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Conditional Covariances for the Signal Lag Measurements in Fluoroscopic Imaging

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Abstract: In fluoroscopic imaging, we can acquire X-ray image sequences using a flat-panel dynamic detector. However, lag signals from previous frames are added to the subsequently acquired images and produce lag artifacts. The lag signals also inflate the measured noise power spectrum (NPS) of a detector. In order to correct the measured NPS, the lag correction factor (LCF) is generally used. However, the nonuniform temporal gain (NTG), which is from inconsistent X-ray sources and readout circuits, can significantly distort the LCF measurements. In this paper, we propose a simple scheme to alleviate the NTG problem in order to accurately and efficiently measure the detector LCF. We first theoretically analyze the effects of NTG, especially on the correlation-based LCF measurement methods, where calculating the correlation coefficients are required. In order to remove the biases due to NTG, a notion of conditional covariance is considered for unbiased estimates of the correlation coefficients. Experiments using practical X-ray images acquired from a dynamic detector were conducted. The proposed approach could yield accurate LCF values similarly to the current approaches of the direct and U-L corrections with a low computational complexity. By calculating the correlation coefficients based on conditional covariance, we could obtain accurate LCF values even under the NTG environment. This approach does not require any preprocessing scheme of the direct or U-L correction and can provide further accurate LCF values than the method of IEC62220-1-3 does.

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1. Introduction

In fluoroscopic imaging, flat-panel (FP) dynamic detectors can acquire X-ray image sequences with frame rates higher than 300 frames per second (fps) [1]. However, the sequentially acquired images have lag signals from previous frames [1,2]. For indirect detectors, trapping charges in the amorphous structure of the thin-film-transistor (TFT) panel and incomplete reads are the main causes of lag signals [3–6]. The lag artifacts appear in the form of temporal blurring and ghosting in fluoroscopic imaging [7–13].

In measuring the noise power spectrum (NPS) for a dynamic detector [14–16], the lag signal lowers the NPS curve. Conventional approaches for correcting the measured NPS are based on using the lag correction factor (LCF) [13,17–21]. Busse et al. [19,20] used a temporal power spectral density (PSD) method to measure LCF using steady-state images based on a moving average model of order L (MA(L)) for the lag signals. This PSD method is adopted in the IEC62220-1-3 standard [22]. Recently, Kim and Lee [21] proposed an LCF measurement method using a series of the Pearson correlation coefficients based on the MA(L) model. Based on an autoregressive model of order 1 (AR(1)), Matsunaga et al. [17] considered LCF measurements. Granfors and Aufrichtig [18], as well as Kim and Lee [13], considered the means of transient decaying images after the X-ray turns off.

Because of the nonuniform temporal gain (NTG) from inconsistent X-ray sources and readout circuits, accurately measuring PSD curves as well as correlations for LCF is difficult. In particular, the PSD method [19] is very sensitive to NTG because the temporal

spectrum at low frequencies has unusually high values. Hence, applying a gain correction is important in obtaining an accurate LCF [19]. Compared to the PSD case, the correlation method [21] is less sensitive to NTG even though a gain correction is required. On the other hand, the mean-based methods are generally insensitive to NTG [13,18].

In this paper, we analyze effects of NTG on the correlation-based methods [21]. Formulating a mathematical NTG model, the effects on measuring the correlations are observed. We also theoretically observe that a scheme called the upper-lower (U-L) correction can efficiently alleviate the NTG problem. We then propose an LCF measurement method based on the correlation method considering estimates from conditional covariances. This algorithm can efficiently measure LCF without applying any gain correction schemes. We extensively conducted experiments using real X-ray images acquired from a dynamic detector.

This paper is organized in the following way. In Section 2, the definitions on LCF for the MA(L) and AR(1) models are introduced. An NTG model is formulated and theoretical analyses are conducted in Section 3, and an LCF measurement algorithm is proposed in Section 4. Experimental results are shown in Section 5, and the paper is concluded in the last section.

2. Lag Correction Factors

In this section, we formulate two lag models based on MA(L) and AR(1) for introducing the LCF definitions, respectively.

For a pixel position $\mathbf{u}$, let $g_n[\mathbf{u}]$ denote a signal that is independent and identically distributed. We first consider a linear lag model of MA(L). The MA(L) model [18] with a signal f_n is defined as

$$f_n[\mathbf{u}] = \sum_{l=0}^L g_{n-l}[\mathbf{u}]h_l, \text{ for } \mathbf{u} \in \{0, \dots, U-1\}^2, \quad (1)$$

where h_ℓ is a causal system, such that $\sum_{\ell=0}^L h_\ell = 1$ with nonnegative h_ℓ . Here, f_n has U^2 pixels and represents the n th X-ray image acquired under specified irradiation conditions with uniform intensity. Let I_f and I_g denote the periodogram means of f_n and g_n , respectively [23]. The periodogram means then satisfy $I_f = r_{MA}I_g$, where r_{MA} is the LCF under the MA(L) model and is defined as $r_{MA} := \sum_{\ell=0}^L h_\ell^2$. By using this LCF, we can asymptotically correct the measured NPS to obtain the true NPS from $NPS_g \approx NPS_f / r_{MA}$ [24], where NPS_g and NPS_f are the NPS curves of g and f , respectively. Let ρ_ℓ denote the autocorrelation of f_n with the frame lag ℓ defined as

$$\rho_\ell := \frac{\text{Cov}\{f_0, f_\ell\}}{\text{Var}\{f_0\}}, \text{ for } \ell = 0, \dots, L. \quad (2)$$

Kim and Lee [21] showed that the LCF of r_{MA} can be a function of ρ_ℓ as

$$r_{MA} = \frac{1}{2 \sum_{\ell=0}^L \rho_\ell - 1}. \quad (3)$$

Obtaining the autocovariance of f_n is enough to calculate the autocorrelation ρ_ℓ of Equation (2) and thus the LCF of Equation (3).

We now consider a linear lag model based on AR(1). The acquired image f_n can be described from the AR(1) model point of view [17] as

$$f_n[\mathbf{u}] = af_{n-1}[\mathbf{u}] + (1-a)g_n[\mathbf{u}], \text{ for } \mathbf{u} \in \{0, \dots, U-1\}^2. \quad (4)$$

From Equation (4), we can obtain a relationship of $I_f[\mathbf{v}] = r_{AR}I_g[\mathbf{v}]$, where r_{AR} is the LCF under the AR(1) model and is defined as $r_{AR} := (1-a)/(1+a)$. We can obtain a of

AR(1) from the autocorrelation as $a = \rho_1$, and the resultant r_{AR} can also be a function of ρ_1 as

$$r_{AR} = \frac{1 - \rho_1}{1 + \rho_1}. \quad (5)$$

Note that we can also obtain a from image means for transient decaying frames after the X-ray turns off. By using the LCF of Equation (5), we can also correct the measured NPS to obtain the true NPS. For relatively small values of ρ_1 , $(1 - \rho_1)/(1 + \rho_1) \approx 1/(2\rho_1 + 1)$ holds. In other words, we can obtain an approximation of $r_{MA} \approx r_{AR}$ for $L = 1$ from Equations (3) and (5) if ρ_1 is relatively small and thus obtain an approximate LCF value using a coefficient ρ_1 , as discussed by Granfors and Aufrichtig [18].

3. Nonuniform Temporal Gain Model

For appropriately measured autocorrelations, measuring LCF from Equations (3) or (5) can be insensitive to various noises compared to the PSD method [19]. In this section, theoretical analyses are conducted on this sensitivity property to NTG.

In calculating ρ_ℓ , various noises, such as NTG, can make accurate LCF measurements difficult. In order to mathematically describe NTG, we introduce a weakly stationary random sequence γ_n with mean $E\{\gamma_n\} = 1$ and variance $\text{Var}\{\gamma_n\}$, and we modify the image model of Equations (1) or (4) considering NTG as [21]

$$q_n[\mathbf{u}] := \gamma_n f_n[\mathbf{u}], \text{ for } \mathbf{u} \in \{0, \dots, U - 1\}^2, \quad (6)$$

where q_n is a signal with NTG, and γ_n is independent of the pixel values f_n . In the NTG model of Equation (6), we can practically assume that $1 \gg \text{Var}\{\gamma_n\}$. Hence, we can obtain an approximation of $E\{\gamma_n^2\} = 1 + \text{Var}\{\gamma_n\} \approx 1$. The signal-to-noise ratio, $E\{\gamma_n\}/\sqrt{\text{Var}\{\gamma_n\}}$, is very high in practical fluoroscopic imaging. Hence, this NTG can be ignored in the mean-based LCF measurement methods [13,17,18]. However, the PSD or correlation-based methods can be very sensitive to NTG even though its strength is quite weak [21].

In estimating the autocovariance $\text{Cov}\{f_0, f_\ell\}$ for ρ_ℓ in Equation (2), we can instead measure $\text{Cov}\{q_0, q_\ell\}$ using the acquired image q_n and can have a bias of $\mu^2 \text{Cov}\{\gamma_0, \gamma_\ell\}$ as derived in Appendix A. Hence, we cannot accurately calculate ρ_n for the LCF values of Equations (3) and (5) if we use the autocovariance of q_n due to the bias from NTG.

To reduce the biases from NTG, we can estimate the gain γ_n from a conditional mean $\gamma_n = E\{q_n|\gamma_n\}/\mu$ and can then directly correct NTG from dividing q_n by γ_n [19]. Instead of this direct correction, we can use a notion from the image differences, as described by Kim [25] and Kim and Lee [26], for a weak gain. For an image q_n with U^2 pixels, the image difference Δq_n between the upper and lower pixels can be calculated as

$$\Delta q_n[\mathbf{u}] := \frac{1}{\sqrt{2}}[q_n[\mathbf{u}] - q_n[\bar{\mathbf{u}}]], \quad (7)$$

for the upper image positions of $\mathbf{u} \in \{0, \dots, U - 1\} \times \{0, \dots, U/2 - 1\}$, where $\bar{\mathbf{u}} := \mathbf{u} + (0, U/2)$ implies the lower image part. In Equation (7), we assume that the pixels that are separated by $U/2$ pixels are mutually uncorrelated or are α -mixing [27]. The autocovariance of Δq_n approximately satisfies $\text{Cov}\{\Delta q_0, \Delta q_\ell\} \approx \text{Cov}\{f_0, f_\ell\}$, as shown in Appendix B. Hence, by conducting a preprocessing on the difference Δq_n , we can accurately estimate LCF regardless of the influence of NTG. Here, we call the preprocessing the U-L correction. The measured autocorrelation can now be given as $\rho_\ell \approx \text{Cov}\{\Delta q_0, \Delta q_\ell\}/\text{Var}\{\Delta q_0\}$ using the difference Δq_n of Equation (7).

4. Estimates Based on Conditional Covariance

In this section, we propose a method based on conditional covariance for an unbiased estimate of ρ_ℓ in Equation (2) to alleviate the NTG problem besides the direct and U-L corrections. The conventional schemes of the direct and U-L corrections are applied to the

input signal as preprocessing approaches. Estimates of $\text{Cov}\{f_0, f_\ell\}$ are next obtained to calculate ρ_ℓ . However, the proposed method can obtain unbiased estimates of $\text{Cov}\{f_0, f_\ell\}$ even for the NTG environment based on conditional covariance.

A conditional covariance $E\{\text{Cov}\{q_0, q_\ell | \gamma_0, \gamma_\ell\}\}$ can be expanded as

$$E\{\text{Cov}\{q_0, q_\ell | \gamma_0, \gamma_\ell\}\} = (E^2\{\gamma_0\} + \text{Cov}\{\gamma_0, \gamma_\ell\})\text{Cov}\{f_0, f_\ell\} \quad (8)$$

for $\ell = 0, \dots, L$. From the assumption on γ_n , $E^2\{\gamma_0\} + \text{Cov}\{\gamma_0, \gamma_\ell\} \approx E^2\{\gamma_0\} = 1$ holds. Hence, we can obtain an approximation for the conditional covariance as $E\{\text{Cov}\{q_0, q_\ell | \gamma_0, \gamma_\ell\}\} \approx \text{Cov}\{f_0, f_\ell\}$ and use the conditional covariance as an unbiased estimate of $\text{Cov}\{f_0, f_\ell\}$ for calculating ρ_ℓ .

We now introduce the proposed algorithm as follows. For N frames of q_n , where $N > L$, we can asymptotically estimate the covariances on q_n from empirical sum of $q_n[\mathbf{u}]$ as N and U increase. Using the N image samples of $q_n[\mathbf{u}]$, an empirical covariance,

$$\frac{1}{U^2(N - \ell) - 1} \sum_{n=0}^{N-\ell-1} \sum_{\mathbf{u}} [q_n[\mathbf{u}] - \mu(\ell)][q_{n+\ell}[\mathbf{u}] - \mu(\ell)], \quad (9)$$

where μ is an empirical mean defined as $\mu(\ell) := U^{-2}(N - \ell)^{-1} \sum_{n=0}^{N-\ell-1} \sum_{\mathbf{u}} q_n[\mathbf{u}]$, can be used as an estimate of $\text{Cov}\{q_0, q_\ell\}$. As mentioned in Appendix A, the estimate from Equation (9) has a bias of $\mu^2 \text{Cov}\{\gamma_0, \gamma_\ell\}$ in estimating $\text{Cov}\{f_0, f_\ell\}$.

On the other hand, the conditional covariance of Equation (8) can be empirically estimated from an empirical estimate:

$$\frac{1}{N - \ell} \sum_{n=0}^{N-\ell-1} \left[\frac{1}{U^2 - 1} \sum_{\mathbf{u}} [q_n[\mathbf{u}] - \nu(n)][q_{n+\ell}[\mathbf{u}] - \nu(n)] \right], \quad (10)$$

where ν is an empirical mean defined as $\nu(n) := U^{-2} \sum_{\mathbf{u}} q_n[\mathbf{u}]$. Based on the conditional covariance of (8) and its empirical estimate Equation (10), we can calculate the autocorrelation ρ_ℓ of Equation (2) and obtain the LCF values from Equations (3) or (5). The proposed LCF measurement method within a framework of the correlation-based methods [17,21] is now summarized as follows.

Conditional Covariance Algorithm:

- (1) For a given X-ray image sequence $q_n[\mathbf{u}]$, calculate the empirical estimate with Equation (10) for the conditional covariance $E\{\text{Cov}\{q_0, q_\ell | \gamma_0, \gamma_\ell\}\}$.
- (2) Using the empirical estimates, calculate ρ_ℓ to obtain the LCF of r_{MA} from (3) or r_{AR} from Equation (5).

From this Conditional Covariance Algorithm with the correlation-based methods, we can obtain accurate LCF values without applying the direct or U-L correction preprocessing.

5. Experimental Results

In this section, we introduce experimental results for analyzing the NTG effect on estimating LCF. We used a CsI(Tl)-scintillator FP dynamic detector (DRTECH Co., Ltd., Sungnam-si, Korea), where amorphous In-Ga-Zn-O thin-film transistors control the photodiode pixels. At frame rates of 10 fps and 30 fps, X-ray image sequences for various incident exposures were acquired under the RQA 5 condition of IEC62220-1-3 [22] and a continuous fluoroscopy mode for a continuous X-ray source.

In Figure 1, we illustrate a comparison of the LCF measurements for the different NTG corrections to show the correction performance of the proposed algorithm. Here, the calculated autocorrelation ρ_ℓ are applied to r_{MA} of Equation (3) to obtain LCF. We can observe that the proposed algorithm yields LCF values that are very close to that of the direct correction as “Conditional mean” and “Direct”. However, without the NTG correction, we can observe from “No correction” that the resultant LCF values have sig-

nificant deviations from those of the corrected cases due to the bias term described in Appendix A. For both cases of frame rates in Figure 1a,b, we can observe similar results on the proposed algorithm.

Figure 1. Comparisons of the LCF measurements for the different NTG corrections. The LCF values are calculated from r_{MA} of Equation (3) derived in the correlation method [21] for $MA(L)$. A sequence has $N = 128$ images and $L = N - 1$. (a) Frame rate of 10fps; (b) Frame rate of 30fps.

6. Discussions

In practical calculations of the conditional covariance $E\{\text{Cov}\{q_0, q_\ell | \gamma_0, \gamma_\ell\}\}$, further careful steps are required. Let the white image imply an image acquired at uniform exposure to the detector and the dark image imply an image acquired without exposures. From Step 1) of the algorithm, we first calculate the conditional covariances for both white and dark sequences, respectively, as “white” and “dark” in Figure 2a. Here, considering the dark conditional covariance is important because it can have negative values. We next calculate their difference as “white-dark” and then obtain a variance offset of the difference for relatively large frame lags of ℓ . The final conditional covariance is obtained by subtracting the variance offset from the difference as “white-dark-offset” in Figure 2a. Note that the variance offset in the conditional covariance occurs because of a fixed pattern noise, which is independent of the image frames [21]. As shown in Figure 3a, extracting the variance offset can guarantee a linearity of the variance curve as “white-dark-off”. The standard deviation of this noise, i.e., the root square of the variance offset, is proportional to the incident dose, as shown in Figure 3b. In Figure 2b, estimated conditional covariances are compared. Without any NTG corrections, the conditional covariance curve shows a bias due to NTG. The proposed algorithm could achieve a curve that is very close to those of the conventional direct and U-L corrections.

Figure 2. Estimated conditional covariances with respect to the frame lag ℓ . The frame rate was 10 fps, and the dose was 1809.5 nGy. (a) Conditional covariances of the white and dark images in the proposed conditional covariance algorithm; (b) Comparison of the NTG correction schemes and the proposed conditional covariance algorithm.

Figure 3. Observation of the photon noise and the exposure leak for a dynamic detector in Figure 1. Both the variance electric noise and variance offset are extracted from the variances of the white images to observe a linearity of the photon noise with respect to the incident dose. (a) Variance curves; (b) Standard deviation curves of the variance offset.

For an image sequence acquired at an incident dose of 1809.5 nGy, we estimated the NTG signal γ_n from the conditional mean $\gamma_n = E\{q_n|\gamma_n\}/\mu$. As shown in Figure 4a, the NTG curve γ_n is noisy and is even slightly increasing. Hence, the temporal PSD of γ_n has high values, especially at low frequencies, as shown in Figure 4b, and these values can produce high errors in obtaining LCF from the PSD method [19,21]. The variance of γ_n satisfies $\text{Var}\{\gamma_n\} \approx 1.509 \times 10^{-6} \ll 1$, and thus, the assumption $E\{\gamma_n^2\} = 1 + \text{Var}\{\gamma_n\} \approx 1$ is practically reasonable. The bias in estimating $\text{Var}\{f_n\}$ from $\text{Var}\{q_n\}$ is $\mu^2 \text{Var}\{\gamma_n\} \approx 344.6$ for a signal mean of $\mu = 15,110$. From the proposed algorithm, the LCF is 0.9416, which is similar to those of the direct and U-L corrections. If we do not apply any NTG correction schemes, then the LCF is given as 0.9025, which is significantly lower than the proposed case. As a comparison, the line-estimate method [13], which is based on means, yields 0.9398.

Figure 4. Example of NTG γ_n (the frame rate is 10 fps, and the dose is 1809 nGy). (a) NTG curve of γ_n with respect to the frame; (b) Temporal PSD curve of NTG.

In Figure 5, a comparison on the NTG correction for the PSD method, which is adopted in IEC62220-1-3, is illustrated. As previously mentioned in the temporal PSD of Figure 4b, we notice that the PSD method is very sensitive to NTG as “PSD (no correction)”. Hence, the direct or U-L correction is required for the PSD method as “PSD (direct)”. Even though the NTG correction is conducted, the PSD method showed deviations from the proposed case. The LCF values from the proposed algorithm are very close to those of the line estimate method [13]. Note that the line estimate method is insensitive to NTG but uses transient decaying image frames after the X-ray tube tunes off from a continuous X-ray source.

Figure 5. Comparison of the LCF measurements. “PSD (no correction)” is the method of IEC62220-1-3 without the NTG correction, and “PSD (direct)” is the PSD method with the direct correction [22]. “Correlation (conditional mean)” is the proposed algorithm, and “Line estimate” is the line estimate method [13]. (a) Frame rate of 10 fps; (b) Frame rate of 30 fps.

7. Conclusions

In this paper, the NTG effect on the correlation-based LCF measurement methods was observed both theoretically and experimentally. A theoretical observation on the U-L correction to alleviate the NTG problem was also performed. We next proposed an unbiased estimate of autocorrelations based on the conditional covariance for the correlation-based LCF measurement method under the NTG environment. The proposed algorithm can be implemented without any preprocessing steps of the conventional NTG corrections and can obtain LCF more efficiently and accurately by replacing the method of IEC62220-1-3.

Author Contributions: E.L. formulated the nonuniform temporal gain model and conducted theoretical analyses and experiments. D.S.K. formulated the issues in deriving the lag correction factors and the proposed algorithm and organized and refined the manuscript. All authors have read and agreed to the published version of the manuscript.

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Abbreviations

The following abbreviations are used in this manuscript:

AR	Autoregressive
FP	Flat panel
LCF	Lag correction factor
MA	Moving average
NPS	Noise power Spectrum
NTG	Nonuniform temporal gain
PSD	Power spectral density
U-L	Upper-lower

Appendix A. Autocovariance of q_n

From the law of total covariance, the autocovariance of q_n can be expanded as

$$\text{Cov}\{q_0, q_\ell\} = E\{\text{Cov}\{q_1, q_\ell | \gamma_0, \gamma_\ell\}\} + \text{Cov}\{E\{q_0 | \gamma_0, \gamma_\ell\}, E\{q_\ell | \gamma_0, \gamma_\ell\}\}, \quad (\text{A1})$$

for $\ell = 0, \dots, L$, and can be rewritten as

$$\text{Cov}\{q_0, q_\ell\} = (E^2\{\gamma_0\} + \text{Cov}\{\gamma_0, \gamma_\ell\})\text{Cov}\{f_0, f_\ell\} + E^2\{f_0\}\text{Cov}\{\gamma_0, \gamma_\ell\}. \quad (\text{A2})$$

From the assumption $E^2\{\gamma_0\} \gg \text{Var}\{\gamma_0\}$, where $E\{\gamma_0\} = 1$, we can obtain an approximation:

$$\text{Cov}\{q_0, q_\ell\} \approx \text{Cov}\{f_0, f_\ell\} + \mu^2 \text{Cov}\{\gamma_0, \gamma_\ell\}. \quad (\text{A3})$$

Note that the bias $\mu^2 \text{Cov}\{\gamma_0, \gamma_\ell\}$ can be removed for signals with a mean of zero.

Appendix B. Autocovariance of Δq_n

From $E\{\Delta q_n\} = 0$, the autocovariance of Δq_n satisfies the following relationship:

$$\begin{aligned} \text{Cov}\{\Delta q_0[\mathbf{u}], \Delta q_\ell[\mathbf{u}]\} &= \frac{1}{2} E\{\gamma_0 \gamma_\ell\} E\{(f_0[\mathbf{u}] - f_0[\bar{\mathbf{u}}])(f_\ell[\mathbf{u}] - f_\ell[\bar{\mathbf{u}}])\} \\ &= (E^2\{\gamma_0\} + \text{Cov}\{\gamma_0, \gamma_\ell\})\text{Cov}\{f_0, f_\ell\}, \end{aligned} \quad (\text{A4})$$

for $\ell = 0, \dots, L$. Hence, we can have an approximation of $\text{Cov}\{\Delta q_0, \Delta q_\ell\} \approx \text{Cov}\{f_0, f_\ell\}$. Note that there is no bias in estimating the autocovariance of f_n from Δq_n .

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