Article

Testing the Anomalous Growth of the Black Hole Radius from AGN

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Received: 18 July 2018; Accepted: 20 September 2018; Published: 3 October 2018

Abstract: We analyze constraints on the anomalous growth of the black hole radius or the black hole spin from the X-rays spectrum data of Active Galactic Nuclei (AGN) in NGC 5506. The anomalous growth of the mass or of the spin of a black hole may be unveiled within the framework of models of alternative gravity, including $f(R)$ -gravity. Our phenomenological analysis is based on an effective parametrization for the black hole Kerr metric, which is inspired by the antievaporating solutions discovered by Nojiri and Odintsov. We find tight constraints on the parameter space of anomalous metrics. Intriguingly, we find that a more than secularly growing solution can better fit current data. Our result opens a pathway towards a new phenomenological approach for testing predictions of general relativity and alternative theories of gravity.

Keywords: alternative gravity; black holes; gravitational instabilities; AGN physics

1. Introduction

There exist several open problems of General Relativity (GR) both in the IR and in the UV regimes that may motivate its extension. It is indubitable that GR has very successfully explained many astrophysical and cosmological observations. However, many puzzles still remain in our comprehension of the universe, including the nature of dark energy and dark matter, and the understanding of the homogeneity of the Cosmic Microwave Background (CMB) from a microphysical point of view, consistent with data we have on particle physics.

One of the simplest extensions of GR is $f(R)$ -gravity, a theoretical scenario that includes the $(R + \zeta R^2)$ Starobinsky's model for inflation [1–5]. It is renowned that $f(R)$ -gravity can be conformally mapped onto scalar-tensor theories [1–4], in unambiguous regular space-time backgrounds. It then becomes crucially important to constrain the space of parameters of the possible alternatives to GR, deploying all the different astrophysical and cosmological observations that are available. A frontier for testing GR is black hole (BH) physics. From the theoretical point of view, many recent analyses have shown the presence of antievaporation instabilities within a large class of BH solutions, appearing universally in extended theories of gravity.

Specifically, the antievaporation phenomenon consists in the growth in time of the radius of the BH horizon. It was discovered and first studied by Bousso and Hawking, within the quantum dilaton-gravity—see e.g., Reference [6], and then further analyzed in References [7–9]. More recently, Nojiri and Odintsov discovered that the same antievaporation effect can also show up in $f(R)$ -gravity, but at the classical level, as pointed out in References [10,11]—see also Reference [12] for further analyses. Antievaporation was also recovered in several other contexts, including Gauss-Bonnet gravity [12,13], $f(T)$ -gravity [14], Mimetic Gravity [15,16], Bigravity [17], string-inspired dyonic BH solutions [18], brane-world cosmology [19–22] and Bardeen De Sitter BHs [23].

Such antievaporation seems also to be related to the information paradox in BH physics—see e.g., [24–27]. In Reference [28], it was shown that classical (anti)evaporation instabilities switch off the emission of Bekenstein-Hawking radiation—through the analysis of the Raychaduri equation of BH closed trapped Cauchy’s surfaces [29–31]. Last but not least, it was demonstrated that antievaporation is directly related to a deformation of the energy conditions in extended theories of gravity [13].

We wish to emphasize that the antievaporation phenomena are not just a mean to recast idealistically unphysical models. Conversely, they are compatible with realistic extensions of GR that have passed the test-bed of cosmological and astrophysical limits. It is then worth noting that, among all the possible $f(R)$ -gravity extensions, higher derivative polynomial extensions beyond Starobinsky’s gravity, namely the popular Hu-Sawicki model, and exponential models of $f(R)$ -gravity universally predict the occurrence of antievaporation phenomena—see Reference [12] for a detailed discussion on these aspects. Finally, we remind that antievaporation is still unexplored for rotating BHs, due to well known technical difficulties of the analysis. Nonetheless, it is an open possibility that in this case not only the mass, but also the BH spin, may exhibit an anomalous growth in time.

In this paper, we will analyze the constraints on the anomalous growth of the BH mass and spin from X-ray spectrum of AGN observations, motivated and inspired by the aforementioned theoretical considerations. Recently, a series of papers discussed the possibility of tests of GR BHs from a multi-epoch X-ray spectroscopy analysis of the galaxy NGC5506 [32–38]. In Reference [32], the data analysis simultaneously combines spectra measured by Chandra, XMM-Newton, Suzaku and *NuSTAR*—for covering 200–2014 time span—using standard models for describing the propagation and scattering of primary continuum by optical matter in the AGN medium. The main result was the determination of the spin of the supermassive BH in AGN, with $0.91 < a < 0.98$ at 90% C.L. In every analysis in References [32–38], a time dependence of the mass and spin parameters was not assumed. Our goal is to relax such an assumption, in order to infer new constraints on anomalous time dependent Kerr BHs. We will display tight constraints on the variation in time of the mass and spin. We will then show a surprising result: the mass and the spin that best fit data corresponds to a more than secularly growing anomalous Kerr solution, instead of a static solution. Nonetheless, we cannot claim that we are able to discriminate whether: (i) a solution simply reflects the variation in time of the local matter flux inside the BH, in the AGN environment; (ii) this solution is a genuine hint in favor of exotic new BHs, beyond GR. These considerations highly motivate further analyses on other AGNs in the future.

2. Anomalous Kerr Metric

We start the analysis by remembering the Kerr metric solution of GR, namely

$$ds^2 = -dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + (r^2 + a^2) \sin^2 \theta d\phi^2 + \frac{2GMr}{\rho^2} (a \sin^2 \theta d\phi - dt)^2, \quad (1)$$

with

$$\Delta(r) = r^2 - 2GMr + a^2, \quad \rho^2(r, \theta) = r^2 + a^2 \cos^2 \theta, \quad (2)$$

in which a is the rotation of the BH and M is its mass. In GR, this metric is static if no extra matter in-falls inside the BH. So that the BH accretion in AGN should be sourced only from gas in-falling in the accretion disk.

In the Anomalous Kerr Metric that we propose as an effective ansatz, the mass and the spin parameter can change in time. Therefore, the metric we will be focusing reads

$$ds^2 = -dt^2 + \frac{\rho^2}{\Delta(r, t)} dr^2 + \rho(r, \theta, t)^2 d\theta^2 + (r^2 + a(t)^2) \sin^2 \theta d\phi^2 + \frac{2GM(t)r}{\rho(r, \theta, t)^2} (a(t) \sin^2 \theta d\phi - dt)^2, \quad (3)$$

with

$$\Delta(r, t) = r^2 - 2GM(t)r + a(t)^2, \quad \rho^2(r, \theta, t) = r^2 + a(t)^2 \cos^2 \theta. \quad (4)$$

Within the effective theory approach, one can parametrize the time dependence of M and a with several different ansätze that are motivated by the different theoretical frameworks that can be advocated from the beginning of the analysis. The antievaporation-inspired ansatz that we propose reads

$$M(t) = M_0(c_1(\cosh \alpha^2 t)^n + c_2(\sinh \beta^2 t)^m), \quad a(t) = a_0(k_1(\cosh \gamma^2 t)^l + k_2(\sinh \delta^2 t)^p), \quad (5)$$

where M_0 and a_0 are the initial mass and the spin parameters at fixed initial time $t = 0$, and finally $c_1, c_2, k_1, k_2, \alpha, \beta, \delta, \gamma, n, m, l$ and p are free real parameters. Notice that GR can be recovered when $n = m = 0$ and $c_1 + c_2 = 1$, and at the same time $l = p = 0$ and $k_1 + k_2 = 1$. The introduction of one or more effective energy-scales will provide, for many analytical examples of $f(R)$ -theories, the dependence that is required in order to express these dimensionless parameters.

3. Best Fit of the Spin Evolution in Time

The data sample that we use combines the spectra observed by Chandra, XMM-Newton and Suzaku. All the data reduction methods are the same as in Reference [32], and the details on these aspects can also be found in that paper. The data we use in our analysis are displayed in the Table 1.

Table 1. The data we used.

obsID	Observatory	Exposure Time [s]	Start Time [MJD]	Epoch Index
0013140101	XMM-Newton	20,007	51,942	E01
0201830201	XMM-Newton	21,617	53,197	E02
0201830301	XMM-Newton	20,409	53,200	E03
0201830401	XMM-Newton	21,956	53,208	E04
0201830501	XMM-Newton	20,411	53,234	E05
701030010	Suzaku	47,753	53,955	E06
701030020	Suzaku	53,296	53,958	E07
701030030	Suzaku	57,406	54,131	E08
0554170101	XMM-Newton	88,919	54,833	E09
1598	Chandra	90,040	51,909	E10

The spectra are fitted through the model in Reference [32]. According to Reference [32], since the combined fitting result of the spin is 0.98, the spin result of each epoch far away from 0.98 can be ignored. Thus the final results are extracted from five XMM spectra and one Chandra spectrum that remain. The effective result is displayed in Table 2.

Table 2. The effective result.

obsID	Time	Spin	Error
0013140101	51,942	0.989993	± 5.97296
0201830201	53,197	0.985785	$\pm 2.72601 \times 10^{-2}$
0201830301	53,200	0.989983	$\pm 3.26932 \times 10^{-2}$
0201830401	53,208	0.979123	± 3.51087
0554170101	54,833	0.990000	$\pm 8.70283 \times 10^{-3}$
1598	51,909	0.986380	± 0.187406

Using the parametrization proposed in Equation (5), the best fit of data is achieved for a tiny time dependent spin profile (see Figure 1), namely

$$a(t) = 0.55733 (\sinh(0.05066 \cdot 10^{-5} t))^{1.10871} + 0.97423 (\cosh(0.13576 \cdot 10^{-5} t))^{1.00531}, \quad (6)$$

which seems to be favored with respect to the null time-dependence hypothesis. In particular, the best fit parameters inferred by our analysis are the following:

$$c_2 = 0.25844 \pm 2.42476, \quad c_1 = 0.48532 \pm 0.04714, \quad (7)$$

$$p = 1.10871 \pm 4.94557, \quad l = 1.00531 \pm 0.20467, \quad (8)$$

$$\gamma^2 \cdot 10^5 = 0.13576 \pm 2.79292, \quad \delta^2 \cdot 10^5 = 0.05066 \pm 0.39751. \quad (9)$$

Figure 1. The best fit spin solution ($a(t)$ in the text) is displayed as a function of time.

4. Mass Fit

The method that we use in order to infer the mass value from the data was firstly proposed in Reference [39], using a correlation law between mass and narrow line. It is well known that the relation between the mass and the standard error of the narrow line is

$$\ln(M/M_{\odot}) = \tilde{\alpha} + \tilde{\beta} \ln(\sigma), \quad (10)$$

where $\tilde{\beta} = 4.02$ in Reference [39]. On the other hand, the BH mass of NGC5506 that we consider as input to Equation (10) is $88 \times 10^6 M_{\odot}$ —see e.g., [40].

In our analysis, we deploy the same model considered for the spin fitting, and a gaussian line profile. We then find a narrow line, in the NGC5506 spectrum, the wavelength of which is between 3.5 Å and 3.8 Å. In the data sample, we ignore the spectrum, the narrow line of which is hard to detect with experimental devices. We emphasize that the standard error of the narrow lines have two different values at different observation times. This is an effect caused by the changing of X-ray status: in X-ray band, the luminosity of the X-ray is much larger than the narrow line generated by the host galaxy light. As a consequence, one has to separate the spectrum in two samples, and then calculate the value of α in Equation (10), with two different combined fit results and the mass of NGC5506. The standard error of the narrow line and the results of the mass are provided in Table 3.

Table 3. The standard error of the narrow line and the results of the mass.

obsID	Standard Error	Error	Mass
0201830401	0.6053	± 0.1143	68.28
0201830501	0.6683	± 0.1201	101.66
701030010	0.6527	$\pm 7.7903 \times 10^{-2}$	92.46
701030020	0.2157	± 0.1049	60.48
701030030	0.6544	± 0.1343	93.45
0554170101	0.2600	$\pm 6.2502 \times 10^{-2}$	128.03

We can now fit the model of mass proposed in Equation (5), with the same parametrization used for the spin fit. The best fit expression, plotted in Figure 2, of the BH mass reads

$$M(t) = (43.5857(\cosh(0.0735094 \cdot 10^{-5}t))^{0.779544} + 43.5857(\sinh(0.0742621 \cdot 10^5t))^{0.400512})M_{\odot} \cdot 10^6, \quad (11)$$

in Table 4.

Table 4. The parameters and values.

Parameters	Values	Errors
$M_0/M_{\odot} \cdot c_1 \cdot 10^{-6}$	43.5857	± 4.75554
$M_0/M_{\odot} \cdot c_2 \cdot 10^{-6}$	43.5857	± 4.75554
m	0.400512	± 18194.9
n	0.779544	± 10049.6
$\alpha^2 \cdot 10^5$	0.0735094	± 4.69429
$\beta^2 \cdot 10^5$	0.0742621	± 0.185221

Let us stress that, in such an analysis, all the time dependence of the narrow lines ($\ln(\sigma)$) is originated from the time dependence of the Black hole mass ($\ln(M/M_{\odot})$). The $\tilde{\beta}$ parameter is assumed as fixed in time, since our modified metric does not introduce any modification on the narrow-lines/mass intercept parameter.

Figure 2. The best fit mass solution $m = m(t)$ in solar mass units M_0 is displayed as a function of time.

Exclusion Plots

Finally, in Figures 3–13, we scan many different subspaces of mass and spin parameters, as level/region exclusion plots in the 12 + 12 dimensional parametrization in Equation (5). Very large regions (displayed as the colored ones) of these latter are already ruled out by data.

Figure 3. Constraints on α, β in the $s = s(t)$ profile are displayed.

Figure 4. Constraints on the initial BH mass and the α^2 are displayed. We assume $c_1 = c_2$ in the mass function.

Figure 5. Constraints on the index p and the γ^2 in spin function are displayed. We assume $p = l$ in our test.

Figure 6. Constraint on the initial BH mass $M_0 / (10^6 M_{\text{sun}})$ and the $(10^5) a^2$ are displayed. We assume $m = n$ in our test.

Figure 7. Constraints on p and γ are displayed. We assume $p = l$ in our test.

Figure 8. Constraints on the initial BH mass and α are displayed. This is the parameter space in mass function. We assume $c_1 = c_2$ in our test.

Figure 9. Constraints on m and α are displayed. We assume $m = n$ in our test.

Figure 10. Constraints on the initial BH mass and m are displayed. We assume $c_1 = c_2$ and $m = n$ in our test.

Figure 11. Constraints on the constant c and γ are displayed. We assume $c_1 = c_2$ in our test.

Figure 12. Constraints on the p and γ are displayed. We assume $p = l$ in our test.

Figure 13. Constraints on γ, θ in the spin function profile are displayed.

5. Conclusions and Remarks

We have analyzed the anomalous Kerr metrics deploying AGN data. This provides a test of Standard BH solutions of General Relativity. Our effective time dependent parametrization of the anomalous Kerr metrics is inspired by the antievaporating solutions that can be recovered within the framework of extended theories of gravity—in particular $f(R)$ -gravity. If a space-time solution with a mass and a spin parameter growing or decreasing in time was found in AGN, then it would imply that there would be a new aspect which is, at least, still missing and not understood in the standard model of AGN accretion disks proposed in literature. In the case of growing mass or spin metrics, such solutions will satisfy Einstein's equation if and only if extra matter is in-falling into the black hole interior. In our case, we already considered possible effects of mass and spin variation from standard accretion disk matter, using the same discrimination methods of References [32–37]. Our aim is to limit solutions in which the mass and spin parameters increase with time from other causes than the accretion disk matter. If an evidence of such an effect was found, it should imply that: (i) a new kind of matter, such as dark matter, is inflating down into the black hole interior, while Einstein's equation is not modified; or (ii) Einstein's equations should be modified into new equations solved by Kerr anomalous metrics. On the other hand, in the case of anomalous Kerr solutions decreasing in time, we cannot imagine any possible explanation which is different from gravity modification: matter cannot go out from the black hole interior and, indeed, it cannot cause any decrease of the black hole mass and spin¹ This highly motivates our analysis as a test of theories beyond General Relativity to alternative theories of dark matter.

Intriguingly, we found tight bounds on the parameters space of the anomalous Kerr metric. This is certainly a phenomenologically healthy signal that such a new paradigm of testing GR from AGN can

¹ The only radiation emission is in the form of Hawking's radiation, which is highly suppressed and undetectable for macroscopic black holes.

achieve a very high precision. At effective time scales, parameters corresponding to $O(1)$ corrections to the standard GR masses and spins occur at an order of $5 \times 10^2 \div 10^3$ years of observations. Thus, the time variation of the mass and spin must be more than secularly growing in time. However, we find that a more-than-secularly growing metric in mass and spin fits better the data than the standard (null hypothesis) Kerr metric. Intriguingly, this may be a hint of a secularly growing BH beyond General Relativity. However, we cannot claim that the significance of our fit is powerful enough to conclude that the variation in time of the mass and spin that was detected was not provoked by a variation of matter in-falling the AGN BH during its cosmological history. In order to clarify this phenomenological aspect, further analyses from AGN data will be crucially important in enforcing the hypothetical hint. Indeed, even if the best fit favored by data seems to point toward time varying mass and spin parameters, such a result is still lying under uncertainties on the accretion disk modeling. At the moment, our best fit is not out of the 3σ of significance from the Standard AGN picture based on General Relativity, i.e., it cannot provide more than a mild hint of new physics. On the other hand, we think that such a mild hint result may be scrutinized by future experiments with higher resolution in the X-ray spectrum.

Finding an anomalous Kerr BH solution in a specific extended theory of gravity—for example in $f(R)$ -gravity—is undoubtedly challenging from a theoretical perspective. However, finding a non-spherically symmetric solution in theories beyond GR, and studying the metric perturbations of it, seems a herculean task, which, for the moment, is missing in literature. Conversely, it is conceivable that these solutions could exist, since spherically-symmetric anomalous solutions were found in many alternative, aforementioned, models of gravity. On the other hand, our phenomenological approach may also have a great impact in our understanding of BH thermodynamics. As in the case of antievaporation instabilities, the BH thermodynamical properties radically change, compared to the Bekenstein-Hawking picture, leading to a suppression of the radiation. In other words, secularly growing BHs may provide new not-paradoxical objects, thus solving the standard information problems of GR BHs.

Author Contributions: Conceptualization, A.A. and A.M.; Formal analysis, C.Q.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

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