

Article

Dimensional Synthesis and Optimization of Leading and Mixed-Leading Double Four-Bar Steering Mechanisms: A Comparative Metaheuristic Approach

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Abstract

This study investigates the dimensional synthesis and optimization of multi-link steering mechanisms—namely, the leading and mixed-leading double four-bar configurations—for front-wheel-drive vehicles. To overcome the accuracy limitations of conventional steering at large angles (up to 70°), a comparative metaheuristic approach is employed, utilizing two popular metaheuristic optimizations, Improved Particle Swarm Optimization (IPSO) and Differential Evolution with golden ratio (DE-gr), to optimize the geometric parameters of these complex eight-bar steering systems. Using a track-to-wheelbase ratio of 0.5, the optimization minimizes a mean-squared structural-error objective function integrated with Grashof mobility constraints. The optimized mechanisms are validated via ADAMS kinematic simulations and further analyzed in MATLAB R2021 regarding steering accuracy, transmission angles, and mechanical advantage. The results reveal a distinct performance trade-off: mixed-leading configurations achieve superior geometric precision and mass reduction due to shorter link lengths, with IPSO yielding the highest accuracy. Conversely, leading-type mechanisms provide a more linear and stable mechanical advantage, ensuring predictable force transmission. While DE-gr exhibits faster convergence across both variants, both algorithms effectively exploit the complex parameter space of multi-link systems. Ultimately, this metaheuristic optimization-based approach offers a superior and robust framework for the dimensional synthesis of high-performance multi-link steering mechanisms, surpassing the constraints of traditional gradient-based methods. Our findings recommend the mixed-leading configuration for precision-focused applications and the leading configuration for scenarios requiring consistent mechanical performance.



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1. Introduction

Conventional Ackermann steering mechanisms are widely used in front-wheel-drive passenger cars due to their simplicity, compactness, and ease of packaging. Although they aim to satisfy Ackermann geometry by aligning the extended front-wheel axes at a common point on the rear axle [1], this Ackermann condition is met exactly only at a few discrete precision points (typically three), rather than continuously across the steering range. As steering angles increase, geometric errors grow, leading to tire slip and uneven wear,

while the limited design parameters further restrict steering accuracy, particularly for large steering angles or high-performance applications. Rack-and-pinion steering is the predominant configuration in modern mass-production small vehicles due to its manufacturing simplicity, cost-efficiency, and compact packaging. Furthermore, its architecture is highly compatible with power-assisted and electronic control systems. Despite these advantages, the system is kinematically equivalent to a simplified six-bar mechanism. The requirement to solve for multiple design parameters, alongside inherent geometric constraints, often compromises steering accuracy—particularly during sharp turns or at large steering angles. Consequently, extensive research has focused on the dimensional synthesis and improvement of steering accuracy for rack-and-pinion mechanisms [2–10]. To overcome the limitations of conventional Ackermann and rack-and-pinion systems, some multi-link steering architectures—such as the Watt-I six-bar (central-lever) mechanisms [7,11–14], double four-bar mechanisms [7], and eight-bar mechanisms [15,16]—incorporate additional links and closed loops and were adopted to provide greater geometric flexibility and design freedom. Comprehensive reviews of the six-bar steering mechanisms are provided in [14], where it was demonstrated that optimized Watt-I (central-lever) configurations—specifically the Type II-2 design—attain near-ideal Ackermann characteristics. Building upon these six-bar architectures, representative studies on eight-bar steering mechanisms are reviewed and synthesized below.

Fahey and Huston [15] introduced the Fahey eight-member mechanism (FEMM) as an alternative to the conventional Ackermann steering linkage. Using a numerical iterative synthesis approach with prescribed initial parameters, they compared the FEMM with two synthesized Ackermann linkages for a vehicle with a track-to-wheelbase ratio of 0.6 and a maximum inner-wheel steering angle of 61° . The FEMM demonstrated superior kinematic performance, achieving a maximum steering error of only 0.03° and closely matching the ideal Ackermann geometry. Its improved end-behavior characteristics allowed extended steering travel while maintaining minimal structural error. Chicurel [16] further extended the steering capability by proposing a mechanism capable of a 180° steering interval (90° maximum wheel angle). The design incorporated angular displacement amplifiers—implemented as gear or chain drives—between the output links of a conventional eight-bar steering linkage and the steering knuckles. Through a three-variable optimization procedure, the link proportions and amplification ratio were tuned to nearly satisfy the Ackermann condition (maximum error 0.61%) while maintaining a minimum transmission angle of 45.7° , thereby achieving a wide steering range with high kinematic precision. Dooner [17] integrated an eight-link mechanism with optimized non-circular gear elements to generate coordinated nonlinear steering motion. The noncircular gear profiles were optimized using a generalized reduced-gradient method, with an initial feasible solution obtained interactively, enabling accurate approximation of the desired nonlinear steering function. De-Juan et al. [7] developed a gradient-based optimization framework for planar rack-and-pinion, four-bar, six-bar, and double four-bar steering systems. Their method minimized steering error by solving nonlinear equations derived from a Jacobian-based formulation with Taylor-series approximation. The leading and trailing double four-bar variants were synthesized using six design variables, with convergence requiring appropriate initial estimates. The optimized designs achieved maximum steering errors of 0.25° and 1.49° , respectively. Topaç et al. [18] proposed an optimal kinematic design methodology for a multi-link steering system for a bus with independent front suspension (IFS). A multibody dynamics model developed in MSC Adams/Car was coupled with a design of experiments–response surface methodology (DOE-RSM) via Adams/Insight. Sensitivity analysis and central composite design were first employed to identify influential tie-rod hardpoints affecting toe-angle variation. Subsequently, the

relay lever geometry was optimized to minimize maximum steering error. The optimized mechanism reduced toe-angle deviation by up to 85.4% and steering error by up to 89.6%, satisfying a maximum steering error within $\pm 0.5^\circ$ over a steering range of $\pm 44^\circ$. Romero et al. [19] employed a continuous genetic algorithm implemented in MATLAB to optimally synthesize a multi-link (leading type double four-bar) steering mechanism for a vehicle with a 2400 mm wheel track, 4800 mm wheelbase, and a steering range of -27° to 40° . Using nine design variables, the optimized mechanism achieved a maximum steering error of approximately 0.8° . Topaç et al. [20] optimized a mixed-leading double four-bar steering system for a passenger bus with IFS. A two-stage optimization was conducted: (i) kinematic optimization using Adams/Insight to minimize steering error and toe-angle deviation while satisfying the Ackermann error limit ($\pm 0.5^\circ$) over an inner-wheel angle range of -20° to 20° , and (ii) structural optimization using ANSYS Workbench to reduce stress and deformation. The final design achieved an approximate 0.75° maximum steering error, along with reductions of 67% in von Mises stress and 55% in deformation. Ağakışi and Öztürk [21] investigated steering-kinematic optimization for a compact passenger vehicle with a McPherson front suspension using a hybrid approach combining DOE, RSM, and a neural network genetic algorithm (NN-GA). A multibody dynamics model developed in Adams/Car and correlated with kinematics-and-compliance test data was employed to minimize Ackermann error and camber variation. The NN-GA approach achieved approximately 14% and 5% greater improvement in Ackermann error and camber variation, respectively, compared with conventional RSM methods.

The dimensional synthesis of planar multi-link steering mechanisms is a complex, nonlinear, and highly coupled optimization problem. Conventional deterministic and gradient-based techniques—such as sequential quadratic programming (SQP), least-squares minimization, and Newton–Raphson—are highly sensitive to initial guesses and frequently converge to local optima due to the multimodal nature of the design space. In contrast, metaheuristic optimization algorithms are population-based and stochastic, offering robust global search capabilities without requiring gradient information or specialized initial solutions. Metaheuristic algorithms are well suited for the highly nonlinear, multi-variable constraints in mechanism synthesis due to their simplicity and convergence stability. Consequently, a wide range of swarm intelligence-based algorithms [22], including PSO, HPSO, MKH, CS, APT-FPSO, and BAS, as well as evolutionary-based algorithms [22], such as GA, GA-KK, GA-CSP, DE, GA-FL, GA-DE, MUMSA, NSGA-II, IOA^{s-at}, ICA-SA, DE-SRT, TS-MBFOA, SAP-TLBO, ICA-GA, TLBO, CMDE, CPF-DE, multi-start, HLIDE, ADELI, DEC, ImHS, GSA, ATLBO-DA, GSEF-IAA, ODSRA + CP, REA, and HCDJ, have been successfully applied to kinematic synthesis tasks [22]. In addition, several variants, including IPSO, HPSO, DE-gr, EPSDE, and L-EPSDE [14,22–24], have demonstrated strong performance in this domain. Recent studies highlight their effectiveness in the dimensional synthesis of Watt-I six-bar steering mechanisms [14], path-generating four-bar linkages [22], spring-actuated VCB mechanisms [23], and multi-axle Ackermann steering mechanisms [24]. Building on these advances, this study extends the algorithms of IPSO and DE-gr to the dimensional synthesis of two eight-bar steering architectures to achieve improved kinematic precision and material efficiency.

This study extends metaheuristic techniques to the dimensional synthesis of 8-bar steering architectures, targeting superior kinematic precision and material efficiency. To address the limitations of conventional linkages, a comparative optimization approach is employed for leading and mixed-leading double four-bar configurations. The framework utilizes a swarm-based Improved Particle Swarm Optimization (IPSO) [25] and an evolutionary-based Differential Evolution with golden ratio (DE-gr) [14,22–24,26]. The optimization objective minimizes mean-squared structural error over a 70° range, subject to Grashof

mobility and geometric feasibility constraints [27]. This ensures near-ideal Ackermann compliance, enhancing maneuverability, cornering efficiency, and tire longevity—factors critical for heavy-duty vehicles, agricultural machinery, and autonomous platforms.

The performance of the two employed algorithms is assessed based on the convergence behavior and solution accuracy. Simultaneously, the optimized mechanisms are evaluated using critical kinematic metrics, including maximum structural error, transmission angle, and mechanical advantage. To ensure statistical robustness, both algorithms are implemented in MATLAB, utilizing a population-based search over 500 iterations, and executed through 100 independent runs per case. The final optimal designs are selected from the best results of ten independent optimization trials to mitigate the stochastic nature of the metaheuristics. These optimal results are further validated through kinematic simulation in MSC-ADAMS v2015 to confirm steering performance, geometric feasibility, and real-world applicability.

The remainder of this paper is organized as follows: Section 2 establishes the theoretical foundation, detailing the Ackermann steering principle and kinematic formulations for the angular position, transmission angle, and mechanical advantage of the steering mechanisms. Section 3 provides an overview of the two metaheuristic optimization frameworks: the IPSO and DE-gr algorithms. Section 4 presents the optimal synthesis results for both the leading and mixed-leading configurations, including a detailed analysis of convergence behavior, steering accuracy, and MSC-ADAMS kinematic validation. In Section 5, a comparative performance assessment of the two steering variants is conducted. Finally, Section 6 summarizes the key findings and provides concluding remarks.

2. Planar Double Four-Bar Steering Mechanisms

2.1. Ackermann Steering Principle

Figure 1 illustrates the schematic diagram of the traditional Ackermann steering mechanism for a front-wheel-drive, two-axle, four-wheeled road vehicle. The mechanism is symmetrically designed along the vehicle's central longitudinal-vertical plane, utilizing equal-length input and output steering arms to ensure identical steering characteristics for both left and right turns. As shown, the bold white arrow indicates the direction of straight-line motion, with the neutral position represented by dashed lines. During a left turn (depicted by solid red lines), the steering angle of the front inner wheel must exceed that of the front outer wheel—and vice versa for a right turn. This geometry ensures that the extended axes of the front wheels intersect at a common point on the rear-axle centerline (Point O), known as the Instantaneous Center (IC) of turning. When the vehicle turns slowly, this alignment allows for smooth cornering around the IC without wheel slip, thereby reducing friction and excessive tire wear. This geometric condition, which governs the relative steering angles of the outer and inner wheels, is known as the Ackermann condition or Ackermann steering principle [1]. It satisfies the following geometric relationship.

$$\cot\delta_{out} - \cot\delta_{in} = \frac{W}{L} \quad (1)$$

Here, δ_{out} and δ_{in} denote the steering angles of the front outer and inner wheels, respectively. W represents the track width, and L is the wheelbase.

2.2. Position Analysis of Double Four-Bar Steering Mechanisms

Figures 2 and 3 illustrate the schematic diagrams of two assembly configurations of planar eight-bar steering mechanisms, referred to as the leading and mixed-leading double four-bar steering mechanism, respectively [7]. In the figures, the bold white arrow indicates the direction of straight-line vehicle motion, and the dashed lines represent the

corresponding initial (or neutral) kinematic phase of the mechanism. In the leading type, the left four-bar linkage ($O_A A_0 B_0 O_B$) is arranged as assembly-I (an uncrossed four-bar linkage) [27], whereas in the mixed-leading type it is arranged as assembly-II (a crossed four-bar linkage). The right four-bar linkage ($O_C C_0 D_0 O_D$) is the mirror image of the left four-bar linkage about the Y_0 axis (the vehicle's central longitudinal axis), which passes through the midpoint of length r_{10} . The geometric relationships among the linkage lengths are defined as $r_2 = r_4 = r_6 = r_8$, $r_7 = r_3$, $r_9 = r_1$, $r_{11} = r_{12} = \frac{r_4}{2}$, $r_{10} = W - 2r_0$. The X-axis of the global X–Y coordinate system is aligned with the initial axes of front wheel. At the initial condition (black dashed line), the input link 2 forms an angle θ_{20} with the X-axis, whereas the output link 8 forms an angle θ_{80} , where $\theta_{80} = \pi - \theta_{20}$. The red solid lines illustrate the phase of the mechanism during a left-turn maneuver. In this state, the front inner wheel rotates through an angle δ_{in} , corresponding to the instantaneous phase in which the input link 2 subtends an angle θ_2 with the X-axis. Simultaneously, the front outer wheel rotates through an angle δ_{out} , and the output link 8 forms an angle θ_8 with the X-axis. Note that $\theta_{10} = 0$, $\theta_{11} = \theta_4$, $\theta_6 = \theta_{12}$, $r_1 = \sqrt{r_0^2 + h^2}$, $\theta_9 = -\theta_1$, and $\theta_1 = \tan^{-1}\left(\frac{h}{r_0}\right)$. Accordingly, the motion of double four-bar steering linkage can be fully described by six independent design parameters, r_2, r_3, r_0, r_5, h , and θ_{20} , which are collectively represented by the design vector $\mathbf{X} = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [r_2, r_3, r_0, r_5, h, \theta_{20}]^T$.

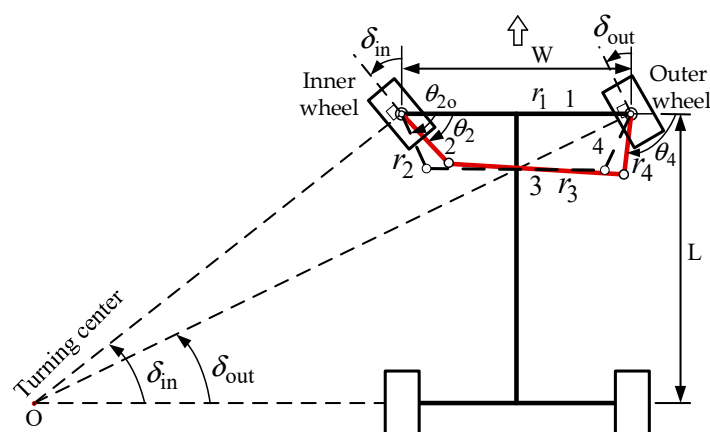


Figure 1. Left-turn motion of a vehicle driven by an Ackermann steering mechanism.

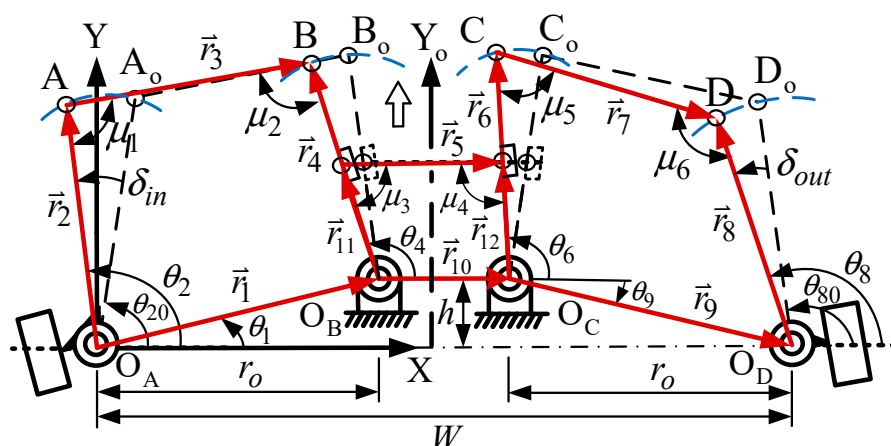


Figure 2. Schematic diagram of the leading double four-bar steering mechanism.

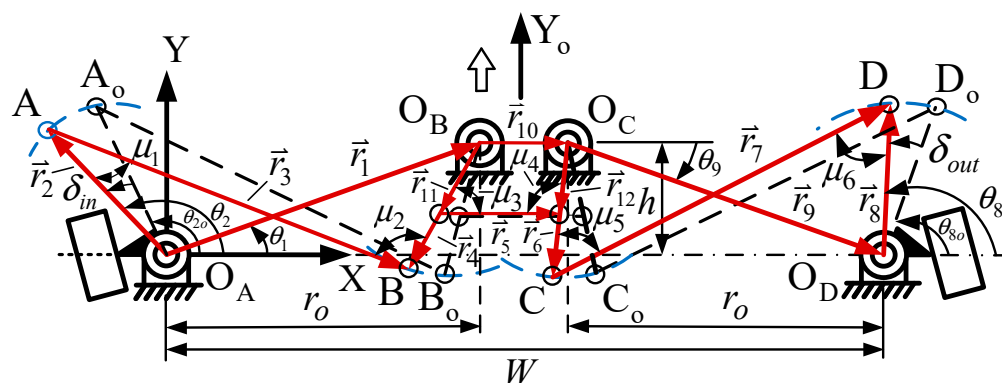


Figure 3. Schematic diagram of the mixed-leading double four-bar steering mechanism.

Three independent vector loops are established for the planar leading and mixed-leading double four-bar steering mechanisms, as illustrated in Figures 2 and 3. The corresponding vector loop equations are expressed as follows:

$$\begin{aligned}\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 &= 0 \\ \vec{r}_{11} + \vec{r}_5 - \vec{r}_{12} - \vec{r}_{10} &= 0 \\ \vec{r}_6 + \vec{r}_7 - \vec{r}_8 - \vec{r}_9 &= 0\end{aligned}\quad (2)$$

The angular position of link 4, θ_4 , in the first four-bar linkage is expressed as [14]

$$\theta_4 = 2 \tan^{-1} \left(\frac{B_1 \pm \sqrt{A_1^2 + B_1^2 - C_1^2}}{(A_1 + C_1)} \right) \quad (3)$$

In which

$$\begin{aligned}A_1 &= 2r_4(r_1 \cos \theta_1 - r_2 \cos \theta_2) \\ B_1 &= 2r_4(r_1 \sin \theta_1 - r_2 \sin \theta_2) \\ C_1 &= r_3^2 - r_1^2 - r_2^2 - r_4^2 + 2r_1r_2 \cos(\theta_2 - \theta_1)\end{aligned}\quad (4)$$

And

$$\theta_3 = \tan^{-1} \left(\frac{r_1 \sin \theta_1 + r_4 \sin \theta_4 - r_2 \sin \theta_2}{r_1 \cos \theta_1 + r_4 \cos \theta_4 - r_2 \cos \theta_2} \right) \quad (5)$$

Similarly, the angular position of link 12, θ_{12} , in the middle four-bar linkage is given as

$$\theta_{12} = 2 \tan^{-1} \left(\frac{B_2 \pm \sqrt{A_2^2 + B_2^2 - C_2^2}}{(A_2 + C_2)} \right) \quad (6)$$

In which

$$\begin{aligned}A_2 &= 2r_{12}(r_{10} - r_{11} \cos \theta_{11}) \\ B_2 &= -2r_{11}r_{12} \sin \theta_{11} \\ C_2 &= r_5^2 - r_{10}^2 - r_{11}^2 - r_{12}^2 + 2r_{10}r_{11} \cos \theta_{11}\end{aligned}\quad (7)$$

And

$$\theta_5 = \tan^{-1} \left(\frac{r_{12} \sin \theta_{12} - r_{11} \sin \theta_{11}}{r_{10} + r_{12} \cos \theta_{12} - r_{11} \cos \theta_{11}} \right) \quad (8)$$

The angular position of link 8, θ_8 , in the third four-bar linkage is expressed as

$$\theta_8 = 2 \tan^{-1} \left(\frac{B_3 \pm \sqrt{A_3^2 + B_3^2 - C_3^2}}{(A_3 + C_3)} \right) \quad (9)$$

In which

$$\begin{aligned} A_3 &= 2r_8(r_9 \cos \theta_9 - r_6 \cos \theta_6) \\ B_3 &= 2r_8(r_9 \sin \theta_9 - r_6 \sin \theta_6) \\ C_3 &= r_7^2 - r_6^2 - r_8^2 - r_9^2 + 2r_6r_9 \cos(\theta_6 - \theta_9) \end{aligned} \quad (10)$$

And

$$\theta_7 = \tan^{-1} \left(\frac{r_9 \sin \theta_9 + r_8 \sin \theta_8 - r_6 \sin \theta_6}{r_9 \cos \theta_9 + r_8 \cos \theta_8 - r_6 \cos \theta_6} \right) \quad (11)$$

Note that the signs “ $\pm$ ” appearing in Equations (3), (6) and (9) are determined by the assembly configurations of the constituent four-bar linkages. In the leading type steering mechanism (Figure 2), all four-bar linkages are configured as assembly I; therefore, the positive sign (“+”) is adopted. In contrast, for the mixed-leading type mechanism (Figure 3), the first and second four-bar loops are configured as assembly II, for which the negative sign (“−”) is applied, while the third loop is configured as assembly I, and thus the positive sign (“+”) is used.

From Figures 2 and 3, the following geometric relationships can be derived:

$$\begin{aligned} \theta_2 &= \theta_{20} + \delta_{in} \\ \delta_{out} &= \theta_8 - \theta_{80} = \theta_8 - \pi + \theta_{20} \end{aligned} \quad (12)$$

Once the output angle of link 8 is determined, the actual steering angle of the outer wheel, δ_{out} , can be obtained using Equation (12). The steering error of the designed mechanism—also referred to as the structural error of the function generator—denoted as δ_s , is defined as

$$\delta_{s,i} = \delta_{out,i} - \delta_{ideal,i} \quad (13)$$

where $\delta_{out,i}$ represents the actual steering angle of the outer wheel produced by the synthesized steering mechanism at i -th steering position, and $\delta_{ideal,i}$ denotes the corresponding ideal steering angle that satisfies the Ackermann condition, as given in Equation (1).

2.3. Transmission Angle

As indicated in Figures 2 and 3, six transmission angles, denoted as $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$, and μ_6 , are defined for the two double four-bar steering mechanisms and are expressed as follows [14,27]:

$$\begin{aligned} \mu_1 &= \cos^{-1} \left(\frac{r_2^2 + r_3^2 - r_1^2 - r_4^2 - 2r_1r_4 \cos(\theta_4 - \theta_1)}{2r_2r_3} \right) \\ \mu_2 &= \cos^{-1} \left(\frac{r_3^2 + r_4^2 - r_1^2 - r_2^2 + 2r_1r_2 \cos(\theta_2 - \theta_1)}{2r_3r_4} \right) \\ \mu_3 &= \cos^{-1} \left(\frac{r_{11}^2 + r_{12}^2 - r_{10}^2 - r_{13}^2 - 2r_{10}r_{12} \cos \theta_6}{2r_{11}r_{12}} \right) \\ \mu_4 &= \cos^{-1} \left(\frac{r_5^2 + r_{12}^2 - r_{10}^2 - r_{11}^2 + 2r_{10}r_{11} \cos \theta_4}{2r_5r_{12}} \right) \\ \mu_5 &= \cos^{-1} \left(\frac{r_6^2 + r_7^2 - r_8^2 - r_9^2 - 2r_8r_9 \cos(\theta_8 + \theta_1)}{2r_6r_7} \right) \\ \mu_6 &= \cos^{-1} \left(\frac{r_7^2 + r_8^2 - r_6^2 - r_9^2 + 2r_6r_9 \cos(\theta_6 + \theta_1)}{2r_7r_8} \right) \end{aligned} \quad (14)$$

2.4. Mechanical Advantage

The mechanical advantage (MA) quantifies the ideal mechanical efficiency of a mechanism. Under the assumption of energy conservation and through application of the instantaneous center theorem, the MA of the leading and mixed-leading double four-bar steering mechanisms can be expressed as follows:

$$MA = \frac{T_8}{T_2} = \left| \frac{\omega_2}{\omega_8} \right| = \left| \left(\frac{\omega_2}{\omega_4} \right) \cdot \left(\frac{\omega_4}{\omega_6} \right) \cdot \left(\frac{\omega_6}{\omega_8} \right) \right| = \frac{(I_{14}I_{24})}{(I_{12}I_{24})} \cdot \frac{(I_{16}I_{46})}{(I_{14}I_{46})} \cdot \frac{(I_{18}I_{68})}{(I_{16}I_{68})} \quad (15)$$

Here, T_i ($i = 2, 8$) denotes the torque applied at link i , while ω_i ($i = 2, 4, 6, 8$) represents the angular velocity of the link i . The term I_{ij} ($j = 2, 4, 6, 8$) refers to the instantaneous center between fixed link and link j , and $(I_{ij}I_{jk})$ ($j = 2, 4, 6; k = j + 2$), ($j = 8; k = 6$) represents the distance between the instantaneous centers I_{ij} and I_{jk} .

For the first four-bar loop, the sine law [14] is used:

$$\frac{(I_{14}I_{24})}{(I_{12}I_{24})} = \frac{r_4 \sin \mu_2}{r_2 \sin \mu_1} = \frac{\sin \mu_2}{\sin \mu_1} \quad (16)$$

Similarly, the following expressions apply to the second and third four-bar loops, respectively.

$$\frac{(I_{16}I_{46})}{(I_{14}I_{46})} = \frac{r_6 \sin \mu_4}{r_4 \sin \mu_3} = \frac{\sin \mu_4}{\sin \mu_3} \quad (17)$$

$$\frac{(I_{18}I_{68})}{(I_{16}I_{68})} = \frac{r_8 \sin \mu_6}{r_6 \sin \mu_5} = \frac{\sin \mu_6}{\sin \mu_5} \quad (18)$$

The MA of the double four-bar steering mechanism can be expressed as

$$MA = \frac{(I_{14}I_{24})}{(I_{12}I_{24})} \cdot \frac{(I_{16}I_{46})}{(I_{14}I_{46})} \cdot \frac{(I_{18}I_{68})}{(I_{16}I_{68})} = \frac{\sin \mu_2 \sin \mu_4 \sin \mu_6}{\sin \mu_1 \sin \mu_3 \sin \mu_5} \quad (19)$$

Although all six transmission angles ($\mu_1, \mu_2, \dots, \mu_6$) influence the MA of the double four-bar steering mechanisms, transmission efficiency during turning is primarily governed by two critical angles, μ_1 and μ_6 , which are associated with the inner and outer steering wheels.

3. Metaheuristic Optimization Algorithms

The dimensional synthesis of multi-link steering mechanisms is a highly coupled, nonlinear problem characterized by a “rugged” landscape of local optima. To navigate this complexity, this study employs two population-based metaheuristic algorithms: Improved Particle Swarm Optimization (IPSO) [25] and Differential Evolution with golden ratio (DE-gr) [14,26]. Unlike traditional gradient-based methods, these metaheuristics do not require differentiable objective functions and are significantly more resistant to premature convergence in local optima.

The selection of these specific algorithms provides a balanced optimization strategy: IPSO excels at exploiting complex local interactions to refine high-precision requirements, while DE-gr utilizes powerful mutation and crossover operators to ensure robust global exploration across broad parameter ranges. Both optimization algorithms have been successfully employed in mechanism dimensional synthesis problems, as reported by the authors in [14,22–24]. By deploying this dual-approach framework, the study systematically explores the design space to yield steering configurations with superior Ackermann compliance and optimized link dimensions across both leading and mixed-leading architectures.

3.1. Improved Swarm-Based Optimization Algorithm

The Particle Swarm Optimization (PSO) algorithm [28] is inspired by the collective behavior of social organisms. In this framework, each particle represents a candidate solution for the optimal design problem, updating its velocity and position based on its individual best performance and the global best solution discovered by the swarm. Building upon the original PSO framework, Shi and Eberhart [29] introduced the inertia weight (w) to modulate the influence of the previous velocity on the current update, effectively balancing global and local search capabilities. To prevent premature convergence and enhance stability, this study adopts the Improved Particle Swarm Optimization (IPSO)

variant proposed by Ratnaweera et al. [25], which incorporates time-varying acceleration coefficients (C_1 , C_2) and linearly decreasing inertia weights. In the IPSO algorithm, the new position $\mathbf{X}_i^{t+1}$ and velocity $\mathbf{V}_i^{t+1}$ of the i -th particle are updated according to the following equations:

$$\begin{aligned}\mathbf{V}_i^{t+1} &= \omega^t \times \mathbf{V}_i^t + C_1^t \times \text{rand}() \times (\mathbf{P}_i^t - \mathbf{X}_i^t) + C_2^t \times \text{Rand}() \times (\mathbf{P}_g^t - \mathbf{X}_i^t) \\ \mathbf{X}_i^{t+1} &= \mathbf{X}_i^t + \mathbf{V}_i^{t+1}\end{aligned}\quad (20)$$

where $\mathbf{P}_i^t$ denotes the personal best position of the i -th particle, and $\mathbf{P}_g^t$ represents the global best position of the entire swarm. The functions $\text{rand}()$ and $\text{Rand}()$ generate a random number uniformly distributed between 0 and 1. Following the empirical recommendations of Shi and Eberhart [30], the inertia weight ω^t is dynamically adjusted according to the following linear decay function.

$$\omega^t = \omega_{\min} + \frac{(N_{\text{iter}} - t)}{N_{\text{iter}}} \cdot (\omega_{\max} - \omega_{\min}) \quad (21)$$

where N_{iter} represents the maximum number of iterations and t denotes the current iteration. $\omega_{\min}$ and $\omega_{\max}$ are the minimum and maximum values of the inertia weight, respectively. $\omega_{\max} = 0.9$ and $\omega_{\min} = 0.4$. Simultaneously, the acceleration coefficients C_1^t (cognitive component) and C_2^t (social component) are varied linearly to encourage exploration in early stages and exploitation in later stages [25]:

$$\begin{aligned}C_1^t &= C_{1\min} + \frac{(N_{\text{iter}} - t)}{N_{\text{iter}}} \cdot (C_{1\max} - C_{1\min}) \\ C_2^t &= C_{2\max} - \frac{(N_{\text{iter}} - t)}{N_{\text{iter}}} \cdot (C_{2\max} - C_{2\min})\end{aligned}\quad (22)$$

where $C_{1\min}$ and $C_{1\max}$ denote the minimum and maximum individual cognitive coefficients, while $C_{2\min}$ and $C_{2\max}$ represent the minimum and maximum social cognitive coefficients of the swarm. The recommended values are $C_{1\max} = C_{2\max} = 2.5$ and $C_{1\min} = C_{2\min} = 0.5$ [25].

This configuration allows the particles to roam the high-dimensional design space of the 8-bar steering mechanism independently at the start, eventually converging toward the global best position $\mathbf{P}_g^t$ as the social influence C_2^t becomes dominant in the final iterations. This dynamic dual-adaptation facilitates a strategic trade-off: intensive global exploration is prioritized during the initial iterations, while fine-grained local exploitation is emphasized in the later stages. Such adaptability significantly improves the robustness and convergence toward the global optimum when optimizing the highly coupled geometric parameters of 8-bar steering systems—specifically link lengths, pivot coordinates, and initial input angular positions. A flow chart of the IPSO algorithm is shown in Figure 4.

3.2. Evolutionary-Based Optimization Algorithm

The Differential Evolution (DE) algorithm [26] is a population-based optimization method that generates new candidate solutions through four vector-based steps, initialization, mutation, crossover, and selection operations. Several improved and hybrid variants of the DE algorithm have been reported in DE survey papers [31–34]. Detailed descriptions of the four basic operations of the DE algorithm are available in these references. For many applications, the population size $N_p = 10 \times D$ is a good choice [35]; D is the dimension of design parameters. After initialization, for each D -dimensional parametric vector (target vector) $\mathbf{X}_{ri}^t$, $\mathbf{X}_{ri}^t = [x_{i,1}^t, x_{i,2}^t, \dots, x_{i,D}^t]$, in the current iteration t , a corresponding mutant vector $\mathbf{V}_i^t$, $\mathbf{V}_i^t = [v_{i,1}^t, v_{i,2}^t, \dots, v_{i,D}^t]$ is generated through the mutation operation. Within the Differential Evolution (DE) family, five mutation strategies, *DE/rand/1*, *DE/best/1*, *DE/rand/2*, *DE/best/2* and *DE/rand-to-best/1* (or called *DE/current-to-best/1*), are commonly recognized [35]. While the “*DE/rand/1*” strategy—often termed the canonical DE

strategy—remains the most widely utilized in the literature, comparative studies in [32,36] indicate that strategies incorporating the “best” individual often outperform the random approach in terms of convergence speed and solution quality. In this study, to ensure both high-order accuracy and robust convergence for the 8-bar steering synthesis, two performance-oriented mutation strategies, *DE/best/1* and *DE/best/2*, are employed. These are formulated as follows:

DE/best/1:

$$\mathbf{V}_i^t = \mathbf{X}_{best}^t + F(\mathbf{X}_{r1}^t - \mathbf{X}_{r2}^t) \quad (23)$$

DE/best/2:

$$\mathbf{V}_i^t = \mathbf{X}_{best}^t + F(\mathbf{X}_{r1}^t - \mathbf{X}_{r2}^t) + F(\mathbf{X}_{r3}^t - \mathbf{X}_{r4}^t) \quad (24)$$

where $\mathbf{X}_{best}^t$ represents the optimal individual vector with the optimal fitness function in the population at iteration t . $(\mathbf{X}_{r1}^t - \mathbf{X}_{r2}^t)$ and $(\mathbf{X}_{r3}^t - \mathbf{X}_{r4}^t)$ are the difference vectors. The mutation scaling factor (or mutation rate) F is a positive control parameter used to scale the difference vector in mutation operations. It typically takes a value in the range [0.4, 0.99].

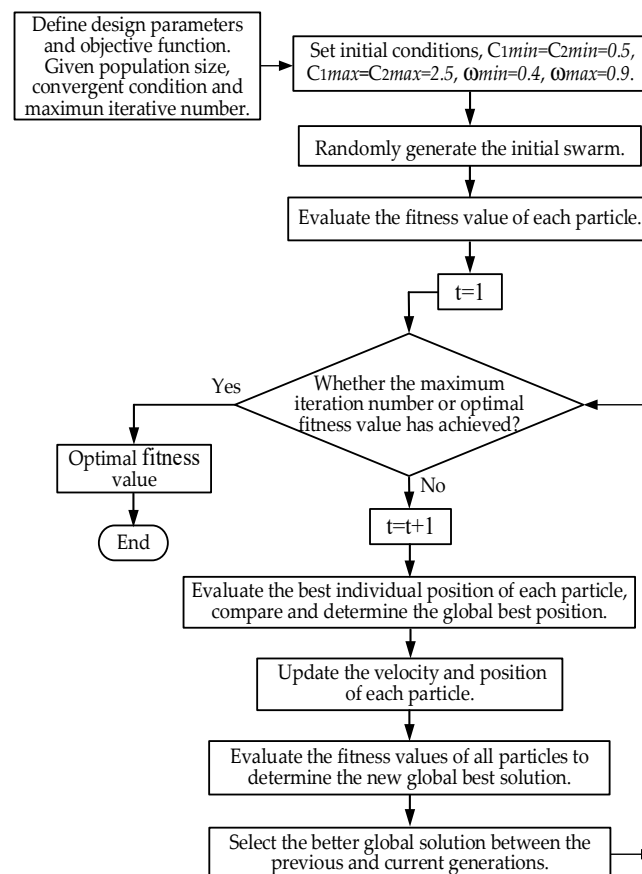


Figure 4. Flow chart of the IPSO algorithm.

Through exponential crossover, the components of the mutant vector were combined with those of the target (parent) vector $\mathbf{X}_{ri}^t$ to form a trail (offspring) vector $\mathbf{U}_i^t, \mathbf{U}_i^t = [u_{i,1}^t, u_{i,2}^t, \dots, u_{i,D}^t]$.

$$u_{i,j}^t = \begin{cases} v_{i,j}^t & \text{if } \text{rand}_{i,j}(0, 1) \leq C_r \text{ or } i = i_r \\ x_{i,j}^t & \text{otherwise} \end{cases} \quad (25)$$

where C_r is a predefined control parameter, referred to as the crossover rate, typically ranging within $[0, 1]$. $rand_{i,j}(0, 1)$ denotes a uniformly distributed random number within the range $[0, 1]$, which ensures that the trial vector $\mathbf{U}_i^t$ includes at least one component from the mutant vector $\mathbf{V}_i^t$. i_r is a randomly selected integer from the set $\{1, 2, \dots, D\}$, where D is the dimension of the problem.

To eliminate the stochastic uncertainty and empirical trial-and-error associated with parameter tuning, this study adopts a golden ratio-based parameterization for the Differential Evolution framework, called Differential Evolution with golden ratio (DE-gr) [22–24]. The parameters are derived from the golden section $\gamma : (1 - \gamma)$ of the unit interval: $\gamma = 0.618$, $1 - \gamma = 0.382$. In the DE-gr variant, the crossover rate (Cr) and the scaling factor (F) are defined by these complementary proportions. Crossover rate (Cr) = 0.618: A relatively high crossover probability ensures that a significant portion of the trial vector is inherited from the crossover operation, promoting the exchange of genetic information and maintaining population diversity. Scaling factor (F) = 0.382: A conservative mutation factor ensures that the differential variations do not lead to excessive step sizes, which is critical for staying within the narrow feasible regions of the highly constrained 8-bar steering design space.

DE employs a greedy selection strategy to determine whether the target (parent) or the trial (offspring) vector survives into the next iteration. The target vector of the next generation, $\mathbf{X}_{ri}^{t+1}$, can be expressed as follows:

$$\mathbf{X}_{ri}^{t+1} = \begin{cases} \mathbf{U}_i^t & \text{if } f(\mathbf{U}_i^t) \leq f(\mathbf{X}_i^t) \\ \mathbf{X}_{ri}^t & \text{otherwise.} \end{cases} \quad (26)$$

where $f(\cdot)$ is the objective function to be minimized.

A flow chart of the DE-gr algorithm is shown in Figure 5.

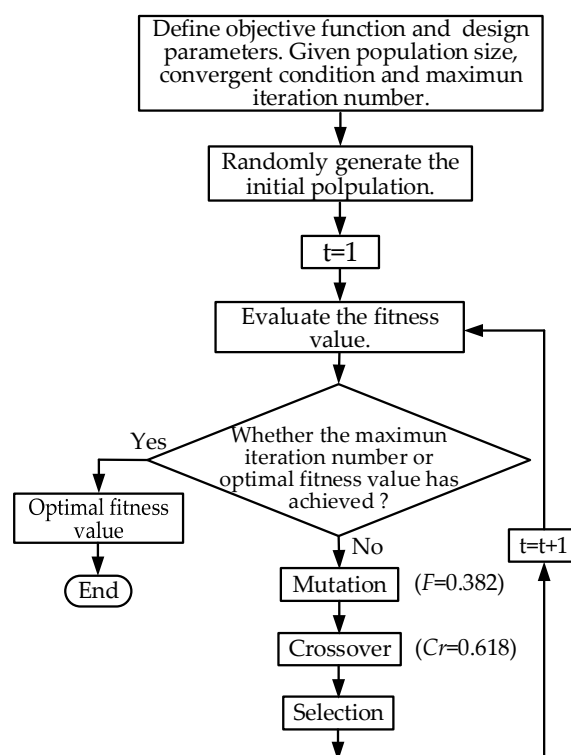


Figure 5. Flow chart of the DE-gr algorithm.

4. Optimal Synthesis of the Double Four-Bar Steering Mechanisms

4.1. Formulation of the Objective Function

The optimization problem is formulated to minimize the mean-squared structural error, defined as the deviation between the actual and ideal Ackermann steering angles of the outer wheel over a set of discrete steering positions [14]. To ensure feasible linkage motion, a penalty function is incorporated to account for violations of the Grashof mobility constraint [27]. The objective function is expressed as follows:

$$\text{Min } f(\mathbf{X}) = \left\{ \sum_{i=1}^N \frac{[\delta_{out,i}(\mathbf{X}) - \delta_{ideal,i}(\mathbf{X})]^2}{N} \right\} + M h_1(\mathbf{X}) + M h_2(\mathbf{X}) \quad (27)$$

where $\mathbf{X}$ denotes the design vector, consisting of the link lengths, height of the pivots' location, and the initial orientation of the input link. N represents the number of selected steering positions. To balance computational efficiency with high-order numerical accuracy, a discrete sampling approach is adopted. The steering range is partitioned into $N = 71$ equidistant points, corresponding to a resolution of 1° per sample over the steering interval $[-30^\circ, 40^\circ]$. The structural error, $\delta_{s,i}$, defined as $\delta_{s,i} = \delta_{out,i} - \delta_{ideal,i}$, as shown in Equation (13), represents the deviation between the actual steering angle of outer wheel produced by the synthesized mechanism and the corresponding ideal steering angle satisfying the Ackermann condition at the i -th steering position. The term $M h_1(\mathbf{X}) + M h_2(\mathbf{X})$ denotes the penalty function applied when the design parameters violate the inequality constraints. The penalty coefficient M is set to 100,000. The function $h_1(\mathbf{X})$ or $h_2(\mathbf{X})$ takes a value of 1 when the corresponding constraint is violated and 0 otherwise.

The inequality constraint ensuring a non-Grashof configuration for the first four-bar linkage is given by

$$h_1(\mathbf{X}) : r_s + r_l > r_p + r_q \text{ and } r_2 = r_s \quad (28)$$

where r_s and r_l represent the shortest and longest link lengths, respectively, and r_p and r_q denote the lengths of the remaining two links. This condition is identical to that of the conventional Ackermann steering mechanism, and guarantees a non-Grashof motion for the linkage.

Similarly, the inequality constraint applied to the middle four-bar loop is expressed as

$$h_2(\mathbf{X}) : r_{sm} + r_{lm} > r_{pm} + r_{qm} \text{ and } r_{11} = r_{sm} \quad (29)$$

where the subscript “ m ” refers to the corresponding links in the middle four-bar loop.

4.2. Design Parameters and Constraint Conditions

The optimal design of the double four-bar steering mechanism involves six independent design parameters, and the corresponding parameter vector $\mathbf{X}$ is expressed as follows:

$$\mathbf{X} = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [r_2, r_3, r_0, r_5, h, \theta_{20}]^T \quad (30)$$

The range of input steering angle of the vehicle's front inner wheel is $-30^\circ \leq \delta_{in} \leq 40^\circ$, wheel track $W = 1480$ mm, and wheelbase $L = 2960$ mm. The allowable ranges of design parameters, defined based on practical geometric constraints and applicable to both steering mechanism types, are summarized as follows:

$$\begin{aligned} 100 \leq r_2 \leq 950 \text{ mm}, & \quad 100 \leq r_3 \leq 950 \text{ mm}, \quad 100 \leq r_0 \leq 700 \text{ mm} \\ 100 \leq r_5 \leq 400 \text{ mm}, & \quad 0 \leq h \leq 400 \text{ mm}, \quad 30 \leq \theta_{20} \leq 150 \text{ deg.} \end{aligned}$$

Non-Grashof inequality constraints of the first and middle four-bar linkages:

$$h_1(X): 2\{\max[r_1, r_2, r_3, r_4] + \min[r_1, r_2, r_3, r_4]\} > \text{sum}[r_1, r_2, r_3, r_4] \text{ and } r_2 = r_s \quad (31)$$

$$h_2(X): 2\{\max[r_{10}, r_{11}, r_5, r_{12}] + \min[r_{10}, r_{11}, r_5, r_{12}]\} > \text{sum}[r_{10}, r_{11}, r_5, r_{12}] \text{ and } r_{11} = r_s \quad (32)$$

4.3. Optimization Parameters

The parameter settings for both IPSO and DE-gr algorithms are summarized in Table 1. To ensure a fair comparison, both algorithms are implemented under identical constraints, including Ackermann compliance and Grashof mobility criteria. To reduce computational cost and improve convergence efficiency, a small population size of 20 and a maximum of 500 iterations were used for each metaheuristic optimization run. This configuration—combining a small population with a moderate iteration limit—balances computational efficiency and solution diversity. The optimization terminates when either the iteration limit is reached or the improvement in the objective function between successive generations falls below 10^{-10} . To ensure convergence robustness and solution repeatability, each steering mechanism was synthesized independently over 100 runs per optimization trial. Performance is evaluated through convergence behavior, while the resulting designs are analyzed for steering accuracy, transmission angles, and mechanical advantage. Ultimately, the best solutions from ten separate trials were selected to ensure mechanical feasibility and to minimize the maximum structural error (Min-Max error).

Table 1. Parameter settings of the two metaheuristic optimization methods.

Algorithm	IPSO	DE-gr
Population size (N_p)	20	20
Iteration times (N_{iter})	500	500
Experimental times	100	100
Mutation method	--	DE/best/1, DE/best/2
Mutation factor (F)	--	0.382
Crossover method	--	Exponential crossover
Crossover rate (C_r)	--	0.618
Selection method	--	Greedy selection
Learning factors (C_1, C_2)	$0.5 \leq C_1, C_2 \leq 2.5$	--
Inertia weighting (w)	$0.4 \leq w \leq 0.9$	--

4.4. Optimal Design of the Leading Type Steering Mechanisms

This section evaluates the optimal design and kinematic performance of the leading double four-bar steering mechanisms. Table 2 details the optimized dimensions; the DE-gr-optimized mechanism achieves a lower Min-Max structural error of **0.1038°** (highlighted in bold). Regarding the physical dimensions, the total link length (excluding the fixed link) is 2770.654 mm for the IPSO-optimized mechanism and 2796.511 mm for the DE-gr-optimized mechanism. While the IPSO method yields a slightly shorter total length, the difference of approximately 25.86 mm indicates that both optimization techniques produce results with similar overall mass and material requirements. As shown in Figure 6, the DE-gr algorithm demonstrates faster convergence compared to IPSO, indicating higher computational efficiency. This advantage is further reflected in the structural error curves (Figure 7), where DE-gr yields lower and more uniformly distributed errors across the entire steering range. Notably, the steering accuracy achieved by both the DE-gr and IPSO algorithms surpasses that previously reported using the continuous genetic algorithm [19].

Table 2. Optimal synthesis results for the leading double four-bar steering mechanisms.

	Obj. Fun	r_0	r_1	r_2	r_3	r_4	r_5	r_{10}	h	θ_{20}	Min-Max. Error
IPSO	2.170×10^{-6}	535.4769	555.2749	225.0147	759.4518	225.0147	351.6916	409.0462	146.9513	129.2072	0.2078
DE-gr	4.220×10^{-7}	506.4085	531.7829	221.4417	755.3782	221.4417	399.9879	467.1830	162.3066	133.1390	0.1038

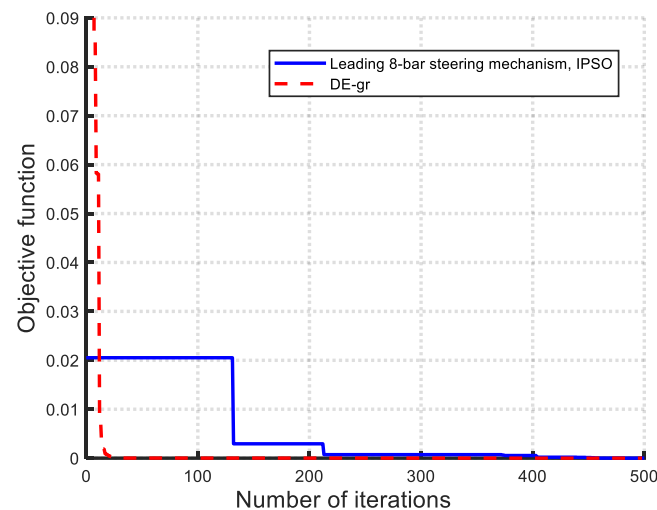
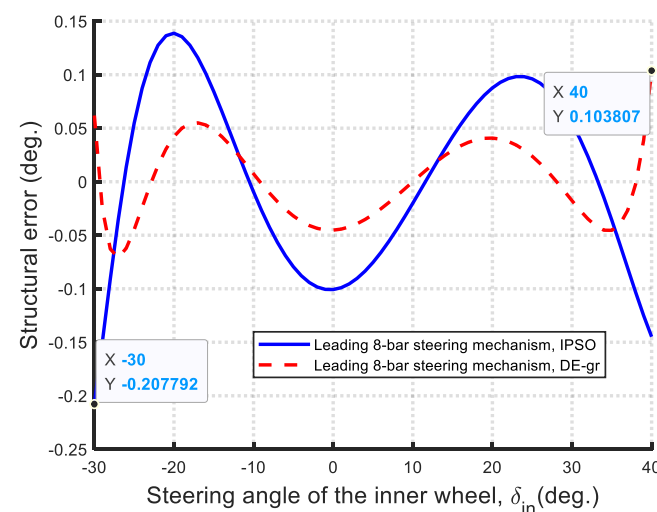
**Figure 6.** Convergence speed curves of the two optimization methods used for the leading 8-bar steering mechanisms.**Figure 7.** Structural error curves for the leading 8-bar mechanisms optimized using IPSO and DE-gr.

Figure 8 illustrates the mechanical advantage (MA) curves for the two optimized leading mechanisms. The IPSO-optimized mechanism displays a nearly linear variation, with only minor curvature at the steering range limits. In contrast, while the DE-gr curve aligns with the IPSO trend between -26° and 30° , it deviates at the extremes—declining beyond 30° and rising slightly below -26° . Moreover, the IPSO-optimized design achieves a maximum MA of 1.6438 and a minimum of 0.6085, providing a more stable and predictable force transmission profile compared to the DE-gr variant.

The kinematic performance of the DE-gr-optimized leading mechanism was validated using ADAMS v2015 (Figure 9) and MATLAB (Figures 10–12). ADAMS facilitated dynamic visualization and kinematic verification, as shown in Figure 9. A supplementary video of the kinematic simulation is available in the Supplementary File. Simulation results confirm

that the 8-bar configurations maintain precise inner and outer wheel coordination throughout the full steering range, validating the physical feasibility of the optimized designs. As shown in Figures 10 and 11, the mechanism's actual steering response closely tracks the ideal Ackermann condition, with the outer-wheel angles and output-link positions remaining nearly indistinguishable throughout the range. The transmission angle curves in Figure 12 assess the mechanism's mechanical feasibility. For the critical transmission angles (μ_1 , μ_6), deviations from 90° remain within the acceptable 45° threshold between -15° and 25° . While performance deviates outside this primary range, the design maintains efficient force transmission during standard steering operations.

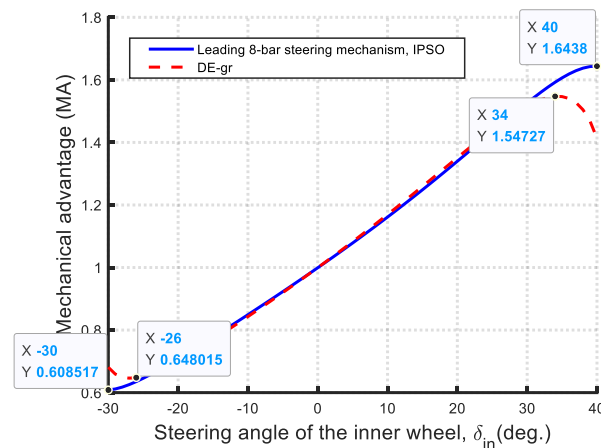


Figure 8. Mechanical advantage (MA) curves of the leading 8-bar mechanisms optimized via IPSO and DE-gr.

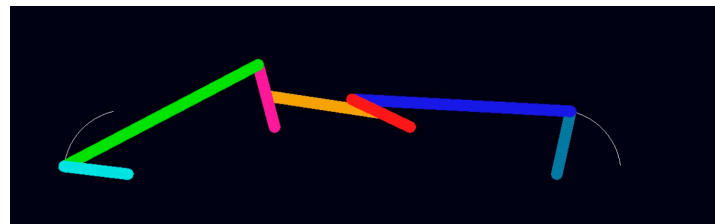


Figure 9. Kinematic simulation of the DE-gr-optimized leading 8-bar steering mechanism performed in ADAMS v2015.

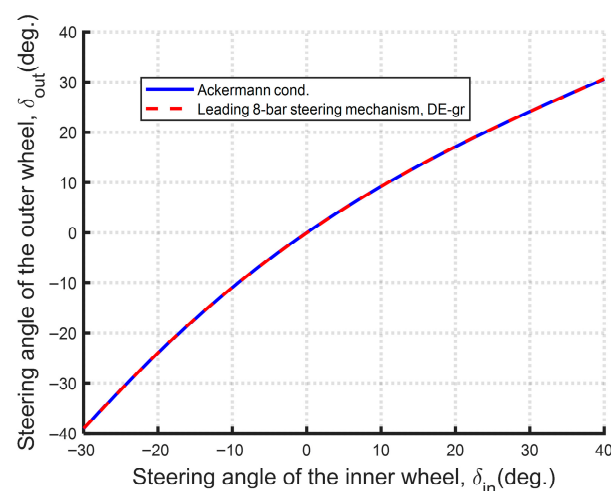


Figure 10. Actual steering angle vs. the ideal Ackermann condition for the outer wheel of the DE-gr-optimized leading 8-bar steering mechanism.

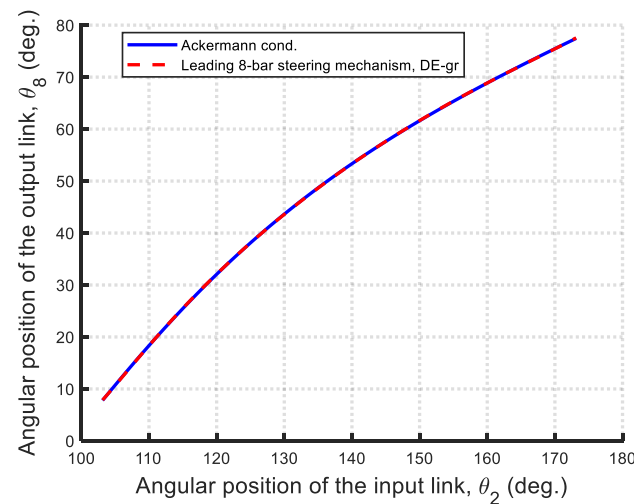


Figure 11. Actual angular position vs. the ideal Ackermann condition of the output link for the DE-gr-optimized leading 8-bar steering mechanism.

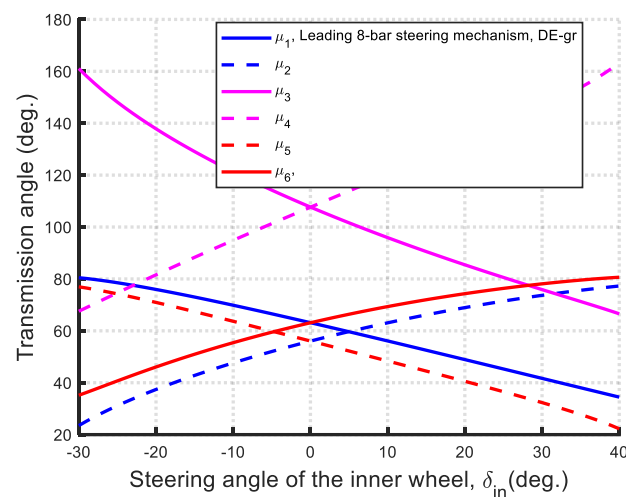


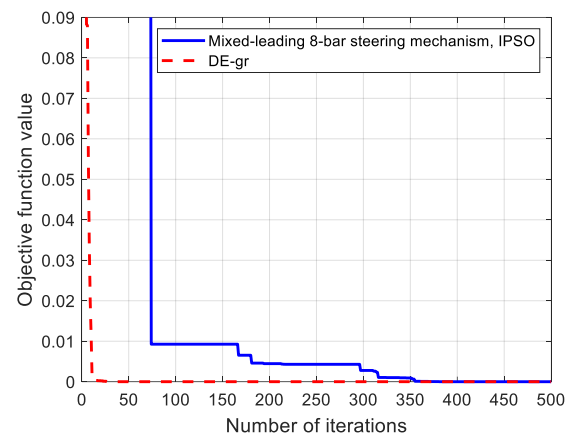
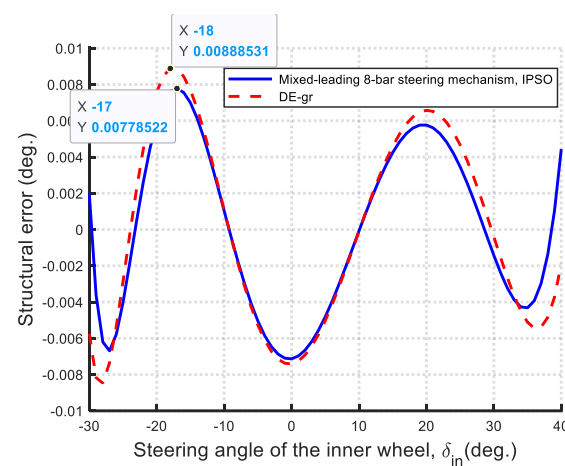
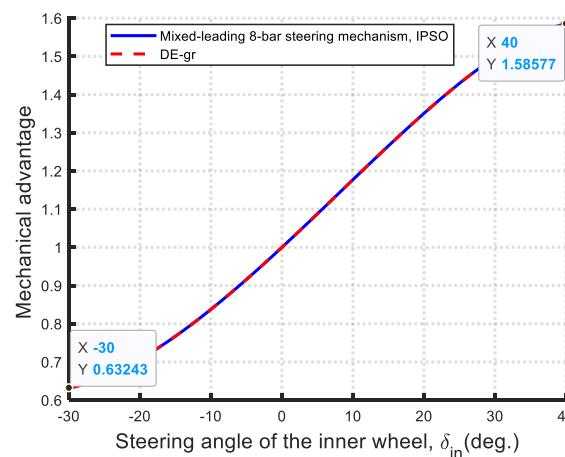
Figure 12. Transmission angle curves of the DE-gr-optimized leading 8-bar steering mechanism.

4.5. Optimal Design of the Mixed-Leading Type Steering Mechanisms

This section evaluates the optimal mixed-leading double four-bar steering mechanisms optimized using the IPSO and DE-gr algorithms. Table 3 details the optimized dimensions, highlighting that the IPSO-optimized mechanism achieves the superior precision with a Min-Max structural error of 0.007785° . Regarding the physical dimensions, the total link length (excluding the fixed link) is 2266.162 mm for the IPSO-optimized mechanism and 2287.438 mm for the DE-gr-optimized mechanism. While the IPSO method yields a slightly shorter total length, the difference of approximately 21.28 mm indicates that both the IPSO and DE-gr algorithms produce results with comparable overall mass and material requirements. While the DE-gr algorithm demonstrates superior computational efficiency with faster convergence (Figure 13), both configurations maintain high accuracy by closely approximating the ideal Ackermann condition across the steering range (Figure 14), with IPSO yielding a lower maximum structural error. Furthermore, the mechanical advantage (MA) trends shown in Figure 15 are nearly identical for both methods. The IPSO-optimized mechanism provides a maximum MA of 1.5858 and a minimum of 0.63243, ensuring stable and consistent force transmission.

Table 3. Optimal synthesis results for the mixed-leading double four-bar steering mechanisms.

	Obj. Fun	r_0	r_1	r_2	r_3	r_4	r_5	r_{10}	h	θ_{20}	Min-Max. Error
IPSO	6.427×10^{-9}	627.8516	660.8471	198.5956	616.1729	198.5956	239.4342	224.2968	206.2067	82.43298	7.785×10^{-3}
DE-gr	8.470×10^{-9}	662.7090	684.4976	204.9646	658.1268	204.9646	151.3256	154.5819	171.3292	75.8162	8.885×10^{-3}

**Figure 13.** Convergence speed curves of the two optimization methods for the mixed-leading 8-bar steering mechanisms.**Figure 14.** Structural error curves of the two optimal mixed-leading 8-bar steering mechanisms.**Figure 15.** Mechanical advantage (MA) curves of the two optimal mixed-leading 8-bar steering mechanisms.

The kinematic performance of the IPSO-optimized mixed-leading mechanism was validated using ADAMS v2015 (Figure 16) and MATLAB (Figures 17–19). Figure 16 displays the kinematic simulation of the optimal mixed-leading steering mechanism. A supplementary video of the kinematic simulation is available in Supplementary File. Figures 17 and 18 demonstrate that the mechanism's steering response closely aligns with the ideal Ackermann condition, with outer-wheel angles and output-link positions remaining nearly indistinguishable throughout the range. Regarding transmission quality, Figure 19 illustrates the six transmission angle curves. The critical transmission angles (μ_1 , μ_6), maintain deviations from 90° of less than 45° , confirming the design's high transmission efficiency and mechanical feasibility.

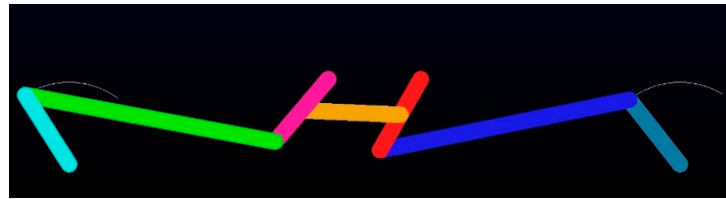


Figure 16. Kinematic simulation of the IPSO-optimized mixed-leading 8-bar steering mechanism performed in ADAMS v2015.

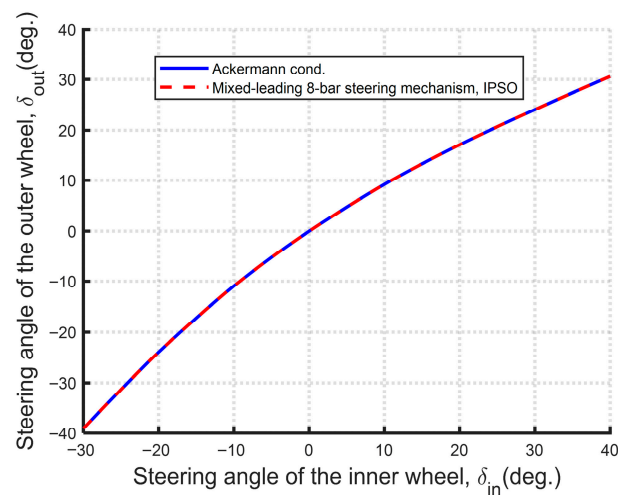


Figure 17. Actual steering angle vs. the ideal Ackermann condition of the outer wheel for the IPSO-optimized mixed-leading 8-bar steering mechanism.

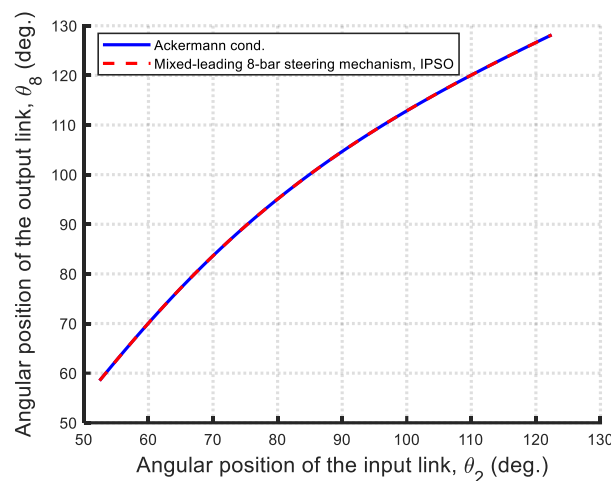


Figure 18. Actual angular position vs. the ideal Ackermann condition of the output link for the IPSO-optimized mixed-leading 8-bar steering mechanism.

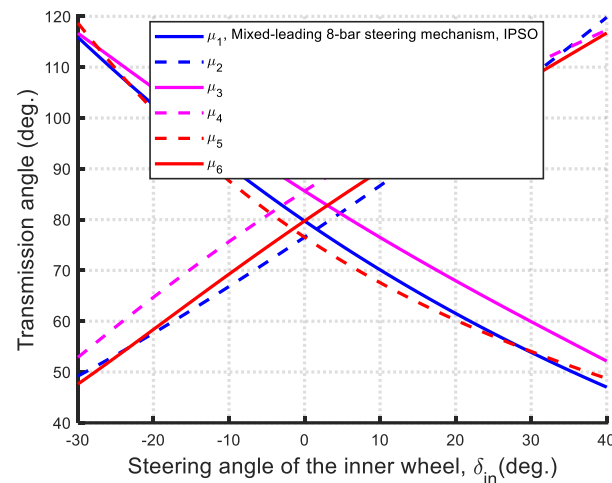


Figure 19. Transmission angle curves of the IPSO-optimized mixed-leading 8-bar steering mechanism.

5. Comparative Analysis of the Two Optimal Steering Variants

5.1. Comparative Performance Analysis

This section provides a comparative analysis of the two optimal double four-bar steering mechanisms, with detailed link dimensions and maximum structural errors presented in Tables 2 and 3. Regarding the physical dimensions, the total link length (excluding the fixed link) is 2266.162 mm for the IPSO-optimized mixed-leading mechanism, compared to 2796.511 mm for the DE-gr-optimized leading mechanism. The IPSO-optimized mixed-leading variant demonstrates a distinct advantage; it achieves a significantly lower overall mass—facilitated by a substantial length reduction of 530.35 mm—while simultaneously maintaining superior kinematic fidelity. Specifically, the IPSO variant further minimizes the maximum structural error of 0.007785° beyond the 0.1038° achieved by the DE-gr model. These results suggest that the IPSO algorithm, when applied to a mixed-leading configuration, yields a more efficient and kinematically precise design solution. According to Figure 20, the IPSO algorithm achieves superior accuracy for the mixed-leading configuration, while DE-gr performs better for the leading configuration. Notably, both algorithms enable the mixed-leading double four-bar mechanism to achieve near-ideal Ackermann behavior (Figure 14), with IPSO yielding the lowest maximum structural error.

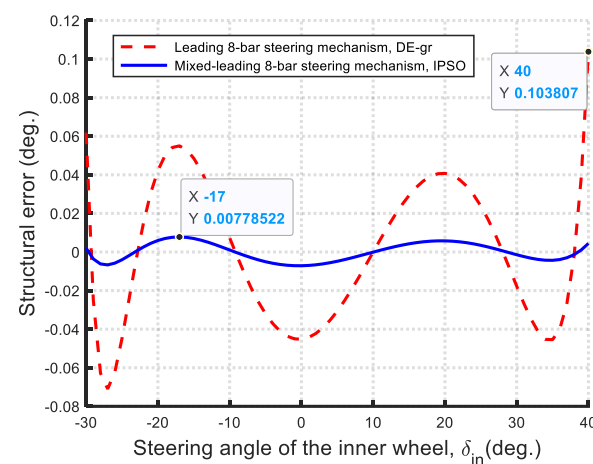


Figure 20. Comparison of structural error curves for the DE-gr-optimized leading and IPSO-optimized mixed-leading 8-bar steering mechanisms.

Figure 21 evaluates the critical transmission angles μ_1 and μ_6 for the DE-gr-optimized leading and IPSO-optimized mixed-leading mechanisms. Within the primary steering

range of -20 to 25 , the IPSO-optimized mixed-leading mechanism demonstrates superior characteristics, with both angles more closely approaching the ideal 90° during turns. However, the DE-gr-optimized leading mechanism exhibits better performance at extreme steering angles (-30° to -20° and 25° to 40°). Additionally, Figure 22 compares the mechanical advantage (MA) of all four optimized mechanisms for the two steering variants. The IPSO-optimized leading mechanism (blue dashed curve) provides the most linear and stable MA variation, ensuring predictable force transmission throughout the steering operation.

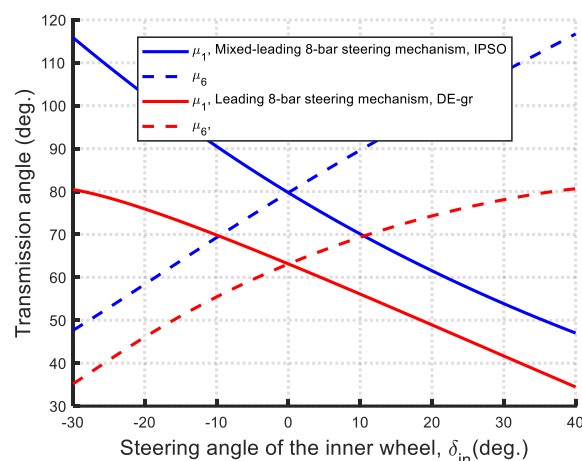


Figure 21. Comparison of critical transmission angle curves for the DE-gr-optimized leading and IPSO-optimized mixed-leading 8-bar steering mechanisms.

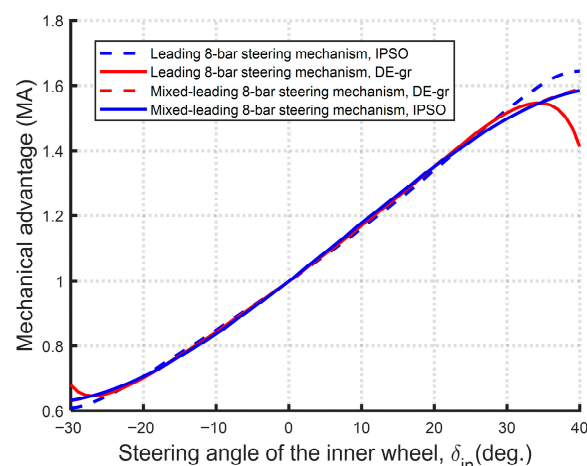


Figure 22. Comparison of mechanical advantage (MA) curves for the optimized leading and mixed-leading 8-bar steering mechanisms.

The findings collectively demonstrate that the IPSO-optimized mixed-leading configuration offers a superior balance between spatial efficiency and kinematic performance. While the reduced link lengths (Tables 2 and 3) contribute to a lighter overall weight, the transmission angle analysis (Figure 19) confirms that this reduction does not compromise mechanical integrity. Specifically, maintaining critical transmission angles (μ_1 , μ_6) within 45° of the 90° ideal ensures that the lighter links are not subjected to excessive toggle-point stresses. Furthermore, the alignment with findings from the optimal dimensional synthesis of central-lever steering mechanisms [14] reinforces that adopting the Assembly II (crossed four-bar) configuration is the primary driver of high steering accuracy. By combining the IPSO algorithm's precision with the inherently efficient mixed-leading architecture,

the design achieves near-ideal Ackermann behavior while remaining lightweight and mechanically feasible for two-axle, front-wheel-drive vehicles.

5.2. Discussion

Figures 6, 7, 13, 14 and 20 compare the convergence behavior and steering accuracy of swarm-based (IPSO) and evolutionary-based (DE-gr) algorithms. The results show that DE-gr converges more rapidly for both variants, reflecting its strong global search capability and superior accuracy for the leading-type mechanism. For the mixed-leading configuration, both algorithms exhibit high accuracy, though IPSO achieves slightly better performance. This suggests that both evolutionary-based and swarm-based methods effectively exploit the complex, coupled parameter interactions inherent in mixed-leading architecture.

The final evaluation compares the performance metrics of the two optimized steering configurations, as shown in Table 4. From a mechanism perspective, the mixed-leading configuration offers two distinct advantages: significantly higher steering accuracy and lighter overall weight due to shorter link lengths. Conversely, the leading-type configuration excels in force transmission, exhibiting a more linear and stable mechanical advantage. Ultimately, for dimensional synthesis in front-wheel-drive vehicles, the mixed-leading (Assembly II/crossed four-bar) configuration is generally preferable for its precision and mass efficiency, provided the mechanical advantage remains within acceptable design limits.

Table 4. Performance comparison between optimized leading and mixed-leading 8-bar steering mechanisms.

Feature	Leading Configuration (Assembly I)	Mixed-Leading Configuration (Assembly II)
Metric	DE-gr	IPSO
Total link lengths (excluding fixed link)	2796.511 mm Longer (standard)	2266.162 mm Shorter (optimized)
Weight/mass	Higher	Lower (lighter)
Maximum steering error/steering accuracy	0.1038° Good (highest with DE-gr)	0.007785° Excellent (highest with IPSO)
Mech. advantage	Highly linear and stable	Moderate
Primary transmission angles (μ_1, μ_6)	Best at near extremes	Best in main range (-20° to 25°)
Design priority	Precision focused	Precision and mass optimized

6. Conclusions

The dimensional synthesis of multi-link steering mechanisms represents a complex, nonlinear optimization challenge. This study successfully implemented a comparative metaheuristic approach, utilizing the IPSO and DE-gr algorithms to optimize planar leading and mixed-leading double four-bar steering systems. By establishing this robust dual-approach framework, the research provides a systematic methodology for advanced multi-link steering design that transcends the limitations of traditional gradient-based methods.

The comparative analysis reveals that while both metaheuristics demonstrate stable convergence, their effectiveness varies by architecture. DE-gr exhibits higher computational efficiency across both configurations and superior performance for leading-type mechanisms. Conversely, IPSO achieves significantly higher geometric precision and mass efficiency for mixed-leading configurations. Furthermore, the results highlight a fundamental trade-off between the two studied eight-bar steering mechanisms:

- (a) Mixed-leading type (Assembly II, crossed): Recommended for applications prioritizing steering accuracy and mass reduction. This configuration offers significantly

higher Ackermann compliance and utilizes shorter, lighter links (specifically reducing total length by 530.35 mm), resulting in improved space and weight efficiency.

- (b) Leading type (Assembly I, uncrossed): Recommended for applications where force transmission consistency is paramount, as it provides more stable and linear mechanical advantage (MA) characteristics compared to the mixed-leading variant.

Although this study focuses on leading and mixed-leading double four-bar configurations, the findings suggest that optimized eight-bar steering systems are inherently capable of achieving superior Ackermann compliance over extended steering ranges (up to 70°). While multi-link systems possess higher structural complexity than traditional Ackermann linkages, their additional design freedom ensures superior precision. Consequently, these optimized mechanisms are highly advantageous for high-precision applications, such as heavy-duty or low-speed electric vehicles.

To build upon this foundation, future research will explore:

- (1) Algorithm Refinement: Implementing a newly developed hybrid metaheuristic that has already demonstrated superior performance compared with DE-gr and IPSO in preliminary testing.
- (2) Constraint Variation: Investigating the impact of varying track-to-wheelbase ratios and spatial packaging constraints on linkage performance.
- (3) Topology Expansion and Multi-Objective Optimization: Extending the proposed framework to trailing and mixed-trailing configurations, and/or evolving the optimization toward multi-objective functions to simultaneously minimize turning radii and maximize transmission efficiency. This will allow for a more comprehensive evaluation of steering performance across a broader range of vehicle architectures.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/machines14040445/s1>, Video S1: Kinematic simulation of the optimal leading 8-bar mechanism (DE-gr optimized leading 8-bar mechanism.mp4), Video S2: Kinematic simulation of the optimal mixed-leading 8-bar mechanism (IPSO-optimized mixed-leading 8-bar mechanism.mp4).

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