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Review

Composite Universal Constants Combining 2–5 Known Constants Reveal Latent Connections Between Disparate Physical Regimes and the Role of Dimensionless Constants in Systems of Units

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Abstract

We introduce a new method of dimensional analysis based on complete systems of units, such as the metric and Planck systems, in which fundamental dimensionless constants arise naturally. In fact, it is the reformulated Planck system that communicates its dimensionless constants to the metric or any other system. The method reveals additional complex dynamical scales and physical effects beyond those amenable to conventional dimensional analysis. We formulate our strategy in simple settings involving pairs of seemingly unrelated constants, and then we extend the analysis to more complicated cases involving combinations of three to five well-known universal constants. In constructions involving several unrelated constants, the method captures increasingly complex effects and places two or more disparate physics areas into a single framework connecting them by never-before-seen combinations of fundamental dimensionless constants, such as the fine-structure constant and the gravitational coupling constant. Thus, this method provides a pathway to blending descriptions of two or more fundamental interactions that have so far eluded a consistent theoretical formulation.

Keywords: coupling constants; entropy; fine-structure constant; gravitation; MOND theory; Planck units

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1. Introduction

1.1. Impetus from Previous Work

This work builds on a puzzling, yet innovative numerical relation discovered between totally unrelated universal constants in metric-system (SI) units [1], viz.

$$\begin{aligned} 10^{-6}\, k_{\mathrm{B}} &= 1.3806\ldots\times 10^{-29}\ \ \mathrm{J\,K^{-1}} \\ e\sqrt{G_{\star}} &= 1.3806\ldots\times 10^{-29}\ \ \mathrm{C^{2}\,kg^{-1}}\ , \end{aligned} \tag{1}$$

where

$$G_{\star} = 4\pi\varepsilon_{0} G\,, \tag{2}$$

and the various universal constants involved in these equalities are listed for convenience in Table 1. For the gravitational constant G, we used here the value determined recently in Ref. [1].

Table 1. Universal physical constants *.

Symbol	Name	Definition	SI Value	SI Unit	SDs
		Reformulated Planck System (RPS)			
e	Elementary charge		$1.602176634 \times 10^{-19}$	C	Exact
m_e	Electron mass		$9.1093837139(28) \times 10^{-31}$	kg	11
k_B	Boltzmann constant		1.380649×10^{-23}	$J\,K^{-1}$	Exact
ε_0	Vacuum permittivity		$8.8541878188(14) \times 10^{-12}$	$F\,m^{-1}$	11
μ_0	Vacuum permeability		$1.25663706127(20) \times 10^{-6}$	$H\,m^{-1}$	12
N_A	Avogadro number		$6.02214076 \times 10^{23}$	—	Exact
f_A	Avogadro factor		10.05530213 . . .	—	Exact
		Composite Universal Constants			
	Vacuum-related				
K	Coulomb constant	$K = 1/(4\pi\varepsilon_0)$	$8.9875517862(14) \times 10^9$	$N\,m^2\,C^{-2}$	11
c	Speed of light in vacuum	$c = 1/\sqrt{\mu_0\varepsilon_0}$	299,792,458	$m\,s^{-1}$	Exact
Z_0	Impedance of free space	$Z_0 = \mu_0 c$	376.730313412(59)	Ω	12
$\mathcal{R}_P$	Planck resistance	$\mathcal{R}_P = Z_0/(4\pi)$	29.9792457960(47)	Ω	12
	Mole-related				
R	Molar gas constant	$R = N_A k_B$	8.314462618 . . .	$J\,K^{-1}$	Exact
F	Faraday constant	$F = N_A e$	96,485.33212. . .	C	Exact
$\aleph_A$	Reduced Avogadro number	$\aleph_A = N_A/f_A$	$5.989020203 \times 10^{22}$	—	Exact
	Gravity-related				
G	Gravitational constant	$\mathcal{N}(G) = \mathcal{N}\left(K(10^{-6}k_B/e)^2\right)$	$6.674015081(1) \times 10^{-11}$	$m^3\,kg^{-1}\,s^{-2}$	10
M_P	Planck mass	$M_P = \aleph_A m_e$	$5.455628310(2) \times 10^{-8}$	kg	10
L_P	Planck length	$L_P = GM_P/c^2$	$4.051264068 \times 10^{-35}$	m	10
h	Planck constant	$h = GM_P^2/c$	$6.62607015 \times 10^{-34}$	$J\,Hz^{-1}$	Exact
$G_\star$	Gravoelectric G	$G_\star = 4\pi\varepsilon_0 G$	$7.425843255 \times 10^{-21}$	$C^2\,kg^{-2}$	Exact
G_B	Gravomagnetic G	$G_B = G\mu_0/(4\pi)$	$6.674015081 \times 10^{-18}$	$m^4\,s^{-2}\,C^{-2}$	10
	MOND-related				
a_0	MOND critical acceleration	$\mathcal{N}(a_0) = \mathcal{N}(4\pi\varepsilon_0)$	$1.1126500562 \times 10^{-10}$	$m\,s^{-2}$	11
$\mathcal{A}_0$	MOND universal constant	$a_0 G = \mathcal{N}(G_\star)$	$7.425843255 \times 10^{-21}$	$m^4\,kg^{-1}\,s^{-4}$	Exact
	Assorted combinations				
R_∞	Rydberg constant	$R_\infty = m_e e^4/(8\varepsilon_0^2 h^3 c)$	10,973,731.568157(12)	m^{-1}	14
σ	Stefan–Boltzmann constant	$\sigma = 2\pi^5 k_B^4/(15h^3c^2)$	$5.670374419 \ldots \times 10^{-8}$	$Wm^{-2}\,K^{-4}$	Exact
Φ_0	Magnetic-flux quantum	$\Phi_0 = h/(2e)$	$2.067833848 \ldots \times 10^{-15}$	Wb	Exact
$\mathcal{R}_K$	von Klitzing constant	$\mathcal{R}_K = h/e^2$	$2.581280745 \ldots \times 10^4$	Ω	Exact
r_c	Electron Compton radius	$r_c = h/(m_e c)$	$2.426310235 \ldots \times 10^{-12}$	m	Exact
	Stoney units				
M_S	Stoney mass	$\sqrt{e^2(G_\star)^{-1}}$	$1.859248778 \times 10^{-9}$	kg	Exact
L_S	Stoney length	$\left(\frac{\mu_0}{4\pi}\right)\sqrt{e^2 G_\star}$	$1.380649000 \times 10^{-36}$	m	10
	RPS coupling constants				
α	Fine-structure constant	$\alpha = Ke^2/(hc)$	$1.1614097321(2) \times 10^{-3}$	—	11
α_w	Weak coupling constant	$\alpha_w = \sqrt{\alpha}$	$3.407946203 \times 10^{-2}$	—	10
α_s	Strong coupling constant	$\alpha_s = (f_A)^2\alpha$	$1.174290938 \times 10^{-1}$	—	10
α_g	Gravitational coupling constant	$\alpha_g = (\aleph_A)^{-2}$	$2.787972231 \times 10^{-46}$	—	10

* The dimensionless function $\mathcal{N}(x)$ denotes the SI magnitude of x apart from units.

Evidently, the numerical equality in Equation (1) reveals the existence of a physical relation in which the factor of 10^{-6} represents a universal constant C_4 with dimensions of

$[\varepsilon_0 G]^{1/2}[k_B/e]^{-1}$, in which each pair of constants represents a well-known composite universal constant [1,2]. Thus, we obtain from Equation (1) the dimensionally balanced equation

$$e\sqrt{G_\star} = C_4 k_B\,, \tag{3}$$

where

$$C_4 = 10^{-6}\ \ \mathrm{F\,K\,kg^{-1}}\,, \tag{4}$$

in SI units. Here, symbol C_4 denotes that this is a composite constant obtained from an amalgamation of four well-known universal constants, viz. $\{\,G,\,K,\,e,\,k_B\,\}$ (Section 4.3), among many such constants examined in this work (Table 1).

Equation (4) is full of surprises:

(a) The magnitude is a precise power of ten; the leading factor is 1 to a precision of at least five significant digits (SDs) when the CODATA value of G is used [1].
(b) Adopting the SI magnitude of C_4 as 10^{-6} exactly, as in Equations (1) and (3), allows for a determination of the Newtonian gravitational constant G with a precision of 10 SDs (see Table 2 in Ref. [1] and the G-entry in Table 1 below).
(c) This is only the second instance of a constant possessing a pure power-of-ten SI magnitude; hitherto, the only other example was the magnetic permeability of the vacuum $\mu_0/(4\pi)$, whose SI magnitude is 10^{-7} to 10 SDs (see Section 2.2.1).
(d) The SI units of C_4 reveal an unexpected combination of dimensions drawn from three seemingly unrelated areas of physics: capacitance [C], temperature [Θ], and mass [M]. Although certain pairs of these quantities appear together in both theoretical and experimental contexts (e.g., kinetic theory of gases or capacitors), we are unaware of any physical effect in which all three quantities appear simultaneously.

The appearance of three disparate SI units in C_4 (Equation (4)) may also be viewed from a broader theoretical perspective. Since C, Θ, and M belong to distinct physical domains, a constant combining all three would naturally be expected to arise in a framework capable of relating these realms in a deeper physical level. In that sense, C_4 may be regarded as suggestive of a physical effect whose full interpretation would result from a more complete unified theory, were such a theory presently available.

1.2. Universal Constants Utilized in This Work

It certainly seems that we are facing complications in both the magnitude and the dimensions of constant C_4 (Equation (4)). Although we are not presently equipped with an inclusive theory to provide an explanation of capacitance related to both temperature and mass, we can still leverage such unusual properties to obtain new empirical results concerning interesting combinations of several universal constants.

The well-known universal physical constants considered in this work are summarized in Table 1. We have searched for multiple independent combinations of 2–5 constants with the following characteristics: (i) each combined constant (or its reciprocal) has a power-of-ten magnitude precise to about five SDs or more; and (ii) the units attached to these constants relate disparate branches of physics that have not been previously studied in unison.

The strategy employed in our search is first demonstrated in three simple cases, each utilizing only two fundamental constants from those listed in Table 1—we have not been able to find a fourth such example. Then, we proceed to analyze in detail 17 additional composite constants that all have nearly pure power-of-ten SI magnitudes (see Table 2 for a summary).

Table 2. Summary of new composite constants.

Symbol	Definition	SI Value	SI Unit	Section	Related Themes in Science
		Section 2. Two Combined Constants			
A_2	K/c^2	10^{-7}	kg m C^{-2}	Section 2.2.1	Biot–Savart law for charges
B_2	$m_e R_\infty$	10^{-23}	kg m^{-1}	Section 2.2.2	Atomic unit of linear mass density
C_2	$a_0/(4\pi\varepsilon_0)$	$10^0 = 1$	m^2 s^{-2} F^{-1}	Section 2.2.3	Capacitors, supercapacitors
		Section 3. Three Combined Constants			
A_3	$e^2 m_e \Phi_0^2$	10^{-97}	kg^3 m^4 s^{-2}	Section 3.1	Electron spin and mass
B_3	$G\Phi_0/k_B$	10^{-2}	m^3 K kg^{-1} s^{-1} C^{-1}	Section 3.2	Neutron stars
C_3	$e^2\Phi_0^2/R_\infty$	10^{-74}	kg^2 m^5 s^{-2}	Section 3.3	Electron spin and Compton radius
D_3	$k_B m_e/\mu_0$	10^{-47}	N C^2 K^{-1}	Section 3.4	Fermi temperature, Fermi gas
		Section 4. Four Combined Constants			
A_4	$c^2 k_B^2/(G e^2)$	10^{19}	kg V^2 m^{-1} K^{-2}	Section 4.1	Fisher information
B_4	$e\,h^2/(G^2\mu_0^2)$	10^{-53}	kg^4 C Ω^{-2}	Section 4.2	Mass–charge scaling $Q \propto 1/M^4$
C_4	$e\sqrt{G_\star}/k_B$	10^{-6}	F K kg^{-1}	Section 4.3	Larmor radius, plasma magnetic rigidity
D_4	$\sigma(hR_\infty)^2/c$	10^{-68}	kg^3 m s^{-4} K^{-4}	Section 4.4	Nambu–Goto strings, Regge trajectories
E_4	$(FK\Phi_0/\sigma)^2$	10^{15}	$(kg\ m^5\ K^4\ C^{-2})^2$	Section 4.5	Plasma current sheets, MHD turbulence, magnetic reconnection, solar wind
F_4	$h^2\sigma/(\Phi_0 R_\infty^2)$	10^{-73}	J^2 C K^{-4}	Section 4.6	Superconducting magnetic fields
G_4	$h^2 G R_\infty/\sigma^2$	10^{-55}	m^6 s^2 K^8 kg^{-1}	Section 4.7	Orbital motion in $-k/r$ potentials
H_4	$h\Phi_0^2\sigma^2/m_e$	10^{-47}	kg^4 m^6 s^{-9} C^{-2} K^{-8}	Section 4.8	Wakefields, accelerators
I_4	$k_B^2 R\sigma/c^2$	10^{-69}	kg^4 m^4 s^{-7} K^{-7}	Section 4.9	Photon ensemble, lasers
		Section 5. Five Combined Constants			
A_5	$G[ce/(k_B K)]^2$	10^{-5}	K^2 s^4 C^6 kg^{-5} m^{-5}	Section 5.1	Johnson–Nyquist thermal noise, inductors
B_5	$G^2 m_e \Phi_0^2/(\mu_0 k_B)$	10^{-51}	m^7 K kg^{-1} s^{-4}	Section 5.2	Entropy density, sound energy, aeroacoustics
C_5	$G^2 c^2 m_e \sigma/\Phi_0$	10^{-26}	m^6 C kg^{-1} s^{-8} K^{-4}	Section 5.3	Zitterbewegung (electron trembling motion)
D_5	$G^2 K m_e \sigma/\Phi_0$	10^{-33}	m^7 C^{-1} s^{-8} K^{-4}	Section 5.4	Zitterbewegung magnetic field, QED

1.3. Outline

The remainder of the paper is organized as follows:

- In Section 2, we summarize the steps of the search method, and we demonstrate the strategy in simple terms using pairs of seemingly unrelated well-known universal constants.
- In Sections 3–5, we present several more complicated cases involving various combinations of three to five unrelated constants, respectively, that point toward new physical scalings and effects.
- In Section 6, we briefly discuss our results and summarize our conclusions concerning pure power-of-ten composite constants.
- In Appendix A, we analyze four additional important pairs of unrelated constants whose SI magnitudes are not pure powers of ten. They show that the same method can be naturally extended to analyze any composite constants in physics irrespective of their magnitudes.

2. Strategy and Simple Demonstrations with Pairs of Fundamental Constants

2.1. Methodology

We search Table 1 for combinations of 2–5 constants with SI magnitudes (or their reciprocals) of $1.0000\ldots\times 10^N$, where N is an integer. We expect the N values to be linear combinations of -7 and -6 because the only known precise powers of ten (10^{-7} and 10^{-6}) occur in the calculations of $\mu_0/(4\pi)$ and $\mathcal{N}(G)$, respectively, where $\mathcal{N}(G)$ denotes the numerical SI magnitude of the Newtonian gravitational constant G (Table 1). In the process,

we avoid using the constant pairs $\mu_0\,\varepsilon_0$ and R/F that can easily produce equivalent relations when substituted for $1/c^2$ and k_B/e [2], respectively.

We also avoid using the reduced Avogadro number $\aleph_A$ [1], which is not a well-known unit at present. We prefer instead to work with the familiar man-made (subjective) Avogadro number N_A, which has also been retained as a fundamental unit in the reformulated Planck system (RPS) of units shown at the top of Table 1. We note that Planck's h is a derived constant in the RPS, and Dirac's $\hbar$ is not at all used in any composite constant (see Ref. [1] for details).

The Stefan–Boltzmann constant σ is the only constant in which $\hbar$ was retained to maintain its well-known SI magnitude; this $\hbar$ was replaced by $h/(2\pi)$ in the definition of σ in Table 1. On the other hand, the definitions of the Planck units, the Compton radius, and the dimensionless coupling constants were given in terms of Planck's h, replacing thus Dirac's $\hbar$ and restoring the correct geometries in these definitions.

This reversion to the original Planck system of units is at the core of the new RPS [1]. Despite its adoption and consistent implementation, the constants $\hbar$ and $\aleph_A$ do occasionally appear in the results, but only in strict 3D and 2D geometries, respectively, where they really belong. The spin of the electron is still $\hbar/2$, and ensembles of particles are naturally described by $\aleph_A$, the number preferred by nature, instead of the familiar man-made number N_A.

The SI units of the new constants are generally expected to be unfamiliar combinations of vacuum, thermal, mechanical, electromagnetic (EM), and quantum mechanical units. It is precisely these unit combinations that may hold new physics or scalings, and we have two distinct ways of analyzing their properties:

- **Part 1.** We use dimensional analysis to uncover relations between disparate or seemingly unrelated physical dimensions. This procedure is not straightforward because the dimensional reductions performed are not unique [3]. From prior experience, we always try to limit the number of fundamental physical quantities involved in a new composite constant to as few as possible: a single reduced quantity produces a constant value, whereas two or three quantities describe scaling relations.
- **Part 2.** We recast the dimensional forms of the combined constants in terms of Planck units. This procedure is straightforward and results in relations that necessarily involve some inherent geometric factors and, more importantly, some of the dimensionless coupling constants listed at the bottom of Table 1. Such combinations of disparate couplings have never before been seen in physics.

The appearance of fundamental physical quantities in Part 1 allows us to determine previously unknown effects or scales (or some known threshold values) that describe interactions stemming from disparate areas of physics. On the other hand, the appearance of dimensionless coupling constants in Part 2 solves a long-standing problem in physics (see, e.g., Refs. [4,5]): it shows that, at the very least, the fine-structure constant (FSC) and the gravitational coupling constant must be included in systems of units other than the RPS, which are otherwise incomplete.[1,2] In our methodology, all unit transformations between the Planck/RPS system and the SI system are carried out by well-known (though little appreciated [4]) dimensionless constants.

All combinations considered below are drawn from subsets of the dimensionful constants listed in Table 1. In Section 2.2, we illustrate the above methodology with three simple examples involving pairs of well-known universal constants. In Sections 3–5, we subsequently analyze more complex cases involving three to five constants, respectively. All composite constants under the present investigation are summarized in Table 2 for convenience.

2.2. Three Simple Combinations of 2 Universal Constants

2.2.1. Subset $A2 := \{ K, c \}$

We consider the constant A_2 such that

$$A_2 = \frac{K}{c^2} = 10^{-7} \ \text{kg m C}^{-2} . \tag{5}$$

In this exceptional case that combines two vacuum constants, it is known that $A_2 = \mu_0/(4\pi)$ [6,7].

Part 1.—The SI units in Equation (5) imply the dimensionally exact relation $ML = A_2 Q^2$. Imagining a particle of mass M and charge Q orbiting in a magnetic field B with azimuthal speed v at radius L, we use the centripetal force $F = Mv^2/L$ to eliminate M and the Lorentz force $F = QvB$ to eliminate F, and we find that

$$B = A_2 \frac{Qv}{L^2} , \tag{6}$$

which is the scalar analog of the well-known nonrelativistic equation (a variant of the Biot–Savart law)

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} \frac{Q(\boldsymbol{v} \times \hat{\boldsymbol{r}})}{r^2} , \tag{7}$$

where the radial unit vector is $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$.

Part 2.—Next, we replace the units in Equation (5) with the corresponding RPS quantities, viz. $\text{kg m C}^{-2} \rightarrow M_\text{P} L_\text{P}/Q_\text{P}^2$, and we find that

$$A_2 = \frac{M_\text{P} L_\text{P}}{Q_\text{P}^2} . \tag{8}$$

In this simple example, the combination of the three Planck units carries both the correct magnitude and the correct SI units of $A_2 = \mu_0/(4\pi)$.[3]

In more complicated cases analyzed below, the RPS units do not carry the correct magnitude, which must then be adjusted by introducing well-known dimensionless constants, revealing thus the presence of additional physics in such composite constants.

2.2.2. Subset $B2 := \{ m_\text{e}, R_\infty \}$

We consider the constant B_2 such that

$$B_2 = m_\text{e} R_\infty = 10^{-23} \ \text{kg m}^{-1} . \tag{9}$$

The nature of B_2 is spectroscopic since the Rydberg constant is involved, but it is doubtful that it has ever been used in atomic physics because B_2 represents a linear mass density. Compared to the corresponding Schwarzschild-type[4] linear density, $M/R_\text{S} \sim c^2/G \sim 10^{27} \ \text{kg m}^{-1}$, the magnitude of B_2 is tiny, which seems to be appropriate for spectroscopy and electronic transitions.

Part 1.—The SI units in Equation (9) imply the dimensionally exact relation $B_2 = M/L$, which describes a unit of measurement (not a physical law). Thus, we proceed to compare B_2 to the Planck/RPS unit of linear mass density c^2/G; we write

$$B_2 = \kappa \left(\frac{c^2}{G} \right) , \tag{10}$$

where the dimensionless scale factor κ takes the value

$$\kappa = 7.425843255 \times 10^{-51} . \tag{11}$$

This scale factor hides a surprise: the leading coefficient 7.425 . . . originally appeared in the magnitude of $G_\star$ (Ref. [1] and Table 1) as a complicated composite constant. Thus, κ can be rewritten as $\kappa = 10^{-30}\,\mathcal{N}(G_\star)$, and using the G-entry in Table 1, as

$$\kappa = 10^{-42}\,\mathcal{N}\big(k_B/e\big)^2\,. \tag{12}$$

Remarkably, factor κ carries entropy and charge information, which then ties into the magnitude of B_2, viz.

$$\mathcal{N}(B_2) = 10^{-42}\,\frac{\mathcal{N}(k_B c)^2}{\mathcal{N}(e^2 G)}\,. \tag{13}$$

Therefore, the apparently simple density magnitude $\mathcal{N}(B_2)$ is determined by the vacuum and entropy, gravity, and EM properties; but the additional ingredients that separate it from the RPS linear mass density c^2/G are entropy and electric charge that are brought in by factor κ (Equation (12)).

The magnitude of the fraction in Equation (13) is $|A_4| = 10^{19}$ to 10 SDs; thus it holds that $\mathcal{N}(B_2) = 10^{-42}\,\mathcal{N}(A_4)$. The subset of the four constants $A4 := \{\,G,\ c,\ e,\ k_B\,\}$ will be analyzed in detail in Section 4.1 below.

Part 2.—Next, we replace the units in Equation (9) with the corresponding RPS quantities, viz. kg m^{-1} $\rightarrow M_P/L_P$, and we find that

$$B_2 = \kappa\left(\frac{M_P}{L_P}\right), \tag{14}$$

which is equivalent to Equation (10) above. Thus, when the starting equation (here, Equation (9)) represents a unit of measurement (rather than a physical law), the analyses in Parts 1 and 2 produce the same result.

Probably the most important conclusion drawn from the above analysis of Equation (9) is that the Planck/RPS unit of linear mass density $\mu_P = c^2/G$ is not appropriate for atomic or subatomic scales, and that the appropriate spectroscopic scale B_2 seems to be influenced by two additional constants, e and k_B, coming from disparate areas of physics (EM theory and thermodynamics, respectively).

Scale Factor κ.—The scale factor κ in Equation (14) admits another interpretation that must rely on known dimensionless constants (and not on dimensionful constants stripped of their units, as in Equation (12)). This is necessary because the ratio M_P/L_P already carries the units of the dimensionful constant B_2, and it also shows clearly why well-known dimensionless constants must be included in systems of units (see discussion in Section 2.1).

In our case, the RPS already includes the various coupling constants listed in Table 1 as derived units. Thus, we express the unitless factor κ (Equation (11)) in terms of geometric and coupling constants as follows:

Combining Equations (9) and (10), we find that $\kappa = G m_e R_\infty/c^2$, and substituting the definition of R_∞ from Table 1, this expression takes the equivalent forms

$$\kappa = 2\pi^2\left(\frac{Ke^2}{hc}\right)^2\left(\frac{Gm_e^{\,2}}{hc}\right) \;=\; 2\pi^2\alpha^2\alpha_g\,. \tag{15}$$

The geometric factor of $2\pi^2$ appears because R_∞ is defined in terms of ε_0, whereas Planck/SI systems use the unit $4\pi\varepsilon_0 = 1/K$ [6–9]. For a reduced Rydberg constant $\widetilde{R}_\infty = m_e e^4/[(4\pi\varepsilon_0)^2 h^3 c]$, the corresponding equation for the reduced $\widetilde{\kappa} = G m_e \widetilde{R}_\infty/c^2$ results in the simple expression

$$\widetilde{\kappa} = \alpha^2\alpha_g\,.$$

In either case, we note that such a remarkable combination of dimensionless coupling constants has not been heretofore seen in physics.

Compton Radius r_c.—One may speculate further that there must be other dimensionless constants expressed in terms of only one or the other coupling constant. The FSC appears prominently in Sections 3–5, so here we visit only the much simpler case of the gravitational coupling constant:

We define the Compton radius of the electron in the RPS as

$$r_c \equiv \frac{h}{m_e c}\,, \tag{16}$$

and we use it to obtain yet another characteristic linear mass density, viz. m_e/r_c, which can be easily cast to the form

$$\frac{m_e}{r_c} = \alpha_g \left(\frac{c^2}{G}\right). \tag{17}$$

This equation is directly comparable to Equation (10) for B_2, with α_g providing the scale here in place of κ. Apparently an atomic-scale constant, the linear density m_e/r_c is nevertheless much larger than B_2 (Equation (9)) by a factor of 3.7544×10^4.

2.2.3. Subset $C2 := \{\, \varepsilon_0,\, a_0 \,\}$

We consider the constant C_2 such that

$$C_2 = \frac{a_0}{4\pi\varepsilon_0} = 1 \ \ \mathrm{m^2\,s^{-2}\,F^{-1}}\,, \tag{18}$$

where a_0 is the critical MOND acceleration (Table 1) [10,11] and F represents the unit of Farad.[5]

Part 1.—The SI units in Equation (18) imply that $\mathrm{m^2\,s^{-2}\,F^{-1}} \to \mathrm{V^2\,kg^{-1}}$, where V represents the unit of Volt. Thus, $C_2 = 1\ \mathrm{V^2\,kg^{-1}}$, establishing thus the dimensional relation [1]

$$C_2 = \frac{\mathcal{V}^2}{M}\,, \tag{19}$$

where $\mathcal{V}$ represents voltage.

This combination of quantities is well-known, though rarely used. It occurs in the energetics of capacitors in vacuum, where the specific energy $\mathcal{E}_M = E/M$ stored in a capacitor by a potential difference $\mathcal{V}$ is

$$\mathcal{E}_M = \frac{C}{2}\left(\frac{\mathcal{V}^2}{M}\right), \tag{20}$$

where $C = Q/\mathcal{V}$ is the capacitance. Recasting this equation in terms of the physically relevant energy density $u_0 = \rho\,\mathcal{E}_M$ (where ρ is the volume density of the material) and using Equation (19), we find that

$$u_0 = \left(\frac{C_2}{2}\right)\rho\, C\,. \tag{21}$$

Apparently, constant C_2 specifies a special constant energy density, since ρ and C are also constants fixed at the design stage of the capacitor.

As an example, we turn to industrial supercapacitors (EDLC; Ref. [12]). Typical EDLC specifications are $\rho = 0.7\ \mathrm{g\,cm^{-3}}$, $M = 30$ g, and $C = 100$ F, for a nominal operating value of $u_{\mathrm{op}} = 8.5\ \mathrm{J\,cm^{-3}}$ at $\mathcal{V}_{\mathrm{op}} = 2.7$ V. According to these values, the special energy density (21) is $u_0 = 0.035\ \mathrm{J\,cm^{-3}}$, only 0.4% of u_{op}. This corresponds to a low operating voltage of

$\mathcal{V}_0 = 0.173$ V, a value that can also be obtained from Equation (19), viz. $\mathcal{N}(\mathcal{V}_0) = \mathcal{N}(\sqrt{M})$ in SI units.

Part 2.—Next, we replace the units in Equation (18) with the corresponding RPS quantities, viz. $\mathrm{m}^2\ \mathrm{s}^{-2}\ \mathrm{F}^{-1} \to L_\mathrm{P}^2/(T_\mathrm{P}^2 C_\mathrm{P}) \to c^2/C_\mathrm{P}$, and we find that

$$C_2 = \frac{a_0}{a_\mathrm{P}}\left(\frac{c^2}{C_\mathrm{P}}\right), \tag{22}$$

where a_P is the Planck acceleration and C_P is the Planck capacitance in the RPS. The leading dimensionless factor has the value $a_0/a_\mathrm{P} = 5.015425001 \times 10^{-62}$. We note that gravity is implicitly present in Equation (22), as the Planck capacitance is defined by $C_\mathrm{P} = G_\star M_\mathrm{P}/c^2$ in the RPS [1].

3. Four Cases Each Involving Three Fundamental Constants

3.1. Subset $A3 := \{\, e,\, m_\mathrm{e},\, \Phi_0 \,\}$

We consider the constant A_3 such that

$$A_3 = e^2 m_\mathrm{e} \Phi_0^2 = 10^{-97}\ \ \mathrm{kg}^3\ \mathrm{m}^4\ \mathrm{s}^{-2}\,. \tag{23}$$

There are two points that need to be made about the quantum of magnetic flux $\Phi_0 = h/(2e)$: (a) the factor of 2 was inserted to reflect the Cooper pairs of electrons [13] that are responsible for superconductivity [14]; and (b) the magnetic flux represented by Φ_0 is missing a geometric factor of 2π because both h and e are intrinsically 3D quantities, whereas flux through a surface is strictly a 2D quantity [1]. This latter issue is clearly resolved below by the reductions themselves, where the factor of 2π appears explicitly. So, there is no need to modify the definition of Φ_0.

Part 1.—The SI units in Equation (23) imply the dimensionally exact relation $A_3 = MJ^2$, where J represents angular momentum. For an electron with spin $s = \pm 1/2$ and $J_z = s\,h/(2\pi)$, we then find that

$$A_3 = (2\pi)^2 m_\mathrm{e} {J_z}^2\,. \tag{24}$$

The geometric factor of $(2\pi)^2$ could be thought as normalization for the term Φ_0^2 embedded in A_3, where it actually imprints Φ_0 by 2π, the signature of 2D surface geometry. Then, $\Phi_0/(2\pi)$ becomes formally a 2D magnetic flux. Furthermore, by combining Equations (23) and (24), we recover the well-known result that $J_z = \pm\hbar/2$ for electronic spins. Here, Dirac's constant is defined, as usual, by $\hbar = h/(2\pi)$, but it is not used in the definitions of RPS units or dimensionless coupling constants.

Part 2.—Next, we replace the units in Equation (23) with the corresponding RPS quantities, viz. $\mathrm{kg}^3\ \mathrm{m}^4\ \mathrm{s}^{-2} \to {M_\mathrm{P}}^3 {L_\mathrm{P}}^4/{T_\mathrm{P}}^2$, and we find that

$$A_3 = \frac{1}{4\aleph_\mathrm{A}}\left(\frac{{M_\mathrm{P}}^3 {L_\mathrm{P}}^4}{{T_\mathrm{P}}^2}\right). \tag{25}$$

The unitless scale factor introduces the reciprocal of the reduced Avogadro number, which effectively represents the gravitational coupling constant, since $\alpha_\mathrm{g} = (\aleph_\mathrm{A})^{-2}$ (Table 1). The factor of $1/4$ is traced back to the Cooper-pair term of $4e^2$ in the flux quantum squared Φ_0^2.

Reduced Avogadro Number $\aleph_\mathrm{A}$.—The appearance of $\aleph_\mathrm{A}$ (as a dimensionless constant that relates Planck/RPS units to SI units in Equation (25)) indicates that this is nature's choice to describe a certain number of particles (contained in approximately 0.1 moles of any substance [1]). This should be contrasted to the man-made (subjective) Avogadro

number N_A, which appears below in the RPS transformations of composite constants only when it is included a priori in the definitions of such constants.

3.2. *Subset* $B3 := \{ G, k_B, \Phi_0 \}$

We consider the constant B_3 such that

$$B_3 = \frac{G\Phi_0}{k_B} = 10^{-2} \ \mathrm{m^3\ K\ kg^{-1}\ s^{-1}\ C^{-1}}, \tag{26}$$

an unusual combination of thermal, EM, and mechanical units. The Coooper charge in $\Phi_0 = h/(2e)$ cannot be eliminated, and this explains the presence of the unit of Coulomb. On the other hand, Planck's constant embedded in Φ_0, defined by $h = GM_P^2/c$, is effectively a mechanical unit in the RPS (see Table 1 and Ref. [1]).

Part 1.—The SI units in Equation (26) imply the dimensional relation $B_3 = L^3\Theta/(MTQ)$. Recalling that L^3 appears prominently in the definition of the magnetic dipole moment

$$\mu_B \equiv \left(\frac{2\pi}{\mu_0}\right) B L^3, \tag{27}$$

and after some manipulations ($F = QvB$, $v = L/T$), Equation (26) takes the surprising form

$$B_3 M^2 = \left(\frac{\mu_0}{2\pi}\right) \mu_B \Theta. \tag{28}$$

This equation relates temperature to mass and magnetic dipole moment.[6] It could turn out to be relevant to some classes of neutron stars, although so far their surface temperatures have not been linked to both properties [16–18].

Part 2.—Next, we replace the units in Equation (26) with the corresponding RPS quantities, viz. $\mathrm{m^3\ K\ kg^{-1}\ s^{-1}\ C^{-1}} \to L_P^3\Theta_P/(M_P T_P Q_P)$, and we find that

$$B_3 = \frac{1}{2\alpha_w}\left(\frac{L_P^3\Theta_P}{M_P T_P Q_P}\right), \tag{29}$$

where we have introduced the weak coupling constant $\alpha_w = \sqrt{\alpha}$ (Table 1), rather than the square root of the FSC. Once again, the factor of $1/2$ is traced back to the Cooper-pair charge of $2e$ in the flux quantum Φ_0 [14].

3.3. *Subset* $C3 := \{ e, R_\infty, \Phi_0 \}$

We consider the constant C_3 such that

$$C_3 = \frac{e^2\Phi_0^2}{R_\infty} = 10^{-74} \ \mathrm{kg^2\ m^5\ s^{-2}}, \tag{30}$$

where the various constants are listed in Table 1.

We have encountered the product $e^2\Phi_0^2$ in Section 3.1 as well. Because it reduces to simply $(h/2)^2$, this combination naturally points to the electron's spin, and this is how we will proceed below to interpret this constant.[7]

Part 1.—The SI units in Equation (30) imply the dimensional relation $C_3 = M^2L^5/T^2$. Similarly to the reductions of A_3 in Section 3.1, this relation takes the form

$$C_3 = (2\pi)^2 R_\infty^{-1} J_z^2, \tag{31}$$

where $J_z = \pm\hbar/2$ is the electron's spin angular momentum.

Constant C_3 can also be expressed in terms of the Compton radius r_c (Equation (16)). Using the relation

$$R_\infty^{-1} = \frac{r_c}{2\pi^2\alpha^2}\,, \tag{32}$$

Equation (31) becomes

$$C_3 = \left(\frac{2}{\alpha^2}\right) r_c J_z^{\,2}\,. \tag{33}$$

The residual factor of 2 can be traced to the factor of 1/2 in the Rydberg energy [6,7]. At a deeper level, this factor originates from the virial theorem for the Coulomb-bound electron in the ground state of the hydrogen atom [20].

Part 2.—Next, we replace the units in Equation (30) with the corresponding RPS quantities, viz. $\mathrm{kg}^2\ \mathrm{m}^5\ \mathrm{s}^{-2} \rightarrow M_P^{\,2} L_P^{\,5}/T_P^{\,2}$, and we find that

$$C_3 = \frac{\aleph_A}{8\pi^2\alpha^2}\left(\frac{M_P^{\,2} L_P^{\,5}}{T_P^{\,2}}\right). \tag{34}$$

Here again, the unitless scale factor introduces the reduced Avogadro number, which effectively represents the gravitational coupling constant since $\alpha_g = (\aleph_A)^{-2}$ (Table 1). Furthermore, the factor of $8\pi^2$ ($= 4 \times 2\pi^2$) results from the 1/2 traced back to the Cooper-pair charge of $2e$ in Φ_0 [14] and the $2\pi^2$ traced to the definition of R_∞, where ε_0 has been introduced without the usual 4π geometric factor (Table 1).

Linear Mass Density B_2.—Eliminating the spin angular momentum J_z between Equations (24) and (31), we find that

$$\frac{A_3}{C_3} = m_e R_\infty \;\equiv\; B_2\,. \tag{35}$$

Thus, the emergence of the linear mass density constant B_2 (Equation (9)) is tied directly to the elimination of the dynamical component (the spin angular momentum) from the two combined equations.

3.4. Subset $D3 := \{ k_B, m_e, \mu_0 \}$

We consider the constant D_3 such that

$$D_3 = \frac{k_B m_e}{\mu_0} = 10^{-47}\ \mathrm{N\,C^2\,K^{-1}}\,, \tag{36}$$

where N represents the unit of force.

It is important to notice the markedly uncommon absence of the 3D geometric imprint of $1/(4\pi)$ from the magnetic permeability. This is bound to generate 4π geometric terms in the analysis that follows.

Part 1.—Equation (36) implies the dimensional relation

$$D_3 = F\left(\frac{Q^2}{\Theta}\right), \tag{37}$$

where F is force, Q is charge, and Θ is temperature. The term in the parentheses points to a degenerate Fermi gas [21]. It appears in the definition of the plasma coupling parameter $\Gamma \equiv K(Q^2/\Theta)/(a\,k_B)$ [22,23], where a is the typical interparticle spacing (or Wigner–Seitz radius) given for all phases of matter by

$$a = \left(\frac{3}{4\pi n}\right)^{1/3}, \tag{38}$$

where $n = N/V$ is the number density of charged particles, and the volume per particle is defined as $V/N = 4\pi a^3/3$.

Using Γ, we can write constant D_3 as $D_3 = \left(k_B \Gamma/K\right) F a$, and assuming that the force F between particles is Coulombic, we find that $\Theta \propto 1/a^2$, which implies that

$$\Theta \;\propto\; n^{2/3}\,. \tag{39}$$

The same scaling can also be obtained from Equation (37) for $F \propto 1/a^2$ and Q = constant.

Thus, temperature Θ exhibits fermionic scaling (like the Fermi temperature $\Theta_F \propto n^{2/3}$) up to a numerical prefactor that does not include h or $\hbar$. This component arises from Coulombic interactions between neighboring charges, with the $n^{2/3}$ scaling resulting from the inverse-square dependence on the Wigner–Seitz radius. Therefore, it reflects a purely EM origin, characteristic of dense classical plasmas [22] and ionic liquids [23].

Part 2.—Next, we replace the units in Equation (36) with the corresponding RPS quantities, viz. $\mathrm{N\,C^2\,K^{-1}} \rightarrow F_P Q_P^2/\Theta_P$, and we find that

$$D_3 = \frac{\sqrt{\alpha_g}}{4\pi}\left(\frac{F_P Q_P^2}{\Theta_P}\right). \tag{40}$$

The factor of 4π appears here to compensate for the missing 3D geometric tag of μ_0 in Equation (36). On the other hand, $\sqrt{\alpha_g} = 1/\aleph_A$, which introduces the reduced Avogadro number $\aleph_A$ (Table 1) in Equation (40), effectively a number of particles. That was indicated by the analysis in Part 1, because the number density of charged particles is a controlling property in degenerate Fermi gases, classical plasmas, and ionic liquids.

An important point to note here is that it is $\aleph_A$ that appears in Equation (40), not the well-known Avogadro number N_A. This, once again, highlights the physical significance of $\aleph_A$ [1] against the importance of the arbitrary man-made constant N_A, which is nevertheless retained in the RPS for continuity and historical reasons (see the RPS section in Table 1).

4. Nine Cases Each Involving Four Fundamental Constants

4.1. Subset $A4 := \{ G, c, e, k_B \}$

We consider the constant A_4 such that[8]

$$A_4 = \frac{c^2 k_B^2}{G e^2} = 10^{19}\ \ \mathrm{kg\,V^2\,m^{-1}\,K^{-2}}\,, \tag{41}$$

where V represents the unit of Volt.

Part 1.—The SI units in Equation (41) are equivalent to the composite units $\mathrm{Pa\,Wb^2\,K^{-2}}$, where Pa and Wb represent Pascal and Weber, respectively. These units imply the dimensional relation

$$A_4 = u\left(\frac{\Phi_B}{\Theta}\right)^2, \tag{42}$$

where u is energy density (or pressure) and Φ_B is magnetic flux. This relation can be rewritten in terms of a special form of the Fisher information $\mathcal{I}(\Theta)$ [24]. For an observable power-law response $\Phi_B(\Theta) \propto \Theta^n$, the sensitivity is $\mathcal{S} = \partial\Phi_B/\partial\Theta = n(\Phi_B/\Theta)$, and then $\mathcal{I}(\Theta)$ takes the form

$$\mathcal{I}(\Theta) = \left(\frac{\mathcal{S}}{\sigma_{\Phi_B}}\right)^2 = \left(\frac{n}{\sigma_{\Phi_B}}\right)^2\left(\frac{\Phi_B}{\Theta}\right)^2, \tag{43}$$

where $(\sigma_{\Phi_B})^2$ denotes the variance of the Gaussian noise associated with the observable flux $\Phi_B(\Theta)$.

Next, combining Equations (42) and (43), we find that

$$\mathcal{S}^2 \;=\; n^2 \left(\frac{A_4}{u}\right), \tag{44}$$

which indicates that the sensitivity of the observable flux to temperature is $\mathcal{S} \propto 1/\sqrt{u}$. This is an unexpected result—that the Fisher information may also be affected by the energy density of the system—and no such property has been reported to date.

If u is assumed to be the energy density of the magnetic field B, then $u \propto B^2$ and $\mathcal{S} \propto 1/B$. Thus, weaker magnetic fields allow for greater sensitivity, and vice versa. Under the additional assumption that the noise level σ_Φ is field-independent, then the Fisher information scales as $\mathcal{I}(\Theta) \propto 1/B^2$. These conclusions are, however, heavily dependent on the initial assumption of a power-law response of Φ_B to temperature. This parameter space needs to be explored by experiment.

Part 2.—Next, we replace the units in Equation (41) with the corresponding RPS quantities, viz. kg V^2 m^{-1} K^{-2} $\rightarrow M_\mathrm{P}\mathcal{V}_\mathrm{P}^2/(L_\mathrm{P}\Theta_\mathrm{P}^2)$, where $\mathcal{V}_\mathrm{P}$ is the Planck voltage, and we find that

$$A_4 = \alpha^{-1}\left(\frac{M_\mathrm{P}\mathcal{V}_\mathrm{P}^2}{L_\mathrm{P}\Theta_\mathrm{P}^2}\right), \tag{45}$$

where [1]

$$\alpha^{-1} \;=\; 861.022576584(132)\,. \tag{46}$$

Once again, we find that the composite constant under consideration is scaled to the corresponding RPS units by a single dimensionless coupling constant—the little-known RPS FSC inverse "861" that deposes the famous yet unphysical number "137" [25] which suffers from an unacceptable mixing of 2D and 3D geometric factors.

4.2. Subset $B4 := \{\, G,\, \mu_0,\, e,\, h \,\}$

We consider the constant B_4 such that

$$B_4 = \frac{e\,h^2}{G^2\mu_0^{\,2}} = 10^{-53}\;\; \mathrm{kg}^4\,\mathrm{C}\,\Omega^{-2}\,. \tag{47}$$

In this case too, μ_0 is not tagged by the 4π geometry of 3D space. But 4π prefactors do not appear in the analysis because it utilizes the impedance of free space $Z_0 = \mu_0 c$, which is also missing its usual 3D tag. Thus, these 4π factors are subject to cancellation. A similar cancellation of 2D geometric factors was outlined in Note 6.

Part 1.—The SI units in Equation (47) imply the dimensional relation

$$M^4 Q = Z_0^{\,2} B_4 \;\equiv\; \mathcal{D}\,, \tag{48}$$

where Z_0 is the impedance of free space (Table 1), and the new hybrid constant is

$$\mathcal{D} \;=\; 1.419257291\times 10^{-48}\;\; \mathrm{kg}^4\,\mathrm{C}\,. \tag{49}$$

Equation (48) is a scaling law that connects disparate physical scales:

(a) $Q = \mathcal{D}/M^4$: For $M = M_\mathrm{P}$, we find that $Q = e = \alpha_\mathrm{w} Q_\mathrm{P}$.—The Planck mass scale M_P produces "naturally" the elementary charge e, which is connected to the Planck charge by the weak coupling constant. Hence,

$$\mathcal{D} = \alpha_\mathrm{w} M_\mathrm{P}^4 Q_\mathrm{P}\,. \tag{50}$$

(b) $M = (\mathcal{D}/Q)^{1/4}$: For $Q = Q_P$, we find that $M = \alpha_w^{1/4} M_P$.—The Planck charge scale Q_P produces a mass scale M, which is a fraction of the Planck mass ($M = 0.430\, M_P$).

Part 2.—Next, we replace the units in Equation (47) with the corresponding RPS quantities, viz. $\mathrm{kg}^4\,\mathrm{C}\,\Omega^{-2} \to \mathrm{M_P}^4\, Q_P/{Z_0}^2$, and we find that

$$B_4 = \alpha_w \left(\frac{{M_P}^4 Q_P}{{Z_0}^2} \right). \tag{51}$$

where the weak coupling constant serves again as the link between B_4 and the corresponding RPS units.

4.3. Subset $C4 := \{ G, K, e, k_B \}$

We consider the constant C_4 introduced in Equation (3), viz.[9]

$$C_4 = \frac{e\sqrt{G_\star}}{k_B} = 10^{-6}\ \ \mathrm{F\,K\,kg^{-1}}\,, \tag{52}$$

where $G_\star = G/K$; thus, $G_\star$ carries two independent constants (see also Equation (2)).[10]

Part 1.—The SI units in Equation (52) imply the unusual dimensional relation $C_4 = C\,\Theta/M$, where C represents capacitance. This relation can be rewritten dimensionally[11] as

$$C_4 = \Theta \left(\frac{Q}{p_\perp} \right)^2, \tag{53}$$

where $p_\perp$ is assumed to represent transverse momentum. We imagine now a thermal plasma at temperature Θ, permeated by a uniform magnetic field B, and a charged (Q) particle injected into this medium with transverse momentum $\boldsymbol{p}_\perp \perp \boldsymbol{B}$, thus executing gyrations about $\boldsymbol{B}$-field lines. The radius of gyration is the so-called Larmor radius [19], viz.

$$R_L = \frac{p_\perp}{|Q|B}\,, \tag{54}$$

and the magnetic rigidity [19] is

$$B R_L = \frac{p_\perp}{|Q|}\,. \tag{55}$$

On the other hand, Equation (53) predicts a magnetic rigidity that depends solely on the temperature of the plasma, viz.

$$B R_L = \sqrt{\frac{\Theta}{C_4}}\,. \tag{56}$$

Experimental tests in magnetized plasmas under controlled conditions are needed to determine whether Equation (56) represents a genuine physical effect (magnetic rigidity $B R_L \propto \sqrt{\Theta}$), or simply a temperature scaling relation involving the new constant $C_4 = 10^{-6}\ \mathrm{F\,K\,kg^{-1}}$.

Part 2.—Next, we replace the units in Equation (52) with the corresponding RPS quantities, viz. $\mathrm{F\,K\,kg^{-1}} \to C_P\,\Theta_P/M_P$, and we find that

$$C_4 = \alpha_w \left(\frac{C_P\,\Theta_P}{M_P} \right), \tag{57}$$

where we have again introduced the weak coupling constant $\alpha_w = \sqrt{\alpha}$ (Table 1), rather than the square root of the FSC.

4.4. Subset $D4 := \{\, \sigma,\, c,\, h,\, R_\infty \,\}$

We consider the constant D_4 such that

$$D_4 = \frac{\sigma}{c}(h R_\infty)^2 = 10^{-68}\ \ \mathrm{kg^3\ m\ s^{-4}\ K^{-4}}\,, \tag{58}$$

where σ represents the Stefan–Boltzmann constant (Table 1).

Part 1.—The SI units in Equation (58) imply the dimensional relation $D_4 = M^3 L/(T\,\Theta)^4$. We take the following steps:

(a) We eliminate the temperature by using the Stefan–Boltzmann law $j = \sigma\,\Theta^4$, where j is the radiant exitance with dimensions of [power][area]$^{-1}$, so that j can, in turn, be replaced by
$$E/(L^2T) \to M/T^3\,,$$
where E represents energy.

(b) By reducing the mechanical quantities, we obtain the relation $D_4/\sigma = M^2 v$, where v is velocity.

(c) Finally, we introduce the dimensional angular momentum $J = MLv$ and linear mass density $\mu = M/L$, and we find the interesting relation

$$\mu J = \frac{D_4}{\sigma}\,. \tag{59}$$

The unusual combination μJ occurs in string theory [26,27], particularly in studies of Regge trajectories. For a rotating open Nambu–Goto (NG) relativistic string—which supports the standard Regge-type relation $J \propto E^2$—with energy per unit length μc^2 and hence string tension $T_0 = \mu c^2$, the angular momentum satisfies $J = E^2/(2\pi c T_0)$ or, equivalently, $J = E^2/(2\pi\mu c^3)$, from which we obtain the relation

$$\mu J = \frac{E^2}{2\pi c^3}\,. \tag{60}$$

Thus, the constant D_4/σ in Equation (59) effectively singles out a radiative/atomic energy scale in the NG framework, viz.

$$E = \sqrt{2\pi}\,\left(h\,c\,R_\infty\right)\ \simeq\ 34.1044\ \mathrm{eV}\,, \tag{61}$$

where the term in the parentheses is the standard Rydberg energy (13.6057 eV in CODATA Refs. [6,7]) and the factor $\sqrt{2\pi} \simeq 2.5$.[12] This is a surprising atomic connection because the rotating NG string is often used as an idealized model of meson-like hadronic states, where it reproduces Regge-type trajectories [27].

Part 2.—Next, we replace the units in Equation (58) with the corresponding RPS quantities, viz. $\mathrm{kg^3\ m\ s^{-4}\ K^{-4}} \to M_\mathrm{P}^{\,3}\, L_\mathrm{P}/(T_\mathrm{P}^{\,4}\,\Theta_\mathrm{P}^{\,4})$, and we find that

$$D_4 = \frac{8\pi^9}{15}\left(\alpha^4\alpha_\mathrm{g}\right)\left(\frac{M_\mathrm{P}^{\,3} L_\mathrm{P}}{T_\mathrm{P}^{\,4}\Theta_\mathrm{P}^{\,4}}\right). \tag{62}$$

In this equation, the composite factor $(8\pi^9\alpha^4/15)$ is traced directly to the geometric factors in Equation (58) introduced by the definitions of σ and R_∞: the Stefan–Boltzmann constant contributes $2\pi^5/15$ (Table 1), while the Rydberg constant contributes $2\pi^2\alpha^2$ (see also Equation (15)).

4.5. *Subset* $E4 := \{ F, K, \Phi_0, \sigma \}$

We consider the constant E_4 such that

$$E_4 = \left(F K \Phi_0 / \sigma\right)^2 = 10^{15} \ \left(\mathrm{kg\ m^5\ K^4\ C^{-2}}\right)^2, \tag{63}$$

where F represents the Faraday constant and K represents the Coulomb constant (Table 1). Because of the overall square in this constant, it is reasonable to consider $\sqrt{E_4}$ below.

Part 1.—The SI units in Equation (63) imply the dimensional relation $\sqrt{E_4} = M L^5 \Theta^4 / Q^2$. We take the following steps:

(a) We eliminate temperature by using the Stefan–Boltzmann law $j = \sigma \Theta^4$, where j is the radiant exitance, so that j can then be replaced by $E/(L^2 T)$, where $E = M v^2$ represents energy.

(b) Next, we notice that the dimensional combination $[Q]^2 [M]^{-1} = [G_\star][M]$, which allows us to replace $M/Q^2 = 1/(G_\star M)$.

(c) From the above substitutions, we deduce the relation

$$\begin{aligned} L^2 v^3 &= \sigma G_\star \sqrt{E_4} \\ &= \tfrac{1}{2} G h N_\mathrm{A} \\ &\equiv C_\mathrm{A} \end{aligned} \ . \tag{64}$$

Although Equation (64) seems to promote a simple fluid-mechanics scaling, it should not be applicable to unmagnetized fluids because the magnetic flux quantum was introduced in the definition of E_4. Instead, this scaling offers a compelling framework for understanding energy transfer in both magnetohydrodynamic (MHD) and kinetic plasma turbulence. We thus outline three potential applications of Equation (64), comparing and contrasting its physical consequences with established classical theories.

1. **Velocity Scaling:** In classical fluid turbulence [29], velocity fluctuations scale as $v \propto L^{1/3}$, implying that energy smoothly decreases as the cascade moves to smaller scales. In contrast, the above scaling implies that $v \propto L^{-2/3}$. This inverse relationship dictates that velocity fluctuations increase at smaller length scales, and this is a signature of energy transfer to highly localized, highly energetic MHD structures in plasma environments.
2. **Constant Power and Current Sheet Discontinuities:** In Electron MHD or incompressible MHD frameworks, applicable to the sub-ion range [30–32], we calculate the kinematic power $\mathcal{P}$ processed by a single localized eddy or plasma filament of length scale L, scale-independent density ρ, and typical turbulent velocity v. We find the scaling relation

$$\mathcal{P} = \rho L^2 v^3 = \rho C_\mathrm{A} = \text{const.} \tag{65}$$

   Thus, according to Equation (64), the power flowing into smaller structures remains scale-invariant. As a result, instead of a smooth Kolmogorov-type energy decay, equal amounts of power $\mathcal{P}$ are funneled into continuously shrinking volumes. Consequently, this framework accurately describes the dynamics of sub-ion range MHD turbulence, magnetic reconnection sites, and the localized powerful explosions that generate sharp discontinuities in thin current sheets within the plasma.
3. **Kinetic and Magnetic Spectra:** Applying such a constant power postulate to Fourier space yields two distinct energy spectra without relying on any constant-dissipation assumptions:

- For the 1D kinetic energy spectrum with wavenumbers $k = 1/L$ and localized velocity fluctuations $v(k) \propto k^{2/3}$, the model predicts a positive-slope spectrum of

$$E_{\rm kin}(k) \;\propto\; v^2 \;\propto\; k^{4/3}\,,$$

  representing an intense spatial intermittency and energy concentration at small scales.
- On the other hand, treating the sub-ion range—where only electrons are current carriers and, from Ampére's law, $\delta B(k) \propto v/k \propto k^{-1/3}$—yields a 1D magnetic-energy spectrum of

$$E_B(k) \;\propto\; (\delta B)^2/k \;\propto\; k^{-5/3}\,.$$

  This spectrum is highly consistent with established results in the gyrokinetic literature [33], Electron-MHD simulations [31], and direct spacecraft observations of the solar wind [32].

Part 2.—Next, we replace the units in Equation (63) with the corresponding RPS quantities, viz. kg m^5 K^4 C^{-2} $\rightarrow M_{\rm P}\, L_{\rm P}^5\, \Theta_{\rm P}^4/Q_{\rm P}^2$, and we find that

$$\sqrt{E_4} = \frac{15}{4\pi^5}\, N_{\rm A} \left(\frac{M_{\rm P}\, L_{\rm P}^5\, \Theta_{\rm P}^4}{Q_{\rm P}^2} \right), \tag{66}$$

where $N_{\rm A}$ is the usual Avogadro number, which appears because it was introduced by Faraday's constant in the definition (63) of E_4. The leading numerical factor can be traced to the numerical factors included in the definitions of Φ_0 and σ (Table 1). We also note the absence of unitless coupling constants in Equation (66). This is because the constant $C_{\rm A}$ defined by Equation (64) does not contain the necessary ingredients (c, e, or $m_{\rm e}$); the two constants G and h are simply not enough.

4.6. Subset $F4 := \{\, h,\, \sigma,\, \Phi_0,\, R_\infty \,\}$

We consider the constant F_4 such that

$$F_4 = \frac{h^2\, \sigma}{\Phi_0\, R_\infty^2} = 10^{-73}\ \ {\rm J}^2\ {\rm C\ K}^{-4}\,, \tag{67}$$

where J represents the unit of Joule.

Part 1.—The SI units in Equation (67) imply the dimensional relation $F_4 = E^2\, Q/\Theta^4$. We manipulate this equation using the Stefan–Boltzmann law $j = \sigma\, \Theta^4$, radiant exitance $j = M/T^3$, energy $E = M v^2$ and $E = h/T$, current $I = Q/T$, magnetic moment $\mu = I L^2$, and action $h = \mu B T$; and we finally find an equation (not a relation) for the magnitude of the magnetic field B, viz.

$$B = \frac{h^2\, \sigma}{F_4} = \Phi_0\, R_\infty^2 = 0.249\ \ {\rm T}\,. \tag{68}$$

Note that this magnetic-field threshold concerns two electrons (the Cooper pair in Φ_0; Table 1), so this threshold could be relevant to superconductivity (i.e., Josephson current magnetic suppression [34–36] and the Shubnikov–de Haas effect in degenerate semiconductors [37–40]).

Part 2.—Next, we replace the units in Equation (67) with the corresponding RPS quantities, viz. J^2 C K^{-4} $\rightarrow E_{\rm P}^2\, Q_{\rm P}/\Theta_{\rm P}^4$, and we find that

$$F_4 = \frac{\pi/15}{\alpha_{\rm g}\, \alpha_{\rm w}^7} \left(\frac{E_{\rm P}^2\, Q_{\rm P}}{\Theta_{\rm P}^4} \right), \tag{69}$$

where the weak coupling constant $\alpha_{\rm w} = \sqrt{\alpha}$ (Table 1).

On the other hand, we define the Planck unit of magnetic field as

$$B_{\rm P} \equiv \frac{F_{\rm P}}{c\,Q_{\rm P}} = \frac{\alpha_{\rm w}\,c^3}{G\,e}\,, \tag{70}$$

and we recast Equation (68) in RPS units as

$$B = \left(2\pi^4\,\alpha_{\rm g}\,\alpha_{\rm w}^{\,7}\right) B_{\rm P}\,. \tag{71}$$

It is rather surprising that the gravitational coupling constant is involved in the above equations, but it is the only dimensionless constant that can effectively drive the enormous RPS value of $B_{\rm P} \sim 10^{53}$ T down to laboratory-scale superconducting field magnitudes ($B \sim 0.1$–1 T).

4.7. Subset $G4 := \{\, h,\, \sigma,\, G,\, R_\infty \,\}$

We consider the constant G_4 such that

$$G_4 = \frac{h^2\,G\,R_\infty}{\sigma^2} = 10^{-55}\ \ {\rm m}^6\ {\rm s}^2\ {\rm K}^8\ {\rm kg}^{-1}\,. \tag{72}$$

Part 1.—The SI units in Equation (72) imply the dimensional relation $G_4 = L^6 T^2 \Theta^8/M$. We manipulate this equation using the Stefan–Boltzmann law $j = \sigma\,\Theta^4$, radiant exitance $j = M/T^3$, energy $E = M\,v^2$, and specific angular momentum $\ell = L\,v$; and we finally find a relation $E(\ell)$, viz.

$$E(\ell) = \frac{h^2\,G\,R_\infty}{\ell^2}\,. \tag{73}$$

This equation represents an invariant of orbital motion in a central $-k/r$ potential [41], where circular orbits obey the $E(J)$ relation $|E| = m\,k^2/(2J^2)$ with total angular momentum $J = m\,\ell$. We rewrite this classical Newtonian equation as $|E| = k^2/(2m\,\ell^2)$ for direct comparison. Thus, constant $h^2\,G\,R_\infty$ in Equation (73) plays the same role as $k^2/(2m)$ in the central potential.

We imagine an electron orbiting in the gravitational field of another mass $M_\star$, in which case $k = G\,M_\star\,m_{\rm e}$, and we equate the above two constants (i.e., $k^2/(2m_{\rm e}) = h^2\,G\,R_\infty$) to determine a typical magnitude of the central mass $M_\star$. Using $R_\infty = 2\pi^2\,\alpha^2/r_{\rm c}$ (Equation (32)), Equation (16) for $r_{\rm c}$, and $M_{\rm P}^2 = h\,c/G$, we find that

$$k = 2\pi\,\alpha\left(G\,M_{\rm P}\,m_{\rm e}\right). \tag{74}$$

Thus, the central mass implied by the $E(\ell)$ relation (73) for an orbiting electron is

$$M_\star = 2\pi\,\alpha\,M_{\rm P} \ \simeq\ 4.0\times10^{-10}\ \ {\rm kg}\,, \tag{75}$$

which turns out to be a small fraction of the Planck mass ($2\pi\,\alpha = 0.73\%$). Such a mass places the orbital dynamics of Equation (73) firmly in the sub-Planckian gravitational regime—but not at subatomic scales, where Newtonian gravity is negligible.

Part 2.—Next, we replace the units in Equation (72) with the corresponding RPS quantities, viz. ${\rm m}^6\ {\rm s}^2\ {\rm K}^8\ {\rm kg}^{-1} \to L_{\rm P}^{\,6}\,T_{\rm P}^{\,2}\,\Theta_{\rm P}^{\,8}/M_{\rm P}$, and we find that

$$G_4 = \frac{1}{2}\left(\frac{15}{\pi^4}\right)^2\left(\frac{\alpha^2}{\aleph_{\rm A}}\right)\left(\frac{L_{\rm P}^{\,6}\,T_{\rm P}^{\,2}\,\Theta_{\rm P}^{\,8}}{M_{\rm P}}\right). \tag{76}$$

The leading numerical factor naturally stems from the definitions of σ and R_∞ (Table 1). Again, the appearance of the reduced Avogadro number $\aleph_A$ indicates that this is a true natural constant, unlike N_A, which is a man-made constant (see Ref. [1] for more details). For the record, the natural constant $\aleph_A$ corresponds to nature's unit of $f_A^{-1} = 0.09945002021$ mol (Table 1), or, approximately, 0.1 mol.

4.8. Subset $H4 := \{ h, \sigma, m_e, \Phi_0 \}$

We consider the constant H_4 such that

$$H_4 = \frac{h \Phi_0^2 \sigma^2}{m_e} = 10^{-47} \ \mathrm{kg^4\,m^6\,s^{-9}\,C^{-2}\,K^{-8}}\,. \tag{77}$$

Part 1.—The SI units in Equation (77) imply the dimensional relation $H_4 = M^4 L^6/(T^9 Q^2 \Theta^8)$. After some straightforward manipulations, we obtain the dimensional equalities

$$\frac{H_4}{h\sigma^2} = \left(\frac{T}{h}\right)\left(\frac{EL}{Q}\right)^2 = E\left(\frac{L}{Q}\right)^2 = \frac{E}{\lambda^2}\,, \tag{78}$$

where the 1D line charge density $\lambda = Q/L$ was introduced in the last step.

The scaling $E \propto \lambda^2$ is not unknown in physics; it is a standard result in accelerator physics, where the energy loss of a charged bunch due to the self-induced "wakefields" is quadratic in the bunch line charge density [42–44]. In the wakefield formalism, this follows from the convolution structure of the wake potential or, equivalently, from the quadratic spectral weighting $|\lambda(\omega)|^2$ of the longitudinal impedance $Z_\parallel(\omega)$ in the Fourier-domain representation. Here, $\lambda(\omega)$ denotes the Fourier transform of the longitudinal charge distribution $\lambda(z)$ extending in the z-direction, and ω is the angular frequency of the Fourier modes.

Casting Equation (78) in the form $E = C_0 \lambda^2$, the proportionality constant turns out to be

$$C_0 = \frac{H_4}{h\sigma^2} = \frac{\Phi_0^2}{m_e} = 4.6940 \ \mathrm{Wb^2\,kg^{-1}}\,, \tag{79}$$

which is rather surprising.

In this setting, the electron mass introduces the fundamental inertial response in an accelerating bunch or beam of electrons [43,44]. Thus, m_e serves as a mechanical impedance to EM coupling. However, because the intrinsic inertia of the electron is so small, the bunch exhibits a high dynamical susceptibility to self-induced wakefields. Consequently, the scaling $C_0 \propto 1/m_e$ in Equation (79) reflects the efficiency with which kinetic energy is transduced into EM energy of the wakefield, as the electrons lack the inertial stiffness required to resist the modulation coming from the magnetic flux Φ_0.

Part 2.—Next, we replace the units in Equation (77) with the corresponding RPS quantities, viz. $\mathrm{kg^4\,m^6\,s^{-9}\,C^{-2}\,K^{-8}} \to M_P^4\, L_P^6/(T_P^9\, Q_P^2\, \Theta_P^8)$, and we find that

$$H_4 = \left(\frac{\pi^5}{15}\right)^2 \left(\frac{\aleph_A}{\alpha}\right)\left(\frac{M_P^4\, L_P^6}{T_P^9\, Q_P^2\, \Theta_P^8}\right). \tag{80}$$

The leading numerical factor naturally comes from the definitions of σ and Φ_0 (Table 1).

On the other hand, we define the Planck unit of magnetic flux [1] as

$$\Phi_P \equiv \sqrt{\frac{Kh}{c}}\,, \tag{81}$$

and we recast Equation (79) in RPS units, viz.

$$C_0 = \frac{\aleph_A}{4\alpha}\left(\frac{\Phi_P^2}{M_P}\right). \tag{82}$$

The leading factor of $1/4$ comes from the definition of Φ_0 (Table 1). Once again, it is surprising that the reduced Avogadro number appears in this scaling in the numerator of the unitless fraction, where it clearly dominates over the FSC, just as we also saw in Equation (34). Evidently, inertial response does play a role in an accelerated bunch or beam of electrons, the least massive free particles known in this universe.

4.9. Subset I4 := { c, σ, k_B, R }

We consider the constant I_4 such that

$$I_4 = \frac{k_B^2 R\,\sigma}{c^2} = 10^{-69}\ \text{kg}^4\,\text{m}^4\,\text{s}^{-7}\,\text{K}^{-7}, \tag{83}$$

where R is the molar gas constant (Table 1).

Part 1.—The SI units in Equation (83) imply the dimensional relation $I_4 = M^4L^4/(T^7\Theta^7)$. After some straightforward manipulations, we obtain the equalities

$$\frac{k_B I_4}{\sigma^2} = \frac{k_B\Theta}{T}\left(ML^2\right)^2 = \frac{E}{T}\left(ML^2\right)^2 = (ET)^3. \tag{84}$$

Before we can interpret this scaling physically, we must analyze the composite constant on the left-hand side. Thus, we solve Equation (84) for E, we simplify the constant, we introduce the frequency $f = 1/T$, and we find that

$$E = \left(\frac{15}{2\pi^5}N_A\right)^{1/3} hf. \tag{85}$$

The simplified constant scales the photon energy hf up to the total energy of a classical ensemble of about 24.5 million photons, a number that is not unrealistic in laser optical technology [45].

As an example, consider a continuous-wave semiconductor laser with a spectral linewidth $\Delta f = 100$ MHz, corresponding to a coherence time $\tau_c = 1/\Delta f = 10$ ns. This coherence time defines the interval over which the optical field maintains a well-defined phase.

For visible light with frequency $f = 6\times10^{14}$ Hz and a modest output power of $\mathcal{P} = 1$ mW, the corresponding photon flux is $dN/dt = \mathcal{P}/(hf) \simeq 2.5\times10^{15}\ \text{s}^{-1}$. Over a single coherence time, the number of photons contained in the phase-coherent portion of the field is $N = \tau_c(dN/dt) \simeq 25$ million. In this regime, the radiation is well described as a classical EM field with a stable phase over the coherence interval, and the photon number fluctuations are correspondingly very small ($\Delta N \simeq \sqrt{N} \simeq 5.0\times10^3$ by Poisson statistics, and then $\Delta N/N \simeq 0.02\%$).

Before closing, we should also interpret the power of $N_A^{1/3}$ in Equation (85). The power of $1/3$ appears because the photon occupation number is obtained from an integral over a 3D phase space, where the number of accessible modes scales as the cube of the characteristic frequency or wavevector. Then, deriving a single characteristic energy scale corresponds to expressing the cumulative 3D phase-space occupation in terms of its associated radial wavevector scale, which necessarily introduces a cubic-root dependence on the total occupation number. On the other hand, such a radial reduction of the 3D phase

space is not encoded in the definition (83) of the constant I_4, nor does it appear in the RPS unit transformation described by Equation (86) below.

Part 2.—Next, we replace the units in Equation (83) with the corresponding RPS quantities, viz. $\mathrm{kg}^4\ \mathrm{m}^4\ \mathrm{s}^{-7}\ \mathrm{K}^{-7} \rightarrow M_\mathrm{P}^{\,4}\, L_\mathrm{P}^{\,4}/(T_\mathrm{P}^{\,7}\, \Theta_\mathrm{P}^{\,7})$, and we find that

$$I_4 = \frac{2\pi^5}{15}\, N_\mathrm{A}\, \frac{\left(M_\mathrm{P}\, L_\mathrm{P}\right)^4}{\left(T_\mathrm{P}\, \Theta_\mathrm{P}\right)^7}\,. \tag{86}$$

The leading numerical factor[13] comes from the definition of the Stefan–Boltzmann constant σ, and the Avogadro number comes from the definition of the molar gas constant R (Table 1).

5. Four Cases Each Involving Five Fundamental Constants

5.1. Subset $A5 := \{\, G,\, c,\, e,\, k_\mathrm{B},\, K \,\}$

We consider the constant A_5 such that

$$A_5 = G\left(\frac{c\,e}{k_\mathrm{B} K}\right)^2 = 10^{-5}\ \ \mathrm{K}^2\ \mathrm{s}^4\ \mathrm{C}^6\ \mathrm{kg}^{-5}\ \mathrm{m}^{-5}\,, \tag{87}$$

where $K = 1/(4\pi\varepsilon_0)$ (Table 1) and K represents the unit of Kelvin. Constant A_5 combines elements of gravity, electricity, the vacuum, and Boltzmann entropy.

Part 1.—The SI units in Equation (87) are equivalent to the composite units $\mathrm{A}^6\ \mathrm{K}^2\ \mathrm{N}^{-5}$, where A and N represent Ampere and Newton, respectively. Eliminating the unit of force from the dimensional relation $F = I^2\mu_0/(4\pi)$ (a combination of the Biot–Savart law and the Lorentz force), where I represents electric current, these units imply the dimensional relation

$$\frac{\Theta^2}{I^4} \;=\; A_5\left(\frac{\mu_0}{4\pi}\right)^5 = 10^{-40}\ \ \mathrm{K}^2\ \mathrm{A}^{-4}\,. \tag{88}$$

Surprising as it may be, this scaling is a precise statement of equipartition between thermal energy per degree of freedom $E_\mathrm{th} = \frac{1}{2}\, k_\mathrm{B}\Theta$ and magnetic energy $E_\mathcal{B} = \frac{1}{2}\,\mathcal{L}_\mathrm{S}\, I^2$ of an inductor with inductance

$$\mathcal{L}_\mathrm{S} = \frac{\mu_0}{4\pi}\, L_\mathrm{S} = 1.380\,649 \times 10^{-43}\ \mathrm{H}\,, \tag{89}$$

where $L_\mathrm{S} = \alpha_\mathrm{w} L_\mathrm{P}$ is the Stoney length and H represents the unit of Henry. Because the vacuum permeability is a lower limit in nature and $L_\mathrm{S} < L_\mathrm{P}$, it certainly seems that the Stoney inductance $\mathcal{L}_\mathrm{S}$ is the vacuum's lower limit of inductive resistance (or "magnetic inertia").

Equation (88) manifests as a fundamental spatial expression of the Johnson–Nyquist thermal noise [46,47]. An ambient thermal bath, characterized by temperature fluctuations Θ, continuously imparts stochastic kinetic energy to the vacuum manifold. These thermodynamic kicks drive a microscopic thermal-noise current I. Spacetime resists this spontaneous flow via its intrinsic permeability μ_0, momentarily capturing the kinetic energy as magnetic energy.

This magnetic storage is strictly transient. As the initial thermal excitation wanes, the vacuum's localized magnetic field collapses and discharges the stored energy back into the surrounding thermal bath. The constant in Equation (88) acts as the fundamental scaling factor that bridges thermal fluctuations and magnetic energy. It dictates that the energy absorbed by the intrinsic magnetic field is balanced by the energy released back out to the bath, thus sustaining energy equipartition at the fundamental length scale L_S [48].

Part 2.—Next, we replace the units in Equation (87) with the corresponding RPS quantities, viz. $\mathrm{K}^2\ \mathrm{s}^4\ \mathrm{C}^6\ \mathrm{kg}^{-5}\ \mathrm{m}^{-5} \rightarrow \Theta_\mathrm{P}^{\,2}\, T_\mathrm{P}^{\,4}\, Q_\mathrm{P}^{\,6}/(M_\mathrm{P}^{\,5}\, L_\mathrm{P}^{\,5})$, and we find that

$$A_5 = \alpha \left(\frac{\Theta_\mathrm{P}^{\,2}\, T_\mathrm{P}^{\,4}\, Q_\mathrm{P}^{\,6}}{M_\mathrm{P}^{\,5}\, L_\mathrm{P}^{\,5}} \right). \tag{90}$$

The same method, applied also to Equation (88), gives the relation

$$A_5 \left(\frac{\mu_0}{4\pi} \right)^5 = \alpha \left(\frac{\Theta_\mathrm{P}^{\,2}}{I_\mathrm{P}^{\,4}} \right). \tag{91}$$

Combining Equations (90) and (91), we obtain an RPS identity for the vacuum permeability, viz.

$$\frac{\mu_0}{4\pi} = \frac{M_\mathrm{P}\, L_\mathrm{P}}{Q_\mathrm{P}^{\,2}} = \frac{L_\mathrm{P}}{G_\star M_\mathrm{P}}, \tag{92}$$

where we used $Q_\mathrm{P}^{\,2} = G_\star M_\mathrm{P}^{\,2}$ [1] to obtain the second leg. In this sense, μ_0 is determined only by mechanical units and a vacuum property, since $G_\star = 4\pi\varepsilon_0 G$.

5.2. *Subset* $B5 := \{\, G,\ m_\mathrm{e},\ \Phi_0,\ k_\mathrm{B},\ \mu_0 \,\}$

We consider the constant B_5 such that

$$B_5 = \frac{G^2\, m_\mathrm{e}\, \Phi_0^{\,2}}{\mu_0\, k_\mathrm{B}} = 10^{-51}\ \ \mathrm{m}^7\ \mathrm{K}\ \mathrm{kg}^{-1}\ \mathrm{s}^{-4}, \tag{93}$$

where, notably, μ_0 alone (without the prefactor $\frac{1}{4\pi} \simeq 0.080$) determines the precise power of ten to 6 SDs. Constant B_5 combines elements of gravity, magnetism, the vacuum, and Boltzmann entropy.

Part 1.—The SI units in Equation (93) imply the dimensional relation

$$B_5 u \ = \ \Theta v^6, \tag{94}$$

where u represents energy density with dimensions $[M][L]^{-1}[T]^{-2}$. On the other hand, the second law of thermodynamics effectively relates u to entropy density s in a locality of temperature Θ, viz. $u = \Theta s$, and Equation (94) reduces to

$$B_5 s \ = \ v^6, \tag{95}$$

an unusual relation scaling the entropy density s to a high power of the velocity v.

Entropy density essentially represents the information capacity of a field or a medium; thus Equation (95) dictates that such a field acts as an immense entropic reservoir. Since $|B_5| \ll 1$, a massive amount of information is required to be stored in the field to support even microscopic kinematic fluctuations.

The high power (v^6) of the kinematic state can be viewed in at least two ways:

(a) Since the specific kinetic energy $E/M \propto v^2$, then $s \propto (E/M)^3$, indicating that the field coupling is very sensitive to kinematic fluctuations driven by fluctuations of the magnetic field.

(b) In Curle's extension of aeroacoustic theory [49,50], sound energy is radiated by dipoles at the boundaries and propagates in the medium- to the far-field, where its intensity scales as v^6. Here, v^6 represents the radiation intensity of these dipole sources; thus, Equation (95) implies that the flow's internal entropy density s acts as the

thermodynamic source term that scales with the power required to sustain and radiate these boundary-induced fluctuations.

Part 2.—Next, we replace the units in Equation (93) with the corresponding RPS quantities, viz. $\mathrm{m}^7\ \mathrm{K}\ \mathrm{kg}^{-1}\ \mathrm{s}^{-4} \rightarrow L_\mathrm{P}^{\,7}\ \Theta_\mathrm{P}\ M_\mathrm{P}^{\,-1}\ T_\mathrm{P}^{\,-4}$, and we find that

$$B_5 = \left(16\pi\,\alpha\,\aleph_\mathrm{A}\right)^{-1} \left(\frac{L_\mathrm{P}^{\,7}\,\Theta_\mathrm{P}}{M_\mathrm{P}\,T_\mathrm{P}^{\,4}}\right). \tag{96}$$

The factor of $(16\pi)^{-1}$ is a combination of $(4\pi)^{-1}$ (the prefactor missing by design from μ_0 in the definition (93) of B_5) and $(1/2)^2$ coming from the definition of $\Phi_0^{\,2}$ (Table 1).[14,15]

5.3. Subset $C5 := \{\, G,\, c,\, m_\mathrm{e},\, \sigma,\, \Phi_0 \,\}$

We consider the constant C_5 such that

$$C_5 = \frac{G^2\,c^2\,m_\mathrm{e}\,\sigma}{\Phi_0} = 10^{-26}\ \ \mathrm{m}^6\ \mathrm{C}\ \mathrm{kg}^{-1}\ \mathrm{s}^{-8}\ \mathrm{K}^{-4}\,. \tag{97}$$

This constant combines elements of gravity, the vacuum, thermal radiation, and quantum mechanics.

Part 1.—The SI units in Equation (97) imply a simple dimensional relation. Using in sequence the standard dimensional conversions $\Theta^4 = j/\sigma$, $j = M/T^3$, $Q = IT$, $\rho = M/L^3$, and $\rho = 1/(GT^2)$, we find that

$$C_5 \;=\; \sigma\,G^2\,I\,, \tag{98}$$

where I represents electric current. The product $G^2 I$ is unique in physics because there is no theory unifying gravity and EM theory and no relation showing that $I \propto G^{-2}$—but it cannot be avoided. A clear intermediate step, viz. $C_5/\sigma = I/(\rho^2 T^4) = \text{const.}$, seems to associate current with mass density and time, which is not palatable. On the other hand, the scaling $\rho T^2 = 1/G$ of free-fall time T is ubiquitous in gravitational dynamics and in astrophysics—although it eliminates both ρ and T simultaneously, resulting in the mere determination of a constant current I.

Substituting the definitions of C_5 (Equation (97)) and Φ_0 (Table 1) into Equation (98), then G and σ both cancel out, and we find that the constant current is

$$I \;=\; \frac{m_\mathrm{e}\,c^2}{\Phi_0} = 2e\,f_\mathrm{c} = 39.6\ \mathrm{A}\,, \tag{99}$$

where $f_\mathrm{c} = c/r_\mathrm{c}$ is the Compton frequency attributed to each of the electrons in the Cooper pair captured by Φ_0. Obviously then, the internal circulatory motion of one electron is equivalent to a current of

$$I_\mathrm{1e} \;=\; e\,f_\mathrm{c} = 19.8\ \mathrm{A}\,. \tag{100}$$

This is a well-known result in relativistic quantum physics [51–56], corresponding directly to the so-called Zitterbewegung or the "trembling motion" of the electron. In the geometric and hydrodynamic interpretations of the Dirac equation [54,56], the electron is modeled as a localized charge e circulating with speed c at the Compton radius r_c. The resulting internal current for a single electron is indeed given by Equation (100).

In the above setting, C_5 acts as a universal repository of disparate physics, encompassing gravitational, thermal, and quantum scales within a single dimensional value. The transition from Equation (98) to (100) represents a systematic decoupling of macroscopic factors: by stripping away the contributions of gravity (G) and thermal state (σ), we have effectively removed the two large-scale or system-wide layers from this constant. What remains is

a precise description of the electron's internal state (I). This implies that the localized, high-frequency circulation of the electron is the irreducible quantum core that persists once the broader (global) gravitational and thermodynamic physics packed into C_5 is removed.

Part 2.—Next, we replace the units in Equation (97) with the corresponding RPS quantities, viz. $\mathrm{m}^6\ \mathrm{C}\ \mathrm{kg}^{-1}\ \mathrm{s}^{-8}\ \mathrm{K}^{-4} \rightarrow L_\mathrm{P}^{\,6}\ Q_\mathrm{P}\ M_\mathrm{P}^{\,-1}\ T_\mathrm{P}^{\,-8}\ \Theta_\mathrm{P}^{\,-4}$, and we find that

$$C_5 = \frac{4\pi^5}{15}\sqrt{\alpha_\mathrm{g}\,\alpha}\,\left(\frac{L_\mathrm{P}^{\,6}\,Q_\mathrm{P}}{M_\mathrm{P}\,T_\mathrm{P}^{\,8}\,\Theta_\mathrm{P}^{\,4}}\right). \tag{101}$$

As usual, the leading factor of $4\pi^5/15$ is a combination of the numerical factors coming from the definitions of Φ_0 and σ (Table 1).

Furthermore, the equivalent current $I_{1\mathrm{e}}$ to the trembling motion of one electron (Equation (100)) takes the following form after G and σ are removed:

$$I_{1\mathrm{e}} = \sqrt{\alpha_\mathrm{g}\,\alpha}\,\left(\frac{Q_\mathrm{P}}{T_\mathrm{P}}\right) \;=\; \sqrt{\alpha_\mathrm{g}\,\alpha}\;\mathcal{I}_\mathrm{P}\,, \tag{102}$$

where $\mathcal{I}_\mathrm{P} = c^2/\sqrt{G_B}$ is the RPS current [1], and the gravomagnetic constant G_B is defined in Table 1.

It is important to note that, at the Planck scale, the current $I_{1\mathrm{e}}$ is determined, in part, by the gravitational coupling constant α_g (Table 1), the fundamental universal constant of gravity that is systematically neglected in quantum physics and in QED (see also Note 15).

5.4. *Subset* $D5 := \{\, G,\, K,\, m_\mathrm{e},\, \sigma,\, \Phi_0 \,\}$

We consider the constant D_5 such that

$$D_5 = \frac{G^2\,K\,m_\mathrm{e}\,\sigma}{\Phi_0} = 10^{-33}\ \ \mathrm{m}^7\ \mathrm{C}^{-1}\ \mathrm{s}^{-8}\ \mathrm{K}^{-4}\,. \tag{103}$$

Just as C_5 above, this constant also combines elements of gravity, the vacuum, thermal radiation, and quantum mechanics. Then, it is not a leap to expect that these two composite constants are related in a specific way.

Part 1.—Constant D_5 can be written in terms of C_5 (Equation (97)) as

$$D_5 = \frac{\mu_0}{4\pi}\,C_5\,, \tag{104}$$

or, using Equation (98), as

$$D_5 \;=\; \sigma\,G^2\left(\frac{\mu_0\,I}{4\pi}\right). \tag{105}$$

The appearance of the term $(\mu_0\,I)$ indicates that D_5 is connected to the magnetic field B generated by the trembling motion of the electron (Section 5.3). Using $B = (\mu_0\,I)/(2r_\mathrm{c})$ at the center of the current loop [57] of radius r_c, Equation (105) produces the trembling magnetic field of a Cooper pair of electrons, viz.

$$B = \frac{e\,c}{\varepsilon_0}\left(\frac{m_\mathrm{e}}{h}\right)^2 = 1.0253\times 10^7\ \ \mathrm{T}\,. \tag{106}$$

Thus, for a single trembling electron, the magnetic field at the center of the current loop of radius r_c turns out to be

$$B_{1\mathrm{e}} = \frac{1}{2}\,B = 5.1265\times 10^6\ \ \mathrm{T}\,. \tag{107}$$

Compared to the QED critical value of $B_{\rm crit} = (m_{\rm e}c)^2/(e\hbar) = 4.4140 \times 10^9$ T (the so-called Schwinger limit [58–60]), the magnitude of field $B_{1\rm e}$ is, of course, much smaller. In fact, the scaling down to the trembling electron magnetic field is controlled entirely by the FSC, viz.

$$B_{1\rm e} = \alpha B_{\rm crit}\,, \tag{108}$$

a fundamental result to be kept in mind—a result precise to 12 SDs (although only 5 SDs are shown here).

This remarkable scaling is revealed only in RPS units. The old $\hbar$-based definition of the FSC obscures the relation by introducing a geometric term that has no place in Equation (108). The same 2π geometric contamination from the $\hbar$-based FSC has also been detected in the first-order correction to the Landé $g_{\rm s}$-factor of the anomalous magnetic moment of the electron, which is simply $(g_{\rm s} - 2)/2 = \alpha$ in RPS units [1].

Part 2.—Next, we replace the units in Equation (103) with the corresponding RPS quantities, viz. $\mathrm{m}^7\,\mathrm{C}^{-1}\,\mathrm{s}^{-8}\,\mathrm{K}^{-4} \rightarrow L_{\rm P}^{\,7}\,Q_{\rm P}^{\,-1}\,T_{\rm P}^{\,-8}\,\Theta_{\rm P}^{\,-4}$, and we find that

$$D_5 = \frac{4\pi^5}{15}\sqrt{\alpha_{\rm g}\,\alpha}\left(\frac{L_{\rm P}^{\,7}}{Q_{\rm P}\,T_{\rm P}^{\,8}\,\Theta_{\rm P}^{\,4}}\right). \tag{109}$$

Once again, the leading factor of $4\pi^5/15$ is a combination of the numerical factors coming from the definitions of Φ_0 and σ (Table 1). The dimensionless coefficient in D_5 is the same as that in C_5 (Equation (101)) because the two constants differ only by a dimensionful factor of $\mu_0/(4\pi)$ (see Equation (104)).

6. Discussion and Conclusions

In this work, we have systematically explored algebraic and dimensional relations constructed from small subsets of 2–5 fundamental constants (Sections 2–5, respectively) carved out of the main set shown in Table 1. The SI values of the composite constants that we analyzed are all precise powers of ten (Table 2).[16] This choice was motivated by constant C_4 with an SI magnitude of 10^{-6} which appeared in previous work [1] and allowed for the determination of the Newtonian gravitational constant G to 10 SDs.[17]

By organizing the analysis of the composite constants listed in Table 2 according to the number of the known constants involved in each synthesis (pairs to quintets), a clear pattern emerges: seemingly disparate physical domains—gravitation, electromagnetism, quantum mechanics, and thermodynamics—admit nontrivial couplings and relations never before seen or contemplated in physics or astrophysics. Thus, this method of analysis constitutes another pathway toward investigating unified descriptions of fundamental interactions.

A central outcome of this investigation is that most of these constructions reduce to scalings that resemble known physical relations or hint at structures already present in established theory. In most cases, the composite constants naturally reproduce quantities with clear physical meaning (e.g., energies, forces, magnetic fluxes, magnetic fields, electric currents, ohmic resistances, or EM characteristic scales), although some suggest less familiar but dimensionally consistent relations. This reinforces the value of dimensional analysis when applied exhaustively and systematically, not merely as a consistency check, but as a tool for uncovering latent connections between disparate physical regimes.

The present analysis was mainly restricted to composite constants with precise power-of-ten magnitudes (Table 2). This is not a limitation of the method; it is a choice reflecting our plan to expand the search beyond the only such constant heretofore known in physics, the vacuum's magnetic permeability $\mu_0/(4\pi)$ tagged by 3D geometry. From a methodological standpoint, the analysis of any other composite constants is also expected to generate additional families of relations and/or scalings. To probe this end, we extend in Appendix A

the analysis to four composite constants listed in Table 1 ($\mathcal{A}_0$, $G_\star$, G_B, and $\mathcal{R}_K$), and we discuss the resulting scalings, which are related to known physics—the Tully–Fisher/Faber–Jackson relations [61,62] in galaxies, the Larmor power Formula [9] for accelerating charges, the collapse of magnetized giant molecular clouds (GMCs) [63,64] in star-forming regions, and the quantum Hall effect [65–67]. These four composite constants are summarized in Table 3.

Table 3. Composite constants (not precise powers of ten) discussed in Appendix A.

Symbol	Definition	SI Value	SI Unit	Section	Related Themes in Science
$\mathcal{A}_0$	$a_0 G$	$7.425843255 \times 10^{-21}$	$m^4\,kg^{-1}\,s^{-4}$	Section A.1	Tully–Fisher/Faber–Jackson relations
$G_\star$	$4\pi\varepsilon_0 G$	$7.425843255 \times 10^{-21}$	$C^2\,kg^{-2}$	Section A.2	Larmor power formula
G_B	$G\,\mu_0/(4\pi)$	$6.674015081 \times 10^{-18}$	$m^4\,s^{-4}\,A^{-2}$	Section A.3	Magnetized GMCs, star formation
$\mathcal{R}_K$	h/e^2	2.581280746×10^4	Ω	Section A.4	Quantum Hall effect

The invariant quantities represented by the composite constants appear far from arbitrary; they instead reflect physically significant couplings of elementary constants across disparate physical domains. We suggest that these invariants are precursors to a fundamental set of constants inherent to unified theories bridging the gaps between mechanics, quantum mechanics, gravitation, thermal physics, and EM theory. By bridging any number of these realms, such extended frameworks will inevitably expose latent symmetries and governing principles currently obscured by the partitioning of modern physics.

As Part 2 of the dimensional method used in this work, we have also systematically transformed the SI units of each composite constant to Planck/RPS units. In most cases, the RPS units must be modulated by a specific set of dimensionless prefactors. These include the dimensionless coupling constants, the reduced Avogadro number, the Avogadro number, and geometric factors involving 2π (a 2D tag) or 4π (a 3D tag) (see Table 1 for their details):

- The appearance of the α, α_w, and α_g couplings in various exotic combinations turns out to be very important. It shows that such constants must be included in all systems of units to provide their services during transformations between systems. Without them, unit transformations exhibit seemingly arbitrary numerical factors that cannot be interpreted physically [4]. Here, the dimensionless constants appeared naturally because they have been included in the RPS long ago [1,5].
- The reduced Avogadro number $\aleph_A$ is nature's chosen number of particles in about 0.1 moles of matter. It also appears naturally, ever since it was discovered [1] and was included in the RPS (via constant f_A in Table 1).
- The well-known Avogadro number N_A appears only if it is implicitly used in the construction of a composite constant. This occurs when the synthesis includes the Faraday constant $F = N_A\,e$ or the molar gas constant $R = N_A k_B$ (see constants E_4 and I_4, respectively; Table 2).
- The geometric and numerical factors that appear in RPS unit transformations to SI units all come from the mixed numerical structures imprinted onto some of the elementary constants of Table 1 (specifically, the man-made constants R_∞, σ, and Φ_0, taken from CODATA Refs. [6,7]).

The same dimensionless factors also appear in transformations of composite constants between Gaussian/cgs units and Planck/RPS/cgs units (although we did not use Gaussian/cgs units in this study). Therefore, the dimensionless constants acting as numerical prefactors in unit conversions are universal units independent of systems of measurement,

and this reinforces our recommendation that they must be included in all systems of units currently employed in the physical sciences.

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Abbreviations

The following abbreviations are used in this manuscript:

CODATA	Committee On DATA
EDLC	Electric Double-Layer Capacitor
EM	ElectroMagnetic
FSC	Fine-Structure Constant
GMC	Giant Molecular Cloud
MHD	MagnetoHydroDynamic
MOND	MOdified Newtonian Dynamics
NG	Nambu–Goto
QED	Quantum ElectroDynamics
RPS	Reformulated Planck System
SDs	Significant Digits
SI	Système International d'unités
1D, 2D, etc.	One-Dimensional, Two-Dimensional, etc.

Appendix A. Additional Composite Constants

Appendix A.1. MOND Universal Constant $\mathcal{A}_0$

We consider the universal constant in MOND theory [11], viz. [1]

$$\mathcal{A}_0 = a_0 G = 7.425\,843\,255 \times 10^{-21}\ \ \mathrm{m^4\,kg^{-1}\,s^{-4}}\,. \tag{A1}$$

Part 1.—The SI units imply the dimensional relation

$$v^4 = \mathcal{A}_0 M\,, \tag{A2}$$

which is a manifestation of the Tully–Fisher/Faber–Jackson relations[18] [61,62] in spiral/elliptical galaxies of mass M, respectively. The velocity v represents orbital velocity of stars and gas in spiral galaxies and stellar velocity dispersion in elliptical galaxies.

Part 2.—Next, we replace the units in Equation (A1) with the corresponding RPS quantities, viz. $\mathrm{m^4\,kg^{-1}\,s^{-4}} \to c^4/M_\mathrm{P}$, and we find that

$$\mathcal{A}_0 = \frac{a_0}{a_\mathrm{P}}\left(\frac{c^4}{M_\mathrm{P}}\right), \tag{A3}$$

where $a_{\rm P}$ represents the Planck/RPS acceleration.

Appendix A.2. Gravoelectric Constant $G_\star$

We consider the gravoelectric constant (Table 1)

$$G_\star = 4\pi\varepsilon_0 G = 7.425843255\times10^{-21}\ \ \mathrm{C^2\,kg^{-2}}\,. \tag{A4}$$

Part 1.—The SI units imply the dimensional relation

$$G_\star = \left(\frac{Q}{M}\right)^2. \tag{A5}$$

The "specific charge" squared $(Q/M)^2$ is implicitly present in Larmor's nonrelativistic power Formula [9]

$$\mathcal{P} \equiv \frac{Q^2a^2}{6\pi\varepsilon_0 c^3} = \frac{F^2}{6\pi\varepsilon_0 c^3}\left(\frac{Q}{M}\right)^2, \tag{A6}$$

where $\mathcal{P}$ is the power radiated by the accelerating charged (Q) particle, a is the acceleration, and $F = Ma$ by Newton's second law of motion.

It seems then that $(Q/M)^2 \propto P/F^2$, an unusual relation. Tracing the term P/F^2, we see that its dimensions are $[\dot{M}]^{-1}$, where $\dot{M}$ represents mass flow rate (commonly "m-dot" in astrophysics), and it holds dimensionally that

$$\frac{P}{F^2} = \frac{E}{TF^2} = \frac{L}{TF} = \frac{v}{F} = \left(6\pi\varepsilon_0 c^3\right)^{-1}\left(\frac{Q}{M}\right)^2; \tag{A7}$$

so, $(Q/M)^2 \propto v/F$ as well, where v is the instantaneous speed of the accelerating particle. The same result can be obtained formally using $P = dE_{\rm kin}/dt = Mva = Fv$.

For an accelerating electron, Equation (A7) predicts that $(v/F)_{\rm e} = 6.88\times10^6\ \mathrm{s\,kg^{-1}}$ [9]. On the other hand, combining Equations (A5) and (A7) results in a scale

$$\left(\frac{v}{F}\right)_\star = G_\star\left(6\pi\varepsilon_0 c^3\right)^{-1} = 1.65\times10^{-36}\ \ \mathrm{s\,kg^{-1}}\,, \tag{A8}$$

some 42 orders of magnitude smaller. This scale is not atomic or even subatomic; even a top-quark (if it were free) would scale to a $(v/F)_{\rm top}$ value that lies 31 orders of magnitude higher.

Naturally then, the scaling in Equation (A8) falls near the Planck/RPS scale. For an accelerating Planck mass $M_{\rm P}$ carrying charge $Q_{\rm P}$, we find that the corresponding $(v/F)_{\rm P}$ scale is

$$\left(\frac{v}{F}\right)_{\rm P} = \left(\frac{v}{F}\right)_\star. \tag{A9}$$

This is smaller by a factor of 2/3 compared to the conventional Planck/RPS unit of $(\dot{M}_{\rm P})^{-1} = G/c^3$ (Note 4), the same numerical factor entering Larmor's power formula in Gaussian/cgs units [9].

Part 2.—Next, we replace the units in Equation (A4) with the corresponding RPS quantities, viz. $\mathrm{C^2\,kg^{-2}} \to (Q_{\rm P}/M_{\rm P})^2$, and we find that

$$G_\star = \left(\frac{Q_{\rm P}}{M_{\rm P}}\right)^2, \tag{A10}$$

as anticipated. In RPS, $Q_{\rm P}^2 = G_\star M_{\rm P}^2$, so $G_\star$ is a bridge between electrostatics and gravity [1].

Appendix A.3. Gravomagnetic Constant G_B

We consider the gravomagnetic constant (Table 1)

$$G_B = G\left(\frac{\mu_0}{4\pi}\right) = 6.674\,015\,081 \times 10^{-18}\ \ \mathrm{m^4\,s^{-4}\,A^{-2}}\,. \tag{A11}$$

Part 1.—This constant carries the same information as $G_\star$ (Equation (A4)), but it is scaled by a different vacuum constant [1], viz.

$$G_B = \left(\frac{Z_0}{4\pi}\right)^2 G_\star\,, \tag{A12}$$

which indicates that G_B is a bridge between EM transport and gravity.

The SI units in Equation (A11) imply the dimensional relation

$$v^4 = G_B\, I^2\,, \tag{A13}$$

but $v^2/I = \Phi_B/M$, where Φ_B represents magnetic flux. Then, in analogy to Equation (A5), we see that

$$G_B = \left(\frac{\Phi_B}{M}\right)^2, \tag{A14}$$

where the presence of Φ_B does indeed indicate the relevance of this scale to magnetic transport phenomena.

The "specific magnetic flux" (Φ_B/M) plays a key role in the gravitational collapse of dense magnetized GMCs in the interstellar medium and star-forming regions [63,64,70,71]. Just like gas pressure, the magnetic pressure can support a cloud against gravitational collapse, so the ratio (Φ_B/M) must decrease—commonly by the process of ambipolar diffusion—below a typical critical SI value of

$$\frac{3\pi}{0.53}\sqrt{\frac{G}{5}} = \frac{3\pi}{0.53}\sqrt{\frac{4\pi\,G_B}{5\,\mu_0}} = \sqrt{6.3\times 10^8\,G_B}\,,$$

for collapse to proceed [63,70]; thus, for comparison purposes, we adopt the squared critical value

$$\left(\frac{\Phi_B}{M}\right)^2_{\mathrm{crit}} = 6.3\times 10^8\,G_B\,, \tag{A15}$$

which is much larger than $(\Phi_B/M)^2$ in Equation (A14). Thus, we expect that the (Φ_B/M) scaling given by Equation (A14) falls in the Planck scale (see below).

Part 2.—Next, we replace the units in Equation (A11) with the corresponding RPS quantities, viz. $\mathrm{m^4\,s^{-4}\,A^{-2}} \to (c^2/\mathcal{I}_\mathrm{P})^2 \to (\Phi_\mathrm{P}/M_\mathrm{P})^2$, and we find that

$$G_B = \left(\frac{\Phi_\mathrm{P}}{M_\mathrm{P}}\right)^2, \tag{A16}$$

which was anticipated, in analogy to Equation (A10) in electrostatics.

Appendix A.4. von Klitzing Constant $\mathcal{R}_\mathrm{K}$

We consider the von Klitzing constant (Table 1)

$$\mathcal{R}_\mathrm{K} = \frac{h}{e^2} = 2.581\,280\,746\times 10^4\ \ \Omega\,. \tag{A17}$$

Part 1.—The unit of Ohm implies right away that $\mathcal{R}_K$ is an electric resistance, which turns out to be the ground state of the macroscopic quantization of the electric resistance in the quantum Hall effect [65–67].

Part 2.—Next, we replace the unit in Equation (A17) with the corresponding RPS resistance, viz. $\Omega \to \mathcal{R}_P$, and we find that

$$\mathcal{R}_K = \frac{1}{\alpha}\,\mathcal{R}_P\,, \tag{A18}$$

which, written in this form, is a new result produced by RPS units. It shows that the von Klitzing constant is but a scaled version of the RPS electric resistance, and the scaling factor is simply $(1/\alpha)$, the inverse of the FSC, as this is defined in the RPS (in terms of Planck's constant h). This relation, written in the alternative form

$$\alpha = \frac{Z_0}{4\pi}\left(\frac{e^2}{h}\right), \tag{A19}$$

allows for an independent high-accuracy determination of the FSC [65].

This is an important technical conclusion. In the conventional Planck/Dirac system of $\hbar$-defined units, a 2π term appears in the right-hand side of Equation (A18), which is obviously incorrect, since the relation between these two electric resistances needs no tagging from a 2D geometric factor—2D geometry is simply irrelevant in this scalar electronic setting.[19] On the other hand, the 3D factor of 4π is absolutely necessary in Equation (A19), since the vacuum always attaches a 4π to its resisting properties ε_0, μ_0, and Z_0 to make them applicable to all directions within the 3D space.

Notes

1 It is important to note that the RPS is complete, as all four coupling constants can be derived from its seven fundamental units (see Table 1).

2 On the other hand, the entire subset of coupling constants $\{\,\alpha,\ \alpha_g,\ \alpha_s\,\}$ is needed to complete the SI system of units, because only the weak coupling constant $\alpha_w = \sqrt{\alpha}$ is thus also introduced by default.

3 Thus, we see that μ_0 is indeed a derived unit in the RPS (as well as in Dirac's $\hbar$-based Planck system of units).

4 The constant $\mu_P = c^2/G$ is an especially simple unit of linear mass density in the RPS. In fact, the simplest RPS units are found in the geometric sequence c^2/G (linear mass density), c^3/G (mass flow rate), c^4/G (force), c^5/G (power). On the other hand, black holes are characterized by the Schwarzschild linear mass density $\mu_P/2$.

5 Strange as it may seem, this result is recent. Constant C_2 does indeed combine (kinematic) speed with (geometric) capacity or, equivalently, voltage and mass, as is shown in the main text. In our times, this is not unheard of (see Equation (A4) in Ref. [1]).

6 Although $B_3 \propto \Phi_0$, Equation (28) does not show a geometric factor of 2π on the left-hand side, as would be expected from the appearance of 2D geometry in Equation (24). Our interpretation is that, before simplifications, Equation (28) takes the form

$$\left(\frac{B_3}{2\pi}\right)M^2 \ \propto\ \left(\frac{\mu_B}{2\pi}\right)\Theta\,,$$

so the Φ_0 terms embedded in B_3 and μ_B are tagged by 2π, thus both represent formally 2D quantities. The reason is that the magnetic moment—much like the spin angular momentum J_z in Equation (24)—is specifically defined over a 2D surface, as can be seen, for instance, in a closed current loop enclosing an area **S** (see, e.g., Ref. [15]). Then, the 2π factors cancel out, leading to Equation (28).

7 We note that simpler alternative interpretations do not work; the SI units in Equation (30) point to a dimensional relation of the form $C_3 = p^2V$, where p represents momentum and V represents volume. But this momentum cannot be due to thermal motions, since C_3 is determined by EM/atomic constants; neither can p be associated with the Debye momentum and electron shielding in a plasma [19], because the calculations lead to physically unacceptable scales.

8 Because of the presence of the well-known constant (k_B/e) [2] in Equation (41), and since $k_B/e = R/F$, the new composite constant A_4 takes the equivalent form $A_4 = (c^2/G)(R/F)^2$, where R is the molar gas constant and F is the Faraday constant (Table 1).

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[9] As for A_4 in Note 8, the constant C_4 also contains the ratio e/k_B that can be replaced by F/R (with F and R listed in Table 1).

[10] We note that C_4 can be naturally recast to $C_4 = (4\pi/\mu_0)L_S/k_B$ using the Stoney length $L_S = \frac{\mu_0}{4\pi} e\sqrt{G_\star}$ (Table 1). This example shows how composite constants can be written in terms of Stoney units, if so desired. Using L_S, e, and $G_\star$, the other units of the Stoney system are $M_S = e/\sqrt{G_\star}$, $T_S = L_S/c$, and $\Theta_S = M_S c^2/k_B$.

[11] The dimensional relation $C/M = (Q/p)^2$, where p represents momentum, is physically relevant: for $p = Mv$ and $C = 4\pi\varepsilon_0 R$ of a spherical conductor of radius R, this relation implies the particular length scale $R = KQ^2/(Mv^2)$. For $Q = e$, $M = m_e$, and $v = c$, then R describes the classical electron radius [6,7,9].

[12] The residual factor $\sqrt{2\pi} \simeq 2.5$ in Equation (61) is traced to the standard string-theory normalization of the Regge slope α', for which the Polyakov action carries the coefficient $1/(4\pi\alpha')$ and the tension of the fundamental string then is $T_0 = 1/(2\pi\alpha')$ [26]. Thus, Equation (61) effectively introduces the standard Rydberg energy E_R of atomic physics to string theory. On the other hand, the value $\Delta E \simeq 2.5E_R$ is well-known in atomic physics; it is the first-order perturbation energy representing the Coulomb repulsion between the two electrons in a helium atom [28].

[13] The inversion of the geometric factor $C = 2\pi^5/15$ between Equations (85) and (86) arises from the transition between the volumetric constant I_4 and the linear scale E. While $I_4 \propto \sigma \propto C$ is a 3D constant, the energy E represents a 1D scale of the 3D ensemble. Obtaining this linear scale requires dividing the occupation number $\propto N_A$ by the volume $\propto C$ to obtain the density of the phase space before taking the cubic root, resulting in the $(N_A/C)^{1/3}$ dependence in Equation (85).

[14] The appearance of the reduced Avogadro number $\aleph_A$ in Equation (96) is not surprising. Entropy (Equation (95)) is an ensemble property, and the constant B_5 characterizes an ensemble of electrons subject to a fluctuating magnetic field. In this framework, $\aleph_A$ acts as the scaling factor for the collective degrees of freedom of the vacuum carrying this ensemble and the field.

[15] It is important to note that $\aleph_A$ appears in Planck/RPS units when the power-law constants describe ensembles of particles. In contrast, Avogadro's number N_A appears only when it is included in the definition of a pure power-law constant (via $R = N_A k_B$ or $F = N_A e$; Table 1). This distinction shows that $\aleph_A$ is indeed nature's chosen number of particles [1], in marked contrast to the man-made (subjective) definition of N_A. It is also important to realize that $\aleph_A = 1/\sqrt{\alpha_g}$ (Table 1); thus, nature's choice is not arbitrary, it is based exclusively on the gravitational coupling constant α_g!

[16] Composite power-of-ten constants that include the proton mass m_p (rather than m_e) do exist, but they cannot be interpreted on dimensional grounds. As an example, $C \equiv hN_A m_p/G = 10^{-26}$ kg^3 s m^{-1} to 5 SDs, but the corresponding dimensional relation shows that $v \propto M^3$, a scaling not encountered in physics. On the other hand, the transformation of C to Planck/RPS units, viz. $C = f_A(m_p/m_e)(M_P^3/c)$, is messy but understandable. It shows that (M_P^3/c) is modulated by the ratio (m_p/m_e) and by the new RPS unit $f_A = N_A/\aleph_A$ (Table 1).

[17] Another new natural constant, the reduced Avogadro number $\aleph_A$ that appeared in the same previous work [1] is not a precise power of ten (Table 1). This constant allowed for the determination of the Planck/RPS mass in classical mechanics, viz. $M_P = \aleph_A m_e$ (Table 1).

[18] The Tully–Fisher/Faber–Jackson relations are commonly described in the context of MOND and varying-G gravity by the relation $v^4 = GMa_0$ [10,11,68,69].

[19] The same type of 2D geometric inconsistency (by a factor of 2π) has also been detected in the first-order correction to the Landé g_s-factor of the anomalous magnetic moment of the electron (see Section 5.4 and also Ref. [1] for more details).

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