

Article

Experimental and Numerical Simulation of the Dynamic Response of a Stiffened Panel Suffering the Impact of an Ice Indenter

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Abstract: At a laboratory scale, the response of a stiffened panel subjected to the impact of an ice indenter was studied by both experimental and numerical means. The experiment was conducted using a Falling Weight Impact Tester, and the impact force and deformation data of the stiffened panel were measured and recorded. The experimental results showed that the ice indenter could cause significant indentation to the stiffened panel and experienced severe crushing and scattering itself. Finite element analysis was performed to reproduce the structural deformations in an appropriate manner, and a constitutive model with a multisurface yield criterion and a dynamic empirical failure criterion for ice material was developed. Good agreement was obtained, and the influences of various parameters in the constitutive model and the performance of other different material models are discussed. The purpose of this study is to present an experimental and numerical study on a scenario of high-energy collision between a hull structure and an ice block, the conclusions of which can be very useful for studying ship-ice collisions and guiding engineering applications.

Keywords: stiffened panel; ice indenter; constitutive model; impact test; numerical simulation



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1. Introduction

The interest in the Arctic is increasing and resource exploitation continuously progresses northward. The rapid reduction in Arctic sea ice in the coming decades will dramatically increase the risk of collisions to ships and offshore structures operating in polar regions [1,2]. As indicated by the ALIE (Abnormal Level Ice Event) or ALS (Accidental Limit State) format, according to the International Standard ISO 19906 (2010), the most important structural safety criterion is to ensure that the hull structure is able to withstand a worst-case event and maintain structural integrity when it is directly exposed to ice load [3]. An accurate prediction of the ice load is therefore crucial when designing and assessing ship structures.

Polar ships will encounter a variety of ice conditions, including level ice, package ice, ice ridges and ice bergs, etc., and many studies have been carried out [4–6]. In most cases, route planning and detection of the ice mass will prevent collision accidents, but accidental ice loads are still unavoidable, as execution of evasive maneuvering is not possible in uncalm seas. The consequences of accidental events can be very serious, especially for the fragile Arctic environment. For engineering calculations, strength, ductility and shared energy design are available, in which shared energy design allows for a more optimized structure, since the energy is absorbed by both the ice and the vessel [7]. However, a proper treatment of the ice material becomes necessary.

There has been considerable research conducted to measure the natural properties of sea ice [8–11]. The aim of early research was to present simple and efficient empirical formulas for determining ice load [12–14], but such formulas are limited in terms of accuracy

and range of application. On the other hand, finite element methods have been widely used for a long time, but the lack of suitable ice materials has always been a major problem in predicting ice loads. In 1977, Hibler proposed a dynamic thermodynamic numerical model for the simulation of sea ice circulation and thickness change over a seasonal cycle [15,16]. The isotropic-elastic-failure model was developed by Carney when collisions between ice and structures occurred at very high speeds [17]. In addition, a designation of a hard “crushable foam” material model was introduced by Gagnon and successfully produced some reasonable values of load, pressure, and impact response for the collision experiment between the CCGS Terry Fox icebreaker (Burrard Yarrows Corporation, Vancouver, Canada) and a bergy bit [18,19]. However, the physical explanation is unclear, and the material model lacks a failure definition. Collision between ice bits and hull structure is actually a very complex process, especially for ice materials, which undergo considerable compression within the contact area and high-pressure zones. A triaxial compression test is particularly useful for revealing the three-dimensional mechanical properties of ice materials [20–24]. On this basis, user-defined constitutive subroutines have been developed and applied in ship-ice collision scenarios and have been proved as feasible and effective approaches [25–28]. A well-established ice material model, however, is still a long way off.

In terms of experiments, there have been few direct measurements of stress distributions or collision force distributions in full-size ship tests, such as that of the CCGS Terry Fox icebreaker [29,30]. Considering the difficulty and cost of ice impact testing, a structure-based test is acceptable and realistic. A series of impact tests was conducted at the Norwegian University of Science and Technology and Aalto University by Kim in 2013 [31,32]. Similar tests carried out by Storheim increased the impact energy to 24 kJ and the impact velocity to 8 m/s [33].

A literature review indicates that collision tests between ice blocks and structures at relatively high energies or high speeds are still relatively rare, and many tests that are more relevant are needed to better understand their interaction in collision process. In this study, an impact test of an ice indenter on a stiffened panel is carried out to simulate a high-energy collision scenario between a ship structure and an ice block; a recording and analysis of the impact process, panel deformation, and ice load are made. Through a subroutine embedded in the finite element software package LS-DYNA (Livermore software R971), numerical simulations were performed based on an elastic-plastic constitutive model with a multisurface yield criterion and a dynamic empirical failure criterion. The influences of various parameters in the constitutive model; the performance of other different material models are discussed, and some insights are provided on how they affect the impact performance.

2. Experimental Details

At a laboratory scale, a wedge-shaped ice indenter was struck against a stiffened panel to evaluate structural deformation and impact response. The panel and stiffeners were made of mild steel. The static material characteristics were obtained by quasi-static tensile tests in which the material was the same plates as the impact stiffened plate. Deaired granular freshwater ice was used in the indenter, which was manufactured using commercially available crushed ice and cooled freshwater. It was hoped to obtain an ice behavior similar to glacial ice and multiyear ice, which have a predominant granular structure and low or no salinity.

2.1. Quasi-Static Tensile Tests

Quasi-static tensile tests of mild steel were conducted at room temperature (15 °C). The tensile test apparatus (AG-IS100KN, Shimadzu Corporation, Kyoto, Japan), test specimen, and dimensions of the test piece are illustrated in Figure 1. The dimensions were selected according to relevant standards (GB/T 228.1 2010) [34]. The loading speed of the actuator was maintained at 1.0 mm/min until fracture occurred. The obtained engineering stress-strain curve is presented in Figure 2a (the green solid line), and the mechanical properties

are summarized in Table 1. In addition, several uniaxial compression tests were also conducted using the same test apparatus.

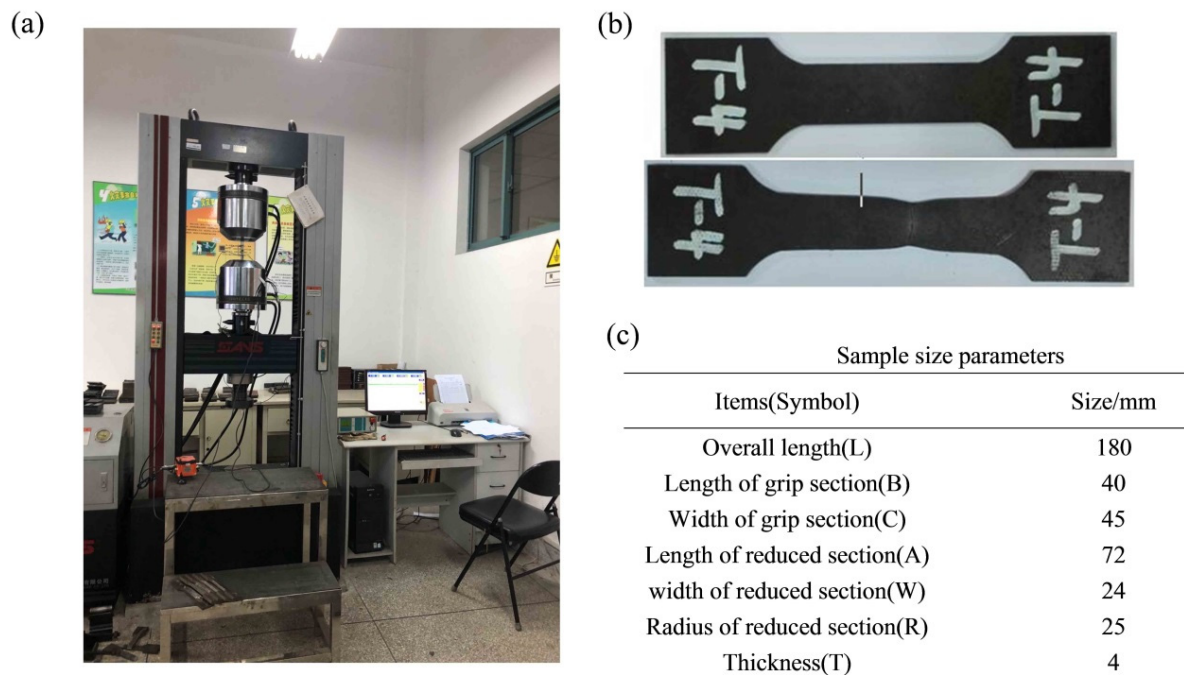


Figure 1. Quasi-static tensile test: (a) Test device; (b) Test specimen; (c) Dimensions of the test specimen.

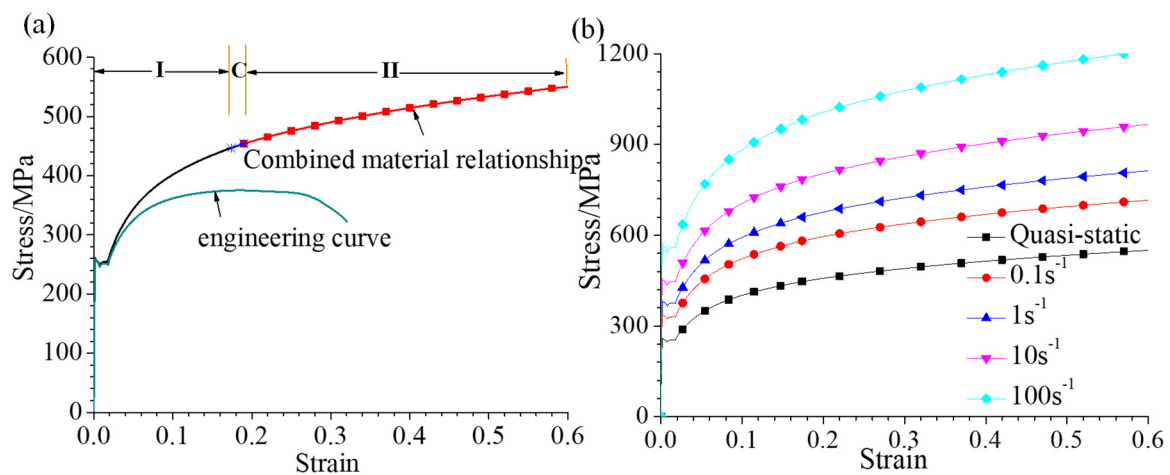


Figure 2. Stress-strain curves for the mild steel: (a) Engineering stress-strain curve obtained by Quasi-static tensile test and true stress-strain curve obtained by the ‘combined material’ relation; (b) Dynamic true stress-strain curve obtained by the Cowper-Symonds constitutive model.

Table 1. Mechanical properties of mild steel and ice material.

Items	Mild Steel	Ice Material	Units
Mass density	7850	910	kg/m ³
Elastic modulus	209	≈2	GPa
Poisson’s ratio	0.3	0.3	-
Yield stress	265	-	MPa
Ultimate tensile stress	359	-	MPa
Fracture strain	0.29	-	-

2.2. Impact Test of Stiffened Panel Struck by an Ice Indenter

As shown in Figure 3, the impact test was carried out by means of the Falling Weight Impact Tester (FWIT) device, which was 6.3 m high, and the maximum lifting height was 3.7 m (impact speed = 8.5 m/s). The wedge-shaped ice indenter can drop from a known variable height between guide rails onto horizontally supported specimens. Figure 4 shows the size of the ice indenter, which is equipped with a counter weight weighing up to 1350 kg. Therefore, the designed maximum impact energy is 50 kJ, which can effectively reflect the high-energy collision between ice blocks and hull structures. In the current experiment, the drop height was set to 3.2 m.

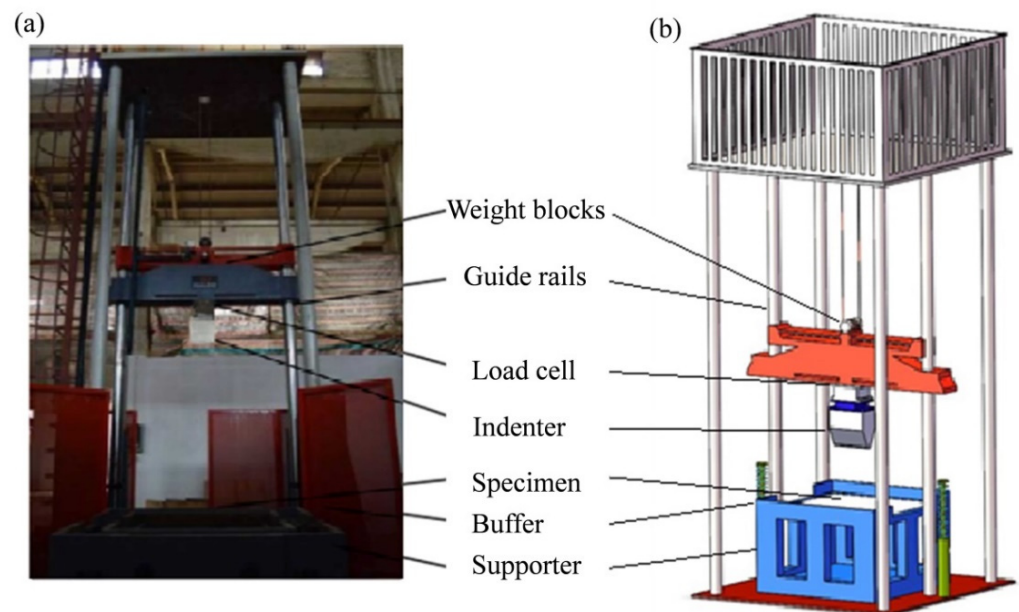


Figure 3. Schematic diagram of the falling weight impact tester: (a) Photograph; (b) Schematic diagram.

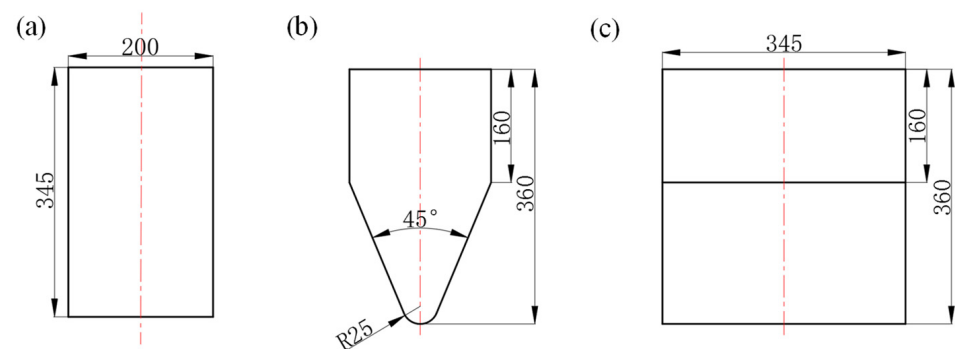


Figure 4. The shape of the ice indenter and its dimensions (Unit: mm): (a) Top view; (b) Side view; (c) Front view.

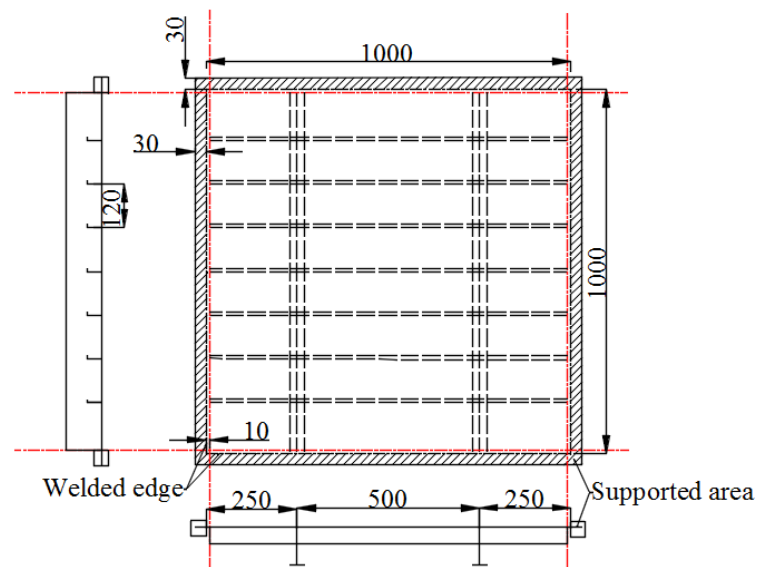
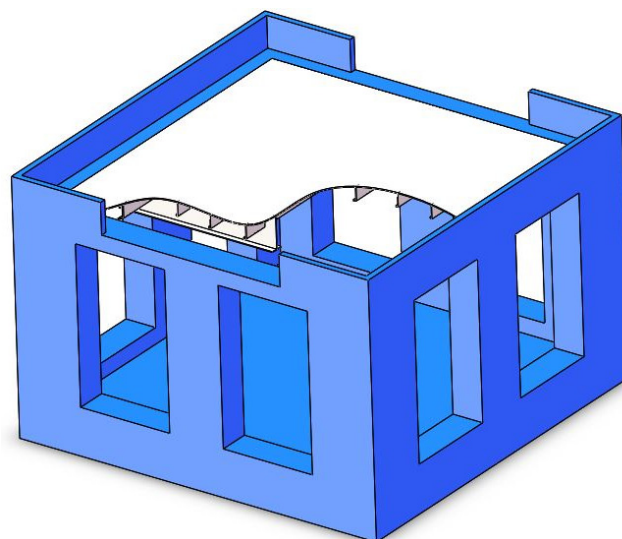
The geometry of the specimen is shown in Figure 5, and its detailed size and thickness details are presented in Table 2. The plate was equipped with seven stiffeners and two ribs. The edges of the stiffened panel were welded onto an additional support to simulate a rigid boundary, as illustrated in Figures 5, 6 and 7a. The specimen was designed with a ratio of 1:4 to the actual size, and the detailed impact parameters for this experiment are listed in Table 3.

Table 2. Dimension and thickness of stiffened panel.

Items	Dimension and Thickness/mm
Plate	2.5
T bar	100 × 2/40 × 2.5
Stiffener	42 × 1.75/8.5 × 3.3

Table 3. Information of falling weight impact test.

Impact Parameters	Values	Unit
Drop height	3.2	m
Impact weight	1340	kg
Impact velocity	7.9	m/s
Impact energy	42.3	kJ

**Figure 5.** The geometry of the specimen (Scale 1:4, Unit: mm).**Figure 6.** The stiffened panel welded on the support.

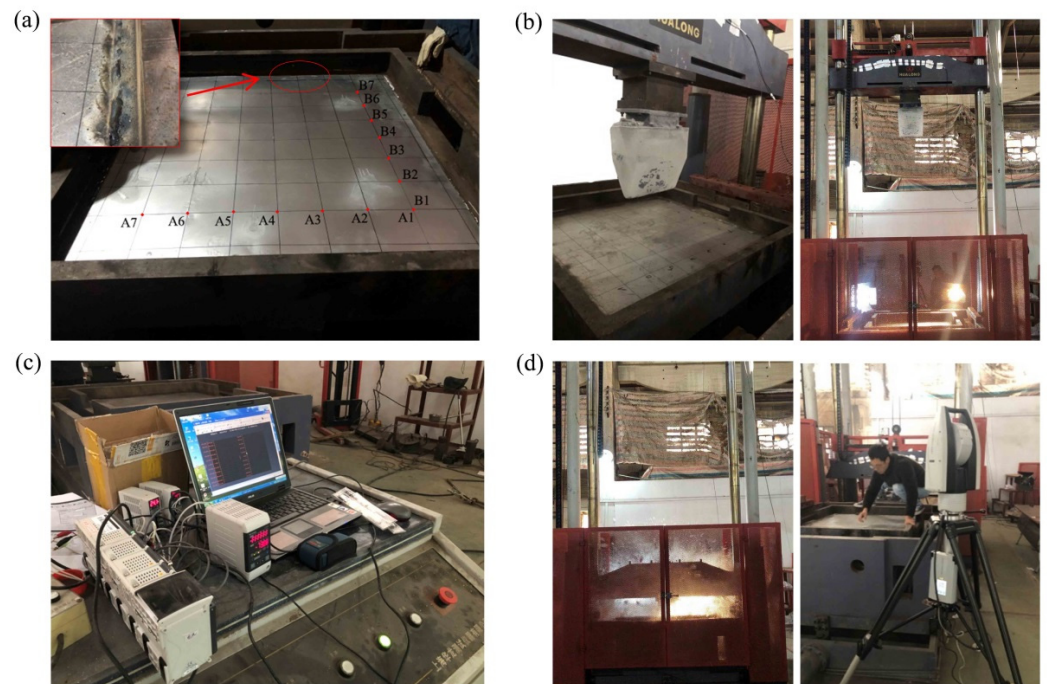


Figure 7. Photos of the test setup: (a) Installing the test piece; (b) Installing the ice indenter and raising it to the target height; (c) Signal acquisition device; (d) Dropping the ice indenter and measuring the deformation.

In the experiment, the drop test process was filmed using a high-speed video camera (FASTCAM SA-Z, Photron Corporation, Tokyo, Japan) at a recording speed of 1000 frames per second and with a resolution of 1024×768 . The displacement of the ice indenter was recorded by the laser displacement sensors and an acceleration sensor was mounted on the holder to calculate the impact force. Before and after the test, the structural deformation (plastic deformation) was calculated by the measurement difference of the grid points with the aid of a Leica absolute tracker device (see Figure 7d). The layout of the grid points is shown in Figure 8. Some photographs of the experiment and equipment are shown in Figure 7.

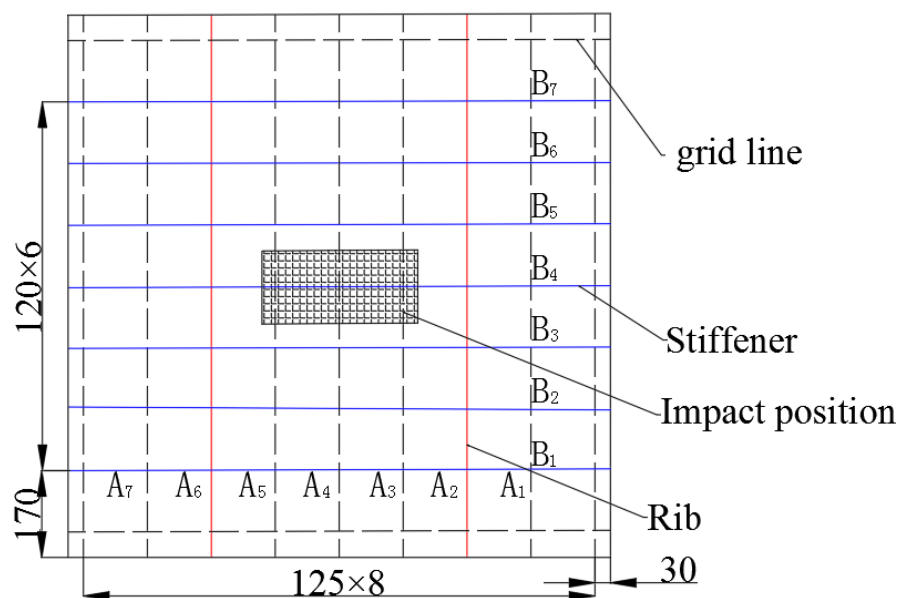


Figure 8. Schematic diagram of the layout of measurement points.

2.3. Experimental Results

2.3.1. Ice Damage

An image sequence of the ice block-stiffened panel interaction process during the impact test is shown in Figure 9. As can be seen, the ice indenter was continually shattered during the impact, and crushed pieces were spread all over the area. At the initial stage of the impact, the upper and lower edges of the ice body, i.e., the contact and connection parts, broke up and failed first (see Figure 9b). As the impact process continued, this part of the ice indenter would consistently break and fail, and the ice indenter no longer retained its original shape until it was entirely destroyed. As a result of its relatively high speed, crushed ice underwent a three-dimensional stress state and a large portion of the crushed ice was unable to spread and eventually became tightly compacted in the depression deformation area, which was difficult to clear and contributed significantly to the deformation of the stiffened panel, as shown in Figure 10a (red arrow).

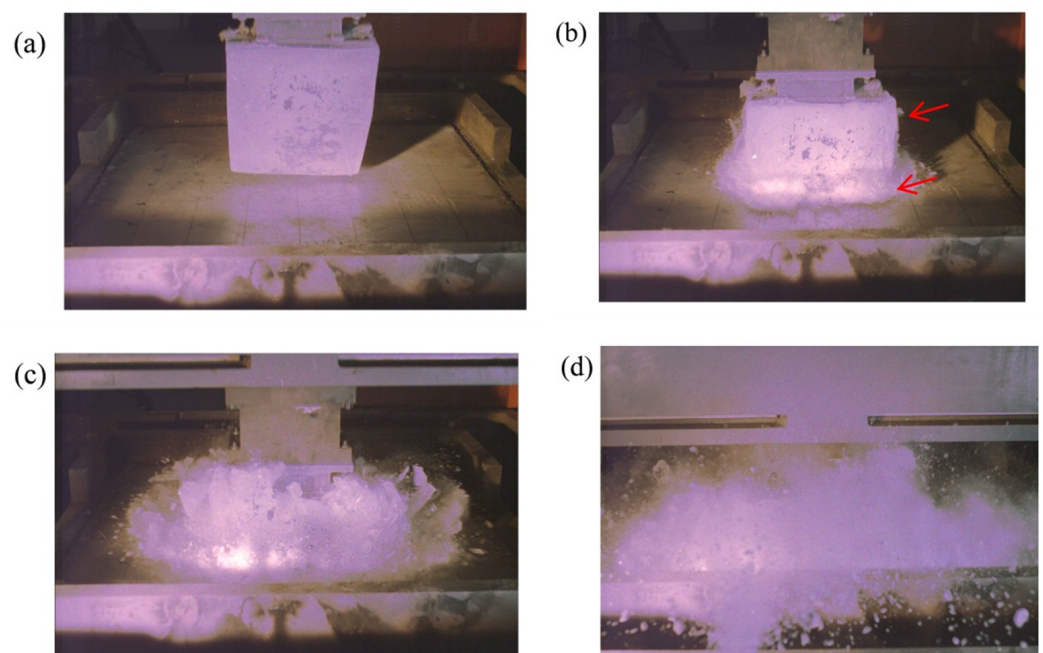


Figure 9. Sequence of images extracted from high-speed video recording showing impact process: (a) Initial contact; (b) Ice extrusion; (c) Ice partially broken; (d) Ice completely crushing.

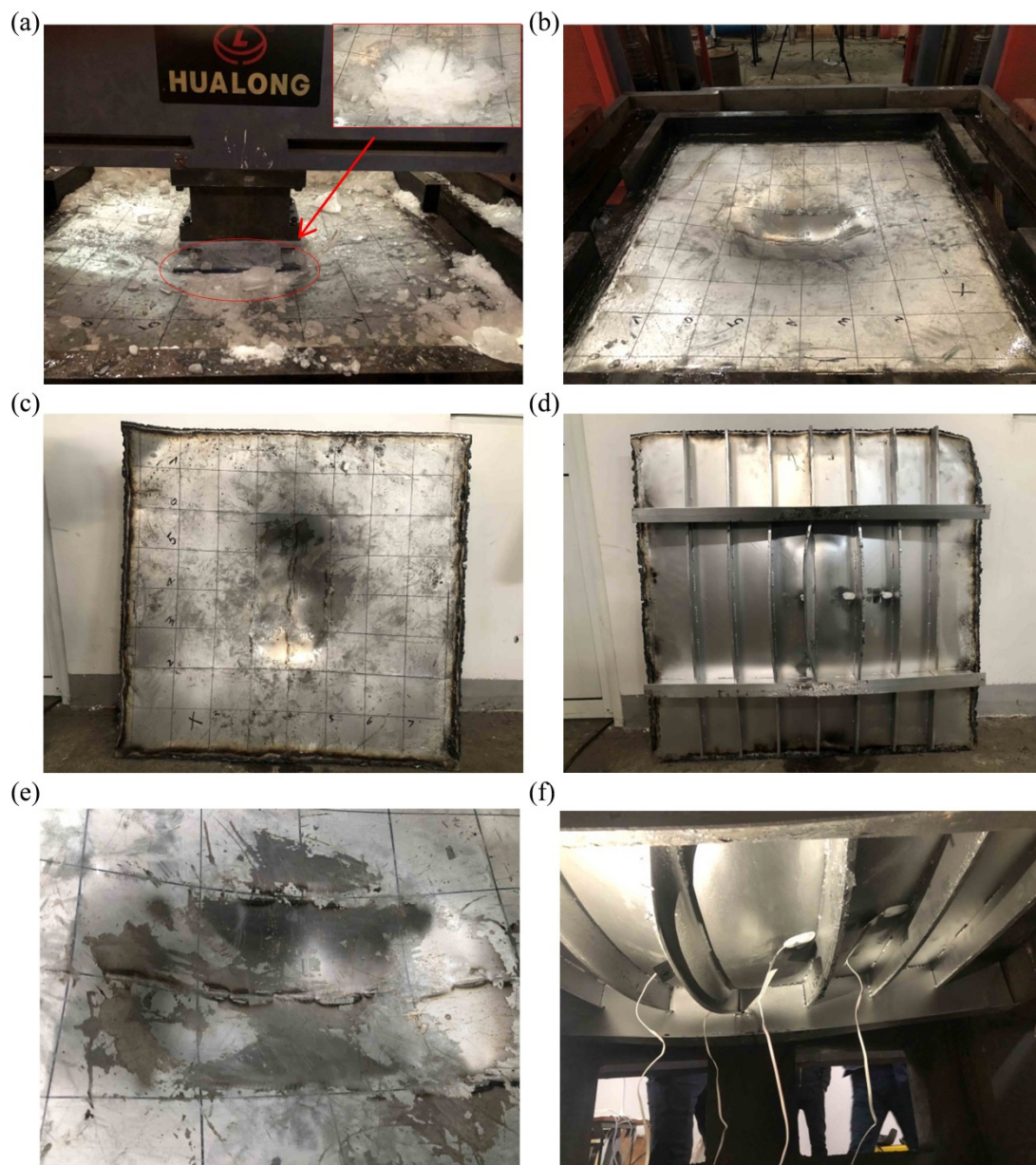


Figure 10. Structural deformation of stiffened panel: (a) A photograph of the plate damage after test; (b) Overall view; (c) Front view; (d) Back view; (e) Local view, (f) Deformation of stiffeners.

2.3.2. Structural Deformation

Photographs of the stiffened panel after the impact test are shown in Figure 10. The main visible deformation occurs in the central area of the stiffened panel, and no evident destruction or structural failure occurs. The deformation of the stiffened panel is localized at the impact area, and the main supporting components (i.e., stiffeners and ribs) suffered substantial buckling deformation due to massive in-plane loads, although the panels and overall structure showed pronounced membrane stretch.

Table 4 lists the quantitatively measured deformation of the stiffened panel. In the table, B_1 – B_7 indicate the measuring points along the grid line parallel to the stiffeners, and A_1 – A_7 indicate the measuring points along the grid line perpendicular to the stiffeners (see Figure 8). The maximum plastic deformation is 80.43 mm. The panel deformation profiles at four reference lines in each direction, namely, LB_1 – LB_4 and LA_1 – LA_4 , are illustrated in Figure 11. Obviously, the stiffened panel deforms in different modes in the two directions and the deformation range and measured values are larger when along the stiffener direc-

tion. In the other direction, the deformation appears more “sharp” and exhibits apparent symmetry, and this is because of the direction of the ice indenter. In this case, the stiffeners have a better inhibition effect on the deformation of the stiffened panel.

Table 4. Measured deformation of the stiffened panel.

Node ID	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇
B ₁	−5.36	−10.15	−13.02	−14.07	−14.11	−11.85	−6.12
B ₂	−9.07	−18.45	−26.56	−29.64	−27.39	−20.16	−10.18
B ₃	−12.55	−28.17	−47.92	−55.29	−50.50	−30.96	−13.96
B ₄	−15.69	−33.32	−66.73	−80.43	−73.85	−37.52	−16.35
B ₅	−12.50	−26.42	−45.35	−52.04	−46.89	−28.92	−13.71
B ₆	−8.61	−17.41	−25.21	−27.60	−25.62	−18.57	−9.74
B ₇	−4.94	−9.60	−11.37	−11.89	−11.66	−10.37	−5.85

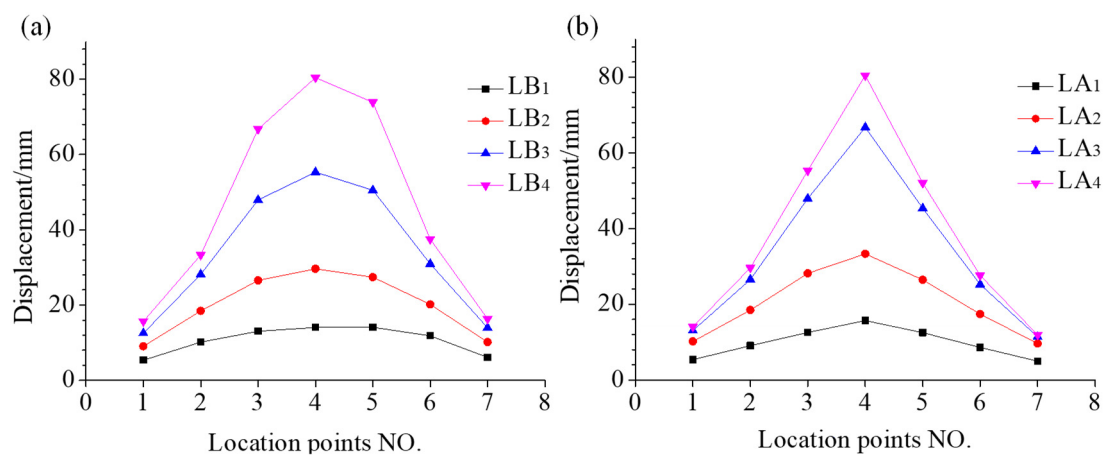


Figure 11. Plastic deformation of stiffened panel at typical locations: (a) Line Ai (LA, measuring points perpendicular to the stiffeners); (b) Line Bi (LB, measuring points parallel to the stiffeners).

2.3.3. Acceleration Response

An acceleration sensor is mounted on the ice indenter, and Figure 12 shows the time history of its value recorded during the experiment. The acceleration curve shows a rapid rise and multiple notable peaks, with a maximum value around 270 m/s². Since the focus is on the force change during the impact process, the data after 0.06 s are of no concern. By multiplying the acceleration curve by the dropping weight, the actual collision force can be determined, which will be discussed below.

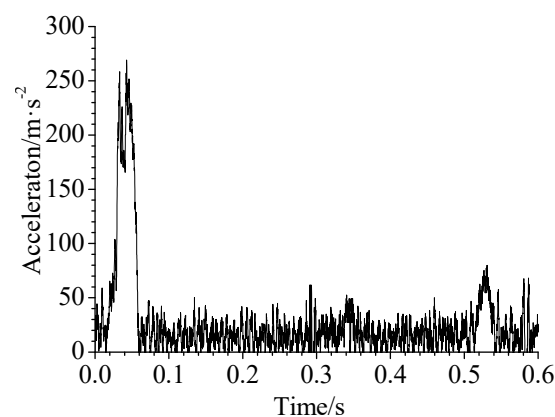


Figure 12. Record of acceleration history of impact tests.

3. Numerical Simulation

3.1. Ice Material Model

3.1.1. Constitutive Model

Single crystals of ice have long been known to deform plastically, and it is imperative to have a good ice material model for numerical simulations to ensure accuracy [35]. Triaxial and uniaxial tests are essential for revealing the three-dimensional stress relationship of ice materials. Derradji-Aouat [36,37] proposed a uniform multisurface yield criterion for isotropic fresh water ice, iceberg ice, and columnar ice, which is widely accepted and suitable for the laboratory-made fresh water ice used in the present study, as expressed in Equation (1).

$$((q - \eta)/q_{\max})^2 + ((p - \lambda)/p_c)^2 = 1 \quad (1)$$

The yield surface is a set of concentric elliptical curves, where η and λ are the coordinates for the center of the ellipse, q is the octahedral shear stress, and p is the hydrostatic pressure $q_{\max}$ and p_c are the minor and major axes of the ellipse, respectively.

The empirical values of parameters p_c , η , and λ for fresh water ice are constants of 55 MPa, 0, and 45 MPa [37]. The values of $q_{\max}$ are temperature- and strain-rate-dependent, as indicated in Equations (2) and (3):

$$q_{\max} = (\dot{\epsilon}/\xi)^{1/n} \quad (2)$$

$$\xi = 5 \times 10^{-6} \exp[-10.5 \times 10^3(1/T - 1/273)] \quad (3)$$

where $\dot{\epsilon}$ is the strain rate, T is the temperature, and $n = 4$. It should be noted that the parameter n is not a constant (reference range 2.8–4.2) [24,38]. The value of parameter n is treated as a constant in this study and, as a result, the yield formula becomes the relation of $q_{\max}$ and p , where p is the independent variable. As $q = \sqrt{(s_{ij} : s_{ij})/3} = \sqrt{2J_2/3}$, the above formula can be converted to

$$J_2 + q_{\max}^2 \times [3 \times (p^2 - 2p\lambda + \lambda^2)/(2 \times p_c^2)] = 3 \times q_{\max}^2/2 \quad (4)$$

where J_2 is the second deviatoric stress tensor invariant. The yield formula can be written in terms of parameters p and J_2 , as follows:

$$f(p, J_2) = J_2 - \frac{3 \times q_{\max}^2}{2} + \frac{3 \times \lambda^2}{2 \times p_c^2} \times q_{\max}^2 - \frac{3 \times 2p\lambda}{2 \times p_c^2} \times q_{\max}^2 + \frac{3 \times p^2}{2 \times p_c^2} \times q_{\max}^2 \quad (5)$$

Parameterizing the above formula, the following can be obtained:

$$f_{\text{yield}}(p, J_2) = J_2 - (a_0 + a_1p + a_2p) = 0 \quad (6)$$

where $a_0 = q_{\max}^2(1.5 - 1.5\lambda^2/p_c^2)$, $a_1 = 3q_{\max}^2\lambda/p_c^2$, and $a_2 = -1.5q_{\max}^2/p_c^2$. As a result, the value of a_i ($i = 0, 1, 2$) depends on the value of $q_{\max}$.

In fact, owing to the discrete nature of the test data, more than one yield surface has been developed. In this study, Equation (6) is used as the yield criterion of an elastic-plastic constitutive model embedded into the finite element software LS-DYNA. The recommended values of a_i ($i = 0, 1, 2$) and the corresponding $q_{\max}$ values are used and listed in Table 5 [24], and the yield equation has been parameterized for further study in a subroutine.

Table 5. Values of a_i ($i = 0, 1, 2$) and $q_{\max}$.

Items	$a_0(\text{MPa}^2)$	$a_1(\text{MPa})$	a_2	$q_{\max}(\text{MPa})$
Values	22.93	2.06	−0.023	6.8

It is important to deal with the strain rate problem of the yield equation in a reasonable manner. For freshwater ice, an expansion of the elliptic curve to the maximum value is observed until the strain rate reaches approximately $1 \times 10^{-3} \text{ s}^{-1}$. Liu [39] pointed out that the ice should be in a brittle failure mode when the impact involves high strain rates (larger than 10^{-3} s^{-1}), and thus the strain rate is not necessarily considered. In this study, the relationship of the value of $q_{\max}$ and strain rate can be described by:

$$q_{\max} = a \times \dot{\varepsilon}^k \quad (7)$$

where a and k are constants for a given strain rate and temperature.

In addition to the yield function, a failure criterion must also be considered. As the stiffness of ice does not significantly change during loading [40], the essence of the failure criterion is to determine the magnitude of the strain before it fails. The formula proposed by Jordaan [41,42] approximates the essential points for deriving design requirements, as indicated in Equations (8) and (9), representing two state variables of ice deformation, the hardening at lower pressures, and the softening at higher pressures.

$$f_1(p) = c_1(1 - p/s_1)^2 \quad (8)$$

$$f_2(p) = c_2(p/s_2)^r \quad (9)$$

where c_1 , c_2 , s_1 , s_2 , and r are constants, and p is an independent variable. The empirical values of these parameters are $c_1 = 0.712$, $c_1 = 37 \text{ MPa}$, $s_2 = 0.1$, $s_2 = 42.8 \text{ MPa}$, and $r = 7$. Since the failure criterion is more empirical, a more simplified failure criterion is used in this study, as

$$\varepsilon_{eq}^p = \sqrt{2(\varepsilon_{ij}^p \cdot \varepsilon_{ij}^p)/3} \quad (10)$$

$$\varepsilon_f = \varepsilon_0 + (p/pr_2 - b)^2 \quad (11)$$

where ε_{eq}^p is the equivalent plastic strain, ε_{ij}^p is the plastic strain tensor, ε_f is the failure strain, ε_0 is the initial failure strain, which is set to 0.01. pr_2 is the larger root of the yield function. b is a constant, and $b = 0.5$ is used in this study [26,39]. When $\varepsilon_{eq}^p > \varepsilon_f$ or $p < p_{cutoff}$, the element fails and p_{cutoff} is the cutoff pressure, which is set to 2 MPa and is the same as uniaxial tensile strength.

The failure criteria are parameterized, and no other failure criteria are defined. This failure criteria focused on the difference in the bearing capacity between the high and low stress zones caused by the hydrostatic pressure. The friction slip and weakening effect of shear failure in collision problems will be ignored for preventing premature failure.

According to the above description, the elastic-plastic constitutive model is quite clear. In the elastic stage, the constitutive model satisfies the isotropic generalized Hooke's law:

$$\begin{aligned} \Delta\sigma_x &= \Delta\sigma_0 + 2G(\Delta\varepsilon_x - \Delta\varepsilon_0), \Delta\tau_{xy} = G\Delta\gamma_{xy} \\ \Delta\sigma_y &= \Delta\sigma_0 + 2G(\Delta\varepsilon_y - \Delta\varepsilon_0), \Delta\tau_{yz} = G\Delta\gamma_{yz} \\ \Delta\sigma_z &= \Delta\sigma_0 + 2G(\Delta\varepsilon_z - \Delta\varepsilon_0), \Delta\tau_{zx} = G\Delta\gamma_{zx} \end{aligned} \quad (12)$$

where $\Delta\varepsilon_0$ is the hydrostatic strain increment, $\Delta\varepsilon_0 = (\Delta\varepsilon_1 + \Delta\varepsilon_2 + \Delta\varepsilon_3)/3$, $\Delta\sigma_0$ is the hydrostatic stress increment, $\Delta\sigma_0 = (\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)/3$, and G is the shear elastic modulus.

As the yield surface of the plastic phase is convex, the plastic strain increment can be expressed as

$$d\varepsilon^p = d\gamma(\alpha f / \alpha\sigma) \quad (13)$$

where $d\gamma$ is a non-negative plastic consistency parameter. The plastic flow rule is adopted, and the total strain increment is

$$d\varepsilon = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p = \frac{1}{2G}d\sigma_{ij} - \frac{3\mu}{E}dp\delta_{ij} + d\gamma \frac{\alpha f^{yield}}{\alpha\sigma_{ij}} \quad (14)$$

where $d\sigma_{ij}$ is the stress increment, and δ_{ij} is the Kronecker symbol with a value of 0 ($i \neq j$) or 1 ($i = j$).

3.1.2. Stress Update Algorithm

In the isoparametric finite element analysis, a generalized rate-independent elastic-plastic constitutive model can be written as [43]

$$\begin{aligned}\varepsilon_{ij} &= \varepsilon_{ij}^e + \varepsilon_{ij}^p \\ \sigma_{ij} &= \sigma_{ij}(\varepsilon^e, qr) \\ \dot{\varepsilon}_{ij}^p &= \dot{\gamma} r_{ij}(\sigma, qr) \\ \dot{q}_\alpha &= \dot{\gamma} h_\alpha(\sigma, qr)\end{aligned}\quad (15)$$

where ε_{ij} , ε_{ij}^e , and ε_{ij}^p denote the total strain, elastic strain, and plastic strain, respectively, σ_{ij} is the Cauchy stress, qr and q_α are both internal variables representing the plastic characteristics, r_{ij} represents the plastic flow direction, $\dot{\gamma}$ is the plastic parameter, and h_α is the plastic modulus.

To update the constitutive formula, a semi-implicit backward Euler return-mapping algorithm is used. The integral method can be reduced as

$$\begin{aligned}\varepsilon_{n+1} &= \varepsilon_n + \Delta\varepsilon \\ \varepsilon_{n+1}^p &= \varepsilon_n^p + \Delta\gamma_{n+1} r_n \\ qr_{n+1} &= qr_n + \Delta\gamma_{n+1} h_n \\ \sigma_{n+1} &= C : (\varepsilon_{n+1} - \varepsilon_{n+1}^p) \\ f_{n+1} &= f(\sigma_{n+1} - q_{n+1}) = 0\end{aligned}\quad (16)$$

where n represents an integral step, ε_n and ε_n^p are, respectively, the strain and plastic strain in a certain integration step, $\Delta\varepsilon$ is the strain increment, $\Delta\gamma_{n+1}$ is the plastic increment parameter, and C is the elastic matrix. While entering the plastic stage, in the next short time period t , the trial stress is obtained by Hooke's law and is updated in the same way as in the elastic stage. If the trial stress exceeds the yield limit, the constitutive equation will enter the process of the iterative algorithm.

It can be seen that an implicit algorithm is applied to calculate the plasticity parameter $\Delta\gamma_{n+1}$, which will be obtained by the Newton iterative method. At the end of each step, the increment of the plasticity parameter is calculated, and the yield condition is strengthened. The plastic strain and internal variables are updated in the plastic stage as

$$\begin{aligned}\Delta\gamma &= f_{n+1}^i / (v_{n+1}^i : C_{n+1}^i : r_{n+1}^i) \\ \sigma_{n+1}^{i+1} &= \sigma_{n+1}^i - \Delta\gamma_{n+1} (C_{n+1}^i : r_{n+1}^i) \\ qr_{n+1}^{i+1} &= qr_{n+1}^i + \Delta\gamma_{n+1} h_{n+1}^i\end{aligned}\quad (17)$$

where $v_{n+1}^i = \alpha f_\sigma^i$ plastic hardening is not considered, $qr_n = 0$. Thereafter, the convergence condition is verified until it converges, otherwise

$$\begin{aligned}\sigma_{n+1} &= \sigma_{n+1}^{i+1} \\ qr_{n+1} &= qr_{n+1}^{i+1} \\ \varepsilon_{n+1}^e &= \varepsilon^e(\sigma_{n+1}, qr_{n+1}) \\ \varepsilon_{n+1}^p &= \varepsilon_{n+1} - \varepsilon_{n+1}^e\end{aligned}\quad (18)$$

The plastic strain increment is $\Delta \varepsilon_{n+1}^p = \varepsilon_{n+1}^p - \varepsilon_n^p = \Delta \lambda_{n+1} r_{n+1}$, and it can be taken into Equation (16) to obtain the value of σ_{n+1} .

$$\begin{aligned}\sigma_{n+1} &= C : (\varepsilon_{n+1} - \varepsilon_n^p - \Delta \varepsilon_{n+1}^p) = C : (\varepsilon_n + \Delta \varepsilon - \varepsilon_n^p - \Delta \varepsilon_{n+1}^p) \\ &= C : (\varepsilon_n - \varepsilon_n^p) + C : \Delta \varepsilon - C : \Delta \varepsilon_{n+1}^p = (\sigma_n + C : \Delta \varepsilon) - C : \Delta \varepsilon_{n+1}^p \\ &= \sigma_{n+1}^{trial} - C : \Delta \varepsilon_{n+1}^p = \sigma_{n+1}^{trial} - \Delta \gamma_{n+1} C : r_{n+1}\end{aligned}\quad (19)$$

In the formula, σ_{n+1}^{trial} is the prediction stress and $-\Delta \lambda_{n+1} C : r_{n+1}$ is the value of plastic correction. The specific steps are as shown in Figure 13, and the corresponding subroutine is generated after the program's execution.

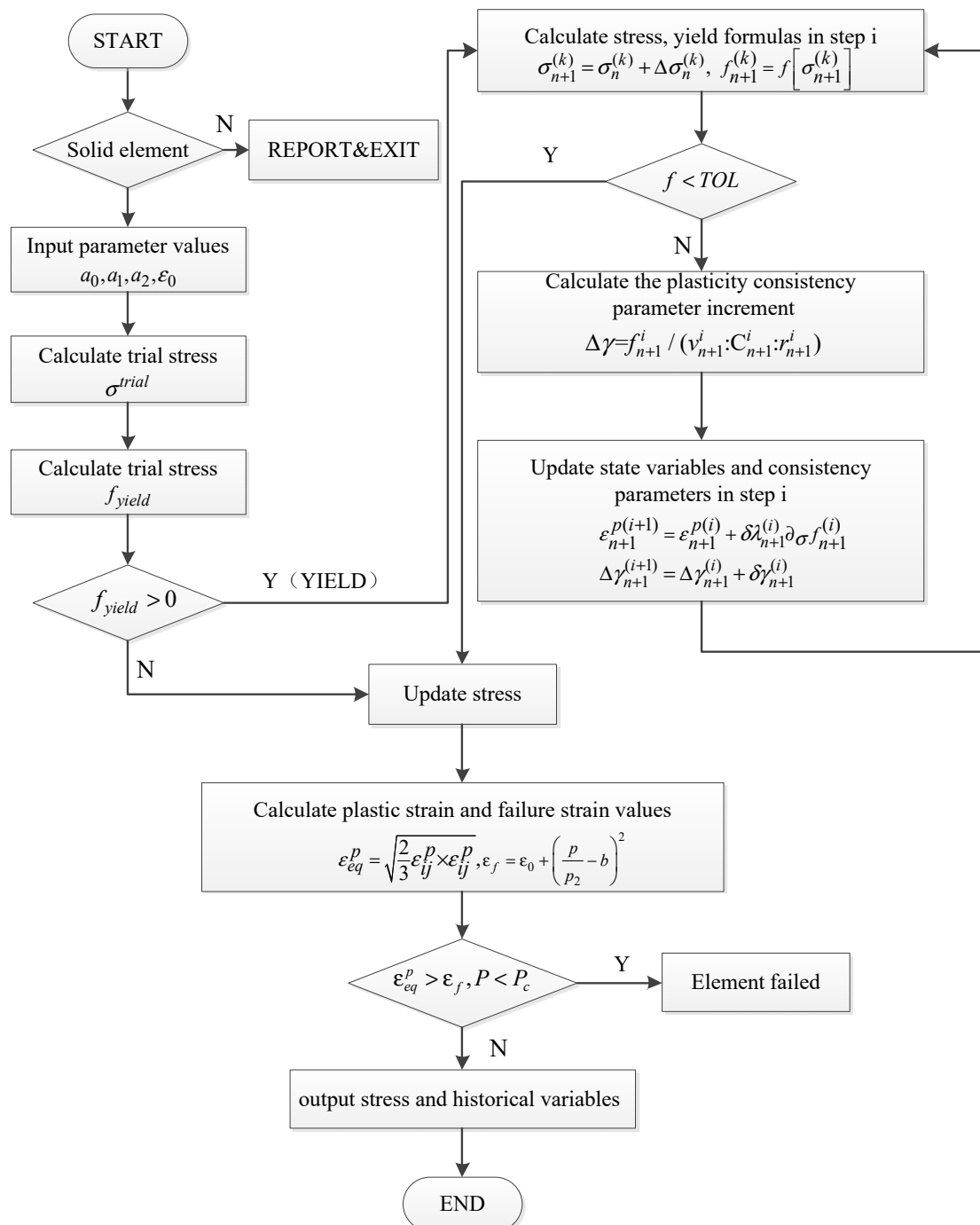


Figure 13. A flow chart of the algorithm of the subroutine.

3.1.3. Calibration of the Material Model

A single unit test was performed throughout the procedures to confirm the correctness of the yield formula and failure criterion. As shown in Figure 14, a single element of hexahedron was used. The bottom of the element is fixed and the compressive and tensile load were applied, respectively. The values of the second deviatoric stress tensor invariant (J_2), hydrostatic stress (P), and failure strain (ε_f) are recorded and output. The relationship between J_2 and P , ε_f and P is illustrated in Figures 15 and 16. It is clear that J_2 and P have a parabolic relationship and that point b is the dividing point between them. On the left side, the output value of J_2 is less than its theoretical value, and the unit is in the elastic stage ($f < 0$); on the other side, the two curves coincide, and the unit is in the plastic stage ($f \geq 0$). Once the input parameters are determined, the coordinates of the curve's vertex can be calculated manually. In Figure 15, the calculated coordinate is (44.78 MPa, 69.05 MPa), which matches the coordinate shown in the figure (point a).

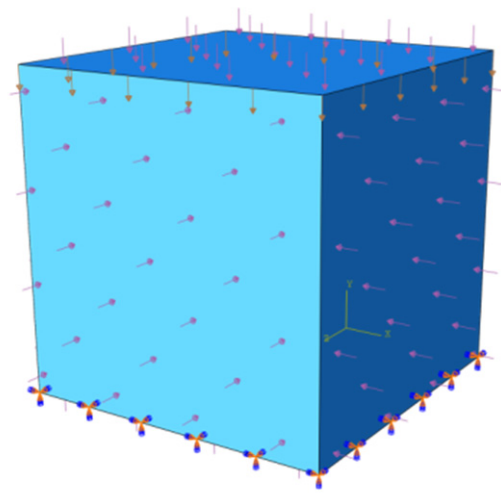


Figure 14. Single unit test and the load condition.

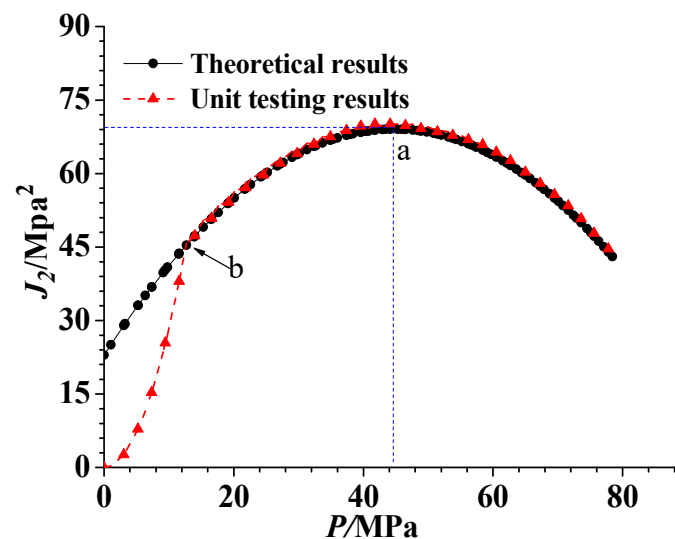


Figure 15. Comparison of $J_2 - p$ relationship between theoretical results and unit testing results.

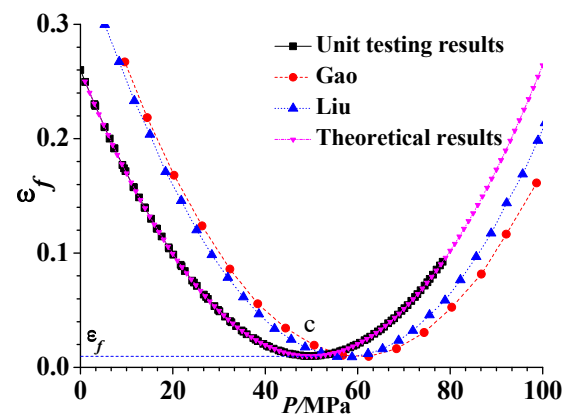


Figure 16. Comparison of $\varepsilon_f - p$ curves obtained from unit testing and other researchers.

Figure 16 shows that the relationship between ε_f and P is also parabolic. It can be observed that the theoretical value is basically the same as the output value; the coordinate of the lowest point is calculated as (49.785 MPa, 0.01) (point c).

A comparison was made between the pressure versus contact area curve and the recommended curves. Two cases are considered, including spherical and truncated cone-shaped ice blocks colliding with a rigid wall at a constant velocity of 1 m/s. A plot of the von Mises stress and damage of ice blocks at 0.5 s is shown in Figure 17. From the figure, a maximum stress of 15 MPa and a minimum value of -2 MPa are observed in both models. Ice spalls are partially simulated, and it is evident that high-pressure zones appear in the central compression position on the contact surface.

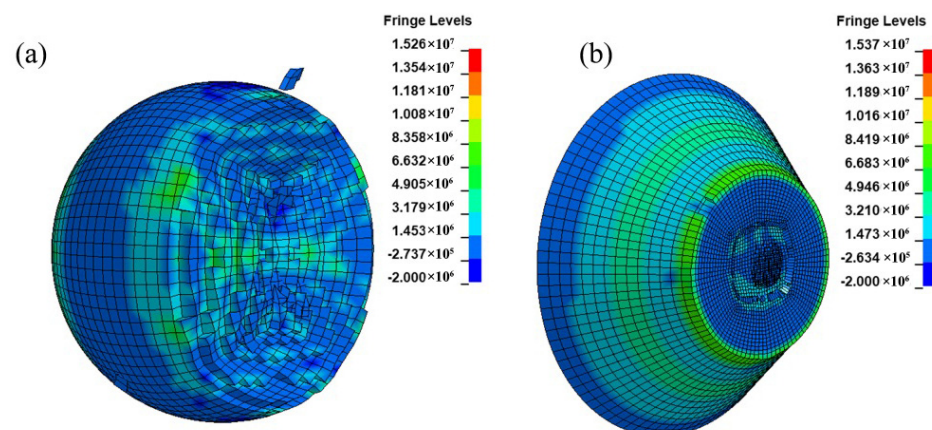


Figure 17. The Von Mises stress and damage to ice blocks at 0.5 s: (a) Sphere; (b) Truncated cone.

Figure 18 shows the obtained contact force versus displacement curves, and the corresponding pressure versus contact area curves are illustrated in Figure 19. It can be seen that the contact force curves exhibit completely different characteristics owing to the finite element mesh arrangement, but the peak values are nearly the same. In general, the curves obtained are consistent with the ISO-ALIE curve, especially when the contact area is greater than 1 m^2 . The curve tends to decrease with increasing area when the contact area is less than 1 m^2 , the curves proposed by different researchers show significant differences [44–46], and the error exists in both of the simulation cases, which is caused by the unrepresentative pressure value. It is necessary to point out that the ISO-ALIE standard curve corresponds to the accidental limit state (ALS) in modern codes. Therefore, it is not considered wrong or unacceptable to obtain a curve value lower than the standard value, especially when the contact area is relatively small. The specific formulae corresponding to the curves used in Figure 19 are listed in Table 6.

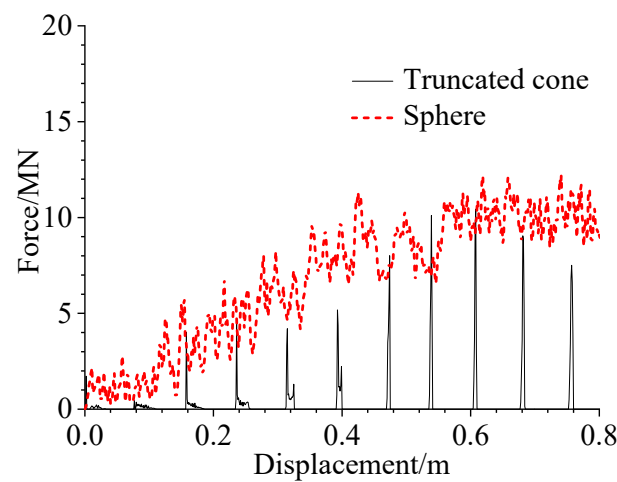


Figure 18. Time history force between ice sphere and rigid plate obtained from the simulations.

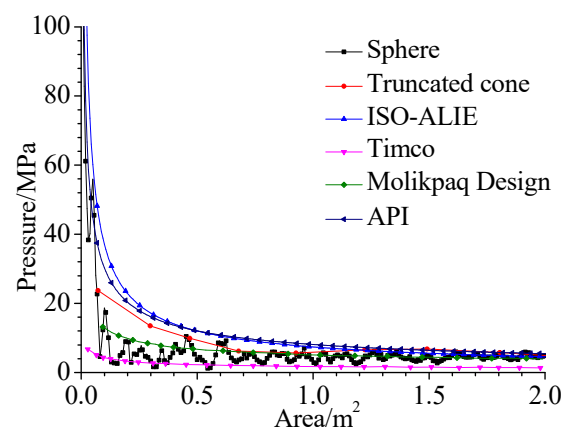


Figure 19. Pressure-area curve obtained in this study and by other researchers.

Table 6. Specific formulas for different pressure versus contact area relationships.

Name	Expression
ISO-ALIE	$P_{ISO} = 7.4 \times A^{-0.7}$
Molikpad Design [37]	$P_{MD} = 5.119 \times A^{-0.4}$
Timco [38]	$P_{Timco} = 1.9 \times A^{-0.37}$
API [39]	$P_{API} = 8.1 \times A^{-0.572}$

3.2. Simulation Details

3.2.1. True Stress-Strain Relationship

The material card ‘Mat.024-Piecewise linear isotropic plasticity’ is used for mild steel in the current study, which requires the input of basic mechanical properties (Table 1) and a true stress-strain curve as an offset Table. The so-called ‘combined material’ relation is used as the strategy to obtain a reasonable true stress-strain curve, as seen in Equation (20), which accurately predicts the plastic response and the fracture strain of quasi-static tensile test [47]. A detailed description can be found in Refs. [48,49].

$$\begin{aligned}\sigma_t &= \sigma_e(\varepsilon_e + 1) \\ \varepsilon_t &= \ln(\varepsilon_e + 1)\end{aligned}\tag{20}$$

where ε_e , σ_e , ε_t , and σ_t are the engineering stress, engineering strain, true stress, and true strain, respectively. Equation (20) is valid until necking occurs (Figure 2a curved segment I).

Beyond necking, the true stress-strain curve is

$$\sigma_t = C \varepsilon_t^n \quad (21)$$

where $n = \ln(1 + A_g)$, $C = R_m(e/n)^n$, $A_g = (0.24 + 0.01395R_m)^{-1}$, and R_m is the ultimate stress [48,49] (Figure 2a curved segment II). The two curves are connected by a straight line (Figure 2a curved segment C), so the material relationship is composed of three parts, as shown in Figure 2, which can be used directly for the input of the true stress-strain. In addition, considering the relatively high speed, the steel material may experience a high strain rate during impact process, the Cowper-Symonds constitutive equation is used, which is included in the material card, and values of $p = 5$ and $D = 40.4$ are adopted for mild steel, as shown in Figure 2b [50].

3.2.2. Finite Element Model

The computations were carried out using the finite element package LS-DYNA version 971. Figure 20 illustrates the numerical model, which consists of a stiffened panel and ice striker. The stiffened panel is modeled by shell elements (Thin shell 163), and a 5.0 mm overall finite element length is adopted. The ice indenter is modeled by solid elements (Solid 164), and a mesh size of approximately 5.0 mm is adopted near the contact area. The four outer edges of the stiffened panel are fixed (Figure 20b), and no constraint is added to the ice indenter. Meanwhile, to save calculation time, the height of the drop is converted into the initial velocity of the ice indenter ($v = 7.92$ m/s). The contact between the ice indenter and stiffened panel is a generic surface-to-surface and eroding surface-to-surface contact algorithm with an assumed static and dynamic friction coefficient of 0.15 [51]. The material used for the mild steel has been described and inputs of ice material are plotted in Table 1.

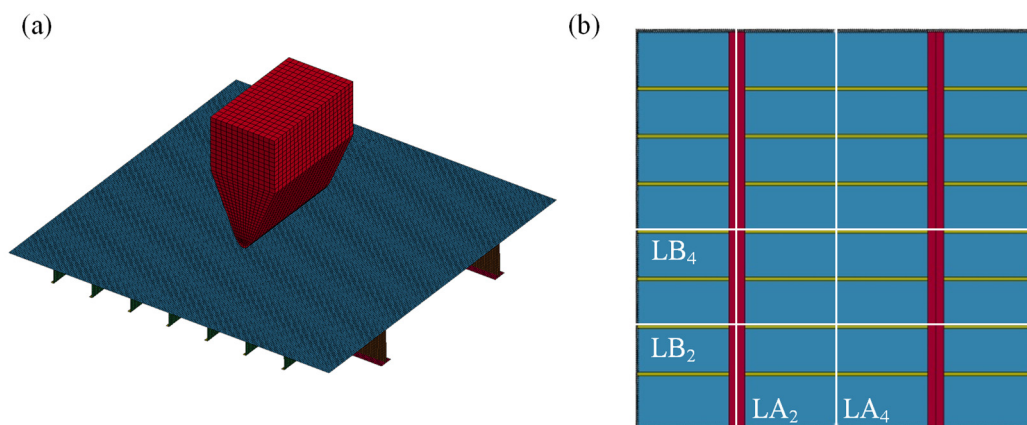


Figure 20. The finite element model of the impact test: (a) Whole model; (b) Stiffened panel and boundary conditions.

3.3. Simulation Results

The comparison of deformation between the experiment and simulation is illustrated in Figure 21, in which the deformation of stiffeners and ribs is examined in more detail. A similar deformation mode is observed between the numerical model and impact test, as both present a rectangular-like dent along the stiffener direction and the main deformations are localized over the impact area. The simulation method also captures the deformation modes of stiffeners and ribs accurately, both of which have a certain amount of torsional deformation. The major difference lies in the degree of deformation, and the difference in rib deformation is more pronounced. It appears that the distortion degree of the ribs is more serious in the experiment compared with that of other components of the structure. This is probably due to the presence of the compacted crushed ice, which may contribute

to the deformation of the panel and is not considered in the simulation. At the same time, the symmetry of deformation is more obvious in the simulation.

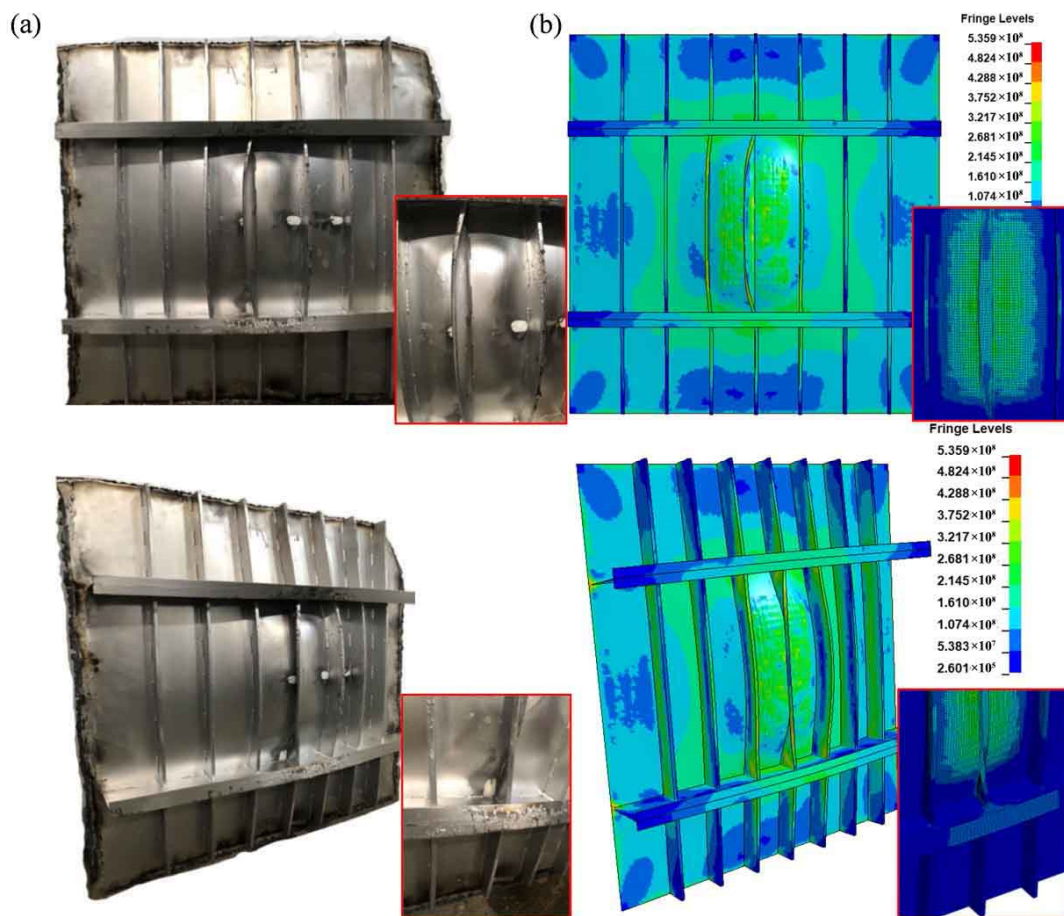


Figure 21. Comparison of the stiffened panel's deformation between the experiment and simulations: (a) Impact test; (b) Numerical simulation.

Table 7 shows the displacement values of the measured points in the simulation. The positions are consistent with those in the experiment. A maximum deformation value of 70.751 mm was obtained, and the relative error was approximately 12.03%. Representative points along the reference lines LA₂, LA₄, LB₂, and LB₄ (Figure 20b and Table 7) are selected and compared, as shown in Figure 22. From the figure, it can be seen that the errors are controlled within a certain range in both directions, and the deformation mode is basically consistent. In the direction of the stiffener, the error is relatively large, and the symmetry of the deformation is different, owing largely to the asynchronous failure of the ice body in both the simulation and the experiment.

Table 7. Displacement values obtained from the simulation.

Node ID	A1	A2	A3	A4	A5	A6	A7
B ₁	4.511	8.452	10.394	11.272	10.294	8.449	4.775
B ₂	6.678	13.943	20.121	23.552	21.161	13.900	6.931
B ₃	9.637	20.584	36.372	45.849	38.208	20.607	9.913
B ₄	11.412	24.179	60.389	70.751	63.422	24.184	13.468
B ₅	9.609	20.640	36.246	44.736	37.452	20.699	9.613
B ₆	7.564	13.986	20.284	23.227	20.271	14.032	6.696
B ₇	4.221	8.450	10.471	11.154	10.384	8.487	4.513

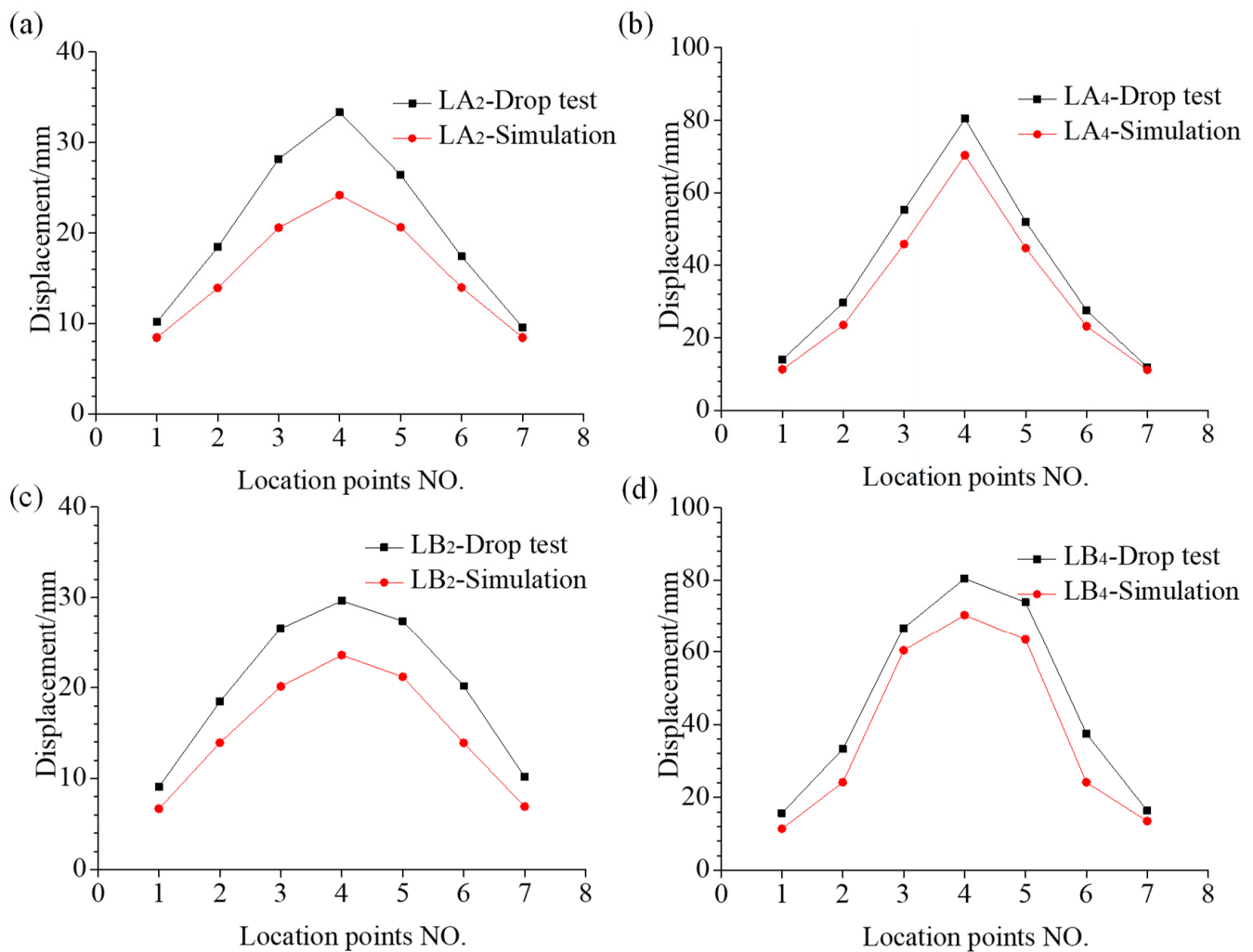


Figure 22. Comparison of deformation profiles between the experiment and simulation: (a) LA₂; (b) LA₄; (c) LB₂; (d) LB₄.

Figure 23 plots the time history of the impact force obtained from the numerical simulation and experiment. The impact force is generally correctly predicted by numerical simulation and the features of the exhibited fluctuations are well reflected. The peak force value is slightly smaller than that in the test, with an error of 14.30% (311.29 kN/363.24 kN), which corresponds to the conclusions of deformation. It should be noted that the proposed ice model focuses on the prediction of ice loads and structural deformation, as it is currently difficult to simulate all the properties of ice materials. Figure 23 shows that the wave form of the impact force is not fully reflected in the simulation, which is mainly because the failure criterion of the constitutive model pays attention to the bearing capacity of ice material under triaxial stress state, but the friction slip and weakening effect of shear failure is ignored for preventing premature failure.

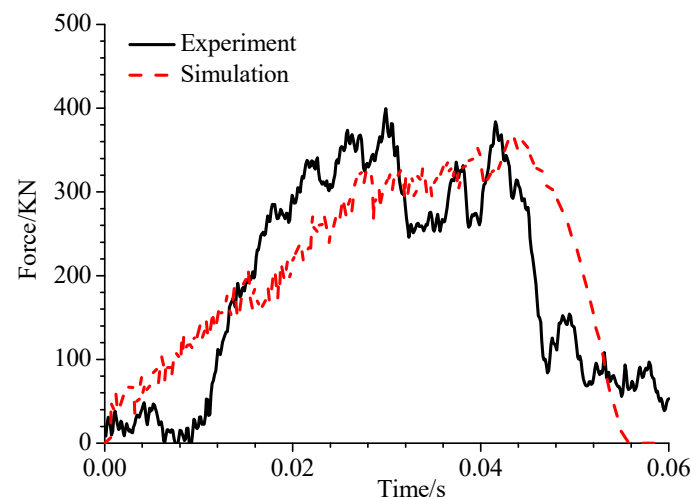


Figure 23. Comparison of the impact force distributions obtained from the experiment and simulation.

It appears that the constitutive model based on multisurface yield and empirical failure criteria is weaker than the ice indenter used in the impact test, as the maximum displacement and the peak force obtained from the simulation are smaller. The reason may be errors in the input parameters and underestimation of the strain rate effect of ice. In addition, the reinforcement of the structure by welding is also neglected. It should be noted that the ice indenter undergoes serious fragmentation in the experiment and the crushed ice was tightly compacted in the deformation area, which plays an important role in the deformation of the panel. In simulation, however, the failed elements will be deleted instead, owing to the inherent disadvantages of finite element methods (based on continuum mechanics). Although the errors are within acceptable limits, and it is clear that the present constitutive model can be improved.

4. Discussions

4.1. Effect of Parameters $q_{\max}$, ε_0 , and b

The definition of the yield equation and failure criterion is crucial for the development of constitutive models and the values of $a_0 = 22.93 \text{ MPa}^2$, $a_1 = 2.06 \text{ MPa}$, $a_2 = -0.023$, $\varepsilon_0 = 0.01$, and $b = 0.5$ are used. The values of a_i ($i = 1, 2, 3$) are determined by $q_{\max}$ (see Equations (5) and (6)), which is monotonically related to the temperature and strain rate (see Equations (2) and (3)).

Five values were chosen for each parameter, as listed in Table 8. The sensitivity study was performed for each variation in one parameter at a time and the deformation and peak force were evaluated. The results are plotted in Figures 24 and 25.

Table 8. Values of a_i ($i = 0, 1, 2$), $q_{\max}$, ε_0 , and b .

Cases	a_0 (MPa ²)	a_1 (MPa)	a_2	$q_{\max}$ (MPa)	ε_0	b
1	49.58	4.46	−0.049	10	0.05	0.3
2	111.57	10.04	−0.112	15	0.1	0.4
3	198.35	17.85	−0.198	20	0.15	0.5
4	309.92	27.89	−0.309	25	0.2	0.6
5	446.28	40.17	−0.446	30	0.25	0.7

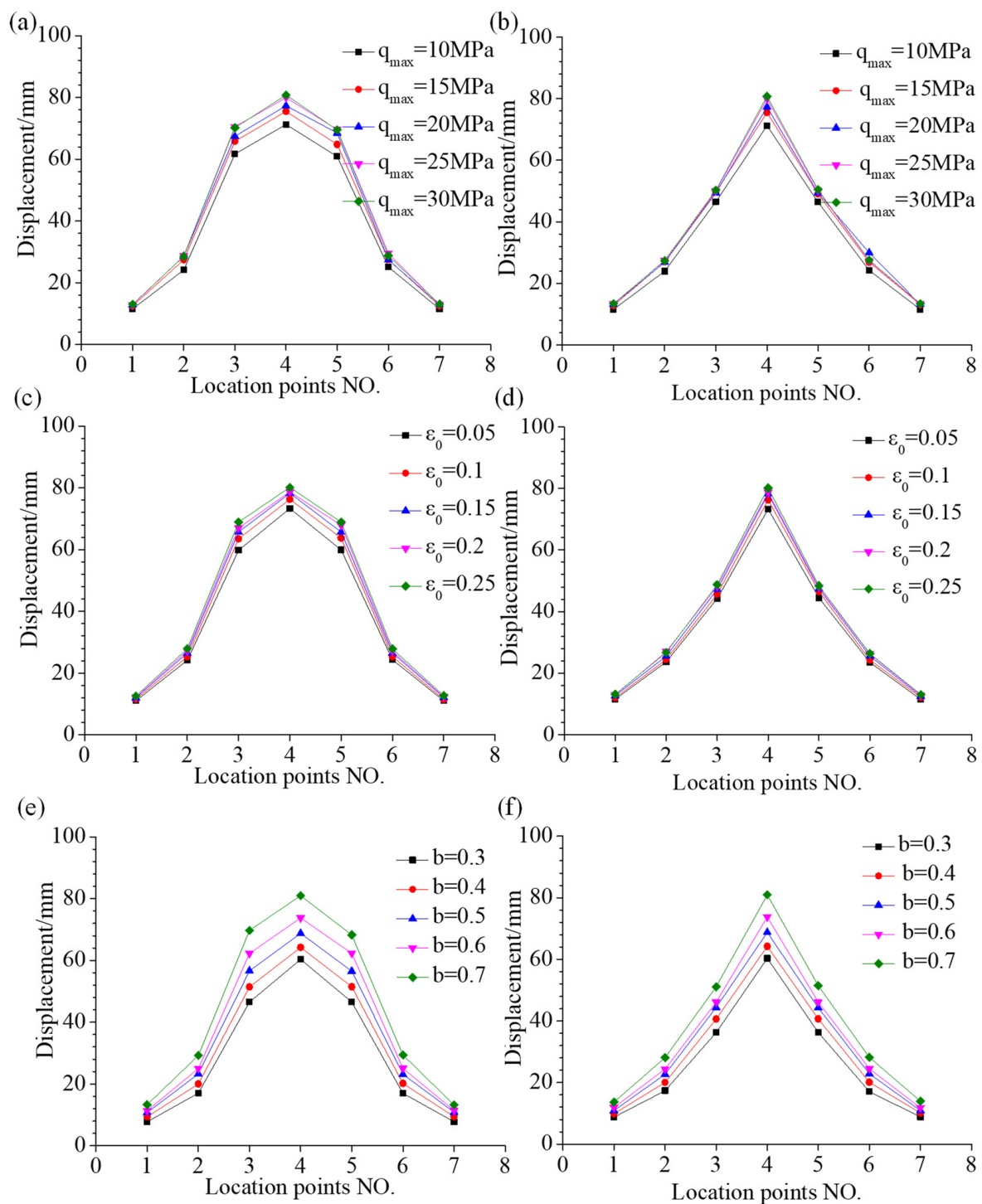


Figure 24. Comparison of deformations under different constitutive parameters: (a) Deformation of line LB₄ under different values of $q_{\max}$; (b) Deformation of line LA₄ under different values of $q_{\max}$; (c) Deformation of line LB₄ under different values of ϵ_0 ; (d) Deformation of line LA₄ under different values of ϵ_0 ; (e) Deformation of line LB₄ under different values of b ; (f) Deformation of line LA₄ under different values of b .

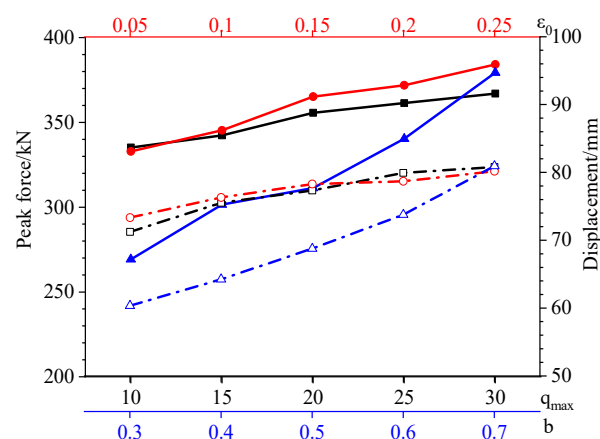


Figure 25. Peak forces and deformations under different parameters.

As can be observed from figures, the panel deformation increases with the increase in q_{max} , ϵ_0 , and b , and obviously the parameter b has the greatest degree of influence. The differences in deformation values under the three types of scenarios are 9.55 mm, 6.83 mm, and 20.58 mm, respectively. The maximum deformation is over 80 mm when the value of q_{max} is 30 MPa or $\epsilon_0 = 0.25$, and $b = 0.7$, which is very close to the experimental results. In the direction along the stiffeners, the effect of the parameter variation is greater, mainly due to the orientation of the ice indenter. As a matter of fact, larger values of q_{max} will strengthen the yield function and larger values of ϵ_0 and b lead to larger failure strains, which will all contribute to the final deformation of the stiffened panel.

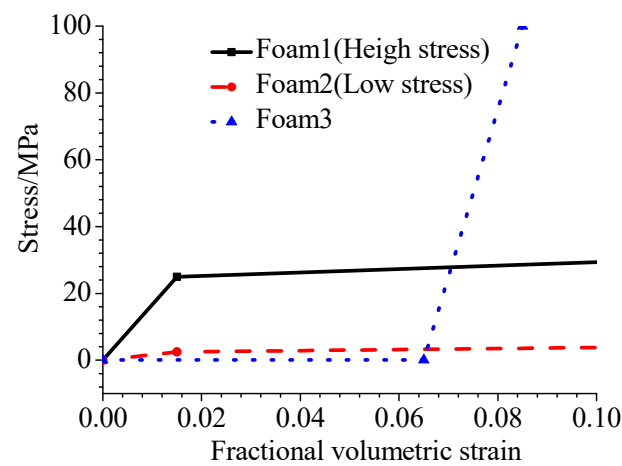
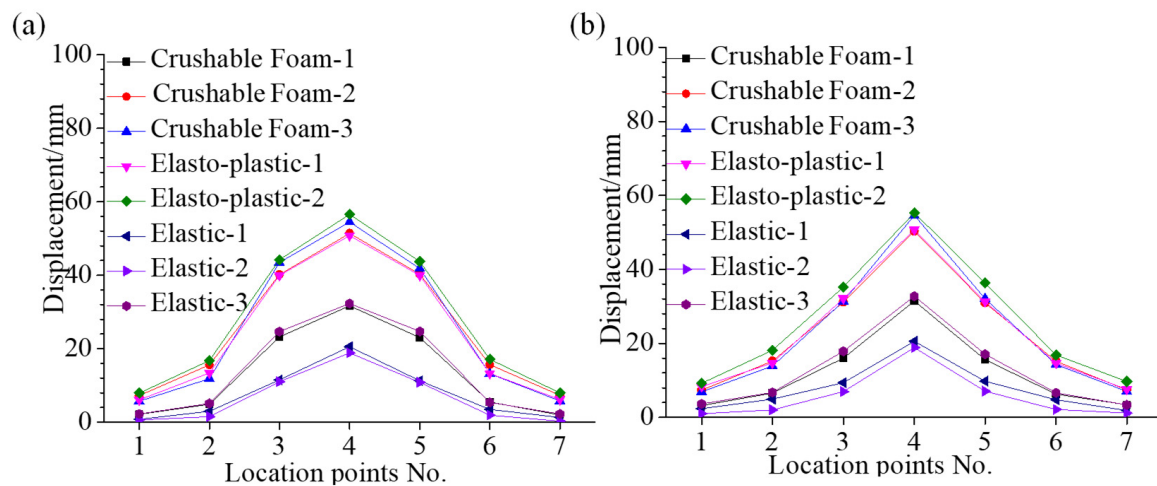
The change in peak forces presents a similar tendency as the parameters q_{max} , ϵ_0 , and b increase. A positive but nonlinear correlation is observed and the influence of parameter b also has the greatest influence. A convincing conclusion is that the definition of the failure criterion has a greater impact on determining the material properties of ice, since the parameter b is directly related to it, which controls the eroding and deletion of the element. Therefore, in practical applications, it is crucial to pay attention to the selection of the constitutive model parameters, especially the failure criterion.

4.2. Comparison with Other Ice Material Models

The main focus of the constitutive model is the three-dimensional stress state of the ice material, but the properties of other material models are also of great interest. Three types of materials, namely, crushable foam, elasto-plastic, and elastic materials are discussed, along with the corresponding definition of failure criteria, as listed in Table 9. The stress versus volumetric strain curves for crushable foam materials (Case 1, Case 2, and Case 3), which have been previously used by Gagnon [18,19], are illustrated in Figure 26. The yield or failure stresses in Table 9 are based on the uniaxial compressive strength of fresh-water ice materials, which is less than 10 MPa most of the time. However, the tests conducted by Jones showed a range of 3.79 to 17.81 MPa for fresh water ice [42]. Therefore, a failure stress value of 10 MPa and 20 MPa is discussed and the results are illustrated in Figures 27 and 28.

Table 9. Detailed information of different material model.

Cases No.	Case Name	MAT No.	Yield Criterion	Failure Criterion	Remarks
1	Crushable Foam 1	MAT 63		10 MPa	Figure 26, curve Foam 1
2	Crushable Foam 2	MAT 63		10 MPa	Figure 26, curve Foam 2
3	Crushable Foam 3	MAT 63		10 MPa	Figure 26, curve Foam 3
4	Elasto-plastic-1	MAT 13	10 MPa	0.01	
5	Elasto-plastic-2	MAT 13	20 MPa	0.01	
6	Elastic-1	User defined		multisurface	
7	Elastic-2	MAT 1		10 MPa	
8	Elastic-3	MAT 1		20 MPa	

**Figure 26.** Stress versus volumetric strain curves for crushable foam materials.**Figure 27.** Comparison of deformations using different ice material models: (a) Deformation along line LB₄; (b) Deformation along line LA₄.

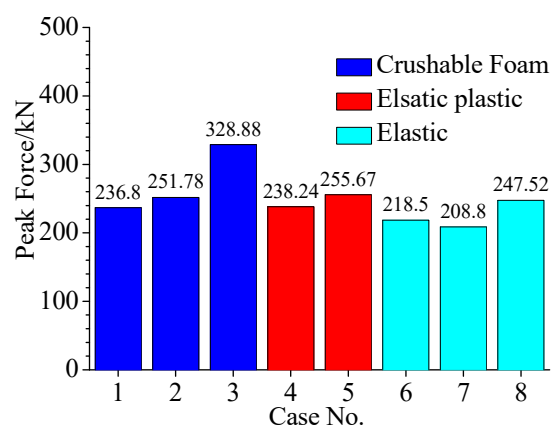


Figure 28. Comparison of peak forces using different ice material models.

According to the figures, the elasto-plastic material provided the best performance among the three types of ice materials, while the elastic material with a simple failure is considered inappropriate. All deformation values are smaller than the experimental values, as is the peak force. Clearly, these material models are not comparable to the user-defined constitutive models used in this study because of the relatively weak yield and ultimate strengths. For crushable foam-2 (low stress), although such a problem is avoided, the stress versus volumetric strain curves are not sufficiently convincing. In addition, it is interesting to note that although the elasto-plastic model is developed from the elastic model, there is a substantial difference between these two ice material models, indicating that the failure criterion is more likely to have a large impact on the results. Additionally, it is clear that when the multisurface envelope is taken as the failure criterion, the expected results cannot be obtained, so a reasonable and meaningful yield equation and failure criterion is both essential and significant.

5. Conclusions

The impact test of an ice indenter on a stiffened panel was carried out, and the deformation of the structure and the damage characteristics of the ice indenter were closely examined. The numerical simulations are performed using an elasto-plastic constitutive model based on a multiple surface yield envelopes and empirical failure criteria. The influences of various parameters in the constitutive model and the performance of other different material models are discussed. The following findings and insights are obtained:

1. The impact test shows that the ice indenter caused a significant indentation and experienced severe crushing and scattering. No obvious destruction or structural failure occurs. The deformation is localized at the impact area, and the main supporting components suffer substantial buckling deformation. The crushed ice was tightly compressed in the depression and experienced three-dimension stress state, which plays an important role in the structure's deformation.
2. The elasto-plastic constitutive model based on the multisurface yield envelope and empirical failure criterion is successfully applied. A similar deformation mode is observed, and the deformation of stiffeners and ribs is captured accurately. The relative errors are approximately 12.03% and 14.30% for the deformation and peak force, respectively.
3. According to the sensitivity study, the stiffened panel increases in both peak force and deformation with increasing values of parameters $q_{\max}$, ε_0 , and b , and the values of b has the greatest degree of influence. Compared to other ice materials, the elasto-plastic material exhibited the best performance. It is confirmed that a reasonable and meaningful yield equation and failure criterion are both significant and indispensable.

It is worth pointing out that, in a specific design stage, the ship's speed, the navigation area, and the general parameters of the ship itself are determined. The ice conditions,

temperature and other navigation information in the target sea area need to be investigated. The simulation method of steel and the ice model proposed in this paper are able to consider the effect of temperature, which can provide a rapid assessment of structural safety and useful technical advice during the design phase.

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