

Review

The Discovery of the Expansion of the Universe

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Abstract: Alexander Friedmann, Carl Wilhelm Wirtz, Vesto Slipher, Knut E. Lundmark, Willem de Sitter, Georges H. Lemaître, and Edwin Hubble all contributed to the discovery of the expansion of the universe. If only two persons are to be ranked as the most important ones for the general acceptance of the expansion of the universe, the historical evidence points at Lemaître and Hubble, and the proper answer to the question, “Who discovered the expansion of the universe?”, is Georges H. Lemaître.

Keywords: cosmology history; expansion of the universe; Lemaitre; Hubble

1. Introduction

The history of the discovery of the expansion of the universe is fascinating, and it has been thoroughly studied by several historians of science. (See, among others, the contributions to the conference *Origins of the expanding universe* [1]: 1912–1932). Here, I will present the main points of this important part of the history of the evolution of the modern picture of our world.

2. Einstein’s Static Universe

Albert Einstein completed the general theory of relativity in December 1915, and the theory was presented in an impressive article [2] in May 1916. He applied [3] the theory to the construction of a relativistic model of the universe in 1917. At that time, it was commonly thought that the universe was static, since one had not observed any large scale motions of the stars. One talked about the fixed stars on the heaven, and this was an expression of the conception that on a cosmic scale, we live in an unchanging universe.

Einstein therefore tried to find a solution to his gravitational field equations describing a static universe model. He discovered that this was not possible because a universe that was originally static would collapse under its own attractive gravity.

In order to be able to construct a static universe model, Einstein introduced a so-called cosmological constant into his equations [4]. From the way he introduced the cosmological constant into his field equations, it follows that it was originally to be interpreted as an expression of a natural tendency of the space to have an accelerated expansion. In the static universe of Einstein, there is equilibrium between the attractive gravity due to matter and radiation and this repulsive tendency of the space.

In 1930, the British astrophysicist Arthur Stanley Eddington showed [5] that Einstein’s static universe model is unstable. If the mass density is less than the equilibrium value, the universe will expand and if it is larger, the universe will collapse.

3. The De Sitter Universe

Already in 1917, the Dutch astronomer Willem de Sitter solved Einstein’s equations with a cosmological constant for an empty universe [6]. He used a reference frame which was not freely

falling. In this system, it looked like he had constructed an empty and static universe model. De Sitter showed that light emitted towards an observer is observed to have a redshift of the spectral lines. The reason for the redshift was that there existed a field of gravity in the de Sitter universe, acting away from an arbitrary observer. This was due to his use of a static reference frame that was not freely falling. The light approaching the observer moved upwards in this gravitational field and was therefore red-shifted. However, de Sitter also wrote that such red-shifts were usually interpreted as a Doppler effect and a sign that the light sources had a velocity away from the observer. He added that such velocities had recently been observed for spiral nebulae.

In 1922, the Hungarian physicist Cornelius Lanczos demonstrated [7] that if one describes the de Sitter universe in a freely falling reference frame, then this universe model appeared as an empty universe with accelerated expansion due to the natural tendency of space to expand.

4. The Friedmann Universe Models

In July 1922 and in 1924, the Russian researcher Alexander Friedmann presented a more realistic class of expanding universe models with matter as solutions of Einstein's field equations with a cosmological constant [8,9]. He wrote more popularly about these models in his book, *The World as Space and Time*, which was published in 1923 (see Belenkyi [10]). These were the first non-empty expanding universe models. They were presented at a time when nearly all thought that the universe was static. Einstein was convinced that this was the case, and he announced in October 1922 that he had found an error in Friedmann's deduction of the solution of his field equations. This turned out, however, not to be the case, and Einstein retracted [11] his criticism in 1923.

Friedmann's articles were remarkable. His equations laid the foundations for modern cosmology and are now called Friedmann's equations. He was the first person to construct relativistic universe models with a beginning at a finite past and an infinitely great spatial extension.

5. The Contributions of Lemaître

In 1927, the Belgian priest and cosmologist Georges Lemaître published an article [12] written in French in a little-known Belgian journal. Translated to English, the heading is, *A Homogeneous Universe with Constant Mass and Increasing Radius Accounting for the Radial Velocities of the Radial Velocities of the Extra-Galactic Nebulae*. He rediscovered one of the solutions in Friedmann's class of solutions. But more than that, as stated by S. van den Bergh [13], in this paper, Lemaître both established the expansion of the universe and interpreted it as a consequence of the general theory of relativity.

In the heading of his article, Lemaître presented an interpretation of the redshift of the remote galaxies that has become the standard interpretation in modern cosmology – namely, that it is due to the expansion of space, and not to the motion of the galaxies through a static space. It is space itself which expands [14]. This was so radical that Hubble, who later became known as the discoverer of the expansion of the universe, was never willing to accept the relativistic conception of an expanding space.

Due to the expansion of the universe, the remote galaxies move away from us with a velocity, v , proportional to the distance, l . This relationship was later called "Hubble's law" and is presently written as

$$v = Hl, \quad (1)$$

where the value of the Hubble parameter as given by Lemaître was $H = 192$ km/s per million light years. With a constant expansion velocity, this value of H means that the universe would be only 1.7 billion years old.

The relationship (1) was originally deduced by Lemaître in 1927 as a consequence of the spatially homogeneous relativistic universe models.

Lemaître followed up by comparing observable consequences of his universe model with observations. He used velocity measurements of 42 nebulae (galaxies) measured by Slipher [15]

and published in 1923 by Eddington [16] and two years later by the Swedish astronomer Gustaf Strömberg [17], who worked at the Wilson-Observatory in California.

Lemaître estimated the distances, l , to the nebulae using the assumption of Hubble that all of them emitted radiation with the same luminosity. The distances could then be calculated from the observed intensities. Lemaître corrected the velocity measurements for the motion of the Earth around the Sun. In this way, he confirmed that the relationship was in agreement with the relationship (1), but with a rather great scattering of the observation points in the velocity-distance diagram (Figure 1).

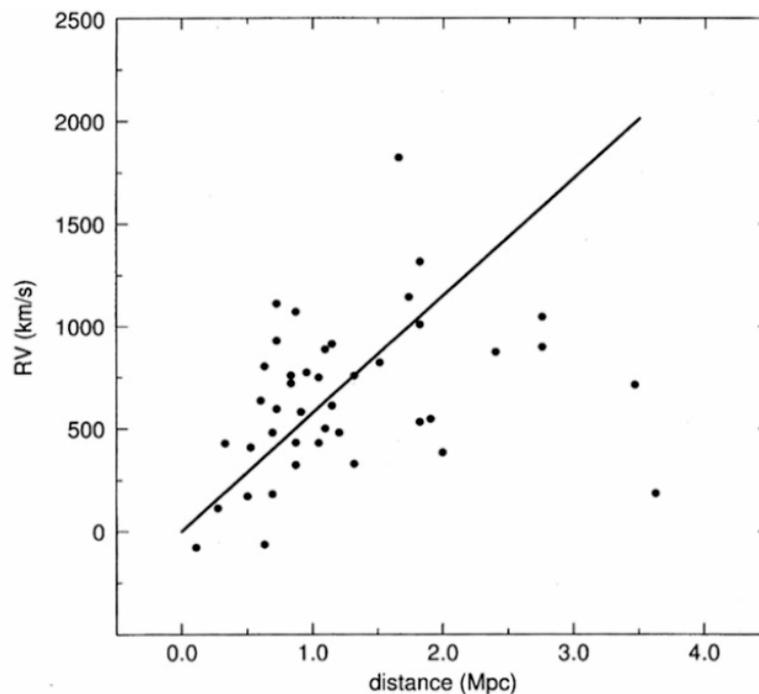


Figure 1. Observational data used by Lemaître and the line [18] fitting best with the data and which was used to calculate the value of H .

One year later, the American cosmologist Howard Percy Robertson [19] found a similar universe model as a solution to Einstein's field equations, but he did not advocate an expanding universe. Over the following years, Richard Chase Tolman [20], Robertson [21], and Arthur Geoffrey Walker [22] worked out the present way of describing relativistic, expanding universe models [23].

Many years later (in 1950), Lemaître wrote [24] about the relation (1) that before the discovery and studies of clusters of galaxies (that were identified in the early thirties by among others Fritz Zwicky), there could be no talking about establishing the Hubble law. What could be done, and what Lemaître actually performed, was to deduce the law as a consequence of the relativistic universe models and then verify that it was in agreement with observational data. Lemaître wrote: "The title of my article leaves no doubt about my intentions". It was to explain observed data in light of an expanding universe model found as a solution to Einstein's field equations.

In November 1931, Lemaître published an article [25] with the title "*L'Expansion de l'Espace*", where he presented a new universe model surprisingly similar to the present standard model of the universe [26]. A detailed analysis of this work has been given by H. S. Kragh and D. Lambert [27]. Lemaître introduced a spherical universe with finite space, but without boundaries, and wrote: "We may imagine that space appears as a first atom and that the appearance of space coincides with the origin of time. The universe came into existence with a vanishing radius. During the subsequent expansion the universe passed through three eras. First a period with rapid expansion, then a period with slow expansion followed by a final period with accelerated expansion (Figure 2). There is no doubt that we presently live in the third era". Hence, Lemaître predicted not only that the universe

expands, but also that there is accelerated expansion at the present time. This was discovered in 1998 and gave the discoverers the Nobel prize in physics for 2011.

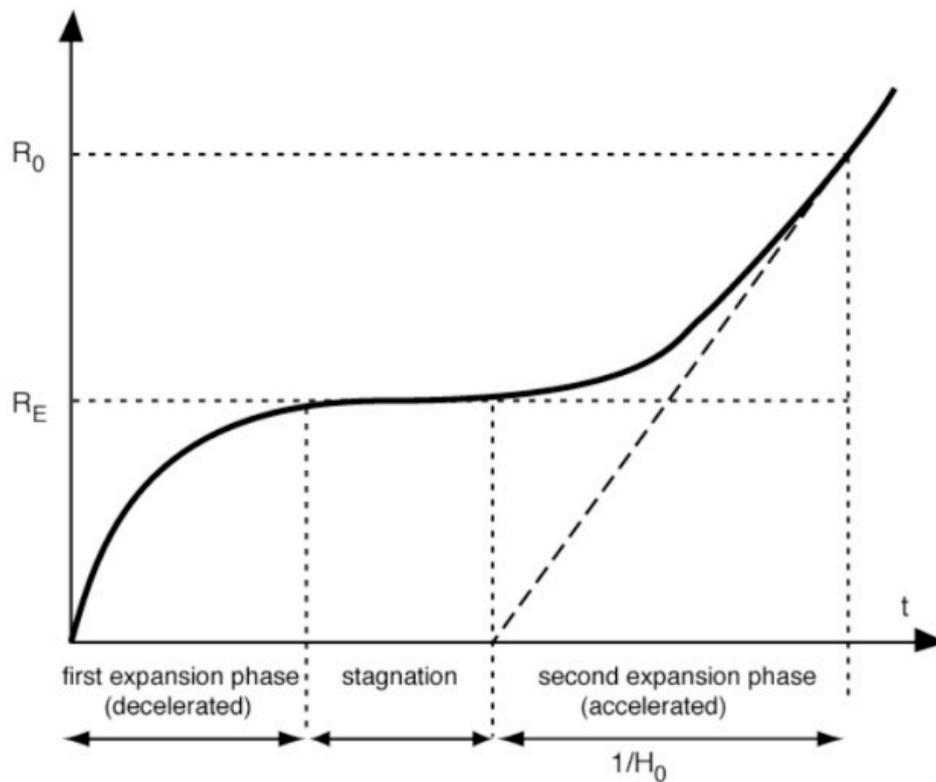


Figure 2. This graph shows how the cosmic distances change as a function of time in the universe model which Lemaître presented in 1931. The quantity $1/H_0$, where H_0 is the present value of the Hubble parameter, is the so-called *Hubble age* of the universe. It is the age which the universe would have had if it expanded with the present velocity during its whole history. The Lemaître-universe started with an explosion of space and went into a first era with rapid expansion. This was then retarded by the attractive gravity due to matter. The universe then entered a period with low expansion. In this era, there is close to equilibrium between the attractive gravity due to matter and the repulsion due to the cosmological constant. The duration of this so-called “hesitating era” could be determined so that the age of the universe model is in agreement with observational data by adjusting the magnitude of the cosmological constant. The universe finally entered the present period with accelerated expansion [25].

The pattern of the evolution of the expansion velocity was due to attractive gravity due to matter and repulsion represented by the cosmological constant. In 1931, the physical interpretation of this repulsion was still that of Einstein: a natural tendency of empty space to expand. However, in 1934, Lemaître introduced a new physical interpretation of the cosmological constant. He wrote [28]: “The theory of relativity implies that when we identify mass and energy, we have to introduce a constant. Everything happens as if the energy of the vacuum is different from zero. In order that it shall be impossible to measure velocity relative to this energy, we have to associate a pressure with the energy density of the vacuum. This is the essential meaning of the cosmological constant”. Lemaître thought that the pressure was positive and the density negative. The present identification is the opposite.

J. P. Luminet [29] has recently given an interesting discussion of the evolution of cosmology during the twenty years from 1917 to 1937, with an emphasis on the contributions of Lemaître.

Lemaître’s interpretation (with a sign correction) has been adopted as the standard interpretation of the cosmological constant. It represents the energy density of a Lorentz Invariant Vacuum Energy (LIVE), which is today often called dark energy. Explicit calculations were later presented

by Zel'dovich [30] and Grøn [31], showing that the requirement that all the components of the energy-momentum tensor of a perfect fluid shall be the Lorentz invariant, leads to the result that this tensor must be proportional to the metric tensor, and hence appears as a cosmological constant in Einstein's field equations. The LIVE causes repulsive gravity.

The reason for the evolution of the cosmic scale factor shown in Figure 2 may now be explained as follows. The universe appears with matter and LIVE in a Big Bang explosion with a great initial expansion velocity. The initial density of matter is greater than that of LIVE. The expansion velocity is then retarded by attractive gravity due to the matter, and there is then a period with a small expansion velocity. This so-called "hesitating era" can last so long that the present age of the universe is much larger than its Hubble age. The density of matter decreases during the expansion, but the density of LIVE is constant. Hence, after a certain time, the repulsive gravity of LIVE will dominate the attractive gravity of matter. Then, the universe enters the present era with accelerated expansion.

6. Observational Results before 1920

In 1912, the American astronomer Vesto M. Slipher reported the results of the first measurements of the radial velocities of the Andromedae nebula using a spectrograph to measure the Doppler shifts of spectral lines [32]. He found that the nebula moves towards the solar system with a velocity of 300 km/s. In 1917, Slipher had measured the velocities and distances of 25 more nebulae (Figure 3). These measurements laid the foundation for finding a velocity-distance relation for cosmic objects.

RADIAL VELOCITIES OF TWENTY-FIVE SPIRAL NEBULAE.

Nebula.	Vel.	Nebula.	Vel.
N.G.C. 221	− 300 km.	N.G.C. 4526	+ 580 km.
224	− 300	4565	+ 1100
598	− 260	4594	+ 1100
1023	+ 300	4649	+ 1090
1068	+ 1100	4736	+ 290
2683	+ 400	4826	+ 150
3031	− 30	5005	+ 900
3115	+ 600	5055	+ 450
3379	+ 780	5194	+ 270
3521	+ 730	5236	+ 500
3623	+ 800	5866	+ 650
3627	+ 650	7331	+ 500
4258	+ 500		

Figure 3. The radial velocities of 25 spiral nebulae published by Slipher [32] in 1917. Minus means motion towards us and plus away. The velocities are given in km/s. (See also O’Raifeartaigh [33]).

It has been emphasized by O’Raifeartaigh [33] and Peacock [34] that Slipher’s 1917 results changed our world picture. Before Slipher presented his results, it was usual to think that the universe was static, with the fixed stars at rest. In his 1917 article, Slipher wrote that on average, the spiral nebulae moved away from the solar system with a velocity of 500 km/s. This meant that the system of spiral nebulae expanded. He noted, however, that the nebulae had a tendency to be collected in clusters, so there was not a global expansion of the system of nebulae. Furthermore, Slipher found a dipole distribution, with velocities away from us in one direction and velocities towards us in the transverse direction. He interpreted this as indicating that we move through a system of spiral nebulae and determined the direction and magnitude of this motion, which turned out to be about 700 km/s. Slipher then wrote [33]: “While the number of nebulae is small and their distribution poor this result may still be considered as indicating that we have some such drift through space. For us to have such motion and the stars not show it means that our whole stellar system moves and carries us with it.

It has for a long time been suggested that the spiral nebulae are stellar systems seen at great distances. This is the so-called ‘island-universe’ theory, which regards our stellar system and the Milky Way as a great spiral nebula which we see from within. This theory, it seems to me, gains favor in the present observations”.

With these results, Slipher introduced a new picture of the world. From a system of stars at rest, the world has become a system of spiral nebulae in motion, and the solar system is within one such nebula—the Milky Way.

It has recently been pointed out by Emilio Elizalde [35,36] that the fundamental contribution of Slipher to the evolution of a realistic world picture has not been properly referred to, in particular not by Hubble, until, in 1953, he finally acknowledged the significance of Slipher’s work in a letter to Slipher, writing, “... your velocities and my distances ...”.

In 1919, Harlow Shapley and his wife Martha Betz Shapley reported the results of an analysis where they had investigated the difference of radial velocities of spherical clusters and spiral nebulae. They found that most of the spherical clusters moved toward us, while most of the spiral nebulae had a rather great velocity away from us. Only three of 25 spiral nebulae moved towards us. Those that moved away from us had a velocity greater than 150 km/s. They concluded that the spherical clusters and the spiral nebulae represented two different populations, and that the one made up of spiral nebulae expands. They wondered whether this motion might be due to some repulsive force, like, for example, radiation pressure. Their observational data also indicated that the most remote nebulae had the greatest velocities.

7. The Years from 1920 to 1925

The so-called Great debate took place as a public discussion on April 26, 1920. It was concerned with how large our universe is, and whether the spiral nebulae were within our own galaxy, the Milky Way, or outside it. Two leading astronomers, Harlow Shapley and Heber D. Curtis, discussed this matter. Shapley argued that the nebulae were inside our galaxy, while Curtis argued that they were far outside it. The question was not answered during the debate, but during the following years, it became clear that Curtis was right. The universe was much larger than imagined by Shapley. Never the less, Shapley had made important contributions to mapping the part of our universe dominated by the Milky Way.

During the years towards 1922, Slipher had measured radial velocities to 41 spiral nebulae. Nearly all of them moved away from us. Slipher did not publish these results in a separate article. They became known by being included in Eddington’s book *The Mathematical Theory of Relativity* [16], which was published in 1923. Eddington wrote: “The most extensive measurements of radial velocities of spiral nebulae have been made by professor V. M. Slipher by the Lowell-observatory. He has kindly prepared for me the following table, containing many unpublished results. It is believed to be complete up to date (Feb. 1922)”.

The Polish-American physicist Ludwik Silberstein determined a velocity distance relation for spherical clusters and for stars in the Magellanic clouds and published his results [37] in 1924. He too found that the spherical clusters move towards us, but he found that the Magellanic clouds move away from us; the Small cloud with a velocity of 150 km/s and the Large one with a velocity of 260 km/s. However, there were great uncertainties in these results, and they were criticized by the Swedish astronomer Knut Lundmark.

In the years 1918–1924, similar results were found by the German astronomer Carl Wilhelm Wirtz [38,39]. He wrote: “A clear large scale motion an expansion of the system of the spiral nebulae with respect to our position and the nebulae closest to us or being most massive have less expansion velocity than the more remote one or those with less mass”.

In June 1924, Lundmark published a velocity-distance relationship for spiral nebulae [40] (Figure 4) based upon Slipher’s velocity measurements and his own distance measurements, and in a subsequent article [41], he wrote: “A rather definite correlation is shown between apparent dimensions and

radial velocity, in the sense that the smaller and presumably more distant spirals have a higher space-velocity.”

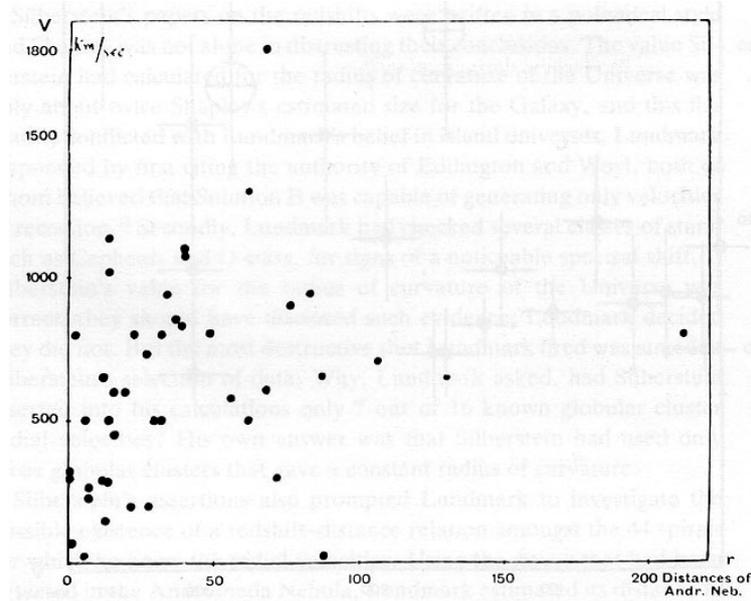


Figure 4. The first published cosmic velocity-distance diagram, 1924.

In 2011, Giora Shaviv [42] made an interesting experiment with the observational data that Lundmark’s diagram is based upon. He took Lundmark’s data for the 43 spiral nebulae and neglected the data for spherical clusters. Then, he assumed that the velocities of the spiral nebulae are proportional to their distance and calculated an average value of the Hubble parameter for spiral nebulae. In this way, he found the following value of the Hubble parameter: $H = 21,8 \pm 17$ km/s per million l. y.. The remarkable result is that this value of the Hubble parameter is in agreement with the present measurements.

Unfortunately, Lundmark did not discover this. However, he was the first to publish a cosmic velocity-distance for spiral nebulae, and his distance measurements had more realistic values than those used by Hubble in 1929 [43].

8. Hubble’s Announcement in 1929

In 1929, Edwin Hubble announced the relationship (1) which now carries his name [44]. Neither Lemaître nor Slipher were mentioned in this 1929 article, although Hubble plotted points in a velocity-distance diagram (Figure 5) where most of the velocities were measured by Slipher, while the distances were measured by Hubble and Milton Humason.

The measurements were more accurate than those of Lundmark, but there were large systematic errors—much larger than the researchers were aware of. The value of the Hubble parameter deduced from Hubble’s diagram was $H = 144 \pm 85$ km/s per million l. y. This value is seven times larger than the present value of H , and leads to an age of the universe of only two billion years. Nowhere in this article does Hubble state that the galaxies move away from us; neither did he write anything about the expansion of the universe [45].

At a Royal Astronomical Society meeting on 10 January 1930, de Sitter presented his own investigations. In two subsequent papers, he presented detailed discussions where he confirmed Hubble’s 1929 results [46] and discussed [47] the observational data in light of the universe model presented by Lemaître in 1927.

Hubble felt a sort of ownership of the relation (1). In a letter cited in [48] to De Sitter dated 21 August 1930, he wrote: “I consider the velocity-distance relation, its formulation, testing and confirmation, as a Mount Wilson contribution, and I am deeply concerned in its recognition as such.”

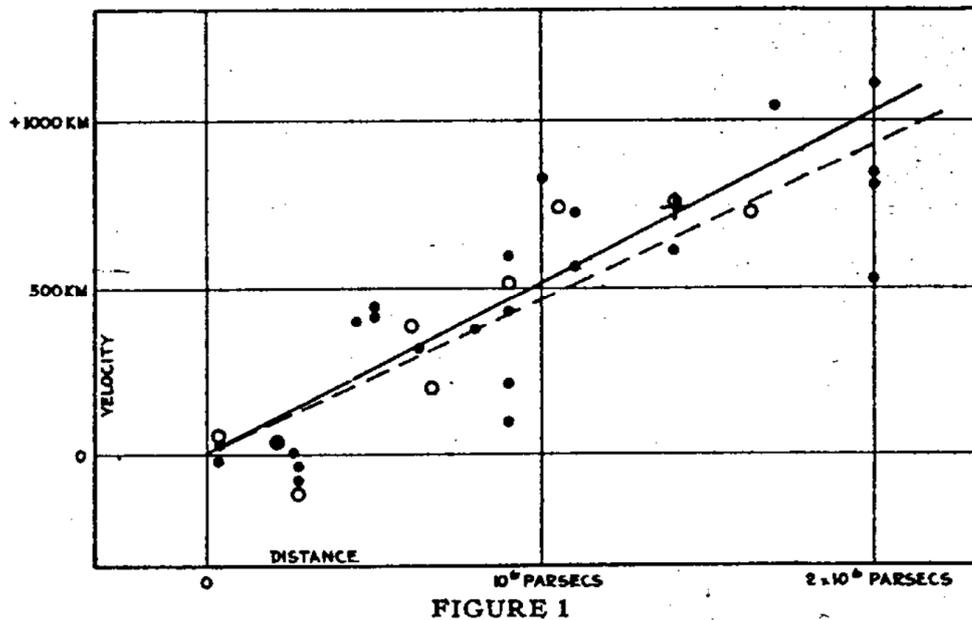


Figure 5. Hubble’s velocity-distance diagram from 1929.

H. Nussbaumer and L. Bieri wrote [48] in 2009 about this relationship: “Its formulation and its central place in cosmology were first given by Lemaître. There is no justification to glorify Hubble’s publication of 1929 as the original discovery of the linear velocity-distance relationship”.

In 2012, O’Raifeartaigh [33] presented the usual statement that “Hubble discovered the expanding universe”. He wrote: “Such a statement confuses *observation* with *discovery*, as a linear relation between recessional velocity and distance for the distant galaxies does not in itself suggest an expanding universe. It is much more accurate to say that the 1929 graph (Figure 5) provided the first experimental evidence in support of the hypothesis of an expanding universe”.

In this connection we must not forget Einstein’s words: “The theory decides what we observe”. Hubble refrained from interpreting the relationship of Figure 5 in the light of any theory, and he never wrote that it supports the conception of an expanding universe. The first one stating this was Lemaître, who interpreted it in light of the general theory of relativity.

The Dutch astronomer Jan Hendrik Oort criticized Hubble for exaggerating the precision of his results. He found, based on other observational data, a value of the Hubble parameter half as large as the value found by Hubble. Hubble and Humason answered this criticism in 1931. They meant that the difference between their results and those of Oorts would lessen with larger observational material. Hubble and Humason had large confidence in the precision of their results and wrote that they had determined the Hubble parameter with 20 % accuracy. However, as mentioned above, the present value is seven times less than the value of Hubble and Humason. Therefore, there was an enormous systematic uncertainty in the data of Hubble, which he was not aware of. Hence it does not seem fair to say that Hubble, who did not derive any theoretical velocity-distance relationship, and whose data turned out not to be correct, discovered the expansion of the universe. Lemaître’s data were not much better, but he interpreted them in the light of the theory of relativity and advocated an expanding universe.

S. van den Bergh [13] has made an interesting point: “The myth that Hubble discovered the velocity-distance relation seems to have originated with Humason [49] (who was at that time acting as Hubble’s assistant). Humason started his 1931 paper with the words “in 1929 Hubble found a relation connecting the velocities and the distances of the extragalactic nebulae for which spectra were then available”.

9. Lemaître’s 1927 Article is Translated to English

In 1930, Lemaître understood that nearly no-one had read his article. He then wrote to the British astrophysicist Arthur Eddington and pointed out the results in his 1927 article. Eddington immediately understood the great significance of Lemaître’s work and decided that the article had to be translated to English, and that it should be published in Monthly Notices of the Royal Astronomical Society (MNRAS). The article was published there in March 1931, but in a slightly shortened version.

The part which was not included in the English translation, was the one concerned with the calculation of the value of the Hubble parameter from observational data. It has been speculated that this part was censored away (Figure 6).

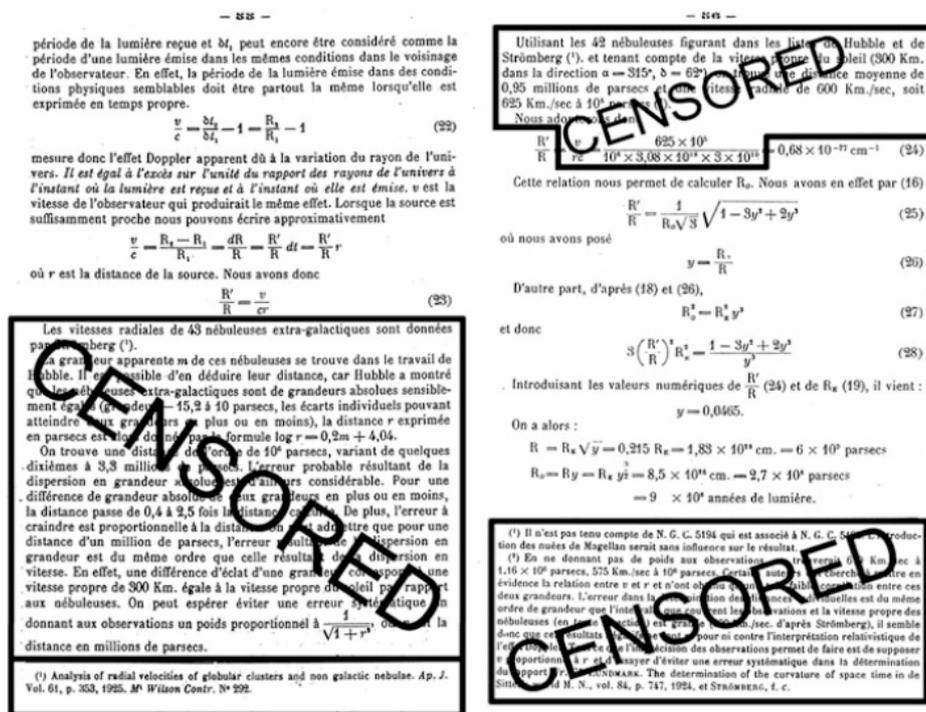


Figure 6. The sections on Lemaître’s 1927 article that were not included in the English translation published in 1931. It is here suggested that the sections were censored away [18], but that is not the case.

It is natural to wonder why the English version of Lemaître’s 1927 article was lacking vital information about how Lemaître tested the velocity-distance relation which he had theoretically deduced from his universe model that he found as a solution to Einstein’s field equations. Surely it is not a translator error of arbitrary character. It is clearly intentional that these data are not included in the translated version of the article. But was it an expression of Lemaître’s intention or that of someone else? Who translated the article, and why was the text concerned with the observational test of the velocity-distance relation, not included in the translation?

Several answers have been suggested. Jean-Pierre Luminet [27] thought that maybe the article was translated by Eddington, and there have been speculations that the changes of the paper were a result of pressure from Hubble. Additionally, it was speculated that some paragraphs in the original

article were removed as a result of some standard practice in MNRAS. A comment [50] on these translation issues was published in Nature 27 June 2011.

What actually happened was clarified a few months later by Mario Livio [51] and reported in an article published in Nature on November 10, 2011. With help from a librarian, Liliane Moens from the Archives Georges Lemaître in Louvain in Belgium, Livio found a letter from the secretary William Marshall Smart in MNRAS to Lemaître. Investigating further in the archives of MNRAS, he subsequently found Lemaître's answer. From this correspondence, it is clear that Lemaître himself translated the article from French to English, and that he was not pressured to omit that part of the text which was not included in the English translation. In the first part of his letter to Smart, Lemaître writes: "Dear dr. Smart! . . . I send you a translation of the paper. I did not find advisable to reprint the provisional discussion of radial velocities which is clearly of no actual interest...".

So we need not wonder who translated the paper and ensured that the information about the test of the velocity-distance relationship against observational data was not included in the English version of the paper. It was Lemaître himself! From what he wrote in the letter to Smart, the reason that he skipped this part of the article seems to be that he considered it superfluous in 1931 in light of the larger amount of observational data that Hubble had at his disposal when he discussed the relation in 1929. It seems that no one at that time had any suspicion about how erroneous these data were.

10. Einstein's Reaction

At the Solvay conference in Brussel in 1927, Lemaître gave Einstein a copy of his new paper about the expansion of the universe [52]. Lemaître has talked about Einstein's reaction after he had read the paper. After a few favorable technical remarks, he concluded by saying that from a physical point of view, this looked, to him, abominable.

Einstein was still convinced that the large-scale universe did not change with time, and that the static universe model he had constructed in 1917 was a realistic model of our universe.

In June 1930, a short time after Eddington had shown that the static universe model of Einstein is unstable, Einstein visited Eddington.

Half a year later, from December 1930, Einstein had a two-month long stay in America at Pasadena, where he, among others, visited the Mount Wilson observatory where Hubble made his observations. Strangely enough, Einstein does not mention Hubble in his diary. However, the work of Richard Tolman on expanding world models is mentioned by Einstein. They have clearly discussed the universe both from a theoretical perspective, and from that relating to what observational data could tell.

During this stay in America, Einstein renounced his belief in a static universe. Nussbaumer has shown [52] that the New York Times reported this from a meeting on February 11, 1931: "Dr. Albert Einstein told astronomers and physicists here today that the secret of the universe was wrapped up in the red shift of distant nebulae. "The redshift of distant nebulae has smashed my old construction like a hammer blow," said Einstein, "The red shift is still a mystery." "The only possibility is to start with a static universe lasting a while and then becoming unstable and starting to expand." Then Einstein adds: "But no man would believe this." Furthermore, he notes: "The rate figured from apparent velocities of recession of nebulae would give too short a life to the great universe. It would only be ten thousand million years old, which is altogether too short a time."" Einstein was still in doubt whether an expanding universe was a correct model of our universe, but he was no longer a dogmatic believer in a static universe.

Einstein had still not renounced the cosmological constant. In a manuscript which was hidden in an archive in Jerusalem until December 2013 [53,54], Einstein constructed an expanding universe model with a non-vanishing cosmological constant. The manuscript is not dated.

However, there exists a hint about when the manuscript was written. In his diary, Einstein wrote on 3 January 1931: "Working at the institute. Doubt about the correctness of Tolman's article about the cosmological problem. But Tolman was right". This may point at the correction Einstein made in the

first equation in Figure 7. This would mean that Einstein worked on the manuscript during the first week of January 1931.

On the first of March, he wrote on a card to his friend Besso: “The nebulae have a Doppler shift of the light proportional to their distance, which by the way follows as a consequence of the theory of relativity (without the cosmological constant)”.

In the recently discovered manuscript of Einstein, he introduced a universe model where the field equations have the form shown in Figure 7.

The image shows a handwritten note on a piece of paper. At the top, it says "Die Gleichungen (1) lauten" in cursive. Below that are two equations. The first equation is $-\frac{3}{4} \alpha^2 + \lambda c^2 = 0$, where the $\frac{3}{4}$ has a horizontal line through it. The second equation is $\frac{3}{4} \alpha^2 - \lambda c^2 = \kappa \rho c^2$.

Figure 7. Einstein’s field equations as they appear in a manuscript from January 1931.

We see from the two terms before the equality signs that Einstein still included the cosmological constant in his equations. This indicates that Einstein removed the cosmological constant from his equations either in the second half of January or in February 1931, before he sent the card to Besso on the first of March.

Back in Germany, Einstein wrote a manuscript about the cosmological problem according to the theory of relativity. The manuscript was received by the journal on 16 April 1931. In this manuscript, he wrote that one may demonstrate by means of Friedmann’s equations that his static universe is unstable, and proceeded: “For this reason alone I tend not to give my static universe model any physical significance, independent of the observational results of Hubble. Under these circumstances one has to ask oneself whether it is possible to construct a realistic universe model without introducing the cosmological constant”.

This was the official declaration of Einstein where he discarded both his static universe model and the cosmological constant.

In 1932, Einstein, together with de Sitter, constructed an expanding universe model which was later called the Einstein-de Sitter universe model. This is the most simple expanding universe model. It is filled with dust only, is spatially homogeneous, and has an Euclidean spatial geometry, i.e., it is ‘flat’. This model was the leading one until 1998, when it was discovered that the cosmic expansion is accelerated, and one needed something like the cosmological constant to explain the accelerated expansion. Thus, the cosmological constant re-entered cosmology, but not as interpreted by Einstein as an inherent tendency of empty space to expand, but with the interpretation introduced by Lemaître [27], representing the energy density of LIVE and causing repulsive gravitation.

11. The Beginning of the Universe

Around 1930, Lemaître, Robertson, and Tolman introduced expanding universe models, and the possibility of a beginning of the universe at a finite past became apparent. Hence, some researchers asked: “How did the universe begin? What caused the expansion?”

Initially the cosmologists thought the expansion might have something to do with radiation pressure. Lemaître assumed in 1927 that the space was positively curved so that light moving freely follows a circular path. If light emitted by a source moved around the universe, it would, after a round trip, return to the emitter position and a very large concentration of radiation would build

up. This would cause a radiation pressure that would cause the universe to expand. The American cosmologist R. C. Tolman speculated in 1930 about a possible transition of matter into radiation as a cause of the expansion.

But it was soon understood that radiation pressure would not cause expansion in a homogeneous universe model since only a pressure gradient causes a net pressure force, and the gravitational action of pressure is attractive and decelerates the expansion. As noted by Kragh and Lambert [27], de Sitter [47] objected in a June 1930 review of Lemaître's theory that "it would not be correct to say that the universe expanded due to the radiation pressure", but rather that the expansion was caused by the repulsive force associated with the cosmological constant.

Lemaître rapidly developed new ideas. In 1931, he wrote a small but visionary note published on 7 March 1931, with the title [55], *The Beginning of the World from the Point of View of Quantum Theory*. Here, he took into account both the quantum theory and the theory of relativity in a speculation as to how the world may have arisen. Some months earlier, Niels Bohr had published a brief article in Nature [56]. There, he argued that space and time exist only as statistical notions according to the quantum theory. Lemaître wrote: "If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning; they would only begin to have a sensible meaning when the original quantum had been divided into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time". In this way, space and time emerged, as well as vacuum energy, with a constant density represented by the cosmological constant. Lemaître was aware that these ideas were rather speculative, but such speculations are inspiring, and they have proceeded to inspire researchers up to our time.

The article [55] was extensively commented on by Luminet [57] in 2011, when it was published as a 'Golden Oldie'.

12. Eddington's 1932-Description of an Expanding Universe

In 19 March 1932, Eddington published in Nature a very nice article [58] based on a lecture from 22 January 1932, in the Royal Astronomical Society, with the title, *The Expanding Universe*. Let us enjoy some of the paragraphs:

"In recent years the line-of-sight velocities of about 90 of the spiral nebulae have been measured. When we survey these data a remarkable state of affairs is revealed. The spiral nebulae are almost unanimously running away from us; moreover, the greater the distance the greater the speed of recession. The law of increase is found to be fairly regular, the speed being simply proportional to the distance. The progression has been traced up to a distance of more than 100 million light-years, where the recession is 20,000 km. per second.

At first sight this looks as though they had a rather pointed aversion to our society; but a little consideration will show that the phenomenon is merely a uniform dilation of the system and is not specially directed at us. If this room were suddenly to expand to twice its present size, the seats separating in proportion, you would notice that everyone had moved away from you. Your neighbor who was 3 feet away has become 6 feet away; the man who was 20 feet away is now 40 feet away. Each has moved proportionately to his distance from you, which is precisely what the spiral nebulae are observed to be doing. The motion is not directed from any one center, but is a general expansion such that each individual observes every other individual to be receding.

Einstein's law of gravitation contains a term called the "cosmical term" which is extremely small in ordinary applications to the solar system, etc., and is generally neglected. The term, however, actually represents a repulsive force directly proportional to the distance; so that however small it may be in ordinary applications, if we go to distances sufficiently great it must ultimately become important. It is this cosmic repulsion which is, we believe, the cause of the expansion of the great system of the nebulae. The repulsion may be to some extent counterbalanced by the ordinary gravitational attraction of the nebulae on one another. This countervailing attraction will become weaker as the expansion increases and the nebulae become farther apart. It seems likely that the universe started with a balance

between gravitational attraction and cosmic repulsion; this equilibrium state is called an “Einstein universe.” But it can be shown that the Einstein universe is unstable; and the slightest disturbance will cause either the repulsion or the gravitation to gain the upper hand so as to topple the system into a state of continually increasing expansion or continually increasing contraction. Apparently expansion won the initial struggle, and as the nebulae spread apart the opposition of gravitation became less and less, until now it is comparatively insignificant.

We see, then, that according to observation the system of the spiral nebulae is expanding, and that relativity theory had foreseen just such an expansion (except that as an alternative it would have been content with an equally regular contraction). What better agreement could we desire? Nevertheless there were some misgivings which I would not by any means condemn as unreasonable. It is true that theory predicted an effect of the kind observed, but it did not say how rapid the expansion would be. It expressed it in terms of an unknown “cosmical constant” λ , leaving λ to be determined by observation. Now the rate of expansion indicated by observation comes to us as a great shock. The universe is expanding so as to double its dimensions every 1300 million years; that is no more than the period of geological time. Astronomers, who had been picturing a slow evolution of the stars extending over billions of years, would scarcely believe our staid old universe capable of such a hustle. In fact it means a cut of something like 99 per cent in our time-scale, which even in these days of economy cuts is not to be accepted lightly by the department concerned. For this reason many have thought that the receding motions of the spiral nebulae cannot be accepted as genuine, and that the whole phenomenon must be explained away as a misinterpretation of the red-shift observed in their spectra. I think, however, that we shall have to accept the expansion”.

This was what Eddington said on 22 January 1932. He did not mention then that the age problem was already solved by Lemaître. Looking at the evolution of the scale factor as shown in Figure 2, we see that there exists a period which may last for a long time where there is close to equilibrium between the gravitational attraction due to matter and cosmic repulsion associated with the cosmological constant. Eddington’s description is based upon Lemaître’s 1927 article, the English translation of which was published in MNRAS in March 1931, while the time evolution shown in Figure 2 is based upon the new universe model Lemaître presented in November 1931.

13. Hubble’s Interpretation of the Observations

The authorship of Hubble indicated that he did not at any time believe in Lemaître’s interpretation of the observational data, that the universe expands, and he had reasons for not doing so. First and foremost, his value of the Hubble-parameter only gave a value of two billion years for the Hubble age, while the British astrophysicist James Jeans estimated that the oldest stars were more than ten times older.

The velocity-distance relation (1) is based upon an interpretation of the observational data. One did not measure velocities, but shifts of wavelengths of spectral lines. The spectral lines were shifted towards longer wavelengths for most of the spiral nebulae, and this was called the cosmic red shift. If these red shifts were interpreted as a Doppler effect, the shifts meant that the sources of the observed light moved away from the observer. In this way, the relation (1) resulted. However, Hubble was far from sure that it was correct to interpret the red shifts as a Doppler effect. Therefore, he was careful to write ‘apparent velocity’ when he wrote about the relation (1).

In 1942, Hubble wrote an article [59] published in *Science*, where he concluded that the red shift is not a velocity effect, but a result of an unknown mechanism acting in the space between the galaxies.

In 1953, Hubble gave the George Darwin-lecture [60], which he titled, *The Law of Red-Shifts*. He divided the topic into three historical parts, and said that the first one ended with the formulation of the law (1) in 1929. Again, he did not mention Lemaître.

14. Summary on the Discovery of the Cosmic Expansion

The possibility of an expanding universe was, for the first time, predicted as a consequence of the theory of relativity by Alexander Friedmann in 1922. Friedmann demonstrated that Einstein's gravitational equations have solutions that represent expanding universe models. This was not a discovery of the expansion of the universe, but a strong motivation for the observers to investigate the possibility that the universe might expand. This consequence of the theory of relativity was rediscovered by Lemaître in 1927. However, he went on and deduced from the theory that the remote galaxies should have a velocity away from us proportional to their distance. Furthermore, he tested this prediction against observations. Lemaître did show, admittedly with limited observational material, that the prediction was in accordance with the observations. Hubble did something else. Hubble used his own observations and those of Humason and Slipher and demonstrated in 1929 that the observations indicated a proportionality between the radial velocities of remote galaxies and their distances. He refrained from interpreting this in light of the theory of relativity.

Eddington wrote in his book, *The Expanding Universe* [61], which was published in 1933: "The investigation of a non-static solution was performed by Friedmann in 1922. His solution was rediscovered in 1927 by Abbe Lemaître, who in a brilliant way developed the astronomical consequences from this solution. His article was published in an unavailable journal, and seems to have been unknown until 1930 when de Sitter and I made it known. In the mean time the solution was discovered a third time by Robertson, and the researchers now began to be aware of it. The astronomical relevance was also rediscovered thanks to Hubble and Humason's observational work with spiral galaxies, but was not worked out so far as in the article of Lemaître".

Briefly, Friedmann was the first one who found expanding universe models as solutions of Einstein's field equations, and Lemaître was the first one who deduced "Hubble's law" for such universe models and calculated the value of the Hubble parameter from observational data.

Who discovered the expansion of the universe? It has been customary to say that it was Hubble. As we have seen above, several historians of science have pointed out that this is not historically correct. Several years before Hubble's 1929 article, Wirtz and Lundmark analyzed observational material on the redshifts and distances of distant spiral nebulae, and concluded that the data indicated that the system of these nebulae expanded with a velocity that was larger the farther away the nebulae were.

Furthermore, Lemaître announced the relation (1) two years before Hubble. He also interpreted it in light of his solution to Einstein's field equation describing an expanding universe model, predicted the expansion of the universe, tested the relation (1) against observational data, and verified that it was in agreement with the available data. He was the first to connect the velocity and distance data to an expanding model for the universe. Mario Livio [53] has given the following summary: "It would seem fair to credit the discovery of the expanding Universe and the tentative existence of a Hubble law to Lemaître; and the detailed confirmation of that law to Hubble and Humason. They were, however, not willing to interpret the relation (1) as a proof of the expansion of the universe".

Holder and Mitton [62] have written: "Lemaître published the original version of Hubble's law, and he gave the first estimate of the Hubble parameter. Also he proposed a "fireworks model" of the universe which was later called the Big Bang theory for the origin of the universe. For this reason he is now popularly regarded as the "father of the Big Bang theory"." This term seems to have been introduced by Jim Peebles who wrote [63] in 1971: "According to the usual criterion for establishing credit for scientific discoveries Lemaître deserves to be called the "Father of the Big Bang Cosmology"".

At the present time, the correct recognition of the discoverer of the expansion of the universe is of particular relevance due to the recent process in IAU of giving a proper name to the law of cosmic expansion [64]. In this connection, we should note a statement in a recent book by D. Lambert [65] concerning the name of this law: "It would be more accurate to call it [Hubble's law] the 'Hubble-Lemaître law'". This has now been decided by IAU.

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