

Article

A Predictive Methodology for Temperature, Heat Generation and Transfer in Gigacycle Fatigue Testing

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Abstract: Recently, a trend in fatigue testing related to increasing excitation frequencies during experiments has been observed. This tendency is a product of both necessity and available technological capabilities. Regarding the last, advances in control and excitation systems made it possible to perform tests at impressive frequencies, beyond the tenths of kHz. Performing fatigue tests much faster is indeed very motivating, representing less time and money spent. On the other hand, such high testing frequencies create some challenges, such as the requirement of measurement systems capable of working with high sample rates and excessive heat generation on the testing samples. The last one is especially critical for fatigue once the mechanical properties, such as the elasticity modulus and yield strength, are highly dependent on the temperature. Therefore, being able to predict and control the sample temperature during fatigue testing is essential. The main goal of the present work is to provide a formulation for estimating the heat generation and specimen temperature during high frequency testing, namely in the ultra-high cycle fatigue (UHCF) regime. Several metallic alloys and specimen geometries were tested, and the model results were compared with experimental temperature measurements. The developed model was able to properly characterize the temperature trend over time. In addition, a script was developed and made publicly available.



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1. Introduction

The ultrasonic fatigue testing (often named gigacycle) was developed based on a need: knowing the fatigue behavior of a material after a few millions of cycles [1] (beyond 10^7 , up to 10^9 or 10^{10}). Indeed, this testing method, also called ultra-high cycle fatigue (UHCF) testing, made it possible to test a specimen for an impressive number of cycles in relatively short time periods. However, this technology is also associated with some challenges. First, because of the operating principle of UHCF, the specimen (and the entire system) must be designed to resonate at a given frequency. This means that it is not possible to have a specimen geometry suitable for all materials since the natural frequency relies on material properties.

In addition, due to the high testing frequencies (the most common testing frequency is 20 kHz, but there are systems working with 30 kHz and higher), the specimen is loaded with thousands of cycles per second, and the strain rates easily reach hundreds 1/s (it is common to find fatigue tests operating at strain rates of 500 1/s or more). This leads to two problems: strain rates directly affect the mechanical properties (yield strength, for example) and excessive heat generation. The second also leads to changes in mechanical properties, because these properties are often temperature dependent.

Temperature influence is so relevant in UHCF that some authors have experienced severe discrepancies on S–N (stress amplitude versus number of loading cycles) results based only on the testing condition [2]. In the referred study, B/M dual phase steel specimens that were subjected to longer interruption testing times presented superior fatigue performance, most probably because their temperatures were lower than their counterparts. The authors concluded that the heat generation and impact of ultrasonic fatigue testing could not be ignored for the analyzed material. An interesting study was developed by Celli et al. [3], where Ti6Al4V specimens were tested using different conditions for ultrasonic tests. Three conditions were tested: continuous excitation (without pauses) with and without compressed air cooling, and an intermittent condition without air cooling. The authors found temperature increases of more than 70 °C during tests and more than 200 °C when cracks were present. A good correlation with the literature was found regarding S–N curves, but major discrepancies between the results from the three different conditions were found. Specimen temperature also has a severe impact on the resonance frequency of Cr–Ni–steel specimens [4], since temperature changes mechanical properties such as elasticity modulus and density, as well as dimensions due to thermal expansion. The authors have observed an increase in surface roughness associated with a higher number of cycles. This phenomenon was related to localized plastic deformations in shear bands. This localized plastic deformation was also associated with increases in heat generation.

In order to have a reliable test output (for example, the S–N curve), it is important to measure the temperature during UHCF tests. Xue et al. [5] monitored UHCF tests using an infrared camera on some steel and a cast iron alloy. On the GS51 cast iron, testing temperatures above 150 °C were recorded at the testing section (center of the specimen). In this work, the authors were able to successfully observe a relation between the temperature evolution on specimen surfaces and the fatigue damage mechanism. Cast iron alloys are often associated with high damping ratios [6], and during the UHCF testing, a large portion of this dissipated energy is converted into heat. Another interesting study regarding heat generation in UHCF was performed by Yang et al. [7], where the effect of pre-load (higher pre-loads lead to higher mean stress) on temperature was studied. A trend was found where increasing the pre-load led to higher measured temperatures. The work encompassed both a numerical model and experimental temperature data. A quadratic relationship was found between the temperature increase and the applied pre-load.

Still focusing on the dissipated energy (but this time not directly related to heat generation and temperature increases), a very comprehensive study was developed for gigacycle fatigue on Gaussian specimens [8] (special geometries which leads to a high volume of material being loaded under the stress levels close to the maximum stress amplitude). This work presented an analytical formulation for energy dissipation based on specimen dimensions, stress level, and material properties. Both analytically and numerically computed dissipated energies were presented. The authors were able to design a specimen with a large risk volume (more than 10 times the typical volume of an UHCF specimen). An important conclusion could be drawn regarding such a large risk volume: since a large volume of material is subjected to large stresses, this specimen design is also associated with large dissipated energy (consequently, large amounts of generated heat).

Another research study regarding vibration and thermal losses was performed by Mihalec et al. [9]. This study was not performed on UHCF specimens, but on a beam. Despite the fact that the test was not ultrasonic fatigue, the operation principle is the same: specimens (beams, bars, or fatigue specimens) are excited on one of their natural frequencies, leading to vibrations and, consequently, heat generation due to damping effects. The authors measured the temperature of a beam using an infrared camera and compared it with analytical calculations. Interestingly, the authors found that, for their polymethyl methacrylate (PMMA) bar, only 30% of the dissipated energy was converted into heat. The authors found good correlation between the numerical and experimental results; areas

subjected to higher stress amplitudes in the numerical models also presented the highest temperature rises during the experimental procedure.

Concerning multiaxial ultrasonic fatigue tests, a very interesting specimen design was proposed by Costa et al. [10], where three specimen designs were developed for different multi-stress conditions. The authors found a good agreement between the computed areas with high stress amplitudes and experimental measurements of temperature. As an overview, they found that the proposed designs were suitable for multi-directional fatigue tests. However, some specimens presented crack nucleation in unexpected regions, which was attributed to surface defects associated with the machining process and parasite vibration modes (modes for which the natural frequency was close to the mode of interest). In order to perform fatigue tests on thin-walled parts (thickness of approximately 0.8 mm), Himmelbauer et al. propose a specimen geometry and testing methodology [11]. An axisymmetric geometry was proposed. The authors found some discrepancies between the designed specimen's natural frequency and the experimental ones, which were attributed to the material damping (which results in a damped natural frequency, smaller than the natural frequency). They also observed a significant reduction in the specimen's natural frequency due to fatigue damage (crack propagation). Moreover, with the proposed geometry, the authors were able to obtain a S–N curve. The crack nucleation occurred in the expected regions (regions of higher stress amplitudes in the numerical model). However, some scatter was found in the obtained data, which was attributed to the influence of machining marks on the specimen's surface.

Regarding dissipated energy, one of the most important material parameters is the loss factor (or other damping counterparts, such as damping ratio or logarithmic decrement). An extensive information regarding damping factors (both longitudinal and transversal ones) is presented by Ono [12]. Results for dozens of metallic materials are presented, encompassing pure iron, low carbon, high carbon, tool and stainless steels, several aluminum and nickel alloys, just to name a few. The author observed that in almost all materials there is a dependency between the attenuation (which is directly related to the material damping) and frequency. A study about the frequency effect on the loss factor for a structural steel was performed by Jung et al. [13]. Two measuring methods were applied and compared, one using a conventional laser vibrometer and the other using a laser beam reflection method. For the frequency range analyzed (100 to 900 Hz), no frequency effect was observed on the loss factor. As a conclusion, the authors found good agreement between the loss factor obtained by the two different methods, validating the proposed methodology.

A study about the influence of stress on the specific damping capacities of some iron and steel alloys was performed by Visnapuu et al. [14]. Very interesting results were found. Some materials presented nearly no influence of the stress levels on the damping parameters, such as the AISI 1045 steel, for example. On the other hand, alloys such as AISI 4140 steel presented damping variations of more than 10 times at high stress levels. Interesting results were found for cast iron as well, where grey cast iron presented large damping variations according to the stress levels, while ductile cast iron variations were much more subtle. They observed that damping ratios were strongly dependent on microstructure features, especially stress levels associated with intergranular boundaries. Moreover, the authors found that damping improved with rougher intergranular contact and with the amount of ferrite in the material.

A dependency between stress level and the damping factor was also observed by Lage et al. [15], this time during UHCF tests for the cooper specimen. The authors observed that, during the free damped vibration (exponential decay), the damping factor would be lower, if more cycles were used for measuring/computing the average damping value. In other words, when a higher number of cycles is used for computing this value (during the exponential decay), lower stress amplitudes will be present. The authors found decreases of more than 25% in the damping factor when utilizing a higher number of cycles (lower stresses) for calculations. It was suggested that this method could be used to monitor the integrity (health) of structures, where damage could be detected due to increases in the

measured damping. Another interesting study regarding damping and material damage due to fatigue was proposed by Guo et al. [16]. In this study, the authors searched for a relationship between the applied stress levels and temperature increases in the material. They observed that this phenomenon could be divided into three stages. The first, called the elastic stage, as the name implies, was characterized by the absence of plastic deformation (micro or macro), and resulted in constant dissipation up to this stress level. The second, referred to as inelastic internal friction, was characterized by a very subtle deviation from the linear portion of the material stress–strain curves. At this second stage, the dissipation increased linearly with the stress level (while in the present stage, it was constant). The third stage was characterized by severe increases in dissipated energy due to higher stress levels. An interesting conclusion was obtained regarding the stress level that characterized the beginning of this third stage, where this stress amplitude was also observed to be the fatigue limit of the material.

Objectives

Since excessive heat generation during UHCF tests is a common challenge, the main objective of the present work is to provide a mathematical formulation and a tool (numerical code) able to estimate the heat generation and specimen temperatures during tests. This tool can help save time and money on materials that present excessive temperature increases during tests, or, at least, warn the user regarding the difficulties of testing certain materials. Additionally, this tool may increase the accuracy of experimentally obtained S–N curves, improving the available information regarding temperature, which directly affects the fatigue behavior of materials.

2. Materials and Methods

The present section aims to provide information regarding both analytical and experimental procedures. Analytical formulations regarding the displacement and stress amplitudes, heat transfer and generation formulations, and determination of the damping parameters will be discussed. The experimental procedure for temperature, displacement, and strain measurements will also be detailed.

2.1. Ultra-High Cycle Fatigue Testing

A brief introduction regarding the UHCF testing will be discussed in this section. Figure 1 presents a standard layout for an ultrasonic fatigue testing equipment. The vibratory system is normally composed of four main parts, namely:

- Converter—typically a piezoelectric (or magnetostrictive) transducer, responsible for transforming the electrical signal into a displacement;
- Booster—this component is responsible for connecting (fixing) the vibration components to the rest of the system. Often, it is also responsible for amplifying the displacement amplitudes;
- Horn—its main function is to amplify the displacement amplitudes and to fix the specimen. There are systems where a single component performs the horn and booster functions;
- Specimen—the highest stress amplitudes are presented at the center of this component, and it is attached to the horn.

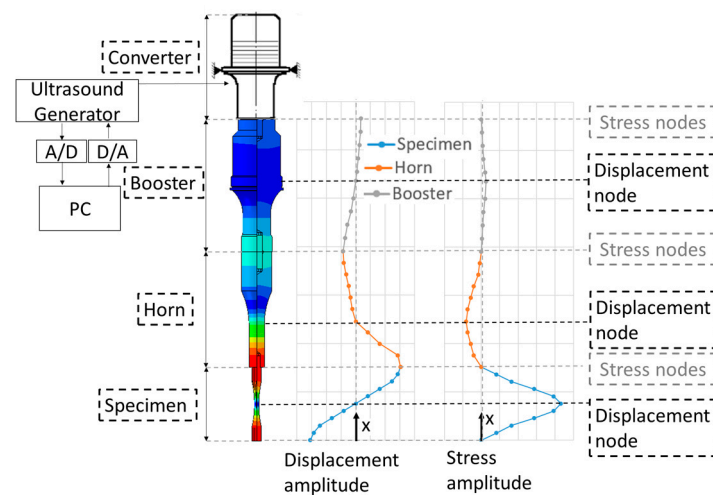


Figure 1. Ultrasonic testing machine layout, stress, and displacement amplitudes, adapted from [1].

The displacements and stress amplitudes along the entire ultrasonic system are also presented in Figure 1. A characteristic of this system is that all components (booster, horn, and specimen) must be designed to have a longitudinal natural frequency equal to the excitation frequency provided by the transducer, in the present case approximately 20 kHz.

For an hourglass specimen excited at its first longitudinal natural frequency, Bathias et al. [1] developed a one-dimensional analytical solution for the displacement, stress, and strain amplitudes along the specimen length. Consider the coordinate system located at the center of the specimen, $|x| \leq L_2$ represents the central section of the specimen and $L_2 < |x| < L$ (being $L = L_1 + L_2$) represents the constant transversal region (specimen ends), as shown in Figure 2.

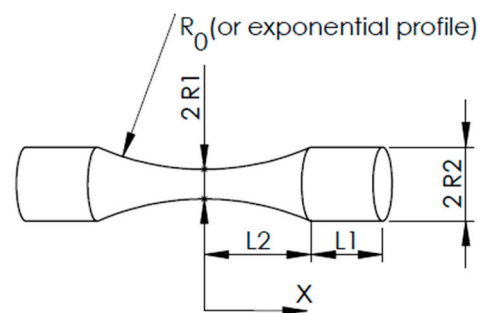


Figure 2. UHCF specimen coordinate system and main dimensions, adapted from [1].

For the central section, $|x| \leq L_2$, the analytical solution [1] for the displacement amplitudes along the x-coordinate can be written as

$$U(x) = A_0 \varphi(L_1, L_2) \frac{\sinh(\beta x)}{\cosh(\alpha x)}, \quad (1)$$

where A_0 is the displacement amplitude at the sample edge ($x = L$) and φ , β , and α are function of specimen's geometry and material, which can be written as

$$\varphi(L_1, L_2) = \frac{\cos(k_c L_1) \cosh(\alpha L_2)}{\cosh(\beta L_2)}, \quad (2)$$

$$\alpha = \frac{1}{L_2} \operatorname{arccosh}\left(\frac{R_1}{R_2}\right), \quad (3)$$

and

$$\beta = \sqrt{\alpha^2 - k_c^2}, \quad (4)$$

where

$$k_c = \frac{\omega}{c}, \quad (5)$$

being ω the natural frequency (which is also the same as the excitation frequency) and c the wave velocity. The stress amplitude at the central section is also known,

$$\sigma(x) = E_d A_0 \varphi(L_1, L_2) \frac{\beta \cosh(\beta x) \cosh(\alpha x) - \alpha \sinh(\beta x) \sinh(\alpha x)}{\cosh^2(\alpha x)}, \quad (6)$$

where E_d is the dynamic elastic modulus.

For the cylindrical section, $L_2 < |x| < L$, the displacement and stress amplitudes are, respectively,

$$U(x) = A_0 \cos(k_c (L - x)) \quad (7)$$

where

$$\sigma(x) = E_d k_c A_0 \sin(k_c (L - x)). \quad (8)$$

With the information regarding all displacement and stress in every position and time step, it is possible to develop an analytical heat generation model based on this information and on the material damping.

Often, due to excessive heat generation during tests, some machines work in intermittent testing mode. Short pulses (frequently smaller than one second) are performed, and during this time the samples are under cyclic loading (and, as a consequence, heating). After, the external excitation ceases, allowing the specimen to cool down for a certain period. For the present work, the pulse duration will be denoted as t_{on} , and the off period will be called t_{off} . It is important to emphasize that during the t_{off} the specimen is not standing still the entire time. Since there is no longer an external excitation, it is experiencing a damped-free vibration.

2.2. Analytical Heat Transfer and Generation Formulation

The equation of motion for a multi-degree of freedom (1 degree-of-freedom per element), can be written as

$$[M] \{\ddot{U}\} + [C] \{\dot{U}\} + [K] \{U\} = \{F\}, \quad (9)$$

where $\{U\}$ is the displacement vector, being $\{\dot{U}\}$ and $\{\ddot{U}\}$ its first- and second-time derivation. $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices, respectively, and $\{F\}$ is the external force. Regarding the three terms of the left-hand side (LHS) of the equation, the first (inertia) and third (stiffness) are conservative; therefore, they do not account for any losses over time. The damping term is non-conservative, and the damping force can be written as

$$\{F_d\} = [C] \{\dot{U}\}. \quad (10)$$

Since both the displacement and dissipative force are known, the dissipated energy can be calculated simply by integrating this force over the displacement. The dissipated energy can be written as a function of the elastic energy,

$$\{E_d\} = 2 \pi \eta \{E_e\}, \quad (11)$$

where η is the hysteretic damping factor and $\{E_e\}$ is the elastic energy. Figure 3a presents an example of the elastic (blue series) and dissipated (red series) energies. For this example, the damping ratio used was typical for high-carbon steels (0.012%), according to [6], who performed an extensive study on loss coefficients for several engineering alloys. It can be noticed in Figure 3a that the dissipated energy (red area) is barely visible. Figure 3b presents a zoomed view of the damping loop. It is important to notice that, compared with the elastic energy available (blue area), the dissipated energy per cycle is very small.

However, when thousands of cycles are performed in one cycle, the total dissipated energy can no longer be neglected.

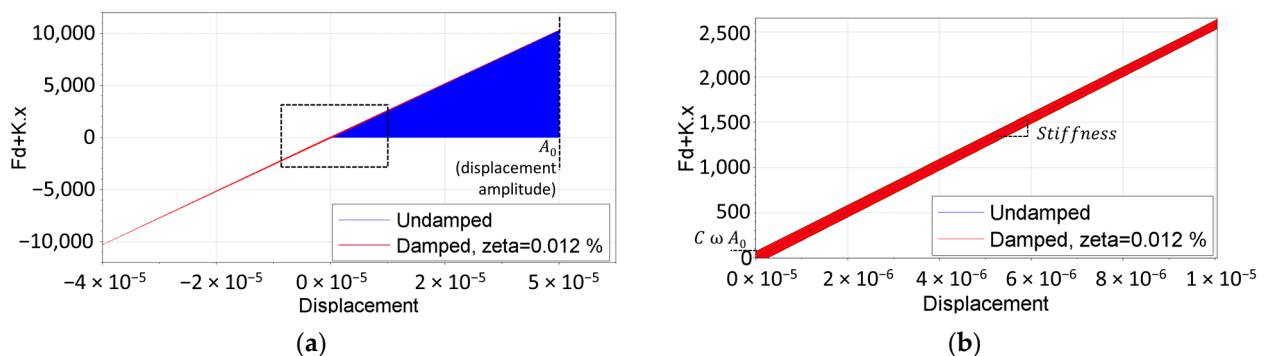


Figure 3. Undamped and damped displacements and energies: (a) Elastic energy and hysteretic cycle; (b) Zoomed in of damping loop (dissipated energy).

At the resonance, it is possible to use a relationship between hysteretic damping factor and the damping ratio, ζ [15],

$$\eta = 2 \zeta. \quad (12)$$

This was performed mainly because the measurement and calculation of the damping ratio are relatively easy using the present experimental setup.

Subdividing the specimen into smaller elements makes it relatively easy to calculate the heat transfer and the energy balance. The law of conservation of energy can be summed up as

$$\Delta E = E_{in} - E_{out}, \quad (13)$$

where ΔE is the energy variation for a given element, E_{in} is the energy added, and E_{out} is the energy output for the same element. For the present study, since there is no phase transition (the material remains a solid during the entire process), all the energy variation is converted to sensible heat,

$$\{\Delta E\} = [M] C_p \{\Delta T\}, \quad (14)$$

where C_p is the specific heat coefficient and $\{\Delta T\}$ is the temperature variation vector.

Figure 4 presents a schematic view of the heat transfer on a specimen. The specimen is subjected to convection, conduction (for the present case, a unidirectional conduction hypothesis was used), and internal heat generation. As in the case presented in Section 2.1, a symmetry condition was used at the specimen center.

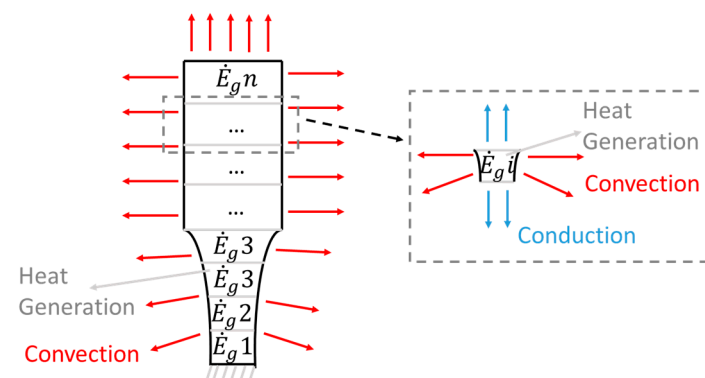


Figure 4. Heat transfer model for UHCF specimen.

For a given element i , Figure 5 shows an energy balance, taking into account the conduction across the adjacent elements, the convection with the external environment, and the heat generated inside this element (associated with the dissipated energy).

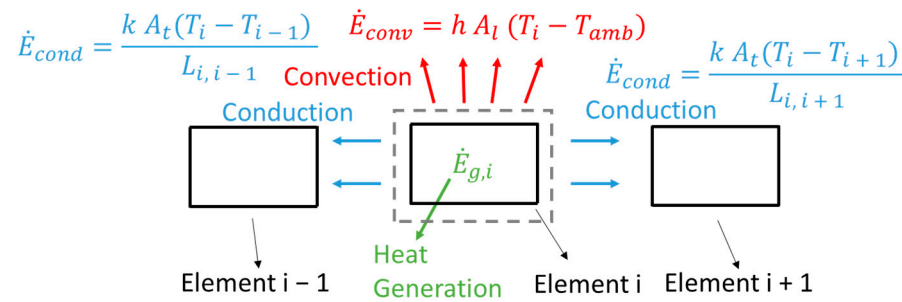


Figure 5. Energy balance for one element, where k is the thermal conductivity, h is the convection coefficient, L is the distance between the centers of two adjacent elements, A_t is the cross-section area, A_l is the surface area, and T_{amb} is the ambient temperature.

Therefore, the governing equation for heat transfer can be written as,

$$\{E_{cond}\} + \{E_{conv}\} + \{E_g\} = [M] Cp \{\Delta T\}, \quad (15)$$

where $\{E_{cond}\}$ is the conduction energy vector, $\{E_{conv}\}$ is the convection vector, and $\{E_g\}$ is the generated energy vector. At this point, the generated energy is still unknown. However, as stated previously in this section, the dissipated energy term is associated with material damping, and it can be said that this dissipated energy is mainly converted into heat, therefore

$$\{E_g\} = \{E_d\}. \quad (16)$$

Now, it is possible to calculate the temperature at any given time step for every element (x-coordinate). For a known temperature vector at a given time step (t), the temperature for the next time step ($t+1$) can be simply computed as

$$\{E_{cond}\} + \{E_{conv}\} + \{E_g\} = [M] Cp (\{\Delta T_{t+1}\} - \{\Delta T_t\}). \quad (17)$$

2.3. Generated Energy Approaches

The dissipated energy presented in Equation (11) is suitable for cases where the material damping is constant (and does not increase according to the stress levels). Moreover, if the damping of a material changes regarding the stress levels, and this relation is known, in the form of

$$\zeta = f(\sigma), \quad (18)$$

the same equation can be used, where the damping ratio will be changed according to the respective stress level, Equations (6) and (8), for that specimen location. For the present work, an approach considering constant damping for calculating the dissipated (and generated) energy will be referred to as Average Damping, or simply Approach 1. On this method, the average damping measured on a sample will be used along the entire specimen.

However, for cases where damping increases regarding stress levels, as previously mentioned in the works of Lage et al. [15] and Vinapuu et al. [14], and its dependency on the stress levels is unknown, the previous approach is not ideal. Based on this challenge, an alternative approach is used on the present work, where the damping ratio is calculated based on the average damping and a weighting according to the stress level (or coordinate) is applied. For this approach, since the damping ratio is a function of the stress, the damping will change along the specimen length (different stress levels according to the coordinate). This approach will be denoted Weighting Damping (or Approach 2). Both approaches are implemented in an algorithm, differing only regarding the heat generation formulation, and the comparison of the results with experimental data will be performed for these approaches in this research paper.

2.4. Experimental Analysis

The present section was developed in order to provide relevant information regarding all the measured physical properties and experimental procedures.

2.4.1. Specimens Geometry and Materials

For the present work, four materials were tested, namely: 316L stainless steel, 51CrV4 spring steel, S690 structural steel, and a nickel superalloy, Inconel 625. The ultrasonic fatigue tests were performed using a Shimadzu USF-2000 machine (Shimadzu, Kyoto, Japan). All tested materials were supplied from conventional manufacturing processes, except for Inconel 625, which was additively manufactured via directed energy deposition (whose manufacturing parameters can be seen in [17]). 316L specimens were manufactured from a bar (tested as supplied, without any heat treatment), while S690 samples' base material was a plate, and 51CrV4 ones were manufactured from railway leaf springs.

Regarding the 316L samples, two specimen geometries were tested, with a testing section diameter (D1) of 6 and 3 mm. The material properties can be seen in Table 1, while specimen dimensions and measured natural frequencies are presented in Table 2. The procedure for the determination of the elasticity modulus and density will be discussed in detail in Sections 2.4.2 and 2.4.3, while specific heat and thermal conductivity values were obtained from literature.

Table 1. Material properties for the studied UHCF specimens.

Material	Elasticity Modulus (MPa)	Density (kg/m ³)	Specific Heat (J/kg.K)	Thermal Conductivity (W/m.K)
316L Stainless Steel	185	7960	498 [18]	13 [18]
51CrV4 Steel	211	7840	460 [19]	48 [19]
S690 Steel	212	7830	460 [20]	19 [20]
Inconel 625 Nickel Superalloy	165	8560	429 [21]	9.8 [21]

Table 2. UHCF specimens' main dimensions and experimental natural frequencies.

Material	L1 (mm)	L2 (mm)	D2 (mm)	D1 (mm)	Natural Frequency (kHz)
316L Stainless Steel (D6)	25.8	20.0	10	6.0	19.94
316L Stainless Steel (D3)	10.4	17.0	10	3.0	20.11
51CrV4 Steel	13.8	23.0	10	4.0	19.89
S690 Steel	16.0	29.5	10	5.5	19.87
Inconel 625 Nickel Superalloy	11.4	21.5	10	4.0	19.79

Due to the intrinsic operation principle of the ultrasonic fatigue testing, the specimen must be designed to resonate at a certain frequency, which the machine is able to operate/excite, for the present case $20,000 \pm 500$ Hz. It is important to emphasize that this natural frequency is dependent on both material properties (elasticity modulus and density) and specimen geometry, and small variations can lead to different frequency values. Even specimens manufactured using the same parameters and from the same base material can present slight changes in their respective natural frequencies. In order to differentiate both specimens' geometries regarding the 316L material, the specimens with D1 = 6 mm and D1 = 3 mm will be referred to as D6 and D3, respectively.

2.4.2. Elasticity Modulus Measurements

The elasticity modulus is extremely important for the present analysis. Both natural frequencies and stress amplitudes during tests heavily rely on this parameter. If the test chosen for elasticity modulus determination was associated with large uncertainties, the specimen geometry selected could lead to natural frequencies outside the machine's oper-

ation range. As previously explained, the specimen natural frequency (first longitudinal mode of vibration) must be inside a narrow operating window of $20,000 \text{ Hz} \pm 2.5\%$.

Taking this into account, the method chosen for dynamic Young's modulus determination was the impulse excitation of vibration. This method consists of loading a specimen with an impulse and measuring its response. If a proper impulse is applied, it will excite the specimen on several frequencies. On the other hand, a specimen under free vibration is prone to vibrate at its natural frequencies.

The procedure was based on ASTM E1876 [22], and an overview of the experimental apparatus can be seen in Figure 6. The test process is simple and straightforward: a cylindrical specimen is suspended by rubber bands positioned as shown in Figure 6 and impacted on its central length. The support position was chosen in order to reduce the fixturing influence on the specimen's natural frequencies, and the position chosen was the vibration nodes for the first natural frequency (bending), which is also presented in Figure 6. The natural frequency of a single degree of freedom system can be written as

$$f_n = \sqrt{K/M}. \quad (19)$$

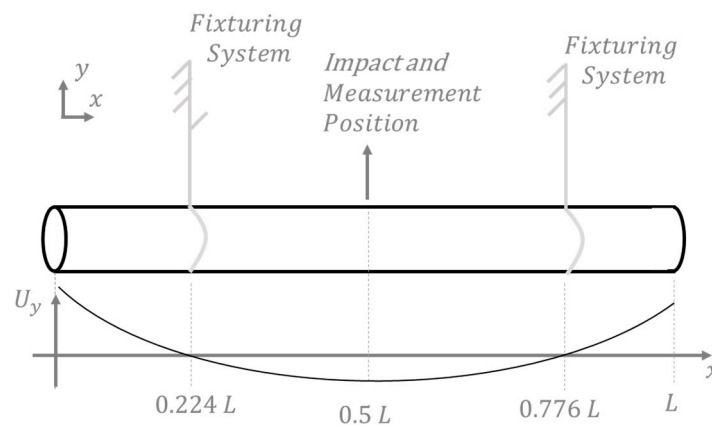


Figure 6. Experimental apparatus for elasticity modulus measurement.

The stiffness is proportional to the elasticity modulus of the material,

$$K \propto E, \quad (20)$$

so, a relationship between the natural frequency and the elasticity modulus can be written as

$$f_n \propto \sqrt{E}. \quad (21)$$

If the natural frequency is measured and the specimen geometry and material density are known, the elasticity modulus can be easily calculated.

For the present work, all specimens used for the tests were cylindrical, being machined with a known diameter and length. Later, these cylindrical specimens were machined again and converted into UHCF specimens. The specimens were impacted by a metallic object, and a microphone was used to measure the specimen output. Even though several frequencies were recorded, just the first one was used to determine the elasticity modulus. The experimental measurements were post-processed, and a fast Fourier transform (FFT) was used to identify the first natural frequency. A finite element method (FEM)-based modal analysis was performed, and the elasticity modulus of the material was determined. This was performed in two steps. A first elasticity modulus guess (E_i) (based on the literature) was used and the corresponding numerical natural frequency was obtained (f_i). After that, the elasticity modulus was changed (E_f) in order to match the numerical

frequency with the one measured experimentally (f_f), according to the following relation, based on Equation (21),

$$\frac{f_i}{f_f} = \sqrt{\frac{E_i}{E_f}}. \quad (22)$$

This procedure was performed for all materials, and their elasticity modulus was determined. The elasticity modulus was used for determining specimen geometry and stresses during UHCF tests.

2.4.3. Density Measurements

Density measurements based on the Archimedes' principle were performed for all four materials. Five measurements were done for each material, and the density was calculated based on the average of the measured values. The experimental procedure was performed using an A&D FR-200 MKII scale. Samples were measured under two conditions, namely "wet" and "dry", corresponding to measurements with the specimen submerged and not submerged in a fluid (whose density is known). The specimen's density can be calculated as

$$\rho = \frac{\rho_f W}{W - W_a}, \quad (23)$$

where ρ_f is the fluid density, W is the "dry" weight, and W_a is the apparent immersed weight ("wet").

2.4.4. Displacement and Strain Measurements

Due to the intrinsic operating principle of the ultrasonic fatigue, it is not possible to use a load cell to calculate the stresses that the specimen is subjected to. Therefore, the most common approach is to compute the stresses based on displacement measurements, using Equations (6) and (8). The displacements were measured for all samples and for each of the stress/displacement levels tested.

For measuring the displacement, an eddy-current sensor was used. This kind of sensor is especially useful for measurements on UHCF specimens, mainly because it is a non-contact sensor and suitable for high-speed measurements. A Lion Precision ECL101 sensor (Lion Precision, Minneapolis, MN, USA) was used. A sampling rate of 200 kHz was used for the data acquisition (approximately 10 times greater than the excitation frequency during tests). An overview of the displacement sensor and other experimental apparatus can be seen in Figure 7a,b.

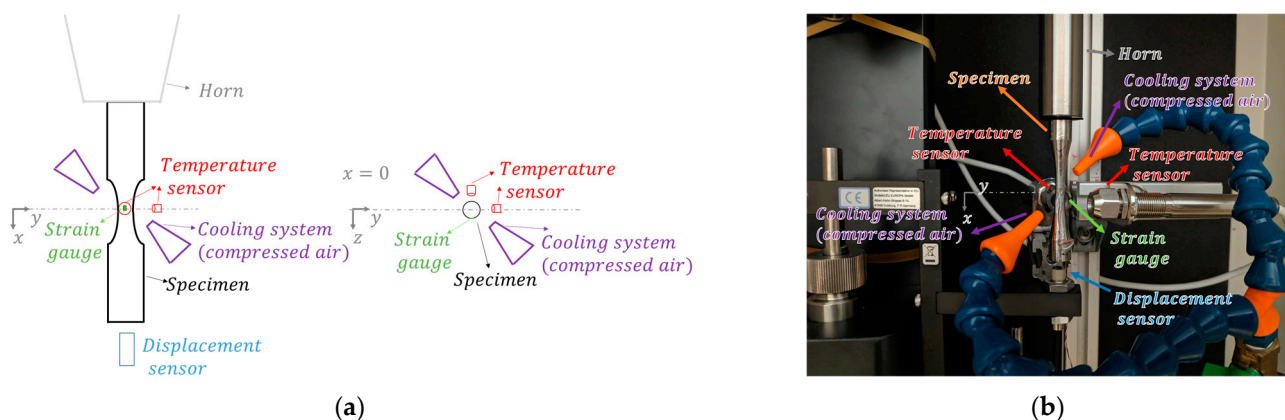


Figure 7. Experimental setup for UHCF testing and measuring systems: (a) Schematic view of experimental apparatus; (b) Experimental apparatus, in-situ.

Additionally, strain measurements were also performed on S690 specimens. This experimental data is especially important for UHCF fatigue, because stresses are usually

the most important output (S–N curves) for this kind of test (as well as the number of cycles to failure). Therefore, to have reliable information regarding the stresses, it is interesting to validate the stress calculations. In other words, comparing the stresses based on the experimental displacement and the experimental strain is relevant and assures that the stress information is trustworthy. The strain measurements were also performed at 200 kHz, and Micro-Measurements MMF402183 strain gauges (Micro-Measurements, Wendell, NC, USA) were used. This strain gauge was selected due to its short active length (0.76 mm), which is required in order to minimize distortion during the strain fixation once the specimen testing section is curved.

2.4.5. Determination of the Damping Ratio

For damping measurements, the displacement data were used in order to calculate the exponential decay. A schematic view of a typical UHCF displacement measurement can be seen in Figure 8. All damping tests, as well as temperature, displacement, and strain measurements, were performed using the already mentioned system, a Shimadzu USF-2000 ultrasonic fatigue testing machine. At the beginning, the machine starts to oscillate (forced vibration, blue series), and after a short period of time, the system reaches a steady state (orange series). After a stipulated time, the forced excitation is ceased, and the specimen experiences a free damped vibration (represented as the grey series in Figure 8). It is important to emphasize that, due to the intrinsic characteristics of all materials, damping and consequently mechanical energy losses and heat generation are always relevant and present when motion exists.

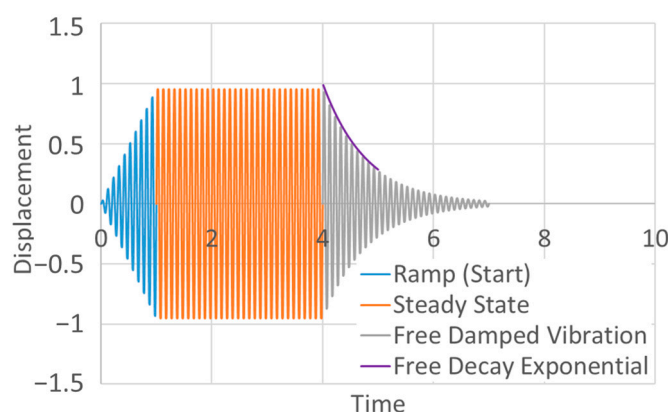


Figure 8. Schematic view of displacement during UHCF test for damping measurements.

The exponential decay can be written as

$$y(t) = A_0 e^{-\zeta \omega t}, \quad (24)$$

where t is the time, A_0 displacement amplitude (steady state), ω is the angular natural frequency, and ζ is the damping ratio, which can then be calculated. It is important to note that the measured damping ratio is an average value due to the fact the specimen is subjected to different stress levels along its length (as shown in Equations (6) and (8) for the central and cylindrical sections, respectively).

For determining the damping ratio, the measurements at the very beginning of exponential decay were used. This information is relevant, because, as previously observed in other studies, such as in the works [14,15], certain materials present damping ratios that are variable with the stress levels that the material is subjected to. Often in these cases, there is a positive correlation, with damping increasing as the stress levels rise.

For each specimen, measurements were taken for several displacement (or stress) amplitudes. Alternatively, instead of performing several tests for each amplitude, it would be possible to perform just one test with the maximum amplitude and take measurements

for exponential decay at different time steps (where the displacement/stress amplitudes are known).

2.4.6. Temperature Measurements

Regarding the temperature measurements, two infrared thermometers were used, Optris OPTCSTCLT15 (Optris, Berlin, Germany). The samples were painted black, in order to have a surface with a known emissivity (around 0.98). The sampling ratio for temperature acquisition was 10 Hz. It was possible to use a much smaller sampling rate when compared with displacement (or strain), once the temperature variations were only noticed after a few hundred cycles, and it was not required to perform several measurements during a single vibration cycle (which has a period of 50 μ s).

Due to the small size of the specimens (the minimum diameter was as low as 3 mm), it was important to have a small spot size. In the present case, a 0.8 mm spot size was used. The sensors were positioned in order to measure the temperatures at the central section of the specimen, where maximum temperatures were expected since this is the region with maximum stress/strain amplitudes. The probes were positioned 90° apart from each other, as shown in Figure 7. The probe positioned behind the specimen will be called Sensor 1, while the one at the side will be denoted as Sensor 2.

In addition, due to an intrinsic characteristic of ultrasonic fatigue testing, there is great variation in stress amplitudes along the specimen length, being maximum at the center and theoretically zero at the edges ($x = L/2$ and $x = -L/2$, according to Figure 2). This will also result in temperature variations along the specimen length; therefore, it is important to have a small spot size, making it possible to perform measurements on an area without major temperature variations.

All measurements were performed on samples that had not been tested before under fatigue loading, meaning that no fatigue damage was present during the tests. This is important to notice because the measurements (especially regarding temperature and damping) would be completely different if major cracks were present. For example, due to the stress concentration at the crack tip, massive temperature increases would be found in these regions.

3. Results and Discussion

The present section will present the obtained results, which will also be discussed. First, results regarding the elasticity modulus determination will be detailed, followed by displacement, strain, and damping measurements. After, temperatures recorded during tests will be shown. Last, with this data available, it is possible to compare the analytical model results for temperature with the experimental data. This last step will provide significant information regarding the quality of the proposed model and the validity of the hypothesis made.

3.1. Elasticity Modulus Measurements

The experimental data regarding the elasticity modulus determination will be presented for the S690 specimen. The procedure performed for the other materials is analogous. A cylinder 207.2 mm long with a diameter of 17.8 mm was machined. After following the procedure described in Section 2.4.2, the specimen was suspended, and an impact was performed. Figure 9 presents one measurement (frequency domain) for the impact test in the S690 specimens. The first natural frequency can be clearly identified.

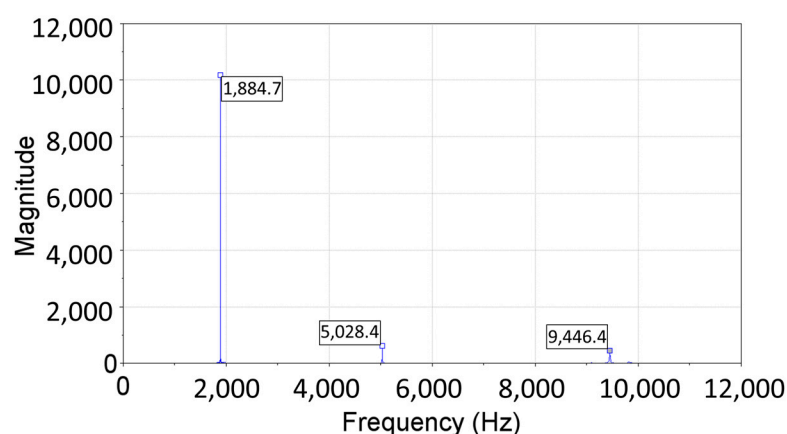


Figure 9. Frequency spectrum of recorded sound, S690 cylinder.

After measuring the natural frequency and knowing the specimen dimensions and density, it was possible to determine the elasticity modulus. A modal analysis was performed. For the first iteration (an initial guess), an elasticity modulus of 200 GPa was used. The experimental and numerical natural frequencies can be seen in Table 3. It is important to emphasize that, at the time of the experimental procedure, it was known that the first mode of vibration was a bending one. However, this information was not available for the remaining modes. In addition, it was not possible to measure the experimental natural frequency of the third mode, torsion.

Table 3. Natural frequency and elasticity modulus.

Natural Vibration Mode	Type	Natural Frequencies (Hz)		
		Experimental	FEM, 1st Iteration, E = 200.0 GPa	FEM, 2nd Iteration, E = 212.7 GPa
1st	Bending	1884.75	1827.60	1884.75
2nd	Bending	5028.38	4877.22	5029.72
3rd	Torsion	-	7527.91	7763.30
4th	Bending	9446.39	9159.97	9446.38

As stated before, if the elasticity modulus is not properly measured, it will result in UHCF specimens with resonance frequencies outside the machine's operating range. When looking at the experimental natural frequencies for the UHCF specimens, presented in Table 2, all the measured values were close to the targeted one, 20 kHz, with a maximum difference of 1%. With that information, it can be stated that the presented approach for elasticity modulus determination was reliable.

3.2. Displacement and Strain Measurements

During all tests and calibrations performed, the displacements were always measured. This was done because, based on the experimental displacement, it was possible to compute the strain and stress for a given specimen. The computed strain was later compared with experimental measurements from the strain gauges.

The strain measurements were performed on the S690 specimen. However, since the analytical formulation for determining both strain and stress is the same for all specimens and materials, the validation procedure can be extrapolated for the remaining specimens, providing reliable data for stress determination during tests.

Figure 10 presents the measurements of displacements and strains along time for an external amplitude of 26.8 μm . The amplitudes for both waves were calculated during the pulse period (steady state).

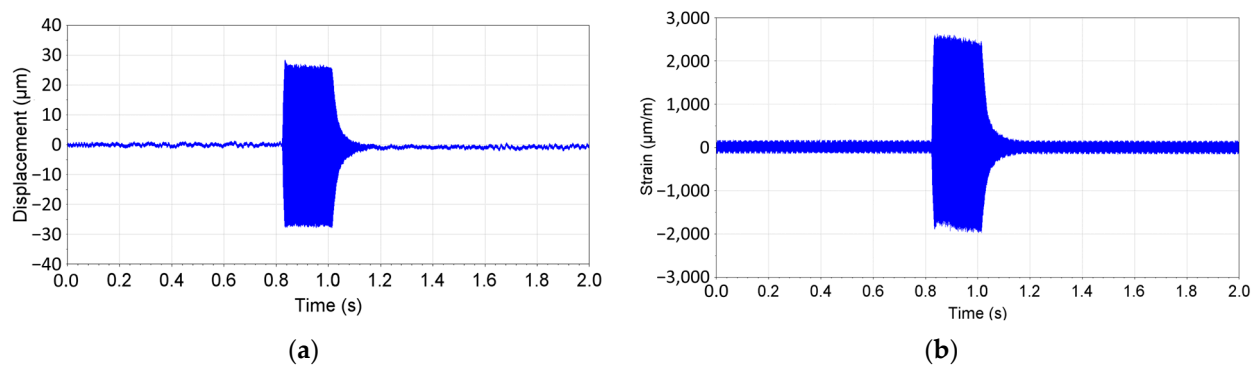


Figure 10. Displacement and strain measurement, UHCF S690 specimen, maximum displacement amplitude of 26.8 μm : (a) Displacement over time; (b) Strain over time.

Analytical strain and stress were computed using the measured displacement amplitude, A_0 , by means of Equation (6) (strain was computed by dividing the stress by the elasticity modulus). The comparison between analytical and measured strain is presented in Table 4. Regarding the two external amplitudes analyzed, an average discrepancy between analytical and experimental data of 6% was found. Therefore, the computed strains and stresses based on the analytical formulation show acceptable reliability.

Table 4. Experimental and analytical strain and stress comparison of UHCF S690 specimen.

Measured Displacement (μm)	Analytical Strain ($\mu\text{m/m}$)	Measured Strain ($\mu\text{m/m}$)	Analytical Stress (MPa)	Computed Stress (MPa)	Strain and Stress Error (%)
13.1	962.4	1021.2	203.6	216.1	6.1
26.8	1964.3	2082.2	415.6	440.6	6.0

3.3. Damping Ratio Measurements

Following the procedure described in Section 2.4.5, damping ratios were measured for all specimens' geometries. Due to differences regarding the fatigue performance of the analyzed materials, it was not possible to test them at the same stress (or displacement) levels.

Some results regarding the measured displacement and the fitted exponential decay are presented in Figures 11–15. Regarding the 316L specimens (Figures 11 and 12), it can be seen clearly that the material presents a high damping ratio if compared with other tested materials. In addition, for higher maximum stress levels, Figure 11b, the damping is so high that the exponential curve's beginning limit looks similar to a vertical asymptote.

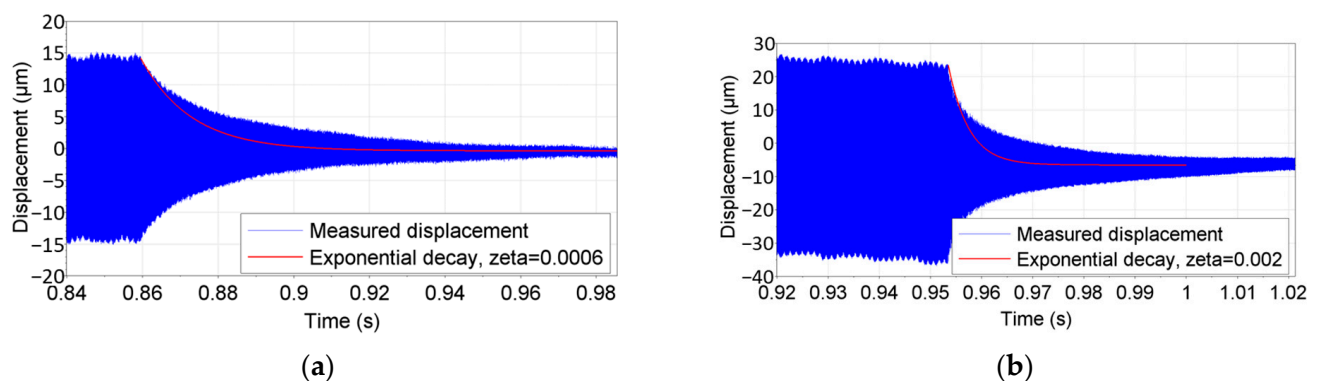


Figure 11. SS 316L D6, UHCF specimen, exponential decay: (a) Maximum stress = 150 MPa; (b) Maximum stress = 300 MPa.

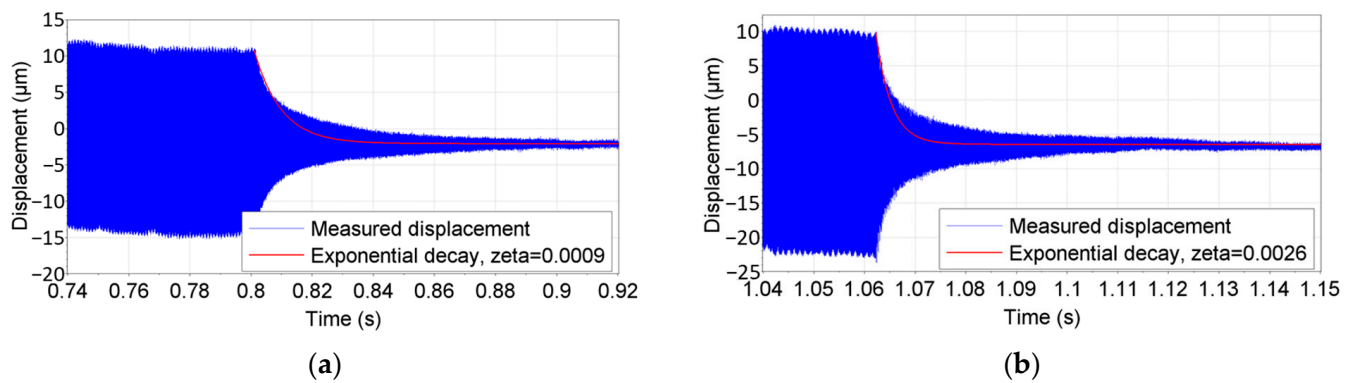


Figure 12. SS 316L D3, UHCF specimen, exponential decay: (a) Maximum stress = 204 MPa; (b) Maximum stress = 350 MPa.

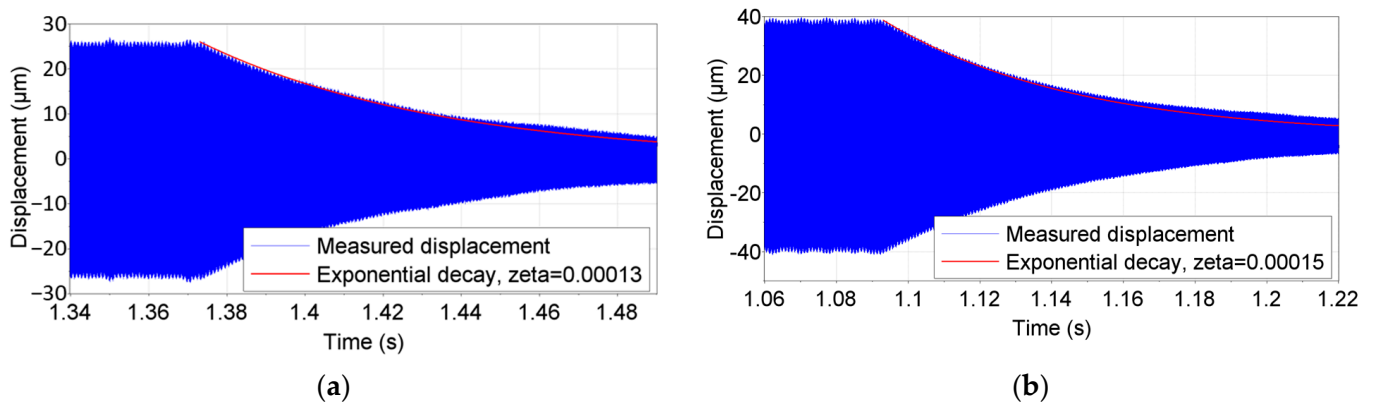


Figure 13. 51CrV4 steel, UHCF specimen, exponential decay: (a) Maximum stress = 400 MPa; (b) Maximum stress = 600 MPa.

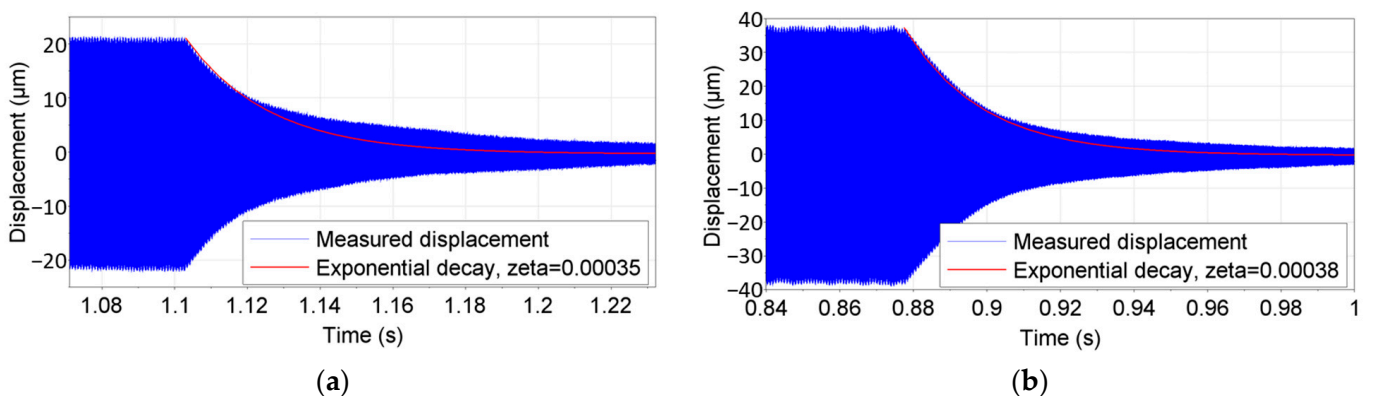


Figure 14. S690 steel, UHCF specimen, exponential decay: (a) Maximum stress = 220 MPa; (b) Maximum stress = 390 MPa.

Another interesting result regarding 316L steel is related to the temperature increase and thermal expansion of the material. It can be seen that the average position for the displacement is not zero (as it was in prior testing), as shown in Figure 12b. Since the specimen experienced a quick temperature increase (as it will be detailed in the next section), the specimen length increased, resulting in the free edge being closer to the displacement sensor (shown in Figure 7).

The 51CrV4 (Figure 13) and the Inconel 625 (Figure 15) were the materials that presented the lowest damping ratio. The S690 (Figure 14) damping properties were located between these and the 316L ones.

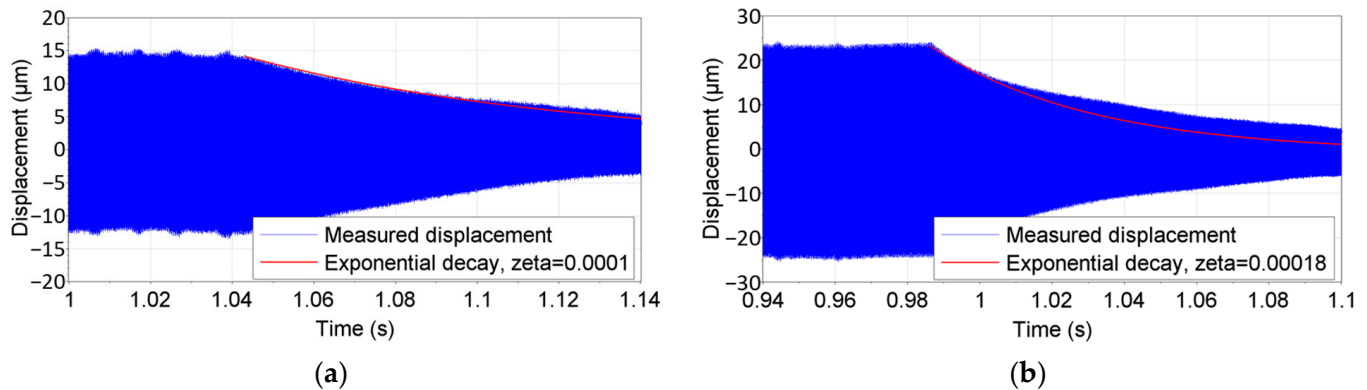


Figure 15. Inconel 625, UHCF specimen, exponential decay: (a) Maximum stress = 200 MPa; (b) Maximum stress = 356 MPa.

The measured damping ratios for all tested conditions are summarized in Figure 16. For every specimen, at least three load levels (displacement or analytical stresses) were tested. It can be noticed that some materials, namely 316L and Inconel 625, present a severe increase in the measured average damping ratio when subjected to higher stresses. On the other hand, the 51CrV4 presented a damping ratio that was nearly independent of the applied stress level. Additionally, both 316L specimens presented a similar relationship between the damping ratio and the maximum stress amplitude. This is a positive result, once it was expected that the damping ratio, as a material property, would not present any dependency of the specimens' dimensions.

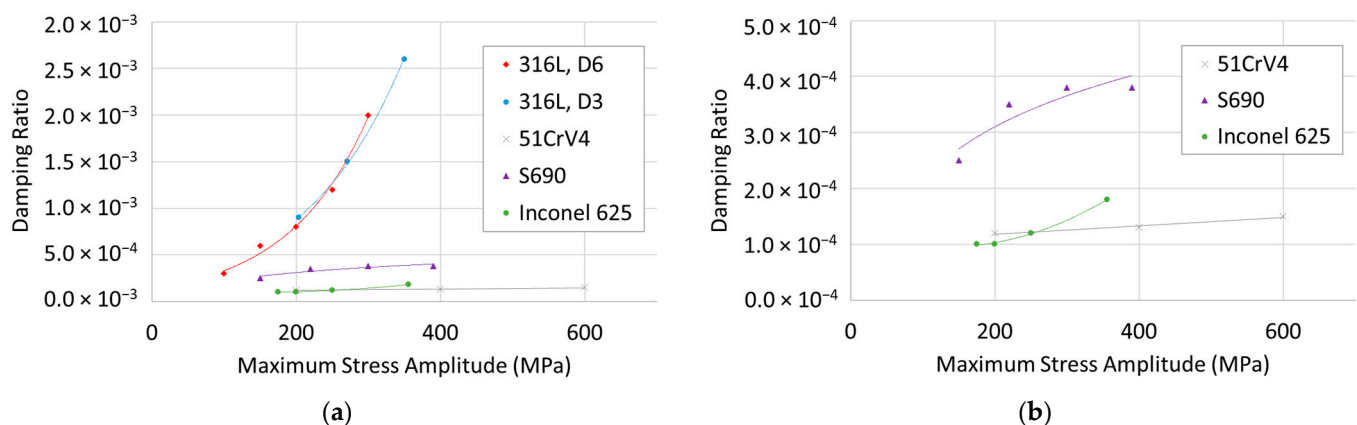


Figure 16. Measured average damping ratios for several stress amplitudes in UHCF specimens: (a) Measured damping for all materials; (b) Closer view, lower damping materials.

It is important to emphasize that, during tests, the specimens were subjected to different stress levels along their lengths. The damping ratio results presented in Figure 16 are displayed according to the maximum stress amplitude during tests (central section of the specimens). Therefore, the measured damping ratio is an average value. Consequently, especially for the materials with great changes in the damping ratio for different stress levels, some regions might be subjected to a higher damping ratio (central section, maximum stress) than others (specimens' edges, null stresses). The measured damping ratio used for the computational model will be selected according to the stress level that the material is subjected to during that test.

3.4. Temperature Measurements

The present section will provide some measured temperature during specific testing conditions and will be essential for the validation and comparison between computational and experimental temperature results. All temperature measurements were performed without active cooling (compressed air), unless mentioned otherwise.

Figure 17a,b presents the results for two measured conditions for the 316L, D3 specimen, being the maximum stress amplitude of 271 and 350 MPa, respectively (maximum displacement of 12.8 and 16.5 μm), for a pulse (t_{on}) of 0.11 s. It can be seen that, for the extreme case, the specimen's central section temperature increased by more than 30 °C for a very short pulse period.

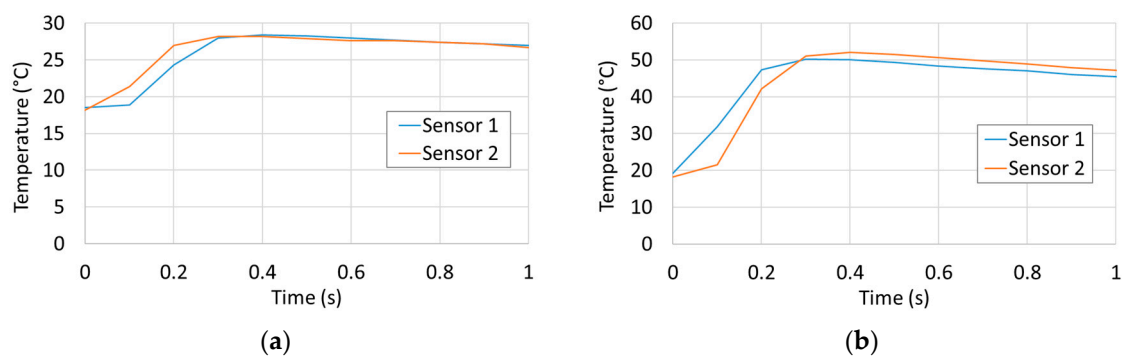


Figure 17. Temperature measurements for 316L, D3, UHCF specimen: (a) Maximum stress = 271 MPa, $t_{\text{on}} = 0.11$ s, 1 pulse; (b) Maximum stress = 350 MPa, $t_{\text{on}} = 0.11$ s, 1 pulse.

Two temperature measurements for the S690 specimen are presented in Figure 18a,b. When comparing the 316L results, it can be seen that the temperature increases per pulse are much smaller, even for superior stress levels and pulse periods. Therefore, it can be said that, regarding the temperatures during tests, the S690 will be a material much easier to perform UHCF tests on when compared to the 316L. The latter will require much more efficient cooling and/or shorter pulse periods and longer resting periods (t_{off}).

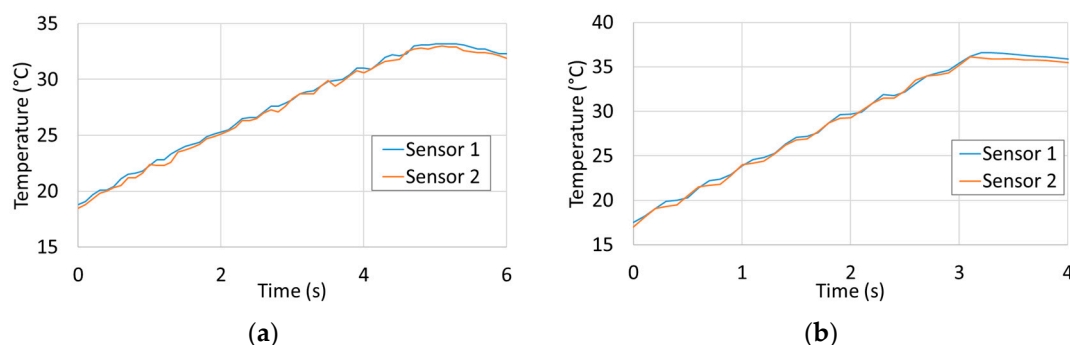


Figure 18. Temperature measurements for S690, UHCF specimen: (a) Maximum stress = 220 MPa, $t_{\text{on}} = 0.2$ s, $t_{\text{off}} = 0.2$ s, 12 pulses; (b) Maximum stress = 390 MPa, $t_{\text{on}} = 0.2$ s, $t_{\text{off}} = 0.2$ s, 8 pulses.

Beside the material properties, stress levels and pulse periods also affect the temperatures during tests; it is also interesting to understand the consequences of different cooling conditions during tests. Figure 19 presents a comparison between two tests where both stresses and pulse periods were the same but with one test (b) having a forced convection at the central section of the specimen. As expected, the specimen cooled by compressed air presented much lower temperatures for the same testing conditions. A steady-state temperature near 24 °C was found. In a practical UHCF testing scenario, this means that, for this stress level, it was possible to increase the pulse period (or decrease the t_{off}), resulting in a much quicker test (more cycles in less time due to smaller idle periods).

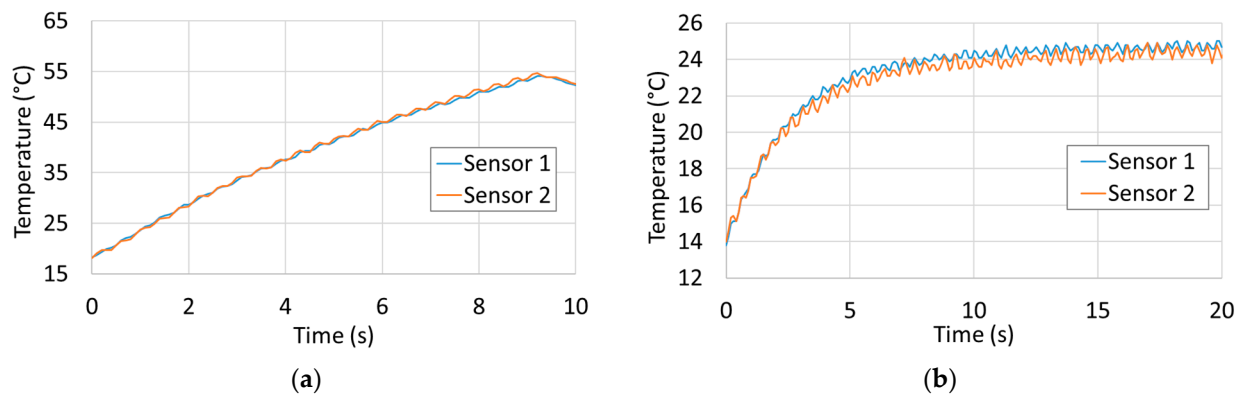


Figure 19. Temperature measurements for Inconel 625, UHCF specimen: (a) Maximum stress = 300 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.2$ s, 23 pulses; (b) Maximum stress = 300 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.2$ s, compressed air cooling, approximately 50 pulses.

3.5. Analytical Model and Experimental Comparison

The present section will provide a comparison between the measured temperatures and the ones estimated by the analytical models. Regarding the weighting damping (or Approach 2), the weighted damping ratio is expressed as a function of the stress level at a given location, and is written as

$$\zeta_w = \zeta_a \left(3.75 \left(\frac{\sigma}{\sigma_{max}} \right)^2 + 0.25 \right), \quad (25)$$

where ζ_a is the average damping, σ_{max} is the maximum stress, and σ is the stress level for the region of interest, as shown in Equation (8). The weighted damping approach was developed based on literature observations of damping ratio being dependent on stress levels, as reported in the works of Vinaspuu et al. [14] and Lage et al. [15]. Additionally, this phenomenon was also observed in some of the studied materials (as shown in Figure 16). The weighted damping ratio will be four times the value of average damping at the central section ($x = 0$), where the larger stresses are present, and will be a quarter of average damping at the free edge ($x = L/2$). The algorithm is presented in Appendix A, where both damping generation approaches are present.

For all tests performed without active cooling (natural convection), a convection coefficient of $15 \text{ W}/(\text{m}^2 \cdot \text{K})$ was used (typical literature references ranges from 10 to $20 \text{ W}/(\text{m}^2 \cdot \text{K})$ for natural convection). Regarding the 316L D3 specimen, the temperature results comparison is presented in Figure 20. Figure 20a presents the temperature increment from a single pulse with a maximum stress amplitude of 200 MPa and duration (t_{on}) of 0.2 s. The approach that presented the best correlation with experimental data was approach 2, Weighting Damping. This result was expected, once the 316L presented significant variation in the damping ratio according to stress level (see Figure 16). Therefore, using the average damping would result in underestimating the heat generation at the testing section, where the stresses are higher, and, in consequence (for this alloy), the damping too. Figure 20b presents the analytical and experimental temperatures for a 270 MPa pulse with the same duration (0.2 s). A greater experimental temperature increase can be noticed (more than 20°C) when comparing with the previous case (approximately 9°C), resulting in more than twice the temperature increase, while the stress increase was only increased by 35% (200 MPa to 270 MPa). This was attributed to the increase in the damping ratio associated with higher stress amplitudes for the 316L material, as shown in Figure 16a.

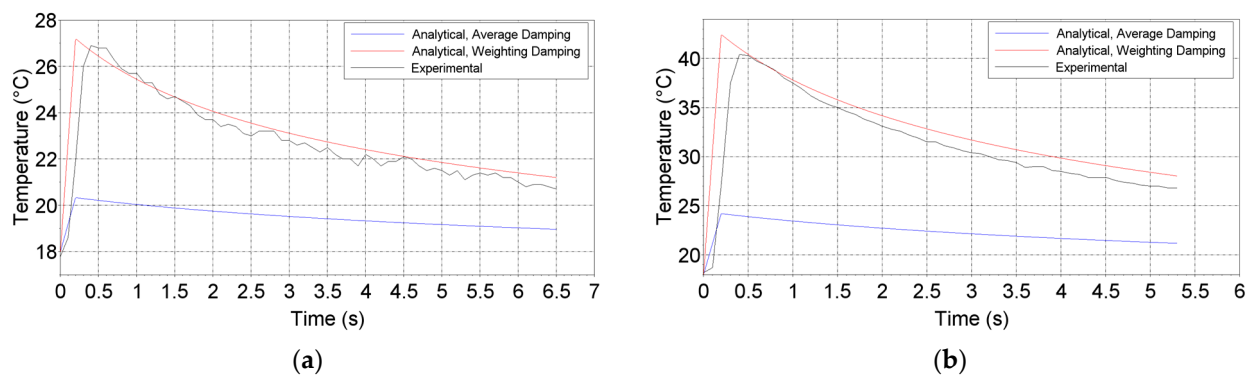


Figure 20. Experimental and computational model temperature comparison, 316L D3 stainless steel, UHCF specimen: (a) Maximum stress = 200 MPa, $t_{on} = 0.2$ s, $t_{off} = 5$ s, 1 pulse; (b) Maximum stress = 270 MPa, $t_{on} = 0.2$ s, $t_{off} = 5$ s, 1 pulse.

A similar behavior was observed for the other 316L specimen (D6). Figure 21a–d presents the results for this specimen geometry, with maximum stress levels varying from 100 to 250 MPa. Figure 21a presents the temperature results for the lowest stress amplitude tested (100 MPa), being applied continuously (no t_{off}). For this case, the analytical prediction of Approach 2 also provided closer results with experimental data but presented a small overestimation of the temperatures.

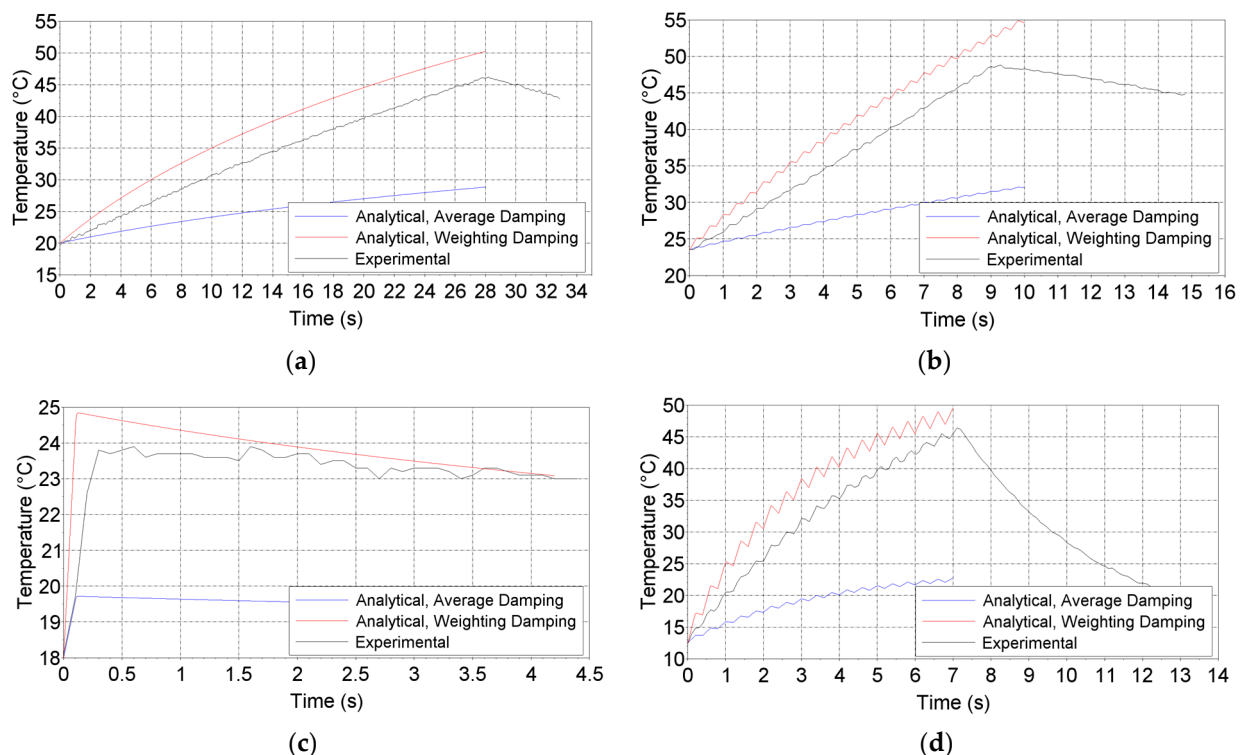


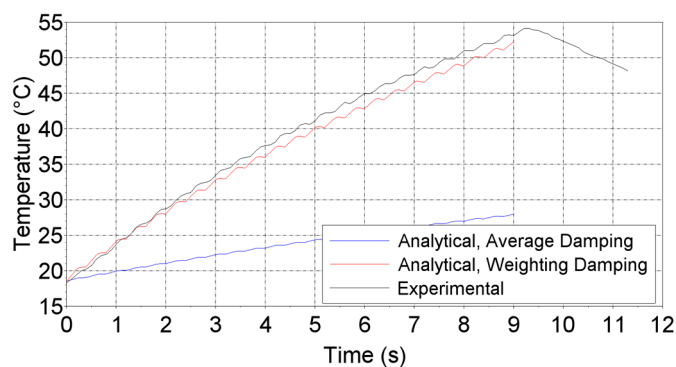
Figure 21. Experimental and computational model temperature comparison, 316L D6 stainless steel, UHCF specimen: (a) Maximum stress = 100 MPa, continuous excitation; (b) Maximum stress = 150 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.2$ s, 23 pulses; (c) Maximum stress = 250 MPa, $t_{on} = 0.11$ s, $t_{off} = 5$ s, 1 pulse; (d) Maximum stress = 200 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.2$ s, forced convection, $h = 1500$ W/(m².K), 18 pulses.

Figure 21a–c presents the results for the specimens under natural convection, while Figure 21d presents temperature results for a specimen cooled by compressed air at the central testing section. In order to estimate the forced convection coefficient, this value was

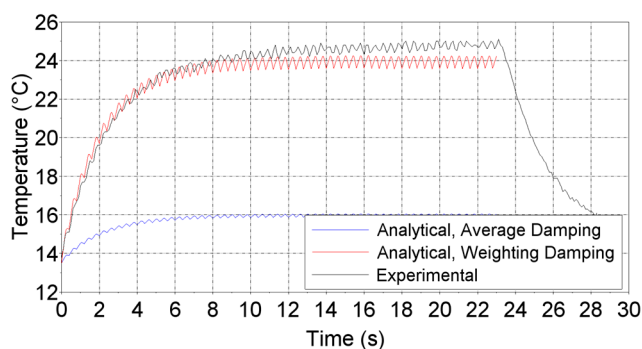
adjusted in order to match the analytical with experimental measured temperatures. A value of $1,500 \text{ W}/(\text{m}^2 \cdot \text{K})$ was found and used for all other tests performed under forced convection. Interestingly, this convection coefficient resulted in an acceptable correlation with experimental data for all tested specimens and conditions.

The temperature comparison for the Inconel 625 specimen is presented in Figure 22a–c. Once more, as it happened to the 316L specimens, Inconel 625 presented a high variation in the damping ratio due to the increase in the stress level, as it can be seen in Figure 16b (although the damping ratios are much lower than those of 316L, and so the temperature increases as well). Consequently, for a second time, the weighing damping approach was the one that better represented the heat generation and temperature increase. Stress levels of 220 and 270 MPa were tested. Figure 22b presents the results for UHCF testing under forced convection, and again, with a convection coefficient of $1,500 \text{ W}/(\text{m}^2 \cdot \text{K})$ was in good agreement with the experimental results.

The temperature results for 51CrV4 steel are presented in Figure 23. The temperature increases observed were much smaller than the ones presented by the other materials, which is attributed to the overall small damping ratio of this material. In contrast to the 316L and Inconel 625 behaviors, the damping ratio of 51CrV4 showed little to no dependence on stress levels. Therefore, the hypothesis that the entire sample is under a constant damping ratio, equal to the measured average damping ratio, is strong. Consequently, it was expected that the most suitable approach would be the first one, average damping, and this was indeed observed. Stresses ranging from 200 to 600 MPa were studied, and, in all cases, the analytical approach of average damping provided great estimations of heat generation and temperature increases. An interesting observation is presented in Figure 23d), where an abrupt temperature increase was found approximately 4.5 s after the test began, regarding crack nucleation and growth. Due to localized stress concentration (especially at the crack tip), excessive heat and a severe temperature increase were observed.



(a)



(b)

Figure 22. Cont.

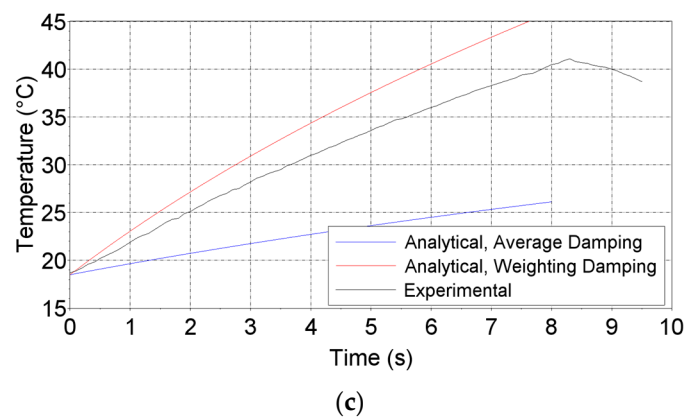


Figure 22. Experimental and computational model temperature comparison, Inconel 625, UHCF specimen: (a) Maximum stress = 270 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.2$ s, 23 pulses; (b) Maximum stress = 270 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.2$ s, forced convection, $h = 1500$ W/(m².K), 58 pulses; (c) Maximum stress = 220 MPa, continuous excitation.

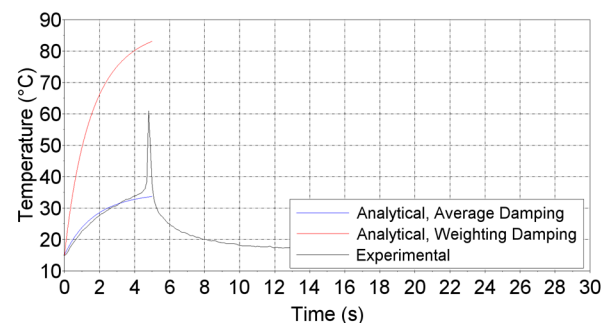
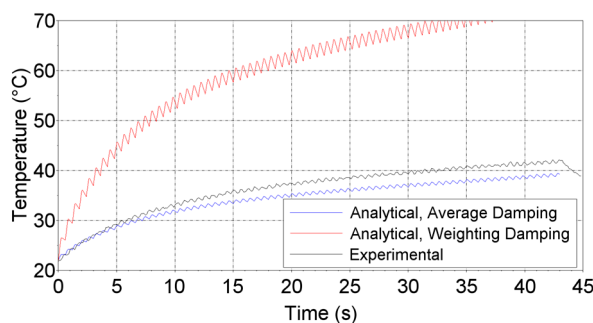
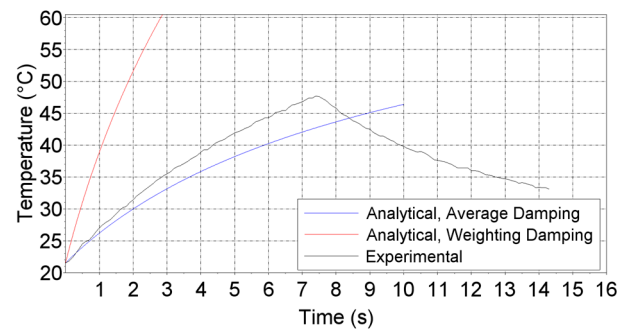
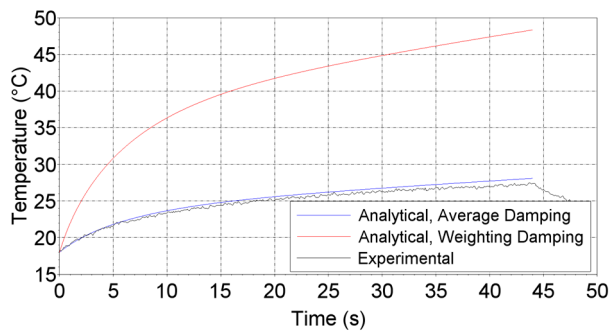


Figure 23. Experimental and computational model temperature comparison, 51CrV4 steel, UHCF specimen: (a) maximum stress = 200 MPa, continuous excitation; (b) maximum stress = 400 MPa, continuous excitation; (c) maximum stress = 400 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.4$ s, 72 pulses; (d) maximum stress = 600 MPa, continuous excitation, forced convection, $h = 1500$ W/(m².K), crack initiation.

Temperature measurements for the S690 specimen and comparison with the computational model are presented in Figure 24a–c. As in the previous specimen (51CrV4), the S690 did not present great damping increases regarding the stress levels. After a certain stress level, the damping ratio tended to a horizontal asymptote (Figure 16). Therefore, the average damping approach was expected to be the most suitable one. As it was observed in the 51CrV4, there was good agreement between the computed temperatures and the reference ones (experimental values). Stress levels between 150 and 300 MPa were tested, varying the working and idle periods (t_{on} and t_{off} , respectively).

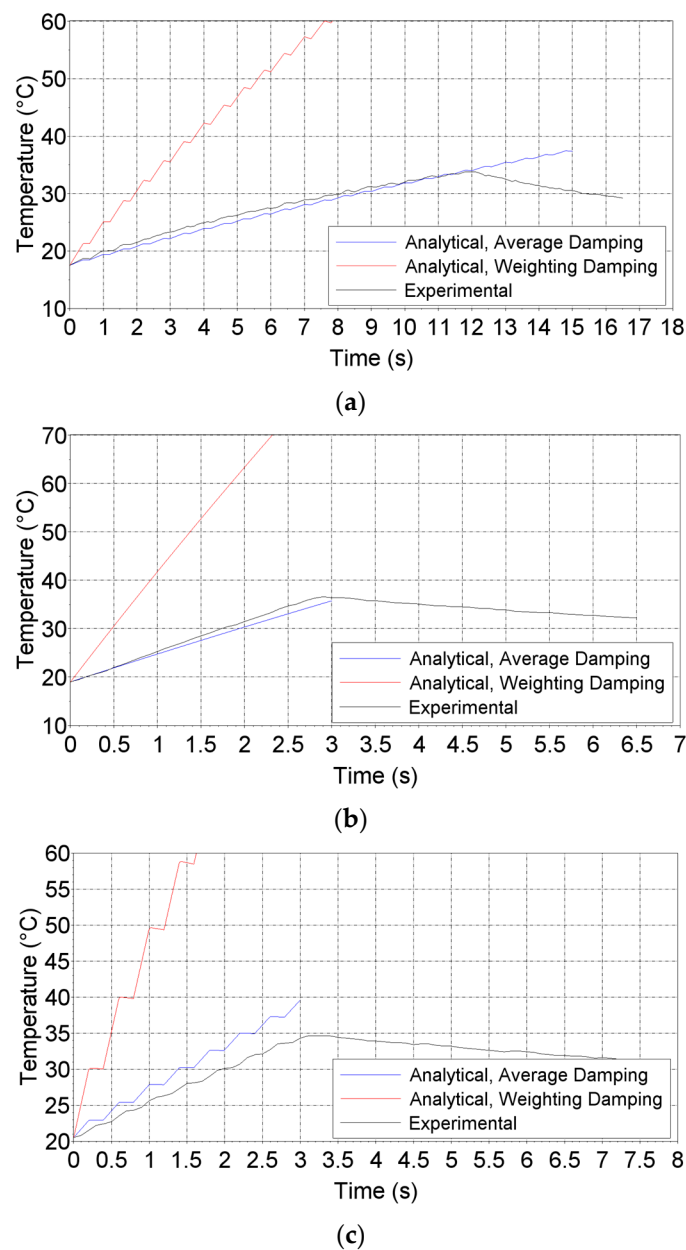


Figure 24. Experimental and computational model temperature comparison, S690 steel, UHCF specimen: (a) Maximum stress = 150 MPa, $t_{on} = 0.4$ s, $t_{off} = 0.2$ s, 20 pulses; (b) Maximum stress = 220 MPa, continuous excitation; (c) Maximum stress = 300 MPa, $t_{on} = 0.2$ s, $t_{off} = 0.2$ s, 8 pulses.

The analyzed testing conditions (material, stress amplitude, excitation, and cooling conditions, as well as the damping ratios) are summarized in Table 5. It can be observed that for materials that presented a significant damping ratio increase (316L and Inconel 625) due to higher stress levels, Approach 2 (weighting damping) presented better results. This can be explained by the fact that this approach takes into account the variations in the damping ratio due to stress amplitudes in the heat transfer model (even though the exact relation between the damping ratio and stress amplitude is unknown). On the other hand, for materials in which little changes were measured for the damping ratio (S690 and 51CrV4 steels), Approach 1 (Average Damping) presented the best agreement between the calculated temperatures and experimental data.

Table 5. Summary of the analyzed results for several materials and the approach which presented better correlation with the experimental data.

Material	Maximum Stress Amplitude (MPa)	Excitation Condition	Convection Condition	Mean Damping Ratio	Maximum Weighting Damping Ratio	Severe Dependency Between Damping Ratio and Stress	Most Suitable Approach
316L Stainless steel (D3)	200	Intermittent	Natural	0.0012	0.0048	yes	2
	270	Intermittent	Natural	0.0018	0.0072	yes	2
316L Stainless steel (D6)	100	Continuous	Natural	0.0003	0.0012	yes	2
	150	Intermittent	Natural	0.00052	0.00208	yes	2
	250	Intermittent	Natural	0.0016	0.0064	yes	2
	200	Intermittent	Forced	0.0012	0.0048	yes	2
Inconel 625 Nickel Superalloy	270	Intermittent	Natural	0.00014	0.00056	yes	2
	270	Intermittent	Forced	0.00014	0.00056	yes	2
	220	Continuous	Natural	0.00012	0.00048	yes	2
51CrV4 Steel	200	Continuous	Natural	0.00014	0.00056	no	1
	400	Continuous	Natural	0.00016	0.00064	no	1
	400	Intermittent	Natural	0.00017	0.00068	no	1
	600	Continuous	Forced	0.00018	0.00072	no	1
S690 Steel	150	Intermittent	Natural	0.0004	0.0016	no	1
	220	Continuous	Natural	0.00048	0.00192	no	1
	300	Intermittent	Natural	0.0005	0.002	no	1

4. Conclusions

The presented approach for heat generation associated with the dissipative energies during UHCF testing presented interesting results. Moreover, the one-dimensional heat transfer model was suitable for the studied cases. In addition, a strong correlation was found between the increase in stress amplitude and damping ratios for some materials. Regarding the tested materials and geometries, the presented approaches proved to be able to properly estimate the temperature history during UHCF tests for several different materials, stress levels, and testing circumstances (for example, active and idle times as well as natural and forced convection conditions).

The main insights can be summarized as:

- The heat generation and temperature increase heavily rely on certain material parameters, namely: damping ratio (or loss factor), thermal conductivity, and specific heat. High thermal conductivity and specific heat are beneficial for fatigue testing (smaller temperature increases). On the other hand, the damping ratio is directly related to heat generation. Therefore, high-damping alloys are more prone to excessive heat generation during tests.
- In addition, specimen geometry plays an important role in determining the specimen temperatures. Specimens with bigger testing diameters (D1) will be cooled less efficiently. This happens due to the smaller surface area-to-cross-section ratio when compared with specimens with smaller diameters. This happens because most of the heat is extracted via convection (proportional to the surface area) and the heat generation is proportional to the cross-section area.
- So far, without the information of the damping ratio for a given stress level ($\zeta = f(\sigma)$), it is not possible to provide an approach that will work for all materials. For materials that present little or no dependency on the damping ratio according to stress levels, the average damping approach works better. On the other hand, for materials showing damping parameters heavily dependent on stress levels, the weighted average damping approach is more suitable.

For future work, it would be a great improvement to have a damping ratio written as a function of the stress. This could be achieved by applying a different test for damping ratio measurements. This would make it possible to more precisely compute the heat generation and temperature increase at every region of the specimen, once the damping information is available. In addition, testing more materials (aluminum or titanium alloys, for example) and specimen geometries would provide a wider view and validation for the proposed approach. Finally, a remarkable improvement for the heat transfer model would be, for example, improving the model from one- to two-dimensional conduction (axisymmetric). This would be especially interesting for specimens with low thermal conductivity and/or cooled by forced convection (air or water, for example), where severe temperature differences are expected between the core and surface of the specimen.

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Appendix A

The current section presents the computational code developed for the heat generation and temperature estimation model, encompassing the displacement, strain, and stress fields along the entire UHCF specimen length. The code was developed in an open-source language, Scilab. Regarding the model, unidirectional bar elements were used. For UHCF fatigue testing, the first longitudinal natural frequency is the one of interest.

```
//program start
clear all
clc
close
//#####Start Inputs#####
//(two slashes) for commenting
//All units are in the International System of Units (SI)
//Temperatures, degree Celsius, °C
Text=13; //[°C] // Ambient Temperature // For convection
Ti=14; //[°C] // Sample Initial Temperature
hext=15; //[W/(m²*°C)] or [W/(m²*K)] // Film coefficient for specimen "external
section" (constant radius section). Often, this is the natural convection coefficient (between
10-20)
hint=500; //[W/(m²*°C)] or [W/(m²*K)] // Film coefficient for specimen "internal
section" (variable radius). In case of no cooling system (compressed air), the input value is
equal to the hext (natural convection)
//Time
Ttot=8; //[s] // Total time
```

```

Dt=0.005; // [s] // Time step. It is important that this value must be lower than the Ton and
Toff.
Ton=0.4; // [s] // Time on (Pulse Period). If testing is continuous, this input can be any value
(bigger than the time increment, equal or smaller than the total time)
Toff=0.0; // [s] // Period . If testing is continuous (not intermittent), Time off=0 (machine is
always on). For example, If Ton=0.2s and T off =0.4s, the machine will work on 0.2s, than
"rest" 0.4s (than, it will repeat)
A0=30E-06; // [m] // The displacement amplitude (on the horn or on the free edge)
// Material properties
zeta=0.00037; // Damping ratio (absolute value, not percentage). This can be a constant
value, or a function of stress amplitude, for example
E=210E9; // [Pa] // Young's modulus (elasticity modulus)
rho=7830; // [kg/m^3] // density
cp=460; // [J/(kg*°C)] or [J/(kg*K)] // Specific heat
k=13.8; // [W/(mK)] // thermal conductivity
// Dimensions
Lext=0.01599; // [m] // Length of the external section (constant radius)
Lint=0.0295; // [m] // Length of the internal section (half the length of this section, due to
symmetry, variable diameter)
Dext=0.01; // [m] // diameter of the external section (maximum diameter)
Dint=0.0055; // [m] // diameter of the internal section (minimum diameter)
// End of Geometry and Material Properties
// number of elements
next=20
; // number of elements on the external section (constant radius). A few hundreds of
elements can be used, although (for the tested cases), a convergence happens after a few
dozens
nint=20; // number of elements on the internal section (variable radius). Since most of the
heat generation occurs on this section (higher stresses/strains), it is interesting to have
more "elements" here
// End of Inputs. After this section, there is no need for changes (unless you know the
formulation well enough)
ntot=next+nint; // total number of elements
nnos=ntot+1; // total number of nodes
// Vectors and matrix
L=zeros(ntot,1); // Length of each element
Al=zeros(ntot,1); // Lateral area (for convection)
At=zeros(ntot-1,1); // Cross section area
Ldist=zeros(ntot-1,1); // Distance between adjacent elements (centre to centre)
D=zeros(ntot,1); // Diameter of each elements
h=zeros(ntot,1); // Convection coefficient vector
fnat=20.0E3; // natural frequency (Hz) //
wnat=fnat*2*pi; // natural angular frequency
Tp=1/fnat; // Oscillating period
Tpulse=Ton+Toff; // Total testing block period
NInc=1+Ttot/Dt; // Total number of steps
Time=zeros(NInc,1); // Time vector
Time(:,1)=[0:Dt:Ttot]';
Temperatures=zeros(ntot,NInc); // Temperature matrix (all elements and time steps)(lines=elements,
column=time step), approach1
Temperatures(:,1)=Ti; // First step, Temperature=Ti
Temperatures2=zeros(ntot,NInc); // Temperature matrix (all elements and time steps)(lines=elements,
column=time step), approach2
Temperatures2(:,1)=Ti; // First step, Temperature=Ti

```

```

Fv=zeros(ntot,1); // "Force" vector
Tv=zeros(ntot,1); // "Old" Temperature Vector (Calculated in the last time step), approach1
Tv2=zeros(ntot,1); // "Old" Temperature Vector, approach2
Tv(:,1)=Ti;
Tv2(:,1)=Ti;
Km=zeros(ntot,ntot); // "Stiffness" matrix - K*T= Fext (Fext is independent of the "present"
temperature (T)
h(:)=hint; // heat coefficient
L(:,1)=Lint/nint; // length of each element
for i=1:next;
    L(i,1)=Lext/next;
    h(i,1)=hext; // heat coefficient (for each external element)
end
for i=1:ntot-1;
    Ldist(i)=(L(i+1)+L(i))/2; // distance between element centres
end
/////....Geometry // Bathias, Claude, and Paul C. Paris. Gigacycle fatigue in mechanical
practice. CRC Press, 2004. [ref]
R1=Dint/2;
R2=Dext/2;
L1=Lext;
L2=Lint;
Lt=L1+L2;
cwave=sqrt(E/rho); // longitudinal wave velocity, 2.5 [ref]
kg=wnat/cwave; // k for gigacycle, 2.23 [ref]
alfa=acosh(R2/R1)/L2; // alfa on gigacycle, 2.25 [ref]
betaa=sqrt(alfa^2-kg^2); // beta, 2.27 [ref]
phi=cos(kg*L1)*cosh(alfa*L2)/sinh(betaa*L2); // phi 2.29 [ref]
X=zeros(ntot,1); // centre coordinate on each element
X(1)=L(1)/2;
for i=2:ntot
    X(i)=X(i-1)+Ldist(i-1);
end
Xaux=zeros(ntot,1); // Similar to X, but the reference now is in the symmetry plane (mini-
mum radius).
Xaux(:)=Lt-X(:);
Y=zeros(ntot,1); // specimen profile (radius)
Y(:,1)=R1*cosh(alfa*Xaux(:)); // internal section (catenary, very similar to a circle)
Y(1:next)=R2; // external section
Al(:,1)=%pi*2*Y(:,1).*L(:,1); // lateral area (for convection)
Afree=%pi*(Y(1,1))^2; // the first element (free edge) has an additional surface area
Al(1,1)=Al(1,1)+Afree;
Mv=zeros(ntot,1); // mass vector
Mv(:,1)=%pi*((Y(:,1))^2).*L(:,1).*rho;
At=zeros(ntot,1);
At(:,1)=%pi*Y(:,1).*Y(:,1); // Cross sectional area
// Here, the displacement, strain and stress
Udisp=zeros(ntot,1);
Udisp(:,1)=A0*cos(kg*(X(:,1))); // 2.28b
edef=zeros(ntot,1);
edef(:,1)=kg*A0*sin(kg*(X(:,1))); // 2.31a
for i=next+1:ntot // Displacement and strain on the variable radius section
    Udisp(i,1)=A0*phi*sinh(betaa*Xaux(i,1))/cosh(alfa*Xaux(i,1)); // 2.28a

```

```

        edef(i,1)=A0*phi*(betaa*cosh(betaa*Xaux(i,1))*cosh(alfa*Xaux(i,1))-alfa*sinh(betaa*Xaux(i,1))*sinh(alfa*Xaux(i,1)))/(cosh(alfa*Xaux(i,1))^2); // 2.30a
    end
    Sigma=zeros(ntot,1);
    Sigma=E*edef; // 2.30b e 2.31b
    Xele=zeros(ntot,1); // displacement for each element (not global coordinates)
    Xele(:,1)=edef(:,1).*L(:,1);
    // Kinetic Energy and Potential elastic "Available"
    Ek=zeros(ntot,1);
    Ep=zeros(ntot,1);
    Ek(:,1)=0.5.*Mv(:,1).*wnat.*Udisp(:,1).*wnat.*Udisp(:,1)
    Ep(:,1)=0.5.*Sigma(:,1).*Sigma(:,1).*At(:,1).*L(:,1)/E;
    Ed1=Ep*4*%pi*zeta; // Dissipative Energy, approach1
    Pow1=Ed1/Tp; // Dissipated Power, approach1
    // Second approach, based on experimental observation (higher values of stress are correlated with higher values of damping.
    zetam=zeros(ntot,1);
    SigM=max(Sigma); // maximum stress
    zetam(:,1)=zeta.*(3.75*(Sigma/SigM)^2+0.25); // Weighting damping (higher at higher stress regions, ranging from 4 to 0.25 times the average damping)
    Ed2=Ep*4*%pi.*zetam(:,1); // Dissipated Energy second approach, using the weighting damping
    Pow=Ed2/Tp; // Dissipated Power second approach, using the weighting damping
    Km(1,1)=h(1,1)*Al(1)*Dt+k*At(1)/Ldist(1)*Dt+Mv(1)*cp; // K for first element (free edge)
    Km(1,2)=-k*At(1)/Ldist(1)*Dt;
    Km(ntot,ntot)=h(ntot,1)*Al(ntot)*Dt+k*At(ntot-1)/Ldist(ntot-1)*Dt+Mv(ntot)*cp; // Km for last element (plane of symmetry)
    Km(ntot,ntot-1)=-k*At(ntot-1)/Ldist(ntot-1)*Dt;
    for i=2:ntot-1
        Km(i,i)=h(i)*Al(i)*Dt+k*At(i-1)/Ldist(i-1)*Dt+k*At(i)/Ldist(i)*Dt+Mv(i)*cp;
        Km(i,i-1)=-k*At(i-1)/Ldist(i-1)*Dt;
        Km(i,i+1)=-k*At(i)/Ldist(i)*Dt;
    end
    for t=2:NInc // Time loop, for calculating the next step temperature
        tx=t*Dt-floor(t*Dt/Tpulse)*Tpulse;
        // if the time period is inside Ton, the power is "100%", else, it is a value between 1 and 0 (exponential decay due to damping)
        if tx<=Ton;
            a=1
        else
            a=exp(-zeta*wnat*(tx-Ton))
        end
        // Generated heat + Sensible heat + Convection
        Fv(:,1)=a*Pow1(:,1)*Dt + Mv(:,1)*cp.*Tv + h(:,1).*Al(:,1).*Text*D;
        Fv2(:,1)=a*Pow2(:,1)*Dt + Mv(:,1)*cp.*Tv2 + h(:,1).*Al(:,1).*Text*D;
        T=Km\Fv; // solving the equation system
        T2=Km\Fv2
        Temperatures(:,t)=T(:,1); // storing the temperature on the global temperature matrix
        Temperatures2(:,t)=T2(:,1)
        Tv=T; // updating the temperature vector
        Tv2=T2
    end
    scnsz = get(0,'screensize_px'); // screen size
    TamanhoScn=[scnsz(3),scnsz(4)]; // screen size

```

```

plot(Time',Temperatures(ntot,:)); // Central node temperature vs. time, Approach 1
plot(Time',Temperatures2(ntot,:), 'red') // Central node temperature vs. time, Approach 2
xlabel("Time (s)", "fontsize", 5); // x label
ylabel("Temperature (°C)", "fontsize", 5); // y label
aa=gca();
aa.font_size=5; // axis font size
aaaa=legend(['Analytical, Average Damping'], ['Analytical, Weighting Damping']); // legend
aaaa.font_size=5; // legend
aaaa.legend_location="in_lower_right"; // legend position
bb=get("current_figure")
bb.figure_size=TamanhoScn // maximizing the image
bb.figure_position=[0,0]
title('Temperatures, Time on= '+string(Ton)+ ' s. Time off= '+string(Toff)+ ' s. hint= '+string(hint)+' W/m^2. Sigmax= ' + string(max(Sigma)/10^6)+' MPa.', 'fontsize', 5); // image title
xgrid(1, 1, 5)

```

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