

Article

Study on Influence of Roller Profile Modification on Wear of Tapered Roller Bearing

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Abstract

Addressing the scientific problem that the profile modification design of tapered roller bearings primarily focuses on contact stress and fatigue life while neglecting its impact on wear evolution, this paper, based on Hertzian contact theory and the Archard wear theory, and considering centrifugal force, gyroscopic effect, and the complex contact state between rollers and raceways, constructed a comprehensive analysis framework integrating a quasi-static model for profiled rollers and a wear depth calculation model. This framework is novel in that it systematically couples roller profile modification parameters with raceway wear evolution under both pure axial and combined radial–axial loads. The validity and effectiveness of the proposed model were verified by comparing the results of the quasi-static model with load distribution data from existing literature and through measurements conducted on a specially designed bearing wear test platform. The main findings are as follows: (1) When the logarithmic modification parameter f_1 increases from 0.7 μm to 3.6 μm , the maximum wear depth of the inner raceway increases by 133% under pure axial load and 144% under combined load, while that of the outer raceway increases by 142% under pure axial load and expands from 0.1–0.2 μm to 0.23–0.52 μm under combined load. (2) Combined load induces significant asymmetric wear on the outer raceway, and the difference between the two wear peaks increases from 0.13 μm to 0.35 μm as f_1 rises from 0.7 μm to 3.6 μm . (3) The wear peak shifts toward the midpoint of the roller generatrix with increasing modification amount. These results provide important guidance for the wear-oriented optimization design of tapered roller bearings.

Keywords: tapered roller bearing; roller profile modification; quasi-static model; wear model



Received: 31 December 2025

Revised: 28 January 2026

Accepted: 30 January 2026

Published: 2 February 2026

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1. Introduction

Tapered roller bearings act as critical core components that find widespread application in vehicle axle housings and various types of speed reducers. However, experimental studies have shown that the “edge effect” occurring at contact positions exerts a significant impact on the bearing’s service life. Roller profile modification is recognized as an effective technical measure to eliminate or mitigate this edge effect, thereby further enhancing the fatigue life of the bearing [1]. Existing roller profile modification design theories primarily focus on performance indicators such as contact stress distribution without

fully considering the subsequent impact of wear evolution on the bearing's long-term operational stability [2–5]. This paper aims to address this research gap by conducting a systematic study on the influence of roller profile modification on the wear behavior of tapered roller bearings.

Wang [6] performed contact analysis on logarithmically modified tapered rollers, and the stress concentration at the ends of the modified tapered rollers was significantly alleviated. Yang [7] investigated the influence of logarithmic profile modification on the radial stiffness of the contact pairs in tapered roller bearings. Wei [8] studied the symmetrical modification of the rollers in the axle box and the tapered roller bearings of high-speed trains, providing guidance for the optimization design of the modification of such bearings. Li [9] conducted research on the modification move towards to the ends of the roller technology for automotive tapered roller bearings and proposed the optimal modification scheme to improve fatigue life. Chen [10] analyzed the elastohydrodynamic lubrication problem of logarithmically modified tapered rollers, and the research showed that the elastohydrodynamic lubrication state of logarithmically modified rollers was significantly improved. He [11] analyzed the influence of modification on contact deformation and vibration, providing a theoretical basis for the design of roller profile modification. Lundberg [12] studied the dry contact problem of finite-length contacts and proposed logarithmic profile modification formulas for cylindrical and tapered rollers. Reusner [13] from SKF analyzed the contact stress and fatigue life of logarithmic rollers, concluding that logarithmic rollers are superior to straight generatrix rollers under heavy loads and skewing conditions, i.e., they exhibit stronger fatigue resistance. Ma [14] established a numerical calculation method for the elastic contact problem of three-dimensional finite-length line contact pairs using the influence coefficient method in structural analysis, discussed the impact of roller profile modification on the contact stress distribution, pointed out the stress concentration problem under finite-length line contact conditions, and provided a basis for the design of roller profile modification. Houpert [15,16] used the slice method to calculate the contact stress distribution of the non-Hertzian contact pairs between rollers and raceways in tapered roller bearings under skewing conditions. Horng TL et al. [17] analyzed the contact stress distribution of cylindrical rollers with different modifications between two flat plates using the finite element method, indicating that exponential curve modification, similar to logarithmic curve modification, can achieve a superior stress distribution. Wang [18] et al. studied the influence of the modification parameters of rollers and raceways on the thermal characteristics of tapered roller bearings. An excessively large modification coefficient leads to an increase in the temperature of the contact area between rollers; when the modification offsets of the rollers and raceways are close, the maximum temperature of the contact area between the rollers and the large rib decreases.

At present, the optimization design results of tapered roller modification are mainly characterized by indicators such as contact stress distribution, radial stiffness, and bearing fatigue life, and none of them consider the impact of raceway wear. Wear affects the above indicators by changing the contact state, aggravating surface damage, and deteriorating lubrication, ultimately shortening fatigue life and thus significantly changing the operating state of the bearings. As one of the early classic research works on wear, Archard [19] proposed the Archard model applicable to adhesive wear and sliding wear phenomena based on the relationship between wear and load, which has been widely cited in the research on bearing wear by Gu et al. [20]. A coupled wear model and dynamic simulation method were established to systematically analyze the influence of wear of angular contact ball bearings on their dynamic characteristics and reveal the key role of operating parameters such as preload, horizontal load, and geometric parameters on wear evolution and nonlinear response. Liu [21] et al. analyzed the fatigue wear problem of the 32,011 tapered roller

bearing used in a crane, and the results showed that the quality of raw materials and heat treatment process are the main causes of bearing failure. Long Risheng [22] et al. studied the tribological and vibration performance of dimple-textured tapered roller bearings under grease lubrication conditions using a universal friction and wear tester. The test results indicated that the dimple texture on the raceway surface helps reduce the friction force and wear loss of the bearing. Hu [23] et al. carried out experiments on Linear Filament Wear (LFW) and Curved Filament Wear (CFW) of the raceways of Elastic Flexible Thin-Wall Bearings (EFTWB), aiming to investigate the polymorphic wear mechanisms and failure characteristics of the bearing raceways. The results revealed that the EFTWB raceways exhibited two types of failure modes, namely complete rolling friction and sliding filament wear, and the axial and tangential residual stresses of the inner and outer ring raceways were all compressive stresses. Rahnejat and Gohar [24] conducted pioneering research on the contact mechanics of profiled tapered roller bearings, revealing that the contact between the roller and raceways, coupled with the rib contact reaction, induces roller misalignment, leading to non-Hertzian contact pressure distributions that deviate from simple Hertzian analysis. Kabus et al. [25] further developed a quasi-static multi-degree-of-freedom model for tapered roller bearings, accurately capturing non-Hertzian contact pressures caused by contact footprint expansion at the edges due to misalignment. However, both studies focused on contact stress and dynamic equilibrium rather than the subsequent wear evolution induced by profile modification.

Despite extensive research on roller profile modification and bearing wear, existing studies have three key shortcomings: (1) Most modification design theories focus on contact stress, radial stiffness, and fatigue life, neglecting the long-term impact of wear evolution on bearing stability. (2) Few studies systematically couple roller profile modification parameters with wear evolution, especially under combined radial–axial loads. (3) Classical Hertzian analysis is often adopted without fully considering the non-Hertzian contact characteristics caused by roller misalignment, and the coupling effects of centrifugal force, gyroscopic effect, and modification on wear are rarely addressed.

This paper strives to overcome these gaps by constructing a coupled quasi-static-wear model, considering non-Hertzian contact characteristics and combined loads, and systematically investigating the influence of modification parameters on wear.

This study constructed a comprehensive analytical framework integrating a quasi-static model for profiled rollers and a wear depth calculation model. The reliability and effectiveness of the proposed model were verified by comparing the calculation results of the quasi-static model with load distribution data from existing literature and through experimental measurements on a specially designed bearing wear test platform. Based on the established model, the influence of different logarithmic profile modification parameters on the contact stress distribution, sliding velocity, and raceway wear depth of the bearing under both pure axial load and combined radial–axial load was investigated. The conclusions drawn provide important guidance for the design of tapered roller bearings.

In addition, the novelty of this study lies in three aspects: (1) constructing a comprehensive analytical framework that couples a quasi-static model, incorporating centrifugal force and gyroscopic effect with an Archard-based wear model, filling the gap that existing roller profile modification designs neglect the coupling between modification parameters and wear evolution; (2) systematically investigating the influence of logarithmic profile modification on wear under both pure axial and combined radial–axial loads, revealing the non-linear and asymmetric wear mechanisms rarely addressed in prior studies; (3) verifying the model through a custom-built test platform, providing experimental evidence for the quantitative correlation between modification amount and wear depth. This study addresses the knowledge gap that the existing literature focuses primarily on

contact stress and fatigue life rather than wear evolution and lacks systematic analysis of the combined effects of load type and modification parameters on wear.

2. Roller Profile Modification Form and Modification Parameters

In the design of tapered roller bearings, the modification design of rollers consists of two parts: modification shape design and modification amount design. The modification shape refers to the geometric shape of the roller surface perpendicular to the direction of motion, i.e., the shape of the roller generatrix; the modification amount refers to the radial drop of the contour surface at a certain distance from the roller ends. Commonly used roller profile modification forms in engineering include circular arc curve modification, partial modification (i.e., straight lines with circular arcs at both ends, which can intersect or be tangent), logarithmic curve modification, and exponential curve modification. Logarithmic profile modification is widely used in tapered roller design and is generally recognized as the optimal modification form. Therefore, this paper mainly studies the influence of logarithmic profile modification on bearing wear.

To characterize the generatrix profile modification form of tapered rollers, a coordinate system is established as illustrated in Figure 1. This coordinate system serves as a fundamental tool for defining geometric parameters and analyzing modification effects accurately in bearing design.

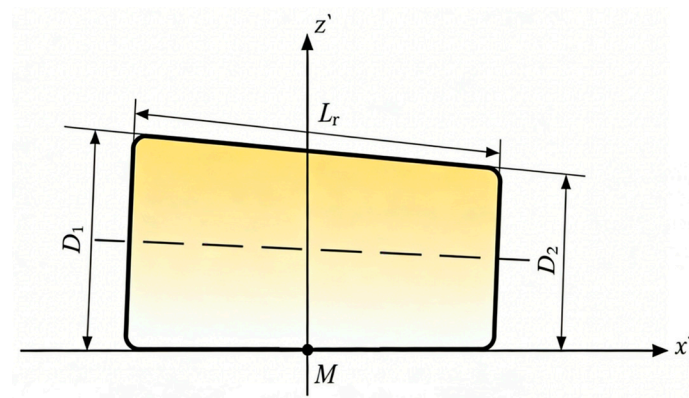


Figure 1. Modified coordinate system of roller profiles.

The generatrix of a logarithmically profiled roller is described by the following equation, which defines the contour shape perpendicular to the direction of motion:

$$z' = f_1 \ln [1 - f_2 (2x' / L_r)^2]^{-1} (|x'| \leq 0.5L_r) \quad (1)$$

where f_1 is the logarithmic profile modification parameter (in units of μm), primarily determining the magnitude of the modification. f_2 is a dimensionless logarithmic profile modification parameter influencing the shape of the curve. This equation provides a mathematical foundation for characterizing the precise geometry of the roller's profile, which is crucial for analyzing its contact behavior and wear performance.

Profile modification parameters refer to [26]:

$$f_1 = 2Q / \pi E' L_r \quad (2)$$

$$f_2 = 1 - 0.3033 \frac{2b}{L_r} \quad (3)$$

where Q represents the total contact load between a single roller and the raceway, integrated load along the contact line. Q is consistent with the contact load definition in Hertzian line contact theory, reflecting the overall load-bearing capacity of the roller–raceway contact pair.

Literature studies [5,27] have shown that under the same load condition, the change in the modification parameter f_2 has a negligible effect on the contact stress along the entire length of the rolling element generatrix and the service life of the tapered roller bearing. Therefore, the influence of the change in f_2 can be ignored; in the subsequent modification design and analysis, f_2 is selected as 0.997, and only f_1 is taken as the core modification design parameter.

3. Geometric Analysis of Tapered Roller Bearings

To establish a precise framework for mechanical and wear analysis, it is essential first to define the key geometric parameters and their interrelationships within a tapered roller bearing. The geometric correlations between components are illustrated in Figure 2. This focus is chosen to optimize the lubrication conditions, heat generation, and wear at the contact interface between the roller's large end and the rib while also considering the need for improved machining accuracy and reduced manufacturing difficulty. The following section provides a brief description of the key geometric parameters involved.

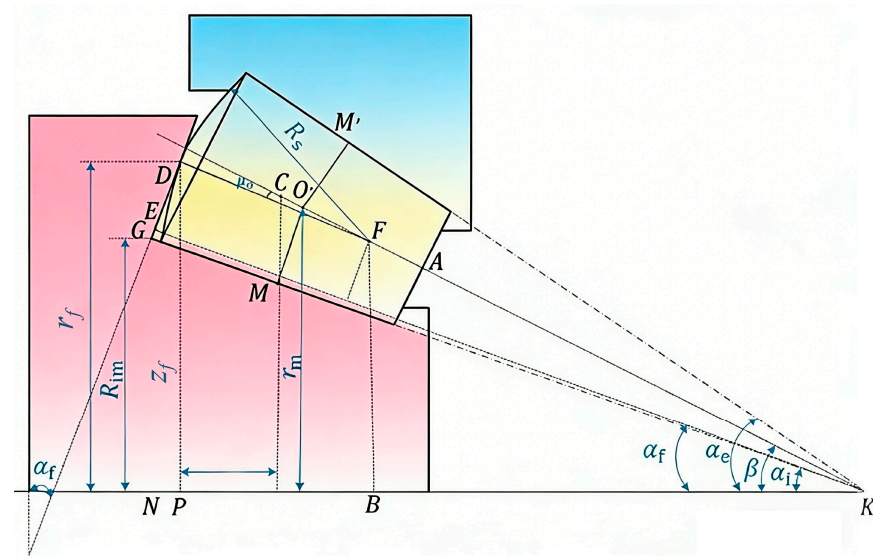


Figure 2. Bearing geometry diagram.

Point K represents the intersection of the raceway generatrix and the bearing axis. A line KE is drawn through point K perpendicular to DG , with its length defined as R_p . The radius of the spherical large end face of the roller is denoted as R_s . Based on geometric relationships, the following parameters are derived:

$$R_p = (R_{im} \tan \alpha_f + R_{im} \cot \alpha_i) \cos \alpha_f \quad (4)$$

The distance DG is calculated as

$$DG = (R_p - R_s) \tan \mu_0 + R_p \tan(\alpha_f - \alpha_i) \quad (5)$$

The radial distance from the contact point D to the bearing axis is

$$r_f = R_{im} + DG \cos \alpha_f \quad (6)$$

The tapered roller can be considered a combination of a conical frustum and a spherical base. Ignoring the spherical base, the effective axial height of the roller, h_r , is determined from its large-end diameter D_1 , small-end diameter D_2 , and the contact generatrix length L_r :

$$h_r = \sqrt{L_r^2 - \left(\frac{D_1 - D_2}{2}\right)^2} \quad (7)$$

The pitch circle radius of the bearing is

$$r_j = (d_i + d_o)/4 \quad (8)$$

The common normal at the contact point between the roller's large end face and the inner ring rib intersects the roller's central axis at point F . The distance l_{FB} is obtained from the geometric relations:

$$l_{FB} = r_m - \left(\sqrt{R_s^2 - \left(\frac{D_1}{2}\right)^2} - \frac{h_r}{2}\right) \sin \beta \quad (9)$$

The horizontal distance from the roller's center of mass, point C , to the contact point D is

$$z_f = R_s \cos(\beta - \mu_o) + \frac{l_{FB}}{\tan \beta} - \frac{r_i}{\tan \alpha_i} \quad (10)$$

4. Quasi-Static Model of Tapered Roller Bearings

To simplify the analytical framework while ensuring the core mechanism is captured, the following assumptions are made: (a) Classical line contact Hertzian analysis is adopted for the roller-raceway contact, with friction neglected in the contact force calculation. (b) The contact surfaces of the roller and raceways are assumed to be smooth, with no surface roughness considered. (c) Thermal effects are ignored in the model, such as temperature-induced material property changes and thermal deformation.

Under external loading, with the outer ring fixed, the inner ring undergoes displacements $\{\delta_{rx}, \delta_{ry}, \delta_{rz}, \beta_{rx}, \beta_{ry}\}$ to equilibrate the external loads $\{F_{rx}, F_{ry}, F_{rz}, M_{rx}, M_{ry}\}$. The loading and displacement state is depicted in Figure 3.

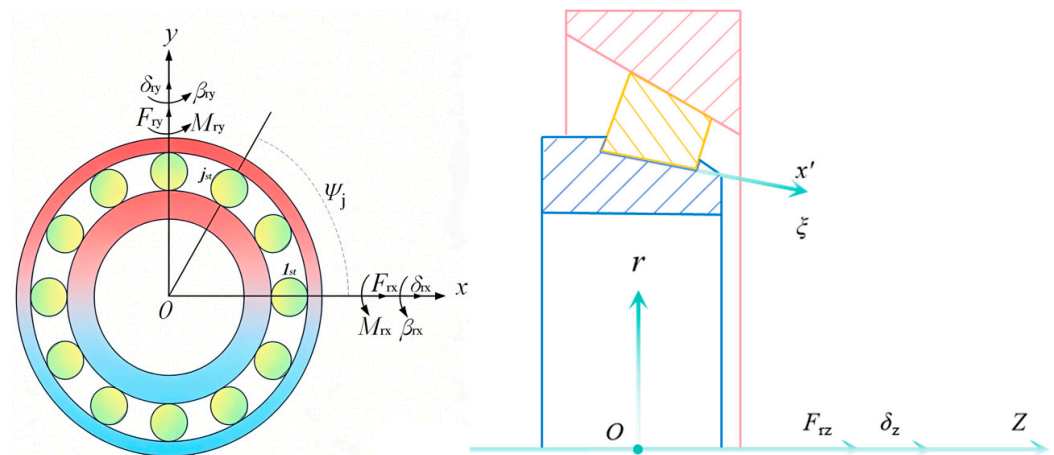


Figure 3. Bearing loads and displacements.

The contact load between the roller and the raceway is non-uniformly distributed along the contact line. For analytical simplification, this distributed load is reduced to an equivalent contact force $Q_{i/e}$ and an equivalent moment $M_{i/e}$ acting at the midpoint M of the roller generatrix. The force analysis for the j th roller, illustrated in Figure 4, includes the normal contact forces from the inner and outer raceways Q_i and Q_e , the contact force with the inner ring rib Q_f , the centrifugal force F_{cd} due to its revolution around the bearing axis, and the gyroscopic moment M_g .

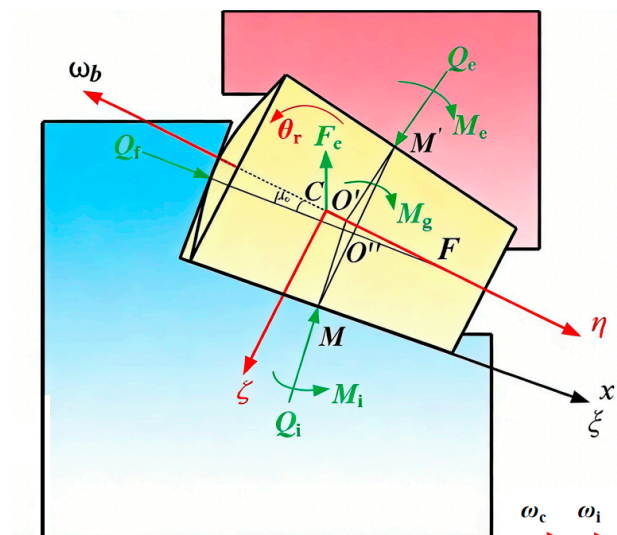


Figure 4. Forces applied on roller.

Under the action of these forces and moments, the roller experiences two translational displacements η , ζ and one rotational displacement θ_r about its center of mass C in the radial plane. To facilitate the calculation of contact deformation, the x' -axis and ζ -axis are established along the contact line between the roller and the inner raceway, with their origin at the generatrix midpoint M . The relationship between the coordinates is $\zeta = x'/L_r$, where L_r is the effective contact length of the roller, and ζ ranges from -0.5 to 0.5 .

The derivation and calculation of the contact deformation between the roller and raceway in this study have been supported by numerous studies [28–30]. This study simplifies the derivation process here and only lists the force and torque formulas used in the subsequent calculation and analysis.

Contact force calculation is based on Hertzian contact theory. For line contact between the roller and the inner/outer rings, the contact force is obtained by integrating the deformation along the contact line:

$$Q_{ij/ej} = K_n \int_{\zeta_1}^{\zeta_2} \delta_{ij\zeta/ej\zeta}^n d\zeta \quad (11)$$

Here, K_n is the contact stiffness coefficient, and for line contact, the exponent $n = 10/9$. The integration limits ζ_1 , ζ_2 are determined by the region where the deformation is greater than zero.

For point contact between the roller and the rib, the contact force is

$$Q_f = K_f \delta_f^n \quad (12)$$

where the exponent $n = 3/2$.

The centrifugal force F_c and the gyroscopic moment M_g of the roller are calculated as

$$F_c = m\omega_c^2 r_m \quad (13)$$

$$M_g = J_b \omega_b \omega_c \sin \beta \quad (14)$$

where m is the roller mass, $m = \rho\pi h_r \left((D_1/2)^2 + (D_2/2)^2 + D_1 D_2/4 \right) / 3$, and J_b is the roller's moment of inertia about its own axis, $J_b = \rho\pi \frac{(D_1/2)^5 - (D_2/2)^5}{10 \tan((\alpha_e - \alpha_i)/2)}$.

Due to the non-uniform load distribution, the load eccentricity $e_{ij/ej}$ is introduced to represent the position of the equivalent point of action on the x' -axis:

$$e_{ij/ej} = \frac{L_r \int_{\xi_1}^{\xi_2} \xi \{V_{ij/ej}(\psi_j) + \Delta V_{ij/ej}(\psi_j, \xi)\}^n d\xi}{\int_{\xi_1}^{\xi_2} \{V_{ij/ej}(\psi_j) + \Delta V_{ij/ej}(\psi_j, \xi)\}^n d\xi} \quad (15)$$

The static equilibrium equations for each roller consist of three force equilibrium equations and three moment equilibrium equations, forming a system of nonlinear equations:

$$\begin{cases} -(Q_{ij} + Q_{ej}) \sin \gamma - F_c \sin \beta + Q_f \cos \mu_o = 0 \\ (Q_{ej} - Q_{ij}) \cos \gamma - F_c \cos \beta - Q_f \sin \mu_o = 0 \\ -F_c CO' \cos \beta + Q_f FO' \sin \mu_o + Q_{ij} e_{ij} - Q_{ej} e_{ej} - M_g = 0 \end{cases} \quad (16)$$

Finally, the forces and moments exerted by all rollers on the inner ring must balance the externally applied loads, constituting the equilibrium equations for the entire bearing system:

$$\begin{cases} F_x = \sum_{j=1}^n (Q_{ij} \cos \alpha_i - Q_f \sin(\beta - \mu_o)) \cos \psi_j \\ F_y = \sum_{j=1}^n (Q_{ij} \cos \alpha_i - Q_f \sin(\beta - \mu_o)) \sin \psi_j \\ F_z = \sum_{j=1}^n Q_{ij} \sin \alpha_i + Q_f \cos(\beta - \mu_o) \\ M_x = \sum_{j=1}^n (Q_{ij}(r_i \sin \alpha_i - e_{ij}) - Q_f \sin(\beta - \mu_o) z_f + Q_f \cos(\beta - \mu_o) r_f) \sin \psi_j \\ M_y = -\sum_{j=1}^n (Q_{ij}(r_i \sin \alpha_i - e_{ij}) + Q_f \sin(\beta - \mu_o) z_f - Q_f \cos(\beta - \mu_o) r_f) \cos \psi_j \end{cases} \quad (17)$$

5. Raceway Wear Depth Model of Roller Bearings

Wear is a critical factor affecting bearing life and reliability [31]. In tapered roller bearings, wear primarily occurs in two regions: the conical surfaces of the inner and outer raceways (contact area with the roller generatrix) and the ends of the roller (near the large/small end faces), where micro-sliding and stress concentration are prominent. Addressing the predominant sliding wear mechanism in tapered roller bearings, based on the Archard model that has been widely verified and recognized in the industry, this study describes the relevant behaviors of the local contact area as the following formula [32]:

Wear model acts as the theoretical foundation. This model establishes the fundamental relationship between the local wear volume dV in the contact area, the local contact load dQ , and the sliding distance dL :

$$dV = k \cdot dQ \cdot dL \quad (18)$$

where dV is the local wear volume, dQ is the local contact load, dL is the local sliding distance, k is the wear coefficient with units of MPa^{-1} , and its value depends on factors such as material pairing and lubrication conditions.

Although the Archard model was initially proposed for sliding friction, it is applicable to tapered roller bearings because pure rolling does not exist in practical rolling bearings—the logarithmic profile modification, centrifugal force, and gyroscopic effect cause micro-sliding between the roller and raceway. This micro-sliding leads to adhesive and abrasive wear, which aligns with the wear mechanism described by the Archard model.

For tapered rollers, the sliding distance dL varies along the generatrix (different for the large and small ends). In Equation (18), dL refers to the instantaneous sliding distance of the local contact point on the roller generatrix. During calculation, we discretize the roller

into N elements, and dL of each element is calculated based on its sliding velocity v and time increment dt .

To translate wear volume into the more intuitive wear depth, both sides of the equation are divided by the contact area dS . Letting $dh = dV/dS$ represent the infinitesimal wear depth and $p = dQ/dS$ represent the local contact stress, the equation simplifies to

$$dh = k \cdot p \cdot dL \quad (19)$$

Furthermore, by differentiating this relationship with respect to time dt , the wear depth rate per unit time at any point on the contact interface is obtained:

$$\dot{h} = \frac{dh}{dt} = kp \frac{dL}{dt} = k \cdot p \cdot v \quad (20)$$

Here, $\dot{h} = kp v$ is the core equation for calculating time-varying wear.

To accurately compute the wear depth at different positions along the raceway generatrix, the roller is discretized into rectangular elements along the generatrix direction, as shown in Figure 5. Each element can be approximated as an independent contact area, allowing the application of the Archard model for local wear calculation [33].

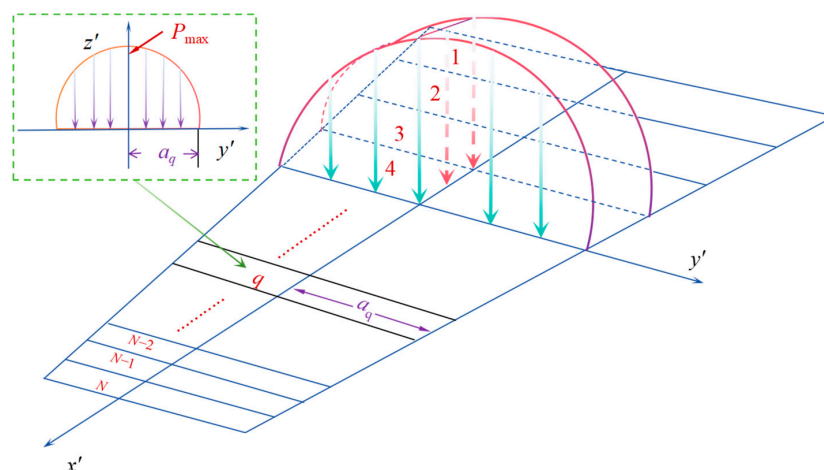


Figure 5. Stress distribution in contact area.

(1) Contact Stress Calculation

According to Hertzian contact theory, the normal contact load Q_{ijq} on the q th rectangular element within the contact area between the j th roller and the raceway is calculated as

$$Q_{ijq} = K_n \delta_{ijq/eq}^n \Delta l / L_r \quad (21)$$

where $\Delta l = L_r / N$ is the element length, and the exponent $n = 10/9$.

The contact half-width $a_{ijq/eq}$ of this rectangular element is

$$a_{ijq/eq} = \sqrt{\frac{D_q \delta_{ijq/eq}}{\cos \gamma}} \quad (22)$$

Consequently, the maximum contact stress at the geometric center of the element is

$$p_{\max, ijq/eq} = \frac{2Q_{ijq/eq}}{\pi \Delta l a_{ijq/eq}} \quad (23)$$

The contact stress at any point (x', y') within the element follows a semi-elliptical distribution:

$$p_{ijq/eq}(x'_{i/e}, y'_{i/e}) = p_{\max, ijq/eq} \sqrt{1 - \left(\frac{y'}{a_{ijq/eq}} \right)^2} \quad (24)$$

(2) Sliding Velocity Calculation

During the operation of the bearing, as shown in Figure 6, taking the bearing cage as the reference coordinate system, the inner ring rotates around the bearing axis with an angular velocity of $\omega_i - \omega_c$, where ω_i is the rotational angular velocity of the inner ring, and ω_c is the orbital angular velocity of the roller (rotational angular velocity of the cage); the roller rotates around its own axis with a rotational angular velocity of ω_b . Roller profile modification reduces the radius of the roller, resulting in a certain difference between the linear velocity of the roller and the linear velocity of the inner and outer rings of the bearing at contact points other than the pure rolling point, leading to sliding.

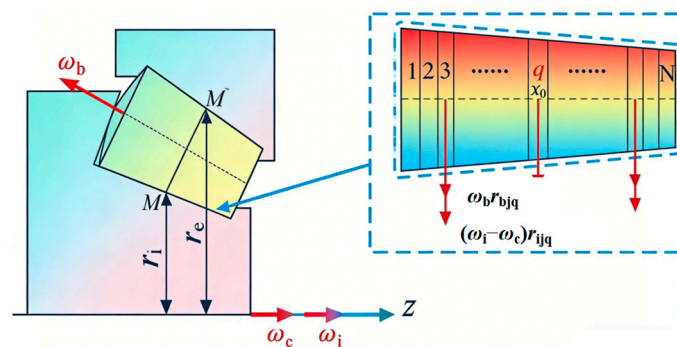


Figure 6. Sliding speed diagram at roller and raceway contact.

The analysis of sliding velocity is based on the kinematics of the bearing, as shown in Figure 6. Considering the variation in roller radius due to profiling, relative sliding occurs between the roller and the inner/outer rings at points other than the pure rolling point.

The roller radius r_{bjq} at the q th element, accounting for the profile modification amount z' , is

$$r_{bjq} = \frac{\xi(D_2 - D_1) + (D_1 + D_2) - 2z'}{2} \quad (25)$$

The distances from the inner and outer raceway centers M and M' to the bearing axis are, respectively,

$$r_i = r_m - \frac{(D_1 + D_2) \cos \beta}{4} \quad (26)$$

$$r_e = r_m + \frac{(D_1 + D_2) \cos \beta}{4} \quad (27)$$

The radius of the inner and outer raceways at the contact point ξ are

$$r_{ijq} = r_i - L_r \xi \sin \alpha_i \quad (28)$$

$$r_{eq} = r_e - L_r \xi \sin \alpha_e \quad (29)$$

Assuming the midpoint M of the roller generatrix is the pure rolling point, the roller's orbital angular velocity ω_e and rotational angular velocity ω_b can be derived:

$$\omega_e = \frac{\omega_i r_i}{2r_m} \quad (30)$$

$$\omega_b = \frac{4(\omega_i - \omega_c)r_i}{(D_1 + D_2)} \quad (31)$$

Finally, the differential sliding velocities at the q th element between the roller and the inner/outer rings are

$$v_{ijq} = (\omega_i - \omega_e)r_{ijq} - \omega_b r_{bjq} \quad (32)$$

$$v_{ejq} = \omega_c r_{ejq} - \omega_b r_{bjq} \quad (33)$$

Wear Depth Calculation is performed by integrating over each discrete element. Substituting the contact stress $p_{ijq}(y')$ and sliding velocity V_{ijq} into the Archard wear rate formula and integrating over the area A_q of the element yields the wear volume per unit time $\bar{V}_{ijq/ejq}$ for that element:

$$\bar{V}_{ijq/ejq} = \int \int_{A_q} k p_{ijq/ejq}(x'_{i/e}, y'_{i/e}) v_{ijq/ejq}(x'_{i/e}, y'_{i/e}) dx'_{i/e} dy'_{i/e} \quad (34)$$

Considering the different loading modes of the inner and outer raceways:

Outer Raceway: As a stationary component, the wear at any point is caused only by the rollers passing over that location. Therefore, the wear depth h_{ejq} of the q th element at the angular position of the j th roller on the outer raceway is

$$h_{ejq} = \frac{Z T \bar{V}_{ejq}}{S_{ejq}} \quad (35)$$

where $S_{ejq} = 2\pi r_{ejq} \Delta l$. Because the load varies with the azimuth angle ψ_j , the wear depth on the outer raceway is non-uniformly distributed.

Inner Raceway: As a rotating component, any point on it periodically contacts all rollers. Therefore, the wear depth h_{ijq} of the q th element on the inner raceway is the accumulation of wear from all Z rollers and is independent of the roller azimuth angle (uniform wear):

$$h_{ijq} = \frac{\sum_{j=1}^Z \bar{V}_{ijq} T}{S_{ijq}} \quad (36)$$

where $S_{ijq} = 2\pi r_{ijq} \Delta l$ is the surface area of the inner raceway corresponding to this element.

6. Results and Discussion

6.1. Model Validity Verification

To assess the accuracy of the established quasi-static model for tapered roller bearings, which incorporates centrifugal forces and gyroscopic effects, along with its computational program, a comparative verification was conducted using the example from reference [34,35]. In this benchmark case, the bearing is subjected to an axial load $F_z = 4500$ N and a radial load $F_x = 4500$ N. Figure 7 illustrates the influence of rotational speed variation on the contact forces between the rollers and the inner/outer raceways, as well as between the rollers and the flange rib.

The comparison reveals that for the vast majority of rollers, the contact forces calculated by the proposed model for the roller-inner raceway, roller-outer raceway, and roller-flange interactions show good agreement with the results from the reference. For a small number of rollers near an azimuth angle of 180° , a certain discrepancy exists between the results of this paper and the reference, but the maximum deviation does not exceed 20%. Figure 8 further analyzes the impact of axial load variation on the aforementioned contact forces.

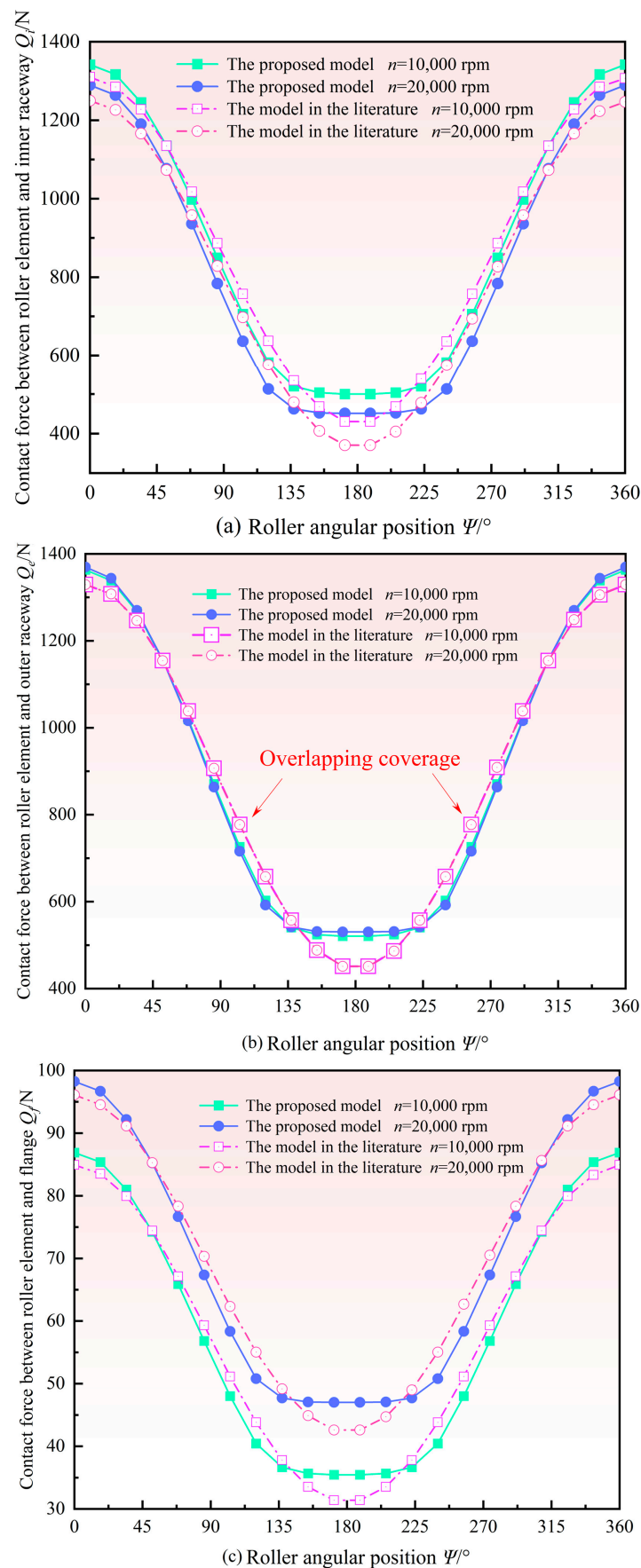


Figure 7. The effect of rotational speed on the contact force between roller and rings. (a) Contact force between roller and inner raceway. (b) Contact force between roller and outer raceway. (c) Contact force between roller and flange.

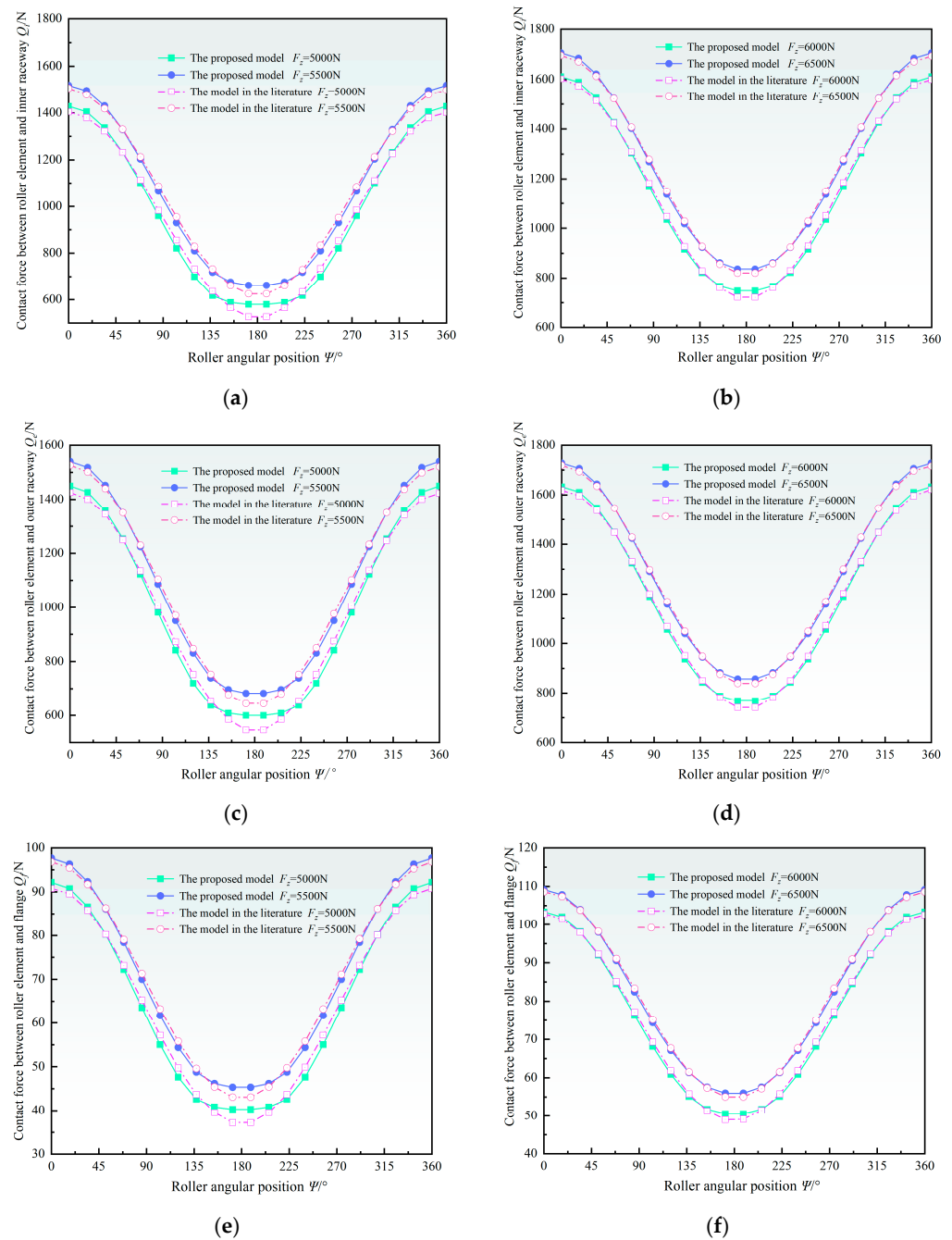


Figure 8. The effect of axial load on the contact force between roller and rings. (a) Contact force between roller and inner raceway. (b) Contact force between roller and inner raceway. (c) Contact force between roller and outer raceway. (d) Contact force between roller and outer raceway. (e) Contact force between roller and flange. (f) Contact force between roller and flange.

The comparative results demonstrate that under different axial loads, the contact forces calculated in this study are in excellent agreement with the reference results, with minimal differences. These comparisons adequately validate the reliability and effectiveness of the proposed tapered roller bearing model and computational program, confirming its suitability for accurate bearing load distribution analysis.

Regarding the direct experimental validation of the wear depth model, it is recognized that bearing wear is a slow evolutionary process, and obtaining sufficient and precise measured data poses significant challenges. However, the authors have previously successfully derived and established a wear model applicable to ball bearings based on Archard's wear

theory. This model has been utilized to study the influence of bearing ring installation angular misalignment on wear dispersion, and the relevant findings have been published [36]. The wear depth model developed in this paper for tapered roller bearings follows a similar analytical approach and derivation methodology as the prior work. Therefore, it is reasonable to consider the validity of this model credible.

6.2. Bearing Model

The key structural parameters of the bearing analyzed in this study are listed in Table 1. The roller profile modification adopts a logarithmic curve form. The core modification parameter f_1 is set to values of 0.7 μm , 1.5 μm , 2.2 μm , 2.8 μm , and 3.6 μm , respectively. Figure 9 visually presents the specific generatrix profiles of the roller under these different f_1 parameters.

Table 1. Tapered roller bearing parameters.

Num.	Name of Bearing Parameter	Parameter Symbol	Parameter Value
1	Inner diameter of bearing (mm)	d	50
2	Outer diameter of bearing (mm)	D	110
3	Average diameter of inner raceway (mm)	d_i	65.4
4	Average diameter of outer raceway (mm)	d_o	94.3
5	Half cone angle of inner raceway ($^\circ$)	α_i	8.9528
6	Half cone angle of outer raceway ($^\circ$)	α_e	12.9528
7	Half cone angle of roller ($^\circ$)	γ	2°
8	Number of rollers	Z	16
9	Diameter of large end of roller (mm)	D_1	15.7060
10	Diameter of small end of roller (mm)	D_2	13.6470
11	Contact angle of large rib of inner raceway ($^\circ$)	α_f	9.4528
12	Density of roller (g/mm^3)	ρ	0.0078
13	Mass of roller (kg)	m	0.0389
14	Arc radius of large end of roller (mm)	R_s	225.018
15	Elastic modulus (pa)	E	2.07×10^{11}
16	Poisson's ratio	ν	0.3
17	Effective length of roller (mm)	L_r	29.5

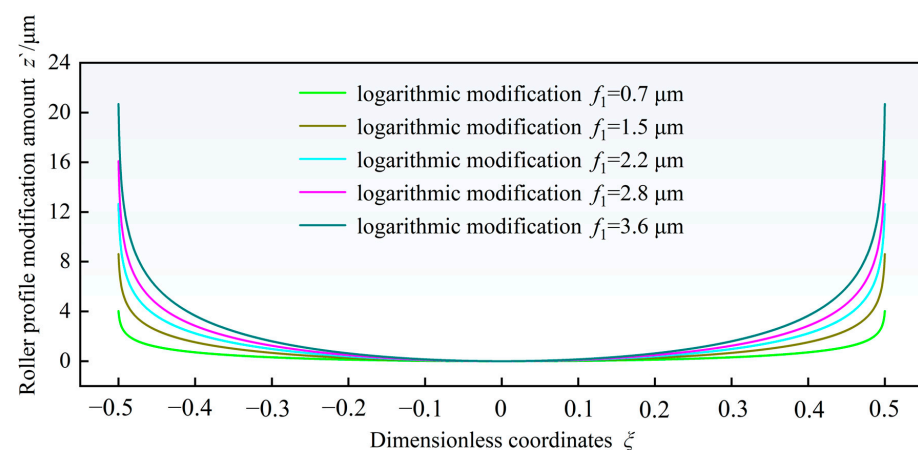


Figure 9. Roller generatrix at different roller profile modification coefficients.

This research focuses on investigating the influence of different roller profiling parameters on bearing wear behavior under two loading conditions:

- ① Pure Axial Load Condition: Axial force is 2600 N.
- ② Combined Radial–Axial Load Condition: Axial force is 2400 N, and radial force is 1000 N.

The key structural parameters of the bearing analyzed in this study correspond to a standard medium-sized tapered roller bearing that is widely used in practical engineering, particularly in medium-light duty vehicle axle housings and industrial speed reducers. This bearing size was selected for three main reasons: (1) Its structural parameters are representative of mainstream tapered roller bearing designs, ensuring that the research findings can reflect the general rules of logarithmic profile modification on bearing wear. (2) It is readily available in the market and easy to process customized modified rollers, which facilitates the construction of the test platform and the implementation of experimental validation. (3) Relevant load distribution data for this size of bearing have been reported in the existing literature, providing a reliable benchmark for model validation.

Regarding the generalizability to larger-sized tapered roller bearings, the proposed analytical framework, integrating a quasi-static model and Archard-based wear model, is fundamentally universal. This is because the model is derived based on Hertzian contact theory, rigid-body kinematics, and Archard wear theory—all of which are general mechanical and tribological principles independent of specific bearing dimensions.

According to the definition method for bearing equivalent load, the equivalent load values for these two conditions are equal, both being 0.026 Cr, which falls within the light load range. The limiting speed of the bearing is 5300 r/min, while the operational speed in this study is set to 4500 r/min, meaning the bearing operates under light-load and high-speed conditions. Assuming the bearing operates two shifts per day, 8 h per shift, with an expected service life of 2 years, the total wear duration amounts to 12,000 h. Considering starved lubrication conditions, the bearing operates in the boundary lubrication regime. Referring to the work of Liu [34], the dimensionless wear coefficient in the wear model is $K = 1.77 \times 10^{-8}$. Given the yield strength of the bearing steel $\sigma_s = 1667$ MPa, the calculated wear coefficient is $k = K / (3\sigma_s) = 3.5e^{-12}$ Mpa $^{-1}$.

6.3. Influence of Roller Profile Modification on Contact Force and Contact Stress

Figures 10 and 11 display the distribution of contact forces between the rollers and the inner/outer raceways under pure axial load and combined radial–axial load, respectively.

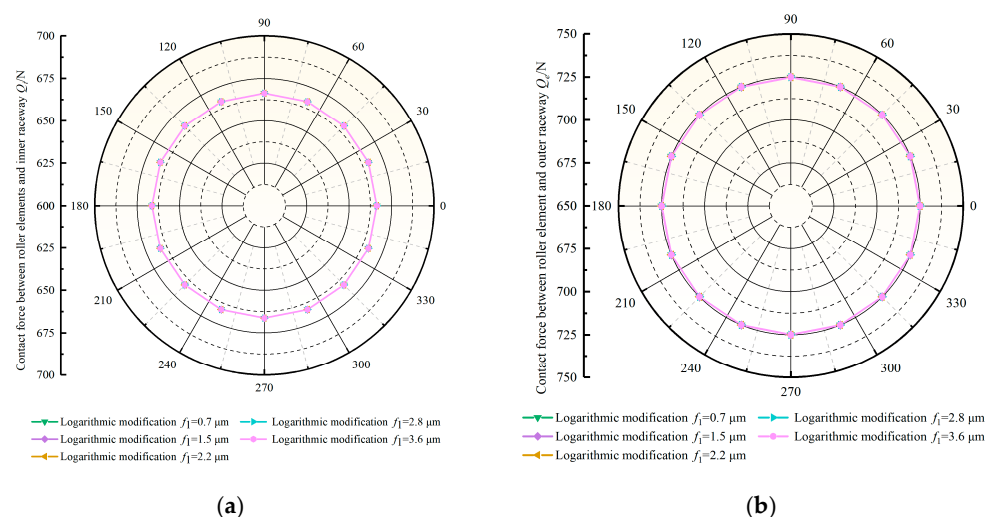


Figure 10. Contact force between rollers and raceway under axial load. (a) Contact force between rollers and inner raceway. (b) Contact force between roller and outer raceway.

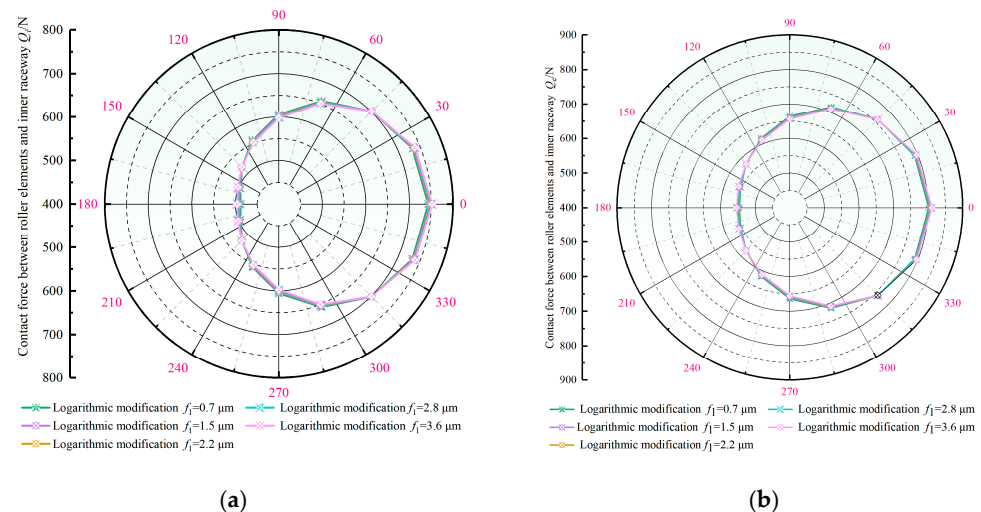


Figure 11. Contact force between rollers and raceway under combined loading. (a) Contact force between rollers and inner raceway. (b) Contact force between roller and outer raceway.

Analysis of Figure 10a,b indicates that under pure axial load, the contact forces between the rollers and the inner/outer raceways are 665 N and 725 N, respectively. These forces remain constant regardless of the roller azimuth angle, showing a uniform distribution. In contrast, under the combined load action (Figure 11a,b), the contact forces exhibit significant periodic fluctuation. The variation ranges for the contact forces between the rollers and the inner/outer raceways are 500 N–750 N and 550 N–800 N, respectively. It is noteworthy that although the pure axial load and the combined load have equivalent values, their contact force distribution characteristics differ markedly. A key observation is that at a speed of 4500 r/min, the centrifugal force generated by a single roller is approximately 58 N, and the difference between the contact forces on the inner and outer raceways is close to this centrifugal force value. The contact force under pure axial load is closer to the maximum value of the fluctuation range observed under combined load.

Crucially, the analysis results demonstrate that the roller profile modification form and amount have a negligible effect on the magnitude of the contact forces between the rollers and the raceways. The contact forces are primarily determined by the type and magnitude of the external loads.

However, roller profiling exerts a decisive influence on the contact stress distribution. Figure 12 shows the contact stress distribution along the contact line direction for different modification coefficients f_1 .

When the bearing is subjected solely to axial load (Figure 12a,b), the contact stress is approximately symmetrically distributed about the midpoint of the roller generatrix, exhibiting only slight skewness due to centrifugal forces, raceway constraints, and rib contact. However, when the bearing is under combined load (Figure 12c,d), pronounced asymmetric skewed distribution characteristics are observed. This is mainly attributed to the radial load component causing an angular displacement (tilting) of the inner ring, leading to a relative rotation angle between the inner ring and the rollers. Consequently, the compressive deformations on either side of the roller generatrix midpoint differ.

An increase in the modification coefficient f_1 significantly accentuates this skewed distribution characteristic. This is because a larger f_1 results in a greater profile modification amount at the roller ends, a shorter effective contact length along the generatrix direction between the roller and the raceways, and a weaker constraining effect of the raceways on the roller, thereby increasing the relative rotation angle. Quantitatively, for $f_1 = 2.2 \mu\text{m}$, the peak contact stress on the highly loaded side exceeds that on the opposite side by approximately 28%. This asymmetric pressure distribution is a critical factor predisposing

the bearing to non-uniform wear and potential premature failure in the heavily loaded region [37]. Furthermore, as f_1 increases, the contact stress distribution becomes more concentrated, and the stress peak rises significantly. This occurs because the increased modification amount reduces the roller's actual load-bearing area.

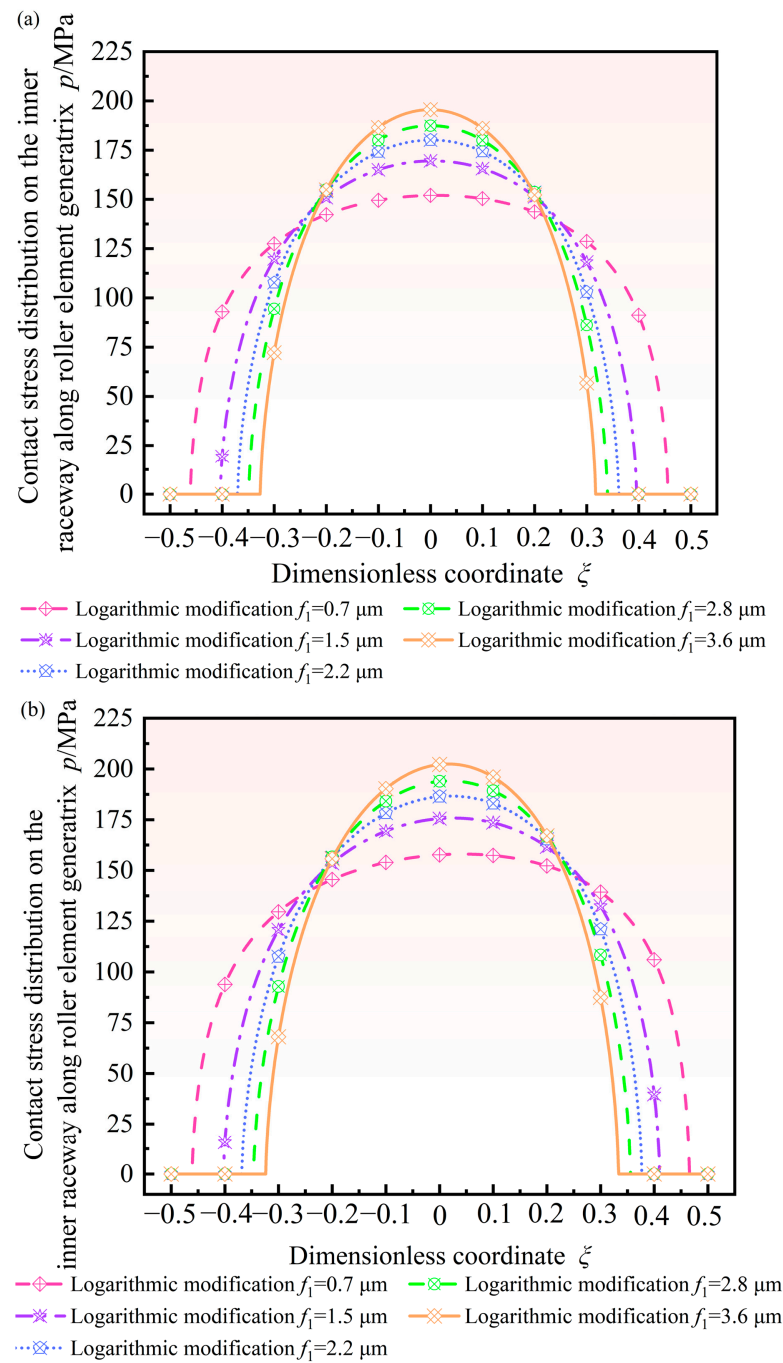


Figure 12. Cont.

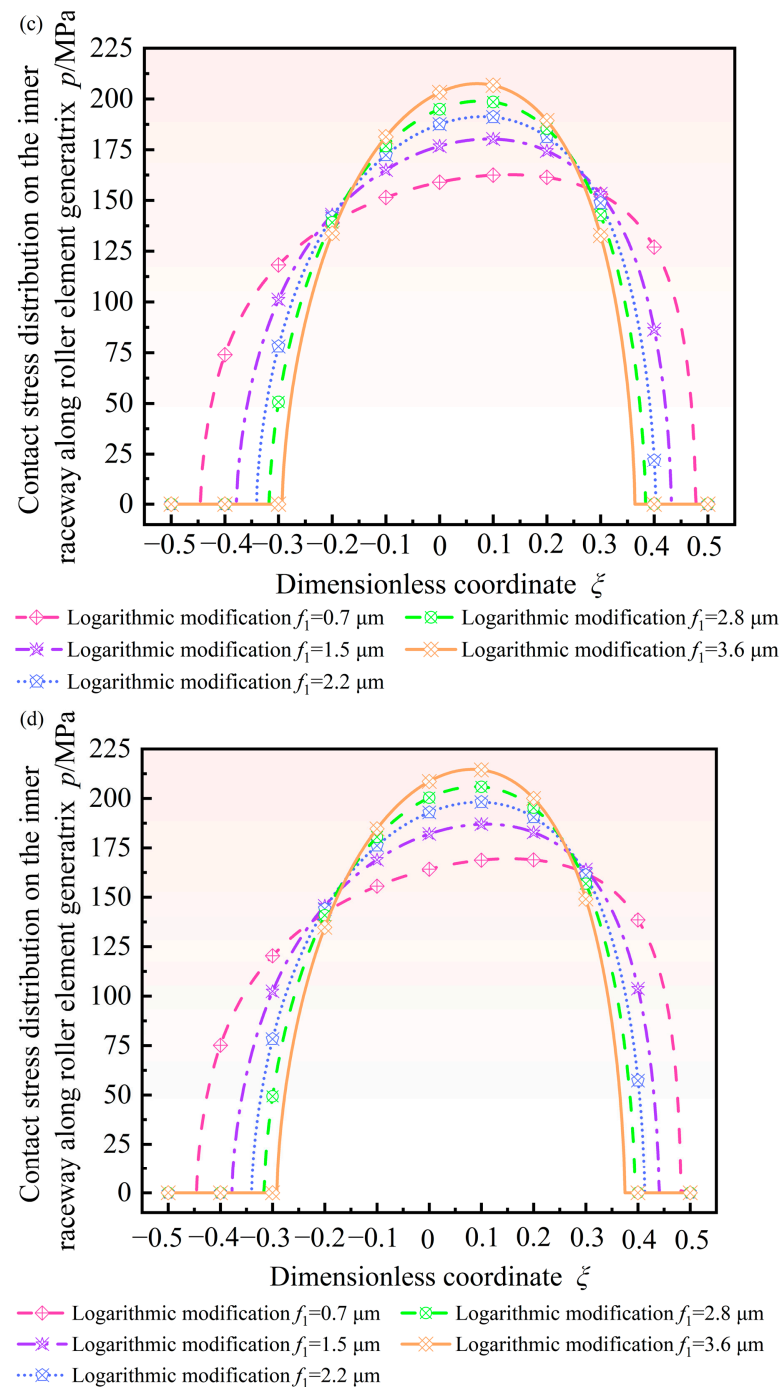


Figure 12. Contact stress on the contact line between rollers and raceway under axial load and combined loading for different f_1 . (a) Contact stress distribution on the inner raceway along roller generatrix under axial load. (b) Contact stress distribution on the outer raceway along roller generatrix under axial load. (c) Contact stress distribution on the inner raceway along roller generatrix under combined load. (d) Contact stress distribution on the outer raceway along roller generatrix under combined load.

6.4. Influence of Roller Profile Modification on Sliding Velocity

The distribution of sliding velocity along the contact lines between the rollers and the inner/outer raceways is illustrated in Figure 13a,b. In the calculations performed for this study, the influence of contact deformation was neglected in the sliding velocity analysis, as it is considered minor compared to the geometric dimensions of the rollers and raceways. The magnitude of the sliding velocity primarily depends on the geometric parameters of

the raceways and rollers, as well as the roller profile modification amount. Consequently, differences in the load applied to the rollers do not alter the fundamental distribution pattern of the sliding velocity.

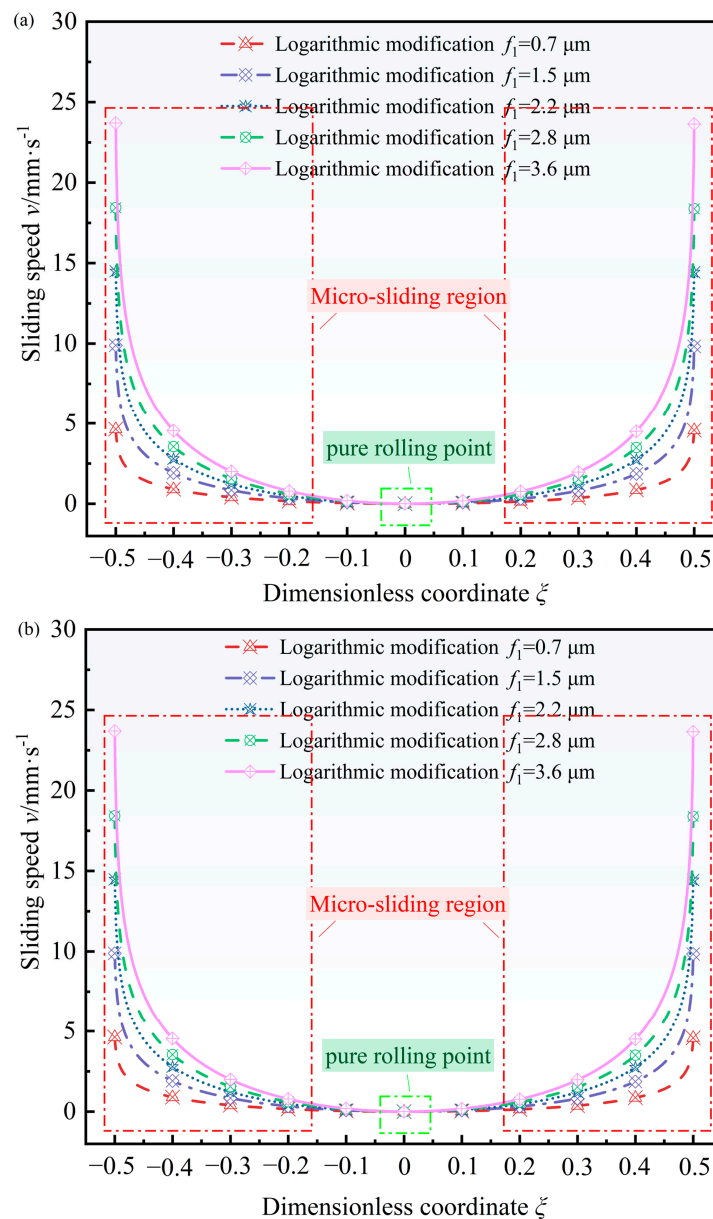


Figure 13. Sliding velocity between rollers and raceway for different f_1 . (a) Sliding speed between roller and inner raceway. (b) Sliding speed between roller and outer raceway.

Analysis of Figure 13a,b reveals that the distribution pattern of the sliding velocity is highly correlated with the roller's profile contour and exhibits symmetry about the midpoint of the roller generatrix. An increase in the modification parameter f_1 directly leads to an elevation in the sliding velocity. It is particularly noteworthy that the sliding velocity values are relatively large in the regions near the roller ends. When the modification coefficient f_1 reaches 3.6 μm , the maximum sliding velocity approaches 24 $\text{mm} \cdot \text{s}^{-1}$, indicating the potential for significant wear in this area. However, referring back to the contact stress distribution results in Figure 12a–d, it is observed that the contact stress is effectively zero at the theoretical boundaries of the roller. This implies that wear will not occur at the roller edges [38,39]. Sliding occurs primarily near the roller ends and regions with geometric

mismatch caused by profile modification. The midpoint of the roller generatrix is the pure rolling point.

6.5. Influence of Roller Profile Modification on Bearing Raceway Wear

The preceding analysis indicates that both the contact stress and sliding velocity between the roller and the raceway vary along the generatrix. This inevitably leads to differences in the wear depth at various points within the contact area on the tapered roller bearing raceways. Figure 14a–d present the maximum wear depth in the contact area between the rollers and the inner/outer raceways at different azimuth angles under pure axial load and combined radial–axial load, respectively.

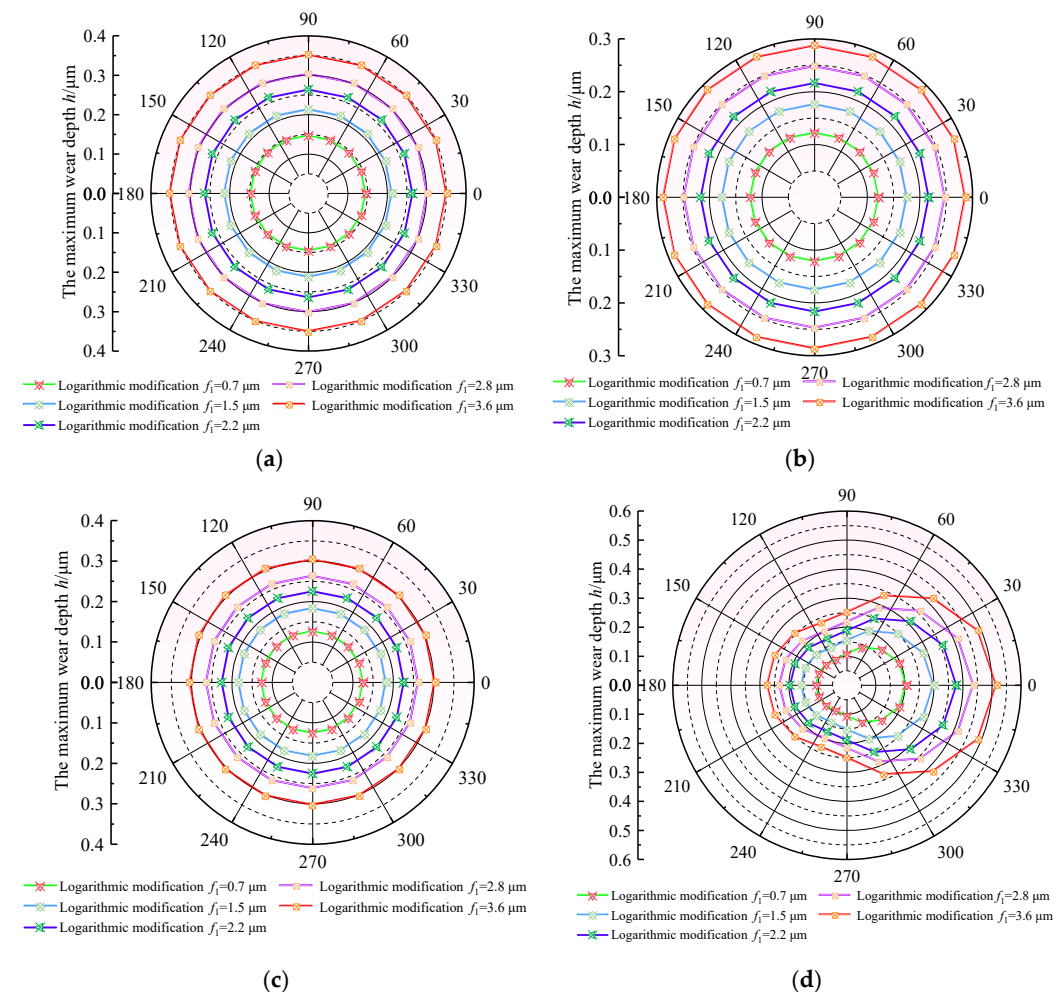


Figure 14. Maximum wear depth at different positions of the raceway under different f_1 . (a) The maximum wear depth at different positions of the inner raceway under axial load. (b) The maximum wear depth at different positions of the outer raceway under axial load. (c) The maximum wear depth at different positions of the inner raceway under combined load. (d) The maximum wear depth at different positions of the outer raceway under combined load.

For the inner raceway, the maximum wear depth remains consistent across different azimuth angles, regardless of the loading condition. This uniformity arises because the rotation of the inner ring subjects its entire circumference to an identical load history, resulting in even wear. In contrast, the wear depth on the outer raceway is strongly dependent on the angular position of the roller. Under pure axial load, the wear depth is the same at all points on the outer raceway. However, under combined load, the significantly different contact load distributions at various position angles, as shown in Figure 11a,b,

lead to distinct contact stress distributions, causing the maximum wear depth on the outer raceway to exhibit considerable variation with the roller's azimuth angle.

A clear trend is observed: the wear depth of both the inner and outer raceways increases monotonically with the increase in the roller profile modification amount f_1 , irrespective of the load type.

Specifically, for the inner raceway, under axial load, when f_1 increases from 0.7 μm to 3.6 μm , the wear depth rises from 0.15 μm to 0.35 μm , an increase of 133%. Under combined load, it increases from 0.125 μm to 0.305 μm , an increase of 144%. For the outer raceway, under axial load, the wear depth increases from 0.12 μm to 0.29 μm (a 142% increase). Under combined load, due to non-uniform wear, the range of wear depth distribution expands from 0.1 to 0.2 μm to 0.23–0.52 μm as f_1 increases.

This data fully demonstrates that the degree of modification is a critical factor affecting bearing wear depth. The fundamental reason lies in the significant alteration of the contact stress distribution between the roller and the raceway caused by profiling, as shown in Figure 12a–d.

Comparing the total wear depth under the two loading conditions reveals that even with equivalent loads, combined load induces significant non-uniformity in the wear depth of the bearing raceways. For instance, when $f_1 = 0.7 \mu\text{m}$, the variation range of the total wear depth under combined load (0.225–0.325 μm) is distributed around the value under pure axial load (0.27 μm). However, when $f_1 = 3.6 \mu\text{m}$, the total wear depth under pure axial load (0.64 μm) is close to the lower limit of the range under combined load (0.535–0.825 μm). This suggests that as the modification amount increases, combined load tends to aggravate bearing wear [26,40].

A deeper analysis of the data in Figure 14a–d shows that the growth of wear depth is not simply linear with f_1 but exhibits an accelerating growth characteristic. For example, when f_1 increases from 0.7 μm to 2.2 μm (a 214% increase), the wear depth of the inner raceway increases by about 47%. Yet, a further increase in f_1 from 2.2 μm to 3.6 μm (a 64% increase) results in an additional 40% growth in wear depth. This non-linear behavior can be explained by the Archard wear model (Equation (29)): wear depth is the product of contact stress p and sliding velocity v . As shown in Figures 12 and 13, larger modification coefficients lead to a reduced contact area and exacerbated geometric mismatch, causing both p and v to increase super-linearly. Ultimately, this results in the wear depth increasing at a faster rate than the increase in f_1 itself, highlighting the trade-off between the benefit of profiling in mitigating edge stress and its detrimental effect on wear resistance [41].

Figure 15a–d provide further details of the wear depth distribution along the roller generatrix. For the combined load, only the distribution at the position of the most heavily loaded roller is shown. Although the peak contact stress is located near the generatrix midpoint, as shown in Figure 12, the wear amount in this vicinity is not the highest due to the relatively small sliding velocity there as shown in Figure 13. The maximum wear depth occurs at the position where the product of contact stress and sliding velocity is greatest. The figures show that the wear depth first increases and then decreases from the generatrix midpoint towards the ends. As the modification coefficient f_1 decreases, the wear peak value decreases, and the location of the peak shifts towards the roller ends. This implies that a smaller modification amount yields a smaller maximum wear depth, but the area of concentrated wear moves closer to the ends.

The shifting of the wear peak location has important implications for bearing life prediction. A wear peak near the midpoint might seem less critical initially, but combined with the higher sliding velocity in that region, it could lead to rapid alteration of the roller profile, potentially inducing new stress concentration points. Conversely, a wear peak near the ends, if accompanied by an optimized profile design that ensures more uniform pressure distribution, might be more manageable. Furthermore, the significant asymmetry of the wear depth under combined load implies that the bearing clearance will evolve asymmetrically. This could define a functional service life shorter than that predicted by traditional fatigue models due to unacceptable vibration and noise occurring well before the average wear depth becomes critical.

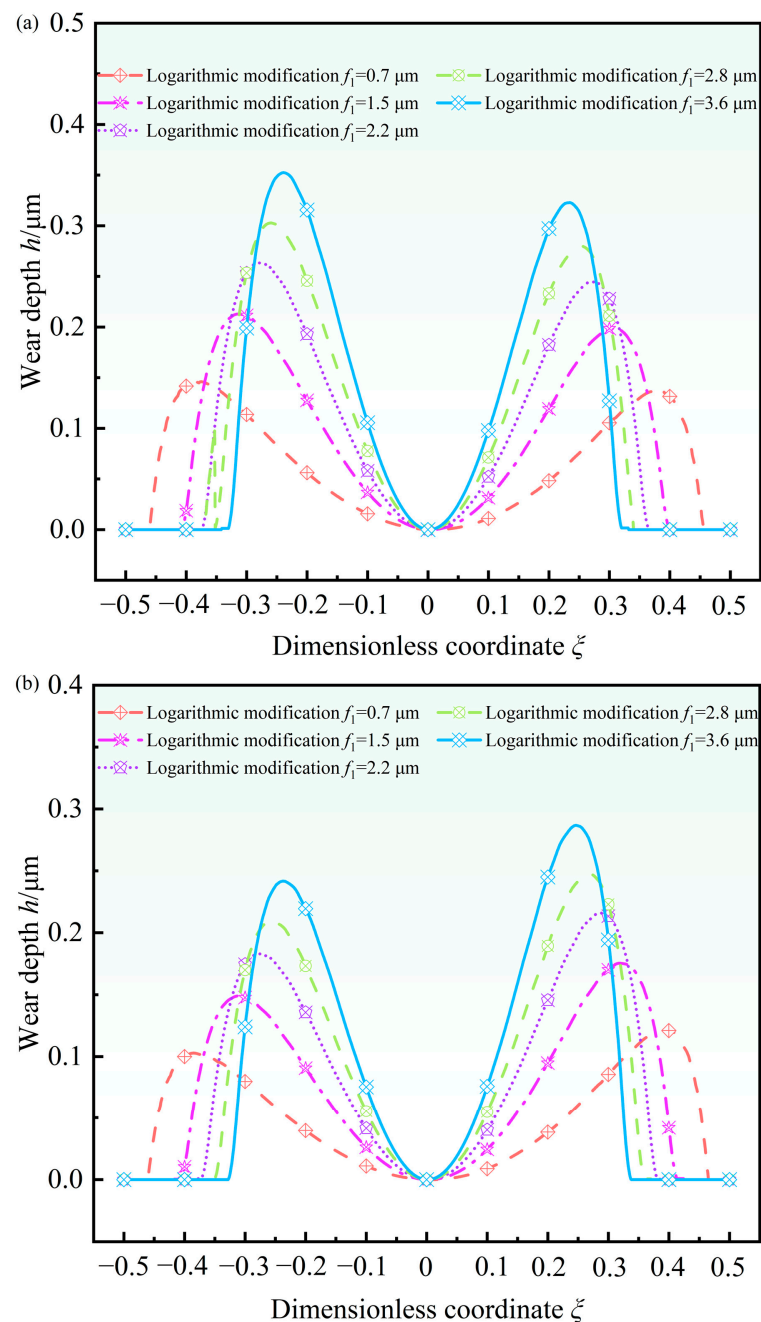


Figure 15. Cont.

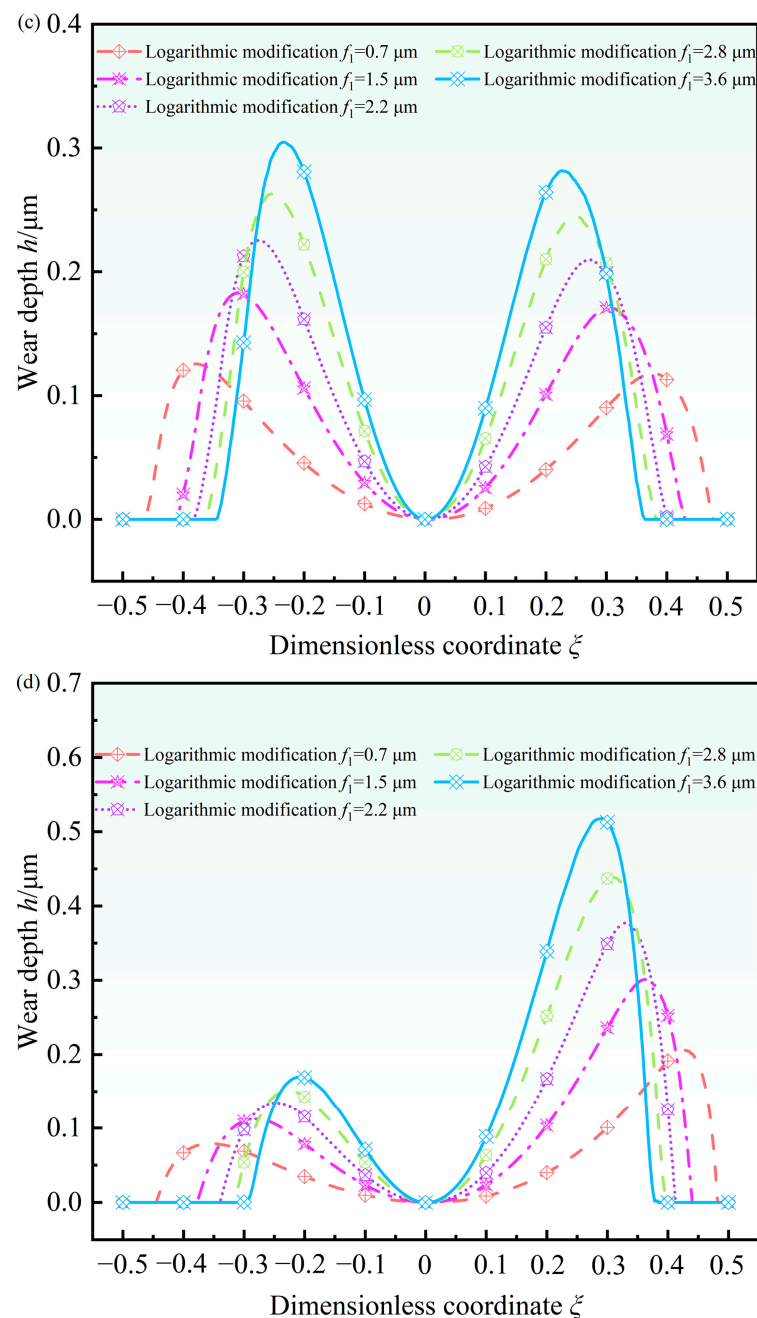


Figure 15. Wear depth along the generating line of the raceway under different f_1 . (a) Wear depth on the inner raceway along roller generatrix under axial load. (b) Wear depth on the outer raceway along roller generatrix under axial load. (c) Wear depth on the inner raceway along roller generatrix under combined load. (d) Wear depth on the outer raceway along roller generatrix under combined load.

Finally, the non-uniform distribution of wear depth along the generatrix is primarily caused by the non-uniformity of the contact stress. Especially under combined load, the tilting of the inner ring leads to a severely skewed contact stress distribution, resulting in a significantly asymmetric wear depth profile. This phenomenon is particularly pronounced on the outer raceway and intensifies with increasing modification amount (e.g., the difference between the two wear peaks on the outer raceway increases from 0.13 μm to 0.35 μm as f_1 increases from 0.7 μm to 3.6 μm under combined load). Such asymmetric wear can worsen the contact stress distribution, increasing heat generation and thermal stress and creating a positive feedback loop that further shortens the bearing's service life.

7. Experimental Validation

7.1. Model Validity

To validate the effectiveness of the constructed wear depth calculation model for tapered roller bearings, specifically focusing on the influence of roller profile modification on the wear depth distribution of raceways, a wear test platform for tapered roller bearings was designed and built, as shown in Figure 16. Under conditions of axial and radial loads, the wear of the inner and outer rings of the tapered roller bearing was tested to verify the wear depth distribution characteristics along the roller generatrix, thereby confirming the model's accuracy [42–46].

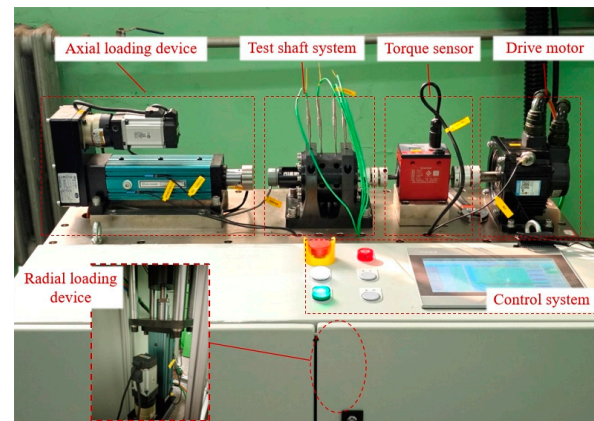


Figure 16. Tapered roller bearing wear test platform.

The test platform consists of a loading system, drive unit, data acquisition module, and lubrication control system. It can simulate actual working conditions under combined radial and axial loads (maximum radial load 1500 N, axial load 3000 N). The platform speed is adjustable in the range of 0–6000 r/min and is integrated with temperature sensors and vibration monitors to record bearing operating status in real time.

The test specimens were customized based on the parameters in Table 1. The roller profile modification adopted a logarithmic form with a modification coefficient $f_1 = 1.5 \mu\text{m}$. The modified tapered roller is shown in Figure 17. Its generatrix profile was calibrated using a coordinate measuring machine, ensuring a modification accuracy within $\pm 0.1 \mu\text{m}$. The inner ring rib of the bearing has a conical structure, the outer ring is fixed, and the inner ring is driven by a motor.

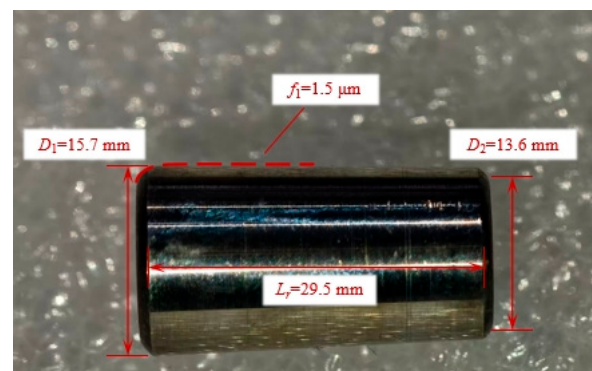


Figure 17. Tapered roller bearing with a profile factor $f_1 = 1.5 \mu\text{m}$.

The experimental load conditions were consistent with the theoretical analysis in Section 5, as shown in Table 2. Considering the case of insufficient lubrication, the bearing is in a boundary lubrication state. Wear depth measurement was performed using a high-

precision surface profile scratch instrument, as shown in Figure 18, with a vertical resolution of 0.01 μm . During measurement, observations were conducted via sample preparation.

Table 2. Test condition parameters.

Num.	Name of Parameters	Parameter Value
1	Axial load (N)	2400
2	Radial load (N)	1000
3	Motor speed (rpm)	4500
4	ISO 16281 [47] Life (h)	21,856
5	Test duration (h)	11,976
6	Shutdown inspection interval (h)	500

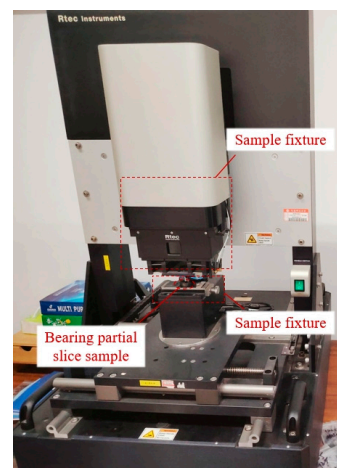


Figure 18. Surface profile scratch tester.

7.2. Results Analysis

The wear morphology of the bearing outer ring after testing is shown in Figure 19a,b. It is evident that the wear distribution is non-uniform, especially under combined loads, where the wear depth at different position angles of the outer ring varies significantly, consistent with the theoretically predicted “asymmetric distribution” characteristic.

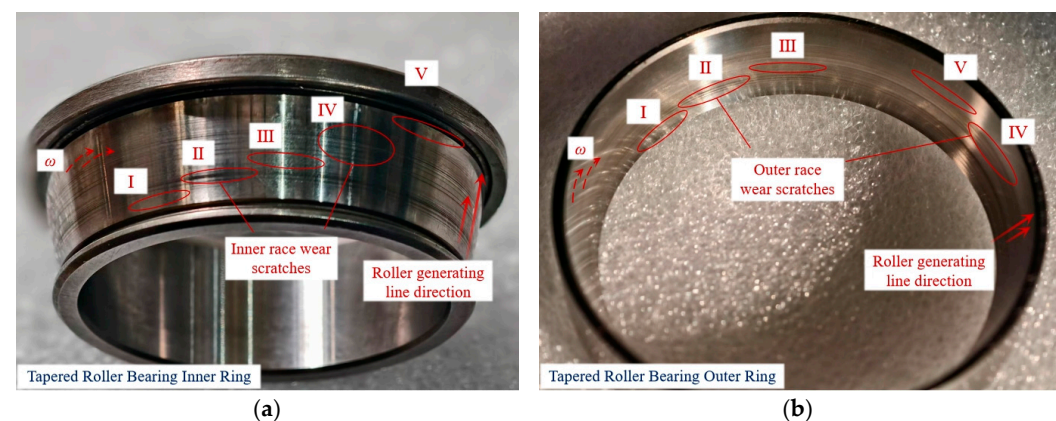


Figure 19. The bearing after testing. (a) Worn inner ring of the bearing after testing. (b) Worn outer ring of the bearing after testing.

The measured wear depth results for the inner and outer raceways under combined loads are shown in Figure 20a,b. Along the generatrix length, the wear depth data curves for both the inner and outer rings can be divided into three segments: areas with concentrated scratches (“I, V” segments), relatively smooth areas (“II, IV” segments), and smooth surface

areas (“III” segments). The red dashed lines in Figure 20a represent the fitted envelope curves of the wear depth data. Comparing with Figure 15a–d, the overall trend of wear depth is generally consistent with the model calculation results, with the maximum wear depth close to the theoretical value of approximately $0.2\text{ }\mu\text{m}$.

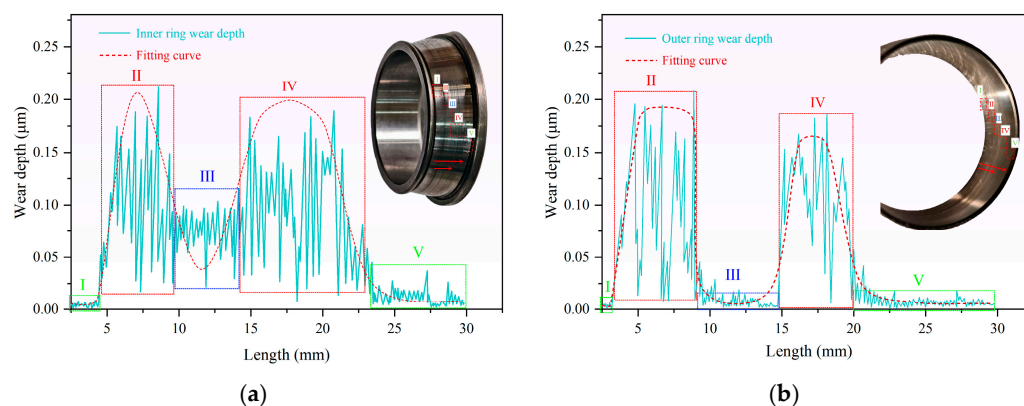


Figure 20. Wear depth of the inner and outer ring surfaces after testing. (a) Inner ring surface. (b) Outer ring surface.

As shown in Figure 20a, the peak wear depth of the inner ring is located at 8 mm along the generatrix, with a depth of approximately $0.22\text{ }\mu\text{m}$. Under combined loads, the wear distribution of the inner ring exhibits a slight skewness. As shown in Figure 20b, the wear depth distribution of the outer ring varies significantly with position, with the peak wear depth located at 8 mm along the generatrix, reaching about $0.21\text{ }\mu\text{m}$. The asymmetry between the inner and outer rings is pronounced. In summary, the experimental results align with the analytical results, validating the model [48,49].

In addition, regarding roller wear, during the 12,000 h test, the roller wear depth was measured using a high-precision surface profiler. The results show that the maximum roller wear depth is less than $0.03\text{ }\mu\text{m}$, which is an order of magnitude smaller than the raceway wear depth. This is because the roller is made of high-hardness bearing steel and the contact stress on the roller surface is more uniform than on the raceways. Furthermore, when multiple rollers are in operation, the force is evenly distributed, so the wear is not so severe. Although roller wear is not zero, it is negligible compared to raceway wear in the short term. For long-term operation, roller wear will accumulate and affect bearing performance, which will be the focus of future research.

8. Conclusions

This study, based on the established quasi-static mechanical model of roller bearings and the derived calculation model for raceway wear depth, systematically investigates the influence mechanism of roller profile modification on the contact force distribution, sliding velocity distribution, and raceway wear depth distribution in tapered roller bearings. The differential effects of two typical loading conditions—pure axial load and combined radial–axial load—on bearing wear characteristics are also comparatively analyzed. Through systematic parametric studies and model calculations, the following main conclusions are drawn:

- (1) Roller profiling exerts a decisive influence on the contact stress distribution. An increase in the modification amount leads to a reduction in the actual effective contact area between the roller and the raceway, resulting in a more concentrated contact stress distribution zone and a significant elevation of the pressure peak. Compared to pure axial load, combined load introduces a significant asymmetric skewness characteristic into the contact stress distribution, primarily stemming from changes in

- roller attitude induced by the angular displacement (tilting) of the inner ring due to the radial load component.
- (2) Roller profiling influences wear through dual pathways: contact stress and sliding velocity. When the logarithmic modification parameter f_1 increases from 0.7 μm to 3.6 μm , the maximum wear depth of the inner raceway increases by 133% under pure axial load and 144% under combined load, while that of the outer raceway increases by 142% under pure axial load and expands from 0.1 to 0.2 μm to 0.23–0.52 μm under combined load. The wear peak shifts toward the midpoint of the roller generatrix with increasing modification amount.
 - (3) Combined load causes asymmetric wear on the outer raceway, with the peak difference increasing by 169% as f_1 increases from 0.7 μm to 3.6 μm . Under conditions of significant profiling, the negative impact of combined load conditions on bearing wear life is more severe.
 - (4) Combined load causes a significantly asymmetric distribution of wear depth along the roller generatrix on the outer raceway. More significant roller profiling, larger f_1 , leads to a greater disparity between these two wear peaks, further deteriorating the uniformity of the wear distribution and posing challenges to the stable operation and long-life design of the bearing.

Author Contributions: Z.L. and D.Z. made equal contributions to this manuscript. Conceptualization, Z.L. and K.Z.; methodology, D.Z.; software, W.Z. and L.H.; validation, Y.Z.; formal analysis, W.T.; investigation, D.Z. and L.H.; resources, K.Z.; data curation, Y.Z. and W.T.; writing—original draft preparation, Z.L.; writing—review and editing, K.Z. and D.Z.; visualization, W.Z. and Y.Z.; supervision, Z.L. and K.Z.; project administration, D.Z. and W.T. All authors have read and agreed to the published version of the manuscript.

Funding: This project was supported by the National Natural Science Foundation of China (52505496), Open Project of Shenyang Key Laboratory of High-end Ceramic Bearing (SJUKLHCB01), and Postdoctoral Fellowship Program of China Postdoctoral Science Foundation (GZC20250909).

Data Availability Statement: The data presented in this study are available on request from the corresponding authors.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

Symbol	Definition (Unit)	Symbol	Definition (Unit)
$a_{ijq/ejq}$	Contact half-width of the q th element between the j th roller and inner/outer raceway (mm)	h_r	Effective axial height of the roller (mm)
α_i	Half cone angle of inner raceway ($^\circ$)	$\dot{h}$	Wear depth rate ($\mu\text{m}/\text{h}$)
α_e	Half cone angle of outer raceway ($^\circ$)	$h_{ijq/ejq}$	Wear depth of the q th element on inner/outer raceway (μm)
α_f	Contact angle of large rib of inner raceway ($^\circ$)	J_b	Roller's moment of inertia about its own axis ($\text{kg}\cdot\text{m}^2$)
β	Average contact angle, $(\beta = (\alpha_i + \alpha_e)/2)$ ($^\circ$)	K_n	Contact stiffness coefficient for line contact ($\text{N}/\text{m}^{10/9}$)
b	Half-width of the contact area (mm)	K_f	Contact stiffness coefficient for point contact ($\text{N}/\text{m}^3/2$)
d	Inner diameter of bearing (mm)	k	Wear coefficient (MPa^{-1})
D	Outer diameter of bearing (mm)	L_r	Effective contact length of the roller generatrix (mm)
D_1	Diameter of large end of roller (mm)	l_{FB}	Distance between point F and B (mm)
D_2	Diameter of small end of roller (mm)	m	Mass of the roller (kg)
d_i	Average diameter of inner raceway (mm)	M_g	Gyroscopic moment of the roller ($\text{N}\cdot\text{m}$)

d_o	Average diameter of outer raceway (mm)	M_{rx}, M_{ry}	External moments (N·m)
D_q	Equivalent diameter of the roller at the q th element (mm)	n	Exponent in contact force equation (-)
$e_{ij/ej}$	Load eccentricity of the j th roller on inner/outer raceway (mm)	p	Local contact stress (MPa)
E	Elastic modulus (Pa)	$p_{max,ijq/ejq}$	Maximum contact stress of the q th element (MPa)
E'	Equivalent elastic modulus, $(E' = E/(1 - \nu^2))$ (Pa)	$Q_{ij/ej}$	Contact force between the j th roller and inner/outer raceway (N)
F_c	Centrifugal force of the roller (N)	Q_f	Contact force between the roller and inner ring rib (N)
F_{rx}, F_{ry}, F_{rz}	External radial and axial loads (N)	Q_{ijq}	Contact load on the q th element of the j th roller (N)
f_1	Logarithmic profile modification parameter (magnitude) (μm)	dQ	Local contact load (N)
f_2	Dimensionless logarithmic profile modification parameter (shape) (-)	R_{im}	Radius of the large end of the inner raceway (mm)
R_p	Length of line KE (mm)	r_{ijq}	Radius of inner raceway at contact point ξ (mm)
R_s	Arc radius of large end of roller (mm)	r_{eqq}	Radius of outer raceway at contact point ξ (mm)
r_{bjq}	Roller radius at the q th element (mm)	r_j	Pitch circle radius of the bearing (mm)
r_f	Radial distance from contact point D to bearing axis (mm)	r_m	Distance from roller center to bearing axis (mm)
r_i	Distance from inner raceway center to bearing axis (mm)	$S_{ijq/ejq}$	Surface area of the q th element on inner/outer raceway (mm^2)
r_e	Distance from outer raceway center to bearing axis (mm)	t	Operating time (h)
T	Total operating time (h)	$\delta_{rx}, \delta_{ry}, \delta_{rz}$	Inner ring displacements (μm)
v	Sliding velocity (mm/s)	β_{rx}, β_{ry}	Inner ring angular displacements (rad)
$v_{ijq/ejq}$	Sliding velocity at the q th element (mm/s)	γ	Half cone angle of roller ($^\circ$)
dV	Local wear volume (mm^3)	μ_0	Angle parameter, $(\mu_0 = \beta - \alpha_f)$ ($^\circ$)
$\bar{V}_{ijq/ejq}$	Wear volume per unit time of the q th element (mm^3/h)	ν	Poisson's ratio (-)
$V_{ij/ej}$	Relative displacement at generatrix midpoint M (μm)	ρ	Density of roller (g/mm^3)
$\Delta V_{ij/ej}$	Additional deformation due to rotation (μm)	ξ	Dimensionless coordinate, $(\xi = x'/L_r)$ (-)
z	Logarithmic profile modification amount (μm)	ω_i	Rotational angular velocity of inner ring (rad/s)
$\delta_{ij\xi/ej\xi}$	Total normal deformation at contact line (μm)	ω_c	Orbital angular velocity of roller (cage angular velocity) (rad/s)
δ_f	Normal deformation of roller-rib contact (μm)	ω_b	Rotational angular velocity of roller (rad/s)

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