

Article

Supersymmetric M-brane Solution in a Dynamical Background

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Abstract: Supersymmetry arises in certain theories of fermions coupled to gauge fields and gravity in a spacetime of 11 dimensions. The dynamical brane background has mainly been studied for the class of purely bosonic solutions only, but recent developments involving a time-dependent brane solution have made it clear that one can get more information by asking what happens on supersymmetric systems. In this proceeding, we construct an exact supersymmetric solution of a dynamical M-brane background in the 11-dimensional supergravity and investigate supersymmetry breaking, the geometric features near the singularity and the black hole horizon.

Keywords: string and brane; supersymmetry; supergravity; classical black holes; singularities in general relativity

1. Introduction

The dynamical p -brane solutions in a higher-dimensional gravity theory were studied by [1–32] and have been widely discussed ever since. However, some aspects of the physical properties, such as supersymmetry and its breaking in the context of string theory, have remained slightly unclear. The motivation for this work is to improve this situation. For this purpose, it is first necessary to construct supersymmetric brane solutions depending on the time, as well as the space coordinates.

In the static background, an M-brane solution in the 11-dimensional supergravity has been constructed, and the properties have been discussed [33]. In the dynamical background, it is well known that there are p -brane solutions with dynamical several p -brane objects in the expanding universe. The first example was found for a D3-brane in the ten-dimensional Type IIB string theory [1], which was generalized for complicated field configurations [6,9]. There has been, however, little success at constructing the dynamical p -brane solution preserving supersymmetry, nor has there been much insight about what kind of geometrical structure might be expected.

The dynamical M2-brane background preserving supersymmetry is a kind of natural extension of the static M2-brane system, which can be described by an analogous Reissner–Nordström solution in the four-dimensional Einstein–Maxwell theory. The existence of supersymmetry in a dynamical background should not come as a surprise, since several analytic solutions in string theories are already known [3,34–36]. In this proceeding, we will find the supersymmetric dynamical M-brane as an exact solution of the supergravity field equations. What we will construct is a time-dependent M2-brane solution preserving supersymmetry in the 11-dimensional supergravity theory. Depending on which ansatz we take, we thus obtain a black hole in the expanding universe. Although it is not necessarily easy, the supersymmetric black hole models governing the dynamics of the universe can be constructed analytically because these are given by the classical solution of field equations.

Different forms of the dynamical brane solution we will be discussing have been obtained by [3] as a supersymmetric solution in a ten-dimensional Type IIB string theory and by [6,9] as a cosmological solution without supersymmetry for an 11-dimensional supergravity model. A class of classical black hole solutions in the expanding universe was found by [13,18]. Time-dependent black hole solutions in lower dimensional effective field theories derived from string theory have been analyzed in [9,20].

Although we will consider in this proceeding the 11-dimensional supergravities, there is also a ten-dimensional version of the supersymmetric dynamical D-brane solutions. It can be obtained by compactifying an internal space. In terms of the dimensional reduction of a dynamical M-brane background to the string in ten dimensions, the solution leads to the dynamical D-brane systems. One starts with an 11-dimensional model, but the resulting ten-dimensional model turns out to have a dynamical D-brane, as in the construction of [16].

This proceeding is organized as follows. We present an exact solution having a quarter of a full supersymmetry for a dynamical M-brane in an 11-dimensional supergravity and discuss how to break supersymmetries in Section 2. In the remainder of the proceeding, we describe some applications of the result, which are the behavior of the geodesic, the analysis of the geometrical structure and the evolution of a time-dependent black hole in the dynamical M2-brane background. In Section 3, we start our discussion of the supersymmetric M2-brane solution by examining the basic features of the background geometry. By solving the radial null geodesic equations, we show that the naked strong curvature singularity appears. Then, we investigate extensions of the solutions inside the horizon and discuss the smoothness at the horizon. In Section 4, we present that the M2-brane background gives a black hole solution in a time-dependent universe and discuss the implications for lower-dimensional effective theories. Section 5 contains some discussions and concluding remarks.

2. Dynamical M-brane Backgrounds

In this section, we will construct the exact solution to the field equations of an 11-dimensional supergravity corresponding to a dynamical M2-brane configuration. The 11-dimensional gravitino (Killing spinor field) equation gives the time-dependent solution with the particular ansatz of fields. We find that the supersymmetric solution depends on the null coordinate along the M2-brane world volume, as well as the coordinates of the transverse space to the M2-brane.

2.1. Supersymmetry in a Dynamical M2-brane

We will start by making an ansatz for an 11-dimensional metric g_{MN} and three-form gauge potential $A_{(3)}$. The 11-dimensional metric and gauge potential are assumed to be

$$ds^2 = A^2(x, y)\eta_{\mu\nu}(X)dx^\mu dx^\nu + B^2(x, y)\delta_{ij}(Y)dy^i dy^j, \quad (1a)$$

$$A_{(3)} = \chi C(x, y)\Omega(X), \quad (1b)$$

where $\mu, \nu = 0, 1, 2$ and $i, j = 3, 4, \dots, 10$, $\chi = \pm 1$, and $\Omega(X)$ denotes the volume form of the three-dimensional Minkowski space (X space). All components of the gravitino ψ_M are zero. The arbitrary functions A , B and C depend on the M2-brane world volume coordinates x^μ , as well as the radial coordinate of the eight-dimensional Euclidean space (Y space)

$$r^2 = \delta_{ij}y^i y^j. \quad (2)$$

Then, the metric of Y space becomes

$$\delta_{ij}(Y)dy^i dy^j = dr^2 + r^2 u_{ab}(Z)dz^a dz^b, \quad (3)$$

where $u_{ab}(Z)$ denotes the metric of the seven-sphere. We will find that three functions A , B and C are reduced to one by the requirement that the metric and gauge field preserve supersymmetry. Then, we find a 11-dimensional Killing spinor ε satisfying

$$\bar{\nabla}_M \varepsilon = 0. \quad (4)$$

Here, $\bar{\nabla}_M$ is the supercovariant derivative appearing in the supersymmetry transformation rule of the gravitino

$$\bar{\nabla}_M = \partial_M + \frac{1}{4} \omega_M^{PQ} \Gamma_{PQ} + \frac{1}{12} (\Gamma_M \mathbf{F} - 3\mathbf{F}_M), \quad (5)$$

in terms of the 11-dimensional γ -matrices Γ^M satisfying

$$\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = 2g^{MN}. \quad (6)$$

\mathbf{F} and \mathbf{F}_M are defined by

$$\mathbf{F} = \frac{1}{4!} F_{MNPQ} \Gamma^{MNPQ}, \quad (7a)$$

$$\mathbf{F}_M = \frac{1}{3!} F_{MNPQ} \Gamma^{NPQ}, \quad (7b)$$

and $F_{(4)}$ is the field strength defined by the three-form gauge potential $A_{(3)}$,

$$F_{(4)} = dA_{(3)}. \quad (8)$$

The notation that has been used here is

$$\Gamma_{MN\dots P} = \Gamma_{[M} \Gamma_N \cdots \Gamma_{P]}. \quad (9)$$

For the background (1a) and (3), it is convenient to introduce γ_μ ($\mu = 0, 1, 2$), γ_r and γ_a ($a = 4, \dots, 10$) by

$$\Gamma^\mu = A^{-1} \gamma^\mu, \quad \Gamma^r = B^{-1} \gamma^r, \quad \Gamma^a = \frac{1}{rB} \gamma^a. \quad (10)$$

Then, γ^μ gives the SO(2, 1) γ -matrices; γ^a provides the γ matrices of Z ; and $(\gamma^r)^2 = 1$. We also define $\gamma_{(3)}$ as

$$\gamma_{(3)} := \gamma_0 \gamma_1 \gamma_2. \quad (11)$$

We take an ansatz for the 11-dimensional metric (1a)

$$A = C^{1/3}, \quad B = C^{-1/6}, \quad C \equiv h^{-1}(x, r). \quad (12)$$

Then, in terms of these γ matrices, the supercovariant derivative in the background with the metric (1a), (3) and field (1b) is expressed as

$$\bar{\nabla}_\mu = \partial_\mu + \frac{1}{6} \partial_\nu \ln h \gamma^\nu{}_\mu - \frac{1}{6} h^{-3/2} \partial_r h \gamma^\mu \gamma^r (1 - \chi \gamma_{(3)}), \quad (13a)$$

$$\bar{\nabla}_r = \partial_r - \frac{1}{12} h^{-1/2} \partial_\nu h \gamma^\nu \gamma^r + \frac{1}{6} \chi h^{-1} \partial_r h \gamma_{(3)}, \quad (13b)$$

$$\bar{\nabla}_a = {}^Z \nabla_a - \frac{r}{12} h^{-1/2} \partial_\nu h \gamma^\nu \gamma_a - \frac{r}{12} h^{-1} \partial_r h \gamma^r \gamma_a (1 - \chi \gamma_{(3)}), \quad (13c)$$

where ${}^Z\nabla_a$ is the covariant derivative with respect to the metric $u_{ab}(Z)$. The number of unbroken supersymmetries is given by the number of Killing spinors ε . The Killing spinor Equation (4) is automatically satisfied provided that the following three conditions are satisfied

$$\varepsilon = h^{-1/6}\varepsilon_0, \quad \partial_\mu h \gamma^\mu \varepsilon = 0, \quad (1 - \chi\gamma_{(3)})\varepsilon = 0, \quad (14)$$

where the sign χ comes from the ansatz of the three-form gauge potential (1b) and ε_0 denotes a constant Killing spinor.

2.2. Einstein Equations

Now, we determine a form of the function $h(x, r)$ in an 11-dimensional supergravity theory, which is composed of the metric g_{MN} and the four-form field strength $F_{(4)}$. The action in 11 dimensions is given by

$$S = \frac{1}{2\kappa^2} \int \left[R * \mathbf{1} - \frac{1}{2 \cdot 4!} * F_{(4)} \wedge F_{(4)} \right] - \frac{1}{12\kappa^2} \int A_{(3)} \wedge F_{(4)} \wedge F_{(4)}, \quad (15)$$

where R denotes the Ricci scalar with respect to the 11-dimensional metric g_{MN} , κ^2 is the 11-dimensional gravitational constant, $*$ denotes the Hodge operator in the 11-dimensional spacetime and $F_{(4)}$ is the four-form field strength defined by (8), respectively.

Let us first consider the gauge field equation

$$d(*F_{(4)}) + \frac{1}{2}F_{(4)} \wedge F_{(4)} = 0. \quad (16)$$

Using the ansatz of fields (1a) and (12), the above equation is reduced to

$$\partial_\mu \partial_r h = 0, \quad \left(\partial_r^2 + \frac{7}{r} \partial_r \right) h = 0. \quad (17)$$

From Equation (17), the function h and the field equation can be expressed as

$$h(x, r) = h_0(x) + h_1(r), \quad \left(\partial_r^2 + \frac{7}{r} \partial_r \right) h_1 = 0. \quad (18)$$

Then, imposing the boundary condition that the 11-dimensional metric is asymptotically vacuum spacetime, we find

$$h_1(r) = \tilde{c} + \frac{M}{r^6}, \quad (19)$$

where \tilde{c} is constant.

Next, we show that Equation (19) is consistent with the Einstein equations and derive the equation for the function h_0 .

The Einstein equations are given by

$$R_{MN} = \frac{1}{2 \cdot 4!} \left[4F_{MABC}F_N{}^{ABC} - \frac{1}{3}g_{MN}F_{(4)}^2 \right]. \quad (20)$$

Using the assumptions (1) and (12), the Einstein equations become

$$-h^{-1}\partial_{\mu}\partial_{\nu}h + \frac{1}{3}h^{-1}\eta_{\mu\nu}\left[\Delta_{\mathcal{X}}h + h^{-1}\left(\partial_r^2 + \frac{7}{r}\partial_r\right)h\right] = 0, \quad (21a)$$

$$\Delta_{\mathcal{X}}h + h^{-1}\left(\partial_r^2 + \frac{7}{r}\partial_r\right)h = 0, \quad (21b)$$

$$R_{ab}(Z) - 6u_{ab}(Z) - \frac{1}{6}r^2u_{ab}(Z)\left[\Delta_{\mathcal{X}}h + h^{-1}\left(\partial_r^2 + \frac{7}{r}\partial_r\right)h\right] = 0, \quad (21c)$$

$$\partial_{\mu}\partial_r h = 0, \quad (21d)$$

where $\Delta_{\mathcal{X}}$ is the Laplace operator on the space of \mathcal{X} and $R_{ab}(Z)$ is the Ricci tensor of the metric $u_{ab}(Z)$.

From Equation (21d), the function h must be in the form

$$h(x, r) = h_0(x) + h_1(r). \quad (22)$$

With this form of h , the Einstein equations reduce to

$$R_{ab}(Z) = 6u_{ab}(Z), \quad (23a)$$

$$\partial_{\mu}\partial_{\nu}h_0 = 0. \quad (23b)$$

In this case, the first equation is automatically satisfied, and the solution for h can be written explicitly as

$$h(x, r) = c_{\mu}x^{\mu} + \bar{c} + \frac{M}{r^6}, \quad (24)$$

where c_{μ} , \bar{c} and M are constant parameters. As seen from supersymmetric Equation (14), the parameters c_{μ} have to obey the relation (14), which is given by $c_{\mu}\gamma^{\mu}\varepsilon_0 = 0$. Therefore, without loss of generality, we shall impose that $\bar{c} = 0$ and $c_{\mu}x^{\mu} = c(t - x)/\sqrt{2}$, where c is a constant.

We comment about the contribution of the source term coming from M2-brane. Let us consider the field equations, which follow from the action

$$S' = S + S_{M2}, \quad (25)$$

where S is the supergravity action (15) and S_{M2} denotes the M2-brane action whose bosonic sector is given by

$$S_{M2} = -\frac{T_{M2}}{2} \int d^3\xi \sqrt{-q} \left[q^{ij}\partial_i X^M \partial_j X^N g_{MN} - 1 \pm \frac{1}{3} \varepsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^P A_{MNP} \right]. \quad (26)$$

Here, T_{M2} denotes the tension of M2-brane. The M2-brane world volume with the coordinates τ, σ^{α} ($\alpha = 1, 2$) is embedded into an 11-dimensional target spacetime using the functions $x^M(\tau, \sigma^{\alpha})$ ($M = 0, \dots, 11$).

The energy-momentum in the Einstein equations receives a contribution from the source term of M2-brane, as well as four-form gauge field strength. If we use a static configuration of the M2-brane [28],

$$t = \tau, \quad x^{\alpha} = \sigma^{\alpha}, \quad x^a = \text{const}, \quad (27)$$

the Einstein equations give

$$M = \frac{\kappa^2 T_{M2}}{3V_{S^7}}, \quad (28)$$

where V_{S^7} is the volume of the seven-sphere and M is M2-brane charge found in (24).

The dynamics of the background does not modify at all with a particular gauge even if we add the source (Wess–Zumino) term. Although the charge of the M2-brane is given by the volume of the seven-sphere, M2-brane tension, the metric is almost the same form as our solution (24).

Moreover, upon setting the configuration (27), the dynamical solution with the source term shares the same supersymmetric property as the solution without the Wess–Zumino term such as supersymmetric breaking, preserving a quarter of maximal supersymmetry. This is a similar result to the static M2-brane system [37].

2.3. Number of Supersymmetry and Supersymmetry Breaking

In this section, we count the number of preserving supersymmetry in the dynamical M2-brane background. An unbroken supersymmetry with respect to each Killing spinor ε has to obey the integrability condition

$$[\bar{\nabla}_M, \bar{\nabla}_N]\varepsilon = 0. \quad (29)$$

From the relation

$$\nabla_M = \partial_M + \frac{1}{4}\omega_M^{PQ}\Gamma_{PQ}, \quad [\nabla_M, \nabla_N] = \frac{1}{4}R_{MNPQ}\Gamma^{PQ}, \quad (30)$$

the commutator of the covariant derivatives in the integrability condition (29) becomes

$$\begin{aligned} [\bar{\nabla}_M, \bar{\nabla}_N] &= \frac{1}{4}R_{MNPQ}\Gamma^{PQ} + \frac{1}{6}\left(\nabla_{[M}\Gamma_{N]}\mathbf{F} - 3\nabla_{[M}\mathbf{F}_{N]}\right) \\ &+ \frac{1}{144}\left[(\Gamma_M\mathbf{F} - 3\mathbf{F}_M), (\Gamma_N\mathbf{F} - 3\mathbf{F}_N)\right]. \end{aligned} \quad (31)$$

In terms of the condition, we count how many supersymmetries exist. We first briefly review the results for the well-known case of the 11-dimensional static background [33,38–40]. For the case in which $h = \text{const}$ or $h = M/r^6$ in the 11-dimensional metric (12), the number of supersymmetries reduces to the number of solutions to the spinor equation, $\bar{\nabla}_a\varepsilon = 0$. In particular, for the 11-dimensional Minkowski spacetime [38,39] and for $\text{AdS}^4 \times S^7$, $\text{AdS}^7 \times S^4$ [40], the background has the full supersymmetry.

Next, we consider the static M2-brane background with $h(r) = \tilde{c} + M/r^6$ ($\tilde{c}M \neq 0$) [33], where \tilde{c} is constant. Then, the μr component of the integrability condition gives

$$0 = [\bar{\nabla}_\mu, \bar{\nabla}_r]\varepsilon = -h^{-1/3}\frac{d^2}{dr^2}\left(h^{-1/6}\right)\gamma_\mu\gamma^r\left(1 - \chi\gamma_{(3)}\right)\varepsilon. \quad (32)$$

Hence, ε have to obey

$$\left(1 - \chi\gamma_{(3)}\right)\varepsilon = 0. \quad (33)$$

Since we can show that this condition and (32) are the only nontrivial integrability conditions, one half of the supersymmetries in the case $\tilde{c}M \neq 0$ is broken in the M2-brane background [33].

Now, we consider the background with $\partial_\mu h \neq 0$. The $[\mu, \nu]$ components of the integrability condition give

$$0 = \tilde{\zeta}^\mu\zeta^\nu[\bar{\nabla}_\mu, \bar{\nabla}_\nu]\varepsilon = -\frac{\eta^{\mu\nu}\partial_\mu h_0\partial_\nu h_0}{18h^2}\tilde{\zeta}_\rho\gamma^\rho\zeta_\sigma\gamma^\sigma\varepsilon, \quad (34)$$

where $\tilde{\zeta}^\mu$ and ζ^ν are linearly independent vectors satisfying the conditions, $\tilde{\zeta}^\mu\partial_\mu h = \zeta^\mu\partial_\mu h = 0$, and we assume that the function $h(x, r)$ obeys

$$h(x, r) = h_0(x) + h_1(r), \quad \partial_\mu\partial_\nu h_0 = 0. \quad (35)$$

Hence, it follows that if $c_\mu = \partial_\mu h_0$ is not null, there exists only a trivial solution to the Killing spinor equation, and the supersymmetry is completely broken. On the other hand, when c_μ is a null vector, the Killing spinor equation leads to (14). For the case

$$h(x, r) = c_\mu x^\mu + \tilde{c} + \frac{M}{r^6}, \quad (36)$$

one quarter of the possible rigid supersymmetries in the maximal case survives.

Here, we check the degree of supersymmetry for the case of $M = 0$. An important simplification occurs if we consider the following special case of vanishing M2-brane charge

$$ds^2 = h^{-2/3}(u) \left[-2dudv + (dy)^2 \right] + h^{1/3}(u) \delta_{mn} dz^m dz^n, \quad (37a)$$

$$h(u) = cu, \quad u = \frac{1}{\sqrt{2}}(t-x), \quad v = \frac{1}{\sqrt{2}}(t+x), \quad (37b)$$

from the dynamical M2-brane to the plane wave background. Here, c is constant, and δ_{mn}, z^m denote the metric and coordinates of eight-dimensional Euclidean space, respectively. The required change of coordinates is $(u, v, z^m) \rightarrow (\bar{u}, \bar{v}, \bar{z}^m)$, where

$$u = \frac{1}{c} \left(\frac{\bar{u}}{\bar{u}_0} \right)^3, \quad v = \bar{v} + f(\bar{u}) \delta_{mn} \bar{z}^m \bar{z}^n, \quad z^m = h^{-1/6}(\bar{u}) \bar{z}^m, \quad (38)$$

which leads to the plane wave metric [35],

$$ds^2 = -2d\bar{u}d\bar{v} + \left(\frac{\bar{u}}{\bar{u}_0} \right)^{-2} \left[-\frac{c^2}{36} \delta_{mn} \bar{z}^m \bar{z}^n (d\bar{u})^2 + (dy)^2 \right] + \delta_{mn} d\bar{z}^m d\bar{z}^n. \quad (39)$$

Here, we used

$$\bar{u}_0 = \frac{3}{c}, \quad f(\bar{u}) = -\frac{c}{12} \left(\frac{\bar{u}}{\bar{u}_0} \right)^{-1}. \quad (40)$$

Setting $M = 0$ in the solution (24), the integrability condition reduces to $c_\mu \gamma^\mu \varepsilon = 0$. Then, the dynamical M2-brane solution with $c_\mu \neq 0$, preserves a half of the maximal supersymmetries. Since the number of unbroken spacetime supersymmetries in the present background must be a half of the full supersymmetries, as in a generic plane wave, our solution is consistent with past results [35].

Next, we comment on the degree of the supersymmetry breaking for the dynamical M2-brane background. The measure of the supersymmetry breaking for the dynamical background is obtained from the consistency condition. The mass scale corresponds to $h^{-2} \eta^{\mu\nu} \partial_\mu h \partial_\nu h$, which could be identified with a kind of induced effective mass scale for the spinor field. The divergence at $h = 0$ means that the degree of the supersymmetry breaking increases as the background approaches the curvature singularity. On the other hand, the supersymmetry breaking becomes negligible near the M2-brane region $r \rightarrow 0$, as h diverges there.

Let us consider the relation between the dynamics of the background and supersymmetry breaking in more detail. Introducing a new time coordinate τ , which is defined by $\tau/\tau_0 = (c_0 t)^{2/3}$, with constant $\tau_0 = (3/2c_0)$, we find the 11-dimensional metric (1a) as

$$ds^2 = - \left[1 + \left(\frac{\tau}{\tau_0} \right)^{-3/2} \left(c_i x^i + \frac{M}{r^6} \right) \right]^{-2/3} \left[-d\tau^2 + \left(\frac{\tau}{\tau_0} \right)^{-1} \delta_{ij} dx^i dx^j \right] + \left[1 + \left(\frac{\tau}{\tau_0} \right)^{-3/2} \left(c_i x^i + \frac{M}{r^6} \right) \right]^{1/3} \left(\frac{\tau}{\tau_0} \right)^{1/2} \left[dr^2 + r^2 d\Omega_{(7)}^2 \right], \quad (41)$$

where x^i ($i = 1, 2$) denotes the space coordinates of the world volume spacetime and the metric δ_{ij} is the spatial part of the three-dimensional Minkowski metric $\eta_{\mu\nu}$. When we set $c_1 = c_2 = 0$, the spacetime

is an isotropic and homogeneous universe with respect to the world volume coordinates, whose supersymmetry is completely broken. On the other hand, the 11-dimensional spacetime becomes inhomogeneous and preserves supersymmetry if parameters c_μ satisfy $c_\mu c^\mu = 0$ and $c_\mu \gamma^\mu \varepsilon = 0$. Thus, in the limit when the terms $c_i x^i$ are negligible, which is realized in the limit $(\tau/\tau_0) \rightarrow \infty$, for small r , we find an 11-dimensional universe without supersymmetry. For concreteness, we discuss the dynamics in the region where the term $c_i x^i$ in the function $h(\tau, x, r)$ is smaller compared to the contribution of the M2-brane charge M/r^6 . In the case of $(\tau/\tau_0) > 0$, we have found that the domains near the M2-brane have supersymmetry. As time increases, the background satisfies $(\tau/\tau_0)^{3/2} \gg c_i x^i$. Then, we find

$$1 + \left(\frac{\tau}{\tau_0}\right)^{-3/2} \left(c_i x^i + \frac{M}{r^6}\right) \rightarrow 1 + \left(\frac{\tau}{\tau_0}\right)^{-3/2} \frac{M}{r^6}. \quad (42)$$

The contribution of the term $c_i x^i$ in the function $h(\tau, x, r)$ eventually becomes negligible in the 11-dimensional metric such that supersymmetries are completely broken, which is guaranteed by the region $c_i x^i \ll M/r^6$. Then, the dynamical M2-brane solution also behaves as a non-supersymmetric cosmological solution in the asymptotic future.

From the integrability condition (31), the spinor mass scale is given by $\eta^{\mu\nu} \partial_\mu h \partial_\nu h / h^2 \sim c_\mu c^\mu / h^2$. This naturally measures for the scale of gravitino mass in our background. Hence, the gravitino mass vanishes in the supersymmetric background because the solution satisfies the integrability condition, $c_\mu c^\mu = 0$. In contrast, the gravitino acquires the mass, $c_\mu c^\mu \neq 0$, when the supersymmetry has been broken. This describes the dynamics of supersymmetry breaking from the viewpoint of the higher-dimensional theory.

Finally, we also comment about a relation between the M2-brane or black hole and plane wave background. Now, we set

$$h(t, x, r) = \frac{c}{\sqrt{2}}(t - x) + \frac{M}{r^6}. \quad (43)$$

In the limit when the term M/r^6 is negligible, corresponding to the far region from the M2-brane, the background changes from the above description to a time-dependent plane wave background (37). Hence, the supersymmetry will enhance from one quarter to a half of the possible rigid supersymmetries in the maximal case when one moves in the transverse space in such a way that $(\tau/\tau_0)^{-3/2} c_i x^i$ remains approximately constant. For fixed x^i , $(\tau/\tau_0)^{-3/2} c_i x^i$ becomes gradually small as time increases. Hence, the solution gives the transition from the plane wave geometry with a half of maximal supersymmetries to the non-supersymmetric (Kasner) universe as time evolves, for which supersymmetry breaking occurs at $r \rightarrow \infty$ or in the case of vanishing M2-brane charge. Although the solution itself is by no means realistic, its interesting behavior suggests an enhancement of the supersymmetry or a possibility that the universe with a quarter of the preserved original supersymmetry began to evolve toward a universe without supersymmetry.

3. Geometry of the Supersymmetric Dynamical M2-brane Solution

As one may expect from the dynamical M2-brane solution, the spacetime with (24) has curvature singularity. For a fixed x , the spacetime asymptotically approaches the anisotropic solution at a large r , while the metric becomes approximately $\text{AdS}_4 \times S^7$ near the M2-brane region (at $r \rightarrow 0$), as we will show in Section 3.3. Now, we investigate the geometric feature near the curvature singularity and discuss the smoothness at the horizon.

3.1. Property of the Solution

We consider the following time-dependent M2-brane solution with the 11-dimensional metric

$$ds^2 = h^{-2/3}(u, r) \left(-2du dv + dy^2 \right) + h^{1/3}(u, r) \left[dr^2 + r^2 d\Omega_{(7)}^2 \right], \quad (44a)$$

$$u = \frac{1}{\sqrt{2}}(t - x), \quad v = \frac{1}{\sqrt{2}}(t + x), \quad (44b)$$

$$h(u, r) = h_0(u) + h_1(r), \quad h_0(u) = cu, \quad h_1(r) = \frac{M}{r^6}, \quad (44c)$$

where c, M are constants. Since the function $h_1(r)$ dominates near $r \rightarrow 0$, the background geometry describes the extremal Reissner–Nordström solution with an infinite throat. The geometry of the dynamical M2-brane is not asymptotically flat, while the extremal Reissner–Nordström solution gives the asymptotically Minkowski spacetime. Near, the M2-brane, the metric becomes $\text{AdS}_4 \times S^7$,

$$\begin{aligned} ds^2 &\approx \frac{r^4}{M^{2/3}} (-2du dv + dy^2) + \frac{M^{1/3}}{r^2} dr^2 + M^{1/3} d\Omega_{(7)}^2, \\ &= \frac{M^{1/3}}{4w^2} (-2du dv + dy^2 + dw^2) + M^{1/3} d\Omega_{(7)}^2, \quad w := \frac{M^{\frac{1}{2}}}{2r^2}, \end{aligned} \quad (45)$$

where $d\Omega_{(7)}^2$ is the line element of the seven-sphere.

Since the square of the four-form field strength diverges at the zeros of the function $h(u, r) = 0$,

$$F_{(4)}^2 = -4! h^{-7/3} (\partial_r h)^2, \quad (46)$$

the curvature of the metric (44) can be singular at $h(u, r) = 0$.

Now, we discuss the cosmological evolution of the spatial geometry in the region $h > 0$ and assume $c < 0$, in the function $h(u, r)$. For $u < 0$, the function h is positive everywhere, and the spatial surfaces are nonsingular unless we treat the negative charge of the M2-brane $M < 0$. They are the asymptotically anisotropic spacetime for a fixed x coordinate. The spatial metric is still regular for $u = 0$ besides the region $r \rightarrow \infty$. As time increases slightly, a singularity appears at $r = \infty$ and moves in from spatial infinity. As u evolves further, the singularity eventually wraps the horizon completely.

3.2. Geodesic Motion

We start by solving radial null geodesic equations for the affine parameter s on the background (44a). As found in [30], the geodesic equations are

$$\begin{aligned} \frac{du}{ds} &= fh^{2/3}, \quad \frac{dv}{ds} = \frac{h^{1/3}}{2f} \left(\frac{dr}{ds} \right)^2, \\ \frac{d^2r}{ds^2} &= -\frac{c}{3} fh^{-1/3} \frac{dr}{ds} - \frac{M}{r^7} h^{-1} \left(\frac{dr}{ds} \right)^2, \end{aligned} \quad (47)$$

where f is a constant.

3.2.1. Geodesic Motion Near the M2-brane

Near the M2-brane, the null geodesic solution of Equation (47) is found analytically. Let us assume that $|u| \ll r^{-6}$ in the limit of $r \rightarrow 0$. Then, the function h takes the simple form

$$h \rightarrow \frac{M}{r^6}. \quad (48)$$

In this approximation, the asymptotic solution is given by

$$u \sim (s_0 - s)^{-1}, \quad r \sim \sqrt{s_0 - s}, \quad (49)$$

near $r = 0$, where s_0 is a positive constant. Note that the assumption $|u| \ll r^{-6}$ is satisfied in this asymptotic solution. Obviously, u becomes infinite as we approach the location of the M2-brane $r \rightarrow 0$ ($s \rightarrow s_0$).

3.2.2. Geodesic Motion near the Timelike Singularity

We now discuss the radial null geodesic near the timelike singularity. For the supersymmetric M2-brane background (44), $h = 0$ hypersurface corresponds to a timelike curvature singularity [30] because $g^{MN}\ell_M\ell_N > 0$ for $\ell_M = \nabla_M h$ near the singularity. Let us then consider the past directed null geodesics, which can hit the curvature singularity within a finite affine parameter length. Now, we set that as $h \rightarrow 0$,

$$h(s) = (s_0 - s)^\alpha, \quad r(s) \simeq r_0 + r_1 (s_0 - s)^\beta, \quad (50)$$

where s_0 denotes the value of s at the singularity, $\alpha (> 0)$, β and r_1 are constants determined later. Near the singularity, the geodesic Equation (47) becomes

$$\frac{du}{ds} = f(s_0 - s)^{2\alpha/3}, \quad \frac{dv}{ds} = \frac{1}{2f}(s_0 - s)^{\alpha/3} \left(\frac{dr}{ds}\right)^2, \quad (51a)$$

$$\frac{d^2r}{ds^2} = -\frac{c}{3}fh^{-1/3}\frac{dr}{ds} - \frac{M}{r_0^7}h^{-1} \left(\frac{dr}{ds}\right)^2 \simeq -\frac{M}{r_0^7}(s_0 - s)^{-\alpha} \left(\frac{dr}{ds}\right)^2. \quad (51b)$$

Here, in the second line, we assumed that the second term on the r.h.s. is dominant. Substituting Equation (50) into Equation (51), we find

$$\beta = \alpha, \quad r_1 = -\frac{r_0^7(\alpha - 1)}{\alpha M}. \quad (52)$$

From Equation (51a), the form of u is given by

$$u(s) = u_0 - \frac{3f}{2\alpha + 3}(s_0 - s)^{1 + \frac{2\alpha}{3}}. \quad (53)$$

For $s \rightarrow s_0$, it follows that $u \rightarrow u_0$, and $r \rightarrow r_0$. Then, expanding h in Equation (44c) around $s = s_0$, we have

$$h(s) = -\frac{3cf}{2\alpha + 3}(s_0 - s)^{1 + \frac{2\alpha}{3}} - \frac{6Mr_1}{r_0^7}(s_0 - s)^\alpha, \quad (54)$$

where we have used $h(s_0) = 0$. From Equations (50), (52) and (54), the constant α becomes $\alpha = 6/5$. Note that this coefficient is consistent with the assumption that $|h^{-1/3}dr/ds| \ll |h^{-1}(dr/ds)^2|$.

We now turn our attention to calculate a geometrical quantity in a parallelly-propagated frame along the null geodesic

$$\Gamma \equiv C_{MPNQ}E_2^ME_2^Nk^Pk^Q, \quad (55)$$

where C_{MPNQ} is the Weyl tensor, k^M denotes the tangent vector of null geodesic and E_2^M is a parallelly-propagated spacelike unit vector orthogonal to k^M . These are defined by

$$k = \frac{du}{ds}\partial_u + \frac{dv}{ds}\partial_v + \frac{dr}{ds}\partial_r, \quad E_2 = h^{1/3}\partial_y. \quad (56)$$

In terms of the metric (44) and 11-dimensional null vectors (51) with $\alpha = 6/5$, we find

$$\Gamma \sim (s_0 - s)^{-2}. \quad (57)$$

The shear σ and the expansion rate $d\theta/ds$ of the congruence along the null vector k^M diverge near the singularity as

$$\sigma \sim \int^s \Gamma ds \sim (s_0 - s)^{-1}, \quad \frac{d\theta}{ds} \sim -\sigma^2 \sim -(s_0 - s)^{-2}. \quad (58)$$

Then, we obtain

$$\int^s \theta ds = \int^s \left(\frac{d}{ds} \ln A \right) ds \sim \ln(s_0 - s), \quad (59)$$

where A is the volume element of the null geodesic congruence. This implies that the timelike singularity is a strong type of curvature singularity [41], as the volume element of any congruence along the radial null geodesic vanishes there.

3.3. Analytic Extension across the Event Horizon

As shown in the previous section, there are null geodesics that terminate a coordinate singularity, $r = 0, t = \infty$ in the metric (44a) within a finite affine parameter distance. Here, we consider an analytic extension across the $(r = 0, t = \infty)$ surface and show that this surface corresponds to a regular null hypersurface (horizon) generated by a null Killing vector field.

In the $c = 0$ case, the metric is static and $r = 0$; the $t = \infty$ surface corresponds to a Poincare horizon in $\text{AdS}_4 \times S^7$. Thus, the near-horizon geometry is clearly regular, and the regular metric in the AdS_4 part is given by

$$ds_{\text{AdS}_4}^2 \simeq -\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_{(2)}^2, \quad (60)$$

by adapting a global coordinate system defined by

$$\begin{aligned} w &= \frac{1}{\cosh \rho \cos \tau + \sinh \rho \sin \theta \sin \varphi}, \\ t &= \frac{\cosh \rho \sin \tau}{\cosh \rho \cos \tau + \sinh \rho \sin \theta \sin \varphi}, \\ x &= \frac{\sinh \rho \cos \theta}{\cosh \rho \cos \tau + \sinh \rho \sin \theta \sin \varphi}, \\ y &= \frac{\sinh \rho \sin \theta \cos \varphi}{\cosh \rho \cos \tau + \sinh \rho \sin \theta \sin \varphi}. \end{aligned} \quad (61)$$

Therefore, we expect that this coordinate system also works even in the $c \neq 0$. For simplicity, we consider the case that c becomes small. Then, expanding the function h with respect to the parameter c and transforming the metric (44a) in terms of the global coordinate (61), we obtain

$$ds^2 = \frac{M^{\frac{1}{3}}}{4} ds_{\text{AdS}_4}^2 + ch_{AB} dx^A dx^B + M^{\frac{1}{3}} \left(1 + \frac{cM^{\frac{1}{2}} u}{24w^3} \right) d\Omega_{(7)} + O(c^2), \quad (62)$$

where $ds_{\text{AdS}_4}^2$ is the AdS_4 spacetime in the global coordinate (60), and h_{AB} ($A, B = 0, \dots, 3$) denotes the four-dimensional metric, which describes the deviation from the AdS_4 geometry in terms of global coordinates. The metric h_{AB} is a complicated function of the global coordinate $(\tau, \rho, \theta, \varphi)$, but each component is regular everywhere. Therefore, the $r = 0$ ($w = \infty$) surface is regular, up to $O(c)$. One can check that the $r = 0$ ($w = \infty$) surface is a null hypersurface since

$$g^{AB} (d\tilde{\xi})_A (d\tilde{\xi})_B \Big|_{\tilde{\xi}=0} = 0, \quad \tilde{\xi} := 1/w = \cosh \rho \cos \tau + \sinh \rho \sin \theta \sin \varphi, \quad (63)$$

up to $O(c)$.

Next, we consider two vectors $N = \partial_t$, $X = \partial_x$ near the horizon. In terms of the global coordinate (61), we obtain

$$N = \frac{\cosh \rho + \sinh \rho \cos \tau \sin \theta \sin \phi}{\cosh \rho} \partial_\tau + \sin \theta \sin \tau \sin \phi \partial_\rho + \frac{\cosh \rho \sin \tau \cos \theta \sin \phi}{\sinh \rho} \partial_\theta + \frac{\cosh \rho \sin \tau \cos \phi}{\sinh \rho \sin \theta} \partial_\phi, \quad (64a)$$

$$X = -\cos \theta \sin \tau \tanh \rho \partial_\tau + \cos \theta \cos \tau \partial_\rho - \left(\frac{\cosh \rho \cos \tau \sin \theta}{\sinh \rho} + \sin \phi \right) \partial_\theta - \frac{\cos \theta \cos \phi}{\sin \theta} \partial_\phi. \quad (64b)$$

Since the vector N is proportional to X and $g(N, N) = g(X, X) = 0$ on the null hypersurface $\xi = 0$, these vectors become null and degenerate on the horizon. Therefore,

$$\partial_v := \frac{1}{\sqrt{2}} (N + X), \quad (65)$$

is also null on the horizon ($r = 0$). Thus, the null Killing vector field ∂_v is also the generator of the horizon, even though the bulk metric is the asymptotically-anisotropic geometry at constant x coordinate.

4. Supersymmetric Black Hole in an Expanding Universe

We can find all sorts of black holes, of any spacetime dimension in the context of the low-energy effective supergravity theories. In fact, solitonic solutions, as well as M2-brane can be seen as black holes in the space transverse to its world-volume. The static M2-brane system describes the microstate of a black hole [42]. Then, it may be natural to apply the present solutions to a time-dependent spacetime with a black hole. As we have presented in the previous section, there is a null Killing vector at the horizon where the M2-brane is located. In the limit $r \rightarrow 0$, the background geometry thus becomes $\text{AdS}_4 \times S^7$. In this section, we discuss the dynamics of a black hole, which is the so-called “black M2-brane” [37,42–44], in the expanding universe on the basis of the results we have obtained in the previous section.

4.1. Black Hole in an 11-Dimensional Background

Here, we give an explicit example of a black hole in the dynamical M2-brane system. The 11-dimensional metric of the supersymmetric M2-brane depends on time,

$$ds^2 = h^{-2/3}(u, r) \left[-2dudv + (dy)^2 + h(u, r) \left(dr^2 + r^2 d\Omega_{(7)} \right) \right], \quad (66)$$

where

$$h(u, r) = cu + \frac{M}{r^6}, \quad (67)$$

with constants c and M . If we introduce a new coordinate \bar{u} , this metric is rewritten as

$$ds^2 = H^{-2/3}(\bar{u}, r) \left[-2d\bar{u}dv + a_{\text{M2}}^{-4/3}(\bar{u}) (dy)^2 + a_{\text{M2}}^2(\bar{u}) H(\bar{u}, r) \left(dr^2 + r^2 d\Omega_{(7)} \right) \right], \quad (68)$$

where $\bar{M}(\bar{u})$, $a_{\text{M2}}(\bar{u})$ denote the effective M2-brane charge depending on \bar{u} and the scale factor, respectively,

$$H(\bar{u}, r) = 1 + \frac{\bar{M}(\bar{u})}{r^6}, \quad a_{\text{M2}}(\bar{u}) = \left(\frac{\bar{u}}{\bar{u}_0} \right)^{3/2}, \quad (69)$$

with

$$\bar{M}(\bar{u}) \equiv \left(\frac{\bar{u}}{\bar{u}_0}\right)^{-3} M, \quad (70a)$$

$$cu = \left(\frac{\bar{u}}{\bar{u}_0}\right)^3, \quad \bar{u}_0 \equiv \frac{3}{c}. \quad (70b)$$

The near M2-brane geometry is the same as the static one because there is a null Killing vector at the horizon, and then, the geometry approaches the static solution. Since it has a horizon geometry, we can regard the present dynamical solution as a black hole. The dynamical M2-brane gives the black hole spacetime while the asymptotic structure in the dynamical M2-brane is completely different from that of a static one. Although the static M2-brane solution has an asymptotically-flat geometry, the dynamical M2-brane solution is a time-dependent anisotropic spacetime at a constant x coordinate.

4.2. Black Hole in the Ten-Dimensional Effective Theory

In this section, we study the dynamics of the M2-brane black hole in the lower-dimensional background after compactifying the internal space. Now, we compactify a one-dimensional M2-brane world volume just as the case of a static black hole and consider the ten-dimensional effective theory. In this case, we find the 11-dimensional metric

$$ds^2 = ds_{10}^2 + ds_1^2, \quad (71)$$

where

$$ds_{10}^2 = h^{-2/3}(u, r) \left[-2dudv + h(u, r) \left(dr^2 + r^2 d\Omega_{(7)} \right) \right], \quad (72a)$$

$$ds_1^2 = h^{-2/3}(u, r) (dy)^2. \quad (72b)$$

The compactification of ds_1^2 gives the effective ten-dimensional spacetime, whose metric in the Einstein frame $d\bar{s}_{10}^2$ is given by

$$d\bar{s}_{10}^2 = h^{-3/4}(u, r) \left[-2dudv + h(u, r) \left(dr^2 + r^2 d\Omega_{(7)} \right) \right]. \quad (73)$$

If we use a new coordinate \tilde{u}

$$cu = \left(\frac{\tilde{u}}{\tilde{u}_0}\right)^4, \quad \tilde{u}_0 = \frac{4}{c}, \quad (74)$$

the ten-dimensional metric (73) can be rewritten explicitly as

$$d\bar{s}_{10}^2 = \tilde{H}^{-3/4}(\tilde{u}, r) \left[-2d\tilde{u}dv + a_{\text{eff}}^2(\tilde{u}) \tilde{H}(\tilde{u}, r) \left(dr^2 + r^2 d\Omega_{(7)} \right) \right], \quad (75)$$

where the function $\tilde{H}(\tilde{u}, r)$ is given by

$$\tilde{H}(\tilde{u}, r) = 1 + \frac{\tilde{M}(\tilde{u})}{r^6}, \quad (76)$$

with the effective M2-brane charge $\tilde{M}(\tilde{u})$, and the scale factor $a_{\text{eff}}(\tilde{u})$,

$$\tilde{M}(\tilde{u}) \equiv \left(\frac{\tilde{u}}{\tilde{u}_0}\right)^{-4} M, \quad (77a)$$

$$a_{\text{eff}}(\tilde{u}) = \left(\frac{\tilde{u}}{\tilde{u}_0}\right)^2. \quad (77b)$$

From Equations (73) and (75) in the limit of $r \rightarrow \infty$, we find

$$\begin{aligned} d\tilde{s}_{10}^2 &= (cu)^{-3/4} \left[-2dudv + cu \left(dr^2 + r^2 d\Omega_{(7)} \right) \right] \\ &= -2d\tilde{u}dv + a_{\text{eff}}^2(\tilde{u}) \left(dr^2 + r^2 d\Omega_{(7)} \right). \end{aligned} \quad (78)$$

Since our solution approaches an asymptotically time-dependent universe with the scale factor $a_{\text{eff}}(\tilde{u})$, we can regard again the time-dependent M2-brane solution as a black hole in the expanding Universe. If we compactify the direction of the world volume coordinate, we find the different power exponent of time in the scale factor, which is also shown in the original 11-dimensional background.

As a result, we always find the different power of time \tilde{u} in the scale factor $a_{\text{eff}}(\tilde{u})$ for a D-dimensional black hole ($d \leq 10$) if we smear the transverse space to the M2-brane. If d_s -dimensions of the transverse space to the M2-brane are smeared, which gives the different power of transverse space coordinates to the M2-brane

$$ds^2 = h^{-2/3}(\tilde{u}, z) \left[-2d\tilde{u}dv + (dy)^2 + h(\tilde{u}, z) \delta_{ab} dz^a dz^b \right], \quad (79a)$$

$$h(\tilde{u}, z) = c\tilde{u} + \sum_l \frac{M_l}{|z^a - z_l^a|^{6-d_s}}, \quad (d_s \leq 7) \quad (79b)$$

in terms of the multi-black-hole coordinates. Here, z^a ($a = 1, 2, \dots, 8$) denote the coordinates of the transverse space to the M2-branes, M_l ($l = 1, 2, \dots, N$) are M2-brane charges and z_l^a ($l = 1, 2, \dots, N$) are the positions of M2-branes. Suppose d_s dimensions of the transverse space to M2-branes are smeared and compactified, where $d_s \leq 7$. If one compactifies the d_s -dimensional transverse space, as well as the $d_M (= 0$ or 1)-dimensional M2-brane world volume, the $d [= (11 - d_M - d_s)]$ -dimensional metric in the Einstein frame is given by

$$d\tilde{s}_d^2 = H^{\frac{d_s-6}{d-2}}(\tilde{u}, z) \left[-2d\tilde{u}dv + (1 - d_M) a_{\text{eff}}^{\frac{2(d_s-6)}{d-2}}(dy)^2 + a_{\text{eff}}^2(\tilde{u}) H(\tilde{u}, z) \delta_{PQ} dz^P dz^Q \right], \quad (80a)$$

$$H(\tilde{u}, r) = 1 + \frac{\tilde{M}(\tilde{u})}{r^6}, \quad (80b)$$

where $\delta_{PQ} dz^P dz^Q$ is the metric of $(8 - d_s)$ -dimensional Euclidean space. The effective M2-brane charge $\tilde{M}(\tilde{u})$, the scale factor $a_{\text{eff}}(\tilde{u})$ and a coordinate \tilde{u} are also given by

$$\tilde{M}(\tilde{u}) = \left(\frac{\tilde{u}}{\tilde{u}_0} \right)^{-\frac{d-2}{3-d_M}} M, \quad a_{\text{eff}}(\tilde{u}) = \left(\frac{\tilde{u}}{\tilde{u}_0} \right)^{\frac{d-2}{2(3-d_M)}}, \quad (81a)$$

$$cu = \left(\frac{\tilde{u}}{\tilde{u}_0} \right)^{\frac{d-2}{3-d_M}}, \quad \tilde{u}_0 = \frac{d-2}{c(3-d_M)}. \quad (81b)$$

This power exponent is obtained for a universe filled by the four-form field strength satisfying the field equation. We may regard the present D-dimensional solution as a time-dependent black hole.

5. Discussions

In the present proceeding, we have constructed the dynamical supersymmetric M2-brane solution for the warped compactification of an 11-dimensional supergravity. The solution is given by an extension of a static supersymmetric M2-branes solution. In the case of a dynamical M2-brane background, a quarter of maximal supersymmetries exists. If the M2-brane charge vanishes, our solution gives a plane wave background that preserves a half of the full supersymmetry. Therefore, in the far region from the M2-brane, the background changes from the dynamical M2-brane to the time-dependent plane wave background. This means that one quarter of the maximal supersymmetry is enhanced to a half of the possible rigid supersymmetries in the maximal case when one moves

in the transverse space to the M2-brane. Although we have mainly discussed the single M2-brane solution in this proceeding, it is possible to generalize it to the solution that describes an arbitrary number of extremal M2-branes in an expanding universe. We have found that the degree of the supersymmetry breaking is strongly related to the dynamics of the background. Then, the time evolution of the geometry is deeply connected with the hierarchy and supersymmetry breaking while the inhomogeneity of the M2-brane world volume coordinates results in preserving the supersymmetry. In the region where the effect of the inhomogeneity of the M2-brane world volume coordinates is smaller compared to the contribution of the M2-brane charge, our supersymmetric solution describes the breaking of the supersymmetry, which is the transition from the supersymmetric universe to a non-supersymmetric one as time evolves.

The dynamical M2-brane solutions can always take a form in the function $h(x, r) = h_0(x) + h_1(r)$, where the function $h(x, r)$ depends on the linear function of the M2-brane world volume coordinates x^μ , as well as the coordinates of the transverse space to the M2-brane. Since the existence of the function $h_0(x)$ implies dynamical instability in the moduli of internal space [3], it would be useful to study the stability of a solution.

Motivated by the construction of a new supersymmetric solution, we have studied the global structure of the dynamical M2-brane background. We have found that the time dependence changes the causal structure of a static M2-brane solution. Since the volume element of any congruence along the radial null geodesic vanishes at the curvature singularity, it turns out that this is a strong version of a timelike singularity. We have studied null geodesics that terminate a coordinate singularity in terms of an analytic extension across there and showed that there is a regular null hypersurface (or horizon) generated by a null Killing vector field. In particular, this null Killing vector field describes the generator of the horizon even if the bulk metric is the asymptotically-anisotropic geometry at a constant x coordinate. Hence, the near horizon geometry in this solution gives the regular spacetime and thus becomes $\text{AdS}_4 \times S^7$.

It is important to explore another analytic solution describing a supersymmetric M-brane or D-brane in the expanding universe. One may present whether supersymmetric dynamical brane solutions affect the formation of the naked singularity. Upon setting an appropriate initial condition, these solutions may allow us to violate the cosmic censorship [30,45].

We can also discuss a dynamical black hole solution whose spacetime gives a time-dependent universe. The near M2-brane region of this black hole in the expanding universe is the same as the static solutions while the asymptotic structures are completely different, giving the anisotropic spacetime at a fixed x coordinate with scale factors for a dynamical universe. The effective M2-brane charge for the supersymmetric background depends on the world volume coordinates of the M2-brane.

The supersymmetric solutions can contain the function depending on null coordinates of the M2-brane world volume direction. The results we have obtained are not unnatural because studies of the supersymmetric plane wave background showed that it is possible to obtain time-dependent supersymmetric solutions with a nontrivial dependence on spacetime coordinates [34,36]. Although this may be a limitation on the applications of our solution, it is interesting to explore if similar more general dynamical and supersymmetric solutions can be obtained by relaxing or extending some of our assumptions for the 10-, 11-, or lower-dimensional backgrounds. We will study this subject in the near future.

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