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Numerical Simulation of Transport Phenomena in Directional Solidification Castings with Changeable Cross-Section and Solidification Interface Control

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Abstract: The roles of traveling magnetic fields (TMFs) within the transport phenomena during the directional solidification of nickel-based superalloys were simulated. The evolution of thermal field, flow field and solid-liquid interface morphology during the solidification process under both natural and forced convection conditions were also simulated and compared. The strength of TMFs window that suppresses the flow of the interfacial front in the melt was quantified. The association between flow velocity at the interface front and defect formation was discussed.

Keywords: numerical simulation; travelling magnetic field; transport phenomena; directional solidification; superalloy; solidification interface

1. Introduction

During the directional solidification process, the natural convection induced by gravity has a significant effect on the transport phenomena and solidification behavior [1,2]. When the convection is at the laminar level, the convection makes the temperature at the front of the solid-liquid interface fluctuate, which can cause the interface to move forward at a disturbed rate [3,4]. This eventually causes the solid phase composition to fluctuate accordingly, i.e., macroscopic segregation occurs [5–7]. Convection promotes the transition of columnar dendrites into equiaxed crystals when it is turbulent. Furthermore, it has an influence on crystal growth direction.

In order to suppress natural convection and obtain high-quality crystals or castings without (with less) defects, low frequency traveling magnetic fields are widely used to control different solidification processes, such as the Bridgman Method, the Vertical Gradient Freezing Method and the Czochralski Method, due to the low frequency fields' convection control properties. Related studies were started in the 1960s by M.C. Flemings [8], P. Rudolph [9–11], and I. Grants [12,13], who simulated the effect of TMFs at different intensities on the flow and solidification interface morphology during the solidification NBof semiconductor materials. The flow patterns associated with various stages of solidification, as well as the evolution of their solidification interface morphology, have been described. Subsequently, T. Duffar [14], P.A. Nikrityuk [15] and G.M. Oreper [16] et al. suggested that the magnetic field could suppress some of the natural convection in the melting of semiconductor single crystals. The simulation results pointed out that the degree of natural convection suppression was dependent on the external magnetic field's strength, geometry and size. Complete suppression of natural convection was very difficult. In the early



Citation: Zhang, Y.; Zhu, B.; Jiang, H.; Tan, L.; Weng, Y.; Yang, Y.; Qin, L. Numerical Simulation of Transport Phenomena in Directional Solidification Castings with Changeable Cross-Section and Solidification Interface Control. *Metals* **2022**, *12*, 1694. https:// doi.org/10.3390/met12101694

Academic Editors: Zhaoyang Hou and Cristiano Fragassa

Received: 9 July 2022 Accepted: 21 September 2022 Published: 10 October 2022

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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). 20th century, after the continuous improvement of magnetic field application techniques, the effects of magnetic field suppression of convective motion during solidification were investigated in different material systems. P. Becla [17] and J. Friedrich [18] et al. made HgMnTe crystals by the Bridgman method, where a 3T axial magnetic field was applied to suppress convective flow in the melt. They found no significant effect on the distribution of Mn in the axial direction. However, when the electrothermal material (Bi,Sb)₂Te₃ [19–22] was directionally solidified in the axial magnetic field, the macro segregation of axially oriented Bi increased with the increase of the magnetic field strength (0–8 T). The relevant simulations and experimental results show the magnetic field influences natural convection flow, and that it is possible to prepare crystalline materials with appropriate structure and uniform composition using a suitable magnetic field. Meanwhile, the magnetic field limits convection flow in different ways depending on the system, material, and method of application. This demonstrates the complexities of the rule of magnetic field effects on liquid phase flow, and far more research is required to determine how to successfully employ magnetic field to control flow during solidification.

Similarly, natural convection plays an important role in the directional solidification process of superalloys. When the density of the liquid phase between the dendrites is less than that at the front of the mushy zone, and natural convection is strong enough to overcome the viscous resistance of the liquid metal, a thermosolutal convection induced by inverse density in the mushy zone is formed. Then, the liquid metal with lower density in the mushy zone flows out of the mushy zone in the form of solute flow. Such flow causes stagnation or fusion of dendrite growth and forms narrow channels. Those fragments of dendrites fused by the liquid metal flow remain partially inside the channels and solidify along with the channels, finally becoming freckle chains in the superalloy casting [23]. These defects, all caused by natural convection, have a significant effect on the mechanical properties of the casting at high temperatures. Natural convection and complicated geometry influence the solidification behavior of superalloy blades during processing. The major feature of this complicated geometry is variable geometric cross section, which differs from the crystal growth investigations of cylindrical semiconductor materials discussed in the earlier references.

For this paper, the roles of TMFs on the flow pattern and solidification interface morphology during the directional solidification of variable-section castings were simulated and their possible influence on defects was explored. For these purposes, the three steps are as follows: (1) The magnetic field strength and Lorentz force density of casting generated by the TMFs were calculated by the finite element method. Subsequently, the flow during solidification was calculated by the finite difference method. A series of user defined functions (UDFs) were developed to deal with data transfer, and a three-dimensional model of the solidification of a casting with a variable cross-section that varies with the TMFs was developed to analyze the associated coupling among the magnetic, thermal, and flow fields. (2) In order to analyze the flow field during solidification under a TMF, first, the relationship between the TMFs and the Lorentz force was established. Thereafter, the distribution characteristics of the Lorentz force generated by the traveling magnetic field were simulated and calculated. (3) Finally, the precise selection of the matching TMFs' strength was the central issue in suppressing natural convection in variable cross-section castings. To this end, the evolutions of the flow field, thermal field and solid-liquid interface morphology at different stages of directional solidification under the action of TMFs of different strengths and orientations were investigated.

2. Results and Discussion

2.1. Magnetic Fields Results and Verification

The TMFs change the flow of liquid metal mainly by generating Lorentz forces of different directions and strengths in the liquid metal. Thus, a clear understanding of the distribution characteristics of the TMFs and Lorentz forces is the basis for regulating the flow of liquid metal. The distribution of magnetic field intensity inside the casting for

different directions of TMFs are shown in Figure 1a,b. Comparing the TMFs in different directions, the characteristics of the magnetic field intensity distribution are basically similar. The larger values of magnetic field intensity all appear at the bottom of the casting, while the smallest values of magnetic field intensity appear in the middle of the bottom of the large section of the casting. In the downward (Figure 1a) and upward TMFs (Figure 1b), the values of magnetic field intensity at the top of the casting are 4.51 mT and 4.43 mT, respectively, and the minimum magnetic field intensity at the middle of the large cross-section is 1.53 mT and 1.51 mT, respectively, while the intensity at both sides of the magnetic field is higher than that at the center of the large cross-section at the same height.



Figure 1. Numerical results computed by Ansys Emag (I = 10 A, f = 50 Hz): (**a**) magnetic field intensity distribution, TMF down; (**b**) magnetic field intensity distribution, TMF up; (**c**) Lorentz force density distribution, TMF down; and (**d**) Lorentz force density distribution, TMF up.

The Lorentz force density distribution inside the casting for different directions of the TMFs are shown in Figure 1c,d. The characteristics of the Lorentz force density distribution are completely different when comparing the TMFs with different directions. In this simulation, the direction of the Lorentz force is controlled by adjusting the phase angle: when the phase angles of the three coils are $-2/3 \pi$, 0, and $-(4/3) \pi$, the Lorentz force generated on both sides of the large cross-section acts upward and at an angle of 45° to the axial direction, i.e., acts as an upward TMF. On the contrary, when the phase angles of the three coils are 0, $-2/3 \pi$, and $-(4/3) \pi$, the Lorentz force generated on both sides of the large cross-section acts downward and at an angle of 45° to the axial direction, i.e., becomes an upward TMF. When the phase angles of the three coils are 0, $-2/3 \pi$ and $-4/3 \pi$, the Lorentz force on both sides of the large cross-section acts downward and is at an angle of 45° with the axial direction, which is the downward TMF. In the different directions of the TMF, the stronger Lorentz force is concentrated in the wall of the large section. Therefore, the influence of the TMF on the melt wall flow is much stronger. The current intensity and current frequency are used to control the magnitude of the Lorentz force; the curves between different current strengths and frequencies and the corresponding Lorentz forces are shown in Figure 2. The maximum Lorentz force within the melt also rises significantly when the frequency and the current increase. The current has a greater effect on the Lorentz force density than does the frequency. The magnetic field intensity at the positions indicated by the dots in (experimental setup) were measured with the Tesla meter, and compared with the simulation results as shown in Table 1. The result shows that the simulation is roughly consistent with the experimental results.



Figure 2. Maximal Lorentz force density as a function of (a) frequency, and (b) amplitude.

Table 1. The magnetic field intensity *B* at the positions indicated by the dots in experimental setup measured with the Tesla meter and the simulation.

Magnetic Field Parameters	P1	P2	Р3	P4	Р5
Experimental measurement (mT)	4.5	4.9	5.2	5.1	5.5
Simulation (mT)	5	5.2	5.4	5.5	5.6
Current intensity (A)	8	8	8	8	8

2.2. Flow and Thermal Fields

2.2.1. Influence of Natural Convection

During directional solidification, the flow pattern at the solidification interface front has a direct influence on the quality of the solidified tissue. Therefore, the analysis of the flow characteristics at the solidification interface front provides a basis for the suppression of natural convection in later chapters. The analysis also facilitates the comparison of the suppression effect of different strengths of traveling magnetic fields on natural convection. In this case, the flow evolution in the melt without a magnetic field is simulated. Figure 3 shows the temperature field, flow field and the corresponding solid-liquid interface morphology at four different stages: 824 s, 900 s, 1040 s and 1280 s.



Figure 3. The contours of temperature at different times: (**a1**) 824 s, (**a2**) 900 s, (**a3**) 1040 s, (**a4**) 1280 s. (**b1–b4**) show the velocity streamlines and the corresponding S/L interface morphology in (**a1–a4**), respectively.

In order to reveal the complex flow pattern inside the casting more clearly, two longitudinal sections at different locations were selected. The selected sections are the surface and central cross sections of the casting. At 824 s, when solidification proceeds to the large cross-section, the solidification interface remains flat. Figure 3(b1) shows a high-speed vortex in the longitudinal section at the front of the interface. Combined with Figure 4(a3), it can be determined that in the central longitudinal section, this vortex flows in a clockwise direction with a maximum flow velocity 8.7 mm/s. Meanwhile, in the surface longitudinal section, the vortex at the front of the solidification interface also flows

in a clockwise direction. However, its flow velocity is weaker than that at the front of the interface in the middle longitudinal section, where the maximum velocity of the vortex is 4.1 mm/s (Figure 4(a5)).



Figure 4. The velocity vector distributions at different times. (**a1–d1**) show 3D velocity vectors at 824 s; 900 s; 1040 s; 1280 s, respectively; (**a2–d2**) show 2D velocity vectors in the mid-vertical plane of (**a1–d1**); (**a3–d3**) show 2D velocity vectors at front of solidification in (**a2–d2**); (**a4–d4**) show 2D velocity vectors in the surface and 2D velocity vectors at front of solidification in (**a4–d4**) as shown in (**a5–d5**), respectively.

Solidification gradually proceeds to a large cross section at 900 s. The right side of the solidification interface buckles upward. In Figure 4(b3), the flow form at the solidification interface front in the central longitudinal section does not change significantly. However, in the surface longitudinal cross-section, the vortex at the solidification interface front changes from clockwise to counterclockwise flow (Figure 4(b5)). At 1040 s, when solidification was fully carried out to the large cross section, the solidification interface morphology transitioned from a unilateral slightly warped concave interface to a flat interface. Meanwhile, the flow pattern of the solidification interface front changed little. In Figure 4(c3), the flow velocity of the solidification interface front gradually decreases in the central longitudinal section. Its maximum flow velocity is 3.5 mm/s. The difference between the central and surface longitudinal sections is small (Figure 4(c5)). At 1280 s, the solidification has proceeded to the middle of the large cross-section. Meanwhile, the solidification interface has completely evolved into a flat interface again. However, the flow at the front of the solidification interface has produced a significant change. The vortex at the front of the interface changes to counterclockwise flow in the cross section. Concurrently, a stronger flow exists at the corner of the casting (Figure 4(d5)). By analyzing the flow forms at the interface fronts of the four different solidification stages, we found that there is always a more intense flow at the solidification interface fronts. This is due to the large longitudinal temperature gradient at the solidification interface front and the fact that the density of the superalloy also decreases with increasing temperature. Therefore, the longitudinal density gradient change significantly at the interface front. This results in greater flow intensity. Such flow causes corresponding fluctuations in solid-phase composition, i.e., macroscopic segregation. Such defect has a significant effect on the high temperature mechanical properties of the casting superalloys.

In addition, combining the above four stages of interface morphology evolution process, we found that the interface morphology at the change of cross-section briefly lost the flat distribution characteristics due to the uneven heat dissipation in the region. In columnar crystal castings, there are often multiple grains with different orientations, and the growth of these grains is in constant competition with each other.

2.2.2. Influence of Magnetic Field

The precise selection of matching traveling wave magnetic field strengths is a central issue in the suppression of natural convection in variable cross-section castings. For this purpose, we analyzed the effect of different strengths of downward traveling magnetic fields on the flow at the solidification interface front. Figure 5 shows the three-dimensional flow field at the solid-liquid interface front, the flow field in the central and surface cross sections, and the corresponding solidification interface morphology for different strengths of traveling magnetic fields as solidification proceeds to the variable cross section. The flow velocity profiles in the middle and surface cross sections at the corresponding solid-liquid interface fronts are shown in Figure 6. Comparing Figure 5a,b, when a traveling wave magnetic field with a current of 8A and a frequency of 50 Hz is loaded, the flow at the solidification interface front in the middle (Figure 5(a3,b3)) and surface (Figure 5(a5,b5)) longitudinal cross-sections is greatly suppressed. The flow velocity at the interface front decreases from 6.8 mm/s to 1.5 mm/s (Figure 6b,e). When the frequency is kept at 50 Hz and the current is increased from 8 A to 25 A, comparing Figure 5b,c, the flow velocity at the solidification interface front increases significantly, and the corresponding flow form within the melt changes significantly. Under the action of the Lorentz force, which is much larger than gravity, only a vortex with flow velocity in the range of 40–80 mm/s and in the counterclockwise direction is formed in the melt (Figure 5(c1)). Meanwhile, the natural convection has little effect in the melt. Thus, the forced convection generated by the TMF determines the direction and size of the flow in the melt. In addition, combining Figure $6c_{,f}$, the influence of the TMF is more pronounced for the flow in the surface longitudinal section than in the central longitudinal section.



Figure 5. The velocity vectors and the corresponding solidification interface morphology: (**a1-c1**) show 3D velocity vectors with I = 10 A, f = 50 Hz; I = 25 A, f = 50 Hz; I = 50 A, f = 50 Hz, respectively; (**a2-c2**) show 2D velocity vectors in the mid-vertical plane of (**a1-c1**); (**a3-c3**) show 2D velocity vectors at front of solidification in (**a2-c2**); (**a4-c4**) show 2D velocity vectors in the surface of (**a1-c1**); (**a5-c5**) show 2D velocity vectors at front of solidification of (**a4-c4**).



Figure 6. Velocity profiles at the front of solidification interface: (a) No TMF (Central); (b) I = 8 A, f = 50 Hz (Central); (c) I = 25 A, f = 50 Hz (Central); (d) No TMF (Surface); (e) I = 8 A, f = 50 Hz (Surface); (f) I = 25 A, f = 50 Hz (Surface).

Figure 7 shows the average flow velocity curves at the interface front in the absence of magnetic field and in the presence of this traveling field. Comparing the two profiles, it was found that under natural convection conditions, the average flow at the interface front increases significantly as the solidification interface gradually moves from a small to a large cross section. The TMF plays a large role in suppressing the flow velocity at the interface front at different stages of the solidification process. The average flow velocity at the solidification interface front was maintained in the range of 0.8–1.1 mm/s as solidification proceeded to the large cross section. The TMF of this strength greatly suppresses the natural convection at the interface front. This provides good theoretical support for the preparation of castings without macroscopic segregation and its associated defects.



Figure 7. Time history of *V* during solidification, *V* refers to the average velocity ahead of the S/L interface. (Downward TMF I = 8 A, f = 50 Hz.)

The morphology of the solid-liquid interface is also closely related to the strength of the TMF. Figure 8 shows the morphology of the solid-liquid interface and the corresponding flow pattern in the melt for different downward traveling magnetic field strengths. Under natural convection conditions, the right side of the solidification interface warps slightly upward as solidification proceeds to the change in cross section. When a TMF with a current of 8 A and a frequency of 50 Hz was loaded, the solidification interface tended to be flat. When the magnetic field strength continues to increase, the flow form in the melt changes from laminar to turbulent flow. The morphology of the solidification interface shows a large degree of distortion, especially on the right side of the solidification interface, where the degree of convexity is severe. Columnar crystals are prone to grains that deviate from the axial direction by a significant angle under such uneven solidification interface conditions. Such conditions also result in substantial transverse concentration gradients at the front of the solidification contact. As a result, segregation flaws occur, leading the mechanical properties of the casting to degrade.



Figure 8. The solidification interface morphology: (a) No TMF; (b) I = 8 A, f = 50 Hz; (c) I = 25 A, f = 50 Hz.

The data reveal that the flow at the solidification interface front can be greatly suppressed only when the downward TMF intensity is within a defined range. Such a magnetic field also improves the uniformity of the solidification interface. These findings indicate that the TMF has the potential to control the natural convection generated during the directional solidification of variable cross-section castings.

In conjunction with the theoretical analysis, we can use the Lorentz force parameter *F* to quantify the influence of the downward traveling magnetic field on the solidification behavior when solidification proceeds to a large cross section. The Lorentz force parameter *F* is given by $F = \sigma \omega k B^2 L^2 / 8$, where σ , ω , *k*, *B* and *L* correspond, respectively, to the conductivity of the high temperature alloy, the frequency of the downward TMF, the number of turns of the traveling magnetic field coil, the strength of the TMF and the characteristic length of the casting. Based on the analysis of the above simulation results, it is found that there is a critical value of *F*, i.e., *F*_{crit}, and when *F* exceeds *F*_{crit}, the flow rate in the melt would continue to rise. When *F* is less than *F*_{crit}, the natural convection within the range of 72.5 N/m³ and 81.4 N/m³, the natural convection within the melt is always suppressed. Therefore, the selection of a suitable Lorentz force parameter is the key to suppressing natural convection during directional solidification.

3. Conclusions

Two simulation software, ANSYS Emag (Ansys, Inc., Pittsburgh, PA, USA) and Fluent (Ansys, Inc., Pittsburgh, PA, USA), were coupled through the UDF interface program to realize the transient coupled calculation of 3D magnetic, thermal, flow and solute fields. The simulation results show that, in the case of natural convection only, the flow rate and interface morphology of the solid-liquid interface front change significantly in the transition region where the cross section changes. When solidification crosses the transition region, the flow rate at the front of the interface suddenly rises, as the interface morphology additionally changes from flat to concave. The natural convection in the melt is greatly suppressed by loading downward directed TMFs of suitable strength. A flat solidification interface is obtained. However, when the strength of this TMF is too high, the flow pattern within the melt changes significantly. As the intensity of TMFs increases, a stronger flow is generated at the front of solidification interface. Thus, the corresponding solidification interface shape is severely distorted.

4. Model Description

4.1. Electromagnetic Field Model

Sine alternating current was fed into the coil. The electromagnetic boundary conditions both on far field elements of the open boundary and at the axis of the model are assumed to be flux parallel. Maxwell equations to solve quasistatic electromagnetic field can be written as follows:

$$\nabla \times \vec{H} = \vec{J} \,, \tag{1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},\tag{2}$$

$$\nabla \cdot \overrightarrow{B} = 0. \tag{3}$$

Electromagnetic constitutive equation in isotropic medium is:

$$\vec{B} = \mu \vec{H},\tag{4}$$

$$\vec{I} = \sigma \vec{E}.$$
(5)

Lorentz Force can be expressed as:

$$\vec{F} = \vec{J} \times \vec{B}$$
(6)

To simplify the solution, equations applied magnetic vector potential A and scalar potential φ is given by

$$\vec{B} = \nabla \times \vec{A}, \tag{7}$$

$$\vec{E} = -\frac{\partial A}{\partial t} - \nabla \varphi, \tag{8}$$

where \vec{H} is the magnetic intensity, \vec{B} the magnetic flux density, \vec{E} the electric field strength, \vec{J} the current density, μ the magnetic permeability, σ the electric conductivity and \vec{F} the Lorentz Force.

4.2. Solidification Model

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0. \tag{9}$$

Momentum equation:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_i} = \mu \frac{\partial^2 u_i}{\partial x_i x_j} - \frac{\partial P}{\partial x_i} + A_c u_i + S_i + \overrightarrow{F}.$$
(10)

Energy equation (conservation of energy):

$$\frac{\partial \rho H}{\partial t} + \frac{\partial (\rho u_i H)}{\partial x_i} = \nabla \cdot (k \nabla T) + S_E, \tag{11}$$

where *u* is the velocity of flow, *P* is the pressure, S_i is the source term in momentum equation, S_E is the source term in energy equation, *T* is the temperature, ρ is the density, \vec{F} is the Lorentz force and *k* is the thermal conductivity.

$$H = h + \Delta H,\tag{12}$$

where

$$h = hh_{ref} + \int_{T_{ref}}^{T} CpdT$$
(13)

where *H* is the total enthalpy, *h* is the sensible enthalpy, ΔH is the latent heat of fusion. The liquid fraction γ is defined as:

$$\gamma = 0 \text{ if } T < T_{solidus} \tag{14}$$

$$\gamma = 1 \text{ if } T < T_{liquidus} \tag{15}$$

$$\gamma = \frac{T - T_{solidus}}{T_{liquidus} - T_{solidus}}, \text{ if } T_{solidus} < T < T_{liquidus}$$
(16)

The latent heat content ΔH in equation can now be written in terms of the latent heat of materials *L*,

$$\Delta H = \gamma L \tag{17}$$

Mushy zone is a semi-solid region existing as an interface between the melted and the un-melted region of superalloys. This region significantly influences the heat transfer and flow characteristics during melting and solidification of alloy. The mushy zone is treated as a porosity area.

The term A_c in the momentum equation is defined as,

$$A_c = -\frac{A_{mush}(1-\gamma)^2}{\gamma^3 + \varepsilon},$$
(18)

where, A_c is the porosity function, A_{mush} is the mushy zone constant, ε is a small computational constant (0.001) to prevent division by zero.

For more governing equations for solute distribution, please see our previous work [24]. Radiative transfer equation: the radiative transfer equation is solved for a set of n different directions, \vec{S}_i , i = 1, 2, ..., n, and the integrals over these directions are replaced by numerical quadratures.

$$\nabla \cdot (I(\vec{r}, \vec{s})\vec{s}) + aI(\vec{r}, \vec{s}) = an^2 \frac{\sigma T^4}{\pi}$$
(19)

where *I* is the radiation intensity, \vec{r} the position vector, \vec{s} the direction vector, σ the Stefan-Boltzmann constant and *a* the wall absorptivity.

4.3. Boundary Conditions and Computational Parameters

In this study, each TMF coil represents a $54 \times 58 \text{ mm}^2$ bunch of 285 windings. The three co-coils, which are coupled in star connection to the variable frequency power source, are supplied by the standard three-phase alternating current. The distance between the coils is $\Delta h = 15 \text{ mm}$. The height of the TMF coil system and the inner diameter are h = 204 mm and d = 185 mm, respectively. The specific dimensions of casting with variable section is shown in Figure 9. A schematic of the enclosure used in this study is shown in Figure 10a. Two typical sections are selected, surface and central cross sections respectively as shown in Figure 10b. The Ni-based superalloy employed was CMSX-4 (Cr 6.2, Co 9.4, Mo 0.7, W 5.7, Al 5.6, Ta 6.5, Ti 1.2, Re 3.0 and Hf 0.08 in wt pct). Tables 2 and 3 show physical parameters of the FEM model. In this process, density changes with temperature, using Boussinesq assumption.



Figure 9. Experimental design: (a) geometry of the setup, and (b) a sketch of the setup.



Figure 10. (a) Schematic representation of the computational domains and the meshes for the furnace heat transfer, and (b) the position of the surface and central cross sections in the casting.

Parameters	Values		
Relative permeability of air	1.0		
Relative permittivity of air	1.0		
Relative permeability of coil	1.0		
Relative permeability of alloy	1.12		
Relative permeability of mold	1.0		
Mold thickness	4 mm		
Frequency	50 Hz		
Electric current	0 A to 25 A		
Graphite bush electrical conductivity	$1.1 imes 10^5~\Omega^{-1}~\mathrm{m}^{-1}$		
Coil electrical conductivity	$4.7 imes 10^7 \ \Omega^{-1} \ { m m}^{-1}$		
Specific heat of alloy	684 J/(kg·K)		
Viscosity of alloy	6×10^{-3} Pa·s		
Density of alloy	$8.710 \times 10^3 \text{ kg/m}^3$		
Electrical conductivity of alloy	$7.69 imes10^5~\Omega^{-1}~\mathrm{m}^{-1}$		
Expansion coefficient of alloy	$1.4776 imes 10^{-4} \ 1/{ m K}$		
Latent heat of alloy	$2.7 imes10^5$ J/kg		
Liquidus temperature of alloy	$1.660 imes 10^3$ K		
Solidus temperature of alloy	$1.580 imes10^3~{ m K}$		
Withdrawal rate	120 µm/s		
Liquid diffusion coefficient of alloy	$3.6 imes 10^{-9} \text{ m}^{-2}/\text{s}$		
Solid diffusion coefficient of alloy	0		
Phase shift	$\pi/3$		
Temperature of heating zone	1500 °C		
Temperature of baffle zone	1330 °C		
Temperature of cooling zone	700 °C		
Temperature of cover zone	1330 °C		
Emissivity of heating zone	0.8		
Emissivity of baffle zone	0.95		
Emissivity of cooling zone	0.7		
Emissivity of cover zone	0.95		

Table 2. Physical parameters of the FEM model.

Table 3. Thermophysical properties of the superalloy and mold materials used for simulation.

Quantity	<i>T</i> (°C)	$\begin{array}{c} CMSX4 \\ (w \cdot m^{-1} \cdot {}^{\circ}C^{-1}) \end{array}$	Mold (w·m ^{-1.°} C ⁻¹)
Thermal Conductivity	30	12	1.92
	200	13.4	2.08
	400	15.2	2.11
	600	18.1	2.15
	800	21.5	2.31
	1000	24.3	2.45
	1200	27.2	2.62
	1400	30.1	2.91
	1600	34.5	33.15

There are some assumptions in the DS process: (1) displacement current is ignored and time-harmonic electromagnetic field is assumed as quasistatic electromagnetic field; (2) flow in melt is laminar flow; (3) electromagnetic force affects fluid flow, but fluid flow has no effect on electromagnetic; and (4) alloy melt is incompressible Newton fluid.

4.4. Solution Procedure

The distribution of magnetic field and Lorentz force field is calculated by ANSYS Emag. In the calculation of TMF, solid236 quadrilateral element was used for coil, graphite bush and casting; and solid236 triangular element was used for the near field. In addition, the far field dissipation was described by using INFIN111 far field element.

The date of force was then introduced into Fluent15.0 in the way of interpolation. During this process, some user defined functions (UDFs) were developed to precisely describe the movement of casting in furnace and data transfer between FDM/FEM.

The calculations of temperature field and flow field were performed by applying the Discrete Ordinates (DO) model and laminar model in software FLUENT, respectively. The DO model was used to describe the radiative heat transfer between furnace wall and castings; the flow in melt in the process was calculated by the laminar flow model. Solidification and melting models were used to describe the DS process of alloy. The Volume of Fluid (VOF) numerical approach is a classic, simple, and widely used robust method for addressing the interface between a 2D and 3D space. In terms of computational efficiency, the VOF excels other methods including the moving grid, level set, and phase field method, because the sample is surrounded by the mold shell, but the top of the melt is in direct contact with the air. The VOF provides tracking of the melt-air interface. Further, we focus more on how air in the furnace or other protective gases affect the thermal field and convection heat transfer. In this way, we can obtain a more reliable thermal field. The alloy was treated as the first phase, while the air as the second phase in VOF model.

Author Contributions: Conceptualization, Y.Z. and L.Q.; methodology, Y.Z.; software, Y.Z.; validation, Y.Z.; formal analysis, Y.Z.; investigation, B.Z., H.J., L.T., Y.W. and Y.Y.; resources, B.Z., H.J., L.T., Y.W. and Y.Y.; data curation, Y.Z.; writing—original draft preparation, Y.Z.; writing—review and editing, Y.Z.; visualization, L.Q.; supervision, L.Q.; project administration, Y.Z.; funding acquisition, Y.Z. and L.Q. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Natural Science Foundation of Chongqing (cstc2019jcyjmsxmX0183 and cstc2020jcyj-msxmX0877), the Science and Technology Research Program of Chongqing Municipal Education Commission (KJQN202001101, KJQN201901117 and KJQN201901118), the Scientific Research Foundation of Chongqing University of Technology, the Natural Science Basic Research Program of Shaanxi (Program No. 2021JM-402).

Data Availability Statement: All data, models, or code generated or used during the study are available from the corresponding author by request.

Conflicts of Interest: The authors declare no conflict of interest.

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