

## Article

# Effect of Alloying Elements on the Mechanical Properties of $\text{Mo}_3\text{Si}$

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**Abstract:** Molybdenum silicides are attractive high-temperature structural materials because of their excellent thermal stability and outstanding oxidation resistance at high temperatures. First-principles calculations were employed to investigate the effect of alloying elements (Cr, Nb, V, W, Al, Ga, and Ge) on the mechanical properties of  $\text{Mo}_3\text{Si}$ . The structural stabilities of doped  $\text{Mo}_3\text{Si}$  were calculated, showing that the Pm-3n structure was stable at the investigated low-doping concentration. The calculated elastic constants have also evaluated some essential mechanical properties of doped  $\text{Mo}_3\text{Si}$ . Cr- and V-doping decreased the elastic modulus, while Al- and Nb-doping slightly increased the shear and Young's modulus of  $\text{Mo}_3\text{Si}$ . Furthermore, V-, Al- and Nb-doping decreased the B/G and Poisson ratio, suggesting that these elements could form strong covalent bonds, and decrease shear deformation and alloy ductility. Based on the three-dimensional contours and two-dimensional projection of the elastic modulus, Cr- and V-doping exhibited a significant influence on the anisotropy of the shear and Young's modulus. According to charge density and density of states, the electronic structures of alloyed  $\text{Mo}_3\text{Si}$  were further analyzed to reveal the doping effects.

**Keywords:**  $\text{Mo}_3\text{Si}$ ; elastic modulus; alloying elements; first principles



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## 1. Introduction

Molybdenum silicides are attractive high-temperature structural materials because of their excellent thermal stability and outstanding oxidation resistance at high temperatures [1–6]. However, their brittleness at room temperature has restricted their engineering applications incredibly. For the Mo-Si system, there are three stable compounds:  $\text{MoSi}_2$ ,  $\text{Mo}_5\text{Si}_3$ , and  $\text{Mo}_3\text{Si}$ . Notably, some studies [1–4] have focused on  $\text{Mo}_5\text{Si}_3$  and  $\text{MoSi}_2$  over the past few decades, while the research about  $\text{Mo}_3\text{Si}$  (another molybdenum silicide) is still limited, attributed to its relatively low fracture toughness and room temperature strength. The low fracture toughness of  $\text{Mo}_3\text{Si}$  is ascribed to its few active slip systems [7]. New doped  $\text{Mo}_3\text{Si}$  with alloying elements should be developed to acquire adequate fracture toughness and ductility.

Alloying elements close to molybdenum in the periodic table, including Nb and W, has attracted great interest in the last few decades. Ray [6] investigated the effects of Nb and W on the stability of  $\text{Mo}_3\text{Si}$  by combining theoretical calculations and experiments. Rosales [7,8] studied the effect of niobium on some mechanical behaviors of  $\text{Mo}_3\text{Si}$  by the arc-melting technique. Their results suggested that Nb can strengthen  $\text{Mo}_3\text{Si}$  due to lattice distortion [7], which is favorable to wear resistance [8]. Xu [9] proposed that Mo addition can improve the anti-damage properties and the hardness of  $\text{Mo}_3\text{Si}$ -based coatings by the double glow-discharge plasma approach. Furthermore, the influences of some alloying elements (including B, Al, and Cr) on the oxidation resistance of  $\text{Mo}_3\text{Si}$  are being investigated [10–12].

First-principles calculations [13–17] can provide a more in-depth explanation and prediction of the relationship between mechanical properties and alloying effect for molybdenum silicides. In our early work [18], the effects of Cr, Nb, and W on the elastic properties of  $\text{MoSi}_2$  were studied by the first-principles. Hu [19] proposed that  $\text{MoSi}_2$  doped with Al enhances the ductility by the first-principles calculations, whereas Ge affects the ductility adversely. However, the alloying influences on the mechanical behaviors of  $\text{Mo}_3\text{Si}$  are still limited, especially these neighboring elements of silicon in the periodic table. Only these lattice constants [20] and some thermodynamic properties [20,21] of  $\text{Mo}_3\text{Si}$  have been investigated using first-principles calculations. This work analyzed the elastic properties of  $\text{Mo}_3\text{Si}$  alloyed with different concentrations of Cr, Nb, V, W, Al, Ga, and Ge from first-principles calculations. Remarkably, Cr, Nb, V, and W substitute preferentially on Mo sites, while Al, Ga, and Ge substitute preferentially on Si sites.

## 2. Computational Methods

These calculations were carried out with the use of Perdew-Burke-Ernzerh of exchange–correlation (XC) functional [22] based on density functional theory (DFT) in the VASP (Vienna *Ab-initio* Simulation Package) [23,24]. The electron-core interaction was described by the projector augmented wave method (PAW). The electronic configurations of elements were: Mo  $4p^6 4d^5 5s^1$ , Si  $3s^2 3p^2$ , Cr  $3d^5 4s^1$ , Nb  $4s^2 4p^6 4d^4 5s^1$ , V  $3p^6 3d^4 4s^1$ , W  $5p^6 5d^4 6s^2$ , Al  $3s^2 3p^1$ , Ga  $3d^{10} 4s^2 4p^1$ , and Ge  $3d^{10} 4s^2 4p^2$ . The energy cut-off was 420 eV, which was considered enough in our early research [18,20].

$\text{Mo}_3\text{Si}$  belonged to an A15 cubic structure (space group:  $\text{Pm}\bar{3}\text{n}$ ). Mo and Si occupied 6c (0.25, 0, 0.5) and 2a (0, 0, 0) of the Wyckoff positions. The crystal structure of  $\text{Mo}_3\text{Si}$  is shown in Figure 1. These  $2 \times 2 \times 1$ ,  $2 \times 1 \times 1$ , and  $1 \times 1 \times 1$  supercells of  $\text{Mo}_3\text{Si}$  were established to study the alloying effects at the concentrations of 3.125, 6.25, and 12.5 at.%, respectively. Cr, Nb, V, and W are doped on the Mo site, while Al, Ga, and Ge are doped on the Si site. Stress-strain method was used to evaluate the elastic constants. Several different strain modes were imposed on the crystal structure and the Cauchy stress tensor for each strain mode was evaluated, and then the related elastic constants were identified as the coefficients in strain-stress relations. The  $16 \times 16 \times 16$  Monkhorst-Pack k-mesh [25] generated for the conventional cell of  $\text{Mo}_3\text{Si}$  and the scaled-down k-meshes were used for the corresponding supercells. The force convergence criterion per atom was  $2 \times 10^{-2} \text{ eV}/\text{\AA}$ .

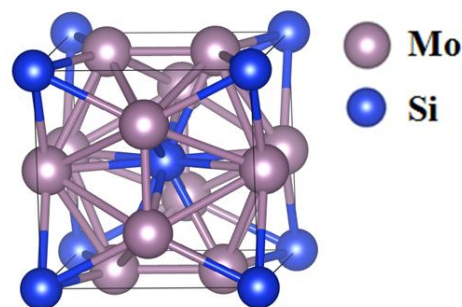


Figure 1. Crystal structure of  $\text{Mo}_3\text{Si}$ .

## 3. Results and Discussion

### 3.1. Structural Stability

It is of great significance to assess the structural stability for a doped system [18,19]. The formation enthalpies of the doped  $\text{Mo}_3\text{Si}$  were calculated to evaluate their phase stability as follows:

$$\Delta H_{\text{Mo}_p\text{Si}_q\text{X}_r} = \frac{1}{p+q+r} (E_{\text{Mo}_p\text{Si}_q\text{X}_r} - p \times E_{\text{Mo}} - q \times E_{\text{Si}} - r \times E_{\text{X}}) \quad (1)$$

where  $p$ ,  $q$ ,  $r$  are the molar fractions of Mo, Si, and alloying elements X in the alloy, and  $E_{\text{Mo}}$ ,  $E_{\text{Si}}$ ,  $E_X$ ,  $E_{\text{Mo}_p\text{Si}_q\text{X}_r}$  are the energies of these pure elements and the corresponding alloy, respectively.

The formation enthalpy of undoped  $\text{Mo}_3\text{Si}$  with the Pm-3n structure was 28.82 kJ/mol-atom, which agreed with the experimental value (−30.5 kJ/mol-atom [26]) and previous DFT calculation (−27.99 kJ/mol-atom [27]).

Figure 2 shows that the formation enthalpies of  $\text{Mo}_3\text{Si}$  doped with Cr, W, Al, Ga, and Ge increase with the increasing doping concentration, while those of  $\text{Mo}_3\text{Si}$  doped with Nb and V decrease with the increasing doping concentration. It is suggested that Nb and V help structural stability, while the other doping elements (Cr, W, Al, Ga, and Ge) decrease structural stability. Ray [6] proposed that Nb substitution ( $\leq 27.5$  at.%) stabilizes  $\text{Mo}_3\text{Si}$ , while W-doping destabilizes  $\text{Mo}_3\text{Si}$ . Our calculations about the formation enthalpy are in accord with Ray's experiments [6].

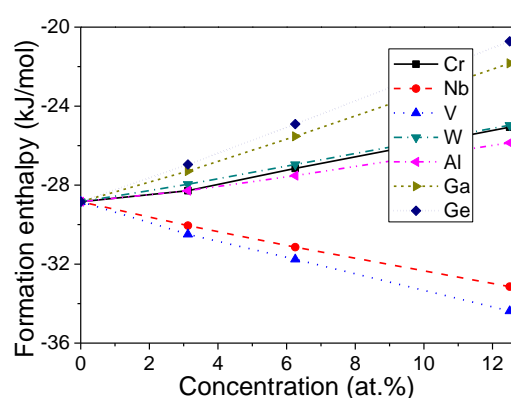


Figure 2. Formation enthalpies of doped  $\text{Mo}_3\text{Si}$ .

These formation enthalpies of doped  $\text{Mo}_3\text{Si}$  are negative, suggesting that doped  $\text{Mo}_3\text{Si}$  with a Pm-3n structure is stable at the investigated low-doping concentration ( $\leq 12.5$  at.%). This work investigated the alloying effects of different elements under low-doping concentrations ( $\leq 12.5$  at.%) on  $\text{Mo}_3\text{Si}$ .

Table 1 shows the elastic constants of  $\text{Mo}_3\text{Si}$ . The calculated elastic constants of  $\text{Mo}_3\text{Si}$  in the present work agree with the previous DFT calculations [9,21,28,29]. For a cubic structure, the mechanical criteria of stability are  $C_{11} > 0$ ,  $C_{44} > 0$ ,  $C_{11} - C_{12} > 0$  and  $C_{11} + 2C_{12} > 0$  [30]. The calculated elastic constants of doped  $\text{Mo}_3\text{Si}$  conform to these stability conditions above, suggesting that doped  $\text{Mo}_3\text{Si}$  is mechanically stable.

Table 1. Elastic constants of  $\text{Mo}_3\text{Si}$ .

Alloys	Method	Ref.	$C_{11}$ (GPa)	$C_{12}$ (GPa)	$C_{44}$ (GPa)
$\text{Mo}_3\text{Si}$	VASP-PBE	This work	473.97	141.28	110.26
	CASTEP-PBE	[9]	457.4	144.7	120.2
	CASTEP-LDA	[21]	514.9	153.6	122.5
	VASP-PBE	[21]	466.6	134.9	114.9
	QUANTUM ESPRESSO-GGA	[28]	438.52	125.1	106.4
	FP-LMTO	[29]	539	147	118

### 3.2. Mechanical Properties

The bulk and shear modulus of a cubic structure can be evaluated based on the Voigt-Reuss-Hill approximation as follows [31,32]:

$$B_V = B_R = \frac{C_{11} + 2C_{12}}{3}, \quad G_V = \frac{C_{11} - C_{12} + 3C_{44}}{5}, \quad G_R = \frac{5(C_{11} - C_{12})C_{44}}{3(C_{11} - C_{12}) + 4C_{44}} \quad (2)$$

where  $C_{ij}$  are the calculated elastic constants of  $\text{Mo}_3\text{Si}$ ;  $B_i$  and  $G_i$  are the isotropised bulk and shear modulus. Here, these subscripts  $i$  ( $V, R, H$ ) represent Voigt, Reuss, and Hill approximation, respectively.

The isotropised Poisson ratio  $\nu$  [33] and isotropised Young's modulus  $E$  are calculated as:

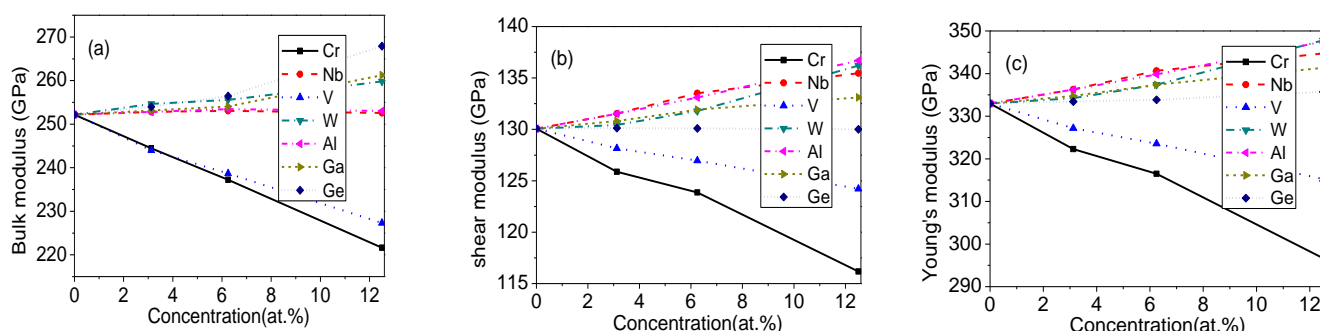
$$E = \frac{9BG}{3B + G}, \nu = \frac{E}{2G} - 1 \quad (3)$$

The Debye temperature  $\Theta_D$  [34] and wave velocities are assessed by these calculated mechanical properties as:

$$\Theta_D = \frac{h}{k_B} \left[ \frac{3n}{4\pi} \left( \frac{N_A \rho}{M} \right) \right]^{\frac{1}{3}} v_m, v_m = \left[ \frac{1}{3} \left( \frac{1}{v_l^3} + \frac{2}{v_t^3} \right) \right]^{-\frac{1}{3}}, v_l = \left( \frac{B + \frac{4}{3}G}{\rho} \right)^{\frac{1}{2}}, v_t = \left( \frac{G}{\rho} \right)^{\frac{1}{2}} \quad (4)$$

where  $v_l$ ,  $v_t$  and  $v_m$  are longitudinal, transverse, and average wave velocities;  $n$  is the atom number;  $\rho$  is the density;  $M$  is the weight;  $N_A$ ,  $h$ , and  $k_B$  are the Avagadro's, Planck's, and Boltzmann's constants, respectively.

According to Equations (2) and (3), the elastic moduli of  $\text{Mo}_3\text{Si}$  doped with elements are calculated and given in Figure 3. The  $B$  and  $G$  of undoped  $\text{Mo}_3\text{Si}$  are 252.1 and 130.1 GPa, respectively, which agrees with Xu's calculation [9] (248.9 and 133.5 GPa) and Zhong's calculation [21] (245.5 and 133.2 GPa).

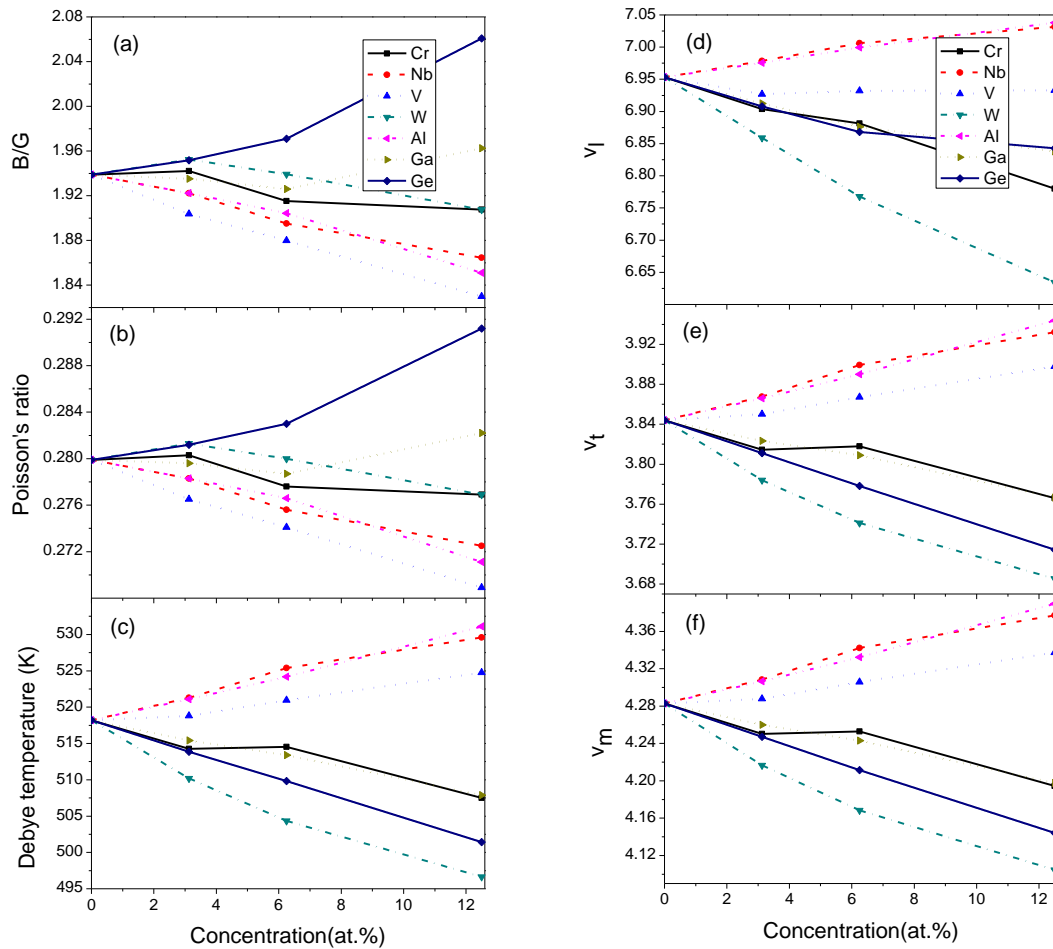


**Figure 3.** Mechanical properties of doped  $\text{Mo}_3\text{Si}$ . (a) bulk modulus; (b) shear modulus; (c) Young's modulus.

In Figure 3, the elastic moduli of Cr- and V-doped  $\text{Mo}_3\text{Si}$  decrease distinctly, while those of W- and Ga-doped  $\text{Mo}_3\text{Si}$  increase moderately. The bulk modulus of Ge-doped  $\text{Mo}_3\text{Si}$  increases obviously, but its  $G$  and  $E$  change little. The  $G$  and  $E$  of Al- and Nb-doped  $\text{Mo}_3\text{Si}$  increase slightly, but their bulk moduli are almost unchanged.

There is an empirical equation ( $G = 6.78H_v$ ) showing that the shear modulus  $G$  is approximately proportional to Vickers hardness  $H_v$ . It is suggested that Al-, Nb-, W- and Ga-doping increase the hardness of  $\text{Mo}_3\text{Si}$ . Unfortunately, there is still a lack of experimental data from the literature about the mechanical properties of doped  $\text{Mo}_3\text{Si}$ . Rosales [7] stated that Nb should enhance the hardness and decrease the toughness according to an arc-melting technique. Our findings are in accord with Rosales' viewpoints [7].

The alloy's ductility or brittleness can be evaluated by  $B/G$  and  $\nu$  [35]. Figure 4 shows the  $B/G$  and  $\nu$  of alloyed  $\text{Mo}_3\text{Si}$ . The  $B/G$  and  $\nu$  of undoped  $\text{Mo}_3\text{Si}$  are higher than 1.75 and 0.26, respectively, suggesting that the ductility of  $\text{Mo}_3\text{Si}$  is not low. The  $B/G$  and  $\nu$  of  $\text{Mo}_3\text{Si}$  decrease with increasing V, Al, and Nb, and increase with the increasing Ge. The effect of Cr, W, and Ga on the  $B/G$  and  $\nu$  of  $\text{Mo}_3\text{Si}$  is relatively small. It is suggested that V-, Al-, and Nb-doping can form strong covalent bonds, and decrease shear deformation and alloy ductility. Rosales [7] proposed that Nb should strengthen the solid solution and reduce the toughness of  $\text{Mo}_3\text{Si}$ , which is in line with our calculations.



**Figure 4.** B/G, Poisson ratio, wave velocities and Debye temperatures of doped Mo<sub>3</sub>Si. (a) B/G (b) Poisson ratio (c) Debye temperatures (d)  $v_l$  (e)  $v_t$  (f)  $v_m$ .

Figure 4 shows the wave velocities and Debye temperature of doped Mo<sub>3</sub>Si. The Debye temperature of doped Mo<sub>3</sub>Si is 518.2 K, consistent with Zhong's calculation (522.2 K) [21]. These alloying elements exhibit different effects on the wave velocities and the Debye temperature of Mo<sub>3</sub>Si. Compared to undoped Mo<sub>3</sub>Si, the wave velocities and Debye temperatures of Nb-, Al- and V-doped Mo<sub>3</sub>Si increase, while those of Cr-, W-, Ga- and Ge-doped Mo<sub>3</sub>Si decrease. Among these alloying elements, Mo<sub>3</sub>Si doped with Al and Nb exhibit high wave velocities and Debye temperatures, and those of W-doped Mo<sub>3</sub>Si reduce strikingly.

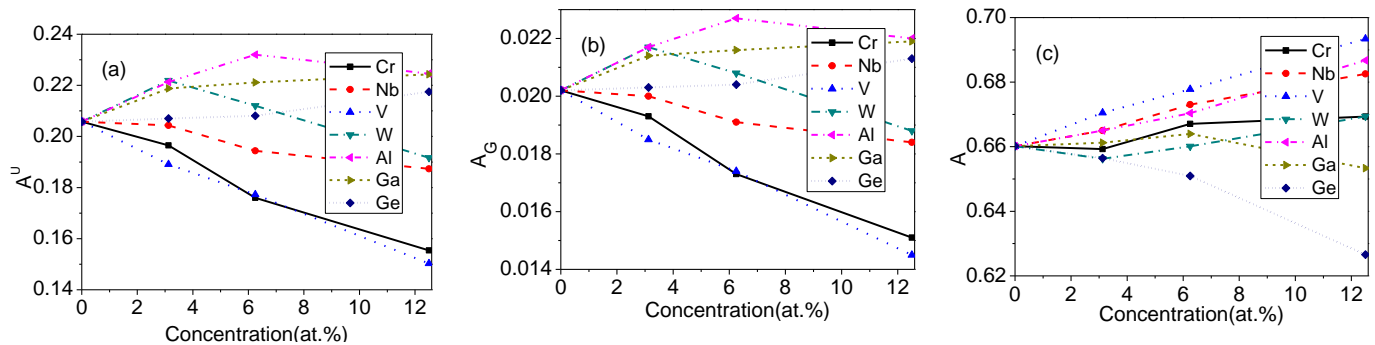
### 3.3. Anisotropic Elasticity

Some mechanical anisotropic indexes of doped Mo<sub>3</sub>Si, such as the universal anisotropic index ( $A^U$ ), the percent anisotropy of the bulk and shear modulus ( $A_B$  and  $A_G$ ), and the Zener's shear anisotropy index ( $A$ ), are denoted as:

$$A^U = 5 \frac{G_V}{G_R} + \frac{B_V}{B_R} - 6 \geq 0, A_B = \frac{B_V - B_R}{B_V + B_R}, A_G = \frac{G_V - G_R}{G_V + G_R}, A = \frac{2C_{44}}{C_{11} - C_{12}} \quad (5)$$

According to Equation (3), the  $A_B$  of doped Mo<sub>3</sub>Si is zero, suggesting that the bulk modulus is elastically isotropic. Figure 5 shows the mechanical anisotropic indexes,  $A^U$ ,  $A_G$ , and  $A$  of doped Mo<sub>3</sub>Si. It is seen that the shear modulus exhibits strong directional dependence. The percent anisotropy of the shear modulus decreases by Mo-site doping (Cr, Nb, and V), but increases by Si-site doping (Al, Ga, and Ge). The  $A_G$  of W-doped Mo<sub>3</sub>Si increases slightly at relatively low concentrations ( $\leq 6.25$  at.%), but decreases at high

concentrations (12.5 at.%). The Zener's shear anisotropy index increases by Mo-site doping (Cr, Nb, and V) and Al-doping, but decreases by Ge-doping obviously. The  $A$  of W-doped  $\text{Mo}_3\text{Si}$  shows a fluctuation, which is similar to the situation of  $A_G$ .



**Figure 5.** Mechanical anisotropic indexes of doped  $\text{Mo}_3\text{Si}$ . (a)  $A^U$ ; (b)  $A_G$ ; (c)  $A$ .

In Figure 5, the  $A_G$  of  $\text{Mo}_3\text{Si}$  doped with Cr and V decrease, that of Al-doped  $\text{Mo}_3\text{Si}$  increases obviously, and that of W-doped  $\text{Mo}_3\text{Si}$  shows a fluctuation. It is also seen from Figure 5 that the  $A$  of  $\text{Mo}_3\text{Si}$  doped with V increases, and that of  $\text{Mo}_3\text{Si}$  doped with Ge clearly decreases. The directionality of the mechanical properties for Cr-, V-, Al and Ge-doped  $\text{Mo}_3\text{Si}$  should be different from that of undoped  $\text{Mo}_3\text{Si}$ , attributed to differential physical properties and electronic structures between the alloying elements (Cr, V, Al and Ge) and these substituted atoms (Mo or Si).

Anisotropic elasticity as a function of crystallographic orientation is studied according to the three-dimensional surfaces of an elastic modulus. The  $B$  and  $E$  [36–39] of a cubic structure in spherical coordinates can be calculated by:

$$\frac{1}{B} = (S_{11} + 2S_{12}) \quad (6)$$

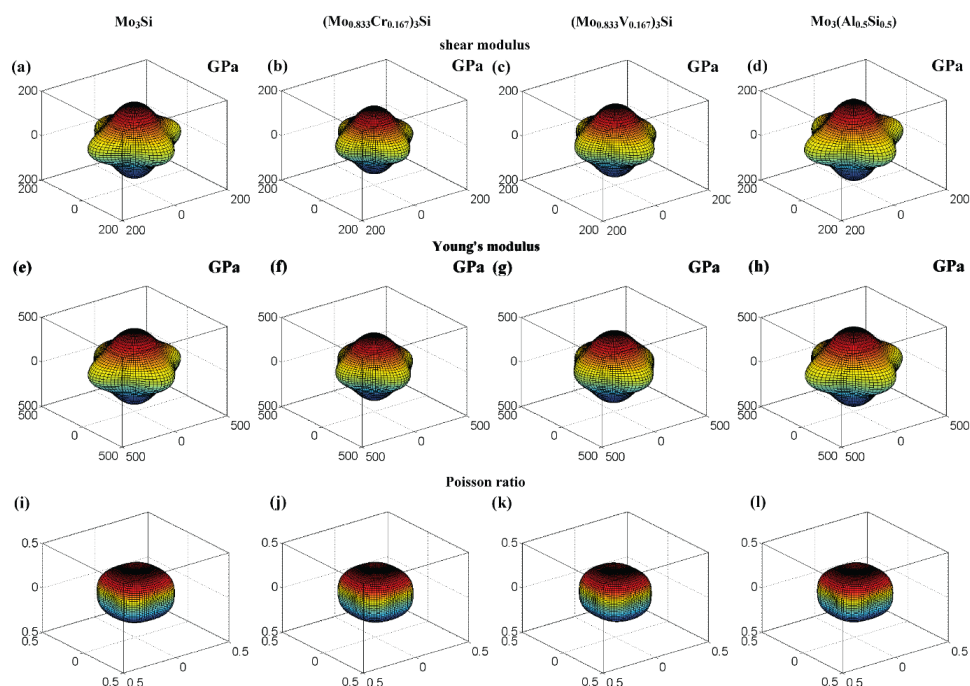
$$\frac{1}{E} = S_{11} - 2(S_{11} - S_{12} - \frac{1}{2}S_{44})(l_1^2 l_2^2 + l_1^2 l_3^2 + l_2^2 l_3^2)$$

where  $S_{ij}$  are the matrix compliance constants which can be obtained by solving the inverse matrix of elastic matrix;  $l_1$ ,  $l_2$ , and  $l_3$  are the direction cosines for three coordinates, respectively.

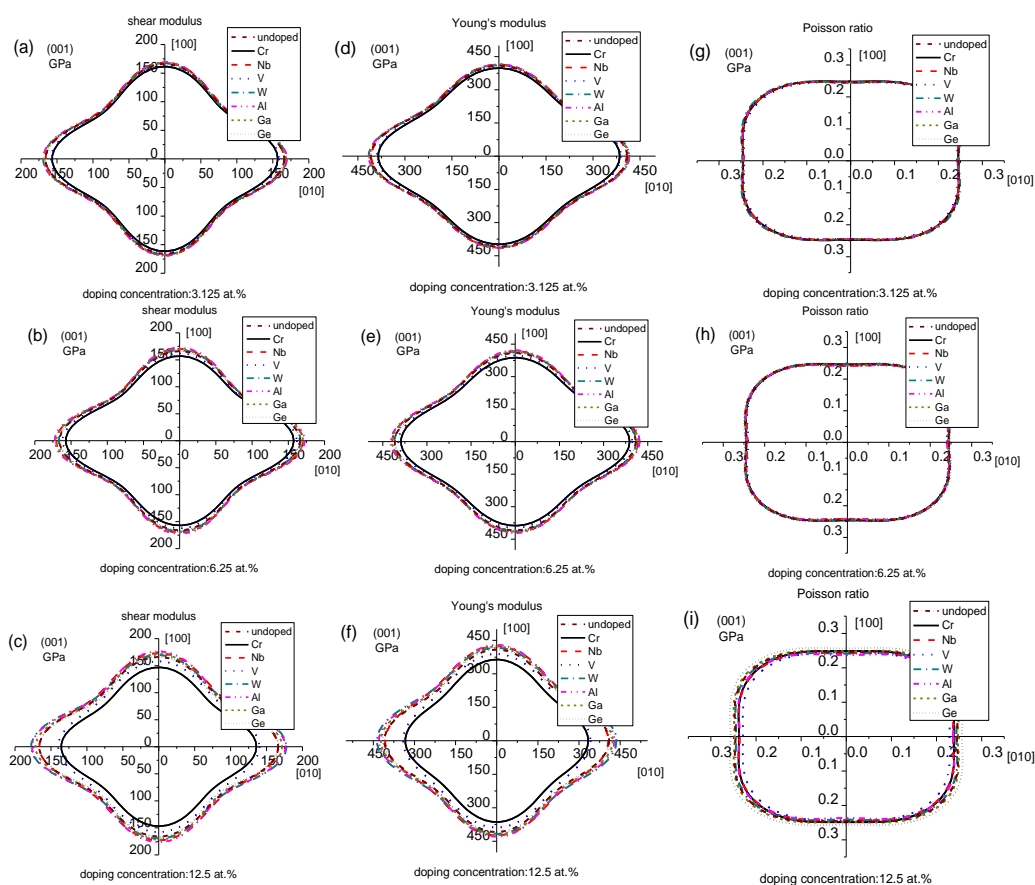
According to Equation (6), the bulk modulus is elastically isotropic. Combined with Equations (3) and (6), the shear modulus and Poisson ratio at different directions can also be calculated using spherical coordinates. Moreover, the calculation formula of the shear modulus and Poisson ratio for a cubic structure can also be referred in Refs. [33,39]. Figure 6 shows the 3D surfaces of the shear modulus, Young's modulus and Poisson ratio for  $\text{Mo}_3\text{Si}$  alloyed with 12.5 at.% Cr, V, and Al. These surfaces of Cr- and V-doped  $\text{Mo}_3\text{Si}$  are different from those of undoped  $\text{Mo}_3\text{Si}$ , showing the significant effect of Cr and V on elastic anisotropy. The anisotropy of shear and Young's modulus for  $\text{Mo}_3\text{Si}$  doped with Al strengthens slightly. The calculations also show that the 3D contours of the Poisson ratio for doped  $\text{Mo}_3\text{Si}$  are very close to those of undoped  $\text{Mo}_3\text{Si}$ , suggesting that the effect of doping with the alloying elements on the anisotropy of Poisson ratio is small.

In order to have a better understanding of the origin of the changes in the mechanical properties along different directions, the 2D projections of the shear modulus, Young's modulus and Poisson ratio at the (001) plane are given in Figure 7. Cr- and V-doping can decrease the shear and Young's modulus for  $\text{Mo}_3\text{Si}$  and affect the anisotropy of the shear and Young's modulus. This trend becomes evident with the increased Cr- and V-doping concentrations, consistent with the above mechanical properties and elastic anisotropy. The anisotropy of  $\text{Mo}_3\text{Si}$  doped with other elements changes little, and that of the shear and Young's modulus for  $\text{Mo}_3\text{Si}$  doped with Al increases mildly. These findings are in accord with the calculated anisotropic elasticity. It is also seen that the  $\nu$  of  $\text{Mo}_3\text{Si}$  doped with 12.5 at.% V, Al, and Nb decrease evidently, and that of  $\text{Mo}_3\text{Si}$  doped with 12.5 at.% Ge increases, which agrees well with the above calculated Poisson ratio in Figure 4.





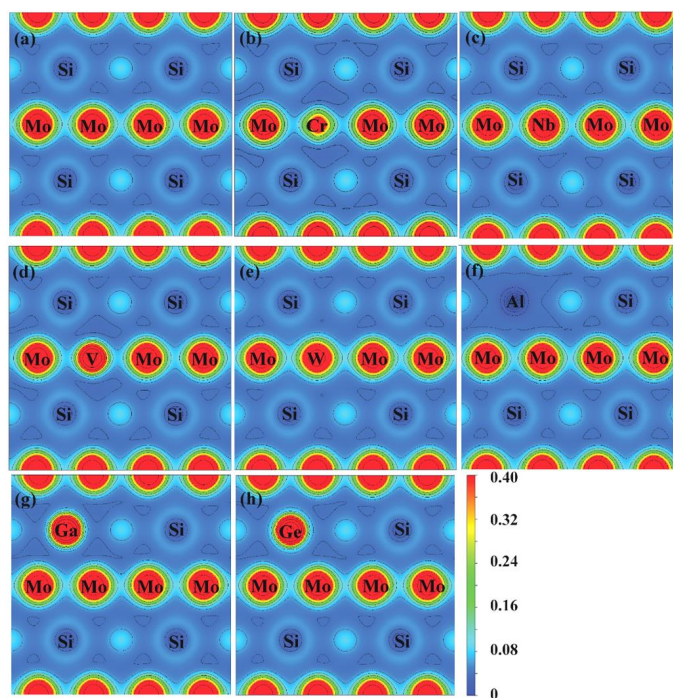
**Figure 6.** Three-dimensional contours of the shear modulus, Young's modulus and Poisson ratio for undoped and doped  $\text{Mo}_3\text{Si}$ . The distance between zero and any point on the surfaces is equal to the elastic modulus or Poisson ratio in that direction. (a–d) Shear modulus; (e–h) Young's modulus; (i–l) Poisson ratio.



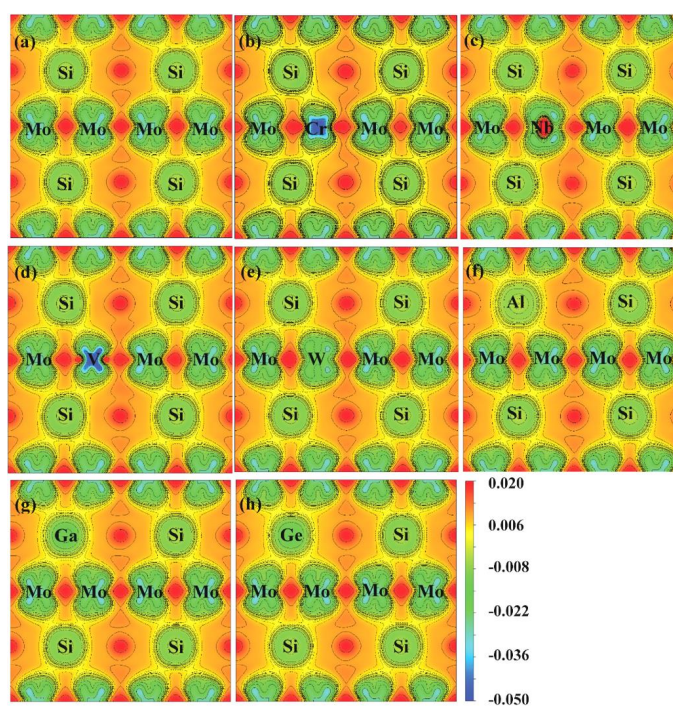
**Figure 7.** Two-dimensional projections of the shear modulus, Young's modulus and Poisson ratio for undoped and doped  $\text{Mo}_3\text{Si}$ . (a–c) Shear modulus; (d–f) Young's modulus; (g–i) Poisson ratio.

### 3.4. Electronic Structures

Figures 8 and 9 show the charge densities and deformation charge densities of undoped and doped  $\text{Mo}_3\text{Si}$ , respectively. The charge densities show the elongated contours between Si and Mo, suggesting a robust covalent bonding generated by hybridizing Si-3p and Mo-4d. Figure 9 shows different charge densities with these doped elements, which affects the mechanical properties of  $\text{Mo}_3\text{Si}$ .



**Figure 8.** Charge densities of undoped and doped  $\text{Mo}_3\text{Si}$ . (a) undoped; (b) Cr; (c) Nb; (d) V; (e) W; (f) Al; (g) Ga; (h) Ge.

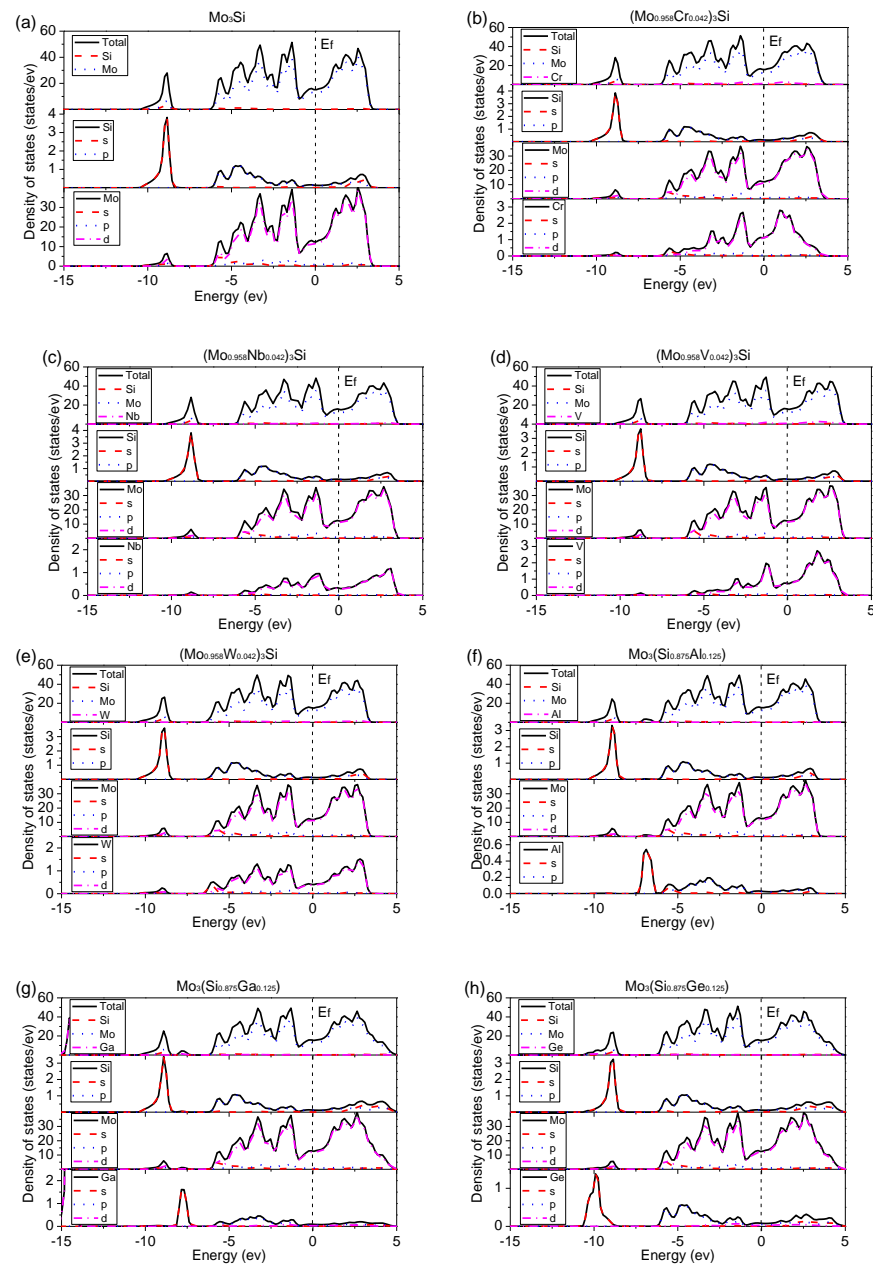


**Figure 9.** Deformation charge densities of undoped and doped  $\text{Mo}_3\text{Si}$ . (a) Undoped; (b) Cr; (c) Nb; (d) V; (e) W; (f) Al; (g) Ga; (h) Ge.



In Figure 9, Cr- and V-doping reduce the charge accumulation between Si and Mo compared to those of undoped  $\text{Mo}_3\text{Si}$ , suggesting their low elastic modulus. Nb-doping increases the charge accumulation slightly, indicating that Nb can strengthen the elastic modulus of  $\text{Mo}_3\text{Si}$ . These findings agree with the above mechanical properties in Section 3.2.

Figure 10 shows the density of states (DOS) of undoped and doped  $\text{Mo}_3\text{Si}$  with alloying elements. In Figure 10a, there is no gap and non-zero DOS at the Fermi level for undoped  $\text{Mo}_3\text{Si}$ , which shows its metallic behaviors. Furthermore, there is an overlap between Mo-4d and Si-3p for undoped  $\text{Mo}_3\text{Si}$ . Two peaks are divided by the Fermi level called the pseudogap. These phenomena suggest a strong covalent bonding between Si and Mo, consistent with the above charge results. The same situations also occur in doped  $\text{Mo}_3\text{Si}$ , suggesting strong covalent bonding caused by the d-p hybridization between Mo-site atoms (Mo, Cr, Nb, V, and W) and Si-site atoms (Si, Al, Ga, and Ge).



**Figure 10.** Density of states (DOS) of undoped and doped  $\text{Mo}_3\text{Si}$ . (a)  $\text{Mo}_3\text{Si}$ ; (b)  $(\text{Mo}_{0.958}\text{Cr}_{0.042})_3\text{Si}$ ; (c)  $(\text{Mo}_{0.958}\text{Nb}_{0.042})_3\text{Si}$ ; (d)  $(\text{Mo}_{0.958}\text{V}_{0.042})_3\text{Si}$ ; (e)  $(\text{Mo}_{0.958}\text{W}_{0.042})_3\text{Si}$ ; (f)  $\text{Mo}_3(\text{Si}_{0.875}\text{Al}_{0.125})$ ; (g)  $\text{Mo}_3(\text{Si}_{0.875}\text{Ga}_{0.125})$ ; (h)  $\text{Mo}_3(\text{Si}_{0.875}\text{Ge}_{0.125})$ .

In Figure 10, the DOS of doped Mo<sub>3</sub>Si is close to that of undoped Mo<sub>3</sub>Si. However, there are some small changes in DOS with different doped elements. A slight shift of the peaks to low energy for V- and Nb-doped Mo<sub>3</sub>Si suggests that V- and Nb-doping increase the structural stability of Mo<sub>3</sub>Si, following the above formation enthalpies in Section 3.1.

In Figure 9c,d, some 4d anti-bonding of V and Nb moves away from the Fermi level and increases the gap between their anti-bonding and bonding states. It is suggested that V and Nb induce dislocation-slip energy, unfavorable to the ductility. As mentioned above, Nb's adverse effect on toughness has been confirmed in Rosales' experiments [7]. Moreover, a high DOS at the Fermi level for Ge- and Ga-doped Mo<sub>3</sub>Si indicates that Ge- and Ga-doping strengthen metallic bonding and the ductility, which conforms to the calculated B/G and  $\nu$  (See Section 3.2).

#### 4. Conclusions

First-principles calculations were employed to investigate the effect of alloying elements (Cr, Nb, V, W, Al, Ga, and Ge) on the mechanical properties of Mo<sub>3</sub>Si. The structural stabilities of doped Mo<sub>3</sub>Si were calculated, which suggested that the Pm-3n structure was stable at these investigated low-doping concentrations ( $\leq 12.5$  at.%). The calculated elastic constants have been used to evaluate some critical mechanical properties of Mo<sub>3</sub>Si doped with alloying elements.

Compared to undoped Mo<sub>3</sub>Si, Cr- and V-doping drastically decreased the elastic modulus, while Al- and Nb-doping slightly increased the shear and Young's modulus. Moreover, V-, Al- and Nb-doping decreased the B/G and Poisson ratio, suggesting that these elements could form strong covalent bonds, and decrease shear deformation and the alloy's ductility. The wave velocities and Debye temperatures of Nb-, Al- and V-doped Mo<sub>3</sub>Si increased, while those of Cr-, W-, Ga- and Ge-doped Mo<sub>3</sub>Si decreased.

Based on the three-dimensional contours and two-dimensional projection of the elastic modulus, Cr- and V-doping exhibited a significant influence on the anisotropy of the shear and Young's modulus. The anisotropy of Mo<sub>3</sub>Si doped with the other elements changed little, and that of the shear modulus for Mo<sub>3</sub>Si doped with Al increased mildly. The electronic structures of doped Mo<sub>3</sub>Si were also investigated according to the charge density and DOS, which helped us gain a deeper understanding of doping behaviors.

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