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A Hydraulic Reciprocating Rod Seal's Life Evaluation Method Incorporating Failure Mechanism Analysis and Test Observation Data

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Abstract: Reciprocating rod seals are widely used in hydraulic systems. Their useful life and reliability affect that of the system. Degradation modeling is necessary to evaluate the useful life of the seal. Seal wear is one of the important forms of hydraulic reciprocating rod seal degradation, yet it is difficult to measure through direct methods. Because seal wear determines the leakage of the seal, we therefore consider the seal leakage as the performance degradation index. Furthermore, the degradation of the seal is always associated with random effects, which cannot be considered by theoretical failure mechanism analysis. Hence, stochastic processes are applied to consider the random effects. Considering the error between the measured value and its real degradation state caused by the measurement environment or other factors, we introduce the measurement error term into the Wiener process model and develop the corresponding Wiener process life prediction model. Finally, the failure mechanism analysis and test measurement data are fused to predict the life cycle of the hydraulic reciprocating rod seals. The effectiveness of the proposed method is verified by comparing the predicted degradation and the experimental observations.

Keywords: hydraulic reciprocating rod seals; failure mechanism analysis; remaining life prediction; data fusion; stochastic process

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1. Introduction

In order to avoid the leakage of fluid and other substances from mechanical parts, as well as to prevent the introduction of external dust and impurities into these parts, the design of special structures in these parts is considered. The seals' construction refers to the structure of the bond between surfaces; such structures are called seals [1,2]. Moreover, in hydraulic systems, the seal is a key component to ensure the safety of the system.

Due to frictional wear, the leakage of the seal gradually increases over time; when this leakage exceeds a specified threshold, the seal is considered to have failed. Moreover, seal failure often causes serious adverse consequences: the seal leakage of hydraulic oil and other media will cause environmental pollution, equipment corrosion, and even economic losses; in addition, seal leakage will cause system pressure loss that may lead the machine to stop working, and even cause casualties and other major accidents. Therefore, making an accurate assessment of the seal's service life and performing scheduled preventive and corrective maintenance would ensure the seal's crucial reliable operation.

Moreover, based on some scholars' experimental research about the mechanisms of the performance degradation of lip seals, wear is a key factor in the failure of dynamic seals [3–6]. However, due to the differences between elastic seal materials and metals, ceramics in friction, wear, and lubrication mechanisms, especially their nonlinear characteristics and their large deformation due to the applied force, it is difficult to establish a complete mathematical analytical model for friction and wear. Therefore, the Finite Element Analysis (FEA) method and the simulation calculation have become the most appropriate method for seal wear research. The main steps consist of the establishment of the Finite Element Model (FEM) and the calculation of the contact pressure using the finite element solution, as well as the calculation of the wear depth to update the seal profile. For instance, Nándor et al. [7,8] used FEA to study the tribological behavior of the reciprocating rod seal. The simulation of wear was performed mainly by removing the mesh of the FEM; however, the disadvantage of this method was that the simulation accuracy decreased when the wear depth did not match the size of the finite element mesh. Moreover, Sui et al. [9] predicted the wear of Poly-Tetra-Fluoro-Ethylene (PTFE) composite seals by FEA assuming that the wear depends only on the contact pressure, and its process is mainly simulated by modifying the seal node coordinates. The advantage of this method is its fast computational speed and high simulation accuracy, but it is difficult to model the continuous wear process and the results are highly affected by the size of the element [8,10]. Li et al. [11] proposed a structure–thermal coupled FEA wear simulation model that considers the effect of temperature, and the authors numerically simulated the wear process of the O-shaped and rectangular section seals. Angerhausen et al. [12] simulated and analyzed seal wear behavior under lubrication conditions by combining the elastohydrodynamic lubrication model, the normal contact model, and the Archard wear model. In addition, Liu et al. [13] proposed the mixed elastohydrodynamic lubrication model with non-Gaussian surfaces and analyzed the effects of sealed fluid pressure on the seal performance. Furthermore, Pend et al. [14] proposed the mixed lubrication model, based on a developed multiple-grid method, to investigate the combined seal at different system pressures and overcome the problem of non-convergence in the mixed lubrication modeling of reciprocating seals. Day [15] considered the viscosity–temperature equation, and constructed the thermal elastohydrodynamic numerical model of the lip seal, studying the coupling mechanism of fluid flow and heat. Guo analyzed the law of influence of the storage process on the performance of lip seals by experiments [16]. Subsequently, the effect of material aging on the degradation of lip seal performance during use was analyzed by placing the lip seal in an oil fluid and controlling it at a constant temperature [17]; based on these experiments, he analyzed the effect of wear on the degradation of the lip seal performance [18]. Furthermore, Liu et al. [19] proposed a multi-scale simulation model for rotating lip seal wear based on lubrication analysis. The hybrid thermoelastic flow lubrication model was used to calculate the hydrodynamic and the rough contact loads, and the Archard model was modified by introducing scaling factors. This methodology is used to calculate the seal lip wear depth under lubrication conditions.

All the above publications studied the failure mechanism and provided a basis for the life prediction of fluid dynamic seals. Moreover, Shao and Kang [20] proposed a method for life prediction of the O-ring based on failure mechanism analysis. By designing an accelerated aging test and establishing a performance degradation model with a permanent compression rate representing the seal degradation performance index, the failure threshold was set to 50%; therefore, the life of the O-ring was evaluated to be 526 days. While the seal degradation process is significantly stochastic due to the influence of the stochastic working environment, it is difficult for the degradation model, based on failure physical analysis, to determine the influence of such stochastic factors. Sun obtained the relationship between fractal parameters and working time by regression using friction and wear test data, obtained the relationship between leakage rate and working time based on the leakage mechanism of mechanical seals, and predicted the wear performance of mechanical seals and evaluated their wear failure life [21]. Zhou considered seal failure caused by wear, used the leakage rate as a performance characterization quantity for mechanical seals, and used the wear data obtained from tests to evaluate their wear life based on neural networks [22]. The stochastic process-based modeling needs to determine the degradation trajectory of the product first, which is mainly obtained due to the failure of physical analysis or a priori experience. When the a priori information regarding the degradation trajectory is not available, the degradation data are usually averaged to estimate the mean degradation trajectory of the product. Obviously, this approach requires at least two sets of degradation data, therefore limiting the use of stochastic process modeling methods. Moreover, Liu et al. [23] obtained the degradation trajectory by analyzing the rotating lip seal wear, and they established the stochastic process analysis model of the seal reliability by a set of degradation data. The gamma process and the inverse Gaussian process were used as candidate stochastic process models, and the model parameters and probabilities were estimated by the Bayesian methods to comprehensively analyze the reliability of the rotating lip seal. Furthermore, Zhang et al. [24] described the monotonic degradation process by gamma process and introduced the reliability demonstration method. Peng constructed the degradation model by an inverse normal-gamma mixture of an inverse Gaussian process and improved the parameter estimation based on the EM-type algorithm [25]. Peng described the time-varying degradation rate by inverse Gaussian process, highlighting the physical meaning of the process parameters [26], and proposed a general Bayesian framework to infer the process parameters [27]. Qin constructed a degradation model based on the inverse Gaussian process and combined the inspection data with the prior distributions to evaluate the parameters by Bayesian method [28].

The above studies concerning the stochastic processes have obvious advantages in life prediction and reliability assessment. However, it is a pity that research related to the life prediction of reciprocating seals based on stochastic processes is rarely found. Moreover, the previously published research cannot utilize both the degradation mechanism and testing data. Therefore, based on the previous analysis of reciprocating seal wear, we propose a hydraulic reciprocating rod seal life prediction method that incorporates the failure mechanism analysis along with the experimental observation data. The main contributions of this research include the following: (1) establishment of the degradation model of the reciprocating seal based on the failure analysis of the reciprocating seal and the Wiener process; (2) consideration of the measurement error in the degradation model of the reciprocating seal; (3) construction of the data fusing and parameter updating method of the model based on Bayesian theory.

The structure of the paper is as follows: Section 2 presents the schematic description of the presented system. Section 3 establishes the Wiener process degradation model with measurement error to describe the degradation process of reciprocating seals. Section 4 gives a method based on wear analysis to obtain the degradation mean function. Section 5 constructs a parameter update and life prediction method based on Bayesian theory. Section 6 shows the flowchart of the proposed method. Section 7 verifies the effectiveness of the constructed life prediction method referring to a case study analysis. Finally, Section 8 gives a summary of the whole paper.

2. System Description

In this paper, a Wiener process-based degradation model is constructed to evaluate the useful life and reliability of reciprocating rod seals by incorporating failure mechanism analysis and test observation data. The schematic description of the presented system is shown in Figure 1. The degradation process of the seal is described based on the Wiener process, $X(t)$, considering the random effects of the seal degradation.

In general, two factors need to be determined when using the Wiener process in reliability and life estimation. The degradation mean function, $\Lambda(t)$, indicates the degradation trajectory and only depends on the failure mechanism of the products. Hence, we obtain the degradation mean function based on the theoretical degradation trajectory by failure mechanism analysis and simulation. The other factor is the model parameters, which include the diffusion coefficient, β, and the drift coefficient, λ. The diffusion coefficient reflects the inherent discreteness of the degradation. The drift coefficient reflects the degradation changing rate of the specific product and is assumed to follow Gaussian distribution, $N(\mu_\lambda, \sigma_\lambda^2)$, considering individual difference. Furthermore, the measurement error is also considered by introducing the measurement error coefficient ε, which is described as Gaussian distribution with zero mean, as $\varepsilon \sim N(0, \sigma^2)$. All the above parameters, $[\beta, \mu_\lambda, \sigma_\lambda, \sigma]$, need to be inferred based on the measured degradation dataset.

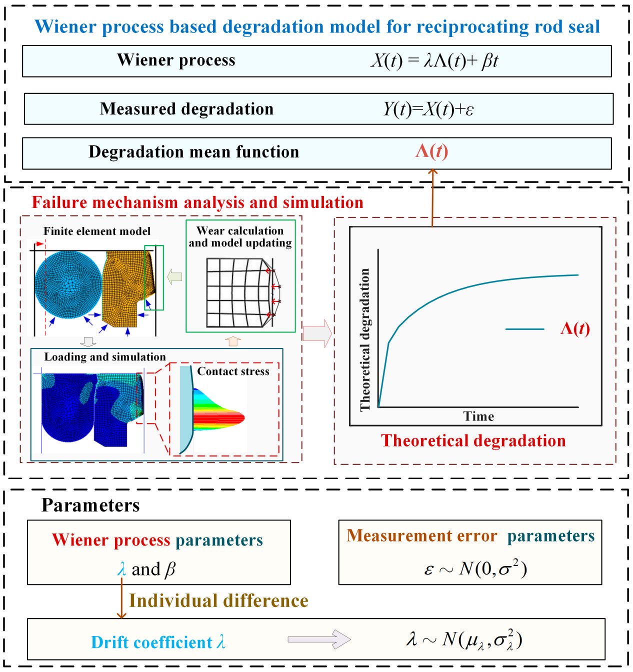

Figure 1. Schematic description of the presented system.

As discussed above, the constructed Wiener process-based degradation model provides a foundation for evaluating the hydraulic reciprocating rod seal's life by making full use of failure mechanism analysis and test observation data, resulting in the improvement of the evaluation accuracy.

3. Wiener Process-Based Degradation Model with Measurement Error

3.1. Wiener Process Degradation Model

The Wiener process is a probabilistic statistical model based on Brownian motion. It is widely used to describe the cumulative degradation failure process of mechanical products. Let $\{X(t), t \geq 0\}$ be the cumulative degradation of the seals at time t; if it obeys the Wiener process, the performance index $X(t)$ can be then expressed as follows:

$$X(t) = X(0) + \lambda t + \beta B(t) \tag{1}$$

where $X(0)$ means the initial performance index of the products; λ is the drift coefficient, which represents the rate of the product performance degradation; $\beta > 0$ is the diffusion coefficient, which represents the time-varying uncertainty of the degradation process; and $B(\bullet)$ is the standard Brownian that is used to characterize the time-varying uncertainty of the performance degradation process itself. To describe the variability of the degradation process among different seals, λ can be usually regarded as a random variable obeying a normal distribution. It is therefore expressed as follows: $\lambda \sim N\left(\mu_\lambda, \sigma_\lambda^2\right)$.

In general, the initial degradation of the seals $X(0)$ can be assumed to be 0 or to change to 0 by transformation, so that the Wiener process is expressed as follows:

$$X(t) = \lambda t + \beta B(t) \tag{2}$$

Through the application of Wiener process, the performance index $X(t)$ has the following characteristics:

(1) $X(t)$ has independent increments, when $t_4 > t_3 > t_2 > t_1$, $X(t_4) - X(t_3)$ and $X(t_2) - X(t_1)$ are independent from each other;

(2) The independent increment $\Delta X(t)$ obeys the normal distribution, $\Delta X(t) \sim N(\lambda \Delta t, \beta^2 \Delta t)$, where $\Delta X(t) = X(t + \Delta t) - X(t)$.

When the product degradation process shows nonlinear characteristics, it needs to be described by a nonlinear Wiener process degradation model. Therefore, the amount of product degradation is expressed as follows:

$$X(t) = \lambda \Lambda(t) + \beta B(t) \quad (3)$$

where $\Lambda(t)$ is a continuous nondecreasing function concerning time t and is called the degradation mean function here. Furthermore, the degradation mean function reflects the theoretical degradation trajectory without considering random effects and measurement errors. Namely, it is determined by degradation mechanism only. Naturally, we determine the degradation mean function of the seal by wear simulation based on the failure analysis in Section 4.

3.2. Measurement Error Analysis

The seals' degradation data is an important factor for life evaluation based on the Wiener process model. However, due to uncertainties, such as the accuracy of sensor equipment and environmental influences, the measured seals' degradation data will inevitably have errors. Such measurement inaccuracies cannot be completely removed, and they also have an impact on how accurately products degrade, which reduces the precision of predicting how long something will last. Therefore, to consider the influence of the measurement error on seals' reliability analysis, this paper introduces the variable ε to represent the measurement error in the Wiener process degradation model, and the real and the observed degradation amounts of the seals have the following relationship:

$$Y(t) = X(t) + \varepsilon \quad (4)$$

where $Y(t)$ means the observed degradation index. $X(t)$ means the true degradation.

Due to the influence of various uncertainties, the measurement error of the degraded data is not fixed but it obeys a certain statistical distribution. Usually, it can be assumed that the measurement error of the product obeys a normal distribution that is defined as $\varepsilon \sim N(0, \sigma^2)$, and its probability density function is the following:

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\varepsilon^2}{2\sigma^2}\right) \quad (5)$$

4. Degradation Mean Function for Reciprocating Rod Seal

Usually, the performance index of the product has a stable change trend, such as monotonically rising or monotonically falling, which reflects the irreversibility of the product degradation process. This changing trend represents the performance degradation trajectory of the product. According to the different degradation rates, the degradation trajectory of the product can be divided into three categories: constant rate degradation trajectory, monotonic rate degradation trajectory, and *S*-type rate degradation trajectory [29]. The degradation rate function is expressed as follows:

$$r(t) = \frac{d\Lambda(t)}{dt} \quad (6)$$

where t denotes the time, $\Lambda(t)$ represents the degradation mean function, and $r(t)$ denotes the degradation rate function.

There are two main methods to obtain the seal degradation mean function: the first one is through seal failure mechanism analysis that generates the seal degradation mean function, whereas the second one consists of obtaining the seal degradation trajectory based on past information of product degradation [30]. At present, there is a lack of sufficient research on the life prediction of hydraulic reciprocating rod seals. Therefore, it is difficult to obtain their degradation mean function using known information. According to the analysis in the previous research, the hydraulic reciprocating rod seals' degradation mean function can be obtained through failure mechanism analysis. Moreover, according to the positive correlation between the seal leakage rate and the seal wear degradation, the leakage rate is used as the seal degradation index.

Seal wear can be simulated and calculated by building a wear model under mixed lubrication conditions [31]. When the seal wear state is predicted, the seal leakage rate, under different wear states, is calculated by the leakage model as a reflection of the seal wear's degradation state. The wear degradation simulation of the hydraulic reciprocating rod seals is shown in Figure 2. The degradation simulation model used in this case is an adaptation of our previously published research [30]. It mainly includes three steps: first, the initial seal leakage rate and the microscopic rough contact pressure are calculated by the multi-field coupled seal mechanism model based on the elastic flow lubrication; second, the seal lip wear depth is determined using the modified Archard wear model according to the microscopic contact pressure, and it updates the seal lip profile to obtain a new seal shape; third, under the new seal shape, the new macroscopic contact pressure is generated by FEA; then it returns to the first step to calculate the seal leakage rate and the surface roughness contact pressure under the new profile, which enters the next simulation cycle.

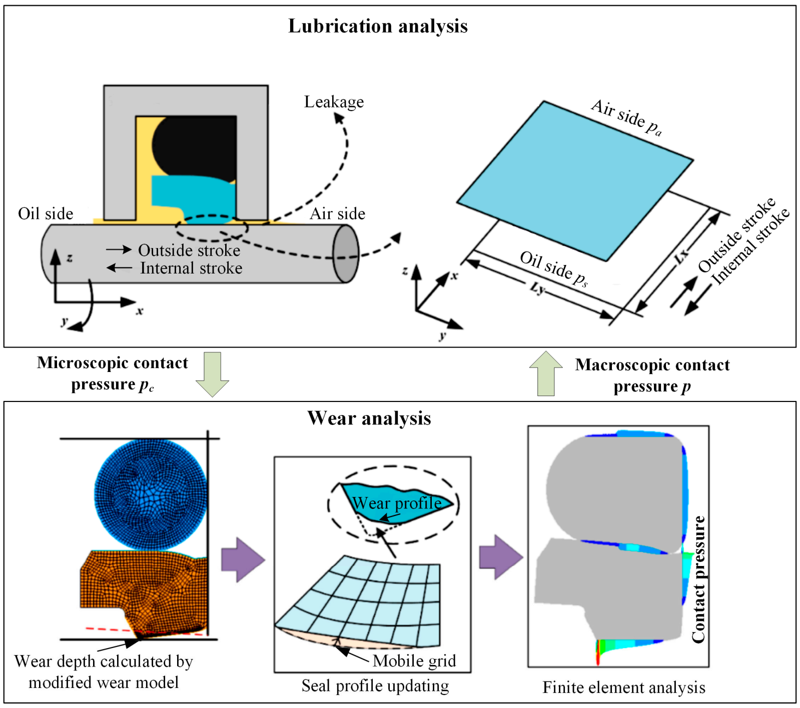

Figure 2. Seal wear degradation simulation method of the reciprocating rod seal [30].

By simulating the wear of the hydraulic reciprocating rod seals, the variation at the level of the seal leakage rate is calculated as shown in Figure 3. Before the wear degradation occurs, the initial leakage rate of the seal is not zero; we converted it to zero by using the leakage rate after the degradation minus the initial leakage rate. By fitting the predicted values of the leakage $q(t)$, the fitted curve is obtained as shown in Figure 3, and the corresponding fitting function is represented as follows:

$$q(t) = 2.2661 \ln\left(\frac{t + 254.2}{253.7}\right) \quad (7)$$

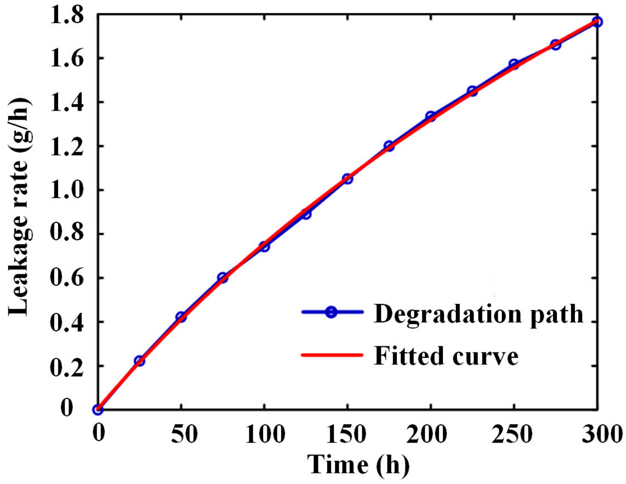

Figure 3. Theoretical degradation trajectory of a hydraulic reciprocating rod seal.

One of the most important things in building a life prediction model for stochastic processes based on the degradation data is to obtain the degradation trajectory of the seals. However, their real degradation trajectory is usually difficult to obtain due to measurement error and the difficulty of obtaining such data. Therefore, through the failure mechanism analysis, the degradation trajectory of the seals can be generated based on the combination with the degradation data obtained from the measurements, and the stochastic process model can be used to build this seal model and then realize its life prediction.

Through the above analysis, the leakage is used as the degradation index and the related degradation mean function of the seal is as follows:

$$\Lambda(t) = 2.2661 \ln\left(\frac{t + 254.2}{253.7}\right) \tag{8}$$

5. Remaining Life Prediction

5.1. Parameter Estimation

For the hydraulic reciprocating rod seals whose degradation process obeys the Wiener process, the measured amount of degradation at time t_i is denoted as Y_i, and the corresponding true amount of degradation is X_i, which is known from the description of the measurement error where $Y_i = X_i + \varepsilon_i$, and $\varepsilon_i \sim N(0, \sigma^2)$(as previously defined). When $i = 1, 2, 3, \ldots, m$, let $\Delta Y_i = Y_i - Y_{i-1}$, $\Delta\Lambda_i = \Lambda_i - \Lambda_{i-1}$, $\Delta t_i = t_i - t_{i-1}$, then $\Delta \boldsymbol{Y} = \{\Delta Y_1, \Delta Y_2, \cdots, \Delta Y_m\}^T$ obeys the multivariate normal distribution $\Delta \boldsymbol{Y} \sim N(\lambda \Delta \boldsymbol{\Lambda}, \Sigma)$, and its joint probability density function is written as follows:

$$P_{\Delta \boldsymbol{Y}|\lambda}(\Delta \boldsymbol{Y}|\lambda) = (2\pi)^{-\frac{m}{2}} |\textstyle\sum|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\Delta \boldsymbol{Y} - \lambda \Delta \boldsymbol{\Lambda})^T \textstyle\sum^{-1} (\Delta \boldsymbol{Y} - \lambda \Delta \boldsymbol{\Lambda})\right] \tag{9}$$

where $\Delta \boldsymbol{\Lambda} = \{\Delta\Lambda_1, \Delta\Lambda_2, \cdots, \Delta\Lambda_m\}^T$, and the covariance matrix $\sum$ is a positive definite tridiagonal matrix represented as follows:

$$\textstyle\sum_{i,j} = \mathrm{cov}\left(\Delta Y_i, \Delta Y_j | \lambda\right) = \begin{cases} \beta^2 \Delta t_i + \sigma^2, & i = j = 1 \\ \beta^2 \Delta t_i + 2\sigma^2, & i = j > 1 \\ -\sigma^2, & i = j + 1 \text{ or } i = j - 1 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

Considering $\lambda \sim N\left(\mu_\lambda, \sigma_\lambda^2\right)$, $\Delta \boldsymbol{Y}$ obeys a multivariate normal distribution with a mean value $\mu_\lambda \Delta \boldsymbol{\Lambda}$ and a covariance $\sum + \sigma_\lambda^2 \Delta \boldsymbol{\Lambda} \Delta \boldsymbol{\Lambda}^T$ [32]. Therefore, it is defined as follows:

$$\Delta \boldsymbol{Y} \sim N(\mu_\lambda \Delta \boldsymbol{\Lambda}, \textstyle\sum + \sigma_\lambda^2 \Delta \boldsymbol{\Lambda} \Delta \boldsymbol{\Lambda}^T) \tag{11}$$

To facilitate the derivation, define the parameters $\overline{\sum} = \sum/\sigma_\lambda^2$, $\overline{\beta}^2 = \beta^2/\sigma_\lambda^2$, and $\overline{\sigma}^2 = \sigma^2/\sigma_\lambda^2$. Supposing that there are N seals for the degradation measurement and the observed value of the degradation of the n-th seal is $\boldsymbol{Y}_n = \{Y_{0,n}, Y_{1,n}, \ldots, Y_{m,n}\}$. Further calculations are needed to obtain the increment of the degradation of the n-th seal $\Delta\boldsymbol{Y}_n$, which represents the unknown parameters in the model as $\boldsymbol{\theta} = [\mu_\lambda, \sigma_\lambda, \overline{\beta}, \overline{\sigma}]$, and where the log-likelihood function is represented as follows:

$$g(\boldsymbol{\theta}) = -\frac{\ln(2\pi)}{2} - \frac{mN}{2}\ln\sigma_\lambda^2 - \frac{N}{2}\ln|\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T| - \frac{1}{2\sigma_\lambda^2}\sum_{n=1}^{N}(\Delta\boldsymbol{Y}_n - \mu_\lambda\Delta\boldsymbol{\Lambda})^T\left(\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T\right)^{-1}(\Delta\boldsymbol{Y}_n - \mu_\lambda\Delta\boldsymbol{\Lambda}) \quad (12)$$

Taking the partial derivatives of $g(\boldsymbol{\theta})$ with respect to μ_λ and σ_λ, respectively, one can obtain the following:

$$\frac{\partial g(\boldsymbol{\theta})}{\partial \mu_\lambda} = \frac{1}{\sigma_\lambda^2}\Delta\boldsymbol{\Lambda}^T\left(\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T\right)^{-1}\sum_{n=1}^{N}(\Delta\boldsymbol{Y}_n - \mu_\lambda\Delta\boldsymbol{\Lambda}) \quad (13)$$

$$\frac{\partial g(\boldsymbol{\theta})}{\partial \sigma_\lambda^2} = -\frac{mN}{2\sigma_\lambda^2} + \frac{1}{2\sigma_\lambda^4}\sum_{n=1}^{N}(\Delta\boldsymbol{Y}_n - \mu_\lambda\Delta\boldsymbol{\Lambda})^T\left(\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T\right)^{-1}(\Delta\boldsymbol{Y}_n - \mu_\lambda\Delta\boldsymbol{\Lambda}) \quad (14)$$

Let the partial derivatives respective to μ_λ and σ_λ be equal to 0, and we can obtain the maximum likelihood estimates for μ_λ and σ_λ as follows:

$$\hat{\mu}_\lambda = \frac{1}{N}\frac{\Delta\boldsymbol{\Lambda}^T\left(\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T\right)^{-1}\sum\limits_{n=1}^{N}\Delta\boldsymbol{Y}_n}{\Delta\boldsymbol{\Lambda}^T\left(\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T\right)^{-1}\Delta\boldsymbol{\Lambda}} \quad (15)$$

$$\hat{\sigma}_\lambda^2 = \frac{1}{Nm}\sum_{n=1}^{N}(\Delta\boldsymbol{Y}_n - \hat{\mu}_\lambda\Delta\boldsymbol{\Lambda})^T\left(\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T\right)^{-1}(\Delta\boldsymbol{Y}_n - \hat{\mu}_\lambda\Delta\boldsymbol{\Lambda}) \quad (16)$$

Substituting the above two formulas into the likelihood function, we obtain the contour likelihood functions of β and σ as follows:

$$g(\beta, \sigma) = -\frac{Nm}{2}\ln(2\pi) - \frac{Nm}{2} - \frac{Nm}{2}\ln\hat{\sigma}_\lambda^2 - \frac{N}{2}\ln|\overline{\sum} + \Delta\boldsymbol{\Lambda}\Delta\boldsymbol{\Lambda}^T| \quad (17)$$

The maximum likelihood estimates of $\overline{\beta}$ and $\overline{\sigma}$ can be obtained by maximizing the contour likelihood function. Therefore, in this paper, using the Genetic Algorithm (GA) to search the estimates of $\overline{\beta}$ and $\overline{\sigma}$, the estimates of $\overline{\beta}$ and $\overline{\sigma}$ are then brought into Equations (15) and (16) to obtain the estimates of μ_λ and σ_λ, and then the estimates β, σ.

5.2. Random Parameter Update

Based on the above parameter estimation method, the initial estimate of λ can be obtained. Namely, the initial estimate is as follows: $\lambda \sim N\left(\hat{\mu}_\lambda, \hat{\sigma}_\lambda^2\right)$, where the corresponding distribution parameters are estimated based on Equations (15) and (16). However, due to the influence of random factors, different individual performance degradation data show variability and randomness. To reduce the estimation uncertainty, the random parameter λ should be updated according to the seals' degradation data to upgrade the prediction of the remaining life. With the initial estimated distribution of λ already obtained, this paper uses a Bayesian approach to update this random parameter (e.g., λ).

When given the degenerate quantity observation vector $\boldsymbol{Y}_{0:k} = \{y_0, y_1, \ldots, y_k\}$ for the seals from instants t_0 to t_k, and assuming that the prior estimate of λ is $\lambda \sim N\left(\hat{\mu}_\lambda, \hat{\sigma}_\lambda^2\right)$, its posterior distribution according to Bayesian theory is defined as follows [32]:

$$p(\lambda|\boldsymbol{Y}_{0:k}) \propto p(\boldsymbol{Y}_{0:k}|\lambda)\pi(\lambda) \propto \exp\left[-\frac{1}{2}(\Delta\boldsymbol{y}_k - \lambda\Delta\boldsymbol{\Lambda}_k)^T\sum\nolimits_k^{-1}(\Delta\boldsymbol{y}_k - \lambda\Delta\boldsymbol{\Lambda}_k)\right]\exp\left(-\frac{(\lambda - \hat{\mu}_\lambda)^2}{2\hat{\sigma}_\lambda^2}\right) \quad (18)$$

where $\pi(\lambda)$ means the prior estimate of λ, $\Delta \boldsymbol{y}_k = \{y_1 - y_0, y_2 - y_1, \cdots, y_k - y_{k-1}\}$ and $\Delta \boldsymbol{\Lambda}_k = \{\Lambda_1 - \Lambda_0, \Lambda_2 - \Lambda_1, \cdots, \Lambda_k - \Lambda_{k-1}\}$. Therefore, $\lambda \mid \mathbf{Y}_{0:k}$ obeys the normal distribution as shown below:

$$\lambda|\mathbf{Y}_{0:k} \sim N(\mu_{\lambda,k}, \sigma^2_{\lambda,k}) \tag{19}$$

where

$$\mu_{\lambda,k} = \frac{\Delta\boldsymbol{\Lambda}_k{}^T \sum_k^{-1} \Delta\mathbf{Y}_k \hat{\sigma}^2_\lambda + \hat{\mu}_\lambda}{\Delta\boldsymbol{\Lambda}_k{}^T \sum_k^{-1} \Delta\Lambda_k \hat{\sigma}^2_\lambda + 1} \tag{20}$$

$$\sigma^2_{\lambda,k} = \frac{\hat{\sigma}^2_\lambda}{\Delta\boldsymbol{\Lambda}_k{}^T \sum_k^{-1} \Delta\boldsymbol{\Lambda}_k \hat{\sigma}^2_\lambda + 1} \tag{21}$$

5.3. Lifetime Prediction

Given the failure threshold w, when the degradation of the seals exceeds the failure threshold for the first time, the corresponding lifetime T_L is defined as the first reach-time and it can be expressed as follows:

$$T_L = \inf\{t|X(t) \geq w\} \tag{22}$$

For the nonlinear Wiener process shown in Equation (8), its first reach-time approximation obeys the inverse Gaussian distribution, and it is expressed as follows:

$$f(t) \approx \frac{1}{\sqrt{2\pi\beta^2 t^3}} [w - \lambda(\Lambda(t) - t\Lambda'(t))] \exp\left[-\frac{(w - \lambda\Lambda(t))^2}{2\beta^2 t}\right] \tag{23}$$

Let the remaining lifetime of the seals at t_k be l_k, so that $l_k = t - t_k$. When the drift coefficient and the degradation at t_k are known, the probability density function of L_k can be approximated as follows:

$$f_{L_k|\lambda_k,X_k}(l_k|\lambda_k, X_k) \approx \frac{1}{\sqrt{2\pi\beta^2 l_k^3}} [w - x_k - \lambda_k(\Lambda(l_k + t_k) - \Lambda(t_k) - l_k\Lambda'(l_k + t_k))] \exp\left[-\frac{(w - x_k - \lambda_k(\Lambda(l_k + t_k) - \Lambda(t_k)))^2}{2\beta^2 l_k}\right] \tag{24}$$

Furthermore, according to Ref. [21]: If $Z_1 \sim N(\mu_1, \sigma_1^2)$, $Z_2 \sim N(\mu_2, \sigma_2^2)$, ω, A, $B \in R$ and C $\in R^+$, then [32]:

$$E_{Z_1}\{E_{Z_2}[(\omega - Z_1 - AZ_2)]\} = \sqrt{\frac{C}{B^2\sigma_2^2 + \sigma_1^2 + C}} \exp\left[-\frac{(\omega - \mu_1 - B\mu_2)^2}{2(B^2\sigma_2^2 + \sigma_1^2 + C)}\right] \left(\omega - \mu_1 - A\mu_2 - \frac{\omega - \mu_1 - B\mu_2}{B^2\sigma_2^2 + \sigma_1^2 + C}\left(\sigma_1^2 + AB\sigma_2^2\right)\right) \tag{25}$$

Based on Equations (24) and (25), an approximate expression for the probability density function of the remaining life of the seals based on $\boldsymbol{Y}_{0:k}$ can be derived as follows:

$$\begin{aligned} &f_{L_k|Y_{1:k}}(l_k|Y_{0:k}) \approx \frac{1}{\sqrt{2\pi l_k^2 F_2}} \exp\left[-\frac{F_1{}^2}{2F_2}\right]\left[F_1 + \mu_{\lambda,k} l_k \Lambda'(l_k + t_k) - \frac{F_1}{F_2} \times \left(\sigma^2_{\lambda,k}(\Lambda(l_k + t_k) - \Lambda(t_k)) + \sigma^2\right)\right], \\ &\text{where} \begin{cases} F_1 = w - y_k - \mu_{\lambda,k}(\Lambda(l_k + t_k) - \Lambda(t_k)) \\ F_2 = \sigma^2_{\lambda,k}(\Lambda(l_k + t_k) - \Lambda(t_k))^2 + \beta^2 l_k + \sigma^2 \end{cases} \end{aligned} \tag{26}$$

When the probability density function of the remaining life of the product is obtained, its expectation is calculated as follows:

$$E(L_k|\mathbf{Y}_{0:k}) = \int_0^\infty l_k f_{L_k|\mathbf{Y}_{0:k}}(l_k|\mathbf{Y}_{0:k}) dl_k \tag{27}$$

6. The Proposed Framework

Figure 4 shows the flowchart of the proposed method. There are three steps that need to be performed when using the proposed model. First, the degradation mean function is obtained based on the theoretical degradation trajectory by failure mechanism analysis and simulation; see Section 4. At step one, the log-likelihood function of the model to the degradation dataset is constructed, as shown in Equation (12). At step two, the maximum likelihood estimation of the hyper-parameters μ_λ and σ_λ are obtained by calculating Equations (15) and (16), which are utilized to set the prior distribution of the drift coefficient when updating the drift coefficient by Bayesian inference at step three. Then, the maximum likelihood estimations of parameters β and σ are also estimated by handling Equation (17) based on genetic algorithm. At step three, the drift coefficient is updated with the accumulation of the collected degradation data based on Bayesian inference, Equation (19), where the prior distribution of the drift coefficient is as Equation (18). Finally, the distribution and expectation of the useful life can be given by calculating Equations (26) and (27).

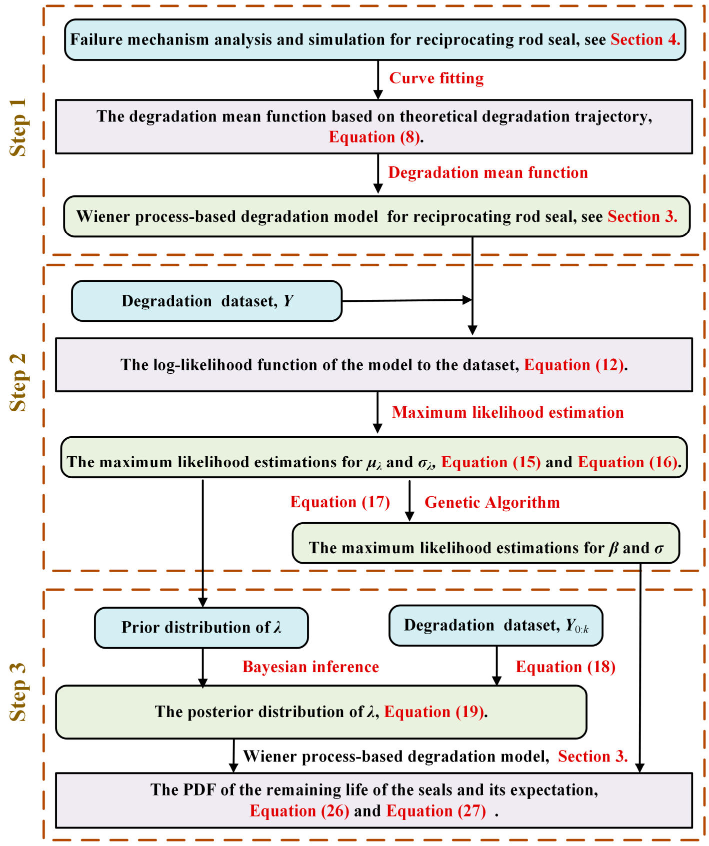

Figure 4. Flowchart of the proposed method.

7. Experimental Study

7.1. Experimental Approach

To verify the proposed method, a hydraulic reciprocating rod seals test platform was designed, and the principle of the test platform is shown in Figure 5. The test platform is mainly composed of two major parts: the driving part and the testing part, including the driving cylinder and the test cylinder. The whole test system is mounted vertically on the test stand with the drive cylinder at the upper end and the test cylinder at the lower end. Among them, the test cylinder is mounted on the sliding guide where one end of the test seal is installed and connected to the piston rod, and the other end is closed and connected to the drive cylinder. The role of the drive cylinder is to test the reciprocating movement of the cylinder up and down while the piston rod remains unmoved. The driving cylinder is fixed to the test stand by a flange connection, and its internal displacement sensor is installed to measure the actuator stroke and to control the change direction. Moreover, the test cylinder is connected to the hydraulic oil source through a hose connection. The hydraulic oil enters the test cylinder from the inlet port, flows through the valve seat from the return port, and the pressure and temperature of the hydraulic oil are measured using the pressure and temperature sensors, respectively, and then the oil returns to the hydraulic oil source. In addition, the tension sensor is installed at the lower end of the piston rod to measure the friction force on the seal. Furthermore, the physical diagram of the test platform is shown in Figure 6.

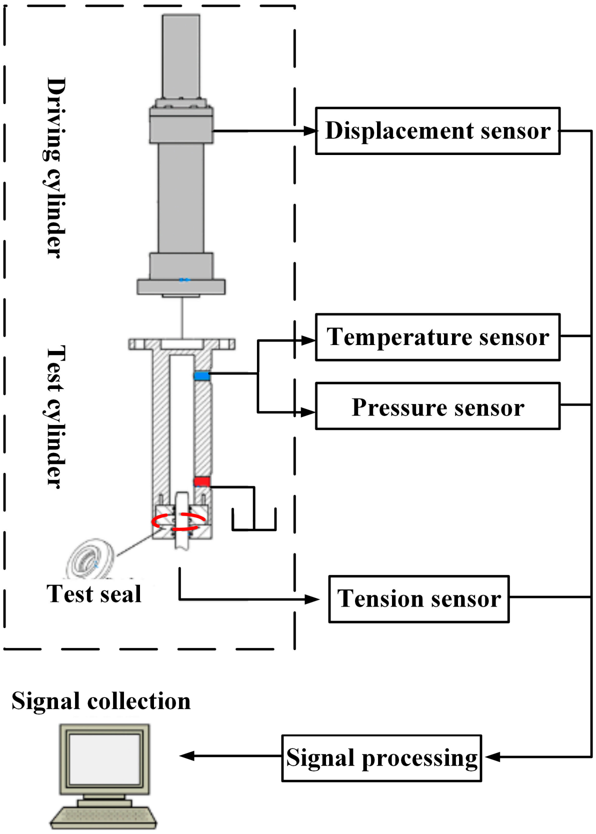

Figure 5. Schematic diagram of the test bench.

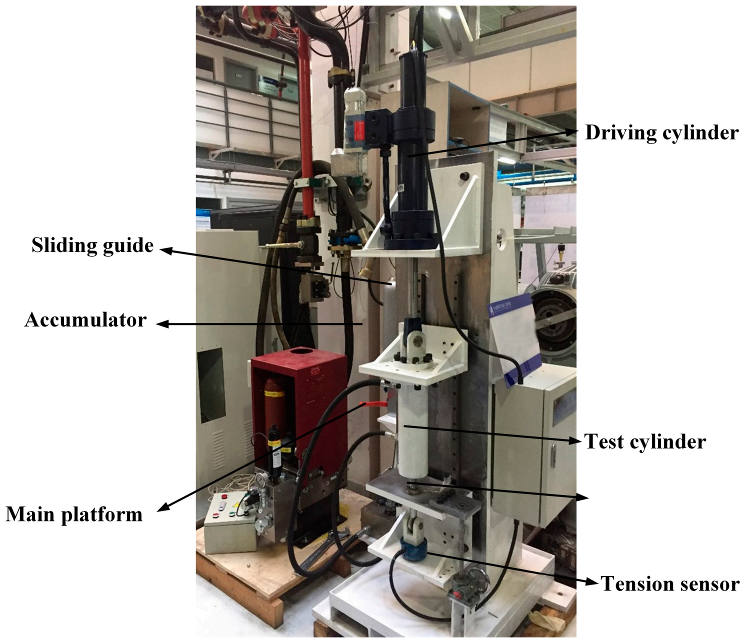

Figure 6. Physical drawing of the experiment table.

A test was conducted on a certain type of rod seal, and the seal leakage was collected on the air side of the seal to measure its degradation status performance. The seal leakage was measured every ten hours by an electronic balance. The electronic balance can measure from 0 to 100 g and the accuracy is 0.001 g. We used the average leakage rate during the measurement interval (ten hours) to indicate the seal degradation test observation. Observations were made at ten-hour intervals and the seal leakage was recorded accordingly. The test was stopped after 300 h, and the seal degradation data were obtained and displayed in Table 1. As shown in Figure 7, the circles mean the observation data every ten hours. It can be seen that the degradation test observation is exponential, so the theoretical degradation mean function is suitable and effective.

Table 1. Hydraulic reciprocating rod seal's degradation test observation data.

Time (hours)	10	20	30	40	50
Observation (g/h)	0.183	0.292	0.351	0.453	0.548
Time (hours)	60	70	80	90	100
Observation (g/h)	0.592	0.701	0.760	0.869	1.059
Time (hours)	110	120	130	140	150
Observation (g/h)	1.089	1.251	1.292	1.403	1.427
Time (hours)	160	170	180	190	200
Observation (g/h)	1.431	1.490	1.563	1.578	1.621
Time (hours)	210	220	230	240	250
Observation (g/h)	1.665	1.746	1.833	1.950	2.001
Time (hours)	260	270	280	290	300
Observation (g/h)	2.052	2.111	2.205	2.271	2.315

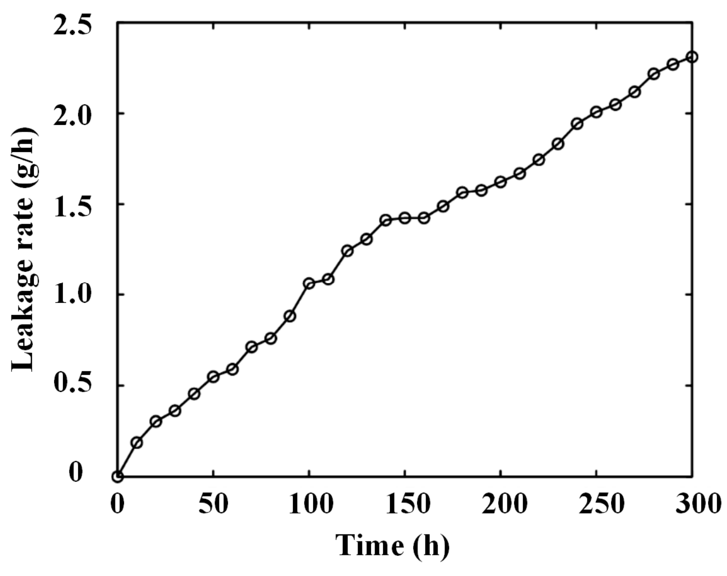

Figure 7. Hydraulic reciprocating rod seal's degradation test observation data.

7.2. Discussions

To illustrate the effectiveness of the method proposed in this paper, we made a comparative study with the method proposed by Si et al. [33] that does not consider the measurement error. As shown in Table 2, the model in this paper is denoted as M_0 whereas the nonlinear degenerate model without considering measurement error is denoted as M_1.

Table 2. The candidate models.

Candidate Models	Measurement Error
M_0	√
M_1	x

The parameters in the models are estimated separately, and the maximum log-likelihood function values, corresponding to the two models, are calculated, as well as the Akaike Information Criterion (AIC) values. The AIC consists of two parts where the first part is the log-extreme likelihood function, which responds to how well the sample information reflects the overall information, and the second part represents the penalty for the complexity of the model. Its calculation formula is as follows:

$$AIC = -2 \times [\max(g)] + 2\overline{p} \tag{28}$$

where $\overline{p}$ denotes the number of parameters to be estimated in the model.

The calculation results are shown in Table 3. The estimated log-likelihood function of model M_0 is slightly larger than the output of model M_1, indicating that the first model is better fitted. However, the model M_0 has one more parameter than the model M_1, which means the complexity of the model M_0 is higher. Finally, the AIC of the calculated model M_0 is larger than the model M_1.

Table 3. Estimated values of initial parameters.

Parameters	μ_λ	σ_λ^2	β^2	σ^2	g	AIC
M_0	1.3126	1.5435×10^{-4}	1.5420×10^{-4}	1.2017×10^{-4}	**51.0560**	−94.1120
M_1	1.3125	1.7801×10^{-4}	1.7798×10^{-4}	-	50.9767	**−95.9534**

It should be noted that when using only the above degradation test data, we cannot predict the remaining life due to the lack of failure data. Furthermore, it is also difficult to predict or estimate the remaining life of the seal by failure analysis alone, because the failure mechanism cannot be perfect enough.

In this paper, the degradation observed at 300 h is taken as the failure threshold with a value of 2.312 g/h. The probability density functions of the remaining life given by the models M_0 and M_1 are calculated, respectively, at 250 h, and the results are shown in Figure 8. Moreover, the probability density functions of the remaining life given by the models M_0 and M_1 are basically the same, and the real remaining life of the seal lies within the range of the probability density functions of the two models; however, the probability density function of M_0 has a sharper angle, which indicates that the model M_0 has a higher prediction accuracy when the measurement error is considered. In general, the proposed methods in this paper are accurate enough to predict the remaining life of the seal.

After updating the estimation of the stochastic parameters by the historical degradation data of the hydraulic reciprocating rod seals, the probability density function of the remaining life can be calculated for each instant. The probability density function of the remaining life of the seal was calculated for 300 h, 250 h, 200 h, 150 h, and 100 h, respectively. The results are shown in Figure 9. Furthermore, the actual lifetimes of the seals all lie within the range of the probability seal function for the remaining life estimates of the corresponding observation times. As the seal degradation intensifies, the calculated probability density function of the remaining life gradually becomes narrower when more degradation data are obtained, indicating that the predicted remaining life of the seal is much more certain.

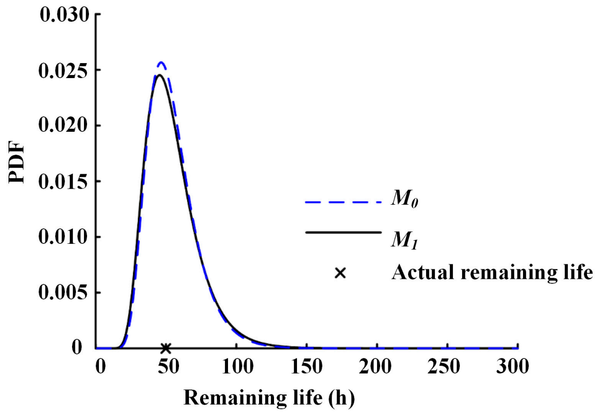

Figure 8. Comparison of the remaining life probability density function (PDF).

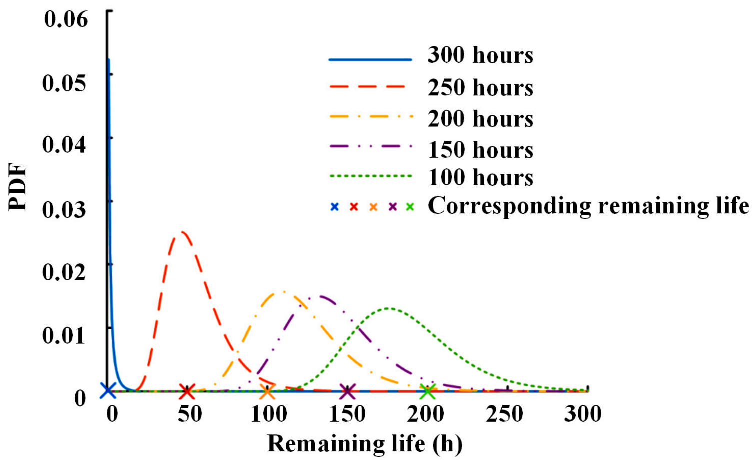

Figure 9. Remaining life probability density function.

Moreover, Table 4 shows the expected and actual remaining life of the seal at different observation times. It is clear that the prediction model proposed in this paper can predict the remaining life of the seal relatively well. At 200 h, the error between the remaining life expectation and the actual life is the largest, reaching 16.2%, whereas at 250 h, the error between both values is the smallest, being equal to 6.4%.

Table 4. Residual life prediction error.

Observation Time	Remaining Life Expectation	Actual Remaining Life	Error
300 h	0.2 h	0 h	-
250 h	53.2 h	50 h	6.4%
200 h	116.2 h	100 h	16.2%
150 h	137.8 h	150 h	8.1%
100 h	180.4 h	200 h	9.8%

As discussed above, the proposed method can provide high enough accuracy on the remaining life prediction even under small degradation data and incomplete mechanism conditions.

8. Conclusions

In this paper, we studied the problem of the hydraulic reciprocating rod seal's life evaluation. Considering the uncertainty of the seal degradation process, a hydraulic reciprocating rod seal's life prediction method was proposed based on a stochastic process model incorporating failure mechanism analysis and experimental observation data.

First, a Wiener process-based degradation model is constructed and provides a foundation for evaluating the hydraulic reciprocating rod seal's life by making full use of failure mechanism analysis and test observation data. The degradation mean function of the seal is predicted through the rod seal failure mechanism analysis. Considering the problem that the wear of the seal lip is difficult to measure, the leakage rate is taken as the observed quantity of the seal wear degradation. The degradation failure process of the rod seal is described by Wiener process. Moreover, we propose the life prediction model of the Wiener process including the measurement error considering the influence of the measurement error. Considering the variability and randomness of different individual items of performance degradation data, the drift parameters are assumed to obey the normal distribution.

Second, based on Bayesian theory, a data fusing and parameter updating method is constructed for applying the Wiener process-based degradation model in engineering practice. The initial values of the fixed and drift parameters in the model are estimated by searching the contour likelihood function through the genetic algorithm, and they are updated referring to the drift parameters by Bayesian theory to update the prediction of the remaining service life.

Finally, the effectiveness of the proposed method is verified by comparing the predicted degradation and experimental observations. The remaining life prediction results of the hydraulic reciprocating rod seals are simulated and calculated by a set of experimental observation data and seal failure mechanism analysis data. Comparing the prediction results of the two models considering the presence and the absence of the measurement error, the proposed methods are accurate enough to predict the remaining life of the seal, and the model considering measurement error has better prediction certainty.

The method in this paper can realize seal life prediction under the condition of a small sample of observation data, and it provides the basis for the reliability design of hydraulic reciprocating rod seals. Future works will focus on how to improve the useful life, reliability, and seal wear by texturing the rod surface.

Author Contributions: X.D.: visualization, writing—original draft. D.L.: conceptualization, methodology, investigation, writing—original draft. S.W.: writing—review and editing, project administration. Y.S.: project administration, writing—review and editing. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement: Since much of the content in this paper is part of the team's ongoing research about seals, the relevant data cannot be shared for the moment.

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Abbreviations

Abbreviation	Definition
AIC	Akaike Information Criterion
$B(\bullet)$	standard Brownian
FEA	Finite Element Analysis
FEM	Finite Element Model
GA	Genetic Algorithm
M_0	The model proposed in this paper
M_1	The nonlinear degenerate model without considering measurement error
N	The number of seals for degradation measurement
$N(\bullet)$	Normal distribution
PDF	Probability density function
PI	Performance Index
PTFE	Poly-Tetra-Fluoro-Ethylene
T_L	First reach-time
$X(t)$	True degradation process
X_i	True amount of degradation at t_i
$\Delta X(t)$	Degradation increment of a degradation process $X(t)$
$Y(t)$	Observed degradation process
$\mathbf{Y}_{0,k}$	The degenerate quantity observation vector for the seals from instants t_0 to t_k
Y_i	The measured amount of degradation at t_i
$\mathbf{Y}_n$	The vector of observed value of the degradation of the n-th seal
$\Delta\mathbf{Y}$	The vector of observed degradation increment
ΔY_i	The observed degradation increment of i-th sample
$f(\varepsilon)$	Probability density function of ε
$\overline{p}$	The number of parameters to be estimated in the model
$r(t)$	The degradation rate
t	Time
t_i	Degradation time
w	The failure threshold
$\Lambda(t)$	The degradation trajectory
$\Gamma(\bullet)$	Gamma function
β	Diffusion coefficient
ε	Measurement error
λ	Drift coefficient
θ	Parameter vector
$g(\theta)$	The log-likelihood function
$\sum$	Positive definite tridiagonal matrix

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