Article

Estimation of Floor Response Spectra for Self-Centering Structural Systems with Flag-Shaped Hysteretic Behavior

Binod Kumar Shrestha , Anil C. Wijeyewickrema *, Hiroki Miyashita and Niraj Malla

Department of Civil and Environmental Engineering, Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro-ku, Tokyo 152-8552, Japan; shrestha.b.ac@m.titech.ac.jp (B.K.S.); miyashita.h.ac@m.titech.ac.jp (H.M.); nirajmalla67@gmail.com (N.M.)

* Correspondence: wijeyewickrema.a.aa@m.titech.ac.jp

Abstract: Spectral floor-acceleration demands are necessary for the seismic design of acceleration-sensitive non-structural components (NSCs). Existing studies to estimate floor response spectra (FRS) using empirical equations are based on elasto-plastic and stiffness degrading hysteretic behavior of the primary structure. In the present study, the FRS for self-centering (SC) structural systems with flag-shaped hysteretic behavior under far-fault ground motions are investigated. The FRS and dynamic amplification factor (DAF) are obtained from nonlinear response history analysis (NLRHA) of SC structural systems with flag-shaped hysteretic behavior. An equation to estimate the FRS is proposed and verified by carrying out NLRHA using a different set of far-fault ground motions. The equation to estimate FRS is shown to predict floor acceleration demands with very good accuracy.

Keywords: dynamic amplification factor; flag-shaped hysteretic behavior; floor response spectra; non-structural components; self-centering system

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1. Introduction

The adoption of seismic design has greatly reduced the damage to the structural elements of buildings. However, damage to secondary structural elements and non-structural components (NSCs) can lead to huge economic losses, due to associated loss of functionality of important facilities and business downtime after a major earthquake (see for eg., EERI [1], Villaverde [2], Dhakal et al. [3], Devin and Fanning [4], Wang et al. [5]). Although existing building codes such as Eurocode 8 [6] and ASCE 7-16 [7] provide expressions to determine the acceleration demand for the estimation of seismic design force for acceleration-sensitive NSCs, several studies have shown that the peak floor acceleration and floor response spectra (FRS) estimated from such codes are not accurate (see for e.g., Sullivan et al. [8], Vukobratovic and Fajfar [9], Aragaw and Calvi [10], Kazantzi et al. [11], Vukobratovic et al. [12]).

Estimation of FRS using empirical equations based on a single-degree-of-freedom (SDOF) system for the primary structure have been reported in previous studies. Oropeza et al. [13] considered primary structures with an elasto-plastic and modified Takeda hysteretic model, while Sullivan et al. [8] also considered primary structures using a modified Takeda hysteretic model. Vukobratovic and Fajfar [14] used the equation of Yasui et al. [15] in the pre- and post-resonance region and an empirically developed expression in the resonance region for primary structures with an elasto-plastic and stiffness degrading model to estimate FRS. Welch and Sullivan [16] modified the equation developed by Sullivan et al. [8] to estimate the FRS.

Several studies have also extended the approach to estimate the FRS using empirical equations based on SDOF systems to multi-degree-of-freedom (MDOF) systems. For instance, the estimation of FRS for reinforced concrete (RC) frame systems by Calvi and Sullivan [17], Vukobratovic and Fajfar [18], and Merino et al. [19], for RC walls by Welch and Sullivan [16] and Vukobratovic and Ruggieri [20], and for base rocking wall buildings by Aragaw and Calvi [10].

The excellent seismic performance of self-centering (SC) structural systems with flag-shaped hysteretic behavior have been reported previously (see for e.g., Kurama [21], Smith et al. [22], Belleri et al. [23], Buddika and Wijeyewickrema [24], Shrestha et al. [25]). In the present study, the FRS for SC structural systems with flag-shaped hysteretic behavior under far-fault ground motions are investigated. The FRS and dynamic amplification factor (DAF) are obtained from nonlinear response history analysis (NLRHA) of SC structural systems with flag-shaped hysteretic behavior. An equation to estimate the FRS is proposed and verified by carrying out NLRHA using a different set of far-fault ground motions.

2. Self-Centering (SC) SDOF System with Flag-Shaped Hysteretic Behavior, Ground Motion Records, and Numerical Modeling

The force-displacement relationship of the self-centering (SC) SDOF system with flag-shaped hysteretic behavior, which is the *primary structure* is shown in Figure 1. The nonlinear response history analysis (NLRHA) of the SC flag-shaped SDOF system is carried out with the following parameters: initial vibration period $T_p = 0.1$ s, 0.3 s, 0.5 s, 0.75 s, 1.0 s, 1.5 s, 2.0 s; response reduction factor $R = 1, 1.5, 2, 3, 4, 5, 6$; post-yield stiffness ratio $\alpha = 5\%$; energy dissipation parameter $\beta = 0\%–100\%$ (increments of 20%) and viscous damping ratio $\xi_p = 5\%$. Even though $R = 6$ or 7 is used for the design of self-centering structural systems using the force-based design method (Smith et al. [22], Buddika and Wijeyewickrema [24]), a wide range of R values are adopted in this study. This is done so that the developed FRS in Section 4 can also be used with the direct displacement-based design method, where the R value depends on the target displacement (Priestley et al. [26], Yang and Lu [27]). It is noted that for SC flag-shaped SDOF systems, a viscous damping ratio $\xi_p = 5\%$ has been used in previous studies (see for e.g., Rahgozar et al. [28], Zhang et al. [29], Dong et al. [30]). The non-structural component (NSC), which is the *secondary structure*, is represented by an elastic SDOF system with vibration period $T_s = 0.1$ s–4.0 s (increments of 0.1 s); and viscous damping ratio $\xi_s = 5\%$.

Figure 1. The force-displacement relationship of the self-centering (SC) SDOF system with flag-shaped hysteretic behavior. Note: k_1 = initial stiffness; α = post-yield stiffness ratio; β = energy dissipation parameter; F_y = yield strength.

For NLRHA, 11 far-fault ground motions are considered from the 22 far-fault ground motion set of FEMA P695 (Table A-4A, FEMA [31]) and given in Table 1.

In the present study, the NLRHA is carried out using OpenSees [32]. The SC flag-shaped SDOF system (primary structure) is modeled using a zero-length element with the *SelfCentering* material model. The NSC (secondary structure) is represented by an elastic SDOF system and modeled using a zero-length element. The numerical integration of the equations of motion is accomplished using the Newmark constant average acceleration method ($\beta_N = 0.25$, $\gamma_N = 0.5$).

Table 1. Far-fault ground motions used for the NLRHA (Table A-4A, FEMA [31]).

EQ. No.	Event	Year	Station	Fault Type	M_w	R_{rup} (km)	PGA Component 1	PGA Component 2	$\bar{v}_s$ (m/s)
1	Northridge	1994	Beverly Hills - Mulhol	Thrust	6.7	17.2	0.42	0.52	356
2	Northridge	1994	Canyon Country-WLC	Thrust	6.7	12.4	0.41	0.48	309
3	Duzce, Turkey	1999	Bolu	Strike-slip	7.1	12.0	0.73	0.82	326
4	Hector Mine	1999	Hector	Strike-slip	7.1	11.7	0.27	0.34	685
5	Imperial Valley	1979	Delta	Strike-slip	6.5	22.0	0.24	0.35	275
6	Imperial Valley	1979	El Centro Array #11	Strike-slip	6.5	12.5	0.36	0.38	196
7	Kobe, Japan	1995	Nishi-Akashi	Strike-slip	6.9	7.1	0.51	0.50	609
8	Kobe, Japan	1995	Shin-Osaka	Strike-slip	6.9	19.2	0.24	0.21	256
9	Kocaeli, Turkey	1999	Duzce	Strike-slip	7.5	15.4	0.31	0.36	276
10	Kocaeli, Turkey	1999	Arcelik	Strike-slip	7.5	13.5	0.22	0.15	523
11	Landers	1992	Yermo Fire Station	Strike-slip	7.3	23.6	0.24	0.15	354

Note: M_w = moment magnitude; PGA = peak ground acceleration; R_{rup} = distance from recording site to epicenter; $\bar{v}_s$ = average shear wave velocity.

3. Floor Response Spectra and Dynamic Amplification Factor from Nonlinear Response History Analysis (NLRHA)

3.1. Floor Response Spectra

The floor response spectra for self-centering (SC) structural systems with flag-shaped hysteretic behavior are determined using the following procedure:

- The NLRHA of the prescribed self-centering (SC) SDOF system with flag-shaped hysteretic behavior (primary structure), for the set of ground motion records and determination of the total floor acceleration response history for each ground motion record. Here, the total floor acceleration response history is the sum of the floor acceleration response history relative to the ground and the ground acceleration response history.
- Linear RHA of the elastic SDOF system (secondary structure), using the set of total floor acceleration response histories determined from step (a), to generate floor response spectra.
- Calculation of the mean floor response spectrum.

The mean normalized floor response spectra (FRS), i.e., A_s/A_p , where A_p , A_s are the peak acceleration demands of the primary structure and secondary structure, respectively, are calculated for the structural parameters stated in Section 2. The ratio A_s/A_p is also referred to as the dynamic amplification factor (DAF). In Figures 2 and 3 the mean normalized floor response spectra are shown only for $T_p = 0.5$ s, 1.0 s, respectively. It can be observed that when the primary structure is elastic, i.e., $R = 1$, a single peak of the mean normalized FRS at $T_s/T_p = 1$ can be observed. When $R > 1$, the primary structure behaves inelastically and rather than a single peak, the maximum value of the mean normalized FRS remains nearly constant over a wide period range and forms a *spectral plateau*. Moreover, the width of the spectral plateau increases with increase in R and decrease in β , which is due to the higher ductility demand on the primary structure. In most cases, the peak value of mean normalized FRS increases when R changes from 1 to 2 and then decreases for $R > 2$.

To investigate the effect of β more clearly, the mean normalized FRS are shown for a SC system with $R = 6$, $T_p = 0.5$ s, 1.0 s, and $\beta = 0\%, 20\%, \dots, 100\%$ in Figure 4. The results show that the mean normalized FRS decreases when β increases. A wider spectral plateau is observed with decrease in β .

Figure 2. Mean normalized floor response spectra for SC systems with $T_p = 0.5$ s, post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\zeta_p = 5\%$; and a secondary structure with viscous damping ratio $\zeta_s = 5\%$: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Figure 3. Mean normalized floor response spectra for SC systems with $T_p = 1.0$ s, post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\zeta_p = 5\%$; and a secondary structure with viscous damping ratio $\zeta_s = 5\%$: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Figure 4. Effect of energy dissipation parameter β on mean normalized floor response spectra for a SC system with $R = 6$, post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\zeta_p = 5\%$; and a secondary structure with viscous damping ratio $\zeta_s = 5\%$: (a) $T_p = 0.5$ s; and (b) $T_p = 1.0$ s.

3.2. Maximum Dynamic Amplification Factor DAF_{max}

Figure 5 shows the maximum values of mean normalized FRS for $T_p = 0.1$ s, 0.3 s, 0.5 s, 0.75 s, 1.0 s, 1.5 s, 2.0 s, also referred to as maximum dynamic amplification factor DAF_{max} . Note that this DAF_{max} will be used in the equation to estimate FRS developed in Section 4. It is observed that with increase in T_p , DAF_{max} increases for $T_p \leq 0.5$ s, and remains nearly constant for $T_p > 0.5$ s and $R > 1.0$. It is also observed that the DAF_{max} decreases with increasing β . Based on the results of NLRHA shown in Figure 5, an empirical equation to estimate the DAF_{max} is developed by carrying out nonlinear least-square regression analysis, where the regression coefficients are obtained using the Levenberg-Marquardt algorithm (Bates and Watts [33]) available in SPSS [34]. The equation for DAF_{max} is,

$$DAF_{max}(T_p, R, \beta; \zeta_s) = \begin{cases} \frac{1}{\sqrt{\zeta_s}} (aT_p^b + c), & R = 1 \\ \frac{0.15(1+\beta)^{-2.6+0.85}}{\sqrt{\zeta_s}} (aT_p^b + c), & R > 1 \end{cases} \quad (1)$$

where a , b , and c are constant coefficients determined from nonlinear regression analysis, separately for each R and given in Table 2. Note that in the present study, the viscous damping ratio of the secondary structure $\zeta_s = 5\%$ (see Section 2). The equation is given for elastic ($R = 1$) and inelastic ($R > 1$) primary structures. For verification purposes, in Figure 6 the DAF_{max} obtained from Equation (1) are compared with the NLRHA results obtained using a different set of far-fault ground motions given in Table 3. The T_p , R , α , β , and ζ_p of the primary structure defined in Section 2 are used for the verification. It can be seen that Equation (1) can estimate the DAF_{max} accurately from the NLRHA results for primary structures with different T_p , R , and β .

Table 2. Regression coefficients of the equation for DAF_{max} .

Coefficients	$R = 1.0$	$R = 1.5$	$R = 2.0$	$R = 3.0$	$R = 4.0$	$R = 5.0$	$R = 6.0$
a	−0.020	−0.001	−0.013	−0.021	−0.017	−0.019	−0.018
b	−1.304	−2.358	−1.574	−1.400	−1.382	−1.258	−1.178
c	1.259	1.330	1.350	1.260	1.175	1.108	1.043

Figure 5. Maximum dynamic amplification factor DAF_{max} for a SC system with post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\zeta_p = 5\%$; and a secondary structure with viscous damping ratio $\zeta_s = 5\%$: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Figure 6. Comparison of the DAF_{max} from the NLRHA results with the estimated DAF_{max} from Equation (1).

Table 3. Far-fault ground motions used for the verification (Table A-4A, FEMA [31]) of DAF_{max} (Equation (1)) and the proposed equation for FRS (Equation (5)).

EQ No.	Event	Year	Station	Fault Type	M_w	R_{rup} (km)	PGA Component 1	PGA Component 2	$\bar{v}_s$ (m/s)
1	Landers	1992	Coolwater	Strike-slip	7.3	19.7	0.28	0.42	271
2	Loma Prieta	1989	Capitola	Strike-slip	6.9	15.2	0.53	0.44	289
3	Loma Prieta	1989	Gilroy Array #3	Strike-slip	6.9	12.8	0.56	0.37	350
4	Manjil, Iran	1990	Abbar	Strike-slip	7.4	12.6	0.51	0.50	724
5	Superstition Hills	1987	El Centro Imp. Co.	Strike-slip	6.5	18.2	0.36	0.26	192
6	Superstition Hills	1987	Poe Road (temp)	Strike-slip	6.5	11.2	0.45	0.30	208
7	Catde Mendocino	1992	Rio Dell Overpass	Thrust	7.0	14.3	0.39	0.55	312
8	Chi-Chi, Taiwan	1999	CHY101	Thrust	7.6	10.0	0.35	0.44	259
9	Chi-Chi, Taiwan	1999	TCU045	Thrust	7.6	26.0	0.47	0.51	705
10	San Fernando	1971	LA - Hollywood Stor	Thrust	6.6	22.8	0.21	0.17	316
11	Friuli, Italy	1976	Tolmezzo	Thrust	6.5	15.8	0.35	0.31	425

Note: M_w = moment magnitude; PGA = peak ground acceleration; R_{rup} = distance from recording site to epicenter; $\bar{v}_s$ = average shear wave velocity.

3.3. Post-Resonance Dynamic Amplification Factor DAF_{pr}

The post-resonance dynamic amplification factor DAF_{pr} will also be used in the equation to estimate the FRS proposed in Section 4. Considering the expression for DAF_{max} given in Equation (1) and the estimation of floor spectra by Welch and Sullivan [16], the equation for DAF_{pr} is expressed by,

$$DAF_{pr}(T_p, R, \beta; T_s, \xi_s) = \begin{cases} \frac{1}{\sqrt{(1 - \frac{T_s}{T_e})^2 + \xi_s^2}} (aT_p^b + c), & R = 1 \\ \frac{0.15(1+\beta)^{-2.6} + 0.85}{\sqrt{(1 - \frac{T_s}{T_e})^2 + \xi_s^2}} (aT_p^b + c), & R > 1 \end{cases} \quad (2)$$

where the effective period T_e associated with the secant stiffness at peak response of the primary structure is given by,

$$T_e(T_p, \alpha, \mu) = T_p \sqrt{\frac{\mu}{1 + \alpha(\mu - 1)}} \quad (3)$$

(see Welch and Sullivan [16]), where α = post-yield stiffness ratio (taken as 5% in this study), and the ductility demand μ is computed using the equation for constant-strength inelastic displacement ratio C_R given by Zhang et al. [29] for SC systems as,

$$\mu(T_p, R, \beta) = \left\{ 1 + (R - 1)^{0.515} \frac{0.184 + 0.119(1 - \beta)^{1.173}}{T_p^{1.478}} \right\} R \quad (4)$$

4. Equation to Estimate Floor Response Spectra and Verification

4.1. Equation to Estimate FRS

In the present study, the following equation is proposed to estimate the FRS for the primary structure as SC systems,

$$\frac{A_s}{A_p}(T_p, R, \beta; T_s, \xi_s) = \begin{cases} 1 + \frac{T_s}{T_p}(DAF_{max} - 1), & 0 \leq T_s < T_p \\ DAF_{max}, & T_p \leq T_s \leq T_e \\ DAF_{pr}, & T_s > T_e \end{cases} \quad (5)$$

Note that an equation of similar form was used by Sullivan et al. [8] for the estimation of FRS for well detailed RC structures.

The acceleration demand of the secondary structure increases linearly in the pre-resonance region $0 \leq T_s < T_p$, remains constant in the resonance region $T_p \leq T_s \leq T_e$ forming a *spectral plateau*, and decreases in the post-resonance region $T_s > T_e$. Note that the effective period T_e increases with the higher inelastic deformation demand of the primary structure, i.e., increase in R and decrease in β , which leads to an increase in the width of the spectral plateau.

4.2. Verification of Proposed Equation to Estimate FRS

For the verification, a primary structure which is a SC structural system with flag-shaped hysteretic behavior, with initial vibration period $T_p = 0.5$ s, 1.0 s, 1.5 s, response reduction factor $R = 2, 4, 6$, post-yield stiffness ratio $\alpha = 5\%$; energy dissipation parameter $\beta = 0\%, 20\%, \dots, 100\%$, and viscous damping ratio $\xi_p = 5\%$; and a secondary structure with viscous damping ratio $\xi_s = 5\%$ is considered. It is noted that both the primary and secondary structures are modeled by SDOF systems.

Here, 11 far-fault ground motions are considered from the 22 far-fault ground motion set of FEMA P695 (Table A-4A, FEMA [31]) and given in Table 3. Note that these 11 far-fault ground motions are not the same ground motions used in Section 2.

The mean floor response spectra are determined from NLRHA using the procedure given in Section 3.1. The mean peak acceleration demand of the secondary structure A_s obtained from Equation (5) and NLRHA are shown in Figures 7–9. The mean normalized peak acceleration demand of the secondary structure A_s/A_p obtained from Equation (5) and NLRHA are shown in Appendix A. For $T_p = 0.5$ s, 1.0 s, the A_s estimated from Equation (5) slightly underestimates the peak acceleration demand of the secondary structure in the resonance region but is mostly conservative in the other regions. For $T_p = 1.5$ s, the A_s estimated from Equation (5) slightly underestimates the peak acceleration demand of the secondary structure in the post-resonance region but is mostly conservative in the other regions. It is also observed that the width of the spectral plateau with varying R and β can be well estimated from Equation (5). In general it can be concluded that the A_s estimated from Equation (5) shows good accuracy when compared with NLRHA results.

4.3. Comparison of the Proposed FRS with Existing Direct Methods

The mean peak acceleration demand of the secondary structure A_s obtained from Equation (5) of the present analysis and existing direct methods of Sullivan et al. [8] and Vukobratovic and Fajfar [14] are compared with the NLRHA results in Figure 10. The equations to estimate the FRS using existing direct methods are given in Appendix B. For the comparison, a primary structure with flag-shaped hysteretic behavior, with initial vibration period $T_p = 1.0$ s, response reduction factor $R = 2$; post-yield stiffness ratio $\alpha = 5\%$; energy dissipation parameter $\beta = 0\%, 20\%, \dots, 100\%$, and viscous damping ratio $\xi_p = 5\%$; and a secondary structure with viscous damping ratio $\xi_s = 5\%$ is considered.

Figure 7. Comparison of the mean floor response spectra from Equation (5) and NLRHA for a primary structure with initial vibration period $T_p = 0.5$ s, post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\zeta_p = 5\%$; and a secondary structure with viscous damping ratio $\zeta_s = 5\%$: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Figure 8. Comparison of the mean floor response spectra from Equation (5) and NLRHA for a primary structure with initial vibration period $T_p = 1.0$ s, post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\xi_p = 5\%$; and a secondary structure with viscous damping ratio $\xi_s = 5\%$: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Figure 9. Comparison of the mean floor response spectra from Equation (5) and NLRHA for a primary structure with initial vibration period $T_p = 1.5$ s, post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\xi_p = 5\%$; and a secondary structure with viscous damping ratio $\xi_s = 5\%$: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Figure 10. Comparison of the mean floor response spectra from Equation (5) of the present analysis, Sullivan et al. [8], Vukobratovic and Fajfar [14], and NLRHA for a primary structure with initial vibration period $T_p = 1.0$ s, response reduction factor $R = 2$; post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\zeta_p = 5\%$; and a secondary structure with viscous damping ratio $\zeta_s = 5\%$: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$. Note: The methods of Sullivan et al. [8] and Vukobratovic and Fajfar [14] are for primary structures with modified Takeda and elasto-plastic hysteretic behavior, respectively.

It is observed that the A_s estimated from Equation (5) of present analysis provides a good estimate in the resonance region and slightly overestimates in the other regions (see Figure 10). The method of Sullivan et al. [8] slightly overestimates in pre-resonance region but underestimates in the resonance region. Moreover, in the post-resonance region, A_s is underestimated for $\beta \leq 20\%$, but shows good accuracy for $\beta > 20\%$. The method of Vukobratovic and Fajfar [14] underestimates A_s in the pre-resonance region for $T_s < 0.6$ s. In the resonance region, underestimation of A_s for $\beta \leq 20\%$, and overestimation of A_s for $\beta > 20\%$ is observed. Furthermore, A_s is underestimated in the post-resonance region.

Note that the proposed equation to compute FRS is developed for SC flag-shaped SDOF system and show dependency with β . As a result, the estimated A_s from Equation (5) of the present analysis shows good accuracy than the existing direct methods when compared with NLRHA results.

5. Conclusions

In this study, the floor response spectra (FRS) for self-centering (SC) structural systems with flag-shaped hysteretic behavior is investigated using nonlinear response history analysis (NLRHA), and an equation to estimate the FRS is proposed. In particular, a primary structure with post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\xi_p = 5\%$; and a secondary structure with viscous damping ratio $\xi_s = 5\%$ is considered. The proposed equation is then verified using a different set of far-fault ground motions. Based on the study, the following conclusions can be drawn:

1. The effect of the primary structure initial vibration period T_p , response reduction factor R , and energy dissipation parameter β on the FRS is studied. A single peak was observed on the mean normalized FRS for $R = 1$, but for $R > 1$, the maximum value of the mean normalized FRS is nearly constant over a wide period range and forms a *spectral plateau*. The width of the spectral plateau increases with increase in R and decrease in β , which is due to the higher ductility demand on the primary structure. In addition, the peak value of mean normalized FRS increases when R changes from 1 to 2 and then decreases for $R > 2$. The reduction in the mean normalized FRS was observed with increase in β . With increase in T_p , the maximum dynamic amplification factor DAF_{max} , increases for $T_p \leq 0.5$ s, and remains nearly constant for $T_p > 0.5$ s and $R > 1$.
2. An empirical equation for DAF_{max} which can be used to estimate the acceleration demand in the resonance region is developed. This equation showed good accuracy when compared with NLRHA results. In addition for the post-resonance region, an equation to estimate the dynamic amplification factor DAF_{pr} is also obtained.
3. An equation to estimate the FRS for SC systems with flag-shaped hysteretic behavior is proposed using the DAF_{max} and the DAF_{pr} . The equation to estimate FRS is then validated using a different set of far-fault ground motions. It was observed that the equation to estimate FRS for SC systems with flag-shaped hysteretic behavior showed good accuracy when compared with the NLRHA results.

The results of this study are useful for the seismic design of acceleration-sensitive non-structural components (NSCs).

The equation to estimate FRS developed in the present study is based on SC structural systems with flag-shaped hysteretic behavior with a primary structure with post-yield stiffness ratio $\alpha = 5\%$ and viscous damping ratio $\xi_p = 5\%$, and a secondary structure with viscous damping ratio $\xi_s = 5\%$. Since the damping ratio of the secondary structure strongly influences the FRS, more damping ratios of secondary structures ξ_s should be investigated and the accuracy of the proposed equation should be examined. Future studies, should also investigate the effect of post-yield stiffness ratio α , and the viscous damping ratio of the primary structure ξ_p , on the FRS for SC structural systems with flag-shaped hysteretic behavior.

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Appendix A. Comparison of the Normalized Floor Response Spectra

The normalized FRS from Equation (5) and NLRHA results for a primary structure with $T_p = 0.5$ s, 1.0 s, 1.5 s; post-yield stiffness ratio $\alpha = 5\%$; viscous damping ratio $\xi_p = 5\%$; and a secondary structure with viscous damping ratio $\xi_s = 5\%$ are shown in Figures A1–A3

Figure A1. Comparison of the mean normalized floor response spectra from Equation (5) and for a primary structure with initial vibration period $T_p = 0.5$ s: (a) $\alpha = 0\%$; (b) $\alpha = 20\%$; (c) $\alpha = 40\%$; (d) $\alpha = 60\%$; (e) $\alpha = 80\%$; and (f) $\alpha = 100\%$.

Figure A2. Comparison of the mean normalized floor response spectra from Equation (5) and for a primary structure with initial vibration period $T_p = 1.0$ s: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Figure A3. Comparison of the mean normalized floor response spectra from Equation (5) and for a primary structure with initial vibration period $T_p = 1.5$ s: (a) $\beta = 0\%$; (b) $\beta = 20\%$; (c) $\beta = 40\%$; (d) $\beta = 60\%$; (e) $\beta = 80\%$; and (f) $\beta = 100\%$.

Appendix B. Existing Direct Methods to Estimate the FRS

B1. Method of Sullivan et al. [8]

The equation of Sullivan et al. [8] to estimate the FRS for a primary structure with modified Takeda hysteretic model is,

$$A_s(T_p, R, \beta; T_s, \xi_s) = A_p \begin{cases} 1 + \frac{T_s}{T_p}(DAF_{max} - 1), & 0 \leq T_s < T_p \\ DAF_{max}, & T_p \leq T_s \leq T_e \\ DAF_{pr}, & T_s > T_e \end{cases} \quad (A1)$$

where $DAF_{max}(\xi_s) = \frac{1}{\sqrt{\xi_s}}$ and $DAF_{pr}(T_p; T_s, \xi_s) = \frac{1}{\sqrt{(1 - \frac{T_s}{T_e})^2 + \xi_s}}$.

B2. Method of Vukobratovic and Fajfar [14]

The equations of Vukobratovic and Fajfar [14] to estimate the FRS for a primary structure with elasto-plastic hysteretic behavior are as follows:

(a) In the pre- and post-resonance region,

$$A_s(T_p, R, \xi_p; T_s, \xi_s, S_e) = \frac{\sqrt{\{A_p(T_p/T_s)^2\}^2 + S_e(T_s, \xi_s)^2}}{|1 - (T_p/T_s)^2|} \quad (A2)$$

where $S_e(T_s, \xi_s)$ is the value at the specific period and damping ratio in the elastic acceleration spectrum of the ground motion.

(b) In the resonance region,

$$A_s(T_p; \xi_s; T_C) = A_p \begin{cases} 18(1 + \xi_s)^{-0.60}, & T_p/T_C \leq 1 \\ 18(1 + \xi_s)^{-0.60}(T_p/T_C)^{-0.20}, & T_p/T_C > 1 \end{cases} \quad (A3)$$

where T_C is the characteristic period of the ground motion.

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