

Article

Vertical–Horizontal Coupling Vibration of Hot Rolling Mill Rolls under Multi-Piecewise Nonlinear Constraints

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Abstract: This study establishes a vertical–horizontal coupling vibration model of hot rolling mill rolls under multi-piecewise nonlinear constraints considering the piecewise nonlinear spring force and piecewise nonlinear friction force constraints of the hydraulic cylinder in the vertical direction of the rolls, the piecewise stiffness constraints in the horizontal direction, and the influence of the nonlinear dynamic rolling force in the rolling process. Using the average method to solve the amplitude–frequency response equation of the coupled vibration system and taking the actual parameters of a 1780 mm hot rolling mill (Chengde Steel Co., Ltd., Chengde, China) as an example, we study the amplitude–frequency characteristics of the mill rolls under different parameter settings. The results show that the amplitude and resonance region can be reduced by appropriately reducing the external disturbance force and the nonlinear spring force of the hydraulic cylinder, appropriately increasing the nonlinear friction force, and eliminating the gap between the bearing seat and the mill housing, to avoid the amplitude jump phenomenon due to piecewise variation. Furthermore, using the singularity theory to study the static bifurcation characteristics of the coupled vibration system, we establish a relationship between the vibration parameters and the topological bifurcation solution of the coupled system. The transition sets and their corresponding bifurcation topological structure in three cases are given, and the steady and unsteady process parameter regions of the rolls are obtained. The dynamic behavior of the coupled vibration system can be controlled by varying the bifurcation parameter. This study provides a theoretical basis for restraining the vibration of hot rolling mill rolls and optimizing the process parameters.

Keywords: hot rolling mill; piecewise nonlinear spring force; piecewise nonlinear friction force; piecewise stiffness; dynamic rolling force; coupled vibration; bifurcation



Citation: Peng, R.; Zhang, X.; Shi, P. Vertical–Horizontal Coupling Vibration of Hot Rolling Mill Rolls under Multi-Piecewise Nonlinear Constraints. *Metals* **2021**, *11*, 170. <https://doi.org/10.3390/met11010170>

Received: 18 November 2020

Accepted: 15 January 2021

Published: 18 January 2021

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1. Introduction

A rolling mill is an important piece of equipment in the field of modern heavy machinery, and its safe and stable operation is vital for ensuring a high-efficiency production of rolling products. However, rolling mill rolls are often unstable during the rolling process because of the high rolling speed and rolling strength of modern mills. This not only affects the quality and precision of the products but may also damage the rolling equipment [1–4].

The vibration problem in hot rolling mills has mainly been studied on the basis of the linear vibration theory in the early stage; however, practical results have shown that a linear analysis can sometimes lead to significant calculation errors. With the development of the nonlinear theory, the nonlinear vibration of hot rolling mill rolls has attracted the attention of researchers, who have studied it from various angles and achieved good results [5,6]. Swiatonowski et al. noticed that when the vertical system of a rolling mill is disturbed by an external disturbance, the stiffness of the workpiece varies periodically, leading to a nonlinear parametric vibration of the rolling mill and causing mill chatter [7]. To study the vibration characteristics of a rolling mill and to determine the vibration source, Fan

et al. established a horizontal-direction friction chatter model of a roller [8]. By analyzing the effect of varying the workpiece thickness and motor speed on the connection angle and roll gap friction force, Shi et al. established a nonlinear torsional vibration model of a rolling mill and showed that reducing the damping coefficient and nonlinear stiffness can help reduce the vibration intensity [9]. The existing modeling and analysis of rolling mills have been conducted by considering a single-vibration direction, such as vertical vibration, horizontal vibration, or torsional vibration; this makes it impossible to accurately capture the characteristics of the rolling process and fully explain the complex phenomena that occur during vibration. In an actual rolling process, the rolling mill vibration has characteristics such as variability, time variation, nonlinearity, and multi-constraint, with most of these factors exhibiting different types of coupling vibrations. Therefore, the key to solving this problem is to analyze the rolling mill vibration from the perspective of coupling vibration [10]. Liu et al. used the Hopf bifurcation theorem to analyze the stability and bifurcation characteristics of rolling mill rolls by subjecting the main drive system of a rolling mill to an electromechanical coupling vibration [11,12]. Through testing and analyzing the vibration signal of a rolling mill in the rolling process, Peng et al. reported many different dynamic motion forms between the rolling mill rolls, strip steel, and rolling parameters; this interaction produces the coupling effect [13]. Wang et al. established a multi-coupling vibration model comprising a rolling interface friction model, a rolling torque model, and a hydrodynamic lubrication model, and studied the interface dynamic characteristics of a high-speed rolling mill work interface based on an unsteady lubrication process [14]. Xu et al. established a coupling dynamic model between a rolling mill gearbox and its various time-varying parameters and explained the dynamic vibration characteristics of the rolling mill under different rolling parameters [15]. In addition, some scholars considered the interaction between the vibration of the rolling mill structure and the workpiece vibration and proposed a coupled vibration model for a roll-rolled piece [16,17]. Qian et al. studied the adaptive fuzzy control problem of a rolling mill vibration system and designed a controller to ensure the stability of the mechanical, hydraulic coupling rolling system [18]. In summary, although some advances have been made in the behavior analysis of the coupled vibration of rolling mills, the complex vibration law of the hot rolling mill itself needs to be further explored and verified from different angles. This can help put forward more effective vibration suppression measures and thereby improve the production efficiency and product quality.

Figure 1 shows a coupling dynamics diagram of a rolling mill. As shown, in the production process, a change in the dynamic parameters of the rolling mill structure model will lead to changes in the process parameters, force, and energy parameters of the rolling process model; the force and energy parameters in turn cause vibrations in the rolling mill structure, which interact with each other and lead to a coupling vibration.

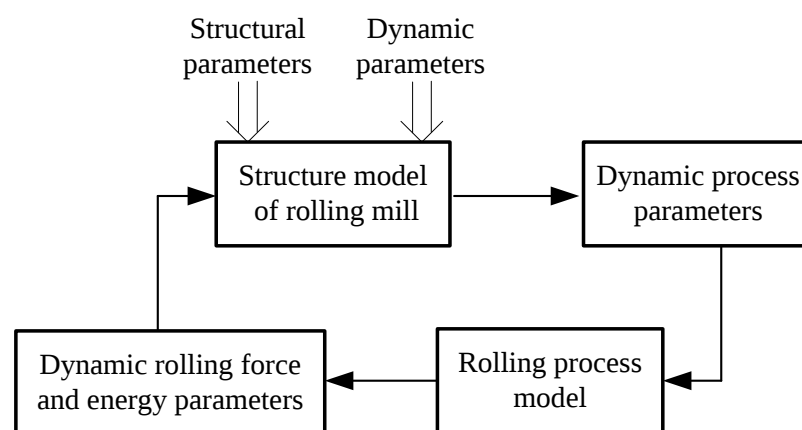


Figure 1. Coupling dynamics diagram of a rolling mill structure and rolling process.

In recent years, the dynamics of mechanical systems with piecewise mechanical factors, such as constraints, friction, and gap, have attracted extensive attention in the engineering field [19,20]. Currently, there are few reports on the vibration of rolling mill rolls under the influence of piecewise nonlinear factors. Therefore, it is necessary to thoroughly study the influence of piecewise nonlinear factors on the vibration of rolling mill rolls. The hydraulic system used in a hot rolling mill is a typical nonlinear system. In the working process of the hydraulic cylinder, a nonlinear hydraulic stiffness is generated because of the compressibility of the oil. When a rolling mill roll vibrates, it exhibits a nonlinear spring force [21], and there is a nonlinear friction between the piston and the cylinder wall [22]. Although the spring force and friction force in the hydraulic system are typically expressed in the form of weak nonlinearity, the nonlinear constraint effect cannot be ignored for high-precision position-controlled rolling mill systems. In the horizontal direction of the rolling mill rolls, there is a gap between the mill housing and the bearing seat, and because of the wear and eccentricity of the rolls, when the rolling mill rolls vibrate, the bearing seat will impact the mill housing, which will change the contact stiffness between the horizontal rolls, thus affecting the smooth operation of the rolling mill.

During rolling, horizontal vibration and vertical vibration often occur in a hot rolling mill. These interact and couple with each other [23]. This study establishes a vertical–horizontal coupling vibration dynamic model of a hot rolling mill under multi-piecewise nonlinear constraints considering a dynamic rolling force model of the rolling process, a piecewise nonlinear spring force model of the hydraulic system, a piecewise nonlinear friction force model, and a piecewise stiffness model in the horizontal direction. The response of the coupled vibration system is solved using the nonlinear method. On this basis, taking the rolling parameters of an actual mill as an example, we study the effects of rolling and process parameters on the amplitude–frequency characteristics and static bifurcation characteristics of the coupled vibration system of the hot rolling mill. This research provides certain guidance for reducing and restraining the vibration of rolling mills.

2. Multi-Piecewise Nonlinear Constraint Modeling of Hot Rolling Mill Rolls

Figure 2 shows the structural diagram of a four-high hot rolling mill, which is mainly composed of a mill stand, backup rolls, work rolls, a hydraulic cylinder, a balance cylinder, and a bending cylinder. The hydraulic cylinder is distributed between the frame and the upper backup roll and acts on the bearing seat of the upper backup roll. The upper backup roll is balanced under the joint restraint of the hydraulic and balance cylinders. The bending cylinder acts on the bearing seat of the upper and lower work rolls. The bending cylinder maintains contact between the work roll and the backup roll, to realize the balance between the work roll and the backup roll. As the main parameters, the work roll body length of the rolling mill is 2080 mm, and the diameter is 840 mm. The backup roll body length is 1780 mm, and the diameter is 1600 mm. The hydraulic cylinder used is a double-acting single-piston servo hydraulic cylinder, the diameter and stroke of which are 1050 mm and 100 mm, respectively.

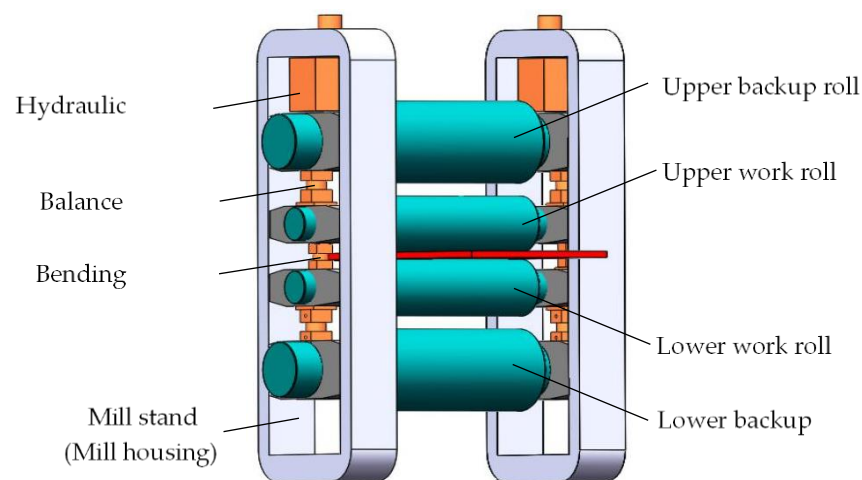


Figure 2. Structure diagram of a four-high hot rolling mill.

2.1. Piecewise Nonlinear Spring Force Model

The hydraulic elastic stiffness refers to the nonlinear elastic stiffness of the liquid when the working chamber of the hydraulic cylinder is completely closed, and the volume of the oil is compressed under the action of external forces [21,23]. The spring force of a hydraulic cylinder can be expressed as:

$$F(y) = \alpha_0 y + \beta_0 y^3 \quad (1)$$

where α_0 and β_0 are, respectively, the linear and nonlinear equivalent stiffnesses of the hydraulic cylinder in motion. The hydraulic cylinder, shown in Figure 2, is a double-acting single-piston servo hydraulic cylinder. With regard to its operation, a change in the piston displacement changes the pressure and oil volume in the two cavities of the hydraulic cylinder, thereby changing the oil stiffness. Therefore, the elastic force of the hydraulic cylinder is a nonlinear function of the vertical vibration displacement. Because of the size of the hydraulic and bending cylinders, the oil bulk elastic modulus, piston stroke, and other factors are different, and so are the equivalent dynamic stiffnesses. To simplify the research, only the upper rolls of the mill are analyzed and simplified into a lumped mass block [7].

In Figure 3, m is the equivalent mass of the upper backup roll, upper work roll, and its bearing seat, and y is the vertical vibration displacement of the rolling mill rolls. e_1 and e_2 are the initial elastic deformations of the hydraulic screwdown cylinder and bending cylinder, respectively, under the action of the preload during steady-state rolling ($e_1 > 0$, $e_2 < 0$). k_{s1} and k_{w1} are the equivalent linear stiffnesses of the hydraulic screwdown cylinder and bending cylinder, respectively, k_{s2} and k_{w2} are the equivalent nonlinear stiffnesses of the hydraulic screwdown cylinder and bending cylinder, respectively.

When the rolling mill is in operation, the downward movement of the rolls exceeds the initial elastic deformation of the bending cylinder, and the elastic stiffness between the hydraulic cylinder and the upper roll system is zero; in this case, the mill is mainly affected by the elastic force of the bending cylinder. Similarly, when the upward movement of the rolls exceeds the initial elastic deformation of the hydraulic cylinder, the elastic stiffness between the balance cylinder and the upper rolls is zero; in this case, the mill is mainly affected by the elastic force of the hydraulic cylinder. When the vertical vibration displacement of the roll system is between the two, the mill is affected by the elastic forces of the hydraulic and bending cylinders. The constraint is piecewise nonlinear, so the piecewise nonlinear spring force in the vertical direction can be expressed as:

$$F_k(y) = \begin{cases} k_{w1}(y + e_1) + k_{w2}(y + e_1)^3 & y > -e_2 \\ k_{w1}(y + e_1) + k_{w2}(y + e_1)^3 + k_{s1}(y + e_2) + k_{s2}(y + e_2)^3 & -e_1 \leq y \leq -e_2 \\ k_{s1}(y + e_2) + k_{s2}(y + e_2)^3 & y < -e_1 \end{cases} \quad (2)$$

Figure 4 shows the variation law of the piecewise nonlinear spring force with the vertical vibration displacement. Evidently, the stiffness changes at e_1 and e_2 .

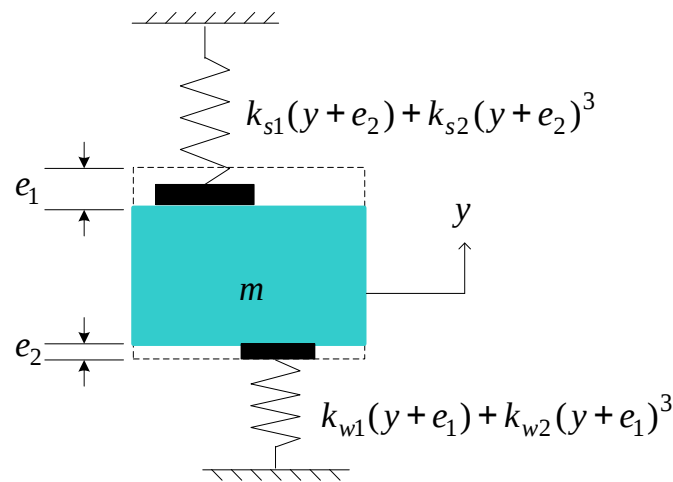


Figure 3. Piecewise nonlinear spring force model of hot rolling mill rolls in the vertical vibration direction.

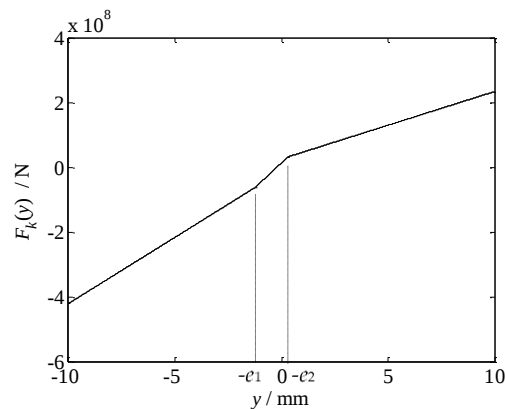


Figure 4. Piecewise nonlinear spring force of hot rolling mill rolls in the vertical direction.

2.2. Piecewise Nonlinear Friction Force Model

While the hydraulic cylinder converts hydraulic energy into mechanical energy, the friction force plays an important role. The main source of the friction force is the friction between the piston rod and the cylinder wall. In the rolling process, when the upper rolls vibrate, the friction force of the hydraulic and bending cylinders is not constant but varies with the periodic change in the vertical vibration speed $\dot{y}$, so the friction force is nonlinear [24]. The law of the friction coefficient between the piston and the cylinder wall of the hydraulic cylinder with the vibration velocity can be expressed as follows:

$$\mu_f = \mu_s \operatorname{sgn}(\dot{y}) - \lambda_1 \dot{y} + \lambda_2 \dot{y}^3 \quad (3)$$

$$\operatorname{sgn}(\dot{y}) = \begin{cases} 1 & \dot{y} > 0 \\ 0 & \dot{y} = 0 \\ -1 & \dot{y} < 0 \end{cases}, \quad \lambda_1 = \frac{3(\mu_s - \mu_m)}{2v_m}, \quad \lambda_2 = \frac{\mu_s - \mu_m}{2v_m^3}$$

where μ_f is the friction coefficient between the piston and the cylinder wall, μ_s is the static friction coefficient, μ_m is the maximum dynamic friction coefficient, and v_m is the vertical

vibration speed when the friction coefficient is μ_m , so the piecewise nonlinear friction force in the vertical direction can be expressed as:

$$F_f(\dot{y}) = p_f \times \mu_f = p_f [\mu_s \text{sgn}(\dot{y}) - \lambda_1 \dot{y} + \lambda_2 \dot{y}^3] \quad (4)$$

where p_f is the pressure between the piston rod and the cylinder wall, and its value is related to the material of the hydraulic cylinder, the tightness of the seal assembly, the hardness of the seal material, and the radial component of the load.

Figure 5 shows the variation law of the friction force with the vibration velocity. The friction force is symmetrical at the origin; when the vertical vibration velocity $\dot{y}$ is positive, the friction force is greater than zero, and when the vertical vibration velocity $\dot{y}$ is negative, the friction force is less than zero.

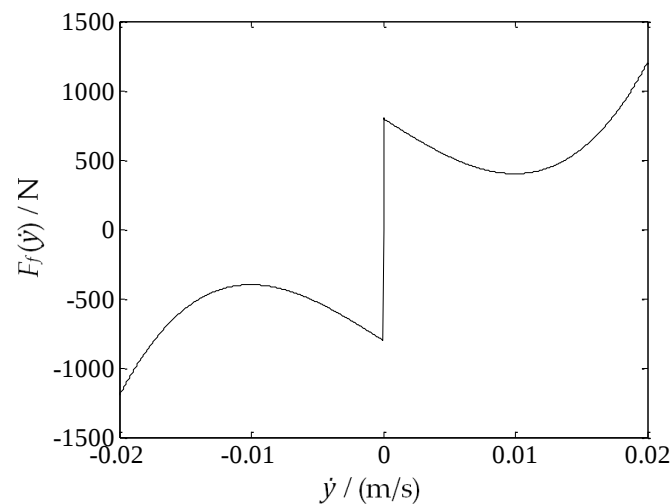


Figure 5. Piecewise nonlinear friction force of hot rolling mill rolls in the vertical direction.

2.3. Piecewise Stiffness Model

Based on the stress characteristics of the roll, when the external disturbance force F_1 in the horizontal direction is high, and there is a gap between the roller bearing seat and the housing, the work roll bearing seat will collide with the mill housing, thus destabilizing the roll system [8]. According to the elastic collision theory and the structural characteristics of the rolling mill, a horizontal collision model of the hot rolling mill rolls is established, as shown in Figure 6.

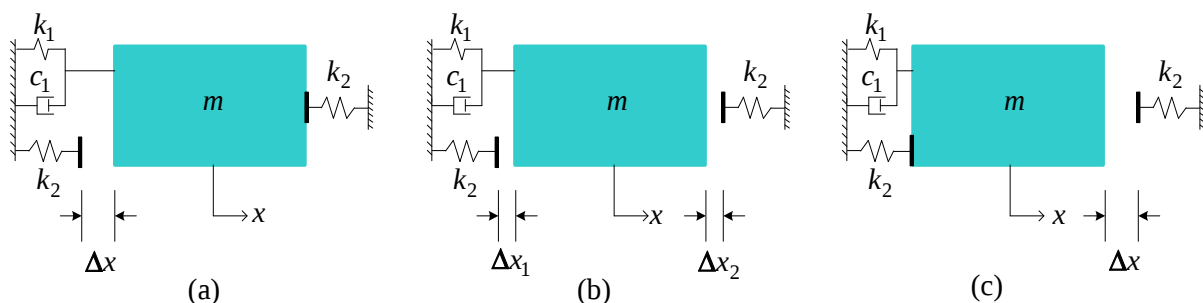


Figure 6. Collision model between the roller bearing seat and the mill housing in the horizontal direction of hot rolling mill rolls. (a) The roller bearing seat is pressed against the right mill housing; (b) The roller bearing seat is between the mill housing; (c) The roller bearing seat is pressed against the left mill housing.

As shown in Figure 6, x is the horizontal vibration displacement of the mill rolls, k_1 and c_1 are respectively the linear stiffness and linear damping from the roll to the left

housing, k_2 is the lateral equivalent stiffness of the housing column, and Δx is the gap between the roller bearing seat and the mill housing.

There is a gap between the roller bearing seat and the liner plate of the mill housing. Therefore, the horizontal stiffness can be modeled as a piecewise function using a force function $F_k(x)$, as expressed in Equation (5); Figure 7 shows the corresponding function curve.

$$F_k(x) = \begin{cases} (k_1 + k_2)x & x \geq 0 \text{ or } x \leq -\Delta x \\ k_1 x & -\Delta x < x < 0 \end{cases} \quad (5)$$

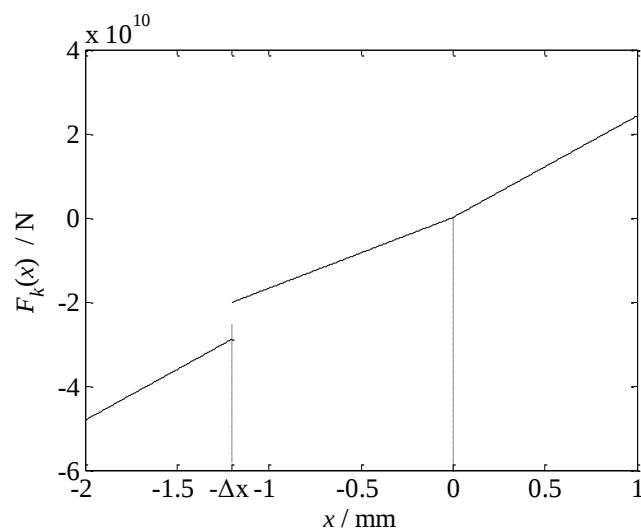


Figure 7. Piecewise stiffness force function of hot rolling mill rolls in the horizontal direction.

3. Dynamic Rolling Force Model of Hot Rolling Mill Rolls

3.1. Determination of Rolling Parameters in Deformation Zone

During rolling, the rolling force is an important factor affecting the vibration of the rolling mill [25,26]. It is important to determine the rolling force correctly to improve the accuracy of the rolling mill vibration model. Figure 8 shows the dynamic deformation process of the workpiece in the rolling process.

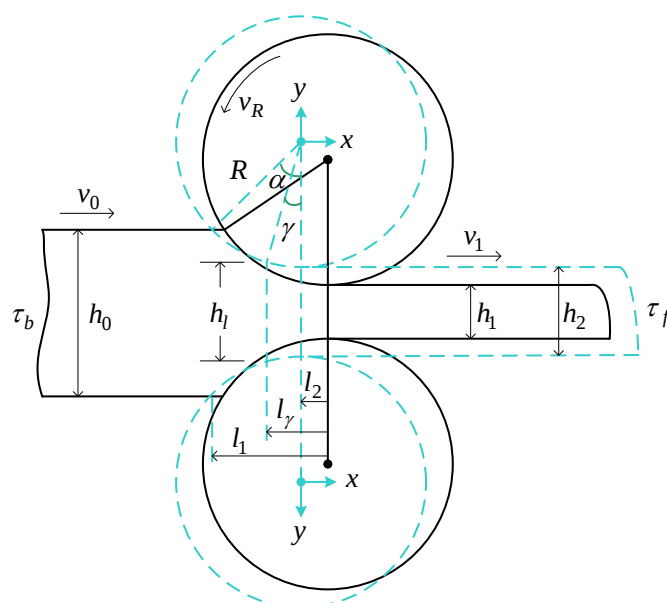


Figure 8. Dynamic deformation process of the workpiece.

In Figure 8, the solid line indicates the roll position in the steady state, and the dotted line indicates the roll position in the vibration state. R is the roll radius, v_R is the roll speed, and α is the bite angle. v_0 and v_1 are the entry and exit velocities of the workpiece in the steady state, respectively, h_0 and h_1 are the entry and exit thicknesses of the workpiece in the steady state, respectively, h_2 is the exit thickness of the workpiece in the vibration state, τ_b and τ_f are the entry and exit tensions, respectively. l_1 and l_2 are the distances from the solid line roller center line to the vibration state at the entrance and exit, respectively.

Assuming that the shape of the roll gap is a parabola [27], when vibration occurs, the section thickness of the workpiece at any position l of the roll gap is as follows:

$$h_l = h_1 + \frac{(l-x)^2}{R} + 2y \quad (6)$$

Due to the vibration, the position l_2 at the exit of the workpiece is the vibration displacement x of the roller center in the horizontal direction. The exit thickness of the workpiece can be obtained from Equation (6).

$$h_2 = h_1 + 2y \quad (7)$$

When the entry thickness h_0 of the workpiece is stable, from Equation (6), we can express the position l_1 at the entrance of the workpiece under vibration as follows:

$$l_1 = x + \sqrt{R(h_0 - h_2)} \quad (8)$$

When the rolling mill rolls vibrate, the entrance speed of the workpiece will be composed of two parts: the velocity v_0 at the entrance of the workpiece in the steady state and the change rate of the position l_1 at the entrance of the workpiece under vibration. This can be expressed as follows:

$$v'_0 = v_0 - \dot{l}_1 = v_0 - \left[\dot{x} - R^{\frac{1}{2}}(h_0 - h_2)^{-\frac{1}{2}}\dot{y} \right] \quad (9)$$

According to Hu's law of the invariance of dynamic volume flows [28]:

$$v_l h_l = v'_0 h_0 - (l_1 - l)\dot{y} - (h_0 - h_l)\dot{x} \quad (10)$$

where $\dot{x}$ is the horizontal vibration velocity of the rolling mill rolls, v_l is the velocity of the workpiece at any section l , and $(l_1 - l)\dot{y}$ and $(h_0 - h_l)\dot{x}$ are the volume flow rates of the workpiece in the vertical and horizontal directions, respectively.

There is a section in the deformation zone, where the horizontal velocity of the workpiece is equal to that of the roller at a particular point; this section is called the neutral surface, and the point on the roller is called the neutral point. Therefore, we have $v_\gamma = v_R \cos \gamma$ at the neutral point l_γ , where v_γ is the horizontal velocity of the workpiece at the neutral point, and γ is the neutral angle. From the geometric relationship shown in Figure 7, we find that the cross-sectional thickness of the workpiece is $h_\gamma = h_2 + 2R(1 - \cos \gamma)$ at the neutral point. Since the bite angle α is small, and $\gamma < \alpha$, it can be approximately considered that $\cos \gamma \approx 1$, $(1 - \cos \gamma) \approx \gamma^2/2$, and $h_\gamma = h_2 + R\gamma^2$. By substituting h_γ and v_γ into Equation (10), we can express the neutral point position l_γ as follows:

$$l_\gamma = x + \left[\sqrt{h_0 - h_2} - \frac{h_0}{\sqrt{h_0 - h_\gamma}} \right] \sqrt{R} + \frac{(v_R h_\gamma - v_0)h_0 + \dot{x}(2h_0 - h_\gamma)}{\dot{y}} \quad (11)$$

$$\gamma = \sqrt{\frac{h_2}{R}} \tan \left(\frac{1}{2} \arctan \sqrt{\frac{h_0 - h_2}{h_2}} - \frac{1}{4\mu_x} \sqrt{\frac{h_2}{R}} \ln \frac{h_0}{h_2} \right)$$

3.2. Determination of Vertical–Horizontal Coupling Dynamic Rolling Force

Assuming that the workpiece is uniformly deformed in the deformation zone and considering that the sign of the friction force between the rolling interfaces changes at the neutral point l_γ , any microelement with the same thickness is selected for analysis in the front and backward sliding areas of the rolling deformation area. Its thickness is dl , under the action of the tensile stress τ_x , rolling compressive stress p , and shear stress τ_s ; Figure 9 shows the stress relation diagram.

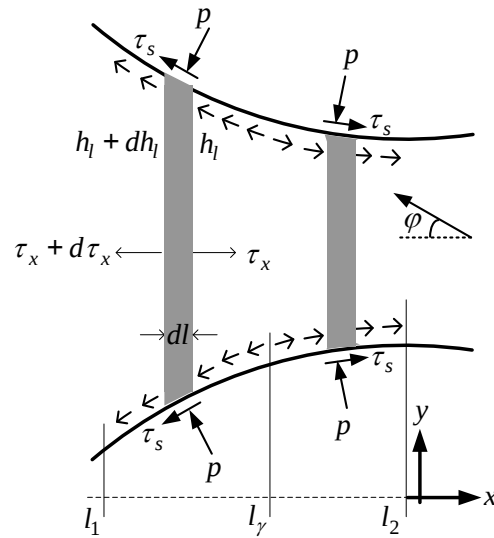


Figure 9. Stress relation diagram at any position in the deformation zone.

Based on the force balance theory provided by Kármán for the deformation zone [29], the force balance distribution equation along the x direction can be obtained as follows:

$$\frac{dh_l}{dl}(p + \tau_x) + h_l \frac{d\tau_x}{dl} \pm 2\tau_s = 0 \quad (12)$$

where the positive and negative signs in “ $\pm$ ” indicate the forward and backward sliding areas, respectively. The Coulomb friction model is used to describe the friction between the rolling interfaces, i.e., $\tau_s = \mu_x \delta$, where μ_x is the friction coefficient between the rolling interfaces, and δ is the shear yield strength.

During rolling, the tensile stress at any point l can be written as:

$$\tau_x = \tau_b + \int_{l_1}^l \frac{2\delta}{h_l} \left(\pm \mu_x - \frac{2(l-x)}{R} \right) dl \quad (13)$$

According to von Mises yield criterion $p = 2\delta - \tau_x$, the unit rolling force can be obtained as follows:

$$p(x) = 2\delta - \left[\tau_b + \int_{l_1}^l \frac{2\delta}{h_l} \left(\pm \mu_x - \frac{2(l-x)}{R} \right) dl \right] \quad (14)$$

At this time, the force F_x of the workpiece in the horizontal direction and F_y in the vertical direction can be expressed as

$$\begin{cases} F_x = - \int_{l_2}^{l_1} \left(2\delta - \left[\tau_b + \int_{l_1}^l \frac{2\delta}{h_l} \left(\pm \mu_x - \frac{2(l-x)}{R} \right) dl \right] \right) \tan \varphi dx + \int_{l_2}^{l_1} \mp \mu_x \delta dl \\ F_y = \int_{l_2}^{l_1} \left(2\delta - \left[\tau_b + \int_{l_1}^l \frac{2\delta}{h_l} \left(\pm \mu_x - \frac{2(l-x)}{R} \right) dl \right] \right) dl + \int_{l_2}^{l_1} \mp \mu_x \delta \tan \varphi dl \end{cases} \quad (15)$$

$$\tan \varphi = \frac{l-x}{\sqrt{R^2 - (l-x)^2}}$$

Since the direction of friction between the rolling interfaces changes at the neutral point, the integral Equation (15) can be divided into two parts, namely $l_2 \sim l_\gamma$ and $l_\gamma \sim l_1$, and can be rewritten as:

$$\begin{cases} F_x = \frac{1}{2}\tau_b(h_0 - h_2) - \delta h_2 \ln \frac{h_0}{h_2} - \mu_x \delta h_2 \sqrt{\frac{R}{h_2}} \left[2 \tan^{-1} \left(\frac{l_\gamma - x}{\sqrt{R h_2}} \right) - \tan^{-1} \left(\frac{l_1 - x}{\sqrt{R h_2}} \right) \right] \\ F_y = \left[2\delta \ln \left(\frac{h_0}{h_2} \right) - 2\delta - \tau_b \right] (l_1 - x) + 4\delta \sqrt{R h_2} \tan^{-1} \left(\frac{l_1 - x}{\sqrt{R h_2}} \right) \\ + 2\mu_x \delta \sqrt{\frac{R}{h_2}} (l_1 - x) \left[2 \tan^{-1} \left(\frac{l_\gamma - x}{\sqrt{R h_2}} \right) - \tan^{-1} \left(\frac{l_1 - x}{\sqrt{R h_2}} \right) \right] + \mu_x \delta R \ln \left(\frac{h_0 h_2}{h_\gamma^2} \right) \end{cases} \quad (16)$$

Equation (16) describes the dynamic rolling force of the hot rolling mill rolls in the horizontal and vertical directions. The expression is complex. To analyze the influence of roll vibration on the rolling mill rolls, the rolling force formula can be expanded under a steady rolling state, i.e., Taylor expansion near $x = y = 0$, $\dot{x} = \dot{y} = 0$. Thus, Equation (16) can be rewritten as:

$$\begin{cases} F_x(x, \dot{x}, y, \dot{y}) = F_x(0, 0, 0, 0) + \Delta F_x(x, \dot{x}, y, \dot{y}) \\ F_y(x, \dot{x}, y, \dot{y}) = F_y(0, 0, 0, 0) + \Delta F_y(x, \dot{x}, y, \dot{y}) \end{cases} \quad (17)$$

where $F(0,0,0,0)$ is the rolling force in the steady state, and $\Delta F(x, \dot{x}, y, \dot{y})$ is the dynamic variation in the rolling force under vibration.

$$\begin{aligned} F_x(0, 0, 0, 0) &= \frac{1}{2}\tau_b \Delta h - \delta h_1 \ln \frac{h_0}{h_1} \\ &\quad - \mu_x \delta \sqrt{R h_1} \left[2 \tan^{-1} \left(\sqrt{\frac{\Delta h}{h_1}} - \frac{1}{\sqrt{h_1}} \frac{h_0}{\sqrt{\Delta h - R \gamma'^2}} \right) - \tan^{-1} \left(\frac{\Delta h}{\sqrt{h_1}} \right) \right] \\ F_y(0, 0, 0, 0) &= \left[2\delta \ln \left(\frac{h_0}{h_1} \right) - 2\delta - \tau_b \right] \sqrt{R \Delta h} + 4\delta \sqrt{R h_1} \tan^{-1} \sqrt{\frac{\Delta h}{h_1}} + \mu_x \delta R \ln \left[\frac{h_0 h_1}{(h_1 + R \gamma'^2)^2} \right] \\ &\quad + 2\mu_x \delta R \sqrt{\frac{\Delta h}{h_1}} \left[2 \tan^{-1} \left(\sqrt{\frac{\Delta h}{h_1}} - \frac{1}{\sqrt{h_1}} \frac{h_0}{\sqrt{\Delta h - R \gamma'^2}} \right) - \tan^{-1} \left(\frac{\Delta h}{\sqrt{h_1}} \right) \right] \\ \gamma' &= \sqrt{\frac{h_1}{R}} \tan \left(\frac{1}{2} \arctan \sqrt{\frac{\Delta h}{h_1}} - \frac{1}{4\mu_x} \sqrt{\frac{h_1}{R}} \ln \frac{h_0}{h_1} \right) \\ \Delta h &= h_0 - h_1 \end{aligned}$$

Because different rolling mills and steel grades have different process parameters, some of the parameters of the Taylor expansion are sensitive to mill vibration, while some can be ignored. The parameters and their specific gravity values are shown in Tables A1 and A2 (Appendix A). Taking the actual parameters of a 1780 mm hot rolling mill (listed in Table 1) as an example, the specific gravity values of the various parameters in the dynamic rolling force variation are calculated and compared, so as to select the appropriate parameters.

Table 1. Parameters of 1780 mm hot rolling mill.

Parameters	Value	Parameters	Value	Parameters	Value
h_0 (m)	0.0141	k_1 (N/m)	7.31×10^9	e_1 (mm)	0.6
h_1 (m)	0.0082	k_2 (N/m)	1.68×10^{10}	e_2 (mm)	−0.2
R (m)	0.42	k_3 (N/m)	1.52×10^{11}	v_m (m/s)	0.01
v_0 (m/s)	2.5	c_0 (N·s/m)	8.85×10^5	p_f (MN)	0.04
τ_b (MPa)	5.5	c_1 (N·s/m)	5.20×10^3	μ_x	0.26
τ_f (MPa)	3.8	c_3 (N·s/m)	7.83×10^5	μ_s	0.02
m (Kg)	1.44×10^5	ks_1 (N/m)	4.09×10^{10}	μ_m	0.01
k_0 (N/m)	2.35×10^{10}	kw_1 (N/m)	2.08×10^{10}	Δx (m)	1×10^{-3}

If the parameters with specific gravity values less than 2×10^{-3} in Tables A1 and A2 are ignored, the change in the dynamic rolling force can be expressed as:

$$\begin{cases} \Delta F_x(x, \dot{x}, y, \dot{y}) = a_1 x + a_2 y + a_3 \dot{y} + a_4 y^2 + a_5 \dot{x} \dot{y} + a_6 y^3 \\ \Delta F_y(x, \dot{x}, y, \dot{y}) = b_1 x + b_2 \dot{y} + b_3 x y + b_4 \dot{x}^2 + b_5 y^2 + b_6 y^3 \end{cases} \quad (18)$$

$$a_1 = \frac{\partial}{\partial x} F_x(0, 0, 0, 0), a_2 = \frac{\partial}{\partial y} F_x(0, 0, 0, 0), a_3 = \frac{\partial}{\partial \dot{y}} F_x(0, 0, 0, 0)$$

$$a_4 = \frac{1}{2} \frac{\partial^2}{\partial y^2} F_x(0, 0, 0, 0), a_5 = \frac{1}{2} \frac{\partial^3}{\partial \dot{x} \partial \dot{y}} F_x(0, 0, 0, 0), a_6 = \frac{1}{6} \frac{\partial^3}{\partial y^3} F_x(0, 0, 0, 0)$$

$$b_1 = \frac{\partial}{\partial x} F_y(0, 0, 0, 0), b_2 = \frac{\partial}{\partial \dot{y}} F_y(0, 0, 0, 0), b_3 = \frac{\partial^2}{\partial x \partial \dot{y}} F_y(0, 0, 0, 0)$$

$$b_4 = \frac{1}{2} \frac{\partial^2}{\partial \dot{x}^2} F_y(0, 0, 0, 0), b_5 = \frac{1}{2} \frac{\partial^2}{\partial y^2} F_y(0, 0, 0, 0), b_6 = \frac{1}{6} \frac{\partial^3}{\partial y^3} F_y(0, 0, 0, 0)$$

Table 2 lists the values of the nonlinear parameters $a_1 \sim a_6$ and $b_1 \sim b_6$ of the dynamic rolling force model.

Table 2. Nonlinear parameters of the dynamic rolling force model.

Parameters	Value	Parameters	Value	Parameters	Value
a_1 (N/m)	5.8×10^6	a_5 (N·s ² /m ²)	3613.40	b_3 (N/m ²)	1.70×10^{10}
a_2 (N/m)	1.46×10^8	a_6 (N/m ³)	-7.47×10^{11}	b_4 (N·s ² /m ²)	−152.85
a_3 (N·s/m)	−1131.23	b_1 (N/m)	6.98×10^7	b_5 (N/m ²)	4.7×10^{11}
a_4 (N/m ²)	2.65×10^{10}	b_2 (N·s/m)	1.37×10^4	b_6 (N/m ³)	-7.90×10^{13}

4. Coupling Vibration Modeling and System Response Solution of Hot Rolling Mill Rolls

The piecewise nonlinear spring force constraint and the piecewise nonlinear friction force constraint of the hydraulic system must be considered in the production process of high-precision rolling mills. Moreover, the piecewise stiffness constraint in the horizontal direction and the influence of the dynamic rolling force on the rolling mill rolls should be considered. In this study, a vertical–horizontal coupling dynamic model of the hot rolling mill under multi-piecewise nonlinear constraints is established as shown in Figure 10. In Figure 10, only the upper part of the mill rolls is shown; this is because many researchers assume that the mechanical structure of rolling mill rolls is symmetrical on and off the rolling line when exploring the vibration of four-high hot rolling mills. Therefore, when constructing the vibration model, they only build the upper part of the rolling mill rolls, which helps reduce the calculation workload and does not affect the accuracy of the vibration model [30].

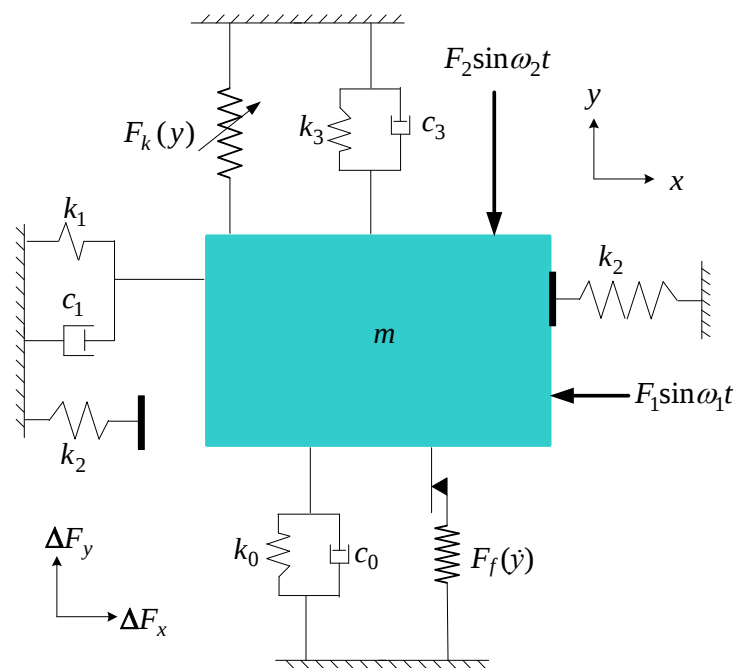


Figure 10. Vertical–horizontal coupling vibration model of hot rolling mill rolls under multi-pieewise nonlinear constraints.

In Figure 10, k_0 and c_0 are the equivalent stiffness and damping of the workpiece, respectively, k_3 and c_3 are the equivalent stiffness and damping of the upper rolls and upper beam of the stand, respectively. The external disturbance excitations of the hot rolling mill in the horizontal and vertical directions are $F_1 \sin \omega_1 t$ and $F_2 \sin \omega_2 t$, respectively. From the Lagrange dissipation equation, the dynamic equation of the hot rolling mill rolls can be obtained as follows:

$$\begin{cases} m\ddot{x} + c_1\dot{x} + F_k(x) + \Delta F_x(x, \dot{x}, y, \dot{y}) - F_1 \sin \omega_1 t = 0 \\ m\ddot{y} + (c_0 + c_3)\dot{y} + (k_0 + k_3)y + \alpha F_k(y) + \beta F_f(\dot{y}) - \Delta F_y(x, \dot{x}, y, \dot{y}) + F_2 \sin \omega_2 t = 0 \end{cases} \quad (19)$$

In Equation (19), the piecewise nonlinear spring force and the piecewise nonlinear friction force are given coefficients α and β , respectively, to study the vibration behavior of the hot rolling mill rolls under different constraint conditions. After simplifying Equation (19), we can obtain the vibration equation of the hot rolling mill rolls as follows:

$$\begin{cases} \ddot{x} + \omega_{10}^2 x + 2\zeta_1 \omega_{10} \dot{x} + F'_k(x) + \alpha_1 y + \beta_1 \dot{y} + \delta_1 y^2 + \eta_1 \dot{x} \dot{y} + \chi_1 y^3 - F'_1 \sin \omega_1 t = 0 \\ \ddot{y} + \omega_{20}^2 y + 2\zeta_2 \omega_{20} \dot{y} + \alpha F'_k(y) + \beta F'_f(\dot{y}) - \alpha_2 x - \beta_2 x \dot{y} - \delta_2 \dot{x}^2 - \eta_2 y^2 - \chi_2 y^3 + F'_2 \sin \omega_2 t = 0 \end{cases} \quad (20)$$

$$\omega_{10} = \sqrt{\frac{k_1 + a_1}{m}}, \quad \omega_{20} = \sqrt{\frac{k_0 + k_3}{m}}, \quad \zeta_1 = \frac{c_1}{2m\omega_{10}}, \quad \zeta_2 = \frac{c_0 + c_3 - b_2}{2m\omega_{20}}$$

$$F'_1 = \frac{F_1}{m}, \quad F'_2 = \frac{F_2}{m}, \quad p'_f = \frac{p_f}{m}, \quad \rho = \frac{k_2}{2m}$$

$$\alpha_1 = \frac{a_2}{m}, \quad \beta_1 = \frac{a_3}{m}, \quad \delta_1 = \frac{a_4}{m}, \quad \eta_1 = \frac{a_5}{m}, \quad \chi_1 = \frac{a_6}{m}$$

$$\alpha_2 = \frac{b_1}{m}, \quad \beta_2 = \frac{b_3}{m}, \quad \delta_2 = \frac{b_4}{m}, \quad \eta_2 = \frac{b_5}{m}, \quad \chi_2 = \frac{b_6}{m}$$

In Equation (20), the expression of $F'_k(x)$ is shown in Equation (21), and the expression of $F'_k(y)$ is shown in Equation (22).

$$F'_k(x) = \begin{cases} 2\rho x & x \geq 0 \text{ or } x \leq -\Delta x \\ 0 & -\Delta x < x < 0 \end{cases} \quad (21)$$

$$F'_f(\dot{y}) = p'_f [\mu_s \text{sgn}(\dot{y}) - \lambda_1 \dot{y} + \lambda_2 \dot{y}^3] \quad (22)$$

$$F'_k(y) = \begin{cases} z_1(y + e_1) + z_2(y + e_1)^3 & y > -e_2 \\ z_1(y + e_1) + z_2(y + e_1)^3 + z_3(y + e_2) + z_4(y + e_2)^3 & -e_1 \leq y \leq -e_2 \\ z_3(y + e_2) + z_4(y + e_2)^3 & y < -e_1 \end{cases} \quad (23)$$

$$z_1 = \frac{k_{w1}}{m}, z_2 = \frac{k_{w2}}{m}, z_3 = \frac{k_{s1}}{m}, z_4 = \frac{k_{s2}}{m}, k_{w2} = \alpha' k_{w1}, k_{s2} = \beta' k_{s1}$$

The horizontal vibration equation contains the vertical vibration displacement y and velocity $\dot{y}$, and the vertical vibration equation also contains the horizontal vibration displacement x and velocity $\dot{x}$, which indicates a coupling between the horizontal and vertical vibrations in the hot rolling mill rolls.

Since the coupled vibration system is weakly nonlinear, the nonlinear term in the system is given a small parameter ε . Moreover, it is assumed that the external excitation frequencies in the horizontal and vertical directions are close to the natural frequencies of the system, i.e., $\omega_1^2 = \omega_{10}^2 - \varepsilon\sigma_1$, and $\omega_2^2 = \omega_{20}^2 - \varepsilon\sigma_2$, where σ_1 and σ_2 are the tuning parameters. When ε is sufficiently small, the motion of the system is closer to a periodic motion; then, Equation (20) can be expressed as:

$$\begin{cases} \ddot{x} + \omega_1^2 x = \varepsilon f_x(x, \dot{x}, y, \dot{y}) + \varepsilon F'_1 \sin \omega_1 t \\ \ddot{y} + \omega_2^2 y = \varepsilon f_y(x, \dot{x}, y, \dot{y}) - \varepsilon F'_2 \sin \omega_2 t \end{cases} \quad (24)$$

$$f_x(x, \dot{x}, y, \dot{y}) = -2\zeta_1 \omega_{10} \dot{x} + \sigma_1 x - F'_k(x) - \alpha_1 y - \beta_1 \dot{y} - \delta_1 y^2 - \eta_1 \dot{x} \dot{y} - \chi_1 y^3$$

$$f_y(x, \dot{x}, y, \dot{y}) = -2\zeta_2 \omega_{20} \dot{y} + \sigma_2 y - \alpha F'_k(y) - \beta F'_f(\dot{y}) + \alpha_2 x + \beta_2 x \dot{y} + \delta_2 \dot{x}^2 + \eta_2 y^2 + \chi_2 y^3$$

The average method is used to solve the system response [31]. Let Equation (24) have the following solution form:

$$\begin{cases} x = a(t) \cos[\omega_1 t - \theta_1(t)] \\ y = b(t) \cos[\omega_2 t - \theta_2(t)] \end{cases} \quad (25)$$

$$\varphi_1 = \omega_1 t - \theta_1(t), \varphi_2 = \omega_2 t - \theta_2(t)$$

where a, b, θ_1 , and θ_2 are slowly varying functions of time t . Through derivation, we can obtain the differential equations of a, θ_1, b , and θ_2 , then integrate them in the period $(-\pi, \pi)$, and finally calculate the average value. The average equation is as follows:

$$\begin{cases} \dot{a} = -\frac{\varepsilon}{2\pi\omega_1} \int_{-\pi}^{\pi} [f_x(x, \dot{x}, y, \dot{y}) + F'_1 \sin \omega_1 t] \sin \varphi_1 d\varphi_1 \\ a\dot{\theta}_1 = \frac{\varepsilon}{2\pi\omega_1} \int_{-\pi}^{\pi} [f_x(x, \dot{x}, y, \dot{y}) + F'_1 \sin \omega_1 t] \cos \varphi_1 d\varphi_1 \\ \dot{b} = -\frac{\varepsilon}{2\pi\omega_2} \int_{-\pi}^{\pi} [f_y(x, \dot{x}, y, \dot{y}) - F'_2 \sin \omega_2 t] \sin \varphi_2 d\varphi_2 \\ b\dot{\theta}_2 = \frac{\varepsilon}{2\pi\omega_2} \int_{-\pi}^{\pi} [f_y(x, \dot{x}, y, \dot{y}) - F'_2 \sin \omega_2 t] \cos \varphi_2 d\varphi_2 \end{cases} \quad (26)$$

Considering that the frequency of the external excitation in the horizontal and vertical directions is the same, there exists $\omega_1 = \omega_2 = \omega$. Moreover, when $\dot{a} = \dot{b} = \dot{\theta}_1 = \dot{\theta}_2 = 0$ in Equation (26), the coupling system has a stable vibration amplitude and frequency.

$$\begin{cases} (2\zeta_1 \omega_{10} a + \beta_1 b)^2 \omega^2 + \left[\left(\frac{\omega_{10}^2 - \omega^2}{\varepsilon} - \rho - \frac{2\rho}{\pi} \arccos \frac{\Delta x}{a} \right) a - \alpha_1 b - \frac{3}{4} \chi_1 b^3 \right]^2 = (F'_1)^2 \\ (2\zeta_2 \omega_{20} b - \beta N_1)^2 \omega^2 + \left[\left(\frac{\omega_{20}^2 - \omega^2}{\varepsilon} + \frac{3}{4} \chi_2 b^2 \right) b + \alpha_2 a - \frac{1}{\pi} \alpha N_2 \right]^2 = (F'_2)^2 \end{cases} \quad (27)$$

$$N_1 = p'_f \left(\lambda_1 b + \frac{3}{4} \lambda_2 \omega^2 b^3 \right) - \frac{4}{\pi \omega} p'_f \mu_s$$

$$N_2 = \left(\frac{1}{2} z_2 e_1^3 + z_1 e_1 + \frac{13}{4} z_2 e_1 b^2 \right) \frac{\sqrt{b^2 - e_1^2}}{b} - \left(\frac{1}{2} z_4 e_2^3 + z_3 e_2 + \frac{13}{4} z_4 e_2 b^2 \right) \frac{\sqrt{b^2 - e_2^2}}{b} \\ + b \left[\left(\frac{3}{4} z_2 b^2 + z_1 + 3 z_2 e_1^2 \right) \left(\pi - \arccos \frac{e_1}{b} \right) \right. \\ \left. + \left(\frac{3}{4} z_4 b^2 + z_3 + 3 z_4 e_2^2 \right) \arccos \frac{e_2}{b} \right]$$

From Equation (27), we find that the amplitude and frequency of the external disturbance, the nonlinear stiffness of the hydraulic cylinder, the nonlinear friction force, and the clearance between the bearing pedestal and the archway affect the vibration of the rolling mill.

5. Analysis of Vibration Characteristics of Coupling System of Hot Rolling Mill Rolls

Table 1 lists the parameters of the 1780 mm hot rolling mill of Chengde Steel Co., Ltd. (Chengde, China) as an example. With the relevant data listed in Table 1, the nonlinear parameters of the dynamic rolling force can be obtained using Equation (18), as listed in Table 2.

5.1. Analysis of Amplitude–Frequency Characteristics

In the following, we mainly study the influences of different parameters on the amplitude–frequency characteristics of the coupled vibration system of the hot rolling mill. When the horizontal and vertical directions are subjected to the same external disturbance, i.e., $F_1 = F_2 = F$, the amplitude–frequency curve of the vertical–horizontal nonlinear coupled vibration system of the hot rolling mill is as shown in Figure 11. There are two resonance peak points in the amplitude–frequency curve in the horizontal and vertical directions. When the external disturbance frequency ω is close to the natural frequency in the horizontal or vertical directions, the coupling system resonates, causing the vibration amplitude to increase and the rolling mill to vibrate violently. The vibration amplitude in the horizontal direction is greater than that in the vertical direction, and the resonance frequency range is 65–155 Hz. In actual production, we can adjust the relevant vibration and process parameters to reduce the amplitude, avoid the resonance region, and suppress and avoid the occurrence of resonance phenomenon.

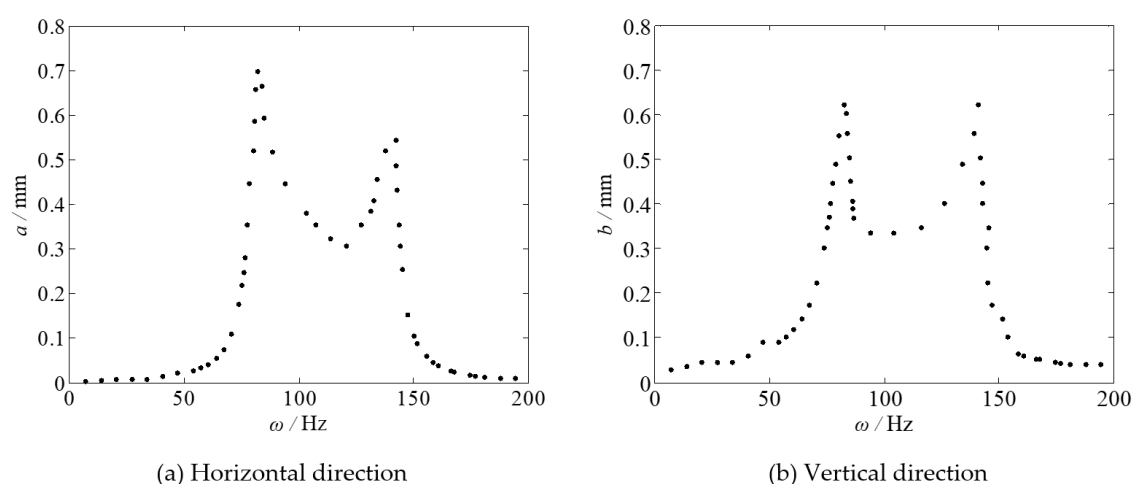


Figure 11. Amplitude–frequency curve of the coupled vibration system of a hot rolling mill. (a) Amplitude–frequency curve of horizontal direction; (b) Amplitude–frequency curve of vertical direction.

With further increase in the external disturbance force, when $F = 4 \times 10^5$ N, the amplitude–frequency curve of the coupled vibration system of the hot rolling mill is as shown in Figure 12. Compared with that shown in Figure 10 ($F = 2 \times 10^5$ N), the vibration amplitudes in both the horizontal and vertical directions increase, and the resonance

becomes more intense. A jump phenomenon is observed in one of the branches of the resonance peak, making the system unstable. Taking the vertical direction as an example (Figure 12b), when the external disturbance frequency increases, the amplitude changes along the AB direction of the curve to point C . When the frequency exceeds point C , the amplitude jumps from C to point D , and the amplitude jumps. When the external disturbance frequency decreases, the amplitude changes along the ED direction to point F , jumps to point B , and then changes along the BA direction. Therefore, the vibration of the coupling system corresponding to the CF section of the amplitude–frequency curve is unstable. The horizontal direction (Figure 12a) also has similar vibration characteristics. Compared with that shown in Figures 11 and 12, it is found that appropriately reducing the amplitude of the external disturbance force can help optimize the control parameters of the vibration system, thus suppressing the vibration of the rolling mill and improving the stability of the system.

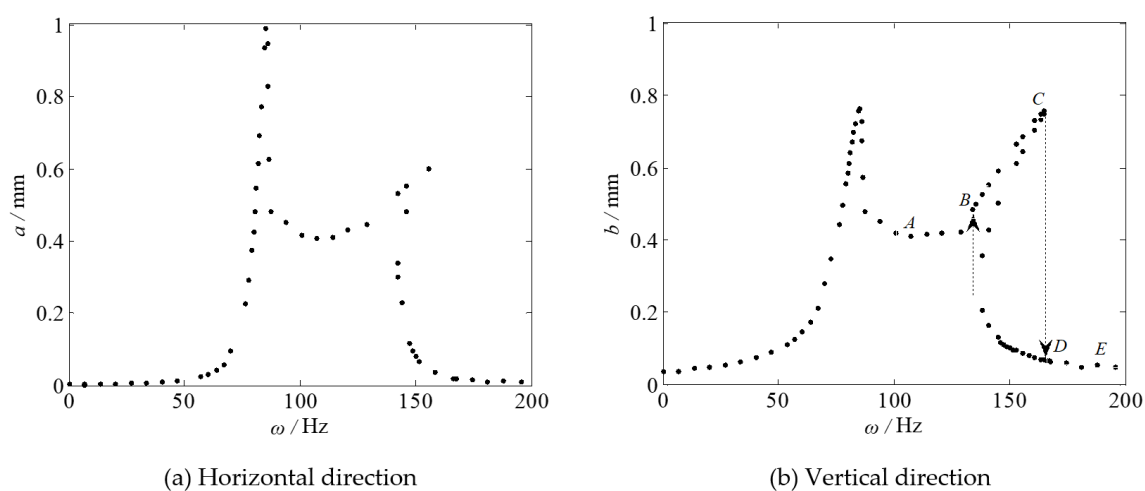


Figure 12. Amplitude–frequency curve of the coupled vibration system of the hot rolling mill with external disturbing force $F = 4 \times 10^5$ N. (a) Amplitude–frequency curve of horizontal direction; (b) Amplitude–frequency curve of vertical direction.

When $\Delta x = 0$ mm, the amplitude–frequency characteristic curve of the coupling system of the hot rolling mill is as shown in Figure 13 (gap $\Delta x = 1$ mm in Figure 11) with the decrease in the gap Δx between the roller bearing seat and the mill housing in the horizontal direction. The vibration amplitudes in the horizontal and vertical directions are significantly reduced, the resonance region is narrowed, and there is only one resonance peak point. This shows that eliminating the gap between the bearing seat and the mill housing can help significantly reduce the vibration of the system and improve the stability of the vibration system. The main reason is that when the gap becomes smaller, the stiffness of the hot rolling rolls varies in the horizontal direction, which changes the natural frequency of the hot rolling mill rolls. In practice, to eliminate the clearance, a hydraulic liner is often added between the bearing seat and the housing of the hot rolling mill rolls.

Figure 14 shows the amplitude–frequency characteristic curve of the coupled vibration system of the hot rolling mill with piecewise nonlinear spring force constraint coefficient $\alpha = 1.5$. Compared with that shown in Figure 11 ($\alpha = 1$ in Figure 11), when the nonlinear spring force increases slightly, the amplitudes in the horizontal and vertical directions increase significantly, one branch of the vibration curve bends and shifts to the right, and the resonance region evidently widens (65–190 Hz). In the vertical vibration curve (Figure 14b), we find evident jumping phenomena at piecewise nodes e_1 and e_2 , the horizontal direction (Figure 14a) also has similar jumping characteristics. Which indicates that the roll coupling system of the hot rolling mill rolls is very sensitive to the piecewise nonlinear elastic constraint of the hydraulic cylinder, because of which the hot rolling mill is often unstable in the rolling process.

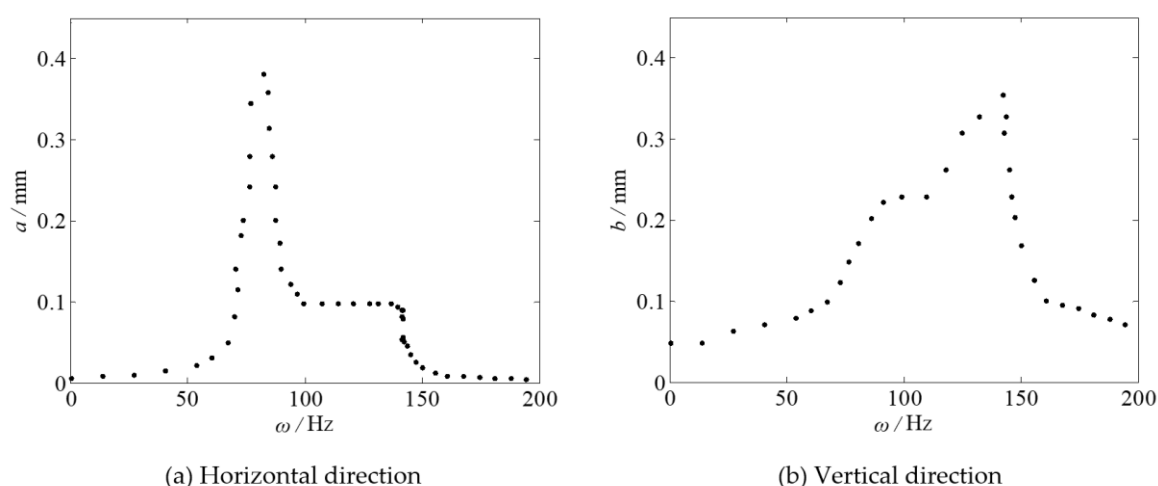


Figure 13. Amplitude–frequency curve of the coupled vibration system of hot rolling mill with gap $\Delta x = 0$ mm. (a) Amplitude–frequency curve of horizontal direction; (b) Amplitude–frequency curve of vertical direction.

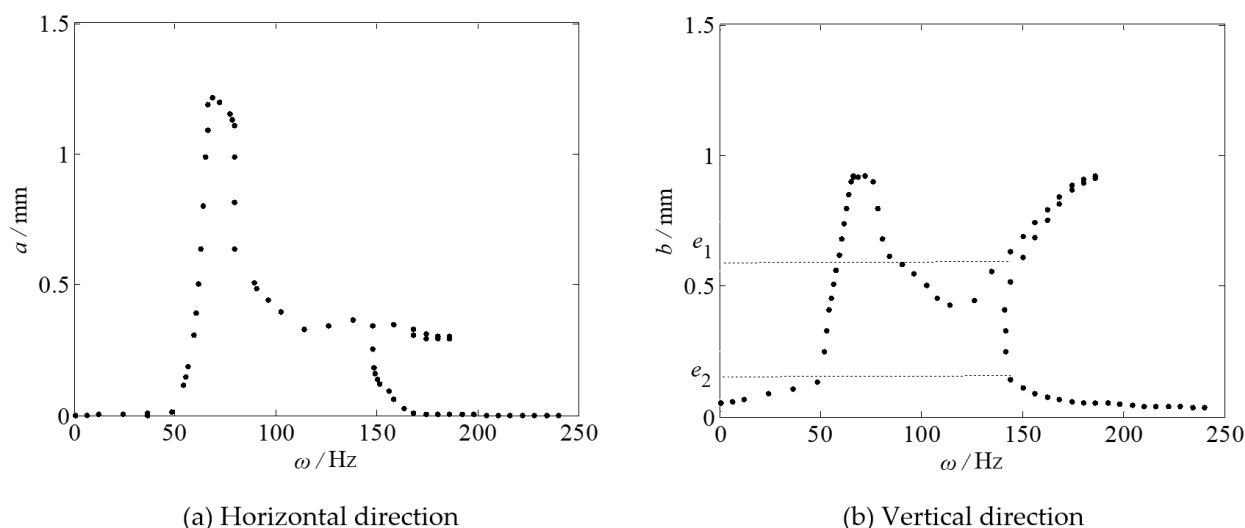


Figure 14. Amplitude–frequency curve of the coupled vibration system of hot rolling mill with piecewise nonlinear spring force constraint coefficient $\alpha = 1.5$. (a) Amplitude–frequency curve of horizontal direction; (b) Amplitude–frequency curve of vertical direction.

Figure 15 shows the amplitude–frequency characteristic curve of the coupled vibration system of the hot rolling mill with piecewise nonlinear friction force constraint coefficient $\beta = 5$. Compared with that shown in Figure 11 ($\beta = 1$ in Figure 11), increasing the nonlinear friction force constraint is equivalent to increasing the damping of the rolling mill rolls, reducing the amplitude of the vibration system, and reducing the resonance area (54–110 Hz). Therefore, appropriately increasing the nonlinear friction force can help effectively suppress the vibration of the system. Another reason for rolling mill instability is the influence of the signum function in the cubic model, which has continuous and discontinuous characteristics.

Figures 11–15 show that the nonlinear stiffness, nonlinear friction, gap between the bearing seat and the mill housing, and external disturbing force affect the amplitude–frequency curve variation of the coupled vibration system of the hot rolling mill. Therefore, the resonance behavior of the coupling system can be tailored by selecting appropriate system parameters to reduce the resonance damage on the rolling mill rolls.

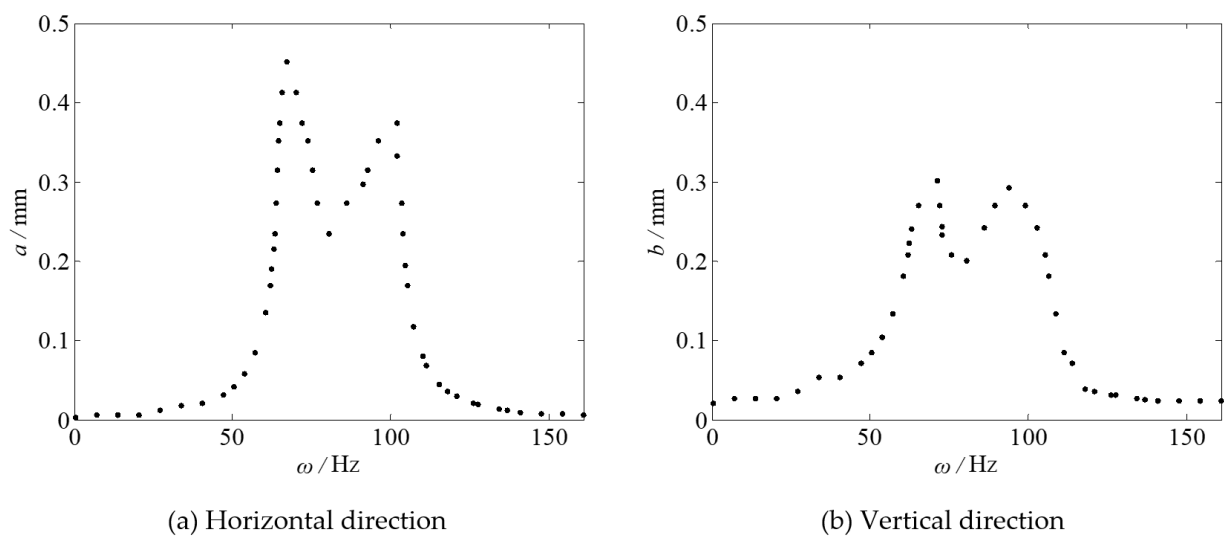


Figure 15. Amplitude–frequency curve of the coupled vibration system of hot rolling mill rolls with piecewise nonlinear friction force coefficient $\beta = 5$. (a) Amplitude–frequency curve of horizontal direction; (b) Amplitude–frequency curve of vertical direction.

5.2. Analysis of Bifurcation Characteristics

The singularity theory can be applied to identify the parameters that reflect the structural stability from a large number of parameters of a vibration system, so as to grasp the overall vibration characteristics of the system and predict the stability. Therefore, we used the singularity theory to analyze the static bifurcation characteristics of the coupled vibration system of the hot rolling mill under different parameters.

After the calculation, the amplitude a in Equation (27) is eliminated, and a Taylor series expansion is carried out at $b = b_0$, with $u = b^2$. After the linear transformation, the static bifurcation equation of the vertical horizontal coupling vibration system of the hot rolling mill rolls can be obtained.

$$u^4 + \lambda + d_1 u + d_2 u^2 + d_3 u^3 = 0 \quad (28)$$

This bifurcation equation is a universal unfolding of GS normal form $u^4 + \lambda$, and the other dimensions are set to 3. Here, λ is the bifurcation parameter, and d_1 , d_2 , and d_3 are the unfolding parameters.

$$\begin{aligned} \lambda &= \frac{T^2}{M} \left(4\zeta_1^2 \omega_{10}^2 \omega^2 + Q^2 \right) \\ d_3 &= \frac{1}{M} \left[Z_1 Z_2 \left(4\zeta_1^2 \omega_{10}^2 + Q^2 \right) - \frac{3}{2} \alpha_1^3 \chi_2 - \alpha_2^2 (F'_2)^2 \right] \\ d_2 &= \frac{1}{M} \left[\left(Z_1^2 + T Z_2 \right) \left(4\zeta_1^2 \omega_{10}^2 + Q^2 \right) - 4\zeta_1^2 \omega_{10}^2 \omega^4 T Z_2 + \alpha_1^2 \beta_1 \right] \\ d_1 &= \frac{T}{M} \left[4\zeta_1^2 \omega_{10}^2 \omega^2 Z_1 + Q^2 + \frac{1}{\pi} p'_f \mu_s \left(16\zeta_1^2 \omega_{10}^2 \omega^2 + 4Q^2 \right) - \alpha_1^2 (F'_1)^2 \right] \\ Q &= \frac{\omega_{10}^2 - \omega^2}{\varepsilon} - \rho - \frac{2\rho}{\pi} \arccos \frac{\Delta x}{a_0} \\ M &= Z_2^2 \left(4\zeta_1^2 \omega_{10}^2 + Q^2 \right) - \frac{9}{16} \alpha_1^2 \chi_1^2 \\ T &= \frac{\alpha}{\pi} \left[\left(\frac{1}{2} z_2 e_1^3 + z_1 e_1 \right) \sqrt{b_0^2 - e_1^2} - \left(\frac{1}{2} z_4 e_2^3 + z_3 e_2 \right) \sqrt{b_0^2 - e_2^2} \right] \\ Z_2 &= \frac{3}{4} \left[\frac{1}{\pi} \left(z_2 \arccos \frac{e_1}{b_0} - z_4 \arccos \frac{e_2}{b_0} \right) - \chi_2 - p'_f \beta \lambda_2 \omega^3 \right] \end{aligned} \quad (29)$$

$$Z_1 = 2\xi_2\omega_{20}\omega - \frac{\omega_{20}^2 - \omega^2}{\varepsilon} - p'_f\beta\lambda_1\omega + \frac{\alpha}{\pi} \left[\frac{13}{4}z_2e_1\sqrt{b_0^2 - e_1^2} - \frac{13}{4}z_4e_2\sqrt{b_0^2 - e_2^2} + (z_1 + 3z_2e_1^2)\arccos\frac{e_1}{b_0} + (z_3 + 3z_4e_2^2)\arccos\frac{e_2}{b_0} \right]$$

Since it is difficult to express the transition set of the coupled vibration system of the hot rolling mill directly in a 3D space, the static bifurcation characteristics on the 2D projection plane in different parameter spaces are discussed in the following three cases.

(1) When $d_1 = 0$, the bifurcation equation is $u^4 + \lambda + d_2u^2 + d_3u^3 = 0$, bifurcation point set $B = \phi$ (ϕ is the empty set), hysteretic point set $H = \left\{ \frac{8}{9} \frac{d_2^2}{d_3^2} = d_2 \right\}$, double limit point set $D = \{d_2 > 0, d_3 = 0\}$, and transition set $\Sigma = B \cup H \cup D$. Figure 16 shows the transition set of the coupled vibration system of the hot rolling mill rolls. Figure 16 shows the corresponding topological structure in the different regions.

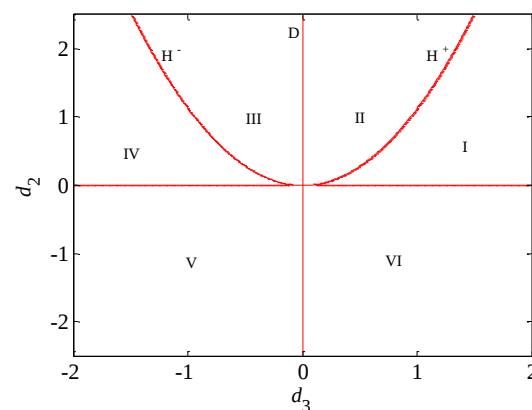


Figure 16. Transition set of nonlinear coupled vibration system of hot rolling mill rolls with $d_1 = 0$.

In Figure 16, the transition set divides the plane composed of unfolding parameters d_3 – d_2 into six regions (I–VI). In the same region, the topological structure is equivalent, i.e., the bifurcation morphology is similar; however, in different regions, the topological structure is not equivalent. Figure 17 shows nine bifurcation modes, which reflect all the information of the coupled vibration system when the unfolding parameter $d_1 = 0$. On the curves of zones I, II, IV, and double limit point set D, the system is stable, and there is no jump phenomenon. On the curves of regions III, V, VI, and set of hysteretic points H, a bifurcation parameter λ corresponds to multiple u , and there is a jump phenomenon. The main reason for the jump phenomenon is the nonlinear elastic force in the vertical direction, nonlinear friction force, and sectional stiffness in the horizontal direction.

(2) When $d_2 = 0$, the bifurcation equation is $u^4 + \lambda + d_1u + d_3u^3 = 0$, bifurcation set $B = \phi$, hysteretic point set $H = \left\{ \frac{16d_1^2}{d_3^2} = -d_1d_3 \right\}$, double limit point set $D = \{d_3=0\}$, and transition set $\Sigma = B \cup H \cup D$. Figure 18 shows the transition set of the coupled vibration system of the hot rolling mill rolls. Figure 19 shows the corresponding topological structure. The transition set divides the plane composed of the unfolding parameters d_3 – d_1 into six regions; on the curves of regions II, VI, and set of hysteretic points H, the system jumps, and other regions are in the steady state.

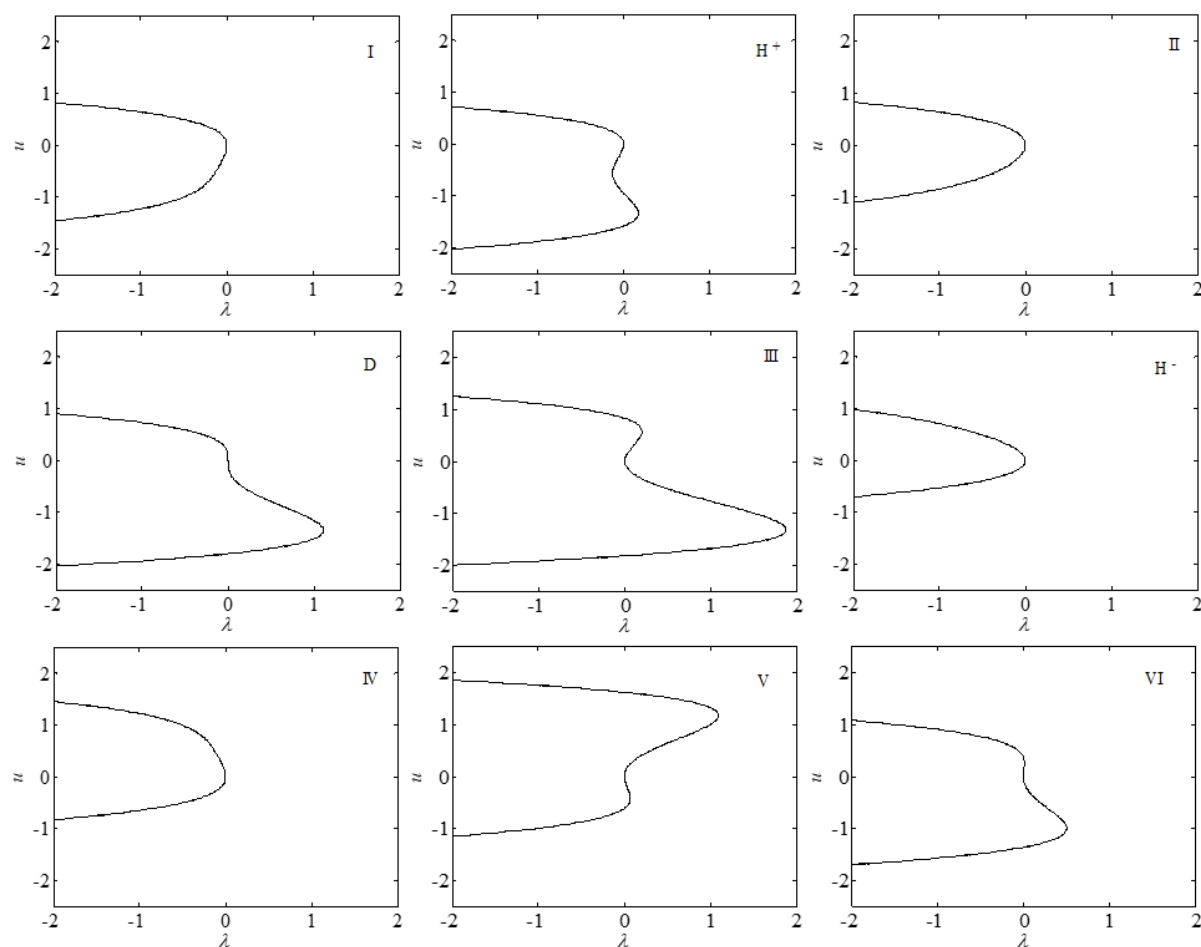


Figure 17. Topological structure of different regions in the transition set when $d_1 = 0$.

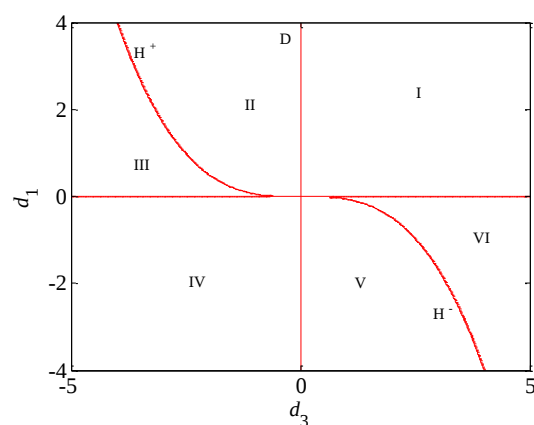


Figure 18. Transition set of a nonlinear coupled vibration system of hot rolling mill rolls with $d_2 = 0$.

(3) When $d_3 = 0$, the bifurcation equation is $u^4 + \lambda + d_1 u + d_2 u^2 = 0$, bifurcation point set $B = \emptyset$, hysteresis point set $H = \left\{ \left(\frac{d_1}{8} \right)^2 = - \left(\frac{d_2}{8} \right)^3 \right\}$, double limit point set $D = \{d_1 = 0, d_2 \leq 0\}$, and transition set $\Sigma = B \cup H \cup D$. Figure 20 shows the transition set of the coupled vibration system. Figure 21 shows the corresponding bifurcation topology. The unfolding plane is divided into four regions, and eight bifurcation modes are given. The rolling mill is in a stable state on the curve of origin O, I, IV, and double limit point set D; in the other regions, the jump phenomenon appears.

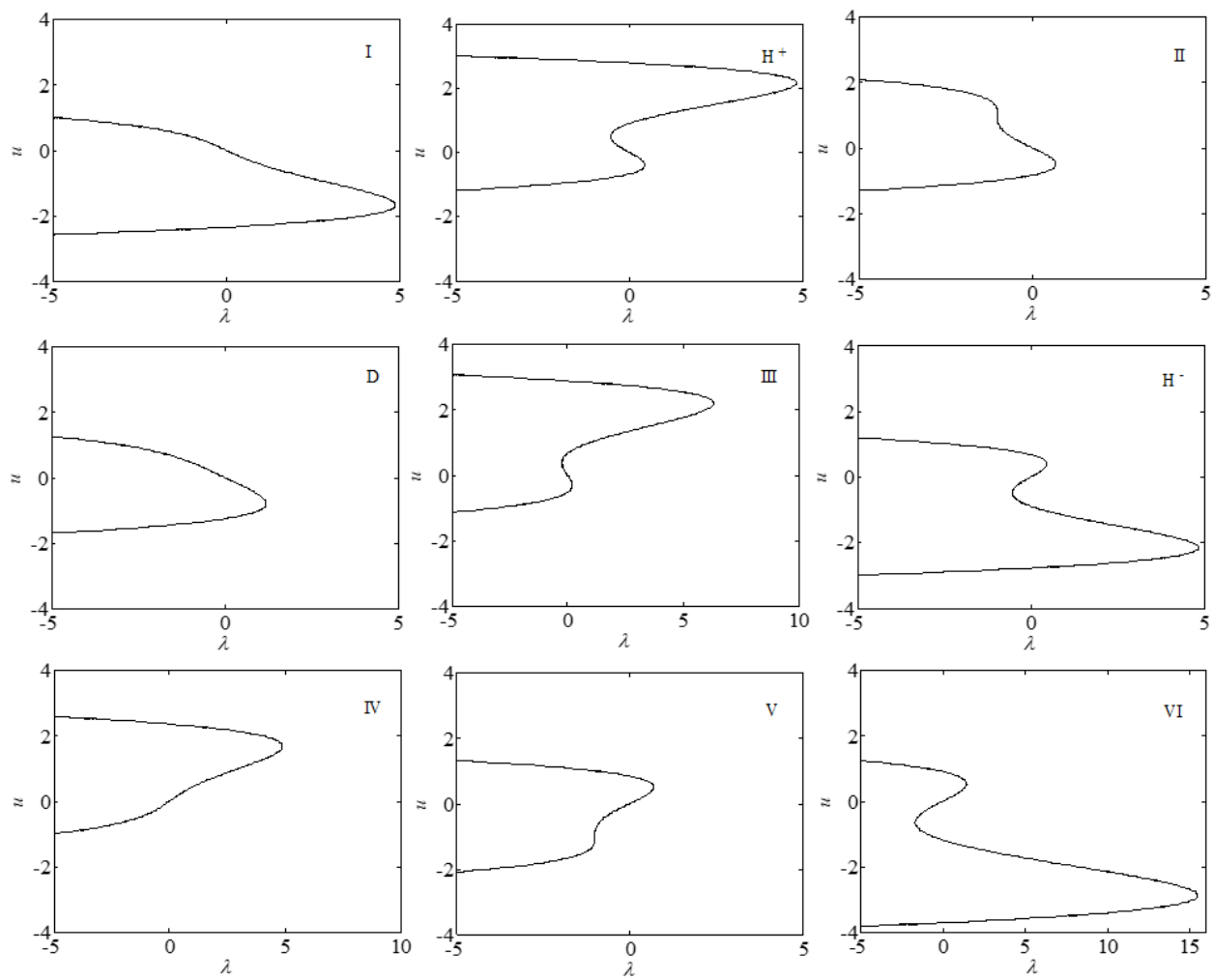


Figure 19. Topological structure of different areas in the transfer set with $d_2 = 0$.

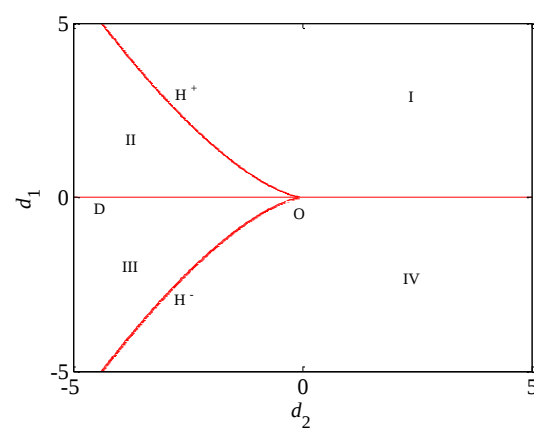


Figure 20. Transition set of nonlinear coupled vibration system of hot rolling mill rolls with $d_3 = 0$.

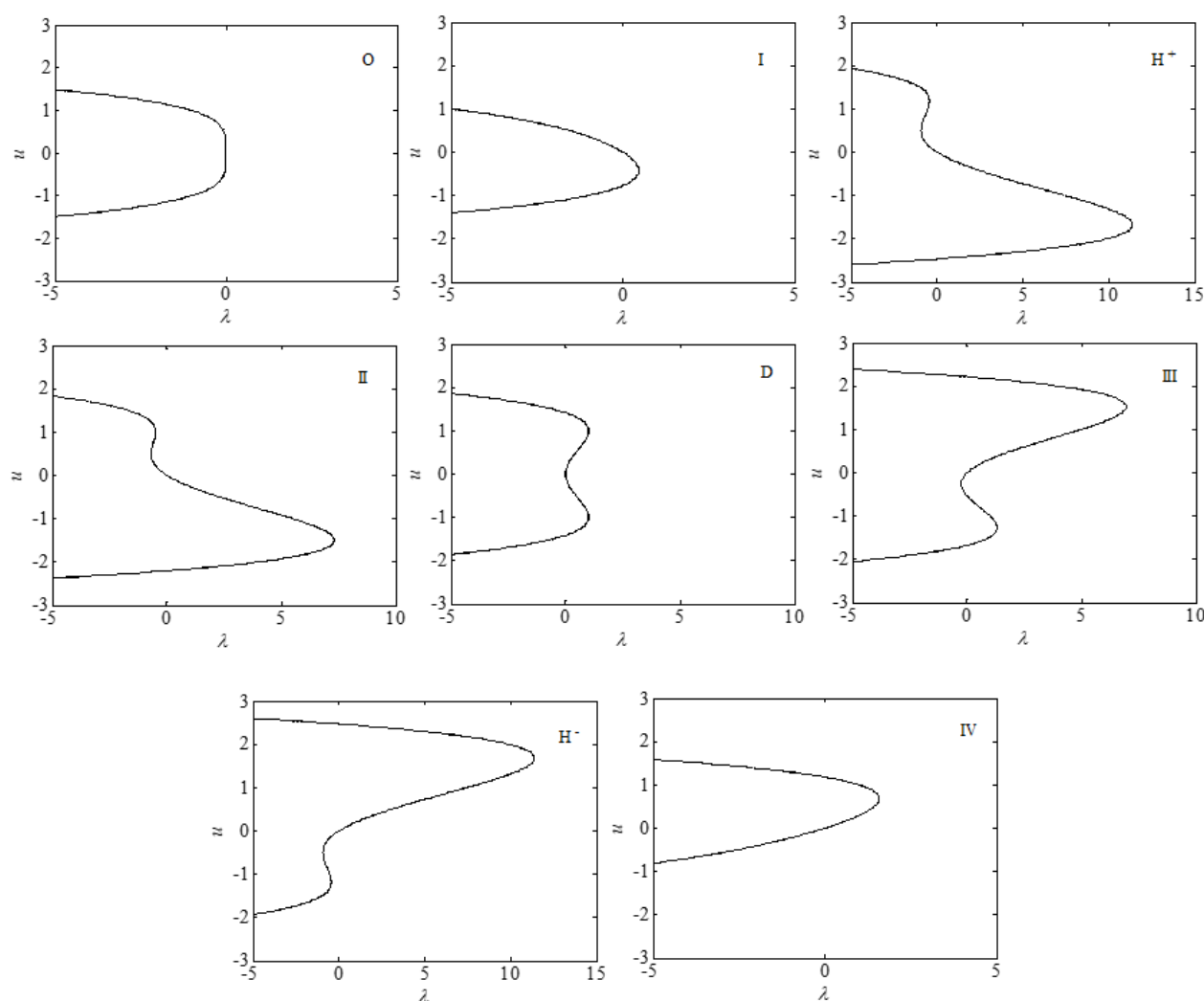


Figure 21. Topological structure diagram of different regions in the transfer set when $d_3 = 0$.

Figures 16–21 show that the vertical horizontal coupled vibration system of the hot rolling mill under multi-piecewise nonlinear constraints has a rich static bifurcation behavior, with different bifurcation characteristics under the influence of different parameters. Through the analysis of the bifurcation behavior, the stable region and unsteady parameter region of the coupling system of the hot rolling mill can be determined. Moreover, changing the bifurcation parameter λ can make the coupling vibration system to exhibit different bifurcation states, thus changing the vibration behavior of the system. This provides a theoretical reference for restraining the vibration of the hot rolling mill rolls.

6. Conclusions

1. To improve the accuracy of the vibration model of a hot rolling mill, the piecewise nonlinear spring force and piecewise nonlinear friction force were used to describe the constraint of the hydraulic screwdown system in the vertical direction of the rolling mill. A piecewise stiffness model was used to describe the impact of the horizontal bearing seat of the rolling mill rolls on the housing. Moreover, we derived an expression for the nonlinear coupling dynamic rolling force, which was mainly affected by the vertical vibration displacement and velocity and horizontal vibration displacement and speed. On this basis, a vertical–horizontal coupling vibration model of the hot rolling mill rolls under multi-segment nonlinear constraints was established.

2. By comparing the effects of different parameters on the amplitude–frequency characteristics of the coupled vibration system of the hot rolling mill, we found an evident

jump phenomenon at the piecewise nodes of the nonlinear elastic force, and the greater the nonlinear elastic force, the greater the vibration amplitude and the wider the resonance region. An increase in the external disturbance force also caused the jump phenomenon, thus destabilizing the vibration system of the hot rolling mill. Reducing the gap between the roller bearing seat and the mill housing and appropriately increasing the nonlinear friction force could help better reduce the amplitude and weaken the influence of piecewise characteristics to suppress the vibration of the hot rolling mill rolls. When rolling a sheet metal with a width of 1500 mm and a thickness of 14.1 mm, through certain measures, we could decrease the horizontal and vertical external disturbance forces to less than 3.5×10^5 N, reduce the gap between the horizontal roller bearing seat and the mill housing to less than 0.5 mm, and increase the friction coefficient between the piston and the cylinder wall of the hydraulic cylinder to greater than 0.6, thus ensuring an efficient and stable operation of the rolling mill.

3. A static bifurcation equation for the nonlinear coupled vibration system of the hot rolling mill rolls was obtained using the singularity theory. By analyzing three different transition sets and their corresponding bifurcation topological structures, we found that the coupled vibration system exhibits varying static bifurcation behaviors, indicating a relatively complex vibration behavior of the coupled system under the influence of multi-piecewise nonlinear factors. Finally, the steady and unsteady process parameters of the hot rolling mill rolls were given, and the stability of the coupled vibration system of the hot rolling mill was predicted from a macroscopic viewpoint.

Author Contributions: Conceptualization, data curation, writing—original draft, R.P.; Supervision, writing—review and editing, X.Z.; Software, project administration, writing—review and editing, P.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant number 61973262, and Natural Science Foundation of Hebei Province, grant number E2019203146, and the Graduate Innovation Funding Project of Hebei Province, grant number CXZZBS2021136.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Taylor series expansion of the dynamic rolling force in the horizontal direction.

First Order Parameter of Taylor Expansion:								
Parameter item	x	$\dot{x}$	y	$\dot{y}$				
Proportion value	0.0033	0.0000	3.6610	0.0028				
Second Order Parameter of Taylor Expansion:								
parameter item	x^2	$x\dot{x}$	xy	$x\dot{y}$	$\dot{x}^2$			
Proportion value	0.0000	0.0000	0.0000	0.0000	0.0000			
parameter item	$\dot{x}y$	$\dot{x}\dot{y}$	y^2	$y\dot{y}$	$\dot{y}^2$			
Proportion value	0.0002	0.0201	1.6583	0.0005	0.0002			
Cubic Order Parameter of Taylor Expansion:								
parameter item	$\dot{x}^2\dot{y}$	$\dot{x}^3$	$x\dot{x}^2$	$\dot{x}\dot{y}^2$	$\dot{y}^3$	x^2y	x^3	$\dot{x}^2y$
Proportion value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
parameter item	$x\dot{x}\dot{y}$	$xy\dot{y}$	y^3	$\dot{x}y\dot{y}$	$y^2\dot{y}$	$y\dot{y}^2$	$\dot{x}y^2$	xy^2
Proportion value	0.0000	0.0000	0.1169	0.0000	0.0000	0.0000	0.0000	0.0013

Table A2. Taylor series expansion of the dynamic rolling force in the vertical direction.

First Order Parameter of Taylor Expansion:								
Parameter item	x	$\dot{x}$	y	$\dot{y}$				
Proportion value	0.0229	0.0000	0.0000	0.0344				
Second Order Parameter of Taylor Expansion:								
parameter item	x^2	$x\dot{x}$	xy	$x\dot{y}$	$\dot{x}^2$			
Proportion value	0.0013	0.0000	0.0532	0.0001	0.0029			
parameter item	$\dot{x}y$	$\dot{x}\dot{y}$	y^2	$y\dot{y}$	$\dot{y}^2$			
Proportion value	0.0001	0.0001	2.9445	−0.0000	−0.0003			
Cubic Order Parameter of Taylor Expansion:								
parameter item	$\dot{x}^2\dot{y}$	$\dot{x}^3$	$x\dot{x}^2$	$\dot{x}\dot{y}^2$	$\dot{y}^3$	x^2y	x^3	$\dot{x}^2y$
Proportion value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000
parameter item	$x\dot{x}\dot{y}$	$xy\dot{y}$	y^3	$\dot{x}y\dot{y}$	$y^2\dot{y}$	$y\dot{y}^2$	$\dot{x}y^2$	xy^2
Proportion value	0.0000	0.0000	1.2374	0.0000	0.0004	0.0003	0.0001	0.0000

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