

Communication

Application of the Taylor Equation to Five-Power-Law Creep Considering the Influence of Solutes

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Abstract: This study determines the feasibility of describing the flow stress within the five-power-law creep regime, using a linear superposition of a dislocation hardening term and a significant solute strengthening term. It is assumed that the solutes are randomly distributed. It was found that by using an energy balance approach, the flow stress at high temperatures can be well-described by the classic Taylor equation with a solute strengthening term, τ_o , that is added to the $\alpha M G b \rho^{1/2}$ dislocation hardening term.

Keywords: creep; microstructural features; constitutive equations

1. Introduction

This paper addresses the theoretical validity of the application of a Taylor equation to five-power-law creep in pure alloys and class M alloys. Previous work on aluminum and stainless steel by the author [1–3] shows that the density of dislocations within the subgrain interior influences the flow stress for steady-state substructures as well as primary creep. The hardening is consistent with the Taylor relation if a linear superposition of solute hardening (τ_o , or the stress necessary to cause dislocation motion in the absence of a dislocation substructure) and dislocation hardening ($\cong \alpha M G b \rho^{1/2}$) is assumed, or

$$\tau = \tau_o + \alpha M G b \rho^{1/2} \quad (1)$$

It appears that dislocation hardening is athermal and the constant, α , is temperature independent. The value of α is consistent with the range of values observed in cases where dislocation hardening is unambiguous. M is the Taylor factor, G is the shear modulus, ρ is the Frank network dislocation density, b is the Burgers vector and τ is the applied stress. Part of the reason that the question of superposition is important is because, historically, the τ_o term is not included or is very small. The question is whether for cases where the τ_o term is large, a linear superposition is fundamentally reasonable. This endeavor complements the earlier work by the author that demonstrated, that, at least phenomenologically, the superposition (i.e., Equation (1)) is effective.

It must be mentioned that the basis for strengthening in five power-law –creep is generally attributed to the Frank dislocation network such as [4–7]. Others have held to the proposition that subgrain walls are associated with the strength of materials within the five-power-law regime [8,9]. This Communication considers the Frank network to be associated with strength and the rate-controlling process for creep.

References [1,2] show for the case of annealed (very low dislocation density) 99.999% pure aluminum, that the yield stress appears to be a significant fraction of the, eventual, steady-state flow stress. This is illustrated in Figure 1. This is also true for 304 stainless steel, a class M (pure metal behavior) alloy. It is assumed that there are no long range internal stresses as consistent with the findings of [10,11]. In both cases the yield stress of the annealed metal is roughly 0.5 at 371 °C for Al

and 0.3 at 750 °C for 304 stainless steel of the steady-state flow stress. Clearly, a description of the stress at steady-state must consider both the solute and the dislocation features of the microstructure. In the author's case, the dislocation feature has been suggested to be the Frank dislocation network within the grain and subgrains of the polycrystalline aggregate. Careful work by the author and coworkers determined that the Frank network of dislocations, rather than the subgrains, is the dislocation feature associated with elevated temperature strength [1,2,6].

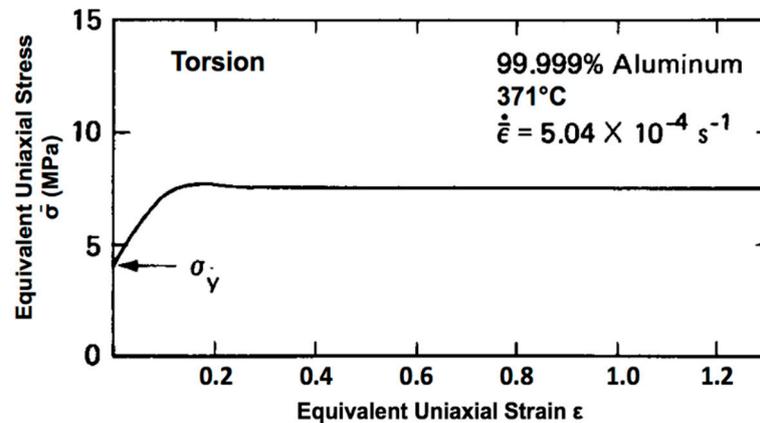


Figure 1. The stress versus strain behavior of 99.999% pure polycrystalline aluminum at 371 °C in torsion.

The yield strength in Figure 1 can only be attributed to the small amount of solute. The grain size of the aluminum in the figure is about 0.5 mm. There are a variety of possibilities to superimpose the strengthening variables (e.g., linear summation, root mean square as in [6], one hardening term exclusively controls the strength, etc.). The current work explores a simple linear superposition such as in Equation (1) that appears phenomenologically effective. The possibility that part of the annealed yield strength could be a lattice friction stress (similar to a Peierls stress) should be acknowledged.

2. Analysis and Discussion

The solutes in the present case are considered to be randomly dispersed. The Frank network coarsens with time at temperature and the link lengths increase. Eventually, some of the links are sufficient in length to operate a multiplication mechanism (e.g., Frank–Read source) and plasticity (creep) ensues. This multiplication also causes network refinement. Thus, there is both recovery (network coarsening) and hardening (network refinement) and steady-state is a balance of these two processes. The yield stress of the starting annealed material would be very low in the absence of the solute as the network strength is presumed to be very low [5]. At the yield stress, then, plastic flow is dictated by the impurities. At 99.999 (wt)% purity, roughly 10 ppm of impurities are present or roughly one in 10^5 host (solvent) atom sites are occupied by impurities. This suggests that the separation between impurities is roughly 50 atomic diameters or 13 nm. This separation is assumed constant with dislocation hardening. Hardening by solutes might occur by elastic interaction between the solute and the dislocations. From the bowing equation,

$$\tau = Gb/r \quad (2)$$

If each solute atom perfectly pins a dislocation then the yield strength is roughly $G/25$, which is much too large. The dislocation must “tear away” from the solute at a much lower stress. The solute atoms do not diffuse to “follow” the dislocation as with three-power-law creep [3]. There are a large variety of solutes that comprise the 0.001% total impurity concentration.

Once the stress has reached the yield stress, dislocation bowing can occur within the Frank network. As the dislocation bows it must (1.) perform work to tear away from the solute atoms that

elastically interact with the stress fields of the dislocation and (2.) also perform work to compensate for the increase in elastic strain energy associated with increasing dislocation line length with bowing. Defining:

- l = bowed dislocation length;
- l_s = distance between solutes;
- r = radius of bowed dislocation links;
- ΔV = difference in volume between solute and solvent;
- P = hydrostatic pressure component of the dislocation stress field;
- k = constant;
- θ = radians

Then, if the loop expands by dr , by the First Law,

$$(\tau bl)dr = \theta dr [Gb^2/2] + dr\{d(P\Delta V)/dr\}(l/l_s) \quad (3)$$

Again, this is just that the work done by the applied stress as the dislocation moves, is equal to the increase in elastic strain energy of dislocation line plus the work done to “tear away” the dislocation from the solutes. Equation (3) leads to,

$$\tau bl = l/r [Gb^2/2] + (l/l_s)\{d(P\Delta V)/dr\} \quad (4)$$

$$\tau = [Gb/2r] + [d(P\Delta V)/dr]/(bl_s) \quad (5)$$

for a typical r , and a simple arrangement of dislocations,

$$r = kl = k/\rho^{0.5} \quad (6)$$

$$\tau = [Gb\rho^{0.5}]/2k + [dP\Delta V)/dr]/(bl_s) \quad (7)$$

In the above, τ is the resolved shear stress on the loop. Therefore, for the applied stress σ ,

$$\tau_a = \tau_o + \alpha MGb\rho^{1/2} \quad (8)$$

or Equation (1), which is the classic Taylor equation, with, in this case, a τ_o that is a significant fraction of the flow stress. Thus, this article fundamentally confirms that dislocation hardening within the five-power-law creep regime, can be described by a classic Taylor equation using a linear superposition of a dislocation hardening term and a solute strengthening term. Because τ_o is a thermally activated term and $\alpha MGb\rho^{1/2}$ is the athermal term, then the constant α is expected to be of a similar value that those cases where dislocation hardening is unambiguous. In the authors earlier work on dislocation hardening in five-power-law creep [1–3], the α value is reasonable for dislocation hardening.

3. Conclusions

This study determined the theoretical feasibility of describing dislocation hardening within the five-power-law creep regime using a classic Taylor equation using a linear superposition of a dislocation hardening term and a solute strengthening term. It was assumed that the solutes are randomly distributed. This assumption and an energy balance approach demonstrated that the high temperature flow stress can be described by the classic Taylor equation with a linearly added solute strengthening term to the dislocation hardening term. The fundamental analysis complements earlier work that showed that the flow stress at steady-state can be satisfactorily described by a summation of a dislocation hardening terms consistent with the Taylor equation and a solute strengthening term.

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Conflicts of Interest: The authors declare no conflict of interest.

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