Article

Hysteresis Envelope Model of Double Extended End-Plate Bolted Beam-to-Column Joint

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Abstract: In this study, a hysteresis envelope mathematical model for the double extended end-plate bolted beam-to-column joint is proposed. The aim of a proposed joint model is to provide a more realistic behaviour of steel frames under seismic loading by using nonlinear static pushover analysis. The hysteresis envelope model defines the ratios between the monotonic properties of the joints and the properties of the joints during cyclic deformation. The proposed models are based on the hysteresis curves of the joints obtained by numerical simulations. The numerical model takes into account the geometric nonlinearity of the connecting elements, preloading of bolts, contacts between plates and bolts, and nonlinear properties of steel. Nonlinear static pushover analyses of steel frames are performed where the behaviour of the joints is described using the proposed hysteresis envelope models. The results are compared with the nonlinear static pushover analyses of steel frames with a trilinear monotonic joint model. Based on the results, the values of maximum peak ground acceleration for moment-resisting frames with the monotonic model of joints and hysteresis envelope model are estimated.

Keywords: hysteresis envelope model; beam-to-column joint; moment-resisting frame; nonlinear static pushover analysis; N2 method

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1. Introduction

Moment-resisting steel frames are widely used in areas of strong seismic activity [1]. It is common practice that the seismic energy introduced into a structure during an earthquake is dissipated by the nonlinear behaviour of the structure, by using its ductile properties [2]. According to the principles of seismic engineering, the structure must be designed to enable the development of plastic deformations or the formation of ductile zones on specific parts of the structure. Ductile zones in the structure must dissipate the seismic energy by hysteresis behaviour. Moment-resisting steel frames are characterized by many dissipative zones, which are located mainly at the ends of beams or in the beam-to-column joints and at the lower ends of columns to the frame supports [3]. After several cycles of seismic action, these zones become plastically deformed parts of the structure. Many scientific studies have shown that joints have a high ability to dissipate seismic energy with high strength and stiffness [1,4–7].

The traditional design approach of moment-resistant steel frames considers the beam-to-column joints as pinned without any resistance and rigidity, or as completely rigid with full resistance. Although the application of this approach significantly simplifies the design calculation procedure, it does not describe its actual behaviour. In reality, both cases are inaccurate and are only boundary cases of real behaviour, where the rotational behaviour of the joint is most common in the area between these two extremes. The effect of semi-rigid joints in regard to rigid or pinned joints is not only the reduction of displacement but also the distribution and magnitude of internal forces in the structure. The joints have their actual stiffness; therefore, they are classified as semi-rigid, and their behaviour has a significant impact on the resistance, stiffness, and stability of the entire structure as well

as on the dissipation of seismic energy that is introduced into the structure during an earthquake. It is therefore necessary to take into account the actual behaviour of the joints in the frame analysis.

From the aspect of seismic design using the nonlinear static N2 method according to [8], Nogueiro et al. [9] analysed the behaviour of steel frames with semi-rigid joints. The frame model takes into account the behaviour of the joints obtained by monotonic testing. The results are compared with those evaluated using nonlinear dynamic analyses. In most cases, horizontal top displacements obtained by the N2 method are larger than in the dynamic analysis. The authors concluded that, for the study of the behaviour of the joints, monotonic methods have some limitations because they give insufficient hysteresis information. Krolo et al., 2014 and 2015 [10–12], provide a comparison of the seismic response of a steel frame using a nonlinear static method for the case when the joints are modelled as rigid and semi-rigid. The actual behaviour of the joints is taken into account in the frame calculation in such a way that a numerical simulation of monotonic bending is performed for the selected type of joint, resulting in a monotonic curve of the relationship between the bending moment and rotation of the joint. It is evident that the behaviour of the joint is different under monotonic and cyclic loading, but the question arises is it possible in some way to define the cyclic behaviour of joints for their application in the nonlinear static pushover analysis.

Analytical models describe the behaviour of the joints, which is shown in the form of the moment-rotation curve ($M - \phi$). To incorporate the $M - \phi$ curve into the frame design, the relationship between the moment and the rotation of the joint must be written in the form of mathematical functions. The joint behaviour plays an important role in the analysis of the frame. Therefore, the accuracy of the mathematical model that interprets its behaviour is very important. Defining the mathematical formulation of the $M - \phi$ curve largely depends on the level of the required precision and is therefore grouped into linear, bilinear, trilinear (multilinear), and nonlinear. Numerous authors have proposed different models to describe the monotonic behaviour of joints, and here we will look only at the nonlinear models.

The first nonlinear mathematical model to describe the behaviour of the stress-strain relationship was proposed by Ramberg and Osgood (1943) [13], which was later used to describe the behaviour of the $M - \phi$ curve. Ang and Morris (1983) [14] first applied the Ramberg-Osgood formulation to describe the behaviour of five types of joints. To describe the $M - \phi$ curve, it is necessary to know three pieces of data: the rotational stiffness of the joint S , and the parameters K and n that define the shape of the $M - \phi$ curve. Richard and Abbott (1975) [15] proposed a mathematical model for predicting the behaviour of the $M - \phi$ curve for semi-rigid joints for which four data are required: initial joint stiffness S ; joint stiffness before failure S_p ; reference bending moment M_0 representing the intersection tangent S_p with coordinate axis M ; a shape factor n that defines the shape of the $M - \phi$ curve. The Ramberg-Osgood and Richard-Abbott models are the most applicable models used to describe the behaviour of different types of joints under monotonic loading. Frye and Morris (1975) [16] and Krishnamurthy et al. (1976) [17] proposed a mathematical model of the $M - \phi$ curve using odd polynomial and regression constants. Yee and Melchers (1986) [18] proposed an exponential formulation, while Wu and Chen (1990) [19] proposed a logarithmic formulation of the $M - \phi$ curve. Both formulations are defined by the geometric and mechanical characteristics of the joints.

Despite the effectiveness of mathematical models determined on the basis of the monotonic response of joints, they cannot realistically describe the properties of the joint during unloading and reloading imposed by the effects of the earthquake. Some authors have provided such formulations to describe the cyclic response of joints [20–22]. Cyclic models of $M - \phi$ curves best describe the behaviour of joints under earthquakes, however, they are not applicable in nonlinear static pushover analysis. Lignos and Krawinkler (2011) [23] proposed the modified Ibarra-Krawinkler deterioration model that establishes strength bounds on the basis of a monotonic backbone curve and a set of rules that define the charac-

teristics of hysteretic behavior. The backbone curve is defined by three strength parameters and four deformation parameters. The rates of cyclic deterioration are controlled by a rule on the basis of the hysteretic energy dissipated when the component is subjected to cyclic loading. The modified Ibarra-Krawinkler model is applied on the beam-to-column connection in which inelastic deformations are primarily concentrated in flexural plastic hinge regions of the beam sections. The primary deterioration mode of the steel components that develop a plastic hinge is local and/or lateral torsional buckling. The modified Ibarra-Krawinkler model is also applied on the specimens that have reduced beam sections in which plastic hinges develop away from the beam-to-column connection. Regression analysis takes into account beam depth, shear span to depth ratio, width/thickness ratio of the beam flange, depth to thickness ratio of the beam web and other parameters describing the influence of the beam on the rotational capacity of the joint. Steel columns such as steel beams dissipate seismic energy. Elkady and Lignos (2014) [24] investigated the cyclic behaviour of wide-flange steel columns. The backbone curve is used to bind the cyclic behaviour of the column. Cravero et al. (2020) [25] also investigated wide-flange steel columns under monotonic and reversed cyclic lateral loading coupled with constant and variable axial load demand. The typical damage of the test specimen involved flexural yielding followed by column cross-sectional local buckling. In order to conduct a nonlinear static analysis, hysteresis behaviour of the column base is defined with the PEER/ATS 72-1 Option 3 [26] modelling option, which represents the first-cycle envelope curve of the structural component subject to symmetric cyclic lateral loading protocol. This curve takes into account the deterioration on a component's strength and stiffness.

The aim of this paper is to propose an analytical model of hysteresis envelope based on the properties of hysteresis behaviour of the double extended end-plate bolted beam-to-column joints. The joints are designed as semi-rigid in such a way that during earthquakes all plastic deformations occur primarily in the joint instead of elements such as columns and beams. The joint resistance is equal to or less than the resistance of the connected beam, as recommended in EN 1993-1-8, Part 1-8 [27]. When designing steel moment-resisting frames, the ratio between the resistance of the columns in relation to the resistance of the beams must be at least 30% higher to satisfy the criterion "strong column-weak beam". The proposed model can be applied in the nonlinear static pushover analysis of moment-resisting steel frames. The model takes into account the effects of cyclic behaviour, such as the reduction in deformations and bending moments during full plastic behaviour of the joint, the degradation of strength and stiffness that the joint can achieve before the fracture. The current nonlinear static analysis of structures that takes into account the monotonic properties of the joints can be considered quite conservative. The development of this model enables a better prediction of the structure's behaviour under seismic action and would approach the behaviour of the structure obtained by nonlinear dynamic analysis. Furthermore, the parameters that correct the curves obtained by monotonic behaviour are defined.

A joint numerical model in ABAQUS software [28] is first developed, which is calibrated with the laboratory test results from the literature. On the basis of the calibrated model, simulations are performed for nine joints exposed to monotonic and cyclic loading. Hysteresis curves obtained by cyclic behaviour are the basis for the development of a hysteresis envelope model. The influence of the proposed joint model on the behavior of the steel moment-resisting frame is carried out by nonlinear static pushover analysis and compared with the behaviour of the steel frame in which the connections are modeled with a trilinear monotonic model.

2. Numerical Model of Joint

Numerical simulations are performed for nine double extended end-plate bolted beam-to-column joints with three different end-plate thicknesses t and three different bolt row spacings p , which are exposed to monotonic and cyclic loading. Calibration of the numerical model is previously performed based on laboratory tests of the authors

Shi et al. [7,29]. Based on the obtained hysteresis curves that describe the relationship between the bending moment M_j and the rotation of the joints ϕ , a mathematical model of the hysteresis envelope is proposed.

2.1. Geometry of Joints

The joints consist of: a steel beam, a column, a double extended end-plate, a column web stiffener, high-strength preload bolts. A 120-cm-long beam made of IPE400 cross section is welded to the end-plate, which are then bolted to a column flange made of HEA360 cross section with 8 high-quality M22 preload bolts with steel grade 10.9. The bolts are arranged symmetrically to the beam flange. The geometry of the beam-to-column EP1_1_M/EP1_1_C joint is shown in Figure 1. The plated parts of the joint are made of European mild S355 steel, and the data are given in Section 2.2. Table 1 lists nine joint models with end-plate thickness values and spacings between bolt rows, while Table 2 provides data on beam and column cross sections. Initial geometric imperfections can affect the buckling behavior of the column and beam, however, their effect on nonlinear cyclic behavior is generally small and is not taken into account in the numerical model.

Figure 1. (a) Geometry of joint EP1_1_M/EP1_1_C; (b) dimensions of end-plates with spaces between bolt rows for $p = 130$ mm.

Table 1. Details of beam-to-column joints.

Group of Joints	FE Model	End-Plate Thickness t (mm)	Bolts Row Spacing p (mm)
1	EP1_1_M/EP1_1_C	15	130
	EP1_2_M/EP1_2_C		140
	EP1_3_M/EP1_3_C		150
2	EP2_1_M/EP2_1_C	17	130
	EP2_2_M/EP2_2_C		140
	EP2_3_M/EP2_3_C		150
3	EP3_1_M/EP3_1_C	20	130
	EP3_2_M/EP3_2_C		140
	EP3_3_M/EP3_3_C		150

The first number in the name of joint model indicates the end-plate thickness 1 = 15 mm; 2 = 17 mm; 3 = 20 mm. The second number in the name of joint model indicates vertical spacing between the rows of bolts 1 = 130 mm; 2 = 140 mm; 3 = 150 mm. Mark M—simulation of monotonic loading. Mark C—simulation of cyclic loading.

Table 2. Beam and column cross section data.

Element/Cross Section	Height h (mm)	Width b (mm)	Flange Thickness t_f (mm)	Web Thickness t_w (mm)
Beam/IPE400	400	180	13.5	8.6
Column/HEA360	350	300	17.5	10

2.2. Material Properties

Two material models are used to define the joint plated elements, one for the case of monotonic loading and the other for the case of cyclic loading. To simulate the monotonic loading, an isotropic multilinear model of the material is applied, which includes the hardening between the yielding of the material and reaching the tensile strength. The plastic properties of steel in ABAQUS are defined as the true stresses $\sigma_t = \sigma_{nom}(1 + \varepsilon_{nom})$ and the true strains $\varepsilon_t = \ln(1 + \varepsilon_{nom})$ where the nominal values of stress σ_{nom} and nominal value of strain ε_{nom} are obtained as mean values of laboratory data given in [30], and are shown in Figure 2. The yield strength is 384 MPa, the tensile strength is 552 MPa, Young's modulus of elasticity is $E = 199,000$ MPa, while the Poisson ratio is equal to $\nu = 0.3$. The damage model for S355 steel is also applied, and the monotonic loading parameters are given in Table 3. The damage model includes damage initiation criteria and a damage evaluation law. Damage initiation criteria are defined as equivalent plastic strain (PEEQ) at the onset of damage $\bar{\varepsilon}_D^{pl}$ in a function of stress triaxiality η and strain rate $\dot{\varepsilon}^{pl}$. Damage evaluation law is defined in ABAQUS in tabular form as a damage variable D and equivalent plastic displacement $\bar{u}_i^{pl}$. The function of PEEQ at the onset of damage on triaxiality is defined according to Rice and Tracey [31].

Figure 2. Plastic properties of steel S355 for monotonic loading.**Table 3.** Parameters of damage model for S355 (monotonic loading).

Damage Initiation Criteria			Damage Evaluation Low	
$\bar{\varepsilon}_D^{pl}$	η	$\dot{\varepsilon}^{pl}$	D	$\bar{u}_i^{pl}$
0.2270	0.32	0.001	0	0
0.2070	0.50	0.001	0.0105	0.0580
0.1945	0.60	0.001	0.0305	0.0965
0.1822	0.70	0.001	0.0625	0.1346
0.1755	0.76	0.001	0.0872	0.1562
0.1676	0.82	0.001	0.1183	0.1788
0.1562	0.90	0.001	0.1701	0.2087
0.1502	0.95	0.001	0.2147	0.2295
0.1480	0.97	0.001	0.2304	0.2355

The Chaboche nonlinear isotropic/kinematic hardening model is used for the simulation of the Von Mises flow rule of steel under cyclic loading. Isotropic hardening defines the evolution of the yield surface size $\sigma|_0$ as a function of the equivalent plastic strain $\bar{\epsilon}^{pl}$ and is expressed as the simple exponential law. $\sigma|_0$ is the initial yield stress at zero plastic strain, while Q_∞ and b are material parameters. Q_∞ is the maximum change in the size of the yield surface, and b defines the rate at which the size of the yield surface changes as plastic strain develops. The parameters ($\sigma|_0$, Q_∞ , b , C , and γ) of S355 steel for use in ABAQUS are given in Table 4. The overall backstress α is computed as a sum of three backstresses as proposed by Chaboche. Calibration of the isotropic and kinematic hardening parameters for steel is verified using the experimental test results and described in detail in [30]. The damage model for S355 steel is also applied and the parameter values are given in Table 5. The procedures of the parameters' calibration of damage model for monotonic and cyclic loading are presented in the doctoral thesis in [32].

Table 4. Material properties of steel S355 for cyclic loading.

Elastic Behaviour		Plastic Behaviour								
		Kinematic Hardening						Isotropic Hardening		
E (MPa)	ν	$\sigma _0$ (MPa)	C_1 (MPa)	γ_1	C_2 (MPa)	γ_2	C_3 (MPa)	γ_3	Q_∞ (MPa)	b
185,000	0.3	386	5327	75	1725	16	1120	10	20.8	3.2

Table 5. Parameters of damage model for S355 (cyclic loading).

Damage Initiation Criteria			Damage Evaluation Low	
$\bar{\epsilon}_D^{pl}$	η	$\dot{\epsilon}^{pl}$	D	$\bar{u}_i^{pl}$
0.63080	0.33	0.001	0.0037	0.0000
0.62491	0.38	0.001	0.0085	0.4719
0.61874	0.43	0.001	0.0243	0.9666
0.61244	0.48	0.001	0.0623	1.4716
0.60607	0.52	0.001	0.1293	1.9823
0.59963	0.57	0.001	0.2386	2.4987
0.59310	0.61	0.001	1.0000	3.0220

The isotropic material hardening model is used to define the material properties of high-strength preload bolts and parameters are adopted by Wang et al. [33] and are given in Table 6.

Table 6. Material properties of high-strength bolts class 10.9 [33].

Stress σ (MPa)	990	1160	1160
Strain ϵ (—)	0.00483	0.136	0.05

2.3. Finite Element Mesh

The column, beam and end-plate are modelled with linear 8-node hexagonal (brick) finite elements C3D8I. Finite elements of 6 nodes C3D6 are used for bolt modelling. The bolt head and nut are modelled as one “part” together with the bolt body and washers on both sides of the bolt. The bolt threads and the extended part of the bolt outside the nut are neglected. In places where contacts between the elements are made, a finer mesh is formed. The mesh size is proposed based on previous models. The finite element mesh for the beam-to-column joint and column is shown in Figure 3a,b, respectively, while the finite element mesh for the bolt and end-plate is shown in Figure 3c,d, respectively.

Figure 3. Finite element mesh for (a) beam-to-column joint; (b) part of column; (c) bolt and (d) end-plate.

2.4. Contact Modelling

The accuracy of numerical models largely depends on the properties of the contacts between the connected elements and bolts. The stresses between the elements of joint connected by preloaded bolts are transmitted by friction between the connected elements, while the stresses between the elements of joint connected by ordinary bolts are transmitted by shear of the bolts. Four contact areas are defined: (1) contact between the end-plate and the column flange; (2) contacts between the bolt washer and the end-plate; (3) contacts between the bolt washer and the column flange; (4) contacts between the bolt and the hole. Friction and normal stresses occur in contact area (1), and the penalty formulation is selected to model friction at contact with friction coefficient 0.44. The friction coefficient corresponds to the measured value on the contact surfaces obtained in the experiment [7], “Hard” contact is taken for modelling normal stresses using Augmented Lagrange formulation. The same contact formulations are applied to modelling contact (2) and (3). The tangential component between the bolt body and the bolt hole in zone (4) is modelled without friction using the “Frictionless” module. The contacts between the bolt and the elements of the joint are shown in Figure 4.

Figure 4. Contact zone in joint. M—“master”. S—“slave”.

2.5. Bolt Pretension

To simplify the numerical model, the diameter of the bolt d and the diameter of the bolt hole d_0 are modelled with equal dimensions ($d = d_0 = 22$ mm). Based on the calibration

model of the joint, it has been found that the application of this simplification gives a good match of numerical and experimental results. The bolt preloading process is performed using the “bolt load” technique in ABAQUS. The preloading is simulated by cutting the bolt body into two equal parts and applying a preloading force to two parallel surfaces. An axis is formed along the bolts to which the preloading force is applied, which is shown in Figure 5a.

Figure 5. (a) Bolt preloading and (b) Mises stress after preloading.

Bolt M22 with steel grade 10.9, cross section area $A_s = 303 \text{ mm}^2$ and ultimate strength $f_{ub} = 1000 \text{ N/mm}^2$ has a value of the preloading force of 212.10 kN. Figure 5b shows the Mises stress in the bolt after the preloading process.

2.6. Boundary Condition and Loading Procedure

The boundary conditions are selected according to the position of lateral restraints of the laboratory joint model according to Shi et al. [29]. Following the example of a laboratory specimen of beam-to-column joint, a numerical model is made which has stiffeners at the ends of the column to prevent deformations due to the action of the load. Horizontal displacements in the direction of the horizontal axes of the stiffeners on the upper and lower sides of the column are restrained, while displacements at the lower edge of the column are restrained in all directions (Figure 6).

Figure 6. Boundary condition and applied force to the FE model (units in mm).

The load is simulated by displacement control at the end of the beam with a 1100 mm distance from the edge of the column flange. The maximum displacement for monotonic loading is 130 mm. Cyclic loading is simulated according to the SAC 2000 [34] loading

protocol, and the vertical displacement values are given in Table 7. At the ends of the column, a vertical force of 500 kN is simulated. The SAC 2000 loading protocol contains gradually increasing deformation cycles as an interstorey drift angle θ i.e., a relative interstorey displacement divided by the floor height. Interstorey drift angle θ is a parameter used to control the loading protocol.

Table 7. Peak deformation and vertical displacement per cycle.

Step	Peak Deformation θ (rad)	Number of Cycles n	Vertical Displacement at the End of the Beam (mm)
1	0.00375	6	3.28
2	0.005	6	4.38
3	0.0075	6	6.56
4	0.01	4	8.75
5	0.015	2	13.13
6	0.02	2	17.5
7	0.03	2	26.25
8	0.04	2	35.0
9	0.05	2	43.75
10	0.06	2	52.5
11	0.07	2	61.25
12	0.08	2	70

2.7. Results of Numerical Simulation

As a result of numerical simulation of the joint, moment-rotation ($M - \phi$) curves are determined, which describe the behavior of the joint under monotonic and cyclic loading. The total joint rotation ϕ is equal to the sum of the column flange rotation ϕ_c and the end-plate rotation ϕ_{ep} , Figure 7. The column rotation ϕ_c occurs due to shear deformations that occur in the panel zone of the column web and the deformation of other column components such as part of the column flange near the joint and the stiffener. ϕ_c is calculated as $\phi_c = (\delta_3 - \delta_2)/z$, while the end-plate rotation is calculated as $\phi_{ep} = (\delta_5 - \delta_4)/z$, where δ_2 and δ_3 are the horizontal displacements of the column flange; δ_4 and δ_5 horizontal displacements of the end-plate due to bending; z is the axial distance between beam flange.

Figure 7. Description of joint rotation.

As a result of monotonic simulations of joints, in the elastic region are recorded the values of bending moment $M_{e,M}$ and rotation $\phi_{e,M}$. The values of the moment $M_{y,M}$ and rotation $\phi_{y,M}$ correspond to the full plastification of the joint when the material has reached the yield strength in the element along the known yield lines. Full plastification is achieved in the end-plates in all joints. The values of the moment $M_{u,M}$ and the rotation $\phi_{u,M}$ correspond to the ultimate strength of the joint when the material has reached the ultimate tensile strength. The obtained values are given in Table 8.

Table 8. Numerical results obtained by simulations of monotonic loading on joint.

Group of Joint	FE Model	$M_{e,M}$ (kNm)	$\phi_{e,M}$ (rad)	$M_{y,M}$ (kNm)	$\phi_{y,M}$ (rad)	$M_{u,M}$ (kNm)	$\phi_{u,M}$ (rad)
1	EP1_1_M	227.72	0.009	333.01	0.023	428.55	0.083
	EP1_2_M	210.2	0.0085	307.63	0.021	401.82	0.078
	EP1_3_M	194.44	0.008	300.19	0.018	394.08	0.071
2	EP2_1_M	237.73	0.009	355.29	0.025	449.28	0.085
	EP2_2_M	220.38	0.0085	340.11	0.023	430.87	0.079
	EP2_3_M	211.88	0.0083	331.05	0.023	415.02	0.075
3	EP3_1_M	257.26	0.009	370.67	0.024	474.47	0.084
	EP3_2_M	237.21	0.0085	359.2	0.023	460.04	0.081
	EP3_3_M	226.5	0.0082	349.68	0.021	451.38	0.081

As a result of cyclic simulations of joints, in the elastic region are recorded the values of bending moment $M_{e,C}$ and rotation $\phi_{e,C}$. The values of the moment $M_{y,C}$ and rotation $\phi_{y,C}$ correspond to the full plastification of joint. Full plastification of the joints is performed on the end-plates when the material reached the yield point. After reaching full plastification, joints under cyclic loading show effects of hardening where the highest values of bending moment $M_{max,C}$ with rotation ϕ_C are detected. Finally, degradations in strength, stiffness, and fracture occur when the bending moments have values of $M_{fr,C}$ with rotations $\phi_{fr,C}$. The values are given in Table 9.

Table 9. Numerical results obtained by simulations of cyclic loading on joint.

Group of Joint	FE Model	$M_{e,C}$ (kNm)	$\phi_{e,C}$ (rad)	$M_{y,C}$ (kNm)	$\phi_{y,C}$ (rad)	$M_{max,C}$ (kNm)	ϕ_C (rad)	$M_{fr,C}$ (kNm)	$\phi_{fr,C}$ (rad)
1	EP1_1_C	227.72	0.009	310.98	0.016	392.44	0.047	331.34	0.055
	EP1_2_C	210.2	0.0085	287.99	0.0152	369.52	0.047	306.08	0.055
	EP1_3_C	194.44	0.008	278.89	0.0135	344.26	0.037	344.26	0.055
2	EP2_1_C	237.73	0.009	330.02	0.0152	429.20	0.048	391	0.055
	EP2_2_C	220.38	0.0085	313.88	0.0148	413.03	0.047	364.69	0.062
	EP2_3_C	211.88	0.0083	310.17	0.0143	399.65	0.047	358.16	0.063
3	EP3_1_C	257.26	0.009	354.13	0.0157	461.86	0.046	451.09	0.063
	EP3_2_C	237.21	0.0085	342.88	0.015	433.93	0.039	424.73	0.063
	EP3_3_C	226.5	0.0082	328.33	0.015	437.07	0.046	414.66	0.063

Figures 8–10 show comparisons of the $M - \phi$ curves obtained by monotonic and cyclic bending of the joints. From the results in Tables 8 and 9, it can be seen that the joints in the elastic region behave equally under monotonic and cyclic loading and have the same initial rotational stiffness. However, in the plastic region, the joints achieve significantly less deformation $\phi_{y,C}$ during cyclic loading compared to the rotations of the $\phi_{y,M}$ joints exposed to monotonic loading. The values of $\phi_{y,C}$ are less than $\phi_{y,M}$ for 33.3% for joints EP1_3_M/EP1_3_C, while the largest difference is obtained for joints EP2_1_M/EP2_1_C and amounts 64.5% (Table 10). Furthermore, the differences are visible in the values of moments between $M_{y,M}$ and $M_{y,C}$ and it is in the range from 4.7% for joints EP3_1_M/EP3_1_C to 8.4% for joints EP2_2_M/EP2_2_C. The ultimate strengths that the joints can achieve under cyclic load are less than those achieved by the joints under monotonic load. The differences range from 3.3% for EP3_3_M/EP3_3_C joints to 14.5% for EP1_3_M/EP1_3_C. Monotonically loaded joints can achieve significantly higher deformations $\phi_{u,M}$ at the ultimate joint strength in relation to cyclically loaded joints ϕ_C , which can be seen from the value of $\Delta\phi_u$ in Table 10. In addition, due to the accumulation of deformations under cyclic loading, the degradation of strength and stiffness occurs, after which fracture occurs in the joints. This effect is not present in monotonically loaded joints. On the basis of the analysis, it can be concluded that the joints behave significantly differently under monotonic and cyclic loading and that all of these neglected cyclical

behaviour effects need to be considered in further analysis. Therefore, a proposal for a hysteresis envelope model that takes into account the effects of cyclic behaviour is given below.

Figure 8. $M - \phi$ curves for joint (a) EP1_1, (b) EP1_2, and (c) EP1_3 for monotonic and cyclic response.

Figure 9. $M - \phi$ curves for joint (a) EP2_1, (b) EP2_2, and (c) EP2_3 for monotonic and cyclic response.

Figure 10. $M - \phi$ curves for joint (a) EP3_1, (b) EP3_2, and (c) EP3_3 for monotonic and cyclic response.

Table 10. Comparisons of results obtained by monotonic and cyclic simulations of joints.

Group of Joint	FE Model	ΔM_e (%)	$\Delta \phi_e$ (%)	ΔM_y (%)	$\Delta \phi_y$ (%)	ΔM_u (%)	$\Delta \phi_u$ (%)
1	EP1_1_M/EP1_1_C	-	-	7.1	43.8	9.2	50.9
	EP1_2_M/EP1_2_C	-	-	6.8	38.2	8.7	41.8
	EP1_3_M/EP1_3_C	-	-	7.6	33.3	14.5	29.1
2	EP2_1_M/EP2_1_C	-	-	7.7	64.5	4.7	54.5
	EP2_2_M/EP2_2_C	-	-	8.4	55.4	4.3	27.4
	EP2_3_M/EP2_3_C	-	-	6.7	60.8	3.8	19.0
3	EP3_1_M/EP3_1_C	-	-	4.7	52.9	2.7	33.3
	EP3_2_M/EP3_2_C	-	-	4.8	53.3	6.0	28.6
	EP3_3_M/EP3_3_C	-	-	6.5	40.0	3.3	28.6
$\Delta M_y = [(M_{y,M}/M_{y,C}) - 1] \cdot 100$		$\Delta \phi_y = [(\phi_{y,M}/\phi_{y,C}) - 1] \cdot 100$					
$\Delta M_u = [(M_{y,u}/M_{max,C}) - 1] \cdot 100$		$\Delta \phi_u = [(\phi_{u,M}/\phi_C) - 1] \cdot 100$					

2.8. Calibration of Numerical Model

To obtain an accurate numerical model of the joint, first is developed a numerical model that is calibrated with the results of laboratory tests conducted by Shi et al. [7,29]. The numerical model has the following properties: beam H cross section H-300 × 200 × 8 × 12 mm, column H cross section H-300 × 250 × 8 × 12 mm, end-plate thickness is 20 mm, while the thickness of the column stiffener is 12 mm. The friction coefficient on the contact surfaces is 0.44. The thickness of the column flange is equal to the thickness of the end-plate in area 100 mm above the upper edge of the end-plate and 100 mm below the lower edge of the end-plate. A description of the experiment and additional information on the study is available in [7].

The specimens are made of Q345B steel. A trilinear stress-strain relationship of steel is chosen to model the monotonic behaviour of the joints. Material properties are taken from [29]. The yield and tensile strength for steel plates thicker than 16 mm are 363 MPa and 537 MPa, respectively, while the value of Young's modulus of elasticity is 204,227 MPa. The yield and tensile strength for steel plates thinner or equal to 16 mm are 391 MPa and 559 MPa, respectively, while the value of Young's modulus is 190,707 MPa. The value of the Poisson's ratio is 0.3, while Young's modulus of elasticity for bolts is 206,000 MPa. An isotropic/kinematic hardening model of steel is chosen to model the cyclic behaviour of the joint, and values are given in Table 11.

Table 11. Plastic properties of steel Q345B [33].

Kinematic Hardening								Isotropic Hardening	
σ_0 (MPa)	C_1 (MPa)	γ_1	C_2 (MPa)	γ_2	C_3 (MPa)	γ_3	C_4 (MPa)	γ_4	Q_∞ (MPa)
363.3	7993	175	6773	116	2854	34	1450	29	21
									b 1.2

High-strength preloaded bolts M20 grade 10.9 are used to connect the end-plate to the column flange. The value of the bolt preload force is 155 kN. Trilinear model is chosen for high-strength bolts, and the parameters are taken from [33] and given in Table 6.

Calibration of the numerical model is performed for monotonic and cyclic loading. The monotonic loading is achieved by a displacement controlled at the end of the beam at the distance of 1200 mm from the column flange, to the maximum displacement of 125 mm. According to the model of laboratory tests, a constant longitudinal force of 485 kN is simulated on the upper and lower edge of the column, which acts on the entire cross section. Cyclic loading is simulated according to the loading protocol shown in Figure 11.

Figure 11. Loading protocol for simulation of cyclic loading.

All the details related to the selection of the finite element, contact modelling, bolt pretension procedure are previously described. The procedure of bolts preloading by two methods in ABAQUS is described in detail in [35].

The behavior of the beam-to-column joint is shown in the form of a moment-rotation ($M - \phi$) curves shown in Figure 12. The red curves represent the results obtained by numerical simulations of the monotonic and cyclic behavior of the joints and are compared with the black curves representing the results of laboratory tests according to [7,29]. The values of the moment resistance of the joint and the corresponding loading capacity are shown in Table 12. The results obtained by numerical simulations for the monotonic response of joints give satisfactory results with slight deviations. The results obtained by simulations for cyclic loading have differences in loading capacity and moment resistance equal to 5.89% and 1.03%, respectively, as compared to the values obtained by laboratory tests. After reaching the ultimate strength, the numerical model cannot describe the degradation of strength that occurs in the real model. This shortcoming of the model is solved in the modelling of new joints in such a way that the material models of steel take into account the damage model.

Figure 12. Comparisons of the $M - \phi$ curves of joints obtained by numerical simulations and laboratory tests for (a) monotonic loading and (b) cyclic loading.

Table 12. Comparison of numerical simulations and laboratory tests results [7,29].

Load Type	Numerical Simulations		Laboratory Test by Shi et al. [7,29]	
	Loading Capacity (kN)	Moment Resistance (kNm)	Loading Capacity (kN)	Moment Resistance (kNm)
Monotonic	256.89	308.28	256.9	308.3
Cyclic	237.89	285.47	251.9	288.4

Figure 13 shows the failure modes of the joints obtained by numerical simulations in monotonic and cyclic loading and compares them with the joints tested in the laboratory. Simulations show satisfactory behavior in relation to experimental results.

Figure 13. Failure modes of joints under monotonic loading (a) Numerical model (b) Test specimen by Shi et al. [29] and under cyclic loading (c) Numerical model and (d) Test specimen by Shi et al. [7].

3. Mathematical Model of Hysteresis Envelope

3.1. Proposal of Hysteresis Envelope Model

The hysteresis curves obtained by the numerical simulations shown in Figures 8–10 are almost symmetric; therefore, the upper right quadrants are chosen for the development of the hysteresis envelope model. Through the peaks of the hysteresis loop is the withdrawn envelope in the form of four lines (Figure 14). The first line ($0 - A_C$) corresponds to the elastic behaviour of the joints, the second line ($A_C - B_C$) corresponds to the area of full plastification of the joint, the third line ($B_C - C_C$) includes the hardening area, while the fourth line ($C_C - D_C$) covers the area of strength degradation after which the fracture occurs in the joint.

Figure 14. Proposal of hysteresis envelope model.

In engineering practice, it is totally impractical to determine the cyclic properties of joints and apply them in the seismic calculations of structures. Such calculations are very exhausting and long-lasting for practical application. Therefore, monotonic properties can be obtained much easier. In addition to laboratory tests and numerical simulations, it is possible to obtain monotonic properties by applying the component method, which is accepted in the European regulations for the design of joints in steel structures [27]. This method is available in numerous commercial software. Hence the goal of the proposed hysteresis envelope model is to provide correction factors to correct the monotonic trilinear model of the joints. Therefore, the hysteresis envelope ($0 - A_C - B_C - C_C - D_C$) is reduced to a trilinear model ($0 - A_C - B_C' - D_C'$). The elastic part does not change under monotonic and cyclic loading, which means that the bending moment and rotation in the elastic region can be adopted from the monotonic behaviour of the joints ($M_{e,C} = M_{e,M}$, $\phi_{e,C} = \phi_{e,M}$). The position of the point B_C' is selected at the place of bending moment at full plastification of the joint ($M_{y,C} = M_{y,M}$), and rotation of the joint during full plastification under cyclic loading $\phi_{y,C}$. The position of the point D_C' has a rotation value corresponding to the fracture of the joint $\phi_{fr,C}$, while the value of the bending moment $M_{fr,C}$ is obtained in such a way that the deformation energy below the proposed hysteresis envelope model ($0 - A_C - B_C' - D_C'$) corresponds to the deformation energy below the hysteresis envelope ($0 - A_C - B_C - C_C - D_C$).

The application of the hysteresis envelope model to the upper right quadrants of hysteresis curves is shown for joints EP1_3, EP2_3 and EP3_3, and is shown in Figure 15a–c. A comparison of the trilinear model and the proposed hysteresis envelope model for joints EP1_3, EP2_3 and EP3_3 is shown in Figure 15d–f. The values of bending moments and rotations to define the individual segment of the hysteresis envelope model are given in Table 13. All values are determined directly from the hysteresis curves obtained by the numerical simulations of the joints.

Figure 15. Application of hysteresis envelope model on joints (a) EP1_3, (b) EP2_3, (c) EP3_3, and comparison of trilinear model and hysteresis envelope model for joints (d) EP1_3, (e) EP2_3, and (f) EP3_3.

Table 13. Parameter of the proposed hysteresis envelope model.

Group of Joint	Model	$K_{e,H}$ (kN/m)	$M_{y,H}$ (kNm)	$\phi_{y,H}$ (rad)	$M_{fr,H}$ (kNm)	$\phi_{fr,H}$ (rad)
1	EP1_1_C_ Hysteresis envelope model	25,302.2	333.01	0.016	370.9	0.055
	EP1_2_C_ Hysteresis envelope model	24,729.4	307.63	0.0152	351.5	0.055
	EP1_3_C_ Hysteresis envelope model	24,305	300.19	0.0135	335.8	0.055
2	EP2_1_C_ Hysteresis envelope model	26,414.4	355.29	0.0152	411.0	0.055
	EP2_2_C_ Hysteresis envelope model	25,927.1	340.11	0.0148	407.11	0.0615
	EP2_3_C_ Hysteresis envelope model	25,527.7	336.05	0.0143	392	0.063
3	EP3_1_C_ Hysteresis envelope model	28,584.4	370.67	0.0157	477.9	0.063
	EP3_2_C_ Hysteresis envelope model	27,907.1	359.2	0.015	456.1	0.063
	EP3_3_C_ Hysteresis envelope model	27,622.0	349.68	0.015	443.5	0.063

3.2. Regression Analysis

Based on the ratios $\phi_{y,H}/\phi_{y,M}$, $\phi_{fr,H}/\phi_{u,M}$ i $M_{fr,H}/M_{u,M}$ are defined as regression functions that allow using the monotonic properties of the joint, which can be determined by one of the known methods such as the Component method [27], and can finally determine the cyclic properties of the joint. To define the functional dependency of the mentioned ratios, multiple nonlinear regression is chosen and a nonlinear exponential regression model is applied. The ratios of rotations and bending moments (Exp. 1) for the proposed hysteresis envelope model and the monotonic model are defined as a function of the independent variables t and p , where p is the thickness of the end-plate, and t is the vertical spacing between the bolt rows.

$$\frac{\phi_{y,H}}{\phi_{y,M}} = f(t, p) \quad \frac{M_{fr,H}}{M_{u,M}} = f(t, p) \quad \frac{\phi_{fr,H}}{\phi_{u,M}} = f(t, p) \quad (1)$$

The following terms define the hysteresis envelope model:

$$\begin{aligned} & (0 - A_C) \left\{ \begin{array}{l} M_{e,H} = M_{e,M} \\ \phi_{e,H} = \phi_{e,M} \end{array} \right. \quad \phi \leq \phi_{e,H} \\ & (A_C - B_C') \left\{ \begin{array}{l} M_{y,H} = M_{y,M} \\ \phi_{y,H} = \phi_{y,M} (0.2341t^{-0.208}p^{0.3293}) \end{array} \right. \quad \phi_{e,H} < \phi < \phi_{y,H} \\ & (B_C' - D_C') \left\{ \begin{array}{l} M_{fr,H} = M_{u,H} (0.5024t^{0.4429}p^{-0.131}) \\ \phi_{fr,H} = \phi_{u,M} (0.0059t^{0.3086}p^{0.8002}) \end{array} \right. \quad \phi_{y,H} < \phi < \phi_{fr,H} \end{aligned} \quad (2)$$

4. Seismic Analysis

To assess the influence of the joint model on the behaviour of the moment-resisting steel frame, a seismic analysis is performed using the nonlinear static N2 method according to Eurocode 8 [36]. Nonlinear static analysis is performed on steel frames for the joint EP1_3, EP2_3, EP3_3, whose behaviour is described by the proposed hysteresis envelope model and the peak ground acceleration 0.3 g. Based on seismic analysis, an estimate of the maximum ground acceleration is made and compared with the results of the behaviour of the steel frame obtained by the nonlinear static analysis with the trilinear model of the joint. Seismic analyses are conducted using the SeismoStruct software [37].

4.1. Description of the Building

To define the effects of seismic action, the relevant moment-resisting frame of the steel structure is taken as shown in Figure 16a. The position of the relevant steel frame is shown on a floor plan of the structure, Figure 16b. The structure consists of five main steel frames at a distance of 6 m that are connected by longitudinal beams. The moment-resistant steel structure is considered as business buildings and designed according to Eurocode 3 [27].

The 18.6 kN/m and 13.68 kN/m are representative values of a dead load acting on the beams on the first three floors and on the last floor, respectively. Representative values of the live load acting on the beams on the first three floors and on the last floor are 18 kN/m and 4.5 kN/m, respectively. In the nodes of the frame act concentrated masses of 10 tons from the main reinforced concrete walls.

Figure 16. (a) 4-storey moment-resisting steel frame, and (b) Floor plan of the steel structure.

4.2. Trilinear Joint Model

To define the monotonic behaviour of the joints, a trilinear model is chosen according to the authors of Wang et al. [38]. The mathematical model consists of three linear segments. The first segment represents the elastic behaviour of the joint with an initial rotational stiffness $K_{e.M}$, and can be obtained according to Equation (3), where $M_{e.M}$ is the elastic capacity of the joint; $\phi_{e.M}$ is the rotation of the joint corresponding to the moment $M_{e.M}$.

$$K_{e.M} = \frac{M_{e.M}}{\phi_{e.M}} \quad (3)$$

The second segment represents the area of plastification of the joint, which, for simplicity, has been replaced by a linear segment, and the rotational stiffness $\alpha K_{e.M}$ can be obtained by Equation (4), where $M_{y.M}$ is the bending moment in the joint that occurs during full plastification of the joint; $\phi_{y.M}$ is the rotation of the joint corresponding to the moment $M_{y.M}$.

$$\alpha K_{e.M} = \frac{M_{y.M} - M_{e.M}}{\phi_{y.M} - \phi_{e.M}} \quad (4)$$

The third segment represents hardening of the joint where the rotational stiffness $\beta K_{e.M}$ can be obtained by Equation (6), where $M_{u.M}$ is the maximum bending moment in the joint; $\phi_{u.M}$ is the rotation of the joint corresponding to the bending moment $M_{fr.M}$.

$$\beta K_{e.M} = \frac{M_{u.M} - M_{y.M}}{\phi_{u.M} - \phi_{y.M}} \quad (5)$$

The values of the bending moments and rotation of the joints required to determine Equations (3)–(5) are given in Table 14. The parameters of the trilinear model are given in Table 14.

Table 14. Parameters of trilinear mathematical model.

Joint Model	$K_{e,M}$ (kN/m)	α	β
EP1_3_M_Tilinear model	24,305	0.435	0.073
EP2_3_M_Tilinear model	25,527.7	0.318	0.063
EP3_3_M_Tilinear model	27,622	0.348	0.061

4.3. Nonlinear Static Pushover Analysis (N2 Method)

To assess the performance of the steel frame for the expected seismic load, the target displacement is determined following the N2 method proposed by [8] and suggested in Eurocode 8—Part 1. The analyses are carried out along one direction of the frame with the trilinear model of the joint and proposed hysteresis envelope model of the joint.

A 5% damping factor is defined for bolted joints in steel structures. The construction has a business purpose and belongs to the II category of importance and the value of γ_I is equal to one. Nonlinear static analysis is performed for the peak ground acceleration $a_{gR} = 0.3 g$. The focus of this research is seismic activities of a higher intensity, then Type I elastic spectra is applied for values of surface magnitudes greater than 5.5. The type of foundation D is selected. The fundamental period of vibration is obtained by modal analysis in the SeismoStruct software and amounts to $T_1 = 1.42s$.

Pushover analyses are performed considering the modal distribution of lateral load proportional to the first mode as recommended in the Eurocode 8—Part 1. The lateral load vector P is determined according to Equation (6), where P is the intensity lateral load, $[m]$ is the diagonal mass matrix, and $\{\phi\}$ is the assumed form of displacement.

$$\{P\} = p \{\psi\} = p [m] \{\phi\} \quad (6)$$

The mass matrix is defined in Equation (7), while the assumed form of displacement is given in Equation (8) and represents the modal distribution. The distribution of lateral forces in the frame model is shown in Figure 17.

$$m = \begin{bmatrix} 51.9 & 0 & 0 & 0 \\ 0 & 70.1 & 0 & 0 \\ 0 & 0 & 70.1 & 0 \\ 0 & 0 & 0 & 70.1 \end{bmatrix} [t] \quad (7)$$

$$\{\phi\}^T = \{0.2 \quad 0.6 \quad 0.9 \quad 1\} \quad (8)$$

Figure 17. Steel frame model with modal distribution of lateral force.

As a result of the pushover analysis, the capacity curve for the steel frames is obtained. The capacity curves for the steel frame with semi-rigid joints modelled with the trilinear joint model (Frame1_M) are presented in Figure 18a. The capacity curve obtained for the multi-degree-of-freedom (MDOF) system is transformed to the equivalent single-degree-of-freedom system. The mass of an equivalent system with SDOF is $m^* = 160.55$ t, and the transformation factor amounts to $\Gamma = 1.35$. Based on pushover analysis, the behaviour of the joints is analysed for each individual steel frame in order to determine the occurrence of plastification. Full plastification is achieved when the rotation of the joints has the values $\phi_{y,M}$ and $\phi_{y,C}$ that are given in Tables 8 and 9. Opening the last plastic hinge in the joint brings the frame system into the mechanism, which defines the maximum displacement of the frame d_m . The procedure is shown for Frame 1_M and peak ground acceleration of 0.3 g, Figure 19. The red dots indicate the occurrence of plastic hinges, the blue dot indicates a fracture in the joint, while the yellow dots indicate the joints that have not been plastically deformed. The occurrence of the limit rotation in joint 314 defines the maximum displacement d_m of the Frame 1_M which amounts to 0.732 m. This displacement occurred before the full plasticization of all joints and is adopted as a displacement in which the system goes into the mechanism. Frame 1_M is finally subjected to a new iteration of the pushover analysis for the value of the top displacement corresponding to the displacement $d_m = 0.732$ m, Figure 18b. A bilinear idealization of the capacity curve is performed, from which the values of F_y^* and d_y^* are defined. Elastic period T^* amounts to 1.65 s, which is larger than $T_C = 0.5$ s and belongs to the range of medium and long periods. Consequently, the inelastic displacement demand is equal to the elastic demand ($S_d = S_{de}(T^*)$). The target displacement $S_{de}(T^*)$ of the SDOF system amounts to 31.18 cm, whereas the corresponding ductility demand amounts to $\mu = 1.51$. In the final step, the target displacement of the SDOF system is transformed back to the target displacement of the MDOF system using the transformation factor $\Gamma = 1.35$. The target displacement amounts to $d_t = 43.44$ cm. From the ratio of the top displacement $d_m = 0.732$ m and the target displacement $d_t = 0.434$ m multiplied by the peak ground acceleration of 0.3 g, the value of the largest earthquake with a peak ground acceleration of 0.51 g is obtained. The selected results of the nonlinear static pushover analysis using the N2 method are shown in Table 15.

Figure 18. Capacity curve with bilinear idealization obtained for (a) First iteration of pushover analysis, and (b) Second iteration of pushover analysis.

Figure 19. Formation of plastic hinges in Frame 1_M.

Table 15. Nonlinear static pushover analysis parameters.

Model	$d_m(A)$ (m)	F_y^* (kN)	d_y^* (m)	T^* (s)	S_{ay} (g)	$S_{ae}(T^*)$ (g)	$S_{de}(T^*)$ (cm)	μ	d_t (cm)	Max $a_g(g)$
Frame1_M	0.73	511.71	0.219	1.65	0.33	0.49	31.18	1.51	43.44	0.51
Frame1_C	0.542	492.9	0.207	1.63	0.31	0.49	31.97	1.59	43.16	0.38
Frame2_M	0.893	526.33	0.224	1.64	0.33	0.49	32.14	1.48	43.39	0.62
Frame2_C	0.632	532.16	0.207	1.57	0.34	0.52	30.74	1.53	41.50	0.46
Frame3_M	0.853	539.48	0.213	1.58	0.34	0.51	31	1.49	41.85	0.61
Frame3_C	0.652	549.72	0.207	1.54	0.35	0.53	30.30	1.51	40.91	0.48

Frame1_M, Frame2_M and Frame3_M: Steel frame with joint model EP1_3_M_Triangular model, EP2_3_M_Triangular model, and EP3_3_M_Triangular model, respectively. Frame1_C, Frame2_C and Frame3_C: Steel frame with joint model EP1_3_C_Hysteresis envelope model, EP2_3_C_Hysteresis envelope model, and EP3_3_C_Hysteresis envelope model, respectively.

5. Conclusions

In this study, numerical models of double extended end-plate bolted beam-to-column joints are developed with three different end-plate thicknesses and three different bolt row spacings. The model takes into account the geometric nonlinearity of the connecting elements, preloading of bolts, contacts between connected plates and bolts, and nonlinear properties of steel. The model is calibrated based on experimental tests available in the literature. Simulations are performed to determine the effect of monotonic and cyclic bending. The nonlinear properties of the material are described using an isotropic/kinematic hardening model and a steel damage model. This model allows the degradation of strength and stiffness and the appearance of a joint fracture after the accumulation of deformations.

On the basis of the numerical results of the cyclic behavior of the joints, a hysteresis envelope model is developed. This joint model implemented in the steel frame model allows a more accurate assessment of the behavior using a nonlinear static pushover

analysis compared to the response of the same frame that takes into account the monotonic model of the joints.

The results of numerical simulations of joints under monotonic and cyclic bending indicate that joints have up to 54.5% lower ability to total deformation capacity, and that at full plastification, they achieve up to 64.5% less rotation compared to monotonic bending, i.e., they have less ductility. Under monotonic loading, the joints have a significant ability of rotation when the ultimate strength of the joint is reached, no sudden degradation of strength was observed. Under cyclic loading, the joints have a hardening effect after full plastification. After reaching the ultimate strength, sudden degradation of strength and stiffness was observed due to the accumulation of deformations under cyclic loading.

Based on the results of the nonlinear static pushover analysis of the steel frame in which the joints are modeled using the hysteresis envelope model, it is estimated that such frames can withstand up to 25.8% less seismic loads compared to the estimates obtained for frames with monotonic joint models. Therefore, it can be concluded that the application of a monotonic joint model can unrealistically overestimate the behavior of the steel frame under earthquake.

In future work, the plan is to investigate the influence of the hysteresis envelope model on the behavior of a moment-resisting frame at higher values of peak ground acceleration, and compare the results with the results of nonlinear dynamic analysis using time history.

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