Article

Comprehensive Investigations of Cutting with Round Insert: Introduction of a Predictive Force Model with Verification

Judit Lukács [†] and Richárd Horváth ^{*,†}

Bánki Donát Faculty of Mechanical and Safety Engineering, Óbuda University, H-1081 Budapest, Hungary; lukacs.judit@uni-obuda.hu

* Correspondence: horvath.richard@uni-obuda.hu

† These authors contributed equally to this work.

Abstract: The observation and prediction of cutting forces are key questions when planning manufacturing technologies. In this paper, all three force components are investigated in the case of turning with round insert. The specific cutting forces were used and adapted for the round insert. Two easily calculable parameters of the undeformed chip cross-section were introduced (h_{eq} —equivalent chip thickness and l_{eff} —total length of the engaged cutting edge). The dependency of the specific force components was examined with regards to h_{eq} and l_{eff} . The main goal was to generate new empirical force models for each component, which helps to estimate their values in a wide range of technological parameters from fine to rough machining. The application of the models was confirmed by a case study where 11SMn30 + C (1.0715) was investigated. The examined interval of cutting parameters was: for feed, $f = 0.12 \dots 0.6$ mm, and for depth of cut, $a_p = 1.2 \dots 4.8$ mm. The novelty of the article is that the introduced new force models based on the geometric parameters characterizing the undeformed chip cross-section of round insert are proper for predicting the cutting force components in the technological process planning with an accuracy high enough—for main cutting force (F_c) $-3.79\% \dots +4.43\%$, for feed force component (F_f) $-5.4\% \dots +5.99\%$, and for radial force component (F_r) $-4.49\% \dots +4.77\%$.

Citation: Lukács, J.; Horváth, R. Comprehensive Investigations of Cutting with Round Insert: Introduction of a Predictive Force Model with Verification. *Metals* **2022**, *12*, 257. <https://doi.org/10.3390/met12020257>

Academic Editor: Umberto Prisco

Received: 20 December 2021

Accepted: 24 January 2022

Published: 29 January 2022

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Keywords: cutting force mode; round insert; calculation of specific force components; total length of the engaged cutting edge; equivalent chip thickness

1. Introduction

The quality requirements of cut parts (e.g., geometrical accuracies, surface roughness parameters, etc.) are strongly influenced by the set cutting parameters, and also by the generated forces (see Figure 1).

Therefore, the investigation of cutting forces and the examination processes constitute an important field of research nowadays. In addition, the key question is to make the calculation methods more accurate. The cutting force and its spatial components are related to further important issues such as the geometric error of the finished workpiece, the energy consumption of the cutting operation, residual stress in the workpiece, adaptive control, and so forth. Carrino et al. [1] highlighted the need for the estimation of cutting force components in the case of turning. With the help of numerical examples, the connection between the workpiece diameter error and the cutting force components were demonstrated. Smith [2] revealed that the feed force component provides a better indication of the cutting edge's condition as a function of tool wear during a machining operation than the torque value. Although Gonzalo et al. [3] studied a clamping process, the main cause of part rejection was also stated: the static deformation and the dynamic vibrations. That is the static deformation, which is strongly affected by the process forces. The residual stress induced in turning was analyzed by Outeiro et al. [4] for workpiece material AISI 316L. It was revealed that the residual stress was sourced from several parts, such as the action of the cutting force components, the temperature generated, and the phase transformation in

the surface layer. So, the knowledge and the investigation of the cutting forces and their components is an important research field in cutting technologies. The chronological order of some important cutting force models is shown in Figure 2.

Figure 1. The force system of turning.

Figure 2. The history of cutting force models adapted from [5].

One of the most widely used force models with reasonable accuracy that can be applied in industrial conditions was presented by Kienzle and Victor [6,7]. The application of specific cutting force components and their equation is still an integral part of the teaching materials in theoretical cutting (books, textbooks, etc.) all over the world [2,8–12]. In the cutting force model of Kienzle and Victor, specific cutting forces k_i were introduced and calculated with the help of empirically determined parameters ($k_{1,1}$, and m). The force is called specific, as it consists of the cutting force divided by the chip cross-section linked to chip width $-b$ and thickness $-h$ (see Figure 3).

Figure 3. The theoretic chip cross-section in case of conventional roughing turning.

It means that, according to Kienzle, the main cutting force (F_c) is equal to the product of the chip cross-section and specific cutting resistance of the workpiece material (see Equation (1)).

$$F_c = A \cdot k_c. \quad (1)$$

To calculate the cutting force for many machining technologies, F_c is determined by this approach. In the case of this method, chip geometry is important. The key factor, which determines the chip cross-section is a cutting edge angle ($KAPR$). Figure 3 shows the relationship between the cutting and the geometric parameters. Since

$$\sin(KAPR) = \frac{h}{f} = \frac{a_p}{b},$$

chip thickness and width can be determined according to Equations (2) and (3).

$$h = f \cdot \sin(KAPR) \quad (2)$$

$$b = \frac{a_p}{\sin(KAPR)}. \quad (3)$$

The area of the undeformed chip cross-section can be calculated with the help of the set cutting parameters (a_p, f) and by the geometric ones (h, b) as well (see Equation (4)).

$$A = a_p \cdot f = b \cdot h. \quad (4)$$

The formula for the specific cutting force in the direction of the main cutting force (F_c) was presented by Equation (5),

$$k = \frac{k_{1.1}}{h^m}, \quad (5)$$

where $k_{1.1}$ is the main value of the specific cutting force (in case of a chip cross-section where $h = 1$ mm and $b = 1$ mm); m is a constant value which is the characteristic slope of the curve showing relation between k and h values on a logarithmic scaled diagram. Both parameters are depending on the type of the material examined.

Nowadays, the precision, the refinement, and the adaptation of the Kienzle formula (Equation (5)), the application of the geometric parameters characterizing the chip cross-section (h, b) are still important fields of research of cutting studies.

Cutting forces are strongly influenced by the set cutting parameters and by the tool geometry that should be involved in the models as correction factors. Korkut and Boy [13] investigated the effect of cutting parameters (v_c, f, a_p) on the generated forces. Günay et al. [14]

examined the influence of rake angle and cutting speed. Similar studies were carried out by Saglam et al. [15]; however, their experiments were expanded with the investigation of entering angle. It was revealed that with the increased positive rake angle, the cutting forces were decreased, which means less power demand. Since the cutting edge enters and leaves the workpiece gradually, the impact of the load is not exerted on it. The optimum entering angle was determined. In addition, it was demonstrated that the greater value of entering angle produces greater feed force but less thrust force.

Several studies deal with the investigations of specific cutting forces. Velchev et al. [16] tested the influence of cutting speed. It was concluded that the presented mathematical formulae can be used for the approximation of the dependency of the specific cutting force on the cutting speed in case of turning in a wide range. Fernández-Abia [17] studied the behavior of austenitic stainless steels at high speed turning using specific force coefficients. A mechanistic model was introduced for cutting force prediction.

In the case of hard turning (workpiece material: AISID3, hardness: 60HRC), Aouici et al. [18] examined the effect of cutting parameters (v_c , f , a_p) on the cut surface roughness, cutting force, specific cutting force, and power.

Cobalt chromium alloys are materials for orthopedic implants mainly due to its unique properties of biocompatibility and wear resistance. Ahearne and Baron [19] reported initial research on cutting biomedical grade cobalt chrome molybdenum (Co-Cr-Mo) alloy, ASTM F75. Based on their results, the determination of the cutting force coefficients, as well as constants of the Kienzle equations, was enabled.

Specific cutting forces are strongly influenced by the chip geometry (Figure 3). Bíró and Szalay [20] presented the effect of the alteration of the chip thickness. Four zones were demonstrated from macro geometry to hypothetical micro-chip thickness based on whose specific cutting forces are characterized in a wide range of h .

Nowadays, one of the main goals with regards to sustainable manufacturing is energy efficiency that is undoubtedly influenced by the generated forces. As a result, several researchers are dealing with this topic. In their studies, cutting force components were estimated with the help of the Kienzle formula. Guo et al. [21] proposed and validated an operation-mode-based approach to decrease the energy consumption of machine tools which incorporates material removal simulation in order to predict the energy demand of machining processes.

The aim of Tapoglou et al. [22] was to construct an architecture for on-board online optimization of the cutting conditions. The main goals were to minimize both cutting time and power consumption at the same time.

The Kienzle equation or its elements are still used and adapted for the process planning procedure of several types of cutting technologies. Gonçalves and Schroeter [23] presented geometric modeling and simulation of helical broach using the equations that characterized the geometry. Traction force and torque acting on the broach shank were calculated by the Kienzle formula. Experiments were carried out with low speed using the longitudinal movement of a CNC lathe to determine the constants of the equation.

A commonly used method to manufacture high precision involute gears is gear hobbing. The cost-efficiency of the manufacturing process and the quality of the produced gear is strongly influenced by the generated forces and the wear of the cutting tool. Tapoglou et al. [24,25] created a novel simulation code that was able to determine the time course of the cutting force components based on the Kienzle equation. With the help of the method presented, the complex movement of gear hobbing was involved, the total cutting forces, as well as the cutting forces in every cutting edge involved in the cutting process, were calculated.

The gear skiving technology is similar to gear hobbing. CAD simulation approach of gear skiving was presented by Antoniadis et al. [26,27]. In their papers, the cutting force components were calculated through the integration of elementary force components by the Kienzle formula. The coefficients of the Kienzle equation were analytically, experimentally determined for the specific tool-workpiece material combination. A similar issue is the

drilling thrust force calculation that requires a large amount of experimental work. Kyratsis et al. [28,29] showed an extensively validated CAD-based approach in their studies.

A 3-Dimensional kinematics simulation of face milling was showed by Tapoglou and Antoniadis [30] in which the cutting forces were calculated according to the Kienzle equation. The effect of cutting geometry on the surface roughness was examined as well.

Furthermore, the specific cutting force plays an important role in the micro-cutting process too. As the specific cutting force is a sensitive parameter, it must be better explored to evaluate the micro-machining processes. Oliveira et al. [31] compared the size effect behavior in micro- and macro-milling regarding the specific cutting force and investigated the effect of the tool edge radius, workpiece roughness, cutting force, and chip formation when milling slots of AISI 104 steel. Zhang et al. [32] established a milling force model for micro-end milling, which was based on the size effect of specific cutting force and validated by a series of experiments. The workpiece material was AISI 1045. It was stated that the specific cutting force of micro-milling increases with the decrease of feed per tooth and axial depth of cut. The helical angle of the tool has a strong effect on the specific cutting force; on the other hand, the cutting tool diameter has a minor effect. The specific cutting force in down-milling is larger than that in up-milling due to the impact of the cutting tool on the workpiece. Anand et al. [33] presented investigations of the size effects on cutting forces in micro-drilling of carbon fiber reinforced plastic composite (CFRP). Specific cutting forces were calculated by dividing cutting force components by chip cross-section. In the mathematical formula presented, the specific cutting force was determined as a function of the ratio of undeformed chip thickness to cutting edge radius. Regression analysis was carried out on the experimental results in order to count the coefficients. Thus, they successfully extended the Kienzle model for both specific thrust and radial forces in the micro-drilling of CFRP.

In the case of turning technologies, the knowledge and the estimation of cutting forces is a key question. Maier and Zaeh [34] showed an approach for the holistic modeling of process effects. In their studies, process heat, cutting forces, and increased load on feed and main drives were involved. The modeling of thermal boundary conditions required static mean values for the cutting process forces that were calculated by the Kienzle equation. Ntziatzias et al. [35] studied a new material—glass fiber reinforced polyamide—and experimentally determined the coefficients of the Kienzle formula for turning under dry conditions.

High productivity and geometrical flexibility, safe chip breaking, and machining of twist-free surfaces are proposed by the turn-milling technology. However, one of the main disturbances is the dynamic profile of process forces. Hertel et al. [36] presented a new process model of orthogonal eccentric turn-milling. Their analysis was concentrated on the process forces and their dynamic behavior. The Kienzle force model was involved in the calculation of specific process forces to show their dependency on the chip thickness.

An analytical force prediction model for the turning process is shown by Weng et al. [37] and Zhuang et al. [38]. The model was based on the discretization of cutting edge and detailed geometric analysis with the consideration of edge effect and size effect and applied to the force distribution of round insert. Their method is developed to divide the uncut chip area of the round insert into many increments, and the local parameters of each increment can be calculated by the classical oblique cutting theory. Finally, cutting experiments were performed on Inconel 718 and Ti6Al4V to verify their model.

Horváth [39] introduced a new model for fine turning forces in which an exact description of cutting on the tool nose radius was presented as a function of the set cutting parameters. With the help of the generated model, cutting force components can be estimated easily with high accuracy in the case of fine turning. For the experimental runs, a new dynamometer system was developed as well [40] that was able to measure cutting forces of low values (0...100 N). Horváth and Lukács [41] verified the applicability of the above-mentioned force model adapted for fine turning [39] for several types of materials

(C45 and KO36 steel types, AS12 die-cast aluminium alloy). Ribeiro Filho et al. [42] adapted the method for turning force prediction of various titanium alloys.

In the past few years, tools with round inserts are widely used in industrial technologies; it has become an important field of research. Pradeep et al. [43] dealt with the modelling of cutting forces in face-milling operation using self-propelling round insert. Even in face-milling technology the proper estimation and analysis of the cutting force plays an important role in the process optimization. To solve this problem Ghorbani and Moetakef-Imani [44] presented a new method to identify the coefficients of the specific cutting force in the case of full immersion face-milling with round insert. An analytical model was introduced by Weng et al. [45] for cutting force prediction based on the precise cutting geometry. Cutting edge discretization was applied in high speed turning operations. Physical and technological points of view were chosen by P. Nieslony et al. [46]. Coated carbide round inserts were used in turning with Self-Propelled Rotary Tool. Feed, depth of cut and cutting speed values were varied to study the changes in cutting forces as a function of the cutting length. A special field of application is turning and re-turning of the rim profile of railway wheels (see Figure 4). In addition, new wheels are either turned with that tool type or face milled. Since, in contrast to conventional tools (see Figure 3), a round insert does not have classic, straight-line-bounded geometry and definable main cutting angle, the wide range of application justifies the need for the adaptation of a new force model for the particular characteristics of the chip cross-section in these cases. In this paper, the new force model is adapted for turning with round insert. Furthermore, with the help of the new equations, the components of the generated spatial force system can be precisely calculated and studied in detail.

Figure 4. Roughing operation of the rim profile of railway wheel (adapted from [47]).

2. Materials and Methods

2.1. Materials and Tool Used

The cutting tests were carried out on 11SMn30+C (1.0715) steel. The overall size of the workpiece was $\varnothing 100 \times 150$ mm. The chemical composition is (in wt %): C = 0.08; Si < 0.05; Mn = 1.1; P = 0.07; S = 0.3. (Hardness: 112...169 HB). Turning studies were implemented with round insert coded RCMT 1204 M0 4325 ($RE = 12$ mm) and the tool holder was SRSCR 2525M 12 (Sandvik Coromant).

2.2. Machine and Device Used

The experiments were performed with Dugard Eagle BNC–1840 CNC lathe. (The main technical parameters are as follows: spindle speed is 100... 4500 1/min, and the machine performance is 7.58 kW). A KISTLER 9257b type dynamometer was used for the force measurements, the measurement range of which was: $F_c = -5 \dots 10$ kN and $F_f, F_r = -5 \dots 5$ kN. The dynamometer was linked to the KISTLER 5019 type charge amplifier. The evaluation of the cutting force components was processed with DynoWare software.

2.3. Adaptation of the Force Model for Round Insert

In the case of turning with round insert, the chip removal takes place on an arc (radius of the insert~tool nose radius). Based on the value of the depth of cut, two types of undeformed chip cross-section can occur: $a_p < RE$ and $a_p > RE$ (see Figure 5). Note that the case of $a_p > RE$ was examined from a theoretical point of view; to compare to the first case in a mathematical sense when determining l_{eff} . It can be stated that both of them significantly differ from the one defined by Kienzle (see Figure 3).

Figure 5. The shapes of undeformed cross-section on the round tool.

As a result, novel parameters characterizing the undeformed chip cross-section are needed [48]. In addition, it is advisable to substitute the chip cross-sections shown in Figure 5 with an equivalent one, the area of which equals the real, undeformed chip cross-section. Thus, h_{eq} , mm equivalent chip thickness, and l_{eff} , mm effective cutting length, the total length of the engaged cutting edge (see Figure 6) were introduced.

Figure 6. The connection between the undeformed cross-section and equivalent one.

The effective cutting length is calculated in two parts, the first of which is behind the contact point of the tool and the workpiece (opposite direction of feed)— l_{eff1} —and the other one is in the feed direction— l_{eff2} (see Equation (6)).

$$l_{eff} = l_{eff1} + l_{eff2}. \quad (6)$$

According to Figure 7, the first component, l_{eff1} is only the function of the feed and the insert radius. In order to calculate its value, the arc length belonging to φ_1 has to be expressed as follows: The circumference of the mentioned sector of the circle is counted as the multiplication of the insert radius and the angle φ_1 (Equation (7)).

$$l_{eff1} = f(RE; f) = RE \cdot \varphi_1. \quad (7)$$

Based on Figure 7 and on the sine trigonometric function, angle φ_1 can be calculated according to Equation (8).

$$\sin \varphi_1 = \frac{f}{2 \cdot RE} \Rightarrow \varphi_1 = \arcsin \frac{f}{2 \cdot RE}. \quad (8)$$

The first part of the total length of engaged cutting edge, shown in Equation (6), can be expressed by integrating Equations (7) and (8).

$$l_{eff1} = RE \cdot \varphi_1 = RE \cdot \left[\arcsin \frac{f}{2 \cdot RE} \right]. \quad (9)$$

The length of the second component (l_{eff2}) only depends on the depth of cut and the insert radius (see Figures 8 and 9). Similarly to the first part, the length of the arc is calculated with the help of the insert radius and the angle belonging to the set value of the depth of cut (Equation (10)).

$$l_{eff2} = f(RE; a_p) = RE \cdot \varphi_2. \quad (10)$$

However, in this case, two different situations occur based on the values of a_p . Firstly, when $a_p > RE$, the sine function is applied on the side angle (φ'_{2a}) of the supplementary triangle shown in Figure 8).

Figure 7. The define of l_{eff1} .

Figure 8. Define l_{eff2} if a_p is bigger than RE .

Figure 9. Define l_{eff2} if a_p is smaller than RE .

With the help of Equations (11) and (12) φ_{2a} is calculated according to Equation (13).

$$\varphi_{2a} = \frac{\pi}{2} + \varphi'_{2a} \quad (11)$$

$$\sin \varphi'_{2a} = \frac{a_p - RE}{RE} = \frac{a_p}{RE} - 1 \Rightarrow \varphi'_{2a} = \arcsin\left(\frac{a_p}{RE} - 1\right) \quad (12)$$

$$\varphi_{2a} = \frac{\pi}{2} + \varphi'_{2a} = \frac{\pi}{2} + \arcsin\left(\frac{a_p}{RE} - 1\right). \quad (13)$$

Having put Equation (13) into (10), Equation (14) shows the formula for calculating the effective cutting length.

$$l_{eff2a} = RE \cdot \varphi_{2a} = RE \cdot \left[\frac{\pi}{2} + \arcsin\left(\frac{a_p}{RE} - 1\right) \right]. \quad (14)$$

Figure 9 shows the second case when the $a_p < RE$ relation is satisfied.

The side angle of the supplementary triangle can be found on the outside of the angle belonging to the insert radius and the set cutting parameter a_p . It is calculated according to Equation (15).

$$\varphi_{2b} = \frac{\pi}{2} - \varphi'_{2b}. \quad (15)$$

With the help of the sine function, φ'_{2b} side angle is determined (Equation (16)).

$$\sin \varphi'_{2b} = \frac{RE - a_p}{RE} = 1 - \frac{a_p}{RE} \Rightarrow \varphi'_{2b} = \arcsin\left(1 - \frac{a_p}{RE}\right). \quad (16)$$

It can be stated that the side angle in both cases is similar (see Equations (12) and (16)).

$$-\arcsin x = \arcsin(-x). \quad (17)$$

Since arcsine is an odd function (its nature is shown in Equation (17)), the conversion demonstrated in Equation (18) is valid. As a result, the formula of φ_{2b} can be transformed according to Equation (18)

$$\varphi_{2b} = \frac{\pi}{2} - \varphi'_{2b} = \frac{\pi}{2} - \arcsin\left(1 - \frac{a_p}{RE}\right) = \frac{\pi}{2} + \arcsin\left(\frac{a_p}{RE} - 1\right). \quad (18)$$

With the help of the above-mentioned connections, the second part of the effective cutting length can be calculated with the same formula in both cases (see Equation (19)).

$$l_{eff2b} = RE \cdot \varphi_{2b} = RE \cdot \left[\frac{\pi}{2} + \arcsin\left(\frac{a_p}{RE} - 1\right)\right] = l_{eff2a} = l_{eff2}. \quad (19)$$

The total length of the engaged cutting length is determined according to Equation (20) by combining Equations (6), (9) and (19).

$$l_{eff} = RE \cdot \left[\frac{\pi}{2} + \arcsin\left(\frac{a_p}{RE} - 1\right) + \arcsin\frac{f}{2 \cdot RE}\right]. \quad (20)$$

It can be seen that the effective, operational cutting length can be calculated from the set cutting parameters (a_p and RE). The other parameter, the equivalent chip thickness, can be determined with the help of Equation (21).

$$A = a_p \cdot f = l_{eff} \cdot h_{eq} \Rightarrow h_{eq} = \frac{A}{l_{eff}} = \frac{a \cdot f}{l_{eff}}. \quad (21)$$

Based on the mechanical analogy, specific cutting force components can be determined by Equation (22) for all directions.

$$k = \frac{F}{A} = \frac{F}{l_{eff} \cdot h_{eq}}. \quad (22)$$

Since the specific cutting force is strongly influenced by the geometric parameters of the undeformed chip cross-section (l_{eff} and h_{eq}), the formerly determined (Equation (5)) was expanded (see Equation (23)),

$$k = C \cdot h_{eq}^m \cdot l_{eff}^n, \quad (23)$$

where C is a constant value, m is characterizing the connection between k and h_{eq} in a logarithmically scaled diagram (expected to have a negative value), and n is the exponent of the total length of the engaged cutting edge.

Finally, in Equation (24) the formula of the force model for turning with round insert is introduced.

$$F = k \cdot A = (C \cdot h_{eq}^m \cdot l_{eff}^n) \cdot (l_{eff} \cdot h_{eq}) = C \cdot h_{eq}^{m+1} \cdot l_{eff}^{n+1}. \quad (24)$$

3. Measurements

The experimental setup (see Figure 10) was developed aiming to investigate possibly the widest technological range of turning with round insert. On the one hand, the suggestions of the manufacturer were taken into consideration when choosing the interval of the input variables. In addition, the required machining performance was observed as well. Due to the above-mentioned requirements, the value of the feed was set in four ($f = 1.2 \dots 4.8$ mm), the depth of cut in nine ($a_p = 0.12 \dots 0.6$ mm) equidistant levels (see marked with blue on Figure 11). In addition, further confirmation measurement points were determined within the investigated interval of technological parameters (marked with red on Figure 11). Several studies have already dealt with the connection between the cutting speed and the force values [13,15,16,18,49–54]. Since either the less influential or a negligible effect was revealed, cutting speed was held on a constant value during the examinations: $v_c = 150$ m/min. In the case of each experimental run, the cutting length was chosen to have a machining time of 3...5 s in order to be able to evaluate the cutting force components with higher accuracy. In the case of each experimental run, the cutting length was chosen to have a machining time of 3...5 s in order to be able to evaluate the cutting force components with higher accuracy. The cutting force components were registered at a sampling frequency of 500 Hz at each measurement point. After filtering the measurement results, the average cutting force values were evaluated at the experimental runs.

Figure 10. Experimental setup.

Figure 11. The location of the experimental and confirmation runs in the cutting parameter range.

4. Results

In the case of turning with round insert, a significance test was carried out for each component in order to determine the influential parameters of the undeformed chip cross-section (h_{eq} and l_{eff}). The results showed that l_{eff} has a non-significant effect on k_c (in line with the Kienzle model [6,7], see Equation (5)). In addition, it can be seen in Figure 12a as well.

Figure 12. Specific force components plotted against l_{eff} and h_{eq} .

Based on Figure 12, in the case of each component, the decreased value of h_{eq} results in increased specific cutting force. Although, the higher value of l_{eff} gives a higher k_f (see Figure 12b). In contrast, the increased k_r is resulted by the lower l_{eff} (see Figure 12c).

As a result, nonlinear regression was fit on the measurement data, and the constants of Equation (23) were calculated. The formulae for the specific cutting force components are shown in Equations (25)–(27):

$$k_c = 693 \cdot h_{eq}^{-0.409} \cdot l_{eff}^0 \quad (25)$$

$$k_f = 10.8 \cdot h_{eq}^{-0.790} \cdot l_{eff}^{1.006} \quad (26)$$

$$k_r = 367 \cdot h_{eq}^{-0.676} \cdot l_{eff}^{-0.326} \quad (27)$$

The conformity of Equations (25)–(27) was examined on probability plots (see Figure 13). It can be stated that the relative residuals of the specific force components follow Gaussian distribution with low values of means. Thus, the introduced formulae are proper to study the generated spatial force system with adequate accuracy in the case of turning with round insert.

Figure 13. The investigation of the residuals on probability plots.

Additionally, based on Equation (24) the force components can be calculated as follows:

$$F_c = 693 \cdot h_{eq}^{0.591} \cdot l_{eff}^1 \quad (28)$$

$$F_f = 10.8 \cdot h_{eq}^{0.210} \cdot l_{eff}^{2.006} \quad (29)$$

$$F_r = 367 \cdot h_{eq}^{0.324} \cdot l_{eff}^{0.674} \quad (30)$$

In Table 1, the experimental measurement data and the calculated ones (by Equations (28)–(30)) are involved. Furthermore, the percentage differences are shown as well.

Figure 14 shows the calculated cutting values as a function of measured ones for each component. The identity function ($x = y$) would mean the perfect fit. It can be stated that Equations (28)–(30) give a good approximation for the cutting forces components. The deviations are as follows: for main cutting force— F_c : $-3.79\% \dots +4.43\%$, for feed force component— F_f : $-5.4\% \dots +5.99\%$, and for radial force component— F_r : $-4.49\% \dots +4.77\%$.

The Model Confirmation

The conformity of the model was proved in five measurement points. These further experimental runs were chosen from the range of the examined technological parameters; however, these significantly differed from the basic measurement points. In Figure 15 (37–41 confirmation runs), the measured and the calculated values of the force components are compared.

It can be stated that the accuracy is high enough; the highest deviations in the three directions are the following: in case of F_c -1.44% – 0.67% , for F_f -0.2% – 3.58% ; and for F_r -1.03% – 4.01% .

Table 1. The comparison of the measured and calculated force values.

Exp. Runs	a_p , mm	f , mm	l_{eff} , mm	h_{eq} , mm	A , mm ²	F_{c_meas} , N	F_{c_calc} , N	ΔF_{c} , %	F_{f_meas} , N	F_{f_calc} , N	ΔF_{f} , %	F_{r_meas} , N	F_{r_calc} , N	ΔF_{r} , %
1	1.2	0.12	5.4723	0.0263	0.144	444	443	−0.13	150	152	1.33	340	355	4.41
2	1.2	0.18	5.5023	0.0393	0.216	571	565	−1.13	173	167	−3.47	414	405	−2.17
3	1.2	0.24	5.5323	0.0521	0.288	675	670	−0.67	188	179	−4.79	458	446	−2.62
4	1.2	0.3	5.5623	0.0647	0.360	734	766	4.43	181	189	4.42	474	480	1.27
5	1.2	0.36	5.5923	0.0772	0.432	826	855	3.56	190	199	4.74	509	510	0.20
6	1.2	0.42	5.6223	0.0896	0.504	914	939	2.73	203	207	1.97	545	538	−1.28
7	1.2	0.48	5.6523	0.1019	0.576	999	1018	1.91	212	215	1.42	571	562	−1.58
8	1.2	0.54	5.6823	0.1140	0.648	1065	1094	2.70	214	223	4.21	588	585	−0.51
9	1.2	0.6	5.7124	0.1260	0.720	1135	1166	2.77	217	230	5.99	600	607	1.17
10	2.4	0.12	7.7820	0.0370	0.288	782	771	−1.39	332	330	−0.60	525	502	−4.38
11	2.4	0.18	7.8120	0.0553	0.432	977	981	0.42	369	362	−1.90	601	574	−4.49
12	2.4	0.24	7.8420	0.0735	0.576	1153	1164	0.99	398	387	−2.76	653	631	−3.37
13	2.4	0.3	7.8720	0.0915	0.720	1290	1330	3.13	407	409	0.49	686	679	−1.02
14	2.4	0.36	7.9020	0.1093	0.864	1445	1484	2.68	425	428	0.71	726	721	−0.69
15	2.4	0.42	7.9320	0.1271	1.008	1597	1628	1.91	439	445	1.37	763	759	−0.52
16	2.4	0.48	7.9620	0.1447	1.152	1765	1764	−0.08	451	460	2.00	791	794	0.38
17	2.4	0.54	7.9920	0.1622	1.296	1912	1893	−0.97	459	475	3.49	820	826	0.73
18	2.4	0.6	8.0220	0.1795	1.440	2050	2018	−1.56	473	489	3.38	849	855	0.71
19	3.6	0.12	9.6048	0.0450	0.432	1081	1068	−1.22	522	525	0.57	632	617	−2.37
20	3.6	0.18	9.6348	0.0673	0.648	1349	1358	0.68	583	575	−1.37	716	704	−1.68
21	3.6	0.24	9.6648	0.0894	0.864	1596	1611	0.97	626	614	−1.92	774	774	0.00
22	3.6	0.3	9.6948	0.1114	1.080	1810	1840	1.68	639	647	1.25	808	833	3.09
23	3.6	0.36	9.7248	0.1333	1.296	2036	2052	0.79	664	676	1.81	855	884	3.39
24	3.6	0.42	9.7548	0.1550	1.512	2260	2250	−0.43	683	702	2.78	896	931	3.91
25	3.6	0.48	9.7848	0.1766	1.728	2534	2438	−3.79	735	726	−1.22	968	973	0.52
26	3.6	0.54	9.8148	0.1981	1.944	2685	2617	−2.55	770	748	−2.86	1015	1012	−0.30
27	3.6	0.6	9.8448	0.2194	2.160	2842	2788	−1.90	807	769	−4.71	1070	1048	−2.06
28	4.8	0.12	11.1875	0.0515	0.576	1366	1347	−1.39	726	733	0.96	698	714	2.29
29	4.8	0.18	11.2175	0.0770	0.864	1712	1713	0.05	800	802	0.25	792	815	2.90
30	4.8	0.24	11.2475	0.1024	1.152	2034	2032	−0.10	856	856	0.00	871	896	2.87
31	4.8	0.3	11.2775	0.1277	1.440	2323	2320	−0.11	869	902	3.80	922	964	4.56
32	4.8	0.36	11.3075	0.1528	1.728	2656	2587	−2.60	925	941	1.73	979	1023	4.49
33	4.8	0.42	11.3376	0.1778	2.016	2891	2836	−1.90	985	977	−0.81	1028	1077	4.77
34	4.8	0.48	11.3676	0.2027	2.304	3149	3072	−2.45	1047	1009	−3.63	1089	1125	3.31
35	4.8	0.54	11.3976	0.2274	2.592	3378	3297	−2.41	1098	1040	−5.28	1155	1170	1.30
36	4.8	0.6	11.4276	0.2520	2.880	3591	3512	−2.20	1129	1068	−5.40	1189	1212	1.93
Conf. Runs	a_p , mm	f , mm	l_{eff} , mm	h_{eq} , mm	A , mm ²	F_{c_meas} , N	F_{c_calc} , N	ΔF_{c} , %	F_{f_meas} , N	F_{f_calc} , N	ΔF_{f} , %	F_{r_meas} , N	F_{r_calc} , N	ΔF_{r} , %
37	1.8	0.15	0.0401	6.7327	0.270	695	697	0.33	243	252	3.58	479	474	−1.03
38	4.2	0.15	0.0604	10.4335	0.630	1367	1376	0.67	654	661	1.10	718	726	1.11
39	3	0.36	0.1220	8.8528	1.028	1795	1769	−1.44	547	551	0.78	810	814	0.49
40	1.8	0.57	0.1478	6.9428	1.026	1546	1554	0.53	341	352	3.26	736	735	−0.17
41	4.2	0.57	0.2249	10.6436	2.394	3093	3054	−1.28	909	907	−0.20	1076	1121	4.01

(a) Main cutting force (F_c) (b) Feed force (F_f)**Figure 14.** Cont.

Figure 14. The graphic comparison of the measured and calculated values of the force components.

Figure 15. Results of the confirmation measurements.

5. Discussion

Within the framework of the research presented above, a Kienzle principle [6,7] based new force model was introduced for turning with round insert that was extended with a new approach with the help of the equivalency of the chip cross-section. The model presented above can be included in and enhance the classical types of cutting force models using mechanical analogy (cutting force/chip cross-section). The model can be used in technological process planning and for further studies of the generated cutting forces in the case of similar tool geometries. The theoretical background and applicability were demonstrated through a comprehensive case study providing promising results for accuracy and conformity. The ways of further development of the introduced force model can be studied from various aspects as follows. The empirical constants in the phenomenological model are mostly dependent on material (hardness and tensile strength of the workpiece material) and further parameters of the tool geometry. It might be useful to generalize them to material categories (see ISO DIN 513 and 3323 VDI, six classes of workpiece material), on which the model can be applied with adequate accuracy. In addition, further studies might reveal more detailed effects resulting from the cutting tool geometry. Furthermore, the application of force models based on the concept of the equivalent chip cross-section can be used in different technologies providing similar cross-section, such as ball end milling [55] or orthogonal turn-milling [56]. Moreover, investigating the detailed mechanistic approach of the chip formation in the case of round tool geometry and also the application of advanced finite element analysis can be another vital research area [57].

6. Conclusions

In this paper, a new force model was created for cutting with round insert. In that case, the geometry of the undeformed chip cross-section strongly differs from the conventional one. Therefore, h_{eq} and l_{eff} parameters were introduced that are characterizing the cut chip cross-section of round insert. Based on the investigations, the following conclusions can be drawn:

- It was demonstrated that the specific cutting force components (k_c , k_f , k_r) could be determined in the form of Equation (23) in a wide range of technological parameters with adequate accuracy;
- It was stated that increasing h_{eq} results in higher values of specific cutting forces for each direction;
- The changes of l_{eff} values have a non-significant effect on k_c , but for k_f and k_r have. The increased level of l_{eff} gives a higher value for k_f but a lower one for k_r ;
- The residuals of the generated specific force models were examined on normality plots. It was concluded that the residuals follow a normal distribution with a mean of approximately zero. In addition, the standard deviation of the relative deviations is a low value. As a result, the accuracy of the models is proper for technological process planning;
- The accuracy of the created force models handling the special geometric features of the undeformed chip cross-section of round insert is high enough for each component (F_c , F_f , F_r) in a wide range of technological parameters from roughing to fine technologies. The deviations in the directions are: for main cutting force (F_c) $-3.79\% \dots +4.43\%$, for feed force component (F_f) $-5.4\% \dots +5.99\%$, and for radial force component (F_r) $-4.49\% \dots +4.77\%$;
- The introduced force model can be easily applied for further material types after preliminarily determining the empirical constant by a few properly selected experimental runs. Thus, the process planning procedure can be improved;
- The strengths of the model include that the introduced model operates with easily calculable parameters. However, it can be said that the empirical constants in the phenomenological models are mostly material dependent (tensile strength-hardness) from the workpiece side; which might prognose further studies.

Author Contributions: Conceptualization, J.L. and R.H.; methodology, J.L. and R.H.; software, R.H.; validation, J.L. and R.H.; formal analysis, J.L.; investigation, R.H.; resources, R.H.; writing—original draft preparation, J.L. and R.H.; writing—review and editing, J.L.; visualization, R.H.; supervision, J.L.; All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The all measured and used data are available in this study.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

Abbreviations

A	chip cross-section, mm ²
a_p	depth of cut, mm
b	chip width, mm
C, m, n	empirical constants of the force model
CNC	Computer Numerical Control
f	feed, mm
F_c	main cutting force, N
F_f	feed force, N
F_r	radial force, N
h	chip thickness, mm
h_{eq}	equivalent chip thickness, mm
$KAPR$	cutting edge angle, °
k_c, k_f, k_r	specific forces, N/mm ²
l_{eff}	total length of the engaged cutting edge, mm
RE	tool radius, mm
v_c	cutting speed, m/min

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