

Article

Interpolation of Pathway Based Non-Destructive Testing (NDT) Data for Defect Detection and Localization in Pre-Baked Carbon Anodes

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Abstract: Producing consistent quality pre-baked carbon anodes for the Hall–Héroult aluminum reduction process is challenging due to the decreasing quality and increasing variability of anode raw materials. Non-destructive testing techniques (NDT) have been developed and recently implemented in manufacturing plants to establish better suited and more efficient quality control schemes than core sampling and characterization. These technologies collect measurements representing effective properties of the materials located along a pathway between two transducers (emitter and receiver), and not spatially-resolved distribution of properties within the anode volume. A method to interpolate pathway-based measurements and provide spatially-resolved distribution of properties is proposed in this work to help NDT technologies achieve their full potential. The interpolation method is tested by simulating acousto-ultrasonic data collected from a large number of 2D and 3D toy examples representing simplified anode internal structures involving randomly generated defects. Experimental validation was performed by characterizing core samples extracted from a set of industrial anodes and correlating their properties with interpolated speed of sound by the algorithm. The method is shown to be successful in determining the defect positions, and the interpolated results are shown to correlate significantly with mechanical properties.

Keywords: pre-baked carbon anodes; non-destructive testing; interpolation; defect detection; acousto-ultrasonic



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1. Introduction

The Hall–Héroult (H–H) process is the only economically viable alternative for aluminum production since its development, and still requires a carbon source for the electrolytic reaction to take place. Consumable pre-baked anodes typically produced in the same facilities are used as the carbon supply. Calcined petroleum coke is mixed with coal tar pitch, formed into a green anode block according to the geometry required by the H–H cell design, and then baked in open ring furnaces. Following the baking, further adjustments may be applied to anode geometry. Then, a metal rod is fitted on its top surface, so that electrical current can travel through its body to reach the electrolytic bath, and produce metal aluminum in electrolytic cells [1].

In the cells, anodes must resist thermal stresses, allow the current to pass without generating excessive heat, and selectively react only with the dissolved oxygen in the bath [1]. Hence, the anodes must have sound mechanical properties, low electrical resistivity, and reactivity to air and CO₂. However, meeting these specifications consistently in spite of significant variations in raw material properties and fluctuations in manufacturing conditions is not straightforward to achieve. The standard anode quality control system currently used in most anode plants is based on sampling a low percentage of the anodes produced, extracting a small core sample from the anode body, and sending it for characterization in the laboratory. A core typically represents less than 0.2% of a full-size industrial anode

volume [2]. Electrical resistivity, Young's modulus, density, air reactivity and chemical composition are just a few examples of the properties measured on core samples. These characterizations are costly and time-consuming, and results are typically available after a few weeks delay [3]. Such a procedure is suitable when incoming raw material properties are stable, which is no longer the case.

Since the last 10–15 years, the quality of coke and pitch materials has been steadily decaying while variability in their properties has significantly increased [4,5]. Since core characterization cannot provide the information needed for quality control in a timely fashion, manufacturing consistent quality anodes is more challenging. This potentially increases the overall carbon and energy consumption required to produce liquid metal.

However, since the 1980's, a number of rapid and non-destructive testing (NDT) methods have been investigated in order to measure either the anode block's electrical resistivity, its density, or some other attributes related with mechanical properties (e.g., speed of sound). The first method available in the public literature is E.J. Seger's [6,7] electrical resistivity measurements performed in multiple baked anode positions and collecting over 30 voltage drops. In the last decade, a few NDT technologies have progressively been introduced into anode plants, and their performance is currently being assessed. For example, Alcoa's 4-point-probe (4PP) [8], SERMA [9–14], and MIREA [15–18] technologies were developed and implemented to measure electrical resistivity in multiple points using a similar concept to Seger's, but using different probe arrangements for application on green and/or baked anodes. Finally, mechanical properties-based measurements were also investigated, namely Modal Analysis (MA) [19], which measures the vibration response of the whole anode body at once after some mechanical excitation, and acousto-ultrasonic (AU) based methods [20–24] applied in multiple anode positions.

Except for MA, the data collected by all of the above-mentioned NDT technologies represent some effective property of the materials contained along a certain pathway between two transducers (i.e., emitter and receiver). The pathways are more or less linear depending on measurement technology and the anode's internal structure. Collecting such measurements at multiple positions on the anode blocks, including different faces, have shown promising results for detecting defects (e.g., cracks, inhomogeneities) [17,19–21,23]. However, since the measurements obtained from these devices are not spatially-resolved in the anode blocks, the information extracted from them is not exploited to its full potential.

To illustrate the point, we use an analogy with X-Ray computerized tomography scan (or CT-scan), an NDT technique used in medicine, but also in many other fields, such as materials inspection. The basic principle is as follows. An X-ray source is located on one side of the object of interest and emits an X-ray fan beam passing through it, and collected by a set of detectors arranged in a semi-circular fashion on the other side of the object. The measured signals represent an averaged X-ray attenuation of the materials along linear pathways between the X-ray source and each detector. The source-detectors assembly rotates around the objects while measurements are continuously collected, and the object is moved so as to scan it partially or completely. An image of the object's internal structure is then obtained by using 2D reconstruction algorithms to interpolate the X-ray attenuation measured along the beams (pathways) in order to obtain a 2D spatially-resolved image [25,26]. Hence, interpolation of the data collected along the pathways leads to richer information for detection and localization of anomalies. Although, X-ray CT-scan was already applied for imaging the internal structure of green and baked carbon anodes [27,28], commercial devices are too small to fit a full-size anode (i.e., needs to be cut in pieces prior to be scanned), and collecting such measurements are highly time-consuming and costly. Thus, X-ray CT-scan cannot be used for on-line quality control of the anodes required in the primary aluminum industry. However, how CT-scan data are processed suggests that more information can be extracted by interpolating data measured along pathways.

To the best of the authors' knowledge, the data collected by all the NDT technologies reviewed previously are analyzed using averaged pathway data, and obtaining spatially-resolved distributions of properties within anodes blocks from these measurements was

not yet investigated. Having access to such distributions may lead to more precise defect detection and localization in the anodes. It may also help diagnose anode internal defects and assign root causes, such as fluctuations in raw material properties or by the manufacturing process. For example, density gradients caused by the forming step or uneven baking. Distribution of properties may also be used to quantify the anode level of homogeneity. This could pave the way to developing more advanced anode quality control schemes, involving feedback adjustments to specific process units and/or anodes sorting procedures.

This work aims at developing an interpolation algorithm using pathway-based signals to infer the spatial distribution of a measured property and use this information to locate the position of defects. This is one of the first techniques applied to baked carbon anodes which automatically detects and locate defects. Efficient interpolation algorithms are important since probes can be expensive to purchase and maintain, and therefore, their number used to sense the anode volume in any technology is limited. For example, only 8 transducers were used in this study to sense the anode volume while off the shelf ultrasonic imaging systems uses arrays of over 64 elements [29]. To test the algorithm's performance, acousto-ultrasonic is used as a candidate NDT technology to illustrate the proposed approach. It was chosen since it is a promising technique for detecting defects in the anode volume [19–21,23]. Furthermore, since the mechanical wave fans out when propagating through a solid body from the emission point, its representation by a set of linear pathways between an emitter and several receivers is a reasonable assumption. A large number of toy examples simulating simplified representations of pre-baked anode internal structures are generated, from which AU results are obtained for different sensor configurations. The simulated data are then used as inputs to the interpolation algorithm as a means to evaluate its capacity to detect and locate defects. Experimental validation of the algorithm is also performed by comparing properties measured from core samples extracted from a set of full-size industrial anodes with the interpolation results obtained in the extracted core position.

2. Materials and Methods

Simulating realistic distributions of properties in baked anodes, which are intrinsically anisotropic, is challenging for many reasons. First, heterogeneity is caused by several complex phenomena, such as fluctuations in raw material properties, uneven distribution of the various components and density gradients within the anode blocks caused by the mixing and forming steps, porosity created by degassing of volatiles and crack formation during the baking step, differences in the properties of coke and cokefied pitch, etc. Second, it is difficult to predict the spatial distribution of heterogeneities in the blocks as some of the phenomena causing them are stochastic. Finally, very few data revealing the internal structure of sets of anodes encompassing typical variability encountered in practice are available in the literature. Some X-ray CT-Scan images of pieces of baked anodes were published in the past since these instruments do not accommodate for samples as large as full-size anodes [27–30]. However, the number of anode samples analyzed using this technique are too few to represent typical variability.

Hence, it was decided to generate toy examples representing simplified distributions of anode internal structures, including randomly distributed heterogeneities to which larger defects are added. These heterogeneities are assumed to cause differences in the speed of sound propagating through the anodes since acousto-ultrasonic measurement technology is used to illustrate the proposed approach. The objective is to assess the capacity of the interpolation algorithm to successfully detect and locate the position of the defects.

Experimental validation is performed according to the following steps. First, a set of baked anodes were scrutinized using an AU setup which collects the measurements at multiple points along the anode's body. Then, core samples were extracted from selected points of interest on the anode's top surface so as to span a wide range of properties, followed by their characterization in the lab. The properties measured on the cores are then

compared with the interpolation results by the algorithm obtained in the positions where the cores were extracted.

The subsections below discuss how toy examples were generated and AU data were simulated. The proposed interpolation algorithm is then presented, followed by the metrics used to assess the method's performance. Finally, the experimental validation procedure is detailed.

2.1. Toy Example Creation

Toy examples were built by combining two layers, as shown in Figure 1a,b presenting a simulated example of a 2D slice of an anode. Each layer consists of applying a grid of a selected density onto a vertical anode slice, so as to span the entire slice area. The mesh is set to 161 points for the anode length, 63 for the height, and 66 for the width. The first layer is a base layer where variations in speed of sound (i.e., proxy for anode mechanical properties) are generated using a normal distribution centered on 3 km/s and a standard deviation of 0.2 km/s. Every point on the mesh is a value randomly sampled from that distribution (Figure 1a). The choice of average and standard deviation in this case was arbitrary. For the second layer, a Bell function is used to create one or several defects in specific positions scaling the maximum value to 2 and minimum to 0 with the covariance matrix having different values to simulate circular defects of 3%, 6% and 12% of the simulated anode length (Figure 1b). The defects simulate localized regions in the anode where the material density changes due to the presence of porosity, cracks, etc. Changes in density affect the speed of sound in the material, and this is how they are detected. However, knowledge of the acoustical properties of the defects are not required to detect changes in speed of sound by AU. The final toy example was generated by subtracting the values from each point in the second layer from the corresponding points in the first layer (Figure 1c). Note that simulated examples have similar dimensions to that of industrial anodes. However, anode dimensions are proprietary information and cannot be disclosed.

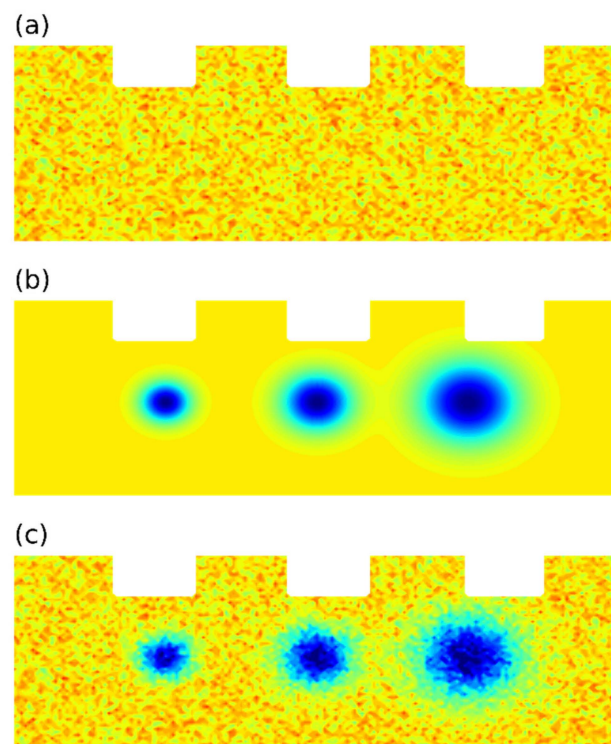


Figure 1. Illustration of toy example generation. Part (a) refers to the base layer built from random sampling, (b) is the defect layer to be subtracted, and (c) is the final toy example resulting from combining (a,b).

A large number of 2D toy examples were generated as shown in Figure 1c by varying the number of defects, their position and size. The decision for the selected defect sizes was taken by visual observation of the 2D resulting toy examples to ensure significant visual size change are obtained. It is understood that the circular/spherical shape of the defects does not match that of typical defects found in industrial anodes (e.g., cracks). However, it was chosen for its simplicity, given that the simulation of a crack by means of an ellipsoidal region involves a larger number of parameters. In addition to its size, the ellipsoid's orientation (horizontal, vertical, or inclined) and its level of eccentricity need to be selected. Furthermore, the objective of the proposed approach is to detect local changes in the properties of the material introduced by defects and not their shape. Thus, in this first study, circular/spherical defects were adopted to assess the method's performance. More accurate defect representation will be considered in the future. Hence, a total of 100,155 2D toy examples were generated, about the same number of examples with 1, 2, and 3 defects. For the one-defect cases, the different positions were obtained by taking all combinations of 183 equidistant points over the length with 183 also equidistant over the height direction points. The 2 and 3 defects examples were generated by sampling randomly 33,333 defect positions 2 and 3 times from the grid. Note that even though the anodes have a 3D structure, presenting interpolation results for 2D examples is relevant for two reasons. First, it provides a simpler way to illustrate and visually appreciate the method's performance in a simpler environment before moving to more complex 3D situations. Second, the proposed interpolation technique is generic, and could be applied to other materials for which a 2D structure can be assumed, such as in [21,22].

For 1-defect cases, the rationale for generating 3D toy examples was the same, but considering 33 possible points for positioning the defect along the anode height, and the same number of points along the height and width, and simulating a full-scale anode. The two and three defect combinations were also randomly sampled from the one defect cases, as with the 2D samples. Finally, 35,937 1-defect samples, 33,333 2-defect samples, and 33,333 3-defect samples were generated. A total of 1,216,548 toy examples were tested encompassing 2D and 3D cases, 3 different defect sizes, and 1, 2, or 3 defects cases.

2.2. Acousto-Ultrasonic Data Simulation

For each of 2D and 3D cases, two arrangements of receivers are simulated. One is a replica of an in-house equipment [24] with receivers placed solely on the bottom of the sample, and for the second a number of sensors are added on the sides to assess whether these additional sensors could enhance the method's capacity to detect and locate the position of defects. The two sensor arrangements are referred to as configurations S1 and S2, respectively, in the remainder of this article.

The 2D cases are built using seven emitter positions on top of the sample. Four of those are on the actual top surface of the anode and the remaining three are inside the stub holes. In the S1 configuration, five receivers are located on the bottom surface of the anodes, equally spaced along the anode's length (Figure 2a). In the S2 configuration, six additional receivers are added on the sides, three on each side (Figure 2b). All the signal pathways, referred to as beams (red lines in Figure 2), are simulated, which means 35 beams per toy example in S1 and 77 in S2.

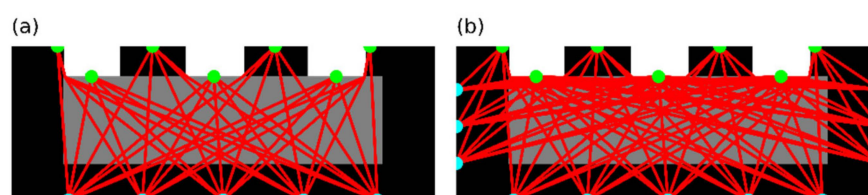


Figure 2. Pathways defining beams in 2D toy examples (a) S1 configuration, and (b) S2 configuration. Emitters are represented by a green dot, while receivers are blue. The gray zone defines the limits for the region referred to as the central region in the anode block.

The position of the transducers in the 3D toy examples are shown in Figure 3. The anode surfaces are labeled T, B, S, and L for the top, bottom, short, and long sides of the anodes, respectively (Figure 3a). The 21 excitation positions on top of the simulated anodes (i.e., emitter positions) are shown in Figure 3b (green dots). Three excitation positions are located in the stub holes (identified by gray circles). For each excitation, a total of 7 receivers are used in S1 configuration. The position of the receivers for all excitation points are shown in Figure 3c (red dots). However, it is important to understand that only 7 receiver positions are active for each excitation. The five receivers located along the center of the anode in Figure 3c are kept fixed for all excitation points. The other two receivers located on each side of the anode (labeled R1 and R7 in the figure) are moved with the emitter. For example, when the emitter is positioned in the top left corner in Figure 3b), the receivers R1 and R7 are positioned as indicated by the red squares in Figure 3c. Another example is provided for an excitation in the central part of the anode (see blue squares in Figure 3b,c). Hence, the letters A–G in Figure 3b,c indicate how the two mobile receivers (R1 and R7) are moved with the excitation points along the length of the anode. The second sensor configuration S2 uses all positions of S1, but adds 9 excitation points on the short side (Figure 3d) and 15 on the long side (Figure 3e). For the S2 configuration, the receivers are positioned vis-à-vis each excitation point, but on the opposite sides of the anode. Hence, the number of resulting beams is 147 for S1 and 1,407 for S2. The relationship between the S1 and S2 configurations in 2D and 3D examples shown in Figures 2 and 3 is as follows. The S1 configuration in 2D (Figure 2a) correspond to the excitation along the center of the anode in the 3D cases (Figure 3b,c). The S2 configuration in 2D (Figure 2b) corresponds to the excitation on the short side in 3D (Figure 3d), but only in the center of the short side. Excitation from the long side of the anode (Figure 3e) does not exist for the 2D examples.

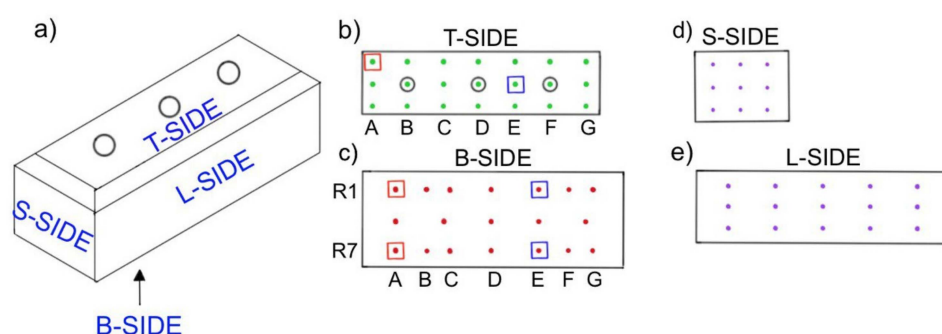


Figure 3. Sensor positioning for the 3D toy examples. (a) Identification of the anode sides, (b) excitation positions (emitter) in S1 configuration (green dots), (c) receiver positions in S1 configuration (red dots), (d) excitation positions on the short side in S2 configuration, and (e) excitation positions on the long side in S2 configuration. Red and blue squares illustrate how receivers R1 and R7 move with the emitter along the length of the anode in S1 configuration.

The simulated ultrasound signal used to represent a speed of sound measurement will be the one which follows the shortest path between the emitter and the receiver, consistent with theory of wave propagation. In that case, the path should be either a straight line, or a pair of lines that minimize the distance between emitter and receiver while avoiding the stub holes. It is traveling a heterogeneous media with varying properties, and thus associated with many different speeds of sound (i.e., see Figure 1c); therefore, the final speed should be a composite of those. To obtain the final effective speed, the shortest path between the emitter and a receiver is first established. For paths that do not cross a stub hole, the shortest path is simply a straight line connecting the emitter to a receiver. If the line crosses a stub hole, it is assumed that the shortest path is formed by two lines: one from the emitter to a point on the stub hole surface, and another from the stub hole surface to the receiver. The exact point on the stub hole surface is determined numerically according to the procedures described in Supplementary Materials (Section S1, Figures S1 and S2). Once the shortest path is identified, it is subdivided in 1600 subsections of equal length.

The speed assigned to each subsection is the speed corresponding to the point on the mesh that is the closest to a given subsection. This is illustrated in Figure 4. Finally, effective speed is calculated by Equation (1):

$$\vec{v}_{avg} = \frac{D}{t} = \frac{D}{\sum_i t_i} = \frac{D}{\sum_i \frac{d_i}{v_i}} = \frac{D}{d_i \sum_i \frac{1}{v_i}} = \frac{D}{\frac{D}{N} \sum_i \frac{1}{v_i}} = \frac{N}{\sum_i \frac{1}{v_i}} \quad (1)$$

where D is the path length, t is the total travel time, t_i is the travel time of a subsection, d_i is the length of a subsection, v_i is the speed in a subsection, and N is the number of subsections. This is represented schematically in Figure 4. The equation is applied to all simulated beams.

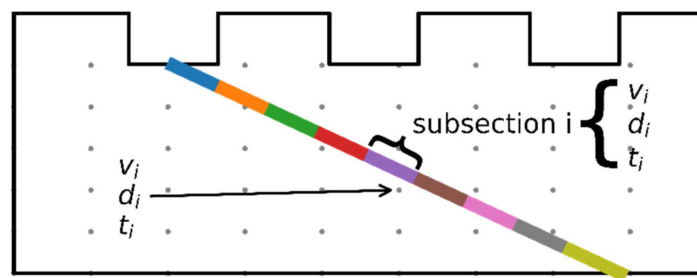


Figure 4. Simplified example showing a beam divided in subsections represented by different colors. How the properties are assigned to a specific subsection based on the closest point on the grid indicated by the arrow.

2.3. Proposed Interpolation Algorithm

Considering a pre-defined mesh, the property in a specific point within the anode block is estimated using the weighted average of the “ k ” nearest beams properties, which is the first hyperparameter in this method. The weight assigned to the properties of each beam is the inverse of the distance of a beam from the point of interest, which forces the algorithm to give more weight to closer beams, but it is limited to a minimum distance (i.e., second hyperparameter), to avoid giving too much weight to only one beam. This algorithm is inspired by the k -nearest neighbors’ algorithm [31], but adapted to use pathways instead of points as the data used to generate the model. For a specific point of interest in the anode volume, the algorithm is as follows:

1. Calculate the distance between the point to all beams;
2. Sort the beams by distance to the point from the closest to the farthest;
3. Compute the weighted average property using the k -closest beams where the weights are $1/\text{distance}$;
4. Repeat the procedure for all desired points.

However, as the sensor arrangement, and consequently the pathways, are always the same, there is no need to completely reproduce this algorithm for every toy example. Hence, for a given emitter-receiver configuration and 2D or 3D grid of points, the distance of each point to the beams is the same in all toy examples. Hence, interpolation results can be obtained by the following simple matrix product:

$$\hat{Y} = X \cdot W \quad (2)$$

In this equation, W is the weight matrix having dimensions of $(p \times m)$, where p is the number of emitter-receiver pairs, and m the number of points of interest on the grid. Each weight is the reciprocal of the distance between a point of interest to a given beam, but those beams not amongst the k -nearest neighbors are assigned a zero value. The measurements collected from each pair of sensors, speeds of sound in this case, are stored in row vector x ($1 \times p$). Finally, $\hat{y}$ ($1 \times m$) contains the interpolation results for each point of interest. For example, in the 2D S1 configuration, x is a (1×35) vector and W

is of dimensions (35×10143) . Using this implementation, the interpolation results are calculated in less than a second using one 3.3 GHz core in the highest dimensional example (i.e., 3D S2 configuration). In the training phase of the algorithm, obtaining the W matrix took approximately 12 h using the same core. However, in a plant implementation, the training step would be performed only when the sensor configuration is changed, and application of the algorithm on the production line would use the fast implementation shown by equation 2. Furthermore, all calculations can be accelerated further if a GPU is used instead of a processor core. Finally, considering the mesh size selected for 3D toy examples, each voxel for which a speed of sound is interpolated corresponds to 0.0001% of the full anode volume.

2.4. Performance Assessment

The defect definition in the simulation part of this work is based on prior knowledge of the toy examples. Knowing that all the defects created in the toy examples have reduced speed of sound to mimic the presence of voids and cracks in the anodes (a choice made by the authors), it is assumed for illustration purposes that all the zones having a value lesser than the average sound speed minus 2.5 standard deviations are considered defect areas. This is applied to both the ground truth (toy examples) and interpolation by the proposed algorithm. Hence, defects detected by the algorithm can be compared with ground truth, and performance metrics calculated. In this work, the Adjusted Balanced Accuracy Score (ABAS) [32] was selected to assess the algorithm's performance in defect detection and localization using the *sklearn 1.0.2* implementation [33]. Moreover, the distance between the center of the ground truth defects and the center of the defects found by the interpolation algorithm is also used as performance criterion.

The ABAS is calculated by comparing every pixel or voxel (2D and 3D, respectively) pairwise between the ground truth and the interpolation. This value is a metric developed to deal with unbalanced datasets, which is the case here. Its value ranges from $-\infty$ to 1, with value 0 representing random performance, and 1 a perfect score. Figure 5 illustrates a bidimensional example showing ABAS scores. One can observe that when the classification is in good accordance the score value is positive.



Figure 5. Examples of ABAS results in different cases of agreement between ground truth and model.

The expression used to calculate ABAS is shown below where positives refer to points considered as belonging to a defect region and negatives to sound material. TP, TN, FP, FN, and C refer to the number of true positives, true negatives, false positives, false negatives, and the number of classes, respectively ($C = 2$ in this work).

$$ABAS = \frac{\left(\frac{TP}{TP+FN} + \frac{TN}{TN+FP} - \frac{1}{C} \right)}{1 - \frac{1}{C}} \quad (3)$$

Assessing the performance of the algorithm when more than one defect is present in the ground truth poses a challenge for computing the distance between defects in the ground truth and interpolation results because the algorithm does not assign different values for each defect. Thus, a k-means clustering [34] algorithm was then used to establish the center of the defects detected by the algorithm. The number of clusters used in the k-means clustering was the number of defects expected and then the distance between all

combinations between the ground truth centers and the clustered centers pairwise was calculated. The combination that minimized the total distance was assumed to represent the correct assignment of which real defect center refer to the estimated defect center, and the average of the distances was used as final score. In practice, the number of clusters would not be known, however ABAS calculations are only intended for off-line performance assessment of the method and is not required for practical on-line implementation.

2.5. Experimental Validation

Finally, as a validation procedure, 34 industrial pre-baked carbon anodes were sampled and analyzed using an apparatus developed for AU testing. The equipment used consists of an adapted manual forklift with all the necessary equipment to move the sensors and acquire the data quickly. It is described in more details elsewhere [24] as well as in the Supplementary Materials (Section S2 and Figure S3).

An anode is positioned over a metal support and the lift is adjusted so that all the sensors are positioned under the anode as planned. Vacuum grease is applied to every ultrasonic transducer to ensure good contact and all the receivers are put into contact with the anode. The emitter is then positioned, and the wave sent through the anode at one excitation position, while all seven receivers are collecting data. This is repeated until it required adjustment of the lateral mobile receivers, or until all 21 excitation positions are tested. The distance between emitter and receiver is calculated using the geometry of the anode, and the time of arrival (or time of flight) is obtained by first estimating the noise level of the signal by calculating the maximum amplitude in the first 100 microseconds, which establishes a threshold, and then finding the first moment that the signal crossed that same threshold. Speed is then calculated as the ratio of that distance to the travelled time. This method is inspired by [35]. The first 100 microseconds time window was selected since it is very unlikely that, based on the anode dimensions, the wave travels between the emitter and a receiver in a shorter time. This was also confirmed by visual examination of the AU signals. An example time of flight and speed of sound calculations can be found in the Supplementary Materials (Section S3 and Figure S4).

The measured data were used as inputs to the interpolation procedure (using $k = 10$ and minimum distance = 1.25% anode length). The interpolation results were then visually inspected, and regions close to the top surface of the anodes showing very different speeds of sound were selected to extract the core samples. An example of interpolation results close to the top anode surfaces and the positions to extract core samples is shown in Figure 6. A total of 60 cores were extracted from the set of 34 anodes. The core samples were then characterized for apparent density, electrical resistivity, and Young's modulus and their resulting values were compared to the average speed of sound of the interpolated points positioned inside the region where the cores were extracted. The hypothesis is that the speed of sound is correlated with these measurements. If the interpolation results indeed correlate well with the measured properties, this would provide evidence supporting that the interpolation method is successful to a certain degree. Note that this validation approach was selected since we did not have access to the full internal structure of the 34 industrial anodes, which is required to assess the method's performance to detect defects.

With the experimental data collected, the hyper-parameters of the algorithm were adjusted in order to maximize the correlation between the interpolated speed of sound in the extracted core positions and their characterization results. The quality of the regression was assessed by reporting the p -value of the slope. To find the optimal values of the hyper-parameters, they were varied between $k = 1$ –20 neighbors and 0.06–2.5% of anode length. The combination that achieved the smallest p -values for all three properties measured were used for final evaluation of the correlation, and validation of the method's performance.

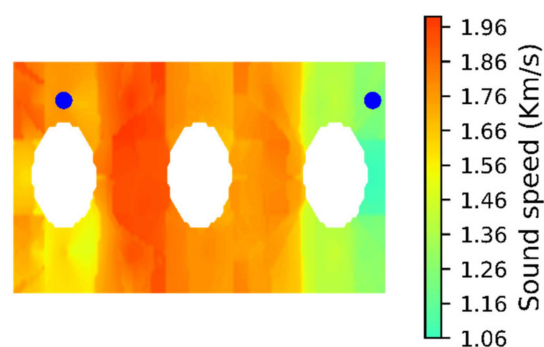


Figure 6. Example of top face interpolation for one industrial pre-baked anode and cored positions indicated by the blue circle. The white circles represent the stub holes, and the color bar is proportional to the interpolated speed of sound.

3. Results and Discussion

3.1. Simulated Toy Examples

The results obtained for 2D toy examples and S1 sensor configuration are shown in Figure 7a,c by means of box plots. In this figure, the various cases are identified by the number of defects (1–3) and their position (centered or uncentered). Defects identified as “centered” are located within the gray area shown in Figure 2, whereas “uncentered” defects are located outside this region. For example, 3-c refer to cases with 3 centered defects whereas 1-uc corresponds to cases with 1 uncentered defect. The combined results for centered and uncentered defects separately for each number of defects are also provided (identified by 1, 2, and 3 defects) as well as the overall results based on all simulated cases. The ABAS values are mostly positive and their medians are all positive. The centered ABAS results are significantly larger when only one defect is present. However, the ABAS results are much more similar in presence of more than one defect. For one defect, the distances between the defect in the ground truth and the detected one are quite similar, but when more than one defect are present, the difference in performance is more important between the uncentered and centered defect positions. It is quite clear that having more than one defect causes the method’s performance to deteriorate using the S1 sensor arrangement (i.e., sensors placed on top and bottom of the anodes).

However, as expected, the additional sensors used in S2 allow for a more robust performance since the ABAS values are higher in all cases in comparison with S1, as shown in Figure 7b. Additionally, for a given number of defects, the difference between centered and uncentered defects is smaller. There is a performance drop with increasing number of defects, but it is much less important than for S1. The distances are also smaller in most cases (Figure 7d). Overall, adding more sensors in the 2D case definitely enhanced the method’s performance for both ABAS and distance measures.

Classification results for some of the 2D toy examples are presented in Figure 8, as well as the impact of defect sizes. First, defect detection and localization for a specific 2D example with a single defect of 12% size are illustrated for both sensor configurations (Figure 8a,b). For one defect only, the addition of sensors (S2) clearly enhances the defect localization performance since the defect region clearly have positive ABAS results.

The impact of defect size on classification performance (i.e., ABAS) is shown in Figure 8c–h. These colormaps represent the spatial distribution of ABAS results, when a single defect of a given size is positioned on each point of the grid (or pixel in the colormap image). An interpolation method leading to nearly perfect classification would lead high ABAS values everywhere in the toy examples. The figures show that ABAS results improve with defect size. Indeed, a defect is easier to detect and locate when it is larger. Furthermore, the colormaps support the higher classification performance of S2 since ABAS results are distributed more homogeneously. The S2 configuration not only increases the average ABAS at each position in the toy examples, but also yields better scores in the right and left edges of the anode since these regions have clearly positive ABAS results (Figure 8c,e,g

compared with Figure 8d,f,h). This is explained by the sensor arrangement in S1 which does not gather any information about those regions. There are still regions where the method is better behaved, namely, the bottom part of the anode. This is probably caused by the distribution of beams in this region which is denser and better distributed.

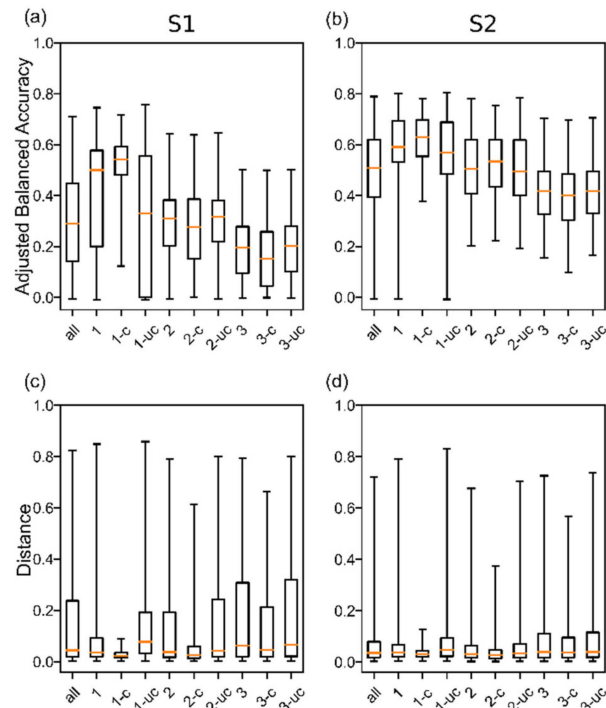


Figure 7. Boxplot distributions of the ABAS (a) and distance (c) metrics for the 2D S1 configuration, and (b,d) for the 2D S2 configuration. The cases are identified by the number of defects and by defect position with “c” referring to centered cases and “uc” to uncentered cases. The distances are normalized values.

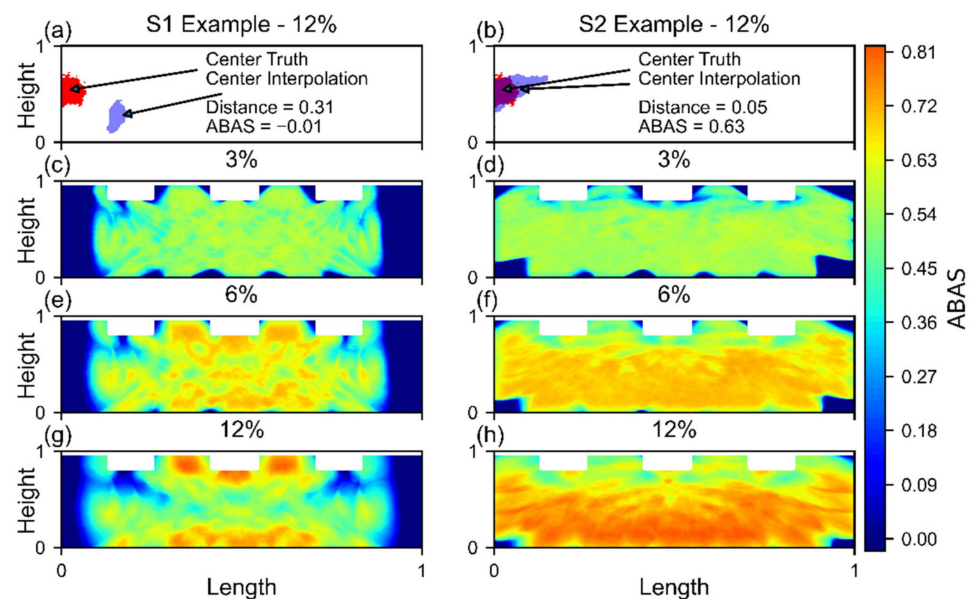


Figure 8. Interpolation and defect detection results obtained with the proposed algorithm. Illustration of a specific 2D toy example for S1 (a) and 2 (b). Colormaps of ABAS results for one-defect cases and S1 with 3% (c), 6% (e), 12% (g) defect sizes and the same for S2 (d,f,h). The color represents the ABAS obtained for the example with a defect centered in the pixel's position. The anode dimensions (height and length) are normalized.

For the 3D toy examples, the ABAS scores are lower compared with the 2D cases (Figure 9a,b), which is expected even if the distances between the true and detected defects do not change significantly. This is explained by the fact that ABAS is calculated using a pairwise pixel/voxel comparison. There is an expected drop in the fraction of correct classifications when adding the third dimension. Consider two overlapping circles of the same radius representing a defect (ground truth) and its predicted location by classification using the algorithm. The ratio of the intersection area to the area of one circle would give a larger value compared with the ratio of the intersection volume to the volume of a sphere of the same radius. Given the ABAS is proportional to these ratios, the 3D ABAS scores are therefore systematically lower. Most of the values are still significantly higher than 0, which is a confirmation of relatively successful positioning (Figure 9), which is an indication that the classification is adequate. Furthermore, centered defects are more easily detected than uncentered ones as in 2D toy examples, although the performance drop is smaller.

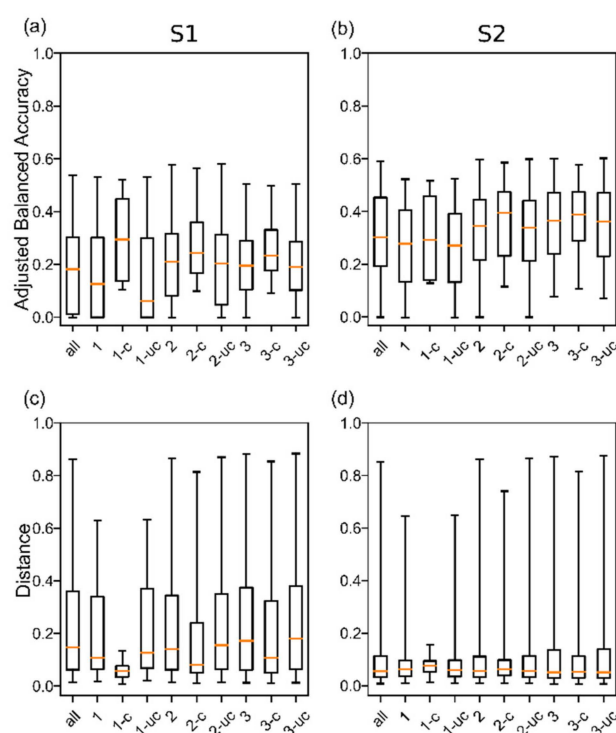


Figure 9. Boxplot distributions of the ABAS (a) and distance (c) metrics for the 3D S1 configuration, and (b,d) for the 3D S2 configuration. The cases are identified by the number of defects and by defect position with “c” referring to centered cases and “uc” to uncentered cases. The distances are normalized values.

The distances between the true and detected defects also increase significantly and the same 2D to 3D rationale can be used to partially explain that loss of performance. Compared to its 2D case counterpart, the major difference in behavior is the distance increase when adding defects and the smaller difference between centered and uncentered in the cases that have multiple defects.

Adding sensors on the sides (S2) again improves overall performance, which confirms what was observed in the 2D case (Figure 7b,d). ABAS scores increase in most cases, centered and uncentered, and with more than one defect. Distances also decrease, and, most importantly, the range of ABAS scores decrease in most cases, making the results more trustworthy than the case with less receivers. However, the impact seems smaller than for 2D toy examples, most likely because the added beams in 3D provide more information about regions not covered by the sensor arrangement of the first configuration instead of increasing beam density in the central region.

The size of the defects also significantly affects the method's performance. This is illustrated in Figure 10 showing the distribution of ABAS scores for the 3D toy examples having a single defect. Smaller defects clearly generate smaller ABAS values in both sensor arrangements (Figure 10). However, virtually no ABAS scores are below zero, which is an excellent indication of the method's performance. The addition of sensors slightly increases the ABAS results for the case of one defect and also reduces their range as shown in Figure 10. The same trend is observed for all sizes, but larger defects showed better enhancements. This behavior might be explained by the fact that the new added pathways in the second sensor configuration in 3D cases are mostly in the uncentered regions, which was not the same in the 2D case where the pathway density in the center increased significantly.

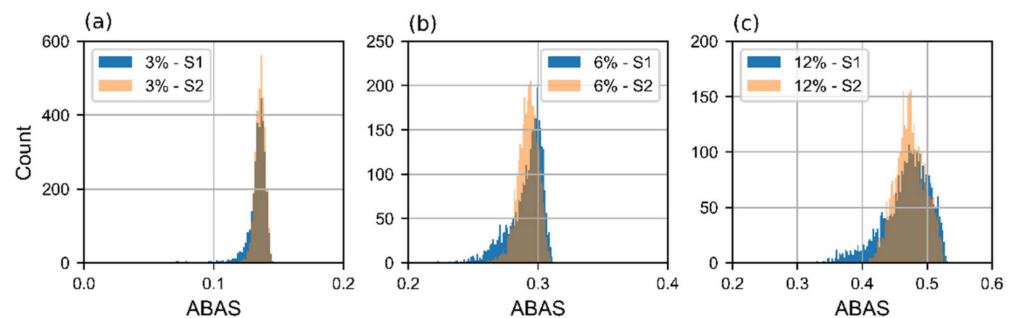


Figure 10. Distribution of ABAS scores for the 3D toy examples having a single defect in both sensor configurations for (a) 3%, (b) 6%, and (c) 12% defect sizes.

3.2. Experimental Validation

The experimental data collected from industrial anodes (i.e., characterization of core samples) provided an opportunity to adjust the interpolation algorithm's hyper-parameters (number of neighbors and minimal distance limit) to maximize the correlation between the interpolated speed of sound in the region where the cores were extracted, and the properties measured from these core samples. The correlation is quantified by the slope of the linear regression line between the measured properties and interpolated speed of sound. More specifically, the p -value of the Student's t -test performed on the regression slope was minimized by adjusting the hyper-parameters. Increasing the number of neighbors leads to higher p -values for the slope of the regression lines for the core apparent density as shown in Figure 11a. The minimum distance limit seems to have the opposite effect, but smaller in magnitude compared with the number of neighbors.

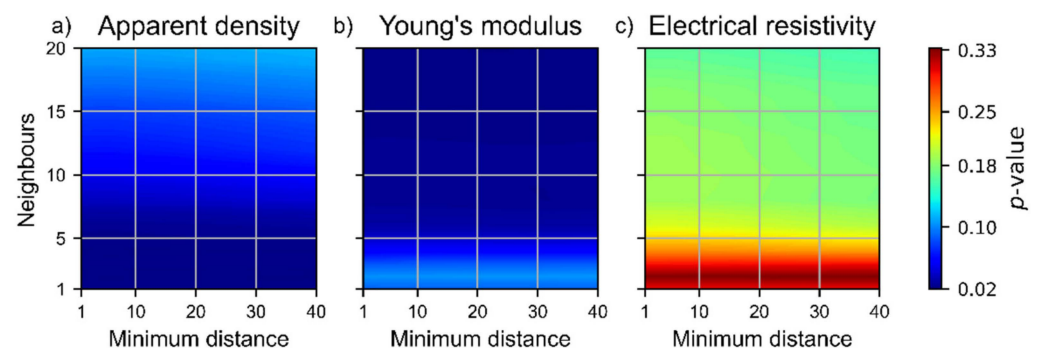


Figure 11. Colormaps for the linear regression slope's p -value using the average estimate of speed of sound speed in the extracted core samples position for (a) Apparent density, (b) Young's Modulus, and (c) Electrical resistivity.

For Young's modulus, the number of neighbors has an opposite trend compared with apparent density (Figure 11b). The fewer the neighbors, the better the correlation is, which

is an interesting trend, for it gives a compromise region where both correlations have a p -value under 0.05. The distance limit, again, does not seem to affect the result much.

Finally, the electrical resistivity (Figure 11c) follows the same trend as Young's modulus, but with a slightly higher impact of the distance limit. The p -values are always above 0.15, which could be explained by the complexity of that property. For better observation and understanding of the correlations, the results of the average interpolated speed of sound using 9 neighbors and 15 mm as the distance limits were plotted in Figure 12. The values are slightly different than the ones used for the toy examples because the objective was not the same. Moreover, collecting acousto-ultrasonic measurements on real industrial anodes is more complex, which might require adjusting the algorithm's parameters. It is expected that for each combination of equipment and anode geometry, the optimal parameters might be different.

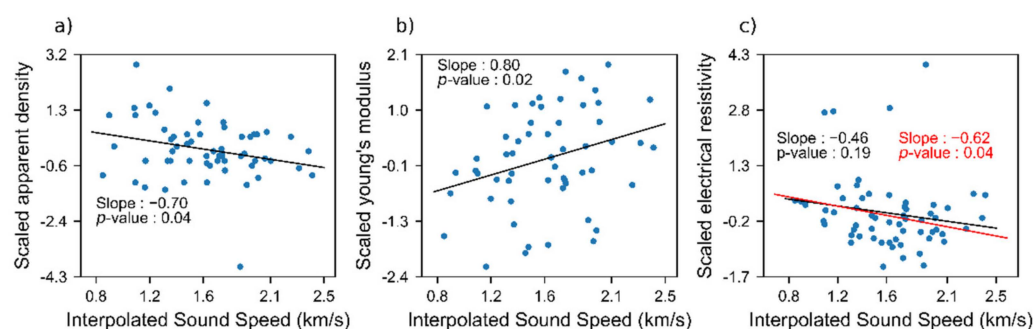


Figure 12. Linear regression results between the anode properties measured on core samples and the average interpolated speed of sound in the vicinity of the extracted core positions for (a) Apparent density, (b) Young's Modulus, and (c) Electrical resistivity. The slopes, as well as their p -values, are indicated in the plots. In (c), the red data refer to the correlation obtained after excluding the farthest outlier.

Overall, the level of errors in the regressions is too high to use the proposed algorithm to make precise predictions for the anode properties. However, they can be used as a way to validate the method in the absence of more detailed information about the anodes internal structure. The correlations for the apparent density (Figure 12a) and Young's modulus (Figure 12b) are both statistically significant at a level of confidence of 5%. This suggests that the speed of sound correctly estimates the distribution of these properties. Regarding the lower correlation for the electrical resistivity, there are four outlying points affecting the regression slope (Figure 12c). These outliers might be caused by measurement error or by the existence of a small, localized defect (i.e., cracks) in the core samples. Excluding just one of them prior to estimating the regression slope already leads to a p -value of 4% as shown in Figure 12c, which is comparable to the other two properties.

The errors in predicting the anode properties may also be caused by the averaging nature of the speed of sound measurements. As it is affected by the entire anode structure along the path between emitters and receivers, any large change outside the average in specific places affect the final measurement, but the change is smoothed by all the other values. The algorithm proposed, as it is based on a weighted average, will, at best, range from the minimum beam value to the maximum beam value which are already smoothed. The resulting values clearly follow a distribution that is relevant, given the relative success that it has in positioning defects, but it cannot deliver values that encompass the same range as those of the raw values. It is probable that an algorithm that can rebuild the original value ranges would enhance these correlations and deliver even more value from the same dataset. It is also to be noted that the method was not developed to estimate local properties accurately, but rather estimate their spatial distribution, which is successfully confirmed as there is a statistically significant correlation between measurements and estimated properties.

4. Conclusions

In the present context of NDT research applied to carbon anodes, where multiple technologies available are based in using a limited number of sensors measuring effective properties of the materials contained along pathways, there is an incentive to develop efficient interpolation algorithms to maximize the information extracted from the data. A new interpolation method providing spatially-resolved estimates of some anode properties using pathway-based measurements was proposed. It was tested on over 100,000 2D and 3D toy examples using two different sensor arrangements. The method's performance is affected by the density of beams used to sense the samples and also by the number of defects in them. The classification performance of the algorithm shows very good potential, with ABAS scores—an objective performance measurement—being almost always over zero. This is a strong indication that the algorithm works well. Additionally, the distances between the true defects in the simulated examples and the defect detected by the algorithm are relatively small, also confirming the method's capacity to correctly locate the defects.

The experimental results also confirm the potential of the interpolation method by showing that the interpolated speed of sound values significantly correlate with anodes' mechanical properties (p -values < 5%). The methods' parameters were optimized to maximize the correlation between the interpolated speed of sound in the vicinity of extracted core samples and the core properties. The proposed approach is not intended for making precise predictions of the anode's properties. However, in addition to detecting and localizing defects, it might be used to assess the anodes level of homogeneity.

Future work will look at applying the interpolation method to a larger set of industrial anodes, and to perform more in-depth validation by, for example, cutting anodes to get access to their internal structure. Ways to assess anode homogeneity will also be explored.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/met12091411/s1>, Figure S1: Top surface view of an anode showing an AU beam pathway crossing a stub hole, for illustration purpose; Figure S2: Simplified scheme to represent the different possible cases to calculate the distance of a point to a pathway comprised of two or more linear subsections. (a) Shows a case where only one distance is true, (b) shows one where more than one is acceptable, and (c) shows a case where none of the distances calculated directly is useful, and one needs to use only the $t = 0$ or 1 distances. The yellow dot represents the point of interest, the dashed line is the continuation of a line defined by the equation at the extremities of the subsection defined by the plain line; Figure S3: Acousto-ultrasonic setup mounted on an industrial carbon anode block. The estimated beam paths between transducer are illustrated for an example excitation position; Figure S4. Example of received raw signal from an acousto-ultrasonic excitation in a baked carbon anode. (a) Shows the raw signal captured by the measurement setup, and (b) shows a zoom in the first 400 microseconds.

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