

Review

Theoretical Foundations of the Hot Stamping of Ultra-High-Strength Boron Steel: The Case of B1500HS with 22MnB5 and BR1500HS as Benchmark Grades

Wei Li and Liming Zhou *

School of Mechanical and Aerospace Engineering, Jilin University, 5988 Renmin Street, Changchun 130025, China; liwei@zybody.com.cn

* Correspondence: zhouliming@jlu.edu.cn

Abstract

The hot stamping of boron-alloyed steel integrates forming with in-die quenching and is a core route to 1500 MPa class automotive safety components, yet its parameter design still lacks a unified theoretical basis. This article critically reviews the theoretical foundations of the process and establishes a framework covering four pillars: analytical parameter-design models (heating time, forming force, springback, and die cooling-channel sizing); hot-plastic-deformation mechanisms interpreted through a molecular dynamics (MD)–discrete dislocation dynamics (DDD)–crystal plasticity finite-element (CPFE) multi-scale hierarchy; phase transformation kinetics extended by a Ginzburg–Landau phase-field description of variant selection; and heat transfer with thermo-mechanical coupling. B1500HS is adopted as the primary grade, with 22MnB5 and BR1500HS serving as benchmarks for cross-validation. The calibrated hyperbolic-sine Arrhenius model yields a deformation activation energy of $245.4 \text{ kJ}\cdot\text{mol}^{-1}$, consistent with dislocation-climb-controlled deformation of austenite, and a worked cooling-channel example (380 kW heat load) confirms engineering applicability. The framework provides quantitative theoretical boundary conditions for process optimization, die design, and quality evaluation of hot-stamped components.

Keywords: hot stamping; B1500HS boron steel; Arrhenius constitutive model; martensitic transformation; phase-field method; thermo-mechanical coupling

1. Introduction

Hot stamping is a key enabling technology for the efficient manufacture of ultra-high-strength steel components, and its theoretical foundations span materials science, heat transfer, and solid mechanics. In this process, a boron steel blank is heated above the austenitizing temperature (A_{c3}), held to transform the initial ferrite–pearlite microstructure into uniform fine austenite, and then rapidly transferred to a water-cooled die, where it is stamped and quenched under pressure to obtain a fully martensitic microstructure. By integrating forming and in-die quench hardening into a single operation, components with tensile strengths above 1500 MPa can be produced, and the process has been widely adopted for automotive lightweighting and safety-critical structural parts [1]. Figure 1 shows the temperature–time profile of the hot-stamping process together with its characteristic microstructural stages (holding, transfer, forming, and in-die quenching).

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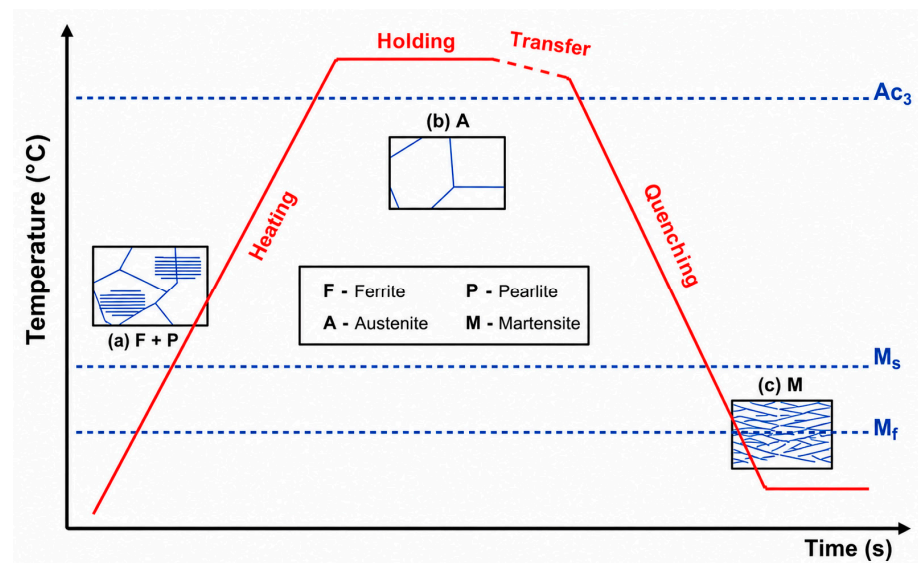


Figure 1. Temperature–time profile of the hot-stamping process with the characteristic microstructural stages (holding, transfer, forming, and in-die quenching). Adapted from Ref. [2].

From a materials science perspective, the essence of hot stamping exploits high-temperature phase transformation and softening to achieve the near-net-shape forming of complex geometries. The blank is heated above the recrystallization temperature (typically 900–950 °C) to homogenize the austenite, at which point the flow stress drops markedly and formability improves substantially; the subsequent rapid cooling in the die (at rates above the critical cooling rate—approximately 27 °C·s^{−1} for B1500HS-class boron steels [3,4]; a lower threshold of about 15 °C·s^{−1} has been reported for 2 mm 22MnB5 sheet [5]) strengthens the material through martensitic transformation to a tensile strength above 1500 MPa, corresponding to a 200–300% improvement over the as-delivered material [5]. Figure 1 summarizes this chain schematically: austenitization above A_{c3} , transfer, forming with in-die quenching above the critical cooling rate, and final martensitic microstructure; each stage maps onto one theoretical pillar of this review.

Austenite transformation kinetics constitutes the theoretical core of the process and encompasses both thermodynamics (driving force and equilibrium conditions) and kinetics (transformation rate, nucleation rate, and growth rate). The transformation behavior of austenite during cooling directly determines the microstructure and properties of the final product and is governed by the coupled influence of temperature, cooling rate, chemical composition, and stress state [6]. Figure 2 shows the time–temperature–transformation (TTT) diagram of a typical boron steel [3,7]. The diagram is read as follows: at a given temperature, the curves mark the onset and completion of the diffusional transformations, so the quenching path must bypass the nose of the ferrite/pearlite region to secure a fully martensitic microstructure; this reading underlies the critical-cooling-rate criterion developed in Section 5.2.

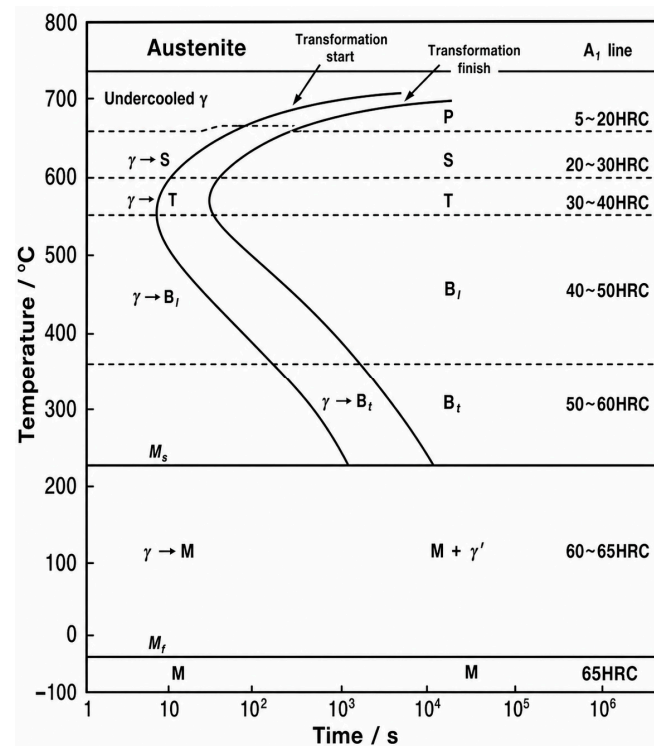


Figure 2. Isothermal transformation (TTT) diagram of austenite. Adapted from Refs. [3,7].

Hot stamping has been reviewed extensively. Karbasian and Tekkaya [4] surveyed the process comprehensively, and Wróbel et al. [8] reviewed the technological–metallurgical aspects and numerical simulation of the hot stamping of advanced high-strength steels; in parallel, landmark studies have characterized the thermo-mechanical behavior and constitutive response of boron steels [9,10] and established thermo-mechanical–metallurgical coupled simulation frameworks for press hardening [3,11,12]. Compared with these works, the present review differs in four respects: (i) it is dedicated to B1500HS boron steel, rather than to 22MnB5 alone, and consolidates its composition, property, and process-window data; (ii) the constitutive parameters are established through a multi-study cross-validation strategy with back-calculation and closed-loop verification, rather than by single-dataset fitting (Section 3.3); (iii) a Ginzburg–Landau phase-field description of martensitic variant selection is integrated into the quenching-process design framework, linking the macroscopic cooling rate to mesoscopic variant selection and macroscopic residual stresses (Section 5.4); and (iv) fully reproducible worked examples—radiative heat-up integration, springback estimation, and a 380 kW cooling-channel design—are provided so that every key theoretical relation is accompanied by a verifiable engineering closure. Despite the extensive literature, a systematic synthesis that connects analytical parameter-design models, multi-scale deformation physics, phase transformation kinetics, and heat transfer theory into a single, quantitatively verifiable framework for B1500HS remains lacking; closing this gap is the motivation of the present review. A structured comparison of the present review with prior reviews is given in Table 1.

Beyond summarizing existing theories, this review applies three explicit evaluative criteria throughout: (i) predictive verifiability—whether a theoretical model can be closed with independent experimental data, as demonstrated by the worked examples of Sections 3.2.2 and 3.2.3; (ii) cross-scale consistency—whether parameters derived at one scale (e.g., activation energy Q) are physically compatible with the mechanisms identified at the adjacent scale, as examined in Sections 3.3 and 4.4; and (iii) engineering transferability—whether a model can be reduced to design formulas usable by practitioners, as in the cooling-channel design of Section 3.2.3. These criteria structure the Critical Discussion

Subsections (Sections 3.1.4, 3.3.3, 4.1.4, 4.3.4, 5.1.3, 5.3.5 and 6.5), which compare competing theoretical approaches rather than merely presenting them, and they underpin the two methodological comparison tables added in this revision (the constitutive-model comparison in Section 3.3.3 and the transformation-kinetic-model comparison in Section 5.3.5).

Alongside these established reviews, research published during the last five years has advanced each pillar of the process. For the constitutive description, Song et al. [13] estimated Johnson–Cook parameters for 22MnB5 produced by the TSCR process, Zhou et al. [14] modeled the flow response of 22MnB5 under isothermal hot tension, and Sun et al. [15] established a high-temperature constitutive model for BR1500HS, while He et al. [16] quantified the hot-forming limit of 22MnB5 under stamping conditions. Regarding the transformation behavior, Hart-Rawung et al. [7] formulated a unified model covering isothermal and non-isothermal phase transformation in the hot stamping of 22MnB5. On the service-integrity side, Wang et al. [17] reviewed the hydrogen embrittlement of press-hardened steels, a key concern for 1500 MPa class components. These recent studies provide the calibrated data and modeling frameworks on which the following sections build.

Table 1. Comparison of the present review with prior reviews of hot stamping.

Reference	Year	Grade Focus	Constitutive Treatment	Transformation & Coupling Coverage	Worked Examples
Karbasian and Tekkaya [4]	2010	22MnB5	Reviewed qualitatively	Process window reviewed	None
Mori et al. [12]	2017	22MnB5 & coated variants	Reviewed qualitatively	Quenching and simulation surveyed	None
Song et al. [13]	2025	22MnB5	Johnson–Cook parameter estimation from a single dataset	Kinetics reviewed	None
Wróbel et al. [8]	2022	AHSS including 22MnB5	Reviewed qualitatively	FE coupling surveyed	None
This review	2026	B1500HS (primary); 22MnB5 and BR1500HS (benchmarks)	Multi-study cross-validation with back-calculation ($Q = 245.4 \text{ kJ} \cdot \text{mol}^{-1}$)	Phase-field description of variant selection integrated with heat transfer coupling	Heating-time integration; springback; 380 kW cooling-channel design

This review is organized as follows: Section 2 describes the methodology of literature selection. Section 3 analyzes the process principles and key parameter-design methods, including a multi-study cross-validation of the high-temperature constitutive model of B1500HS steel. Section 4 elaborates the hot-plastic-deformation mechanisms at three levels: intragranular slip, grain-boundary sliding, and diffusion creep, together with their multi-scale simulation counterparts. Section 5 discusses the phase transformation processes— austenitization, quench transformation, and martensite formation—and introduces a phase-field description of martensitic transformation. Section 6 establishes the heat transfer theory and the thermo-mechanical coupling model, revealing the interaction between the temperature field and the stress field. Section 7 critically discusses cross-cutting issues—data consistency, model maturity, and industrial applicability—and Section 8 summarizes the main conclusions. A roadmap of the review structure is provided in Figure 3.

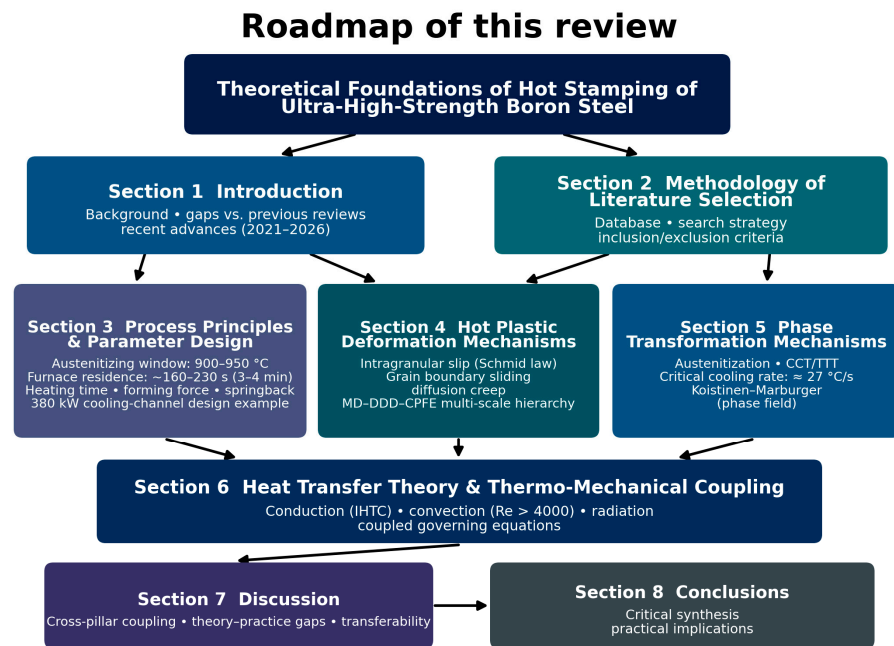


Figure 3. Roadmap of the review structure. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Reconstructed prompt: “Create a hierarchical flowchart roadmap for a review paper on the theoretical foundations of hot stamping of ultra-high-strength boron steel: a title box at the top; Section 1 Introduction and Section 2 Methodology of Literature Selection on the second level; Section 3 Process Principles & Parameter Design, Section 4 Hot Plastic Deformation Mechanisms, and Section 5 Phase Transformation Mechanisms on the third level; Section 6 Heat Transfer Theory & Thermo-Mechanical Coupling below them; and Section 7 Discussion and Section 8 Conclusions at the bottom, all connected by arrows.”

2. Methodology of Literature Selection

This review follows a structured literature selection procedure. The Scopus, Web of Science, and CNKI databases were searched for the combined terms (“hot stamping” OR “press hardening”) AND (“boron steel” OR “22MnB5” OR “B1500HS” OR “BR1500HS”), covering the period 2000–2026; seminal theoretical works older than 2000 (e.g., the Koistinen–Marburger and Nabarro–Herring models) were exempted from the time window and cited directly.

The search returned 612 records. After removal of duplicates, 437 records were screened by title and abstract, leaving 196 full-text articles for eligibility assessment. Of these, 88 articles passed the full-text review, and the final synthesis cites 66 references, of which 30 (45.5%) were published after 2021. Selection prioritized (i) experimentally calibrated constitutive and transformation data for B1500HS, 22MnB5, and BR1500HS; (ii) primary theoretical sources of the models analyzed; and (iii) recent reviews used to position the present work. The remaining 22 articles that passed the full-text review but are not cited served as background sources for framing the Critical Discussion subsections and did not contribute primary data or models directly analyzed in this review. The complete search log—including the exact query strings, the databases searched, the search dates, and the number of records excluded at each screening stage together with the reasons—has been archived by the authors and is available from the corresponding author upon reasonable request, ensuring the full traceability of the screening funnel reported above. Of these 66 references, 59 resulted from the screening funnel above, and seven verified sources ([18–23] and the standard [24]) were incorporated in the present revision in

response to the reviewers' comments, so that the funnel and the final list remain fully consistent.

The selected literature was classified into the four pillars of the review—process principles and parameter design, hot-plastic-deformation mechanisms, phase transformation kinetics, and heat transfer and coupling theory—and each cited model or dataset was checked for internal consistency (units, symbol definitions, and parameter ranges) before inclusion.

3. Process Principles and Parameter Design

3.1. Process Mechanism Analysis

3.1.1. Austenitizing Temperature

The austenitizing temperature is the primary control parameter of the hot-stamping process, directly determining the completeness of phase transformation and grain refinement effect [25]. For hypoeutectoid boron steels, the critical transformation temperatures are defined as follows: A_{c1} (approximately 727 °C) is the start temperature of the pearlite-to-austenite transformation; A_{c3} is the finish temperature of complete austenitization of ferrite, typically 50–80 °C above A_{c1} in low-carbon boron steels. A_{cm} applies only to hypereutectoid steels and is therefore not used for the parameter design of hypoeutectoid B1500HS.

Austenitization serves three main purposes: (i) obtaining a uniform austenitic microstructure as the prerequisite for the subsequent martensitic or bainitic transformation; (ii) refining the grain size by controlling the austenitizing temperature and holding time, thereby improving the strength-toughness balance; and (iii) relieving residual stresses introduced by preceding operations such as cold working or welding. The A_{c3} temperature can be estimated by the Andrews formula [26], shown Equation (1):

$$A_{c3} = 910 - 203\sqrt{C} - 30Mn + 44.7Si - 15.2Ni + 104V + 31.5Mo + 13.1W + 11Cr + 20Cu - 700P - 400Al - 120As - 400Ti \quad (1)$$

where the element symbols (e.g., C , Mn , Si) denote the mass fractions (%) of the corresponding elements and A_{c3} is the austenitization finish temperature (°C). Taking representative values within the typical composition ranges of B1500HS ($C = 0.225\%$, $Mn = 1.35\%$, $Si = 0.2\%$, and $Cr = 0.2\%$, with the remaining alloying elements set to zero) and substituting them into Equation (1), we get $A_{c3} \approx 784$ °C. For boron steels, the practical austenitizing temperature must additionally compensate for the temperature non-uniformity of industrial heating furnaces and for the heat loss during blank transfer from the furnace to the die, so the superheat allowance amounts to $\Delta T \approx 120 - 170$ °C; the practical austenitizing temperature is then determined by Equation (2), i.e., $T = A_{c3} + \Delta T \approx 900 - 950$ °C:

$$T = A_{c3} + \Delta T \quad (2)$$

where T is the austenitizing temperature (°C) and ΔT is the superheat allowance, typically 120–170 °C for the hot stamping of boron steels (covering furnace temperature non-uniformity and transfer heat loss).

3.1.2. Austenite Transformed Volume Fraction

The austenite transformed volume fraction f characterizes the volume proportion of austenite during heating or cooling and directly affects phase constitution and mechanical properties. It is governed by the combined effects of carbon content, alloying elements, heating temperature and time, and cooling rate. Higher carbon content widens the austenitizing temperature range; alloying elements influence the austenite fraction by retarding carbon diffusion and altering the transformation kinetics; and above A_{c3} , the austenite

fraction increases with temperature and holding time. The transformed volume fraction is commonly calculated using the Avrami equation [27], shown in Equation (3):

$$f = 1 - \exp(-k_{Avrami} \cdot t^{n_{Avrami}}) \quad (3)$$

where f is the transformed volume fraction; k_{Avrami} is the rate constant, which depends on temperature and composition; n_{Avrami} is the Avrami exponent reflecting the transformation mechanism; and t is the holding time (s).

3.1.3. High-Temperature Flow Stress of Austenite

The high-temperature flow stress of austenite is the core parameter characterizing the resistance of the material to plastic deformation during hot deformation; it is closely related to chemical composition, deformation temperature, strain rate, and microstructural evolution. Increasing the temperature reduces the flow stress markedly, whereas increasing the strain rate raises it severalfold. Alloying elements raise the flow stress through solid-solution strengthening and the suppression of dynamic recovery; austenite grain refinement, by contrast, effectively lowers the flow stress. The constitutive relation of the high-temperature flow stress is commonly described by an Arrhenius-type equation. The complete Arrhenius equation, in Equation (4), adopts the hyperbolic-sine form and covers the entire stress range (low, medium, and high stress) [12]:

$$\dot{\varepsilon} = A[\sinh(\alpha\sigma)]^n \exp\left(-\frac{Q}{RT}\right) \quad (4)$$

where $\dot{\varepsilon}$ is the strain rate (s^{-1}), σ is the flow stress (MPa), A is the Arrhenius material constant (s^{-1}), α is the stress-level parameter (MPa^{-1}) controlling the transition from the low-stress to the high-stress regime, n is the stress exponent, Q is the deformation activation energy (reported in $kJ \cdot mol^{-1}$ and converted into $J \cdot mol^{-1}$ in the exponential term), R is the gas constant ($8.314 J \cdot mol^{-1} \cdot K^{-1}$), and T is the absolute temperature (K).

In the low-stress regime ($\alpha\sigma < 0.8$), the hyperbolic-sine function reduces to the power-law form of Equation (5):

$$\dot{\varepsilon} \approx A' \sigma^n \exp\left(-\frac{Q}{RT}\right) \quad (5)$$

where $A' = A \cdot \alpha^n$.

In the high-stress regime ($\alpha\sigma > 1.2$), the hyperbolic-sine function reduces to the exponential form of Equation (6):

$$\dot{\varepsilon} \approx A'' \exp(\beta\sigma) \exp\left(-\frac{Q}{RT}\right) \quad (6)$$

where $A'' = A/2^n$ and $\beta = n\alpha$.

On the basis of the Arrhenius equation, a strain-hardening correction term ε^{m_h} is introduced as a proposed extension to describe the effect of strain on the flow stress, yielding the corrected flow-stress expression of Equation (7):

$$\sigma = \frac{1}{\alpha} \ln \left\{ \left(\frac{Z}{A} \right)^{1/n} + \left[\left(\frac{Z}{A} \right)^{2/n} + 1 \right]^{1/2} \right\} \cdot \varepsilon^{m_h} \quad (7)$$

where $Z = \dot{\varepsilon} \cdot \exp(Q/(RT))$ is the Zener–Hollomon parameter, representing the combined effect of temperature and strain rate. In Equation (7), ε is the true strain, and m_h is the strain-correction exponent (dimensionless) that weights the strain-hardening contribution to the flow stress.

Note: Equation (4) is the standard hyperbolic-sine form of the Arrhenius equation and applies over the whole stress range; Equations (5) and (6) are its asymptotic approximations in the low- and high-stress regimes, respectively. This study adopts the parameters reported by Li et al. [28] (2020) for the peak flow stress of B1500HS: $\alpha = 5.41 \times 10^{-3} MPa^{-1}$, $n = 6.294$, and $Q = 245.4 kJ \cdot mol^{-1}$; the pre-exponential factor $A \approx 6.2 \times 10^{10} s^{-1}$ was back-calculated in this work from the peak-stress data of Li et al.'s study at two strain

rates (0.1 and 0.01 s^{−1} at 900 °C), which yield mutually consistent values (Section 3.3). Equation (7) is therefore retained as a proposed form pending experimental calibration and is not used for the numerical cross-validation in this review. The strain-correction exponent m_h of Equation (7) is not reported in that study and requires dedicated experimental calibration; the cross-validation in Section 3.3 therefore relies on the peak-stress form of Equation (4).

This work focuses on B1500HS high-strength boron steel and systematically analyzes its hot stamping behavior. Table 2 lists the chemical composition of B1500HS, Table 3 summarizes the functions of the main alloying elements, and Table 4 compares the mechanical properties in the as-delivered and quenched states; the typical engineering stress–strain curves are shown in Figure 4.

Table 2. Chemical composition of B1500HS boron steel (mass fraction, %).

Item	C	Si	Mn	P	S	Cr	Mo	Ti	B	Al	N
Min	0.20	—	1.00	—	—	—	—	0.02	—	0.01	—
Max	0.25	0.40	1.40	0.025	0.010	0.35	0.35	0.05	0.005	0.06	0.008

Note: Composition data are taken from the delivery technical specification of B1500HS (VDA 239-100); “—” denotes an element that is not specified or present at a negligible level.

Table 3. Functions of the main chemical elements in boron steel.

No.	Element	Function
1	C	Increases strength and hardness; excessive content degrades toughness and weldability
2	Si	Deoxidizer; improves hardenability and tempering stability
3	Mn	Increases strength and hardenability; improves toughness
4	Cr	Improves hardness, wear resistance, and corrosion resistance
5	Mo	Increases strength and toughness; suppresses temper embrittlement
6	Ti	Microalloying element; refines grains and improves strength–toughness balance
7	B	Markedly improves hardenability (effective at trace levels)
8	Al	Deoxidizer; refines grains

Table 4. Mechanical properties of B1500HS boron steel.

Condition	Tensile Strength, MPa	Yield Strength, MPa	Elongation, %
As delivered	≥450	280–450	≥20
Quenched	1300–1700	950–1250	≥5

Note: Mechanical properties were measured according to GB/T 228.1-2021 [24] at a crosshead speed of 2 mm·min^{−1}; the as-delivered condition is hot-rolled and annealed, and the quenched condition corresponds to austenitization at 900 °C followed by in-die quenching.

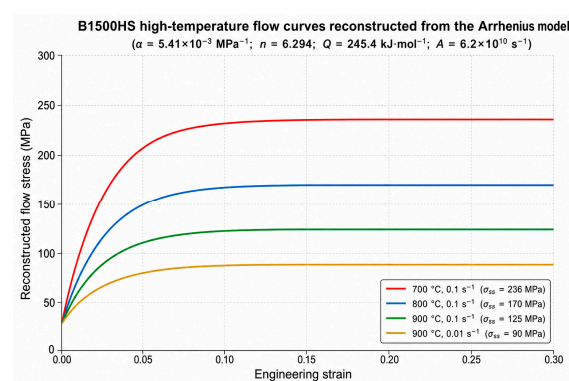


Figure 4. Engineering stress–strain curves of B1500HS. Illustrative curves reconstructed by the authors: the steady-state (saturation) stresses are computed from the Arrhenius model of Equation (4)

with the constitutive parameters adopted by Li et al. ($\alpha = 5.41 \times 10^{-3} \text{ MPa}^{-1}$, $n = 6.294$, $Q = 245.4 \text{ kJ} \cdot \text{mol}^{-1}$, and $A = 6.2 \times 10^{10} \text{ s}^{-1}$; Section 3.3.2), and the hardening transient is described by a Voce-type law; no new experimental data are presented. All four curves lie within the calibration window of Li et al. (700–900 °C and $0.005\text{--}10 \text{ s}^{-1}$) and therefore involve no extrapolation; the moderate extrapolation associated with the upper part of the hot-stamping window (900–950 °C) is discussed in Section 3.3.2. The figure was drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Reconstructed prompt: “Plot illustrative engineering stress–strain curves for B1500HS boron steel at several temperatures and strain rates, each curve showing an elastic stage, a hardening transient described by a Voce-type law, and a steady-state (saturation) stress plateau, with labeled axes and a legend.”

3.1.4. Critical Discussion: Limitations of Isothermal Transformation Kinetics for Industrial Continuous Cooling

The austenite transformation kinetics outlined above provides the basic framework for hot-stamping process design, yet several limitations remain. First, the classical Avrami equation (Equation (3)) assumes homogeneous nucleation and isotropic growth, whereas in practice, the stress field and temperature gradients strongly influence the heterogeneous spatial distribution of nucleation sites. Second, most existing models are based on isothermal transformation assumptions, whereas industrial hot stamping involves non-isothermal continuous cooling, and the mapping between CCT and TTT kinetics is non-trivial. In addition, the mechanical influence of deformation on transformation—such as deformation-induced ferrite transformation (DIFT)—cannot be ignored but remains inadequately described. Developing non-isothermal transformation kinetic models that couple stress, temperature, and composition is therefore a key direction for improving microstructure-prediction accuracy. Beyond these physics-based limitations, the growing availability of high-throughput experimental and simulation data has motivated data-driven approaches. Machine learning methods (e.g., neural network surrogates and Gaussian process regression) are increasingly applied to transformation kinetic prediction [29], compensating for the deficiencies of physical models under complex coupled conditions in a data-driven manner. However, the black-box nature of machine learning models limits physical interpretability, and hybrid modeling that embeds physical constraints into the loss function (physics-informed neural networks, PINNs) represents a promising future direction [30]. Building on the present theoretical framework and on the constitutive parameters calibrated by multi-study cross-validation in Section 3.3, future work within this research program will implement multi-objective collaborative optimization of the process parameters by combining orthogonal experimental design with Kriging–NSGA-II intelligent optimization [31]. For B1500HS specifically, this implies that the isothermal Avrami parameters of Section 3.1.2 should be used only for furnace-heating design, whereas the in-die quenching stage must be analyzed with the non-isothermal, stress-coupled kinetics identified as future work in Section 6.5.

3.2. Process Parameter Calculation Models

3.2.1. Classification of Hot-Stamping Processes

According to the sequence of the forming and heating operations, hot-stamping processes are divided into two categories: direct hot stamping and indirect hot stamping.

In direct hot stamping [4], the as-received blank is heated directly to the austenitizing temperature and held, and then it is transferred to the hot stamping die for press forming and press quenching. Its advantages are that no preforming die is required, reducing tooling cost, and that the regularly shaped blank before heating allows high space utilization of the furnace, favoring productivity. However, direct hot stamping is difficult for parts with complex shapes, and trimming/piercing must be performed by laser cutting after hot

stamping, increasing downstream processing cost. Figure 5 shows the process flow of direct hot stamping.

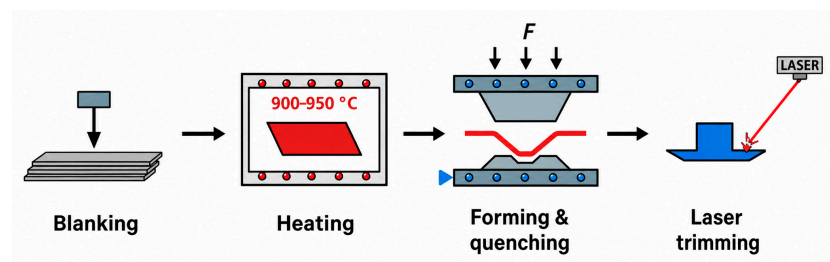


Figure 5. Process flow of direct hot stamping. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Black arrows indicate the process sequence; the blue arrow indicates the cooling-water inlet; the red arrow indicates the laser beam. Prompt used: “Create a horizontal process-flow diagram of direct hot stamping with four steps — blanking, heating in a furnace at 900–950 °C, forming and quenching in a cooled die under a force F , and laser trimming — connected by black arrows, in a clean flat illustration style.”

In indirect hot stamping, the blank is first preformed by cold stamping to a near-final preform, which is then heated, transferred, and quenched in the die. This route suits most automotive ultra-high-strength steel parts: it avoids the non-uniform heating caused by blanks whose width exceeds the furnace dimensions, and the reduced size of the pre-formed blank improves heating efficiency and temperature uniformity. Figure 6 shows the process flow of indirect hot stamping.

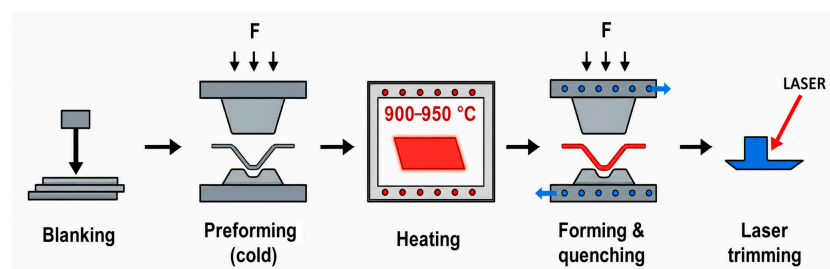


Figure 6. Process flow of indirect hot stamping. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Black arrows indicate the process sequence; the blue arrows indicate the cooling-water inlet and outlet; the red arrow indicates the laser beam. Prompt used: “Create a horizontal process-flow diagram of indirect hot stamping with five steps — blanking, preforming (cold) in a die under a force F , heating in a furnace at 900–950 °C, forming and quenching in a cooled die with cooling-water channels, and laser trimming — connected by black arrows, in a clean flat illustration style.”

Table 5 systematically compares the characteristics and application ranges of the two process routes[8]. Data from Ref. [8]

Table 5. Comparison of direct and indirect hot-stamping process flows.

Item	Direct Hot Stamping	Indirect Hot Stamping
Process feature	Blank heated directly, then formed and quenched in die	Cold preforming, then heated and quenched in die
Tooling cost	Lower (no preforming die)	Higher (preforming die required)
Suitable parts	Simple to moderately complex shapes	Complex shapes, deep-drawn parts
Heating efficiency	Higher (regular blank geometry)	Lower (large preform size)
Dimensional accuracy	Moderate (post-stamping trimming)	Higher (datum established by preforming)
Typical applications	Door beams, bumpers	A/B-pillars, rocker reinforcements

3.2.2. Calculation Methods for Key Process Parameters

The rational setting of the process parameters decisively influences product quality and production efficiency [32,33]. The parameters span multiple dimensions—temperature, pressure, and time—and must be optimized comprehensively according to the material properties, part geometry, and production conditions.

Heating time. The heating time must ensure complete austenitization of the blank and is calculated by considering the heating mode, material properties, blank thickness, and target temperature. Different heating modes call for different calculation methods.

For radiation heating (resistance or gas-fired furnaces), the heating time is derived from energy conservation and the Stefan–Boltzmann law [34]:

$$t = \frac{\rho_s \cdot c_p \cdot d \cdot \Delta T}{\eta_{rad} \cdot \varepsilon_{eff} \cdot \sigma_{SB} \cdot (T_f^4 - T_0^4)} \quad (8)$$

where ρ_s is the material density ($\text{kg}\cdot\text{m}^{-3}$), c_p is the specific heat capacity ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$), d is the blank thickness (m), ΔT is the target temperature rise (K), ε_{eff} is the effective emissivity (combined emissivity of the blank and furnace walls, typically 0.7–0.9), σ_{SB} is the Stefan–Boltzmann constant ($5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$), T_f is the furnace temperature (K), T_0 is the initial temperature (K), and η_{rad} is the radiative heating efficiency (0.6–0.8, accounting for wall heat losses and temperature non-uniformity). Efficiency appears in the denominator because lower radiative heating efficiency implies larger wall heat losses and therefore a proportionally longer required heating time.

For convection heating (forced-air circulation furnaces), the heating time follows from the inverse of Newton's law of cooling [34]:

$$t = \frac{\rho_s \cdot c_p \cdot d}{\eta_{conv} \cdot h} \ln \left(\frac{T_f - T_0}{T_f - T_{target}} \right) \quad (9)$$

where h is the convective heat transfer coefficient ($\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$; typically 20–100 $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ for forced convection), T_{target} is the target temperature (K), and η_{conv} is the convective heating efficiency (0.7–0.9). The efficiency likewise enters the denominator, so a lower convective heating efficiency prolongs the required heating time.

For conduction heating (contact heating), the heating time is based on the analytical solution of the transient heat conduction equation (one-dimensional semi-infinite-body approximation) [34]:

$$t = \frac{s^2}{\alpha_d} \cdot Fo \quad (10)$$

where t is the heating time (s); $s = d/2$ is the half-thickness of the plate (m); $\alpha_d = \frac{k}{\rho_s c_p}$ is the thermal diffusivity ($\text{m}^2\cdot\text{s}^{-1}$), with k the thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$), ρ_s the density ($\text{kg}\cdot\text{m}^{-3}$), and c_p the specific heat capacity ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$); and Fo is the Fourier number, with $Fo \approx 1.3 - 2.5$ when the center temperature reaches 95% of the target temperature (≈ 1.3 for an ideal constant-surface-temperature boundary, $Bi \rightarrow \infty$; $\approx 2.3 - 2.5$ for $Bi \approx 2 - 3$, representative of practical contact heating with finite thermal contact resistance).

Engineering note. In production lines, radiation and convection heating are combined. For a B1500HS blank ($d = 1.2 \text{ mm}$, $\rho_s = 7850 \text{ kg}\cdot\text{m}^{-3}$, and $c_p \approx 600 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$) heated from room temperature to 900–930 °C in a radiation furnace (furnace temperature of 950 °C and $\varepsilon_{eff} = 0.8$), Equation (8) evaluated at the initial heat flux yields a lower-bound heat-up time of about 50 s at the ideal-efficiency limit, which rises to about 70 s when the radiative-efficiency factor (0.7) of Equation (8) is applied. Because the net radiative flux decays as the blank temperature approaches the furnace temperature, integration along the heating path (using $T_f^4 - T^4$ as the instantaneous driving potential) gives a more realistic heat-up time of about 100–130 s, the spread reflecting the radiative-efficiency range $\eta_{rad} = 0.7 - 0.8$. Adding the soaking time required for complete austenitization and carbon homogenization (about 60–100 s for 1.2 mm blanks, scaling approximately with d^2), the total furnace residence time amounts to approximately 160–230 s (about 3–

4 min), consistent with industrial practice (typically 3–5 min). Equation (8) therefore provides a defensible basis for furnace temperature-control design in future work.

Integrated estimate. The transient energy balance is $\rho_s c_p d \cdot \frac{dT}{dt} = \eta_{rad} \varepsilon_{eff} \sigma_{SB} (T_f^4 - T^4)$. Its analytical integral is $t = \frac{\rho_s c_p d}{\eta_{rad} \varepsilon_{eff} \sigma_{SB}} \times \left[\frac{\arctan(\frac{T}{T_f}) + \frac{1}{2} \ln(\frac{T_f + T}{T_f - T})}{2T_f^3} \right]$ evaluated from T_0 to T_{target} . For $d = 1.2$ mm, $\rho_s = 7850$ kg·m⁻³, $c_p \approx 600$ J·kg⁻¹·K⁻¹, $T_f = 1223$ K, $T_0 = 293$ K, $T_{target} = 1173 - 1203$ K, $\varepsilon_{eff} = 0.8$, and $\eta_{rad} = 0.7 - 0.8$, numerical evaluation gives approximately 94–131 s, supporting the rounded 100–130 s heat-up interval used above.

Forming force. Hot-stamping force is the central basis for process design and equipment selection; its calculation must account for the material flow stress, friction conditions, and part geometric complexity. The basic formula is [12,16]:

$$F = K \cdot \sigma_s \cdot A_{form} \quad (11)$$

where F is the forming force (N), K is a correction factor (1.2–2.0) covering friction and shape complexity, σ_s is the flow stress (MPa), and A_{form} is the effective forming area (mm²).

$$\sigma_s = \sigma_0 \cdot \left[1 + C \cdot \ln \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right] \quad (12)$$

where σ_0 is the reference flow stress, C is the strain-rate sensitivity coefficient (0.01–0.1), $\dot{\varepsilon}$ is the actual strain rate (s⁻¹), and $\dot{\varepsilon}_0$ is the reference strain rate (s⁻¹).

Springback. Springback is a key factor affecting the dimensional accuracy of hot-stamped parts and depends on yield strength, elastic modulus, sheet thickness, bending radius, and bending angle. For small-curvature bending in hot stamping (e.g., the fillet region of an A-pillar panel), the springback angle can be estimated by analytical formulas based on beam-bending theory.

Under pure bending, the moment–curvature relation is [12]:

$$M = \frac{EI}{r} \quad (13)$$

where M is the bending moment (N·mm); E is the elastic modulus (MPa); I is the second moment of area (mm⁴), with $I = t_s^3/12$ for a rectangular section of unit width; and r is the bending radius (mm).

When the stress of the outermost fiber reaches the yield strength σ_y , the section enters the elastic–plastic state, and the elastic-core height h_e is related to the total thickness t_s by:

$$h_e = \frac{2\sigma_y \cdot r}{E} \quad (14)$$

where σ_y is the yield strength (MPa), r is the bending radius (mm), and E is the elastic modulus (MPa).

For an elastic–perfectly plastic sheet under pure bending, the bending moment per unit width at the limit of the elastic core is $M = \sigma_y(t_s^2/4 - h_e^2/12)$, and elastic unloading follows $\Delta\kappa = M/(EI)$, with $I = t_s^3/12$; the springback angle caused by elastic recovery over the bending arc length L is therefore:

$$\Delta\theta = \frac{3\sigma_y L}{Et_s} \left(1 - \frac{h_e^2}{3t_s^2} \right) \quad (15)$$

where $\Delta\theta$ is the springback angle (rad); L is the bending arc length (mm), where $L = r \cdot \theta_0$, with θ_0 the bending angle (rad); t_s is the sheet thickness (mm); and h_e is the elastic-core height (mm) given by Equation (14). Equation (15) increases with the yield strength and the arc length and decreases with the elastic modulus and the sheet thickness, consistent with physical expectation. All quantities in Equations (13)–(15) are defined per unit width. For numerical evaluation, the elastic modulus quoted in GPa is converted into MPa

(1 GPa = 10^3 MPa) so that all stress-like quantities in Equations (13)–(16) share a consistent unit.

Two limiting cases of Equation (15) are of practical interest. In the fully plastic limit of tight bending ($h_e \rightarrow 0$), the springback angle approaches the upper bound $3\sigma_y L / (Et_s)$. In the opposite limit of thin-sheet, small-curvature bending ($h_e \geq t_s$; i.e., the whole section remains elastic), Equation (15) reduces to Equation (16):

$$\Delta\theta_{elastic} = \frac{2\sigma_y L}{Et_s} = \frac{L}{r} \quad (16)$$

where $\Delta\theta_{elastic}$ is the fully elastic springback angle (rad) and the identity $\Delta\theta_{elastic} = L/r$ reflects complete elastic recovery upon unloading. For typical hot-stamping conditions ($\sigma_y \approx 100 - 150$ MPa at 700–800 °C, $E \approx 100 - 150$ GPa, $t_s = 1.2$ mm, $r = 10$ mm, and $\theta_0 = 90^\circ$), Equation (15) predicts $\Delta\theta \approx 1.5 - 2.8^\circ$ when σ_y and E are paired at consistent temperatures (both decrease as the temperature rises), consistent with the small springback observed in hot-stamped panels.

It should be emphasized that the analytical formulas above account only for the elastic-recovery component of springback. In hot stamping, two additional mechanism-level factors dominate the final shape deviation. First, the quench martensitic transformation induces transformation plasticity: the volume expansion accompanying the FCC→BCT lattice change (about 3–4%) interacts with the die constraint and the through-thickness stress gradient, partially relaxing—or locally amplifying—the residual-stress state that drives springback. Second, temperature gradients between the blank and the cooled die surface produce non-uniform plastic deformation and differential thermal contraction, which superimpose on the elastic recovery after the die constraint is released. Accordingly, engineering compensation strategies fall into three categories: displacement compensation (reverse-crowning the die surface by the predicted springback displacement), force compensation (increasing the binder or bottoming force to enlarge the plastified zone), and process compensation (adjusting the holding pressure, quenching dwell time, and die temperature to homogenize the stress state); among them, the simulation–trial iterative die-surface correction strategy is currently the most effective path in engineering practice [35].

It must be recognized that the analytical framework of Equations (13)–(16) is strictly valid for small-curvature bending with limited plastic penetration. In practice, hot-stamped structural parts such as A/B-pillar reinforcements and door rings frequently undergo large plastic deformation (equivalent strains of 0.2–0.5), under which the assumptions underlying Equations (14) and (15) break down in three respects. First, substantial strain hardening during large deformation redistributes the residual-stress field, so that springback can no longer be estimated from the bending-moment balance alone and implicit finite-element springback analysis becomes mandatory [18]. Second, experimental and numerical studies on U-shaped and box-shaped hot-stamped parts show that springback under large deformation remains sensitive to initial blank temperature, blank-holder force, punch–die clearance, and die shoulder radius: springback increases markedly as the blank temperature decreases or as the die clearance and shoulder radius increase [19,20]. Third, in indirect hot stamping, the preforming step introduces prior cold work that modifies the stress path during subsequent press quenching, and the resulting springback may either be partially cancelled or amplified by transformation plasticity depending on the local constraint [19]. These findings indicate that for large-deformation hot stamping, die compensation should be based on coupled thermo-mechanical–metallurgical finite-element springback simulation with iterative die-surface correction, rather than on the analytical estimates of Equation (15), which should be reserved for rapid first-pass screening of fillet regions.

3.2.3. Cooling-Channel Parameters

The design of the die cooling system directly affects quenching efficiency and the microstructure and properties of the part [36]. The cooling-channel parameters include the Reynolds number, the flow rate, and the channel diameter. The Reynolds number is the dimensionless criterion for the flow regime; turbulent flow must be ensured to enhance heat transfer, and engineering practice generally requires [34]

$$Re = \frac{\rho_w \cdot v \cdot D}{\mu} > 4000 \quad (17)$$

where ρ_w is the cooling-water density ($\text{kg}\cdot\text{m}^{-3}$), v is the flow velocity ($\text{m}\cdot\text{s}^{-1}$), D is the channel diameter (m), and μ is the dynamic viscosity (Pa·s).

The cooling-water flow rate is calculated from the heat balance. The heat to be removed, Q_r (W), is first computed [34]:

$$Q_r = \dot{m}_{steel} \cdot c_{p,steel} \cdot \Delta T_{steel} + L_m \cdot \dot{m}_M + \eta_{form} \cdot P_{form} \times 10^3 \quad (18)$$

where $\dot{m}_{steel}$ is the mass flow rate of the sheet ($\text{kg}\cdot\text{s}^{-1}$), $c_{p,steel}$ is the specific heat capacity of the steel ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$), ΔT_{steel} is the temperature drop of the sheet (K), L_m is the latent heat of martensitic transformation ($\text{J}\cdot\text{kg}^{-1}$; approximately $2.5 \times 10^5 \text{ J}\cdot\text{kg}^{-1}$), $\dot{m}_M$ is the mass flow rate undergoing transformation ($\text{kg}\cdot\text{s}^{-1}$), η_{form} is the fraction of forming work converted into heat (0.9–0.95), and P_{form} is the forming power (kW), multiplied by 10^3 so that Q_r is expressed in W.

$$Q_v = \frac{Q_r}{\rho_w \cdot c_{p,w} \cdot \Delta T_w} \quad (19)$$

where Q_v is the volumetric flow rate ($\text{m}^3\cdot\text{s}^{-1}$), ρ_w is the density of water ($\text{kg}\cdot\text{m}^{-3}$; approximately $998 \text{ kg}\cdot\text{m}^{-3}$ at 20°C), $c_{p,w}$ is the specific heat capacity of water ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$; approximately $4180 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$), and ΔT_w is the inlet–outlet temperature difference of the cooling water (K; typically 5–10 K).

In engineering, the hourly flow rate Q_h ($\text{m}^3\cdot\text{h}^{-1}$) is commonly used:

$$Q_h = 3600 \cdot Q_v = \frac{3600 Q_r}{\rho_w \cdot c_{p,w} \cdot \Delta T_w} \quad (20)$$

The cooling-channel diameter D (m) is determined from the volumetric flow rate and the flow velocity:

$$D = \sqrt{\frac{4Q_v}{\pi \cdot v}} \quad (21)$$

where v is the cooling-water velocity ($\text{m}\cdot\text{s}^{-1}$; typically $1.5\text{--}3.0 \text{ m}\cdot\text{s}^{-1}$ under turbulent conditions).

Unit-conversion note. Equation (19) outputs $\text{m}^3\cdot\text{s}^{-1}$, and Equation (20) is its hourly-unit counterpart. Equation (21) requires $\text{m}^3\cdot\text{s}^{-1}$ as the input unit; if Q_h ($\text{m}^3\cdot\text{h}^{-1}$) is known, it must first be converted into $Q_v = Q_h/3600$ ($\text{m}^3\cdot\text{s}^{-1}$).

Design example. For the hot stamping of an A-pillar panel, with sheet mass flow rate $\dot{m}_{steel} = 0.5 \text{ kg}\cdot\text{s}^{-1}$, temperature drop $\Delta T = 800 \text{ K}$, and $c_{p,steel} \approx 600 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$, the sensible-heat term of Equation (18) amounts to 240 kW; adding the latent-heat contribution of about 125 kW ($L_m \cdot \dot{m}_M$ with $\dot{m}_M \approx \dot{m}_{steel}$) and the converted forming power of about 14 kW ($\eta_{form} = 0.9$ and $P_{form} = 15 \text{ kW}$), the total heat load is $Q_r \approx 380 \text{ kW}$. Taking $\Delta T_w = 8 \text{ K}$, Equation (19) gives $Q_v \approx 1.13 \times 10^{-2} \text{ m}^3\cdot\text{s}^{-1} \approx 41 \text{ m}^3\cdot\text{h}^{-1}$. For a single channel at $v = 2.5 \text{ m}\cdot\text{s}^{-1}$, Equation (21) gives $D \approx 0.076 \text{ m}$, corresponding to the standard channel size DN80. In die-design practice, the flow is instead distributed over several parallel channels—for example, four parallel DN40 channels ($D \approx 38 \text{ mm}$ each) at $v \approx 2.5 \text{ m}\cdot\text{s}^{-1}$ —for which $Re \approx 9.5 \times 10^4 \gg 4000$, so that the turbulent-flow requirement of Equation (17) is comfortably satisfied [37].

Temperature-uniformity criterion. Beyond the flow and capacity requirements above, the uniformity of the die-surface temperature is quantified in this research program by the temperature-uniformity coefficient $\xi_T = (T_{\max} - T_{\min})/T^- \times 100\%$, where $T_{\max}$, $T_{\min}$, and T^- are the maximum, minimum, and area-averaged die-surface temperatures under steady production conditions, respectively. The engineering design target is $\xi_T \leq 5\%$, which constitutes the temperature-uniformity constraint restated as the cooling-system design criterion in item 6 of the Conclusions (Section 8).

3.3. Multi-Study Cross-Validation of the Hot-Deformation Behavior of B1500HS Steel and Applicability Analysis of the Constitutive Model

The accuracy of the Arrhenius constitutive model of Equation (4) must be verified against independent experimental data. Given the similar chemical compositions of B1500HS and the related grades 22MnB5 and BR1500HS (cf. Table 2), this section adopts a multi-study cross-validation method to assess model applicability and determine the material constants from published data.

3.3.1. Literature Data Compilation and Statistical Verification

Gleeble hot-tension tests on 22MnB5 by Zhou et al. [14] (500–950 °C and 0.01–10 s^{−1}) produced a modified hyperbolic-sine Arrhenius model with correlation coefficients of 0.98–0.997 for the individual regression fits and a deformation activation energy of $Q = 196.8$ kJ·mol^{−1}. Sun et al. [15] investigated BR1500HS by Gleeble hot-tension tests (550–850 °C and 0.01–10 s^{−1}) and obtained $Q = 343.2$ kJ·mol^{−1} with a correlation coefficient $R = 0.982$ and an average absolute relative error (AARE) of 3.58%. The hot-compression study of B1500HS by Li et al. (700–900 °C and 0.005–10 s^{−1}) further confirmed the applicability of the Arrhenius equation to boron steels, yielding $Q = 245.4$ kJ·mol^{−1} and a correlation coefficient $R = 0.998$ between the predicted and measured flow stresses. Taking 900 °C/0.1 s^{−1} as an example, Zhou et al. reported a peak stress in the order of 100 MPa for 22MnB5, and the flow curves by Li et al. give a peak stress of about 125 MPa for B1500HS; with the parameters adopted in this study (Section 3.3.2), Equation (4) predicts a peak stress of about 125 MPa for B1500HS under the same condition, in quantitative agreement with the experimental value. The peak stresses of the three boron steels are of the same order of magnitude and follow consistent temperature and strain-rate trends, supporting the cross-material transferability of the model.

3.3.2. Parameter Determination and Literature Cross-Validation

Table 6 summarizes the Arrhenius-model parameters reported in the major literature sources and provides the basis for the parameter selection of this study.

Table 6. Summary of literature data on the high-temperature deformation behavior of boron steels.

Source	Steel	Test Mode	T (°C)	$\dot{\epsilon}$ (s ^{−1})	α (MPa ^{−1})	n	Q (kJ·mol ^{−1})	Fit Quality
Zhou et al. [14]	22MnB5	Hot tension	500–950	0.01–10	0.0038	9.67	196.8	R = 0.98–0.997
Sun et al. [15]	BR1500HS	Hot tension	550–850	0.01–10	—	—	343.2	R = 0.982; AARE = 3.58%
Li et al. [28]	B1500HS	Hot compression	700–900	0.005–10	5.41×10^{-3}	6.294	245.4	R = 0.998
This work (adopted from Li et al.)	B1500HS	—	800–950	0.005–10	5.41×10^{-3}	6.294	245.4	—

Note: The pre-exponential factor adopted in this work, $A \approx 6.2 \times 10^{10}$ s^{−1}, was back-calculated in this work from the peak-stress data published by Li et al. at multiple conditions using Equation (4): the data at 900 °C/0.1 s^{−1} ($\sigma_p \approx 125$ MPa) and at 900 °C/0.01 s^{−1} ($\sigma_p \approx 90$ MPa) yield mutually

consistent values of 6.20×10^{10} and $6.11 \times 10^{10} \text{ s}^{-1}$, respectively (reported as $A \approx 6.2 \times 10^{10} \text{ s}^{-1}$ after rounding to two significant figures), supporting the robustness of the adopted A ; “—” denotes a quantity not reported in the cited source or not applicable. The activation energies reported for the three boron steels span 197–343 $\text{kJ}\cdot\text{mol}^{-1}$, a range typical of the hot deformation of low-alloy steels; the value for B1500HS (245.4 $\text{kJ}\cdot\text{mol}^{-1}$) is close to the activation energy for self-diffusion in γ -Fe (about 270 $\text{kJ}\cdot\text{mol}^{-1}$), indicating that the hot deformation of austenitized B1500HS is controlled by dislocation climb. The spread among the reported Q values arises from the different test modes (tension vs. compression), fitted temperature windows, and chemical compositions (e.g., the higher Mn content in BR1500HS). The parameters of Li et al.’s study are adopted as the baseline of the present study; because their calibration window ends at 900 °C, application to the upper part of the hot-stamping window (900–950 °C) involves moderate extrapolation, and verification by independent Gleeble experiments is recommended. The last row of Table 6 applies these parameters over the hot-stamping process window of 800–950 °C considered in this study; they do not represent new experimental measurements.

Basis for parameter selection

Material-homology priority: Li et al. [28] conducted Gleeble-3800 hot-compression tests directly on B1500HS; their temperature window (700–900 °C) covers most of the hot-stamping window of this study (800–950 °C), and their strain rates (0.005–10 s^{-1}) cover the full range of interest, so the data are direct and reliable. The parameters adopted by Li et al. are therefore taken as the baseline values.

Verification with related grades: 22MnB5 and BR1500HS belong to the same boron steel family and have compositions similar to that of B1500HS (carbon difference < 0.05%). The Q values reported by Zhou et al. [14] (196.8 $\text{kJ}\cdot\text{mol}^{-1}$, hot tension) and Sun et al. [15] (343.2 $\text{kJ}\cdot\text{mol}^{-1}$, hot tension) bracket the 245.4 $\text{kJ}\cdot\text{mol}^{-1}$ of Li et al.’s work [28] (hot compression); the spread is consistent with the known influence of test mode, fitted temperature window, and Mn/Cr content on the apparent activation energy. Since Li et al. provide direct B1500HS data under conditions closest to the present study, their parameters are preferred.

The finally adopted parameters are:

$$\alpha = 5.41 \times 10^{-3} \text{ MPa}^{-1}, \quad n = 6.294, \quad Q = 245.4 \text{ kJ} \cdot \text{mol}^{-1}, \quad A \approx 6.2 \times 10^{10} \text{ s}^{-1}$$

where A was back-calculated from the peak-stress data of Li et al.’s study as described in the note to Table 6. With these parameters, Equation (4) reproduces the measured peak stresses of B1500HS over 700–900 °C (e.g., about 125 MPa at 900 °C/0.1 s^{-1} and about 90 MPa at 900 °C/0.01 s^{-1}), and the activation energy is consistent with the physical mechanism of dislocation-climb-controlled creep of austenite.

3.3.3. Model Limitations and Correction Strategy

Combining the data of the studies by Li et al. and Sun et al., the Arrhenius model exhibits systematic deviations (about 10–15%) under low-temperature/high-strain-rate combinations (<850 °C, >5 s^{-1}). Sun et al. measured adiabatic temperature rises of 35–50 °C at 800 °C/10 s^{-1} , accompanied by enhanced dynamic-recrystallization softening, neither of which is accounted for by the classical model. Accordingly, numerical simulations in future work should adopt a Johnson–Cook correction (introducing an adiabatic heating factor and a dynamic-recrystallization softening factor) to improve predictive accuracy under extreme conditions. To place the adopted Arrhenius model in the broader methodological landscape, Table 7 compares the four principal constitutive modeling approaches applied to hot-stamping steels—Arrhenius-type, Johnson–Cook, Zerilli–Armstrong, and machine learning surrogates—in terms of physical basis, applicable range, extrapolation capability, microstructural interpretability, calibration cost, and suitability for B1500HS.

Table 7. Methodological comparison of high-temperature constitutive modeling approaches for hot-stamping steels.

Model	Physical Basis	Applicable Range	Extrapolation Capability	Microstructural Interpretability	Calibration Cost	Suitability for B1500HS	Representative Sources
Arrhenius (hyperbolic sine)	Phenomenological; thermally activated dislocation glide/climb	Calibrated window (e.g., 700–900 °C and 0.005–10 s ⁻¹)	Good within window; moderate beyond	Low (no explicit micro-structure)	Moderate (α , n , Q , and A)	Adopted as the baseline ($Q = 245.4$ kJ·mol ⁻¹ ; Section 3.3.2)	[12,14,15,27]
Johnson–Cook	Phenomenological; decoupled strain-rate–temperature product	Wide, including high strain rates	Poor above $\approx 0.5 T_m$ without modification	Low	Low–moderate	Useful for high-rate extrapolation; requires hot-stamping-specific calibration	[13]
Zerilli–Armstrong	Physically based dislocation mechanics; distinct bcc/fcc forms	High strain rates; thermal activation built-in	Intrinsic thermal activation aids extrapolation	Medium (grain-size dependence explicit)	Moderate	Transferable in principle; the austenitic state requires the fcc form	[21]
Machine learning surrogate/PINN	Data-driven, optionally physics-constrained	Limited to the training envelope	Poor outside the envelope unless physics-informed	Low (black box) to medium (PINN)	High (large data demand)	Promising for cross-grade transfer; interpretability still limited	[29,30]

4. Hot-Plastic-Deformation Mechanisms of Hot-Stamping Steels

Hot plastic deformation is the theoretical foundation of the hot-stamping process; understanding it is essential to interpreting the high-temperature flow behavior, predicting the forming limit, and optimizing the process parameters. When a sheet metal is heated above the recrystallization temperature, atomic thermal vibration intensifies and the interatomic spacing increases, markedly reducing the resistance to dislocation glide, climb, and cross-slip; meanwhile, the number of active slip systems increases, and grain boundaries obstruct dislocation motion less effectively, so plastic deformability is greatly enhanced.

The hot plastic deformation of hot-stamping steels proceeds mainly through three mechanisms: intragranular slip, grain-boundary sliding, and diffusion creep. These mechanisms compete and interact synergistically under different temperature and strain-rate conditions and jointly determine the high-temperature deformation behavior. A deep understanding of these mechanisms provides important theoretical guidance for establishing accurate constitutive models and optimizing the forming parameters.

4.1. Intragranular Slip Mechanism

4.1.1. Physical Nature of Intragranular Deformation

At elevated temperatures, intragranular slip remains the dominant mode of plastic deformation in metals. This is because the increased interatomic spacing and intensified thermal vibration reduce the resistance to dislocation motion, while the increased number of slip systems improves slip flexibility and the pinning effect of grain boundaries on

dislocations weakens. These factors act synergistically, making intragranular slip the governing mechanism of hot plastic deformation. Figure 7 illustrates the principal forms of intracrystalline dislocations.

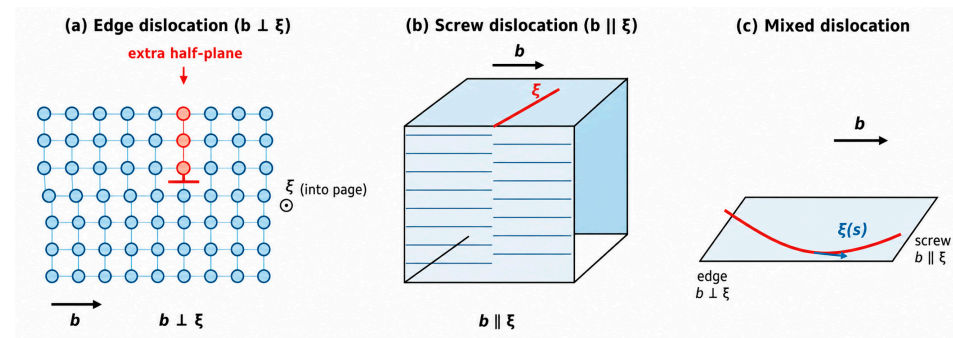


Figure 7. Schematic diagram of intracrystalline dislocation forms. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors; the Burgers vector b and the dislocation line ξ are explicitly identified in each panel. Re-constructed prompt: “Create a schematic diagram of intracrystalline dislocation forms in a metal crystal — dislocation lines, loops and tangles inside grains — with the Burgers vector b and the dislocation line ξ explicitly identified in each panel, in a clean textbook illustration style.”

4.1.2. Main Influencing Factors of Intragranular Slip

The activation degree and coordination efficiency of intragranular slip are influenced by several factors.

Deformation temperature. A higher temperature activates more slip systems and promotes the coordination of intragranular slip; however, excessive temperature may trigger dynamic recrystallization and soften the material, whereas at too-low temperatures, intragranular slip is constrained and larger external forces are needed to sustain plastic deformation.

Strain rate. Under high-speed deformation, the dislocation multiplication rate exceeds the slip-coordination capacity, aggravating work hardening; low-speed deformation favors dislocation rearrangement and annihilation, improving slip uniformity.

Grain size and orientation. In fine-grained materials, the increased grain-boundary fraction makes dislocation pile-up at boundaries more pronounced, requiring higher stresses to drive slip. When the misorientation between adjacent grains exceeds approximately 15° —the conventional limit separating low-angle from high-angle grain boundaries (to be distinguished from the Brandon criterion $\Delta\theta \leq 15^\circ/\sqrt{\Sigma}$, which quantifies the maximum allowable deviation from an exact coincident-site-lattice misorientation for a boundary to retain its special character [22])—the slip-transfer resistance rises sharply, because the geometric and orientational mismatch prevents the direct transmission of incoming slip dislocations and forces the activation of new dislocation sources in the neighboring grain [23]; under such conditions, multipass deformation parameters must be adjusted to coordinate dislocation motion across grain boundaries.

Stress state. A triaxial compressive stress state suppresses the initiation of intergranular damage and makes intragranular slip the dominant mechanism, raising the limit deformation by 30–50% [38]; tensile stresses, by contrast, promote the separation of dislocations on slip planes and readily induce microcracks.

4.1.3. Calculation Methods for the Main Parameters of Intragranular Slip

The theoretical analysis of intragranular slip is based on crystal plasticity; two core calculation methods are introduced below. Figure 8 illustrates the geometric relationship between the applied stress axis and the slip system for intragranular slip.

Schmid's law. Schmid's law is the classical theory describing the onset of plastic deformation in single crystals. Plastic slip begins when the resolved shear stress on the slip plane along the slip direction reaches a critical value (the critical resolved shear stress, CRSS), which is an intrinsic material property independent of crystal orientation. The resolved shear stress is calculated as [39]:

$$\tau = \sigma \cdot \cos\phi \cdot \cos\lambda \quad (22)$$

where τ is the resolved shear stress (MPa), σ is the axial stress (MPa), ϕ is the angle between the slip-plane normal and the loading axis, and λ is the angle between the slip direction and the loading axis. The product $\cos\phi \cdot \cos\lambda$ is the Schmid factor m , whose maximum value of 0.5 occurs at $\phi = \lambda = 45^\circ$.

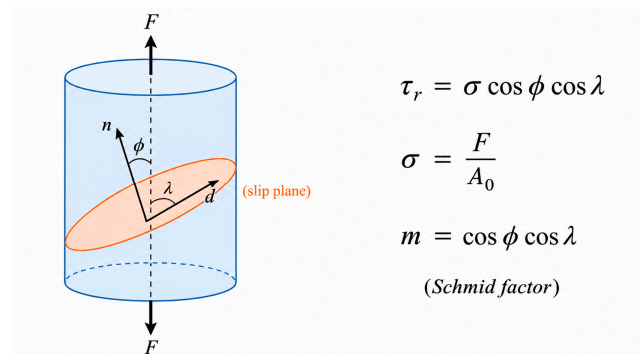


Figure 8. Schematic diagram of the geometric relationship of intragranular slip. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Reconstructed prompt: “Create a schematic diagram of the geometric relationship of intragranular slip in a single crystal, showing the slip plane, the slip direction, the applied tensile axis, and the angles used in Schmid's law, with labeled vectors and planes.”

Dislocation motion and plastic strain. At the microscopic level, plastic deformation manifests as the motion and multiplication of dislocations. The plastic strain rate is related to the mobile dislocation density, the Burgers vector, and the average dislocation velocity by [39]:

$$\dot{\epsilon}_p = \rho_m \cdot b \cdot \bar{v} \quad (23)$$

where $\dot{\epsilon}_p$ is the plastic strain rate (s^{-1}); ρ_m is the mobile dislocation density (m^{-2}); b is the magnitude of the Burgers vector (m); and $\bar{v}$ is the average dislocation velocity ($m \cdot s^{-1}$).

The dislocation velocity is governed by the resolved shear stress [39]:

$$v = v_0 \cdot \left(\frac{\tau}{\tau_r} \right)^{m_v} \quad (24)$$

where v_0 is the reference velocity ($m \cdot s^{-1}$); τ_r is the critical resolved shear stress (MPa); and m_v is the rate-sensitivity exponent.

4.1.4. Critical Discussion: Validity of Schmid's Law and Dislocation Models Under Hot-Deformation Conditions

Schmid's law and dislocation-density-based plasticity theory [40] provide a solid physical basis for understanding intragranular slip, but their practical application to hot stamping has limitations. On the one hand, these theories assume single crystals or ideal polycrystals, whereas B1500HS is a multiphase microstructure (containing small amounts of cementite and Ti carbonitrides), and the pinning and cutting effects of second-phase particles on dislocations are not fully incorporated. On the other hand, dynamic recovery and dynamic recrystallization—important softening mechanisms in hot deformation—involve microstructural evolution processes (dislocation rearrangement, subgrain

formation, and grain-boundary migration) that simple constitutive equations cannot fully describe. Crystal plasticity finite-element (CPFE) methods developed in recent years can account for grain orientation and grain-boundary effects, but their computational cost remains prohibitive for full-scale industrial part simulation. Establishing meso-scale constitutive models that balance computational efficiency and physical fidelity therefore remains an important challenge. Recent developments in high-resolution digital image correlation (HR-DIC) and in situ transmission electron microscopy (in situ TEM) offer new experimental means of verifying intragranular slip theories, enabling the real-time tracking of dislocation trajectories at the nano-scale. In addition, strain gradient plasticity theory accounts for the contribution of geometrically necessary dislocations (GNDs) to size effects, providing a more refined framework for understanding the deformation heterogeneity of micro-regions in hot stamping. These advances provide the micro-mechanical basis for analyzing the local mechanical behavior of the heat-affected zone (HAZ) in future resistance spot welding studies. For the present research program, the practical consequence is that Schmid-factor analysis remains adequate for ranking the slip activity of B1500HS austenite in process-level finite-element models, whereas particle–dislocation interactions and dynamic-recrystallization softening must enter through the phenomenological correction terms discussed in Section 3.3.3 rather than through first-principles crystal plasticity.

4.2. Grain-Boundary Sliding Mechanism

4.2.1. Basic Characteristics of Grain-Boundary Sliding

Compared with cold plastic deformation, the grain-boundary strength decreases in the hot state, and grain-boundary sliding proceeds more readily, so its contribution to the total deformation increases significantly. Nevertheless, under conventional hot-plastic-deformation conditions, the deformation contributed by grain-boundary sliding remains small relative to intragranular slip. Figure 9 schematically illustrates grain-boundary sliding.

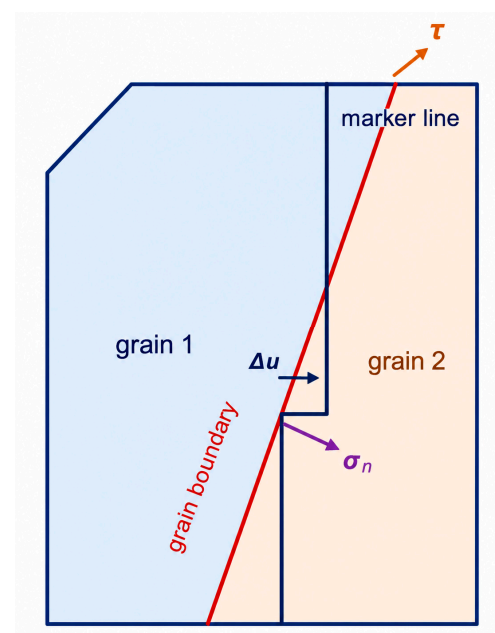


Figure 9. Schematic diagram of grain-boundary sliding. Adapted from Ref. [39].

4.2.2. Main Influencing Factors of Grain-Boundary Sliding

The occurrence and development of grain-boundary sliding are controlled by several factors: increasing temperature reduces the atomic bonding strength at grain boundaries, facilitating sliding; the stress state is the most important factor, with sliding initiating when the applied stress reaches the critical shear stress; the smaller the grain size, the larger the number of grain boundaries and the stronger the resistance to sliding; and the higher the strain rate, the faster the intragranular stress accumulation and the more readily grain-boundary sliding is triggered.

4.2.3. Calculation Methods for the Main Parameters of Grain-Boundary Sliding

Strain rate due to grain-boundary sliding. The strain rate associated with grain-boundary sliding can be estimated by the following boundary diffusion-controlled accommodation expression [41,42]; it is mathematically identical in form to the Coble creep equation (Equation (29)) and is adopted here as an order-of-magnitude estimate of the sliding-accommodation contribution:

$$\dot{\epsilon}_{GB} = \frac{40 \sigma \Omega \delta D_{GB}}{d^3 k_B T} \quad (25)$$

where δ is the grain-boundary thickness (m), d is the grain size (m; numerical values commonly quoted in μm), D_{GB} is the grain-boundary diffusion coefficient ($\text{m}^2\cdot\text{s}^{-1}$), σ is the applied stress (Pa), Ω is the atomic volume (m^3), k_B is the Boltzmann constant ($1.38 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$), and T is the absolute temperature (K).

Grain-boundary sliding rate. The grain-boundary sliding rate can be described by an Arrhenius-type activation law [39]:

$$v = \tau \cdot A_{GB} \cdot \exp\left(-\frac{Q}{R \cdot T}\right) \quad (26)$$

where τ is the shear stress along the grain boundary (MPa), A_{GB} is a material constant and Q is the activation energy for grain-boundary sliding (conventionally tabulated in $\text{kJ}\cdot\text{mol}^{-1}$ and converted into $\text{J}\cdot\text{mol}^{-1}$ in the exponential term), and R is the gas constant ($8.314 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$).

4.3. Diffusion Creep Mechanism

4.3.1. Physical Mechanism of Diffusion Creep

Diffusion creep is a plastic-deformation mechanism caused by the directional migration of vacancies under stress; its contribution becomes more significant at higher deformation temperatures, finer grain sizes, and lower strain rates. Diffusion creep not only contributes directly to plastic deformation but also plays an important accommodating role for grain-boundary sliding. Figure 10 schematically illustrates diffusion creep.

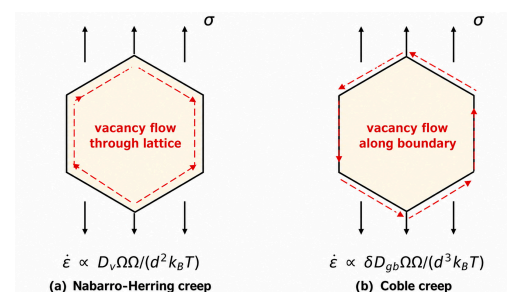


Figure 10. Schematic diagram of diffusion creep. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Black arrows indicate the direction of the applied stress σ ; red dashed arrows indicate the vacancy flow paths. Reconstructed prompt: “Create a schematic diagram of diffusion creep in a polycrystalline material, showing grains deforming under an applied tensile stress σ , with black arrows for the applied stress and red dashed arrows showing the vacancy flow paths between grain boundaries.”

4.3.2. Main Influencing Factors of Diffusion Creep

The diffusion creep rate is mainly influenced by temperature, stress, grain size, and chemical composition. Higher temperatures accelerate atomic diffusion, causing the creep rate to grow exponentially; the stress level directly determines the creep driving force; and the finer the grains, the larger the grain-boundary area and the shorter the grain-boundary diffusion paths, so diffusion creep occurs more readily.

4.3.3. Main Theoretical Calculation Methods for Diffusion Creep

General creep-rate equation. Creep behavior over wide temperature and stress ranges can be described by [39]:

$$\dot{\epsilon} = A_{creep} \cdot \sigma^{n_c} \cdot \exp\left(-\frac{Q}{R \cdot T}\right) \quad (27)$$

where A_{creep} is a material constant, n_c is the creep stress exponent (typically 3–8 for dislocation creep), and Q is the creep activation energy ($\text{kJ} \cdot \text{mol}^{-1}$; converted into $\text{J} \cdot \text{mol}^{-1}$ when used with R).

Nabarro–Herring creep (lattice diffusion-dominated). When creep is dominated by lattice diffusion, the strain rate is inversely proportional to the square of the grain size [43]:

$$\dot{\epsilon}_{NH} = \frac{10 \sigma \Omega D_V}{d^2 k_B T} \quad (28)$$

where D_V is the lattice diffusion coefficient ($\text{m}^2 \cdot \text{s}^{-1}$).

Coble creep (grain-boundary diffusion-dominated). When creep is dominated by grain-boundary diffusion, the strain rate is inversely proportional to the cube of the grain size [41]:

$$\dot{\epsilon}_C = \frac{40 \sigma \Omega \delta D_{GB}}{d^3 k_B T} \quad (29)$$

where δ is the grain-boundary thickness (1–2 atomic layers) and D_{GB} is the grain-boundary diffusion coefficient. Although Equation (29) is formally similar to Equation (25), the two describe different physical boundary conditions: Equation (25) gives the strain-rate contribution of grain-boundary sliding as an accommodating deformation mode, whereas Equation (29) is the steady-state creep rate sustained by grain-boundary diffusion; the former applies to the hot-stamping strain-rate regime and the latter to the high-temperature, low-stress creep regime.

In summary, within the hot-stamping temperature range, intragranular slip is the dominant deformation mechanism, grain-boundary sliding plays a supporting role, and diffusion creep contributes to the total deformation under high-temperature, low-strain-rate conditions. The synergistic action of the three mechanisms endows metals with far greater plastic deformability in the hot state than in the cold state.

4.3.4. Critical Discussion: Applicability of Diffusion Creep Models at Hot-Stamping Strain Rates

Diffusion creep theory provides the classical framework for understanding superplastic forming [42] and high-temperature steady-state creep, but its actual contribution during hot stamping is often underestimated. On the one hand, the strain rates of conventional hot stamping ($0.1\text{--}10 \text{ s}^{-1}$) are far higher than those of typical creep experiments ($10^{-6}\text{--}10^{-4} \text{ s}^{-1}$), and whether the diffusion creep contribution is negligible under such conditions still requires experimental verification; on the other hand, in ultrafine-grained boron steels (grain size $< 5 \text{ }\mu\text{m}$), the greatly increased grain-boundary area may raise the relative contributions of grain-boundary sliding and diffusion creep. Moreover, the Nabarro–Herring and Coble models consider lattice diffusion and grain-boundary diffusion independently, whereas in actual deformation, the two diffusion mechanisms are simultaneously active

and mutually coupled. Establishing a unified diffusion creep constitutive model and quantitatively evaluating its contribution under hot-stamping conditions are important topics for completing the theoretical system of high-temperature plastic deformation. For the present research program, this implies that diffusion creep can be safely neglected in finite-element models of B1500HS hot stamping at strain rates above 0.1 s^{-1} , but its potential contribution in the coarse-grained heat-affected zone of resistance spot welds merits separate evaluation.

4.4. Application of Multi-Scale Simulation in Plastic-Deformation Research

The continuum-mechanics framework presented above describes the macroscopic deformation behavior, whereas thermally activated dislocation motion, grain-boundary responses, and intergranular heterogeneity must be revealed by cross-scale simulation. This section establishes a three-level “atomistic–mesoscopic–macroscopic” theoretical framework that provides a micro-mechanical interpretation of the phenomenological equations in Sections 4.1–4.3 and lays the theoretical foundation for future constitutive modeling and HAZ analyses.

4.4.1. Molecular Dynamics (MD) Simulation: Thermally Activated Dislocation Mechanisms

MD tracks atomic trajectories by solving Newton’s equations of motion, revealing the dynamic processes of dislocation nucleation and glide. Because hot-stamping deformation takes place in the austenitic (γ -Fe) state, whereas well-validated interatomic potentials for austenitic Fe–C alloys are scarce, the atomistic evidence cited here is drawn from the ferritic (α -Fe) model system; the embedded-atom-method (EAM) potential developed by Mendeleev et al. [44] provides the most widely used interatomic description of ferritic iron and is used here only to establish mechanistic trends, not quantitative austenite properties (quantitative γ -Fe simulations require dedicated Fe–C austenitic potentials, e.g., of the Hepburn–Ackland type). MD setups of the type employed by Matsumoto et al. [45] and by related atomistic studies of dislocation motion [44,45] typically use simulation cells with approximately 10–20 nm edge lengths (in the order of 10^5 – 10^6 atoms) and an integration time step of about 1 fs under periodic boundary conditions to examine single-crystal shear over temperatures of 600–1100 K (i.e., within the ferritic stability range, below the $\alpha \rightarrow \gamma$ transition of pure iron at $\approx 1183 \text{ K}$) and strain rates of (10^8 – 10^{10} s^{-1}). The parameter values quoted here are order-of-magnitude estimates based on such typical MD setups, not the results of a dedicated simulation campaign. These simulations indicate that the Peierls stress τ_p of screw dislocations decays approximately exponentially with temperature, falling, as an order-of-magnitude estimate, to roughly 10–20% of its room-temperature value at 900 °C, consistent with the decreasing critical resolved shear stress predicted by Schmid’s law (Equation (22)); meanwhile, high temperature promotes the transition of dislocations from glide to climb, and the thermal dissolution of Cottrell atmospheres markedly reduces the dislocation-motion resistance [39]. This provides an atomistic interpretation of the physical essence of the activation energy $Q = 245.4 \text{ kJ}\cdot\text{mol}^{-1}$ in the Arrhenius equation (Equation (4))—a value close to the activation energy for self-diffusion of γ -Fe ($\approx 270 \text{ kJ}\cdot\text{mol}^{-1}$) and therefore consistent with a thermally activated process controlled by dislocation climb. Because MD-accessible strain rates (10^8 – 10^{10} s^{-1}) are far above industrial hot-stamping rates (0.1 – 10 s^{-1}), these results are used here only to clarify mechanisms and trends; quantitative rate-sensitive parameters must be transferred through DDD/CPFE bridging or experimental calibration.

4.4.2. Discrete Dislocation Dynamics (DDD) Simulation: Grain-Boundary Effects and Mesoscopic Mechanisms

DDD describes the motion, multiplication, and annihilation of dislocation lines at the meso-scale; its core governing equations are the dislocation velocity law and the Peach–Koehler force equation [39]:

$$v = M \cdot f_{PK}, \quad f_{PK} = (\sigma \cdot b) \times \xi \quad (30)$$

where M is the dislocation mobility, b is the Burgers vector, and ξ is the tangent vector of the dislocation line. For polycrystalline B1500HS, DDD simulations reveal that dislocations pile up severely at grain boundaries at low temperatures, and transmission across the boundary must overcome a high activation-energy barrier [46,47]; at high temperatures, grain-boundary sliding and dislocation-absorption capacity are enhanced, and grain-boundary transmission efficiency η_{GB} increases significantly. Order-of-magnitude estimates based on Equation (25) and the reported DDD literature suggest that the contribution of grain-boundary sliding to the total strain may rise from a few percent at room temperature to roughly 10–20% at 900 °C, which delineates the applicability boundary of the grain-boundary-sliding theory; these figures are indicative estimates rather than the results of a dedicated simulation campaign.

4.4.3. Crystal Plasticity Finite-Element (CPFE) Method: A Cross-Scale Bridge

By embedding crystal slip laws into the finite-element framework, CPFE couples the microstructure to the macroscopic response [48]. Its constitutive equation is based on a rate-dependent slip-system hardening model:

$$\dot{\gamma}^\alpha = \dot{\gamma}_0 \left| \frac{\tau^\alpha}{\tau_c^\alpha} \right|^n \text{sgn}(\tau^\alpha) \quad (31)$$

where $\dot{\gamma}^\alpha$ is the shear rate of slip system α , τ^α is the resolved shear stress, and τ_c^α is the critical resolved shear stress. Through statistical averaging over a representative volume element (RVE), CPFE maps grain-scale heterogeneity into an equivalent macroscopic constitutive law, providing micro-scale parameter calibration for the Johnson–Cook correction of Equation (4).

4.4.4. Theoretical Linkage to Future Studies

Although the three-level framework does not serve as a direct input to current process simulations, it provides key theoretical support for future work: the $\tau_p(T)$ relation revealed by MD forms the physical basis of the Johnson–Cook thermal-softening term; the η_{GB} quantified by DDD provides the grain-boundary damage criterion for the HAZ softening tolerance (hardness reduction $\leq 18.4\%$) in future resistance spot welding studies; and the RVE homogenization method of CPFE offers a methodological paradigm for macro–micro cross-scale analysis. This section thus serves as a theoretical reserve supporting the microstructure–property correlation analyses of subsequent studies.

Before turning to phase transformation, it should be emphasized that deformation and transformation do not act independently in hot stamping: prior deformation alters the defect state of austenite and thereby the transformation kinetics, while the latent heat and volume changes in transformation feed back on the temperature and stress fields. This interaction between deformation and phase transformation provides the physical basis for the coupling equations established in Section 6.

5. Phase Transformation Mechanisms of Hot-Stamping Steels

Precisely mastering the phase transformation behavior of boron steels during hot stamping is the key to ensuring product quality and property consistency. Hot stamping involves complex solid-state phase transformations, including austenitization during heating and pearlite/bainite/martensite transformations during cooling. This section systematically elaborates the physical mechanisms of each transformation stage and their influencing factors. Figure 11 shows the Fe–Fe₃C metastable equilibrium phase diagram [49].

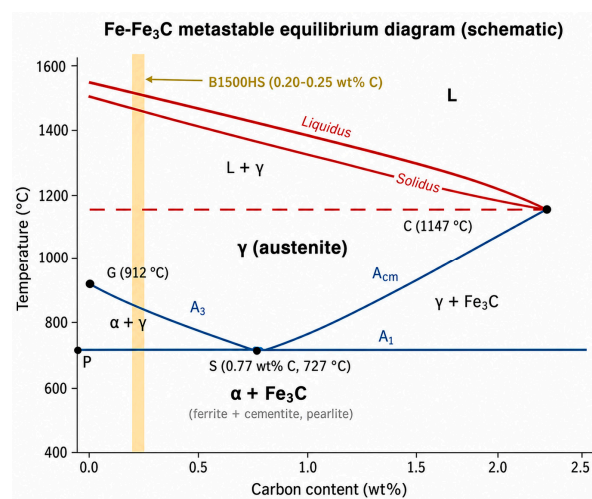


Figure 11. Fe–Fe₃C metastable phase diagram. Adapted from Ref. [49].

5.1. Austenitization Process

5.1.1. Microstructural Transformation Characteristics of Boron Steel

Figure 12 shows the microstructural evolution of boron steel during heating. Below A_{c1} , the steel is in the annealed state, and its microstructure consists mainly of ferrite and cementite; as the temperature rises above A_{c1} , pearlite (a lamellar mixture of ferrite and cementite) begins to transform into austenite; with further heating, the remaining ferrite continuously transforms into austenite; and above the A_{c3} line, the microstructure becomes fully austenitic. If the heating temperature is too low, the transformation is incomplete and a fully martensitic microstructure cannot be obtained after quenching; if it is too high, austenite grain coarsening occurs, and the final mechanical properties deteriorate.

Microstructural evolution of boron steel during heating

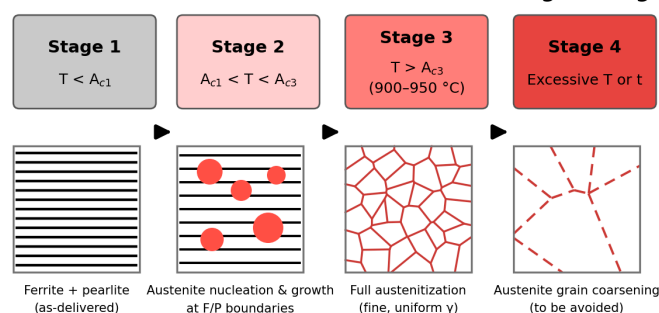


Figure 12. Microstructural phase evolution of boron steel during heating. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Horizontal arrows indicate the evolution sequence with increasing temperature. Reconstructed prompt: “Create a four-stage schematic of the microstructural evolution of boron steel during heating: Stage 1, ferrite + pearlite lamellae ($T < A_{c1}$); Stage 2, austenite nucleation and growth at ferrite/pearlite boundaries ($A_{c1} < T < A_{c3}$); Stage 3, full austenitization with fine uniform grains ($T > A_{c3}$); Stage 4, austenite grain coarsening (to be avoided).”

($T > A_{c3}$, 900–950 °C); Stage 4, austenite grain coarsening (excessive T or t), with colored stage boxes above each panel and arrows between the panels.”

5.1.2. Formation Mechanism of Austenite

Austenite formation is a complex solid-state transformation involving carbon diffusion, crystal-structure reconstruction, and compositional homogenization; its core mechanisms are thermally activated diffusion and phase-boundary migration. The process comprises four stages.

Austenite nucleation. Austenite nuclei preferentially form at the ferrite/cementite phase boundaries, where the carbon-concentration fluctuations are large, the defect density is high, and the energetic state is favorable, so that the thermodynamic and kinetic conditions for nucleation are readily satisfied.

Growth of austenite nuclei. After nucleation, austenite grows into ferrite and cementite mainly through carbon diffusion and interface migration; the growth process involves Fe_3C dissolution, carbon diffusion, and the $\alpha \rightarrow \gamma$ transformation.

Dissolution of residual cementite. After the ferrite has disappeared completely, the residual cementite gradually dissolves into the austenite as carbon continues to diffuse.

Homogenization of austenite. After complete cementite dissolution, the carbon distribution in the austenite remains non-uniform, and prolonged holding or higher temperatures are required to promote sufficient carbon diffusion and obtain a compositionally uniform austenite.

5.1.3. Critical Discussion: Control Precision of Austenitization Under Industrial Furnace Conditions

Austenitization theory provides the basic principles for the design of the heating stage, but challenges remain in precise control. First, the temperature non-uniformity during practical heating (the temperature difference within an industrial furnace commonly reaches ± 10 – 20 °C) leads to inconsistent austenitization degrees across the blank, affecting the final microstructural uniformity. Second, the redox characteristics of the protective atmosphere (N_2 or N_2+H_2) in industrial furnaces strongly influence surface decarburization and scale formation, whereas existing theoretical models rarely account for the coupling between atmosphere and interfacial reactions. In addition, the dissolution and precipitation behavior of microalloyed (Ti, Nb) carbonitrides during austenitization is crucial to grain refinement, but the relevant kinetic data remain insufficient. Developing austenitization models that couple the temperature field, atmosphere, and interfacial reactions is an important research direction for precise microstructural control in hot stamping. For B1500HS production lines, this translates into a concrete control requirement: the furnace set-point must ensure that the coldest point of the blank still exceeds A_{c3} (≈ 784 °C; Section 3.1.1) after transfer, which—given the ± 10 – 20 °C furnace non-uniformity—fixes the practical set-point window at about 930 ± 10 °C rather than the nominal 900 °C often quoted.

5.2. Quench Transformation Process

At the end of hot stamping, the part is fully austenitic and is then rapidly cooled and quenched in the closed die. The transformation type and product are controlled by the cooling rate: upon slow cooling, austenite decomposes below A_{c1} in the high-temperature regime into lamellar pearlite of alternating ferrite and cementite; in the intermediate-temperature regime it transforms into bainite; and when the cooling rate exceeds the critical cooling rate, austenite is undercooled below the M_s point and undergoes martensitic transformation. Figure 13 shows the continuous cooling transformation (CCT) diagram of a typical boron steel. For B1500HS, the landmark values reported in the literature and confirmed by the analysis of this review—the martensite start temperature $M_s = 400$ °C

(estimated by Equation (34) in Section 5.3) and the critical cooling rate of approximately $27\text{ }^{\circ}\text{C}\cdot\text{s}^{-1}$ introduced in Section 1 [3,4]—are marked on the diagram: cooling paths steeper than this critical rate bypass the ferrite/pearlite and bainite noses and secure a fully martensitic microstructure. The martensite finish temperature is deliberately not marked on the diagram: it is not constrained self-consistently by the Andrews formula and the Koistinen–Marburger law (Equation (32)), reported values depend strongly on the experimental definition adopted (for example, the retained-austenite criterion used in dilatometry), and no verified value for B1500HS is currently available.

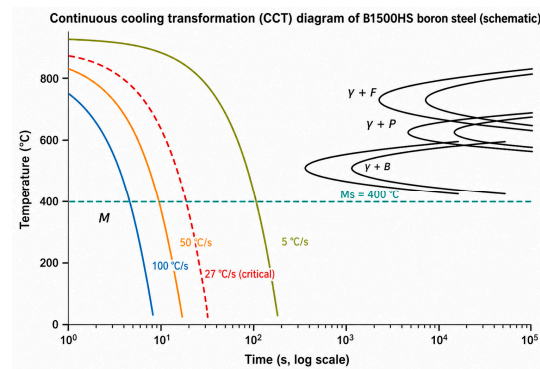


Figure 13. Continuous cooling transformation (CCT) diagram. Adapted from Refs. [3,7].

5.3. Martensite Formation Mechanism

Martensitic transformation is a diffusionless, shear-type phase transformation in which carbon atoms do not diffuse over long ranges; instead, the crystal structure changes from face-centered cubic (FCC) austenite to body-centered tetragonal (BCT) martensite through a lattice shear mechanism. The transformation is extremely fast—typically completed on a microsecond timescale—and is therefore also termed “shear transformation” or “diffusionless transformation”.

5.3.1. Formation Process of Martensite

Martensitic transformation involves the following key steps: austenite is undercooled below the M_s point and its stability decreases; the FCC→BCT structural change proceeds by the shear mechanism, with carbon atoms retained at their original sites; already-formed martensite promotes the transformation of adjacent austenite through autocatalysis; and martensite plates grow along specific crystallographic directions until the austenite is exhausted. Owing to the incompleteness of the transformation, a small amount of retained austenite may persist even after cooling to room temperature, and its content and distribution strongly affect part performance.

5.3.2. Main Factors Affecting Martensite Formation

Martensitic transformation is influenced by several factors: the faster the cooling, the more complete the martensitic transformation and the less the retained austenite; increasing carbon content lowers the M_s point and raises martensite hardness but reduces toughness; among alloying elements, Co raises the M_s point, whereas Mn, Ni, Cr, Si, and Mo all lower it and retard the transformation (consistent with the negative coefficients of Equation (34)); and the finer the austenite grains, the faster the martensitic transformation and the finer the martensite plates, which favors toughness.

5.3.3. Martensitic Transformation Kinetics

Martensite volume fraction. The martensite volume fraction f_m is the key indicator of quenching effectiveness and can be calculated by the Koistinen–Marburger equation [50]:

$$f_m = 1 - \exp[-k_{KM} \cdot (M_s - T)] \quad (32)$$

where f_m is the martensite volume fraction; k_{KM} is the Koistinen–Marburger empirical constant, with the classical value $k_{KM} \approx 0.011 \text{ } ^\circ\text{C}^{-1}$, applicable for $T < M_s$; M_s is the martensite start temperature ($^\circ\text{C}$); and T is the current temperature ($^\circ\text{C}$).

Critical cooling rate. The critical cooling rate V_k is the minimum cooling rate required to avoid the decomposition of austenite into pearlite or bainite and is the design basis for the die cooling system [3,12]:

$$V_k = \frac{dT}{dt} > \frac{T_{aus} - T_{nose}}{\Delta t} \quad (33)$$

where T_{aus} is the austenitizing temperature, T_{nose} is the nose temperature of the TTT curve, and Δt is the residence time in the nose-temperature region. Equation (33) is a conservative engineering estimation formula that does not resolve the actual transformation kinetics at the CCT-curve nose; more accurate values require CCT-based kinetic calculations. In practice, the continuous-cooling transformation is usually analyzed by discretizing the cooling curve into a series of isothermal steps and applying the Scheil additivity rule to the incubation periods read from the TTT diagram—transformation is assumed to start when the sum of the fractional incubation times reaches unity—which provides the kinetic basis for determining the critical cooling rate from measured CCT data.

Martensite start temperature. Martensite start temperature M_s is a key parameter for assessing quenching-process feasibility and can be estimated empirically from the alloying-element contents [25,51]:

$$M_s = 539 - 423C - 30.4Mn - 17.7Ni - 12.1Cr - 11.0Si - 7.5Mo \quad (34)$$

where the element symbols denote the corresponding mass percentages. This is the Andrews empirical formula [26], applicable to low- and medium-carbon alloy steels with 0.1–0.5% C; carbon has the most significant influence on M_s , with each additional 0.1% C lowering M_s by approximately 42 $^\circ\text{C}$. For the nominal B1500HS composition of Table 2 (C = 0.23%, Mn = 1.2%, Si = 0.25%, Cr = 0.15%, and Mo = 0.05%), Equation (34) predicts $M_s \approx 400 \text{ } ^\circ\text{C}$, in line with the experimentally reported range of approximately 380–420 $^\circ\text{C}$ for 22MnB5-type boron steels. Because the B1500HS carbon range (0.20–0.25%) lies near the lower boundary of the 0.1–0.5% C calibration interval, the predicted M_s should be interpreted as an estimate with an uncertainty of approximately $\pm 20 \text{ } ^\circ\text{C}$ until confirmed by dilatometric measurements on B1500HS.

5.3.4. Main Morphologies of Martensite

According to their morphological characteristics, martensite is classified into two main categories: lath martensite and plate martensite.

Lath martensite occurs mainly in low- and medium-carbon steels (C < 0.3%); it consists of parallel arrays of lath packets separated by low-angle grain boundaries, and its substructure is dominated by high-density dislocations; hence it is also called dislocation-type martensite—with a favorable strength–toughness combination. Figure 14 shows the morphology of lath martensite.

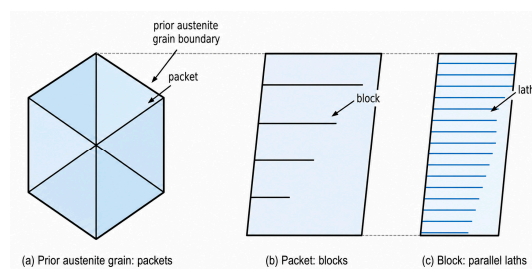


Figure 14. Morphology of lath martensite. Adapted from Refs. [52,53].

Unlike lath martensite, plate martensite forms when the carbon content is higher and the transformation mechanism changes significantly. Plate martensite is common in high-carbon steels ($C > 1.0\%$); its three-dimensional shape is a doubly convex lens, appearing acicular or leaf-like in metallographic sections, and its internal substructure is dominated by high-density twins—hence it is also called twinned martensite. Plate martensite possesses extremely high hardness and strength but is brittle. Figure 15 shows the morphology of plate martensite.

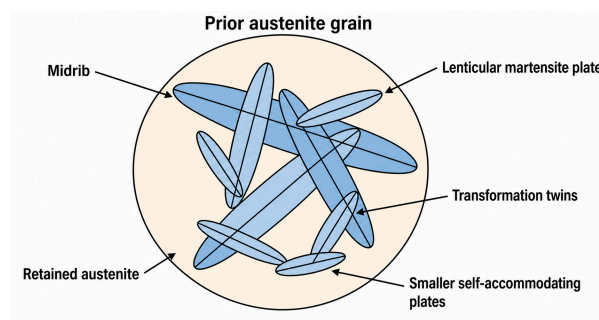


Figure 15. Morphology of plate martensite. Adapted from Ref. [52].

To systematically compare the microstructural characteristics and mechanical properties of the two martensite morphologies, the above analysis is summarized in Table 8.

Table 8. Comparison of lath martensite and plate martensite characteristics.

Feature	Lath Martensite	Plate Martensite
Carbon content range	$<0.3\%$	$>1.0\%$
Three-dimensional morphology	Elongated laths	Doubly convex lens/acicular
Substructure	High-density dislocations	High-density twins
Mechanical properties	High strength, good toughness	Extremely high hardness, high brittleness
Formation temperature	Just below M_s (small undercooling)	Well below M_s (large undercooling)

5.3.5. Critical Discussion: Predictive Limits of the Koistinen–Marburger Equation and M_s Formulas

Martensitic transformation theory provides the core theoretical basis for quench hardening in hot stamping, but existing models still have room for improvement in predictive accuracy and applicability. The Koistinen–Marburger equation (Equation (32)) is empirically fitted, and the dependence of its material constant k on alloy composition and stress state remains unclear; the calculation of the critical cooling rate (Equation (33)) relies on accurately determined TTT nose parameters, whereas the CCT behavior of B1500HS in localized regions such as the welding HAZ differs from that of the base metal [54]; and the applicability of the M_s formula (Equation (34)) to multicomponent alloys still requires verification. Fundamentally, existing models focus on predicting the martensite volume

fraction but lack the ability to quantitatively describe the key microstructural parameters—martensite plate size, dislocation density, and retained-austenite distribution—that determine the mechanical properties. Developing multi-scale predictive models coupling transformation kinetics with microstructural evolution is the key to precise property control of hot-stamped parts. For B1500HS, the immediate implication is that the Koistinen–Marburger coefficient and the M_s estimate of Equation (34) should be treated as grade-specific calibration targets—obtainable from a single dilatometric campaign—before the kinetic model is used to predict heat-affected-zone microstructures in the resistance spot welding studies of this research program.

To make these methodological differences explicit, Table 9 compares the four transformation kinetic approaches used in this review—the JMAK/Avrami equation, the Koistinen–Marburger equation, Scheil-additivity analysis of TTT data, and the Ginzburg–Landau phase-field model—in terms of their cooling-path capability, stress coupling, microstructure-prediction capability, computational cost, main limitations, and applicable scenarios.

Table 9. Comparison of transformation kinetic modeling approaches for hot stamping.

Model	Isothermal/Non-Isothermal Capability	Stress Coupling	Microstructure (Variant Selection) Prediction	Computational Cost	Main Limitation	Applicable Scenario
JMAK/Avrami (Equation (3)) [27]	Isothermal natively; non-isothermal via Scheil additivity	No	No	Very low	Homogeneous-nucleation and isotropic-growth assumptions	Furnace-heating and austenitization design
Koistinen–Marburger (Equation (32))	Athermal; inherently non-isothermal	Indirect (empirical M_s shift only)	No	Very low	Empirical kKM; Martensite fraction vs. temperature dependence unresolved	Martensite fraction vs. temperature during in-die quenching
Scheil additivity on TTT data (Section 5.3.3)	Converts isothermal kinetics into continuous cooling	No	No	Low	Additivity breaks down for non-isokinetic transformation paths (e.g., bainite)	Critical-cooling-rate estimation from CCT data
Ginzburg–Landau phase field (Section 5.4)	Non-isothermal capable	Yes, via elastic-energy coupling	Yes (variant selection, morphology, and residual stress)	Very high	Interface mobility, gradient-energy, and Landau coefficients uncalibrated for boron steels	Mesosopic mechanism analysis; qualitative design guidance

5.4. Application of Phase-Field Simulation in Martensitic Transformation Research

Although classical transformation theories such as the Koistinen–Marburger equation describe the macroscopic kinetics of martensitic transformation, they offer limited insight into nucleation-site selection, variant growth competition, and self-organization behavior. As a meso-scale theoretical simulation tool, the phase-field method (PFM) [55,56] describes the competitive evolution of different phases/variants through continuous order-parameter fields, providing a unique theoretical perspective for understanding the microscopic physics of martensitic transformation. This section presents the theoretical framework of the phase-field model, analyzes the microscopic mechanisms it reveals, and discusses its theoretical implications for quenching-process design in hot stamping.

5.4.1. Theoretical Framework of the Phase-Field Model

The phase-field method is founded on Ginzburg–Landau phase-transition theory [57]; its core idea is to describe the spatial distribution and evolution of different phases or variants through continuous order-parameter fields $\eta(r, t)$. For martensitic transformation, the order parameters η_i ($i = 1, 2, \dots, N$) characterize the volume fraction of the i -th martensitic variant at position r . For the FCC→BCT martensitic transformation in B1500HS, 24 variants are adopted to represent the crystallographically equivalent martensitic variants, corresponding to the 12 orientation pairs of the Kurdjumov–Sachs (K–S) orientation relationship [58] combined with the two signs of the Bain strain, following the reported phase-field convention [59].

The total free-energy functional of the system consists of the chemical free energy, the gradient energy, and the elastic energy (Equation (35)) [55]:

$$\mathcal{F} = \int_V [f_{chem}(\eta_i) + f_{grad}(\eta_i) + f_{el}(\varepsilon, \eta_i)] dV \quad (35)$$

The chemical free-energy density f_{chem} is described by a Landau 2–4 double-well polynomial (Equation (36)), which drives the system toward the martensitic or austenitic stable states:

$$f_{chem} = \sum_{i=1}^N A(T) \eta_i^2 (1 - \eta_i)^2 + \sum_{i < j} B_{ij} \eta_i^2 \eta_j^2 \quad (36)$$

where $A(T)$ is the temperature-dependent height of the chemical energy barrier and B_{ij} is the inter-variant interaction coefficient. The gradient-energy density f_{grad} penalizes abrupt spatial changes of the order parameters at phase interfaces (Equation (37)):

$$f_{grad} = \sum_{i=1}^N \frac{\kappa_i}{2} |\nabla \eta_i|^2 \quad (37)$$

The elastic-energy density f_{el} is based on Khachaturyan's microelasticity theory [60] (Equation (38)) and accounts for the coupling between the transformation strain and external constraints:

$$f_{el} = \frac{1}{2} C_{ijkl} (\varepsilon_{ij} - \varepsilon_{ij}^0(\eta_i)) (\varepsilon_{kl} - \varepsilon_{kl}^0(\eta_i)) \quad (38)$$

where $\varepsilon_{ij}^0(\eta_i)$ is the transformation strain tensor coupled to the order parameters, determined from the characteristic Bain strains of the 24 K–S variants through crystal rotation. The elastic energy is the principal driving force for martensitic variant selection and self-accommodating microstructural arrangement [59].

The temporal evolution of the order parameters follows the time-dependent Ginzburg–Landau (TDGL) equation (Equation (39)):

$$\frac{\partial \eta_i}{\partial t} = -L_i \cdot \frac{\delta \mathcal{F}}{\delta \eta_i} \quad (39)$$

where L_i is the interface mobility and $\delta \mathcal{F} / \delta \eta_i$ is the variational derivative of the total free-energy functional with respect to the order parameter. In addition, the order parameters satisfy the physical constraint $\eta_0 + \sum_{i=1}^N \eta_i = 1$ ($0 \leq \eta_0, \eta_i \leq 1$), with η_0 denoting the austenite fraction and η_i the fraction of the i -th martensitic variant. The austenitic state corresponds to $\eta_0 = 1$ and $\sum \eta_i = 0$, whereas the fully martensitic state corresponds to $\eta_0 = 0$ and $\sum \eta_i = 1$. This volume-fraction normalization remains valid for self-accommodating microstructures in which several variants are simultaneously activated.

For B1500HS (carbon content of about 0.23%), the low-carbon character renders the martensite nearly body-centered cubic ($c/a \approx 1.01$), with weak Bain strain anisotropy, which favors the simultaneous activation of most of the 24 variants and the formation of

a fine lath-packet microstructure—the microstructural origin of the excellent strength–toughness combination of low-carbon lath martensite.

5.4.2. Microscopic Mechanisms Revealed by the Phase-Field Model

Nucleation-site selection and self-accommodating variant arrangement. Phase-field theory predicts that martensite nucleates preferentially at grain boundaries and triple junctions. The physical essence is that the elastic-anisotropy mismatch between adjacent grains at the boundary causes stress concentration, which effectively lowers the chemical barrier to nucleation through the elastic-energy coupling term. As an order-of-magnitude estimate, the equivalent stress at grain boundaries may reach roughly 1.5–2.5 times that within the grain interior, a level qualitatively consistent with microstructural observations of quenched boron steels such as 22MnB5 and B1500HS in the hot-stamping literature. The activation of the 24 variants under the K–S orientation relationship follows the principle of elastic-energy minimization: neighboring variants form specific variant pairs to reduce the long-range elastic strain energy, with six variants on the same $\{111\}_{fcc}$ close-packed plane forming a packet and three variants within the same Bain correspondence forming parallel laths. In B1500HS, since c/a is close to unity, the elastic-energy driving effect is relatively weak and most variants can be activated simultaneously, forming a fine interwoven lath-packet microstructure.

Regulation of microstructural morphology by the cooling rate. The phase-field model reveals the physical mechanism by which the cooling rate regulates the martensitic microstructure. Under rapid cooling ($>50\text{ }^{\circ}\text{C}\cdot\text{s}^{-1}$), the large undercooling and strong driving force activate multiple variants simultaneously, producing small inter-lath spacings (about $0.1\text{--}0.3\text{ }\mu\text{m}$) that favor toughness, consistent with the reported die-quenched microstructures of B1500HS. At intermediate cooling rates ($20\text{--}50\text{ }^{\circ}\text{C}\cdot\text{s}^{-1}$), variant selection effects intensify, and a few dominant variants grow preferentially, producing coarse packets. When the cooling rate falls below the critical value (about $27\text{ }^{\circ}\text{C}\cdot\text{s}^{-1}$ for B1500HS; cf. the CCT diagram in Figure 13, Section 5.2), diffusional transformations begin to compete and the order-parameter evolution exhibits multiphase coexistence. Notably, the cooling-rate–microstructure relationship predicted by the phase-field model provides a mesoscopic physical interpretation of the Koistinen–Marburger equation (Section 5.3): the empirical constant k therein essentially reflects the combined outcome of variant competitive growth and autocatalytic effects.

5.4.3. Theoretical Implications for the Hot-Stamping Quenching Process

Mesoscopic interpretation of the critical cooling rate. The phase-field model reveals the physical essence of the critical cooling rate V_k at the meso-scale: V_k depends not only on the kinetic competition at the CCT-curve nose but also on whether martensitic variants can complete sufficient nucleation and growth before diffusional transformation initiates. When the cooling rate is high enough, a large number of variants are activated, and the martensite plates interweave and fill the austenite grains before diffusional transformation begins; conversely, diffusional transformation occurs preferentially at grain boundaries, forming soft-phase films that hinder the through-growth of martensite plates. Within this framework, the meso-scale trend corresponds to a critical cooling rate in the order of $25\text{--}30\text{ }^{\circ}\text{C}\cdot\text{s}^{-1}$ for B1500HS-class steels; because the phase-field parameters have not yet been calibrated specifically for B1500HS (Section 6.5), this range should be read as an order-of-magnitude estimate, and it is consistent with the experimentally reported value of about $27\text{ }^{\circ}\text{C}\cdot\text{s}^{-1}$ (Sections 1 and 5.2).

Theoretical constraints for die cooling-system design. The phase-field model provides micro-scale theoretical constraints for the computational fluid dynamics (CFD)-based optimization of die cooling systems in future die-design studies: (a) the cooling-

channel layout must ensure that the local cooling rate everywhere exceeds the critical value to guarantee full-section martensitization; (b) the turbulent design ($Re > 4000$) must maintain the die-surface temperature below the M_s point; and (c) the temperature gradient should be controlled within a reasonable range to avoid non-uniform variant selection and microstructural anisotropy caused by local undercooling differences.

In addition, the cross-scale linkage between the phase-field model and the macroscopic CCT curve can be realized by statistical averaging over a representative volume element (RVE): $\langle \eta_i \rangle = \frac{1}{V_{RVE}} \int \sum_i \eta_i^2(r) dV$. This mapping enables micro-scale mechanism predictions to be incorporated into the macroscopic thermo-mechanical–metallurgical coupled simulation framework.

6. Heat Transfer Theory and Coupling Mechanisms

Hot stamping involves complex heat transfer problems: during the transfer of the blank from the furnace to the die, heat is exchanged with the air by convection and radiation; during press forming, conductive heat transfer occurs at the blank–die contact interface; and the die cooling system removes heat by convection. Accurately describing these heat transfer processes is essential to predicting the temperature-field distribution, controlling the transformation progress, and ensuring product quality. Figure 16 illustrates the three basic modes of heat transfer.

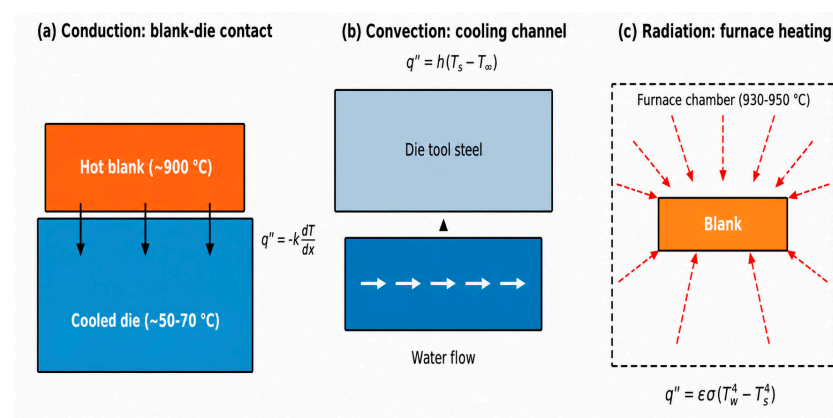


Figure 16. Three basic modes of heat transfer. Adapted from Ref. [34]. Black arrows indicate the heat-flow direction; white arrows indicate the cooling-water flow; red dashed arrows indicate radiative heat transfer from the furnace walls to the blank.

6.1. Heat Conduction Mechanism

Heat conduction is the transfer of heat from high-temperature to low-temperature regions through the interaction of microscopic particles (molecules, atoms, and electrons) within a material. In hot stamping, conduction occurs mainly at the blank–die contact interface and is the dominant heat transfer mechanism during in-die quenching [61]; the thermal conductivity of the die material directly affects the blank cooling rate and the temperature-distribution uniformity. The theoretical calculation methods are as follows.

The three classical formulations used in hot-stamping heat transfer analysis—Fourier’s law, one-dimensional steady-state conduction, and transient conduction—are presented together in the following, with their sources and applicability limits.

The fundamental law of heat conduction is Fourier’s law (Equation (40)) [34]:

$$q = -k \cdot \nabla T \quad (40)$$

where q is the heat flux density (W/m^2); k is the thermal conductivity ($\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$); and ∇T is the temperature gradient ($\text{K} \cdot \text{m}^{-1}$).

For die contact heat transfer, a one-dimensional steady-state model can be used for estimation (Equation (41)) [34]; the interfacial heat transfer across the blank–die contact interface is illustrated schematically in Figure 17:

$$Q_{cond} = \frac{k \cdot A_{ht} \cdot \Delta T}{d} \quad (41)$$

where Q_{cond} is the total heat conduction rate (W), A_{ht} is the heat transfer area (m²), d is the material thickness (m), and ΔT is the temperature difference (°C).

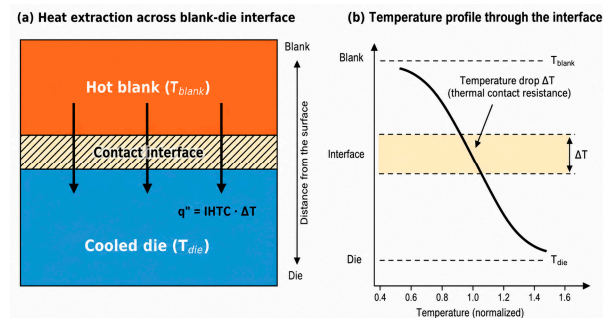


Figure 17. Schematic diagram of interfacial heat transfer across the blank–die contact interface: (a) heat extraction across the interface governed by $q'' = \text{IHTC} \cdot \Delta T$; (b) temperature profile through the interface, showing the temperature drop ΔT caused by the thermal contact resistance. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors; panel (b) uses a normalized temperature scale for schematic illustration. Reconstructed prompt: “Create a two-panel schematic of interfacial heat transfer across the blank–die contact interface in hot stamping: (a) heat extraction from a hot blank (T_{blank}) across a contact interface into a cooled die (T_{die}), with downward heat-flow arrows and the relation $q'' = \text{IHTC} \cdot \Delta T$; (b) the temperature profile through the interface, showing the temperature drop ΔT caused by the thermal contact resistance.”

Since the temperature field in hot stamping varies with time, a transient formulation is required (Equation (42)) [34]:

$$\rho_s \cdot c_p \cdot \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) \quad (42)$$

where ρ_s is the material density (kg·m^{−3}), c_p is the specific heat capacity (J·kg^{−1}·K^{−1}), and t is the time (s).

6.2. Heat Convection Mechanism

Heat convection is the transfer of heat by the macroscopic motion of fluid particles. In hot stamping, convective heat transfer occurs mainly inside the die cooling channels (forced convection) and inside the heating furnace (natural convection). The convective heat transfer coefficient is directly related to the fluid velocity, temperature gradient, and contact area. The theoretical calculation methods are as follows.

Convective heat transfer is treated in three classical steps: Newton’s law of cooling, the forced-convection coefficient, and the natural-convection coefficient.

The basic equation of convective heat transfer is Newton’s law of cooling (Equation (43)) [34]:

$$Q_{conv} = h \cdot A_{ht} \cdot \Delta T \quad (43)$$

where Q_{conv} is the convective heat transfer rate (W), h is the convective heat transfer coefficient (W·m^{−2}·K^{−1}), A_{ht} is the heat transfer area (m²), and ΔT is the temperature difference (K).

For turbulent forced convection, the Dittus–Boelter correlation can be used (Equation (44)) [34]:

$$h = 0.023 \cdot \frac{k}{D} \cdot Re^{0.8} \cdot Pr^n \quad (44)$$

where $Re = \rho v D / \mu$ is the Reynolds number, $Pr = \mu c_p / k$ is the Prandtl number, and the exponent n is 0.4 for the heating and 0.3 for the cooling of the fluid.

The natural-convection heat transfer coefficient can be approximated by Equation (45) [34]:

$$h = 1.42 \cdot C_{nc} \cdot \left(\frac{\Delta T}{L_c} \right)^{0.25} \quad (45)$$

where C_{nc} is a shape factor ($C_{nc} = 1$ for an isothermal vertical plate) and L_c is the characteristic length (m); the coefficient $1.42 \text{ W} \cdot \text{m}^{-7/4} \cdot \text{K}^{-5/4}$ is the value of the simplified correlation for laminar natural convection of air at atmospheric pressure along an isothermal vertical plate [34].

6.3. Thermal Radiation Mechanism

Thermal radiation is the transfer of energy by electromagnetic waves and is an important heat exchange mechanism between the high-temperature blank and its surroundings during hot stamping. Because the radiative heat transfer rate scales with the fourth power of the surface temperature, radiative heat loss dominates at high temperatures (after the blank leaves the furnace). The theoretical calculation methods are as follows.

Radiative exchange is described first by the Stefan–Boltzmann law and then by the net radiative heat flux between two surfaces.

The total radiative power of a blackbody is proportional to the fourth power of its absolute temperature (Equation (46)) [34]:

$$P = \varepsilon \cdot \sigma_{SB} \cdot A_{ht} \cdot T^4 \quad (46)$$

where P is the radiative power (W), ε is the surface emissivity ($0 \leq \varepsilon \leq 1$), σ_{SB} is the Stefan–Boltzmann constant ($5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$), and A_{ht} is the radiating surface area (m^2).

The net radiative heat flux between two surfaces is given by Equation (47) [34]:

$$Q_{net} = \varepsilon \sigma_{SB} A_{ht} (T_1^4 - T_2^4) \quad (47)$$

where A_{ht} is the heat transfer area (m^2), and T_1 and T_2 are the temperatures of the hot and cold surfaces (K), respectively.

6.4. Thermo-Mechanical Coupling Model

Thermo-mechanical coupling is the core theoretical issue of the hot-stamping process [62] and involves the strong interaction between the temperature and stress fields: temperature changes cause thermal expansion/contraction and generate thermal stresses; plastic-deformation work converts into heat and affects the temperature field; and the stress state influences the transformation behavior and the thermophysical properties of the material. This interaction requires the temperature and stress fields to be solved simultaneously [63]. Figure 18 schematically illustrates the thermo-mechanical coupling in hot stamping.

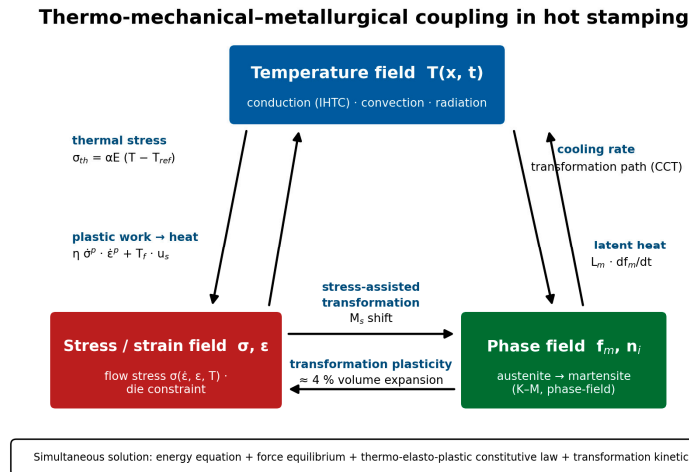


Figure 18. Schematic diagram of thermo-mechanical coupling in hot stamping. Original schematic drafted by the authors with AI assistance (Moonshot AI, 2026) and then redrawn, verified, and finalized by the authors. Reconstructed prompt: “Create a triangular coupling diagram for hot stamping with three boxes — Temperature field $T(x, t)$, Stress/strain field σ, ϵ , and Phase field f_m, n_i — connected by bidirectional arrows labeled thermal stress, plastic work $\rightarrow$ heat, cooling rate (transformation path, CCT), latent heat, stress-assisted transformation (M_s shift) and transformation plasticity ($\approx 4\%$ volume expansion), with a bottom bar reading: Simultaneous solution: energy equation + force equilibrium + thermo-elasto-plastic constitutive law + transformation kinetics.”

6.4.1. Influence of Deformation on the Temperature Field

Plastic-deformation work is released as heat (Equation (48)), constituting an internal heat source in the temperature field [64]; frictional heating at the die–blank interface also contributes. The volumetric heat-generation rate in the workpiece is:

$$\dot{q} = \eta \cdot \sigma : \dot{\epsilon}^p \quad (48)$$

where η is the Taylor–Quinney coefficient (typically 0.9–0.95), representing the fraction of plastic work converted into heat.

Frictional heat generation at the contact surface contributes to the temperature field (Equation (49)) [12]:

$$q_f = \tau_f \cdot u_s \quad (49)$$

where τ_f is the frictional shear stress (Pa) and u_s is the relative sliding velocity ($\text{m} \cdot \text{s}^{-1}$).

6.4.2. Influence of the Temperature Field on Deformation

The temperature field alters the deformation behavior through its influence on the flow stress (Equation (50)); the flow stress is a function of strain, strain rate, and temperature:

$$\sigma = f(\epsilon, \dot{\epsilon}, T) \quad (50)$$

6.4.3. Governing Equations of Thermo-Mechanical Coupling

Energy equation. The energy equation describing heat transfer and conversion is Equation (51) [12]:

$$\rho_s c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \eta(\sigma : \dot{\epsilon}^p) + Q_{other} \quad (51)$$

where ρ_s is the material density ($\text{kg} \cdot \text{m}^{-3}$), c_p is the specific heat capacity ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$), T is the temperature (K), t is the time (s), k is the thermal conductivity ($\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$), η is

the Taylor–Quinney coefficient (typically 0.9–0.95), σ is the Cauchy stress tensor (Pa), $\dot{\varepsilon}^p$ is the plastic strain-rate tensor (s^{-1}), and Q_{other} represents other internal heat sources, such as the latent heat of phase transformation (W/m^3) [6].

Force equilibrium equation. Under the quasi-static assumption, the force equilibrium equation is Equation (52):

$$\nabla \cdot \sigma + b = 0 \quad (52)$$

where $\nabla \cdot \sigma$ is the divergence of the stress tensor ($N \cdot m^{-3}$) and b is the body-force vector (e.g., gravity) ($N \cdot m^{-3}$).

Thermo-elastic–plastic constitutive relation. The total strain is decomposed into elastic, plastic, and thermal parts:

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^{th} \quad (53)$$

and the thermal strain is calculated by Equation (54):

$$\varepsilon^{th} = \alpha_{th}(T - T_{ref})I \quad (54)$$

where α_{th} is the coefficient of thermal expansion (K^{-1}), T_{ref} is the reference temperature (K), and I is the identity tensor.

Plastic-flow closure. The plastic part is closed by the temperature-dependent von Mises yield function $f(\sigma, \varepsilon^p, T) = \sqrt{3s:s/2} - \sigma_y(T, \varepsilon^p) \leq 0$ and the associated flow rule $d\varepsilon^p = d\lambda(\partial f / \partial \sigma)$, where s is the deviatoric stress tensor and $d\lambda$ is the plastic multiplier. These relations complete the thermo-elastic–plastic constitutive description in Equations (53) and (54).

In summary, thermo-mechanical coupling in hot stamping is a complex process involving the strong coupling of the temperature, stress, and phase fields. By establishing the above mathematical models and solving them numerically, the physical mechanisms of hot stamping can be understood in depth, providing theoretical guidance for process-parameter optimization and die design.

6.5. Limitations and Future Directions

The framework presented in this paper is theoretical in nature, and several limitations should be acknowledged. First, the constitutive parameters adopted for B1500HS are taken from a single literature source; although they have been cross-validated against the related grades 22MnB5 and BR1500HS, independent experimental verification under non-isothermal, industrially relevant conditions is still required. Second, the multi-scale simulation evidence cited in Section 4.4 is drawn from published studies and order-of-magnitude estimates rather than from a dedicated, fully coupled simulation campaign, and the quantitative transfer of parameters across the MD–DDD–CPFE–continuum hierarchy remains an open methodological challenge. Third, the phase-field and thermo-mechanical coupling models discussed in Sections 5.4 and 6.4 have not yet been validated against instrumented industrial hot-stamping trials, and their predictive accuracy under production scatter (e.g., furnace temperature non-uniformity of ± 10 – 20 °C) remains to be quantified.

Fourth, the phase-field parameters adopted in Section 5.4—including interface mobility, the gradient-energy coefficient, and the Landau polynomial coefficients—are taken from published studies on analogous ferrous systems and have not been calibrated specifically for B1500HS; quantitative prediction of martensite morphology therefore awaits dedicated experimental calibration. Fifth, the interfacial heat transfer coefficient (IHTC) at the blank–die interface is treated through nominal engineering ranges, whereas in practice, it depends strongly on contact pressure, surface coating, and temperature, introducing uncertainty into the cooling-system design equations.

Sixth, the framework is validated primarily for the boron steels B1500HS, 22MnB5, and BR1500HS; extrapolation to other press-hardening grades or coating systems (e.g., Zn-based coatings and higher-Si/Cr grades) requires re-validation. Seventh, phenomena such as oxidation and decarburization during heating and hydrogen embrittlement of the quenched microstructure [17] are outside the scope of the present theoretical treatment, although they can affect in-service performance. Finally, because this is a theoretically oriented review and reproduction permissions were not sought, no original experimental micrographs of the investigated grades are presented; the martensite morphologies are instead illustrated by original schematics redrawn from the cited literature (Figures 14 and 15), and acquiring original micrographs of B1500HS is identified as future work.

An analysis of the literature cited in this review reveals three discernible research trends over the last two decades. First, the center of gravity of constitutive modeling is shifting from phenomenological fitting toward physically constrained and data-driven hybrid approaches: whereas the Arrhenius-type models of the 2000s [9,10] were calibrated on single datasets, recent work increasingly cross-validates parameters across steels and embeds physical constraints into machine learning surrogates [29,30]. Second, transformation kinetic modeling is migrating from macroscopic volume-fraction equations toward mesoscopic field models capable of resolving variant selection and residual-stress evolution (Section 5.4), although a validated bridge between the two remains open. Third, die design is evolving from empirical rules of thumb toward CFD/FE-based optimization with explicit uniformity constraints, as reflected in the dual-constraint criterion ($Re > 4000$, $\xi T \leq 5\%$) proposed in Sections 3.2.3 and 8. On this basis, five unresolved issues are identified: (1) the lack of B1500HS-specific non-isothermal constitutive data; (2) the absence of stress-coupled non-isothermal transformation kinetics validated for press-hardening strain paths; (3) the unresolved transferability of MD/DDD parameters across the strain-rate gap of seven to eleven orders of magnitude to industrial forming rates; (4) the absence of phase-field models calibrated against instrumented industrial quenching trials; and (5) the limited physical interpretability of machine learning transformation models, which physics-informed approaches [30] have begun but not yet fully resolved.

Future work will proceed along three directions: (i) independent Gleeble calibration and uncertainty quantification of the Arrhenius constitutive model for B1500HS, including non-isothermal deformation paths; (ii) the development of stress-coupled, non-isothermal transformation kinetic models [7] and their experimental validation by dilatometry and quenching microscopy; and (iii) the integration of physics-informed machine learning with the present theoretical framework, so that data-driven surrogates can accelerate the coupled simulations while respecting the governing equations. Tailored-property hot stamping with heated and cooled tooling [65] will also be incorporated, for which strain-rate-sensitive constitutive descriptions of quenched boron steel have already been established [66]. These topics constitute the planned follow-up work of this research program.

7. Discussion

The four pillars developed above are not independent. The austenitizing window (Section 3.1) sets the initial condition for the flow-stress response (Section 4); the deformation heating and strain rate in turn shift the effective transformation kinetics (Section 5); and the die cooling system (Section 6) closes the loop by imposing the thermal boundary conditions that determine whether the critical cooling rate is met. The coupling diagram of Figure 18 schematically summarizes these interactions, with the temperature field and the martensite volume fraction as the two principal quantities exchanged between the pillars.

Compared with prior reviews (Table 1), the distinguishing contribution of this work is quantitative closure: every pillar terminates in either a calibrated parameter set (the Arrhenius constants for B1500HS) or a worked engineering example (heating-time integration, springback estimate, and the 380 kW cooling-channel design). This comes at a cost—the conclusions inherit the uncertainty of the underlying literature data, which for B1500HS is still limited to a small number of Gleeble campaigns.

For industrial practice, the most mature outcomes are the closed-form design relations of Sections 3 and 6, which can be applied directly within the stated parameter ranges. The least mature outcome is the phase-field description of variant selection (Section 5.4), whose parameters await calibration for B1500HS; we therefore recommend treating its predictions as qualitative guidance until dedicated calibration experiments become available.

8. Conclusions

This review has systematically established the theoretical foundations of the hot-stamping process for ultra-high-strength boron steel, covering process principles and parameter design, hot-plastic-deformation mechanisms, phase-transformation kinetics, and heat transfer with thermo-mechanical coupling. The main conclusions, together with a critical appraisal of their maturity and limitations, are as follows:

1. Constitutive modeling of B1500HS with multi-study cross-validation. A hyperbolic-sine Arrhenius constitutive model for the high-temperature deformation of B1500HS was established, with material constants $\alpha = 5.41 \times 10^{-3} \text{ MPa}^{-1}$, $n = 6.294$, $A \approx 6.2 \times 10^{10} \text{ s}^{-1}$ (back-calculated from the data of Li et al.'s study), and deformation activation energy $Q = 245.4 \text{ kJ} \cdot \text{mol}^{-1}$. The activation energy is consistent with a dislocation-climb-controlled thermally activated process and falls within the reasonable range of values ($197\text{--}343 \text{ kJ} \cdot \text{mol}^{-1}$) reported for the related boron steels 22MnB5, BR1500HS, and B1500HS by Zhou et al., Sun et al., and Li et al., verifying the universality and reliability of the Arrhenius model for boron steel hot deformation and providing a solid materials science basis for process parameter design. The multi-study cross-validation strategy—the triple consistency checking of activation energy, stress exponent, and flow-curve morphology—enhances the credibility and extrapolation applicability of the constitutive model relative to conventional single-dataset fitting. It should nevertheless be noted that this parameter set rests on a single Gleeble campaign for B1500HS and that its use in the $900\text{--}950 \text{ }^{\circ}\text{C}$ range remains moderate extrapolation; independent hot-compression verification under non-isothermal conditions is still required before the set can be regarded as definitive.
2. Thermo-mechanical–metallurgical coupled simulation framework. A finite-element theoretical framework coupling the temperature, stress, and phase fields is presented, and the simultaneous-solution strategy linking the heat-conduction governing equation, the elastic–plastic mechanical equilibrium equation, and the martensitic transformation kinetic equation (Koistinen–Marburger) is elaborated, with the latent-heat source term embedded into the thermo-mechanical model. The calibration methods for the interfacial heat transfer coefficient, friction coefficient, and transformation latent heat in thermo-mechanical coupling analyses are clarified, providing a complete theoretical toolchain for process simulation. The predictive value of this framework, however, is currently bounded by the nominal engineering ranges adopted for the interfacial heat transfer coefficient, the friction coefficient, and the transformation latent heat, all of which depend on contact pressure, coating, and temperature; instrumented industrial trials are needed to close this gap.
3. Springback mechanism and compensation strategy. From the thermo-mechanical–metallurgical coupling perspective, three dominant factors of springback in hot-

stamped parts were identified: transformation plasticity induced by quench martensitic transformation, elastic recovery after release of the die constraint, and non-uniform plastic deformation caused by temperature gradients. The applicability boundaries of displacement compensation, force compensation, and process compensation were compared, and the simulation–trial iterative die-surface correction strategy was identified as the most effective springback-control path in current engineering practice. These conclusions remain qualitative at the mechanism level, however: the analytical estimates capture only the elastic-recovery component, and quantitative springback prediction for a specific part still requires part-level thermo-mechanical simulation validated against physical die try-out. The analytical estimates of Equations (13)–(16) are mainly applicable to small-curvature regions; large-deformation parts require finite-element-based iterative die-surface compensation (Section 3.2.2).

4. Multi-scale simulation methodology. A multi-scale simulation framework is presented, spanning macroscopic continuum mechanics, mesoscopic crystal plasticity finite-element (CPFE) modeling, and microscopic molecular dynamics (MD) and discrete dislocation dynamics (DDD). The physical basis, spatiotemporal applicability range, and cross-scale information-transfer bottlenecks of each method are clarified, and the capability of CPFE-based representative-volume-element homogenization to map grain-scale heterogeneity into equivalent macroscopic constitutive laws is emphasized, providing a methodological reserve for the microstructure analyses of future studies. The quantitative transfer of parameters across the MD–DDD–CPFE hierarchy remains an open methodological challenge, however, and the evidence cited here is drawn from published studies and order-of-magnitude estimates rather than from a dedicated simulation campaign for B1500HS.
5. Martensitic transformation kinetics and phase-field theory. The Koistinen–Marburger equation quantitatively describes the relationship between the martensite volume fraction and undercooling; combined with the Ginzburg–Landau free-energy functional and Allen–Cahn-type kinetics, the phase-field method is shown to offer theoretical advantages in predicting martensite morphology, variant selection, and residual-stress distribution. The transformation driving force, interfacial-energy anisotropy, and elastic interaction energy are identified as the three key factors determining the martensitic microstructure, and a cross-scale control concept—regulating the quenching cooling rate (macro-scale) → altering the martensitic variant selection mode (meso-scale) → optimizing the residual-stress distribution and mechanical properties (macro-scale)—is proposed, breaking through the limitations of the conventional black-box “process–property” correlation. This control concept is presently at the proof-of-concept stage, however: the phase-field parameters have not yet been calibrated for B1500HS, so the variant selection predictions should be read as qualitative guidance until dedicated calibration experiments become available.
6. Design criteria for die cooling systems. Engineering design formulas for the cooling-channel Reynolds number (Re), volumetric flow rate (Q_v), and flow velocity (v) are established, and the influence of channel diameter, spacing, and layout on quenching uniformity is analyzed. A cooling-channel optimization design criterion with dual constraints—turbulent flow ($Re > 4000$) and a die-surface temperature-uniformity coefficient below 5%—is proposed, providing quantifiable theoretical guidance for engineering die design. The criterion is derived for steady production of panel-type geometries, however, and its applicability to complex parts with strongly varying section thicknesses remains to be verified by instrumented die trials.

Critically appraised, the conclusions differ in maturity. The analytical design relations for heating time, forming force, and die cooling (Section 3), together with the calibrated constitutive model, rest on well-established physical laws and multi-study cross-

validation and can be applied in engineering practice with the stated parameter ranges. By contrast, the phase-field description of martensitic variant selection and the multi-scale MD-DDD-CPFE hierarchy remain at the proof-of-concept level for hot stamping: their parameters are not yet calibrated for B1500HS, and their predictive value is currently qualitative rather than quantitative. In practical terms, the framework already provides die designers with closed-form sizing criteria (the 380 kW worked example) and provides process engineers with defensible windows for austenitizing temperature, transfer time, and quenching rate; its principal value for researchers lies in identifying exactly where experimental calibration efforts should be concentrated next.

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Nomenclature

Symbol	Meaning
Latin symbols	
A	Arrhenius material constant (s^{-1})
A_0, A''	pre-exponential constants of the low- and high-stress asymptotes in Equations (5) and (6): $A_0 = A \cdot \alpha^n$, $A'' = A/2^n$
A_{c3}	austenitization finish temperature ($^{\circ}C$)
A_{form}	effective forming area (mm^2)
AGB	material constant for grain-boundary sliding in Equation (26)
A_{creep}	creep material constant in Equation (27)
$A(T)$	temperature-dependent chemical energy-barrier height in Equation (36) ($J \cdot m^{-3}$)
A_{ht}	heat transfer (radiating) surface area in Equations (41), (43), (46) and (47) (m^2)
b	magnitude of the Burgers vector (m); body-force vector in Equation (52) ($N \cdot m^{-3}$)
B_{ij}	inter-variant interaction coefficient in Equation (36) ($J \cdot m^{-3}$)
C	strain-rate sensitivity coefficient in Equation (12) (–)
C_{nc}	shape factor for natural convection in Equation (45) (–)
C_{ijkl}	elastic stiffness tensor in Equation (38) (Pa)
cp	specific heat capacity ($J \cdot kg^{-1} \cdot K^{-1}$)
$cp_{steel/cp,w}$	specific heat capacity of the steel sheet/of the cooling water ($J \cdot kg^{-1} \cdot K^{-1}$)
D	cooling-channel diameter (m)
DGB	grain-boundary diffusion coefficient ($m^2 \cdot s^{-1}$)
DV	lattice (volume) diffusion coefficient ($m^2 \cdot s^{-1}$)
d	blank thickness (m); grain size in Section 4 (m; numerical values commonly quoted in μm)
$d\lambda$	plastic multiplier in the associated flow rule; Section 6.4.3 (–)
E	elastic modulus (GPa)
F	forming force (N)
f	transformed volume fraction (–); f_m , martensite volume fraction (–)

fchem/fgrad/fel	chemical, gradient and elastic free-energy densities in Equations (35)–(38) ($\text{J}\cdot\text{m}^{-3}$)
Fo	Fourier number (–)
fPK	Peach–Koehler force per unit length on a dislocation in Equation (30) ($\text{N}\cdot\text{m}^{-1}$)
h	convective heat transfer coefficient ($\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$)
he	elastic-core height (mm)
I	second moment of area (mm^4); identity tensor in Equation (54) (–)
K	correction factor for forming force (–)
k	thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$)
kB	Boltzmann constant ($1.38 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$)
kAvrami	Avrami rate constant
kKM	Koistinen–Marburger coefficient (K^{-1})
L	bending arc length (mm)
Lc	characteristic length (m)
Lm	latent heat of martensitic transformation ($\text{J}\cdot\text{kg}^{-1}$)
Li	interface mobility of order parameter i in Equation (39) ($\text{m}^3\cdot\text{J}^{-1}\cdot\text{s}^{-1}$)
M	bending moment per unit width ($\text{N}\cdot\text{mm}$); dislocation mobility in Equation (30) ($\text{m}^2\cdot\text{N}^{-1}\cdot\text{s}^{-1}$)
m	Schmid factor (–)
mh	strain-correction exponent (–)
Ms	martensite start temperature ($^{\circ}\text{C}$)
msteel	mass flow rate of the sheet ($\text{kg}\cdot\text{s}^{-1}$)
mM	mass flow rate of the sheet undergoing martensitic transformation in Equation (18) ($\text{kg}\cdot\text{s}^{-1}$)
mv	rate-sensitivity exponent of dislocation velocity (–)
N	number of martensitic variants in Equations (35)–(39) ($N = 24$ for the K–S orientation relationship)
n	stress exponent (–); nAvrami, Avrami exponent (–)
nc	creep stress exponent (–)
P	radiative power (W); Pform, forming power (kW)
Pr	Prandtl number (–)
Q	deformation/creep activation energy ($\text{kJ}\cdot\text{mol}^{-1}$) in Equations (4), (26) and (27)
Qcond	heat conduction rate (W) in Equation (41)
Qconv	convective heat transfer rate (W) in Equation (43)
Qr	required heat-removal rate (W) in Equation (18)
Qv	volumetric flow rate ($\text{m}^3\cdot\text{s}^{-1}$); Qh, hourly flow rate ($\text{m}^3\cdot\text{h}^{-1}$)
Qnet	net radiative heat transfer rate (W)
Qother	other internal heat sources in Equation (51), e.g., transformation latent heat ($\text{W}\cdot\text{m}^{-3}$)
q	heat flux density ($\text{W}\cdot\text{m}^{-2}$)
qf	frictional heat flux at the die–blank interface in Equation (49) ($\text{W}\cdot\text{m}^{-2}$)
R	gas constant ($8.314 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$)
Re	Reynolds number (–)
r	bending radius (mm)
s	half-thickness of the blank (m); deviatoric stress tensor in Section 6.4.3 (Pa)
T	temperature (K or $^{\circ}\text{C}$, as indicated); Tf, furnace; T0, initial; Ttarget, target
Taus/Tnose	austenitizing temperature/nose temperature of the transformation diagram in Equation (33) ($^{\circ}\text{C}$)
T [−]	area-averaged die-surface temperature in the uniformity coefficient ξT (K)
Tref	reference temperature (K)
Tmax/Tmin	maximum/minimum die-surface temperature in the uniformity coefficient ξT (K)
t	time (s)
ts	sheet thickness (mm)
us	relative sliding velocity ($\text{m}\cdot\text{s}^{-1}$)
v	flow velocity ($\text{m}\cdot\text{s}^{-1}$); dislocation velocity ($\text{m}\cdot\text{s}^{-1}$) in Section 4
v0	reference dislocation velocity ($\text{m}\cdot\text{s}^{-1}$)
Vk	critical cooling rate ($^{\circ}\text{C}\cdot\text{s}^{-1}$)
VRVE	volume of the representative volume element (m^3)
Z	Zener–Hollomon parameter (s^{-1})
Greek symbols	
α	stress-level parameter (MPa^{-1}); α_d , thermal diffusivity ($\text{m}^2\cdot\text{s}^{-1}$)
α_{th}	coefficient of thermal expansion (K^{-1})
β	$=n\alpha$, exponential-regime parameter (MPa^{-1})
$\gamma^{(\alpha)}/\dot{\gamma}_0$	shear rate of slip system α and reference shear rate in Equation (31) (s^{-1})
ΔT	temperature rise/difference (K)

$\Delta T_{\text{steel}}/\Delta T_w$	temperature drop of the sheet in Equation (18)/inlet–outlet temperature difference of the cooling water in Equations (19) and (20) (K)
$\Delta\theta$	springback angle (rad)
$\Delta\theta_{\text{elastic}}$	fully elastic springback angle in Equation (16) (rad)
δ	grain-boundary thickness (m)
ε	true strain (–); $\dot{\varepsilon}$, strain rate (s^{-1})
$\varepsilon_{\text{GB}}/\varepsilon_{\text{NH}}/\varepsilon_{\text{C}}$	strain rates of grain-boundary sliding, Nabarro–Herring creep and Coble creep in Equations (25), (28) and (29) (s^{-1})
ε_{eff}	effective emissivity (–)
$\dot{\varepsilon}_{\text{p}}$	plastic strain rate (s^{-1})
$\varepsilon_{ij}^0(\eta_i)$	transformation (Bain) strain tensor of variant i in Equation (38) (–)
$\eta_{\text{rad}}/\eta_{\text{conv}}$	radiative/convective heating efficiency (–)
η	Taylor–Quinney coefficient (–)
η_{form}	fraction of forming work converted into heat (–)
η_0	order parameter of the austenite phase (–)
η_i	phase-field order parameter of variant i (–)
η_{GB}	grain-boundary dislocation-transmission efficiency from DDD simulations (–)
θ_0	bending angle (rad)
κ_i	gradient-energy coefficient of variant i in Equation (37) ($\text{J}\cdot\text{m}^{-1}$)
φ/λ	angles between the slip-plane normal/the slip direction and the loading axis in Equation (22) ($^\circ$)
μ	dynamic viscosity ($\text{Pa}\cdot\text{s}$)
ξ	dislocation line (tangent) vector in Equation (30) (–)
ξT	temperature-uniformity coefficient (%)
ρ_{s}	steel density ($\text{kg}\cdot\text{m}^{-3}$)
ρ_{w}	cooling-water density ($\text{kg}\cdot\text{m}^{-3}$)
ρ_{m}	mobile dislocation density (m^{-2})
σ	flow/applied stress (MPa); σ_{y} , yield strength; σ_{s} , flow stress; σ_0 , reference flow stress
σ_{SB}	Stefan–Boltzmann constant ($5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$)
τ	shear stress (MPa)
τ_{f}	frictional shear stress (Pa)
τ_{r}	critical resolved shear stress (MPa)
τ_{p}	Peierls stress of screw dislocations; Section 4.4.1 (MPa)
$\tau^{(\alpha)}/\tau_{\text{c}}^{(\alpha)}$	resolved and critical resolved shear stress of slip system α in Equation (31) (MPa)
Ω	atomic volume (m^3)
$\mathcal{F}$	total free-energy functional (J)
Abbreviations	
B1500HS/22MnB5/BR1500HS	hot-stamping boron steel grades studied
CCT/TTT	continuous cooling/time–temperature transformation
CPFE	crystal plasticity finite-element method
DDD	discrete dislocation dynamics
IHTC	interfacial heat transfer coefficient
JMAK	Johnson–Mehl–Avrami–Kolmogorov kinetics
MD	molecular dynamics
TDGL	time-dependent Ginzburg–Landau equation
UHSS	ultra-high-strength steel

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