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On the Behaviour of 316 and 304 Stainless Steel under Multiaxial Fatigue Loading: Application of the Critical Plane Approach

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Abstract: In this work, the multiaxial fatigue behaviour of 316 and 304 stainless steel was studied. The study was based on the critical plane approach which is based on observations that cracks tend to nucleate and grow in specific planes. Three different critical plane models were employed to this end, namely Fatemi–Socie (FS), Smith–Watson–Topper (SWT) and the newly proposed Sandip–Kallmeyer–Smith (SKS) model. The study allowed equi-biaxial stress state, mean strain and non–proportional hardening effects to be taken into consideration. Experimental tests including different combinations of tension, torsion and inner pressure were performed and were useful to identify the predominant failure mode for the two materials. The results also showed that the SKS damage parameter returned more conservative results than FS with lower scatter level in both materials, with prediction values between FS and SWT.

Keywords: critical plane model; multiaxial fatigue; non–proportional loading; 316 stainless steel; 304 stainless steel

1. Introduction

Fatigue failure is a common problem for a wide range of industries. Since the first reported study, new materials and advanced methods to predict the number of cycles until failure has appeared. Uniaxial or bending rotation cyclic tests are often conducted to characterise the fatigue behaviour of different metals [1]. However, most mechanical applications imply more complex scenarios, real service loads are usually variable and designs include complex profile shapes instead of just flat or cylindrical surfaces. As a consequence, different stresses/strain distributions appear on the real structures subjected to cyclic loads [2–4]. For characterising such complex scenarios there exists more sophisticated methods such as the critical plane approaches as an alternative to the classical models [5]. Critical plane models have been successfully applied for different materials and service loads. For example Chu observed improvements using these methods for AISI 1045 steel under complex loading conditions [6]. Sharifimehr employed critical plane methods to predict the fatigue life of a brittle and a ductile material under variable amplitude loads [7]. Llavori also used the critical plane methods to study the fatigue performance of a welded joint of S275JR, and was able to achieve better predictions as compared to classical methods [8]. One of the main strengths of critical plane methods is that they take into consideration the physical mechanisms involved in the nucleation and growth of the fatigue crack [9,10]. Nevertheless, there exists other alternative approaches that allow more accurate predictions to be



achieved. One such approach is the Strip Yield Model (implemented in Nasgro software [11]). Besides the total fatigue life, these cycle by cycle models can also describe with certain accuracy the propagation stage until final failure takes place [12]. Depending on the material and the loading conditions, certain mechanisms will show a dominant presence along the fatigue process. For example, brittle materials tend to show a dominant Mode I crack growth along the fatigue process while ductile materials tend to have a dominant Mode II crack growth [13]. Critical plane methods are based on defining the plane where the highest damage takes place. This also means that they allow the crack growth angle to be predicted. This has been shown by Reis et al. who assessed the crack path initiation and growth for several structural steels [14,15]. The procedure requires evaluating the damage along the cycle from some stress and strain components. In some cases, obtaining such stress and strain components can be difficult and might introduce an additional source of error. Depending on the type of material, different critical plane models have been proposed. Models that include only stress variables are more useful in the high cycle regime but often fail at computing the fatigue damage in the low-cycle regime based on S-N curves. Stress values used in such models are frequently unrealistic and different to the actual stress experienced by the specimen due to the material behaviour above yield stress. Models that include strain variables are more robust in that sense.

To date, there is not a universal critical plane model that is valid for all the types of materials and all loading conditions. A very comprehensive review of different critical plane models can be found in the literature [16].

Well established models such as the Fatemi–Socie [17] or Smith et al. [18] were thoroughly studied, showing good results for ductile and brittle behaviour materials, respectively. Usually these models are chosen as a benchmark to propose new damage parameters [19,20]. In some cases the new model will return better results for the studied material and load condition [21,22], considering it more appropriate for those scenarios.

In this work a newly proposed critical plane damage parameter, called Sandip–Kallmeyer–Smith (SKS) was assessed based on its excellent performance on a low carbon steel [22]. First, the collapse capacity of the newly proposed damage parameter was evaluated. This was done by fitting the model to a set of experimental data with different loading paths both under proportional and non-proportional loads. Then, the fitted curve was used to predict fatigue lives for different multiaxial cases. The study was conducted on two stainless steels, namely 316 stainless steel and 304 stainless steel. The efficacy of the SKS damage parameter was compared with Fatemi–Socie model and Smith–Watson–Topper model.

2. Materials and Methods

The different models were tested on 316 and 304 stainless steels that are widely used in the industry. Previous studies observed better results with Mode II/III dominant critical plane methods for 316 stainless steel and with Mode I for 304 stainless steel [19,23].

All the tests were carried out on hollow cylindrical samples with 8.5 mm gauge length, 14 mm outer diameter and 12 mm inner diameter. The specimens were carefully polished to a surface roughness of approximately 0.3 μ m both in the external and the internal surface. An in-house built fatigue machine allowed axial loads as well as inner pressure to be applied, thus allowing a very wide range of loading paths to be applied (see Figures 1 and 2). All tests were conducted in air. More details about the biaxial loading rigs, as well as additional details about the experiments can be found elsewhere [24,25].

The experimental tests employed for fitting the models and to evaluate the collapse capacity of the models on the 316 and the 304 stainless steel are shown in Tables 1 and 2, respectively [26]. Both experimental sets include proportional and non-proportional (out-of-phase between axial and shear strain) fatigue tests, as described in Figure 1. There are no mean stress tests among the experimental tests used for the 316 stainless steel. This is because no significant effect of the means stress on the fatigue life was observed for this material [27]. A comparison of the equivalent tests between 316 and 304 stainless steel (Tables 1 and 2) indicates that 304 stainless steel presents a higher hardening level.



Figure 1. Proportional and non-proportional loading paths.



Figure 2. Proportional multiaxial loading paths applied on the 316 stainless steel specimens.

Table 1.	Summary of th	e experimental	data used to	o fit the model	parameters for	316 stainless steel.
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Path	ID	ε_a	γa	σ_a	$ au_a$	N_{f}
Р	1	0.0025	-	225.5	-	25,100
	2	0.0035	-	252.5	-	8750
	3	0.005	-	278	-	4220
	4	0.0075	-	326	-	2200
NP	1	0.0015	0.0026	219.5	138.5	32,400
	2	0.0025	0.0043	344.5	219.5	4780
	3	0.0035	0.0061	412.5	238.5	3660
	4	0.0050	0.0087	474	299.5	1360
	5	0.0075	0.0130	615	388	410

Table 2. Summary of the experimental data used to fit the model parameters for 304 stainless steel.

Path	ID	\mathcal{E}_{a}	γa	σ_a	$ au_a$	N_f
Р	1	0.0025	-	265	-	49,000
	2	0.0033	-	290	-	23,400
	3	0.004	-	315	-	7100
	4	0.005	-	365	-	1500
NP	1	0.002	0.0035	300	168	50,000
	2	0.002	0.0035	307	176	45,000
	3	0.0035	0.0061	457	256	3730
	4	0.0035	0.0061	477	267	3560

The tests were used to fit the SKS model that was subsequently used to predict fatigue lives for both materials under a range of multiaxial loading conditions. The predictions given by the SKS model are valid for the range of fatigue lives covered in Tables 1 and 2 for 316 and 304 stainless steel, respectively.

Fifteen different tests were conducted on 316 stainless steel to evaluate the different critical plane models, which are described in Table 3. The loading path used for the 316 stainless steel are shown in Figure 2. The load control mode was used to conduct the tests. It was possible to produce a triaxial stress state at the inner surface of the specimen and a biaxial stress state at the outer surface. On the

outer surface, cases 1, 2 and 3 produced a uniaxial stress state, cases 4 and 7 a biaxial stress state and cases 5 and 6 an alternating pulsating stress state in perpendicular directions. For all the cases, a high level of ratchetting was observed [27]. The total reverse stress was applied in the case 3 (Figure 1) which promoted a non-zero mean strain probably because of the real stress asymmetry caused by the high load levels. Accordingly, a biaxial stress condition was induced on the outer surface. Since the principal stress directions are constant in time, all loading paths can be considered as proportional, given that the main slip plane does not change along the load cycle. The load ratio for Path 3 in Figure 2 is R = -1. The rest of tests had a zero load ratio, R = 0. These tests will be used in Section 4 to evaluate the accuracy of the different models, as well as the response of the models under mean stress loads and biaxial conditions.

Path	$\Delta\sigma_z$	$\Delta\sigma_{ heta}$	$\Delta arepsilon_z$	$\Delta arepsilon_{ heta}$	N_{f}
1	445.15	1.8277	0.0022	0.0007	159,600
2	2.3931	450.32	0.0016	0.0055	29,300
2	1.8885	420.25	0.0013	0.0037	24,800
2	1.6815	366.46	0.0005	0.0033	53,000
3	1,024.3	1.7169	0.0406	0.0059	208
3	884.49	1.4092	0.0245	0.0042	393
4	399.2	424.1	0.0006	0.0051	3560
4	373.95	413.12	0.0033	0.0036	8400
5	6.5677	346.09	0.0023	0.0058	14,486
5	400.92	393.85	0.0057	0.0016	5300
5	376.34	383.25	0.0023	0.0045	14,486
6	399.76	450.21	0.0016	0.0048	25,770
6	442.64	511.01	0.0018	0.0049	13,542
6	375.8	335.22	0.0022	0.0027	31,400
7	389.49	465.73	0.0019	0.0019	24,700

Table 3. Summary of the 316 stainless steel experimental data used to evaluate the fitted models.

The experimental tests used to evaluate the different models on the 304 stainless steel are shown in Table 4. The loading path used for studying the 304 stainless steel are shown in Figure 3. That is 29 experimental tests, three for path 0, and two for each of the other paths in Figure 3. These tests will allow the different models to be evaluated in terms of their capacity to take into account the fatigue damage produced by the hardening caused by non-proportional loads. For the same range of applied strains, increasing the non-proportionality in the loads requires increasing stresses to conduct the test. Previous results showed a high-hardening level for the 304 stainless steel [26]. Cases 1 and 6 can be considered proportional as the principal stress direction are constant along the cycle. The maximum non-proportionality factor appeared for cases 9, 10, 11, 13 and 14 [28]. Further experimental details are available elsewhere [25,26,28].

The coordinates adopted in this work are shown in Figure 4a. The radial and axial directions are defined as *R* and *Z*, respectively. The hoop direction θ is defined as being perpendicular to both other directions. The plane φ is defined by the normal vector \vec{n} Figure 4b. This vector forms an angle α between its projection over the plane [θR] and the direction *R*. It also forms an angle β between \vec{n} and the *Z* direction. The vector \vec{p} , parallel to the intersection between φ and [θR] is defined to consider the shear values. In addition, another vector \vec{s} contained on φ and perpendicular to \vec{p} is also defined for handling the shear component.

For the 316 stainless steel loading paths, the stresses and strains are computed at different planes φ . This is done by evaluating α and β angles in 15° increments in the range 0° to 90° [10]. For the 304 stainless steel loading paths, the hoop and radial strain should be the same on the surface (i.e., $\varepsilon_{\theta} = \varepsilon_R$). The maximum strain values are found on planes perpendicular to the surface ($\alpha = 90^\circ$), with β ranging between 0° and 180°.

Once the strain and stress values are defined on each plane, a cycle counting process was performed using the rainflow method [29]. For dominant Mode II models, the shear strain cycles were counted and for dominant Mode I models, the normal strain cycles were counted. Mean and amplitude values for shear strain and shear stress were obtained using the circumscribed theory proposed by Papadopoulus [30]. Finally the damage was computed following Miner's linear rule [31].

Path	$\Delta arepsilon_z$	$\Delta \gamma_{ heta z}$	$\Delta\sigma_z$	$\Delta au_{ heta z}$	N_f
1	0.0113	0	730	730	1700
1	0.012	0	805	805	690
1	0.015	0	825	825	540
2	0.005	0.0087	685	685	9500
2	0.008	0.0139	950	950	1400
3	0.005	0.0087	670	670	20,000
3	0.008	0.0139	860	860	2100
4	0.005	0.0087	670	670	2400
4	0.008	0.0139	975	975	820
5	0.005	0.0087	790	790	3400
5	0.008	0.0139	1010	1010	900
6	0.005	0.0087	485	485	17,500
6	0.008	0.0139	590	590	3200
7	0.005	0.0087	500	500	9700
7	0.008	0.0139	670	670	2600
8	0.005	0.0087	530	530	18,000
8	0.008	0.0139	735	735	1700
9	0.005	0.0087	760	760	2050
9	0.008	0.0139	1055	1055	470
10	0.005	0.0087	780	780	2950
10	0.008	0.0139	1075	1075	660
11	0.005	0.0087	765	765	2600
11	0.008	0.0139	1060	1060	320
12	0.005	0.0087	570	570	14,400
12	0.008	0.0139	850	850	1200
13	0.005	0.0087	660	660	4750
14	0.008	0.0139	940	940	710
14	0.005	0.0087	655	655	3200
	0.008	0.0139	965	965	1000

Table 4. Summary of the 304 stainless steel experimental data used to evaluate the models.



Figure 3. Proportional and non-proportional multiaxial loading paths applied on the 304 stainless steel specimens.



Figure 4. (a) Coordinates adopted in this work and (b) definition of the plane ϕ that was used to evaluate the critical plane.

3. Critical Plane Models

Critical plane models are based on observations of the nucleation and growth of fatigue cracks [10]. They are based on a damage parameter (DP) which incorporates stress and/or strain information that is subsequently used to predict the fatigue life. The plane where the DP is maximised is called the critical plane. The DP is defined for each cycle extracted along the entire loading block. For the sake of computational speed, the damage below 25% of the maximum damage along the loading block was not taken into account in the algorithm. This is because the effect of such low damage values on the fatigue life is negligible. Subsequently, a damage accumulation rule was used to obtain the number of cycles until the failure. In this work, three different critical plane models were used to characterise the multiaxial fatigue behaviour of the 316 and 304 stainless steels. The Fatemi—-Socie (FS) critical plane model is normally employed for materials prone to shear failure [17]. The Smith–Watson–Topper (SWT) critical plane model gives accurate predictions for materials with predominant tension failure [21]. In addition, a newly proposed critical plane model by Suman, Kallmeyer and Smith (SKS) was also used to investigate the two materials. By studying the two materials with the FS and SWT models, it will be possible to identify the predominant failure mechanism for each of the materials. In addition, the study will also be useful to assess the predictive capabilities of the newly proposed model via comparison with two widely used critical plane models.

3.1. Fatemi–Socie model (FS)

The Fatemi–Socie model defines a strain type DP (Equation (1)) [17]. The model is based on that proposed by Brown and Miller [1]. They suggested substituting the normal strain component by a normal stress component. The DP is defined on the plane φ^* that maximises the shear strain range, $\Delta \gamma$.

$$DP_{FS} = \frac{\Delta \gamma_{max}}{2} \left(1 + k \frac{\sigma_{n,max}}{\sigma_y} \right) \tag{1}$$

where $\Delta \gamma_{max}/2$ is the maximum shear strain amplitude, $\sigma_{n,max}$ is the maximum tensile stress at φ^* , σ_y is the yield stress and k is a material parameter. The values for the yield stress were set to 260 MPa and 290 MPa for 316 and 304 stainless steel, respectively [26].

The strain hardening effect is considered with the $\Delta \gamma_{max}$ to be 2 times the $\sigma_{n,max}$ product. The mean normal stress effect is also considered via $\sigma_{n,max}$.

The parameter k represents the sensitivity of the material to normal stresses. This parameter can be estimated from the fatigue life N_f [10], through Equation (2).

$$k = \left[\frac{\frac{\tau'_{f}}{G}(2N_{f})^{b_{\gamma}} + \gamma'_{f}(2N_{f})^{c_{\gamma}}}{(1+\nu_{e})\frac{\sigma'_{f}}{E}(2N_{f})^{b} + (1+\nu_{p})\varepsilon'_{f}(2N_{f})^{c}} - 1\right]\frac{\sigma'_{y}}{\sigma'_{f}(2N_{f})^{b}}$$
(2)

where v_e and v_p are the Poisson's ration in the elastic and plastic regimes, respectively, *E* the Young modulus, σ'_f the fatigue strength coefficient, *b* the fatigue strength exponent, ε'_f the fatigue ductility coefficient, *c* the fatigue ductility exponent, σ'_y the cyclic yield stress, *G* the shear modulus, τ'_f the shear fatigue strength coefficient, b_γ the shear fatigue strength exponent, γ'_f the shear fatigue ductility coefficient and c_γ the shear fatigue ductility exponent.

Figure 5 shows the k values for 316 and 304 stainless steel against fatigue life N_f . For the 316 stainless steel, there is little variation of the k parameter with respect to the fatigue life. In addition the *k* parameter is very small (around 0.1) throughout the entire life. Figure 5 indicates that *k* parameter is much more sensitive to the fatigue life for the 304 stainless steel, with values ranging between ~0.5 and ~1.25. It is noted that the sensitivity parameter increases with the fatigue life for both materials, although with a much greater gradient for the 304 steel. For the cases where little information is gathered at either low fatigue lives or high fatigue lives, it is possible to use a constant sensitivity factor [10,19]. Nevertheless, in general this might reduce the accuracy of the fatigue predictions.



Figure 5. Fatemi–Socie normal stress sensitive factor k for 304 and 316 stainless steel.

3.2. Smith–Watson–Topper Model (SWT)

The Smith, Watson and Topper model [18] defines a strain energy density type DP (Equation (3)). The DP considers the normal strain and stress acting on the critical plane φ^* . The DP is defined on the plane φ^* that maximises the normal strain range, $\Delta \varepsilon$.

$$DP_{SWT} = \frac{\Delta\varepsilon}{2}\sigma_{n,max} \tag{3}$$

where $\Delta \varepsilon/2$ is normal strain amplitude, $\sigma_{n,max}$ is the maximum tensile stress at φ^* .

The strain hardening effect is considered in the SWT model through the $\Delta \varepsilon/2$ and $\sigma_{n,max}$ product. The mean normal stress effect is also taken into account via $\sigma_{n,max}$.

3.3. Sandip-Kallmeyer-Smith Model (SKS)

The multiaxial fatigue behaviour of the two steels is also evaluated with the Suman, Kallmeyer and Smith newly proposed critical plane model [21]. The SKS model defines a stress type DP (Equation (4)). Stress based models, such as Findley [32] and McDiarmid [33] normally give worse predictions for low-cycle fatigue due to lack of real stress information under such conditions. This is overcome with SKS model because it includes a strain component, in a similar way to the FS [17] and SWT [18] models. By using shear strain and shear stress elements, the SKS damage is more suitable for ductile

failing materials. Following Sines compilation of ductile failing materials [34], such an effect is more predominant in the low-cycle regime. The DP is defined on the plane φ^* that maximises the shear strain range, $\Delta \gamma$.

$$DP_{SKS} = (G\Delta\gamma)^w \tau_{max}^{(1-w)} \left(1 + k \frac{(\sigma \cdot \tau)_{max}}{\sigma_0^2} \right)$$
(4)

where *G* is the shear modulus, $\Delta \gamma$ is the shear strain range, τ_{max} is the maximum shear stress, $(\sigma \tau)_{max}$ is the maximum shear and tensile stress product value, σ_0 is a factor used to maintain unit consistency, w and k are material fitting parameters. The values for the shear modulus were set to 75 GPa to 316 and 304 stainless steel [26]. A value of 500 MPa was set to σ_0 , following the suggestions given by the authors [21]. The σ_0 parameter in the SKS damage parameter (Equation (4)) currently does not have a physical meaning, other than consistency of the units. Its value is corrected with the value of k in the fitting.

The strain hardening effect that takes place in the LCF regime is considered by $\Delta \gamma$ and τ_{max} values. The mean shear stress effect in the high cycle fatigue (HCF) regime is also considered by the shear ratio τ_{min}/τ_{max} . The parameter w weights the hardening and mean shear stress effects. The product $(\sigma \tau)_{max}$ introduces the detrimental effect over fatigue life observed when sub-cycle load peaks are applied simultaneously. The parameter k gauges the interaction effect between the shear and the normal stresses.

Unlike the FS model, there is not an equation to define the w and k parameters (Equation (4)). w parameter is tuned by fitting the experimental data with a mean shear stress effect. w incorporates the mean shear stress effect and the strain hardening effect. Unlike in the FS model, the w parameter has a constant value for all fatigue lives. By using tests for the fitting of the model with a wide range of lives, the w parameter cross the different fatigue regimes.

3.4. Fitted Models

Models damage parameter DP_{exp} (Equations (1), (3) and (4)), are related to the fatigue life N_f using the same double exponential curve Equation (5). All the material parameters used in the fitting were obtained from previous experimental data, Tables 1 and 2 [26]. The parameters were obtained with an optimisation process based on a least square error minimisation between DP_{exp} and DP_{calc} [9]. As the number of experimental data used to fit the parameter were relatively low, the expected difference between the minimisation of the DP instead of the fatigue life N_f should be negligible over the fitted values.

$$DP_{calc} = AN_f^b + CN_f^d \tag{5}$$

where *A*, *b*, *C* and *d* are material dependent parameters and N_f is the fatigue life in cycles. When fitting the models, N_f is the experimental value of each test. Thus, utilisation of SWT requires evaluating those four parameters (*A*, *b*, *C* and *d*). FS requires those four parameters plus the sensitivity factor described in Section 3.1. SKS requires those four parameter plus the two material parameters described in Section 3.2. Fatigue live data are required in order to fit the material parameters for SKS model. Since the SKS model has six fitting parameters, in order to obtain a deterministic (or an over-deterministic) system of equations, six fatigue tests (or more than six tests) were required. These tests should be conducted in conditions as general as possible, to make it as versatile as possible. Accordingly, both proportional and non-proportional tests with a wide range of fatigue lives were employed. In our case we observed an improvement by using an over-deterministic system of equations (Nine and eight tests for 316 and 304 steels, respectively, as shown in Tables 1 and 2).

The collapse capacity of the different models was evaluated by studying the mean and the standard deviation of the error [35]. The error is defined as the difference between the predicted and the experimental life in logarithmic scale (Equation (6)).

$$\operatorname{error} = \log_{10}(N_{mod}) - \log_{10}(N_{exp})$$
(6)

where N_{mod} is the fatigue life predicted by the fitted model and N_{exp} is the experimental fatigue life.

Table 5 includes the mean and standard deviation of the error values observed in the fitting. Negative mean values are indicative of conservative results and vice versa. A better fitting is obtained with mean values as close as possible to zero. In a similar way, a better fitting is also obtained with the standard deviation value as close as possible to zero. It is observed that the lowest mean values for both materials were obtained with the SKS critical plane model, followed by FS. The mean values in Table 5 indicated that the SWT model appears to produce the least accurate predictions for the type of experiments under study. For both materials, SKS returns the best fit and SWT the worst fit, probably because of the different number of material parameters used in the different models. Materials that normally exhibit a ductile behaviour were more sensitive to damage mechanisms caused by shear stress rather than by normal stress. Materials that normally present a brittle behaviour were more sensitive to damage mechanisms caused by normal stress [16]. Accordingly, the Fatemi–Socie model will be more appropriate for ductile materials failing predominantly under shear mode (Mode II and III); and Smith–Watson–Topper for brittle materials failing predominantly under tension mode (Mode I).

Table 5. Statistical	analysis for	the comparison of	the models	collapse	capacity.
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Statistical Values	FS	SWT	SKS
316 stainless steel	-	-	-
Mean value	0.0091	0.0308	0.0022
Standard deviation	0.0130	0.0370	0.0014
304 stainless steel	-	-	-
Mean value	0.0020	0.0069	0.0016
Standard deviation	0.0026	0.0065	0.0019

4. Results and Discussion

The fatigue life predictions of each fitted model are shown in Figures 6 and 7 for 316 and 304 stainless steels, respectively. The experimental fatigue life N_{exp} is defined in the horizontal axis and the predicted fatigue life N_{mod} in the vertical axis. Logarithmic scale is used in both Figures 6 and 7. The points falling along the solid line present coincidence between N_{mod} and N_{exp} . The values along the dashed lines have a factor 2 deviation between N_{mod} and N_{exp} , that is the prediction given by the model that is twice or half of that measured experimentally [9,36]. The estimations of FS are shown by the blue crosses, SWT by green circles and SKS by purple diamonds on both Figures 6 and 7. It was observed that most of the predictions fall within the factor 2 band deviation.

Figures 6 and 7 show that the predictions returned by SKS are mostly between those of FS and those of SWT. The SKS predictions appear to be overall closer to the FS predictions. For the 316 stainless steel, better results were obtained with SKS and FS models (Figure 6). This is in agreement with previous research that indicated that dominant Mode II critical plane models appear to be more suitable for 316 stainless steel [37]. Figure 6 also shows that both SKS and FS models produced less conservative predictions for the square-shape and equi-biaxial tests (cases 4 and 7 in Figure 2). These cases correspond to the two FS points in Figure 6 where the predictions were beyond the twice fatigue life bound. In these tests, the simultaneous application of the loads in the different directions highly restrict the deformation of the material. As a consequence the range of strains were reduced considerably as compared to the equivalent uniaxial loading test. This in turn reduced the value of the damage parameter thus decreasing the accuracy of the predictions towards the non-conservative side [38,39]. The most conservative prediction by the SKS in Figure 6 has $N_{exp} = 159,600$ cycles. This is indeed the most conservative prediction given by the SKS model. Since the 316 study was conducted with nearly half the samples of the 304 study, the relative weight of this single prediction is larger on the 316 than on the 304 material. It is not surprising that that point produces the longest fatigue life, since it corresponds to the simplest loading case: Pure uniaxial cyclic tension, as shown in Table 3 and Figure 2. The SWT model appears to yield the most conservative predictions, in agreement

with previous research [40] where a different steel with tendency to ductile failure was subjected to proportional loadings.



Figure 6. Fatigue life predicted by each model, N_{mod} versus experimental fatigue life, N_{exp} for 316 stainless steel.



Figure 7. Fatigue life predicted by each model, N_{mod} versus experimental fatigue life, N_{exp} for 304 stainless steel.

Figure 7 shows that most of the predictions given by the different models on the 304 steel are inside the factor of 2 deviation. Comparison between Figures 6 and 7 indicate the best predictions were overall achieved on the 304 steel. In general the best results were obtained by SWT, as it was observed for this material by Socie [23]. Previous analysis showed that only torsion tests promoted predominant Mode II crack growth and only axial loading tests promoted predominant Mode I cracking on 304 stainless steel, for a range of fatigue lives below 10⁵ cycles [10]. As mentioned previously, SWT should then produce more accurate predictions for only axial loading tests and FS present better accuracy for purely torsional tests. Loading cases 2 and 3 in Figure 3 pose a challenging problem in this sense because each loading block is formed of alternating cycles of pure Mode I load and pure Mode II load. That is pure axial load and pure torsional load applied but non-simultaneously. Accordingly, the

predictions given by FS and by SWT should be similar. This is indeed observed for the loading cases 2 and 3 in Figure 7.

As it can be seen in Figure 5, the sensitivity parameter k in FS model changes from ~0.5 to ~1.25 in the range from 10^2 to 10^5 cycles. If not enough experimental data in the entire range were available, it would be possible to take a constant value of 1 for the FS sensitivity parameter [10]. Nevertheless, the effect in the accuracy of the predictions would be detrimental, producing more conservative predictions in the lower range of the fatigue life (between 10^2 and 10^3 cycles in Figure 5) and non-conservative predictions in the higher range of the fatigue life (between 10^5 and 10^6 cycles in Figure 5).

FS and SWT models allow the additional hardening of the material caused by the non-proportionality of the loads to be taken into account because their damage parameter includes both stress and strain variables. SKS also includes the additional hardening cause by the non-proportionality because its damage parameter has both stress and strain information. This is clear for the loading cases with high non-proportionality (cases 9, 10, 11, 13 and 14 in Figure 3) where the three critical plane models yielded predictions within the factor 2 error bound. Tests with the loads applied proportionally (cases 6, 7 and 8 in Figure 3) were also handled satisfactorily by the three models.

In order to assess numerically the overall performance of the different models, the probability density function (PDF) of the error (Equation (6)) was computed [35]. The results of the PDF for the 316 material are shown in Figure 8a and the results for the 304 material are shown in Figure 8b. In addition, Table 6 summarises the mean value and standard deviation for both materials. The PDF curves closer to a zero mean error and with lower deviation indicated a better accuracy of the model.

For the 316 stainless steel, Table 6 shows a slightly higher mean value for SKS than for FS but a slightly smaller standard deviation for SKS than for FS, thus indicating a similar performance of SKS and FS for the loads analysed on the 316 stainless steel. It was also noted that the mean SKS values were negative while the mean FS values were positive. That is because the SKS predictions are overall more on the conservative side while the FS predictions are more on the non-conservative side for the 316 steel. The larger mean and standard deviation values observed for the SWT indicate overall worst predictions as compared to SKS and FS models. In addition, this is symptomatic of the SKS model being more appropriate for predominantly shear mode failing materials.

On the other hand, for the 304 steel SWT shows the lowest values of both the mean and the standard deviation. This suggests that 304 fails predominantly under tensile mode for the tests described. In addition, the FS the mean value is lower than the SKS, while for the SKS model the standard deviation is lower than that of FS model. The performance of both SKS and FS appears to be similar for the 304 steel but again, the SKS model tends to yield predictions on the conservative side and FS more on the non-conservative side.



Figure 8. Probability density function of prediction error for (a) 316 stainless steel and (b) 304 stainless steel.

Statistical Values	FS	SWT	SKS
316 stainless steel	-	-	-
Mean value	0.0763	-0.2490	-0.0836
Standard deviation	0.2432	0.2945	0.2315
304 stainless steel	-	-	-
Mean value	-0.0299	0.0291	-0.0595
Standard deviation	0.2919	0.2339	0.2681

Table 6. Statistical analysis for the comparison of the model prediction errors.

Table 6 also indicates that the three mean values are smaller for the 304 than for the 316 material, thus indicating that the predictions obtained for the 304 steel are slightly more accurate than for the 316 steel.

The different performance of the different types of models were useful for identify the predominant failure mode of the two materials. The 316 steel appears to fail predominantly under shear mode because FS produces better estimations. Conversely, the 304 steel appears to fail predominantly under tension mode since SWT generated the best predictions. This appears to hold for the wide range of multiaxial loads analysed. Moreover, the alignment of SKS with FS in terms of predictions suggests that SKS model is most suitable for predominantly shear mode failing materials.

5. Conclusions and Future Works

The multiaxial fatigue behaviour of two widely used materials, 316 and 304 stainless steels, was studied by means of the critical plane approach. The analysis has been performed on a wide range of experimental tests including different combinations of tension, torsion and inner pressure. Three different critical plane models were used, namely FS, SWT and the newly proposed SKS model. First, the collapse capacity of the different models were evaluated. The larger number of material parameters of SKS model appeared to offer a higher flexibility in this sense, thus producing the best fitting. Nevertheless SKS did not offer any expression to define the k and w parameters included in the damage parameter. In addition, the σ_0 parameter also included in the SKS damage parameter did not have any physical meaning. Producing analytical expressions relating the k and w parameters to different fatigue properties of the material remain as challenging prospective research activities. The SKS could also be improved by relating σ_0 to another characteristic material property, to promote a more uniform use of the model. Unlike the FS model, the parameter k and w parameters take the same value across the whole range of fatigue lives. This can be a weakness for design situations where a very wide range of conditions and very different fatigue regimes are studied. In addition, the SKS model should also be applied to other different materials, to evaluate its performance for other types of alloys.

The efficacy of the different models has also been analysed in terms of their accuracy for predicting the fatigue life. To this end, the damage parameter was correlated with the fatigue life using a double exponential curve. The fitted curves for 316 and 304 stainless steel were used to predict fatigue life under different loading path for the same materials. Most of the predictions given by the three models fall in the band defined by the factor of 2 deviation. For cases with a higher level of hardening, the critical plane models have shown to also generate satisfactory predictions. SKS and FS produced the best predictions for the 316 material while SWT generated the best predictions for the 304 material. Such a trend has been useful to identify the predominant failure mode of the two materials: 316 fails predominantly under the shear mode loading and 304 material fails predominantly under the tension mode loading. The results also indicated that the SKS model appears to be most suitable for shear mode failing materials. For the experimental tests described, SKS has proven to generate predictions on the conservative side and FS on the non-conservative side. This suggests that SKS could be more suitable from a design point of view.

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