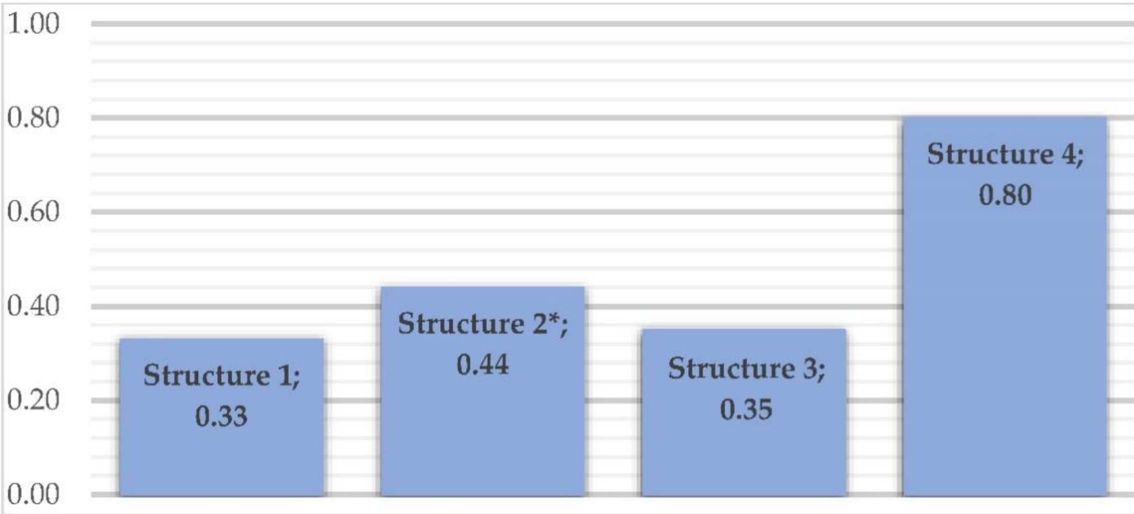

Figure 7. The final result of each structure. * Structure 2 did not meet the investor's conditions.

It is clear from Figure 7 that, with the priorities given in Table 4, structure 4 will benefit the investor the most. Structure 2 was second. Structure 2, which, according to Table 3, did not meet the investor's requirement for the strength of the masonry but was left here for explanation, is now the second best, according to Figure 7. This is because the requirement for the strength of the masonry was not given much importance. From this principle, it is clear how this software uses the Saaty method to indicate the weights of individual user requirements and thus define their importance. It follows that if the user is not really forced to state specific requirements for the technical parameters of the composition sought, he should not do so. If the user gave more importance to such a requirement, it could cause unnecessary elimination of the selection. In other words, the user should only state the requirements for which he is convinced of their importance. It can also be seen from the case study that the variance of the results after the WSM recalculation (Table 5) is quite large. This is due to the small number of compositions in the case study. With the thousands of combinations, the differences between the individual compositions will be in the hundreds to the thousands. Therefore, the overall result will not be as diametrical as in this case.

There are many studies that deal with optimization tools that focus on improving various metrics in building design. However, most of these studies deal with the optimization of a specific building [24] or only a narrower part of the design [22,27], such as the position of the building relative to the cardinal points, the size of the windows that affects the internal thermal comfort of the residents, etc. Only a few studies also implement the cost of designed optimization measures into the design [32]. In the work [24], optimization was focused on changes in the insulation thickness of the walls and roof, the type of windows, the thickness of an internal thermal mass wall, and the night ventilation air change rate. Compared to our work, the optimization of this work concerns only a specific house in Sydney, so it is not a general optimization tool for any building. The work [22] solved building optimization only in terms of its energy costs and thermal comfort. Unlike our work, it does not include construction costs and other criteria, such as static or acoustic requirements, etc. The study [27] sought ideal solutions based only on the physical properties of the materials. Again, the price level of the materials used was not reflected in the calculation at all. The focus of work [32] is the closest to the focus of our work. However, compared to our work, there is a difference in that the given structures in this work were compared only within the framework of Spanish legislation. The algorithm always works with only six layers of the structure, and, above all, it does not take into account the different weights of individual criteria, so it only works with a variant of the equal importance of each criterion. Our article works with multiple criteria and gives them different weights of importance. Unlike the above-mentioned works, the price is implemented in the work [25]. However, price indicators include only the estimated prices for the province in China where the case study building is located. In addition, the tool of this work only works with Revit. Another difference from our work is that their assessed structures were selected only from the construction library and formulated in accordance with the design Atlas of Engineering Construction Standards and the budgetary estimate quota.

4. Conclusions

This work focused on the design of an algorithm to find the composition of external walls. This algorithm can have a great impact on building design and on finding the ideal ratio of physical and environmental properties, as well as the financial costs for buildings. At present, there is no software in the Czech Republic that could compare the composition of external walls, especially in terms of technical parameters vs. price. There is a large selection of special software on the Czech market, however, that specializes only in the narrower area of designing a given construction issue. None of the software even deals marginally with the issue of building material prices. The proposed algorithm also inserts a financial aspect into the external wall design process. In the construction practice, the design of the external wall composition itself is influenced by a large number of factors. However, the final decision is almost always based on the financial capabilities of the investor. The price thus always becomes a decisive factor influencing the final decision of the investor. Situations where the investor does not address the price and only the characteristics of the composition that are a priority for him are rather exceptional. The contribution of the proposed algorithm can be especially expected in the pre-project preparation of buildings and studies. In this phase, an important step is the design of the external wall and thus the future spatial dimensions of the building. The algorithm also finds its application in the case of the unavailability of some materials and efforts to find the best alternative that does not worsen the physical properties of the wall. Non-deterioration in physical properties can be an important factor in obtaining various grant titles such as the New Green Savings Program (subsidy program in the Czech Republic for the thermal insulation of buildings and for increasing the share of renewable energy sources in buildings), where physical properties must be the same or improved. In addition, an algorithm can be used to specify the criteria of the structures sought to match the construction practice, and the best compromise can be found. A compromise solution is necessary to find the ideal composition. It often happens that, due to the improvement of one physical property, another, such as a thermal or strength property, usually deteriorates. The algorithm uses multi-criteria analysis to find the optimal compromise solution. Thanks to multi-criteria optimization, the algorithm designs compositions that support sustainable construction. This is one of the requirements of the United Nations and its Agenda 2030 document for sustainable development [45]. This document is implemented worldwide in the laws of individual countries. The proposed algorithm also allows for the consideration of the environmental friendliness of materials from raw materials sourcing and production to transport. This requirement for materials will certainly become more important in the coming years. It will be necessary to design constructions not only from the point of view of their price and ecological demands on operation but also from the point of view of the ecological and price demands of their construction. The proposed algorithm is adapted to Czech standards, but it is not difficult to adjust the algorithm formulas according to the relevant national standards of individual countries. After the successful testing of this algorithm, its use will be extended to the composition of roof structures and floor structures on the ground. Furthermore, our intention is to convert the algorithm into software (app). The main findings can be summarized as follows:

- The algorithm finds the best structure of external walls with the best benefit for the user.
- The algorithm designs wall structures that meet the requirements of technical standards.
- The algorithm designs a part of the building envelope—in particular, an external wall—in an ideal ratio of price vs. technical parameters.
- The algorithm offers a choice of up to nine technical and environmental criteria that the user assigns importance.

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