

Article

Hybrid-Mechanism Deep Learning Modeling for Machine Tool Thermal Error: Robust Prediction via Few-Sample Learning

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Abstract

To address spindle thermal error in precision machining, this study proposes a hybrid modeling method. It combines a physical model for linear deformation with a GAT-LSTM network. Experiments show the hybrid model achieved RMSE/MAE of 4.6/4.0 μm under full training (12 conditions), 5.5/4.9 μm under 3 training condition and 4.8/4.3 μm under 1 training condition, substantially reducing the data requirements for thermal error modeling. The compensation experiment conducted using a high real-time surrogate-model-based architecture reduced thermal error by 78% (from 54 μm to 12 μm), demonstrating high precision and minimal data requirements suitable for real-time applications.

Keywords: machine tool thermal error; spindle thermal error; GAT-LSTM; hybrid model

1. Introduction

Spindle thermal error presents a major challenge to machining precision. To mitigate its effects, researchers have developed various modeling and compensation methods. Mechanism-based models—such as those employing thermal resistance networks [1,2] or finite element analysis [3,4]—offer strong physical interpretability and robustness. Nevertheless, their predictive accuracy is often constrained by simplifying boundary conditions, geometric assumptions, and thermal loads. For instance, Ma et al. developed a motorized spindle simulation model that accounts for contact thermal resistance. Although this model significantly improved prediction accuracy, it still exhibits temperature errors on the order of 1–2 °C [5]. Li et al. [6] reviewed various thermal error models and noted that although mechanism-based models are robust and offer strong physical interpretability, they often suffer from limited prediction accuracy due to simplifications in boundary conditions, geometry, and thermal load assumptions.

Conversely, data-driven models have gained popularity due to their high computational efficiency and ability to capture nonlinear relationships [7]. However, the robustness of these data-driven models depends on datasets that cover all operating conditions, thereby requiring large volumes of high-quality data [8,9]. Given the high sensitivity of thermal error to working conditions and the high cost of data collection often requiring hours or days per experiment, data-driven models exhibit poor generalization under data-scarce conditions due to their heavy reliance on extensive training data and the sensitivity of thermal error to operating conditions [10–14]. Currently, researchers have attempted to employ more sophisticated data-driven models for thermal-error prediction under small-sample



Academic Editor: Khalil Khanafer

Received: 13 March 2026

Revised: 1 April 2026

Accepted: 3 April 2026

Published: 6 April 2026

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conditions and have achieved promising results [15,16]. Nevertheless, given the practical difficulty of acquiring sufficient data in real engineering applications, further research in this direction remains necessary.

Researchers have also explored hybrid modeling strategies. For instance, Zheng et al. abstracted the spindle box as a multi-link mechanism, decomposing its behavior into linear and nonlinear portions modeled, respectively, through mechanistic relations and an LSTM network [17]. Nevertheless, such approaches are predominantly limited to single-component analysis and do not adequately account for the influence of other machine components (e.g., column, bed).

To overcome these limitations, this paper presents a hybrid modeling method that integrates a whole-machine mechanistic model with a GAT-LSTM residual network. The core contributions are: (1) an engineering-oriented linear thermal expansion model for the entire machine tool, calibrated via FEA; (2) a GAT-LSTM network that captures spatial dependencies among temperature sensors and temporal dynamics to predict the mechanistic model's residual; and (3) a hybrid framework that maintains high prediction accuracy under few-sample conditions by leveraging the mechanistic model as a robust baseline.

To overcome these limitations, this paper presents a hybrid modeling method integrating physical mechanisms with deep learning. The following sections detail: thermal characteristic analysis and experimental investigation (Section 2), hybrid model development (Section 3), and model evaluation (Section 4). Finally, Section 5 concludes that the hybrid model proposed in this study is capable of accurate prediction and effective compensation for spindle thermal errors in vertical CNC machine tools.

2. Thermal Characteristic Analysis and Experimental Investigation

2.1. Machine Tool Thermal Characteristic

This study focuses on a vertical five-axis machining center (model HMC-200i), which is equipped with an HNC-10 intelligent CNC system (see Figure 1). The key parameters of the machine tool are listed in Table 1. During spindle operation, the main heat sources include the spindle motor, the electrical control cabinet, and the axis-drive motors. Notably, the spindle itself contributes substantial heat input, especially under high-speed conditions. Although active cooling and ambient convection dissipate a portion of the generated heat, a significant amount of thermal energy is conducted into the bed, column, and spindle box, resulting in non-uniform expansion and structural deformation.



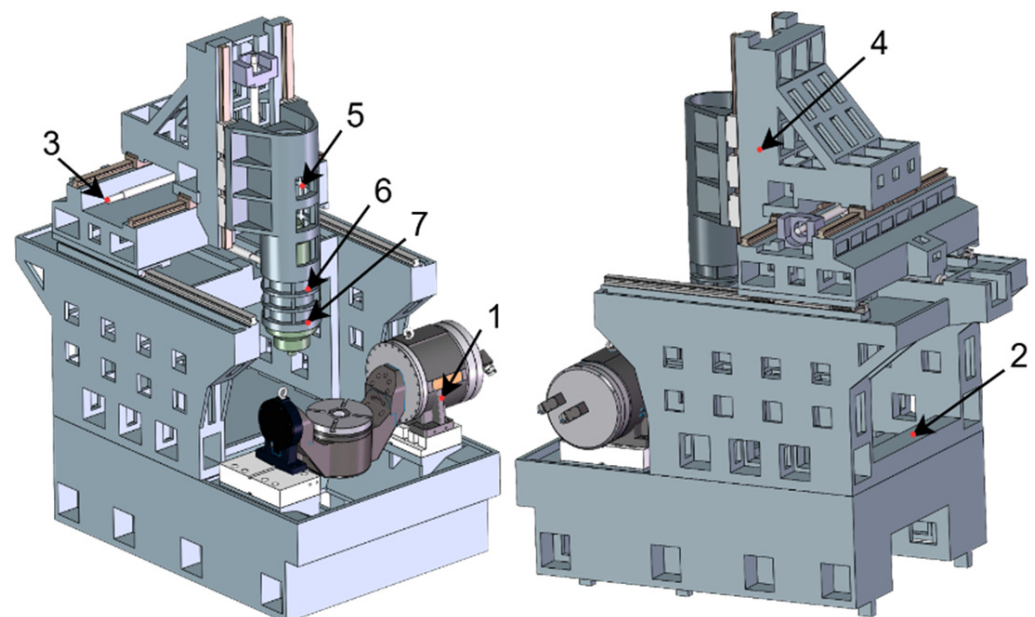
Figure 1. HMC-200i machining center.

Table 1. Key parameters of experimental equipment.

Parameter Type	Value	Parameter Type	Value
Rated Spindle Power	11 kw	Temperature Sensor Accuracy	$\pm 0.1\text{ }^{\circ}\text{C}$
Rated Spindle Speed	10,200 rpm	laser displacement sensor	25–30 mm
Coolant Flow	18 L/min	Measurement Range	
Coolant Temperature	21 $^{\circ}\text{C}$	laser displacement sensor	$\pm 1\text{ }\mu\text{m}$
Temperature Sensor Model	PT1000	Measurement Accuracy	
		Travel of the Sensors used in the Experiment	$\pm 150\text{ }\mu\text{m}$
		Sensor Resolution	0.05 μm

For a uniform temperature field, the relationship between temperature and thermal deformation can be described by a thermal-expansion function. To address the non-uniform temperature field of the machine tool, we decompose it into a combination of multiple uniform temperature fields. This decomposition allows us to approximate the relationship between the machine's temperature field and its thermal deformation using the thermal-expansion function, thereby enabling the construction of a mechanistic prediction model for spindle thermal error.

A temperature sensor layout was designed based on finite element analysis results, as shown in Figure 2. To ensure that the temperature sensor data accurately represents the overall temperature of the current region and minimizes the impact of sensor placement, we will make every effort to ensure that the maximum temperature difference in this region in the FEA simulation does not exceed 1 $^{\circ}\text{C}$. The corresponding simulation methods are elaborated in Section 3.2.

**Figure 2.** Schematic diagram of temperature sensor layout positions.

2.2. Experimental Setup

A custom-developed non-contact measurement system was used to capture the machine's Z-axis thermal error. This system employs a laser displacement sensor to continuously acquire deformation data during spindle rotation. The full experimental configuration is shown in Figure 3.

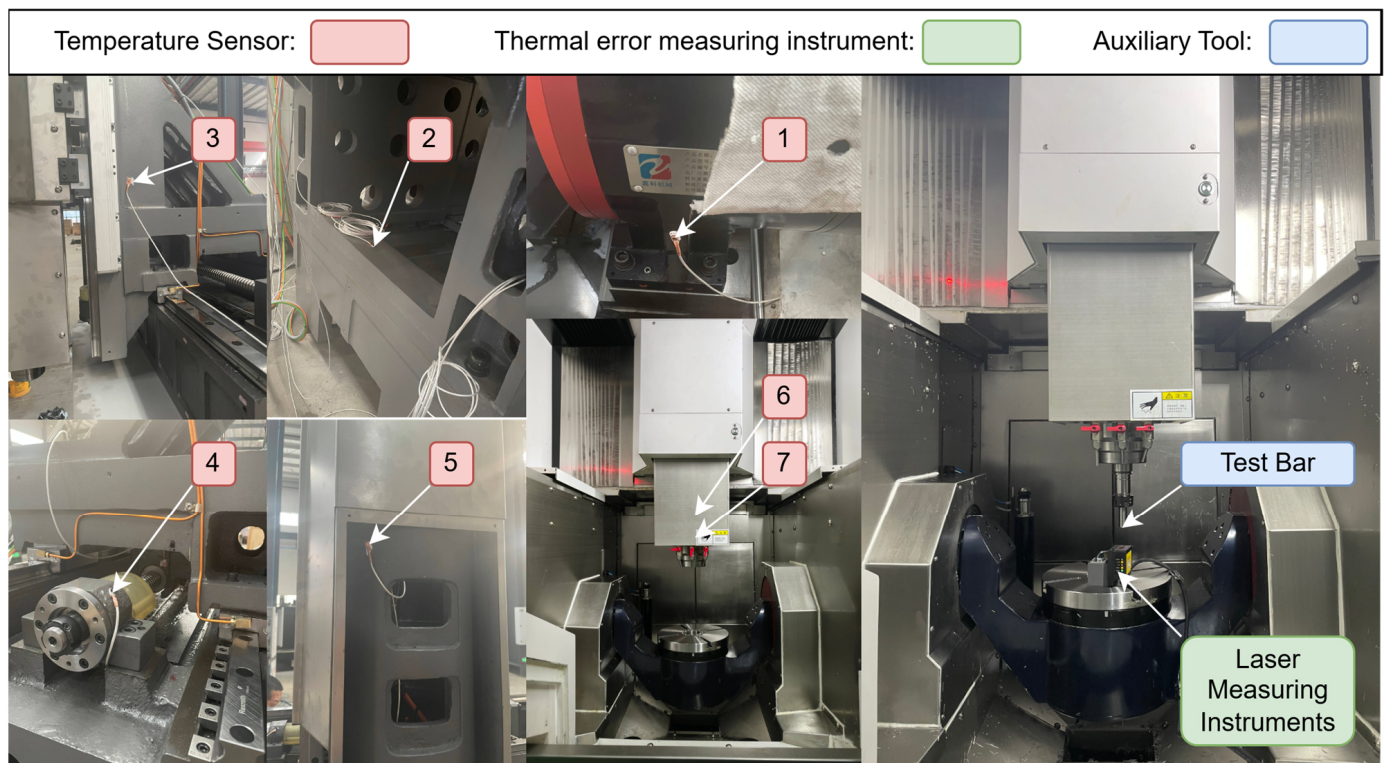


Figure 3. Schematic diagram of experimental setup configuration.

3. Hybrid Model Modeling

3.1. Hybrid Modeling Framework

Figure 4 illustrates the hybrid modeling framework, comprising three main components:

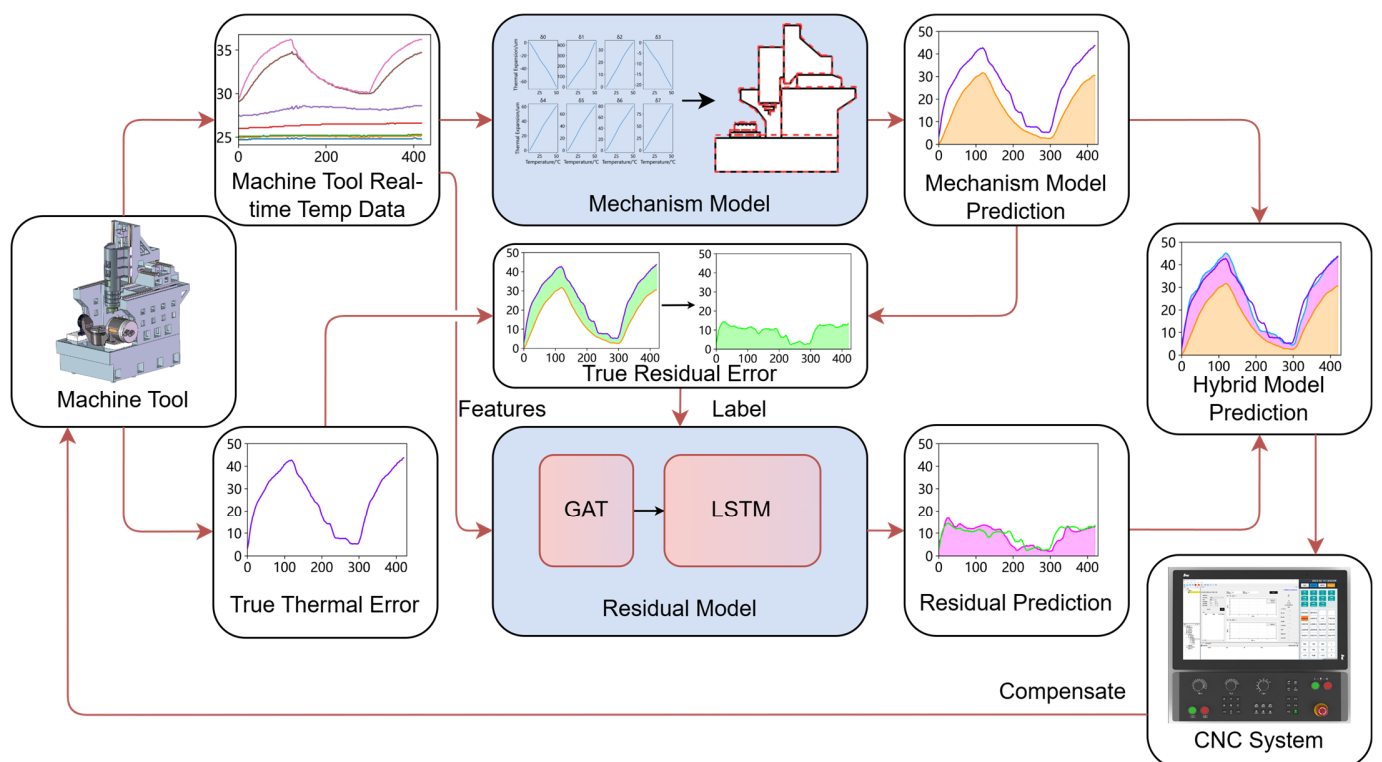


Figure 4. Structure of hybrid modeling method.

Mechanism Model Prediction: Predicts the next time-step thermal error using current temperature data.

Residual Prediction: The GAT-LSTM residual model predicts the mechanism model's next time-step error residual using current temperature data.

Fusion Output: The mechanism model prediction and the residual prediction are additively fused to yield the final hybrid thermal error prediction.

The introduction of a data-driven residual model may reduce the robustness provided by the mechanistic model. However, as the residual model can only correct a small fraction of the overall error, as shown in Figure 4, the impact on the model's overall robustness is minimal.

3.2. Mechanism Model Development

Z-direction thermal errors primarily stem from uneven deformation of key structures, particularly the spindle box. The modeling process is shown in Figure 5, and modeling involves:

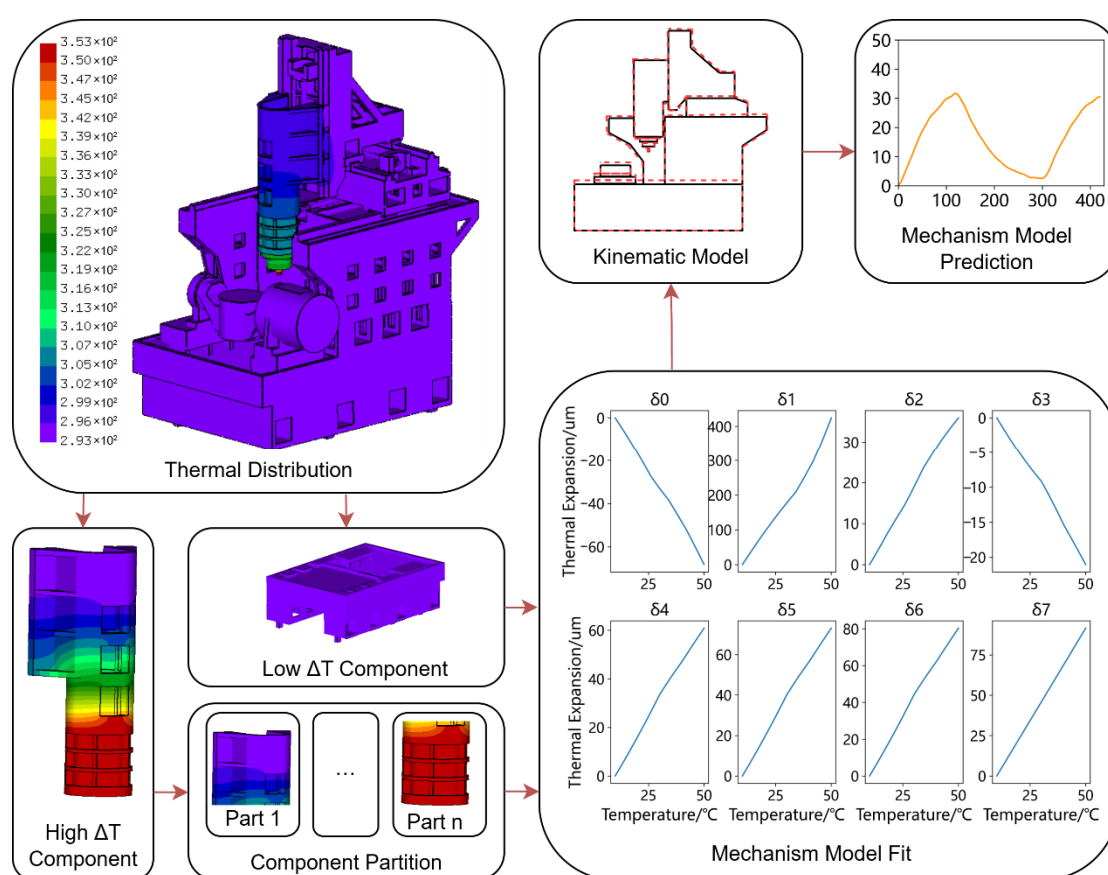


Figure 5. Flowchart of thermal expansion mechanism model simplification.

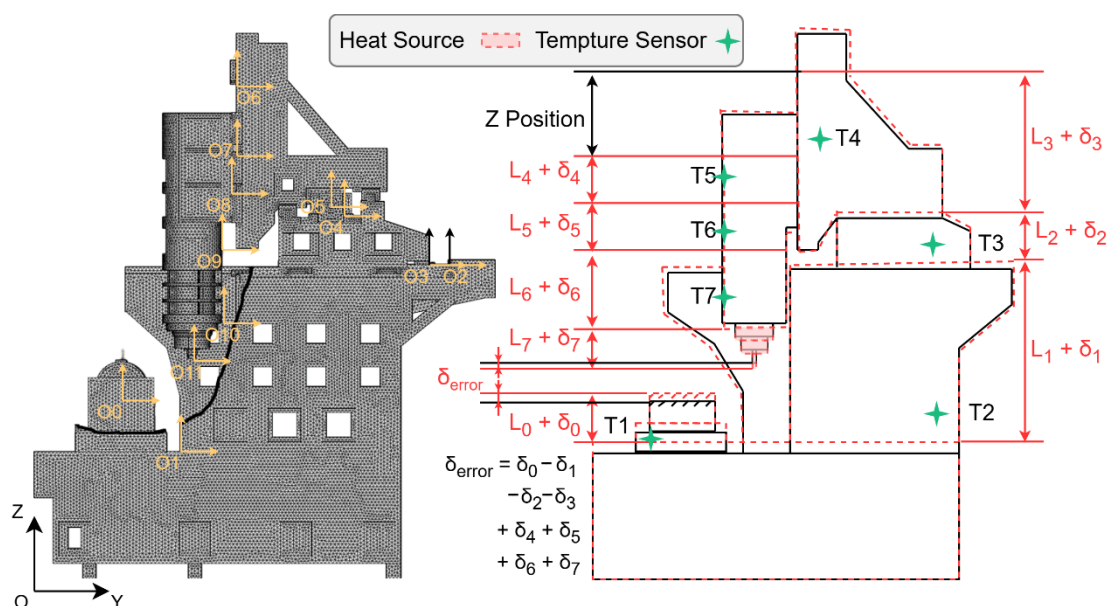
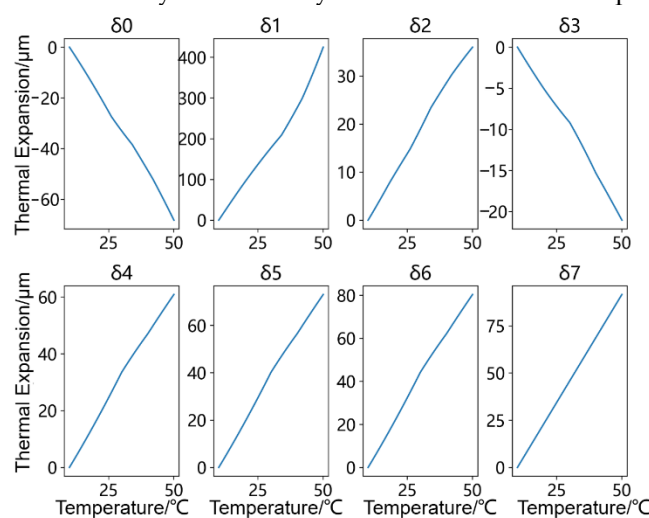
Component-Level Thermal Deformation: A transient thermal analysis was performed using CalculiX [18]. The material parameters employed in the simulation are listed in Table 2, and the boundary conditions applied to the finite element model are summarized in Table 3. The simulation results after 8 h of spindle operation at 10,000 rpm are shown in Figure 6b.

Table 2. Material parameters.

Component Type	Density (kg/mm ³)	Thermal Conductivity (W/(mm·K))	Specific Heat Capacity (J/(kg·K))
Casting	7.3×10^{-6}	1.17×10^{-2}	500
Spindle	7.98×10^{-6}	1.4×10^{-2}	500

Table 3. CalculiX setting.

Parameter Type	Value
Natural convection heat transfer coefficient	$h = 5\text{--}15 \text{ W}/(\text{m}^2 \cdot \text{K})$, $T_\infty = 20 \text{ }^\circ\text{C}$
Ambient temperature	$20 \text{ }^\circ\text{C}$
Emissivity	0.3
Temperature boundary	$0\text{--}50 \text{ }^\circ\text{C}$

**(a)** Coordinate systems for key machine structural components**(b)** Simulation results**Figure 6.** Equivalent thermal expansion coefficient setting and result.

Kinematic Chain Modeling: Captures deformation propagation through the machine structure.

Based on the simulation results (Figure 6b), components are categorized by their thermal gradient. From the simulation results presented in Figure 6b, it can be concluded that the thermal expansion of most components can be approximated as linear thermal expansion. While also considering computational efficiency, this study adopts the linear thermal expansion model as the mechanistic component within the hybrid model framework. High-gradient parts are partitioned into quasi-isothermal zones, enabling low-order thermal expansion modeling (Equation (1)), where the coefficient k is calculated

based on the thermal expansion coefficient and the actual equivalent length, as shown in Equation (2). Component-specific coordinate systems were defined (Figure 6a). Using material parameters from Table 1 and initial ambient temperature of 20 °C, Z-direction thermal expansion simulations were conducted under specified temperature rises.

Finite element simulation results for Z-direction thermal expansion under isothermal conditions (10–50 °C temperature rise range) are shown in Figure 6b. Equivalent thermal expansion coefficients K (listed in Table 4) were derived through a least squares regression of the FEA results. Since the coordinate systems of the components in the FEA process were not defined according to the machine tool coordinate system, the fitting results only consider their absolute values. In the linear thermal expansion model, the sign of the coefficients depends on the relationship between the component's thermal expansion direction and the spindle thermal error. Incorporating kinematic relationships, component deformation models were combined to establish the mechanism model for overall Z-direction thermal error, expressed as Equation (3):

$$\delta = K \cdot \Delta T \quad (1)$$

$$K = \alpha \cdot L \quad (2)$$

$$\begin{aligned} \delta_{err} = & -0.00254\Delta T_1 - 0.00854\Delta T_2 \\ & -0.00265\Delta T_3 - 0.00581\Delta T_4 + 0.0037\Delta T_5 \\ & + 0.00281\Delta T_6 + 0.00321\Delta T_7 + 0.00165\Delta T_8 \end{aligned} \quad (3)$$

where

δ —Z-direction thermal expansion,

α —thermal expansion coefficient,

ΔT —temperature rise,

δ_{err} —total Z-direction thermal expansion.

Table 4. Fitted expansion coefficients for machine components.

Parameter Type	Value	Parameter Type	Value
K0	2.54×10^{-3}	K4	3.7×10^{-3}
K1	8.54×10^{-3}	K5	2.81×10^{-3}
K2	2.65×10^{-3}	K6	3.21×10^{-3}
K3	5.81×10^{-3}	K7	1.65×10^{-3}

We performed a sensitivity analysis on the calculated equivalent thermal expansion coefficients (K), with the results shown in Figure 7. Specifically, Figure 7a presents the total-order sensitivity indices of each coefficient on the predicted thermal error when the temperature at every measurement location rises by 0.1 °C. Figure 7b shows the weighted total-order sensitivity indices, which incorporate the actual temperature variation ranges recorded during experiments. The weighting is calculated using Equations (4) and (5).

As observed in Figure 7, the coefficient associated with the column has the strongest influence when temperature variation ranges are not considered. Once the actual experimental temperature ranges are accounted for, however, the coefficients corresponding to the spindle and the end of the spindle box exhibit the most pronounced impact.

$$\Delta T_i = \max(\Delta T_i(t)) - \min(\Delta T_i(t)) \quad (4)$$

$$SQ_i = \frac{S_i \cdot \frac{\Delta T_i}{\max(\Delta T_1, \Delta T_2, \dots, \Delta T_7)}}{\sum_{j=1}^n (S_j \cdot \frac{\Delta T_j}{\max(\Delta T_1, \Delta T_2, \dots, \Delta T_7)})} \times 100\% \quad (5)$$

where

t —time,
 T_i —temperature rise range of the i -th sensor,
 ΔT_i —peak-to-peak temperature rise of the i -th sensor,
 SQ_i —weighted total-order sensitivity index of the i -th parameter K_i ,
 S_i —total-order sensitivity index of the i -th parameter K_i ,
 n —total number of parameters.

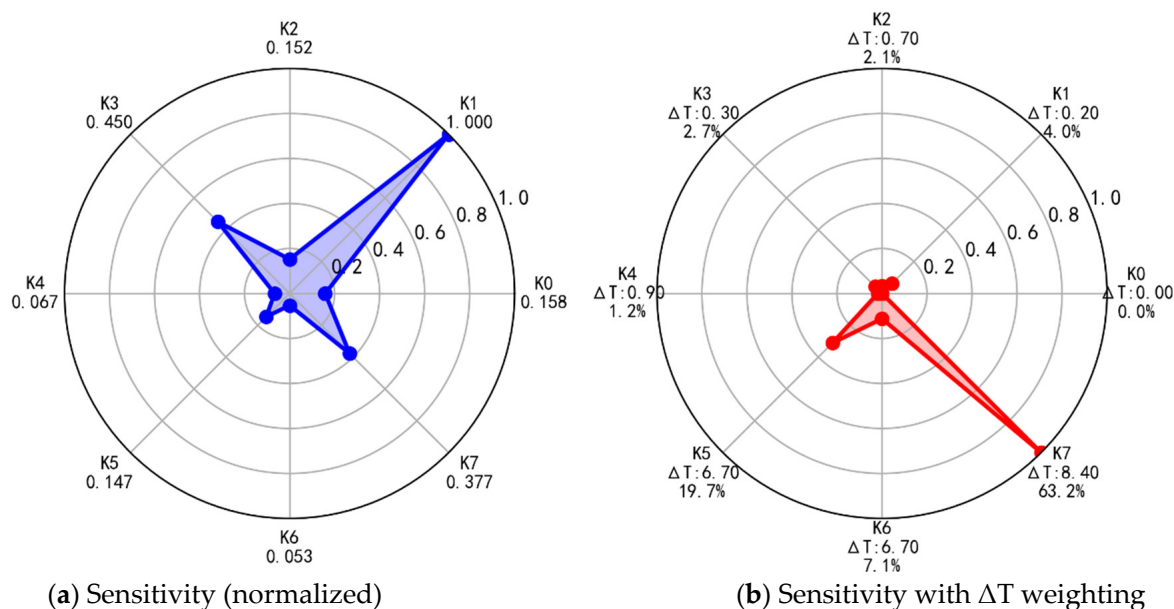


Figure 7. Sensitivity result.

3.3. Residual Model Development and Hybrid Fusion

To address dynamic effects unmodeled by the mechanism model, the residual model employs a GAT-LSTM network [19]. The model structure is shown in Figure 8. As shown in Figure 9, due to the simplification of the complex thermal expansion process in the linear thermal expansion model used in this study, direct predictions using this model result in a residual error of approximately 20 μm . The model combines Graph Attention Networks (GATs) for spatial correlation and LSTMs for capturing temporal patterns.

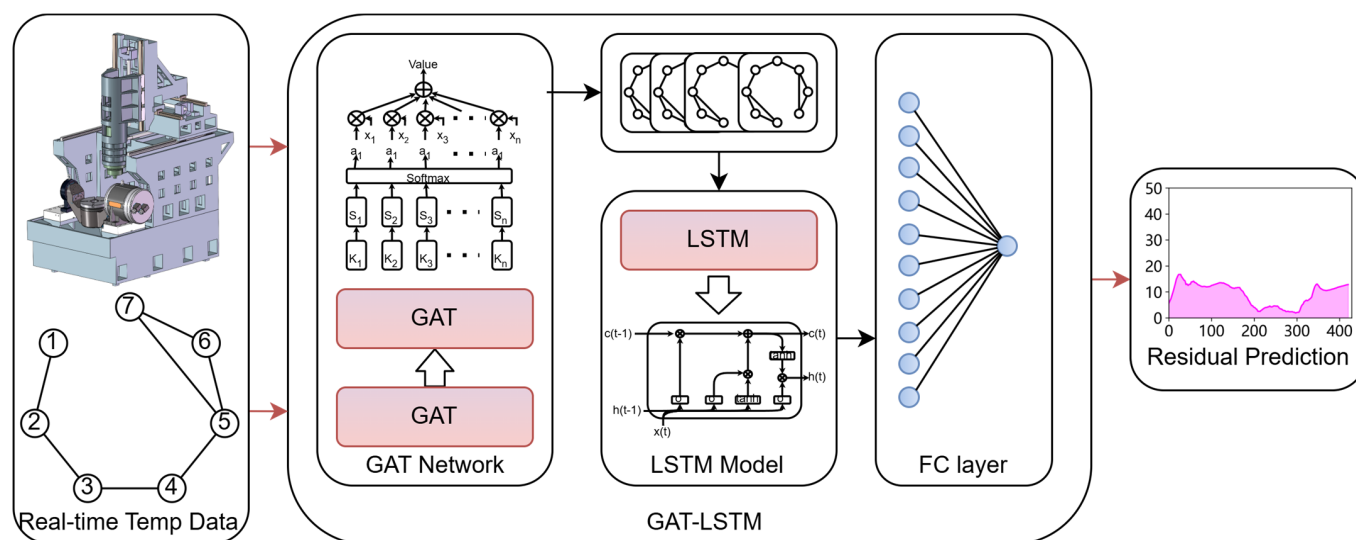


Figure 8. GAT-LSTM residual model structure.

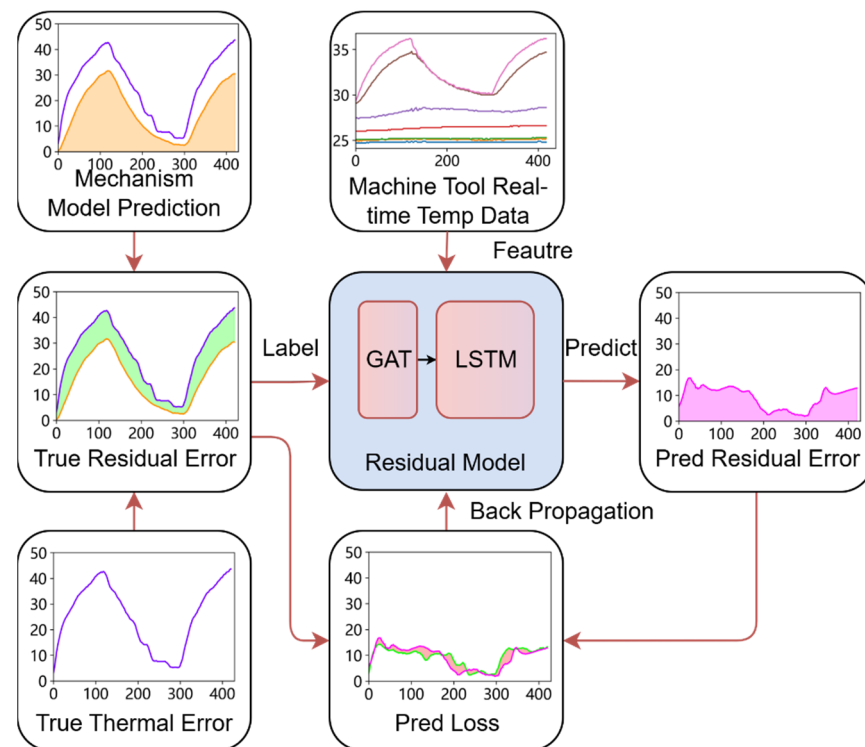


Figure 9. Residual model training process.

The sensor layout defines the graph structure (Figure 8), where nodes correspond to temperature sensors. GATs extract spatial features from sensor readings at each time step; LSTMs process these to capture temporal dependencies, outputting the predicted residual. Figure 9 depicts the training process. The hyperparameters used for training the model in this study are listed in Table 5. The final hybrid prediction is obtained by adding the residual prediction to the mechanism model output, defined as:

$$\delta_Z(t+1) = \delta_{Z_mech}(t+1) + r(t+1) \quad (6)$$

where

- $\delta_Z(t+1)$ —the fused thermal error prediction,
- $\delta_{Z_mech}(t+1)$ —the mechanism model prediction,
- $r(t+1)$ —the residual prediction.

Table 5. Hyperparameter settings of the residual model.

Parameter Type	Value	Parameter Type	Value
Attention layer hidden dimension	16	optimizer	Adam
Number of attention heads	4	learning rate	0.0015
Number of nodes	7	activation function	ReLU
LSTM hidden dimension	64	edge weighting strategy	Uniform Edge Weighting

4. Model Training and Experimental Validation

4.1. Model Training and Verification Method

Various spindle operating conditions were designed to simulate realistic thermal loading, including constant- and variable-speed modes. In practical machining cycles, machine tools often undergo re-tooling after extended idle or cooling periods. As a result, the dominant thermal errors stem from extended heating periods during each cycle. Each condition is divided into primary and secondary heating stages (Figure 10). Corresponding spindle speed

histories are shown in Figures 11 and 12. The data for each operating condition was sampled every 2.5 min, with a single condition lasting approximately two hours.

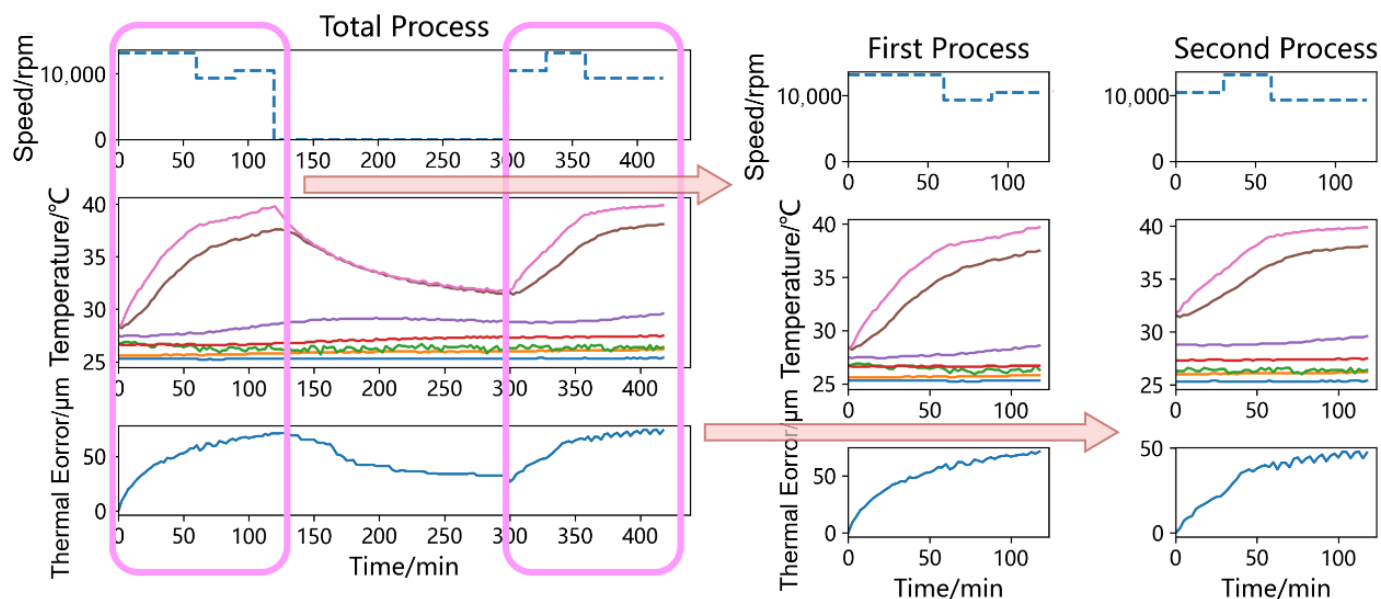


Figure 10. Process divided.

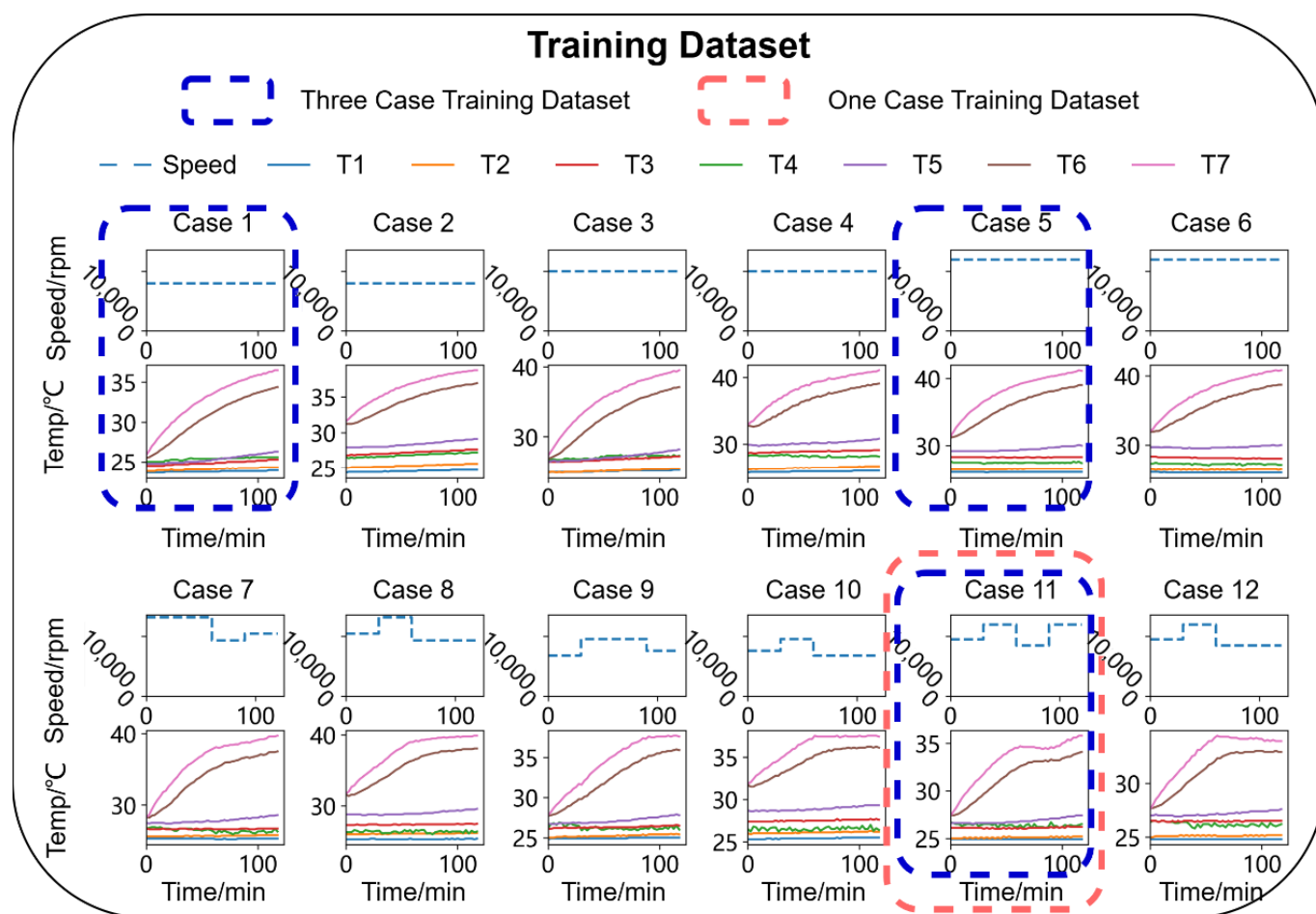


Figure 11. Spindle speed curves for training set.

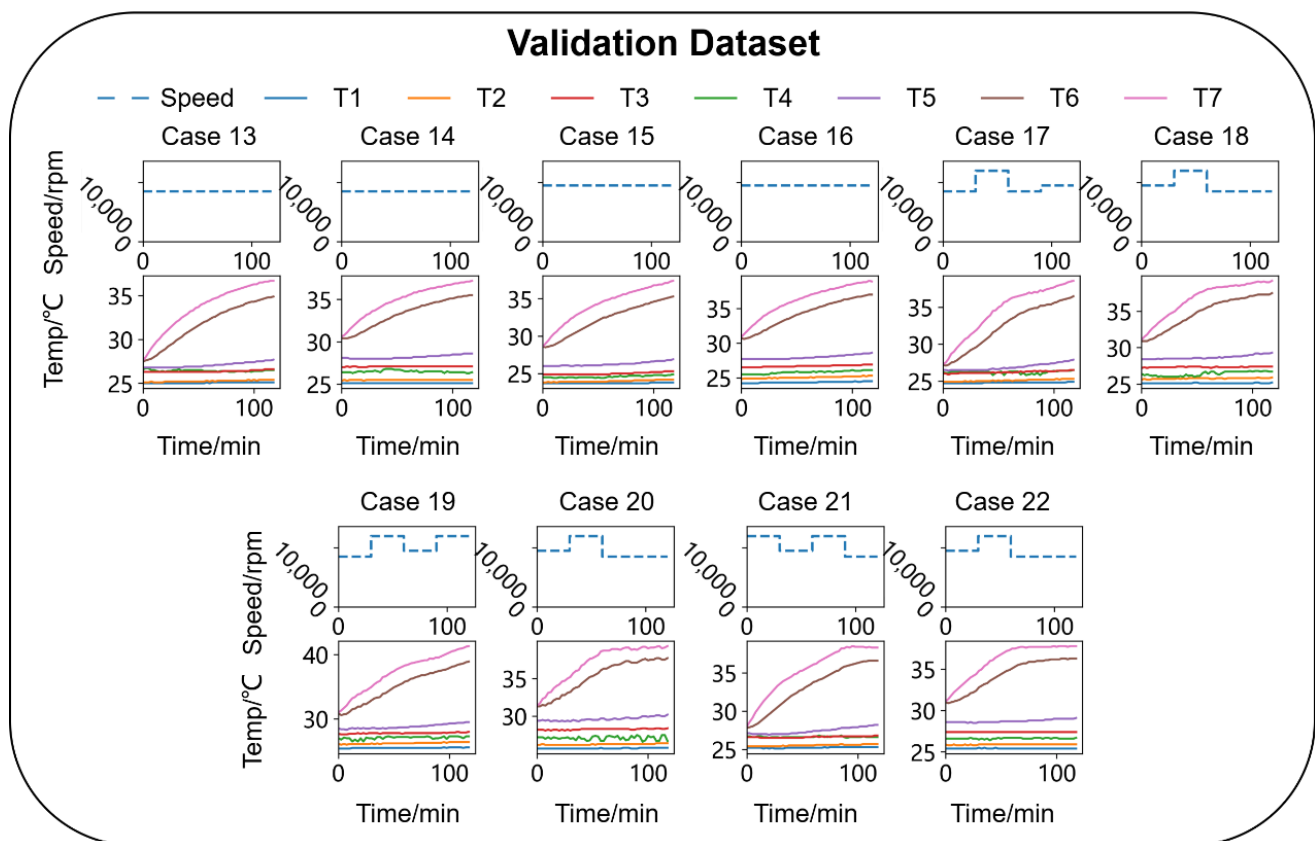


Figure 12. Spindle speed curves for validation set.

A complete training set comprising 12 operating conditions and a separate validation set containing 10 distinct conditions were constructed, ensuring no overlap between the two datasets. Additionally, a minimal training set of three samples was created by randomly selecting two constant-speed conditions and one variable-speed condition. To encompass a broader range of rotational speeds and to contrast with the results from the three-sample training set, the variable-speed condition (Case 11) included in the three-sample set was also adopted as the single-condition training set.

To evaluate the prediction performance of the proposed mechanism-data fusion model, the following models were selected as comparative baselines:

- (1) GAT-LSTM: Shares the residual model's architecture and training (Figures 8 and 9) but operates independently and solely data-driven without the mechanism model. All models shared datasets and hyperparameters; the model with the lowest validation loss after 1000 epochs was selected.
- (2) LSTM: This model shares an identical architecture to the LSTM used in the residual model. All models shared datasets and hyperparameters; the model with the lowest validation loss after 1000 epochs was selected.
- (3) Hybrid-GP: This variant uses the same hybrid framework as the proposed model but replaces the GAT-LSTM component with a Gaussian process (GP) model. The GP model includes a constant mean function, an RBF kernel with learnable lengthscale and output scale, and a Gaussian likelihood with learnable noise variance.
- (4) BPNN: A BPNN model constructed with four hidden layers, each containing 64 neurons.
- (4) Multiple Linear Regression (MLR): Established thermal error as a linear function of temperature increase.

$$\delta_{MLR} = 0.07995\Delta T_1 - 0.00389\Delta T_2 - 0.02363\Delta T_3 + 0.00438\Delta T_4 + 0.0372 \quad (7)$$

where

ΔT —temperature increase,

δ_{MLR} —total Z-direction thermal expansion.

All models shared datasets; the model with the lowest validation loss after 1000 epochs was selected.

A quantitative comparison of training time, inference latency, and deployment constraints for each model is shown in Table 6.

Table 6. Comparison of training time, latency, and deployment constraints.

Model	Training Time (s/1000 Epochs)	Latency (ms/Sample)	Memory Footprint (MB)	Deployment Feasibility
Hyb-GAT	320	2.8	18	Medium
GAT-LSTM	290	2.5	16	Medium
LSTM	180	2.1	12	High
Hyb-GP	950	15.6	45	Low
BPNN	140	1.7	10	High
MLR	5	0.3	<1	Very High

4.2. Full-Sample Training Performance

The performance of each model on the full training set is shown in Figures 13–16. Specifically, Figures 13 and 15 present the prediction results of each model under the corresponding operating conditions. Due to the large number of conditions considered in this study, detailed prediction results for each condition are provided in Appendix A. Figures 14 and 16, respectively, summarize the performance of each model across all conditions in the training set and the validation set. Table 7 summarizes the overall performance in terms of RMSE, MAE, and P–V.

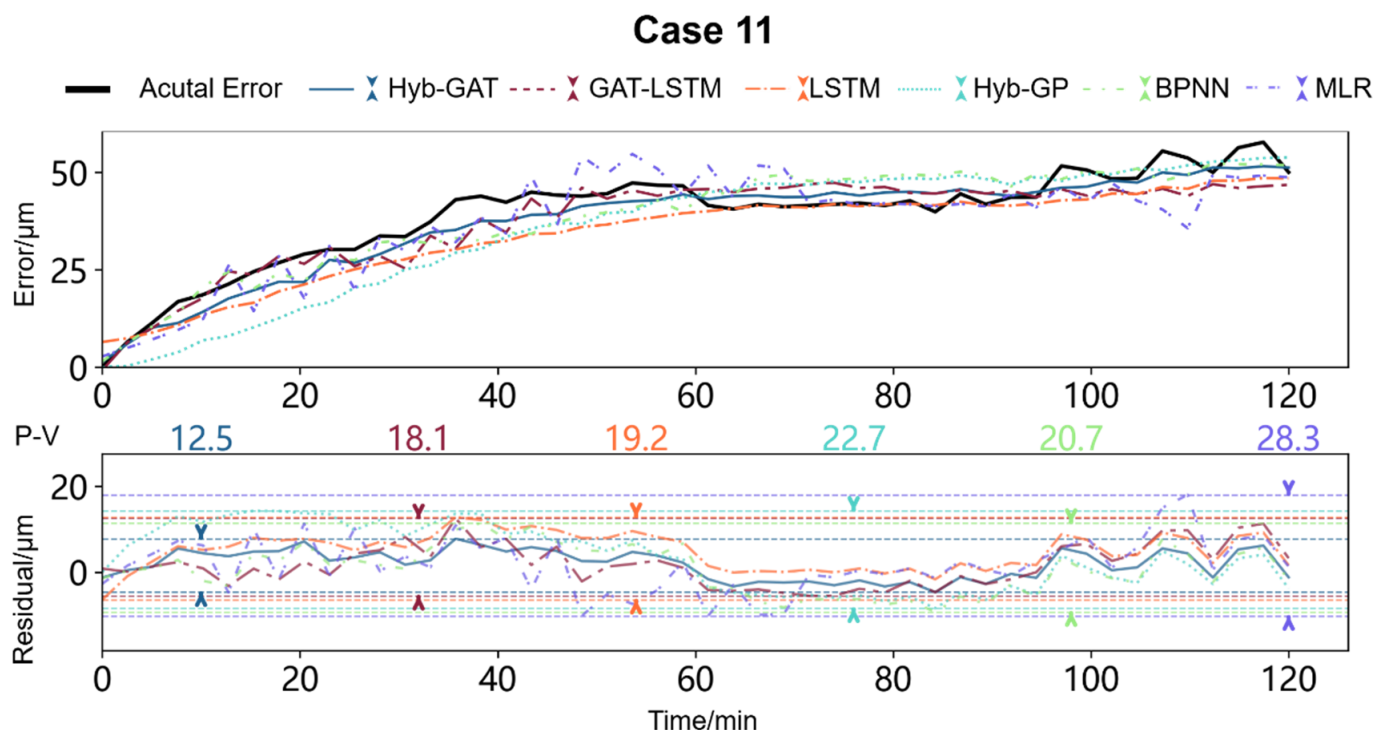


Figure 13. Fitting and residuals on the training set.

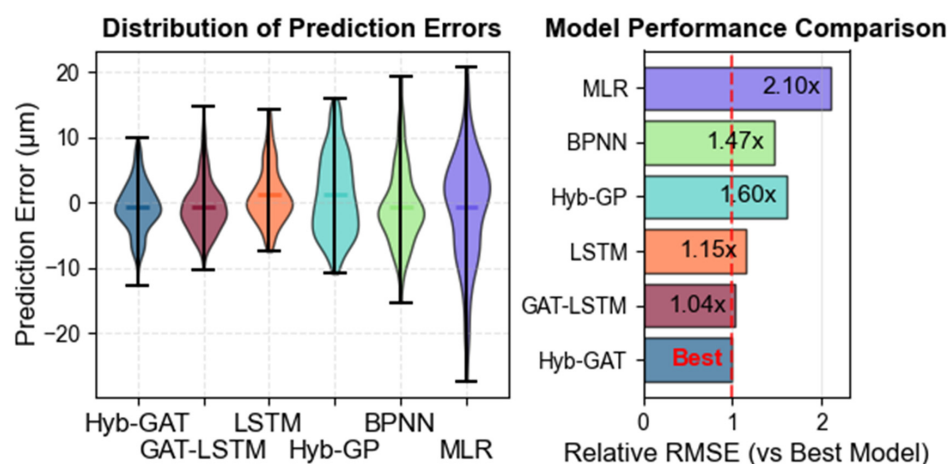


Figure 14. Statistical analysis results on training set.

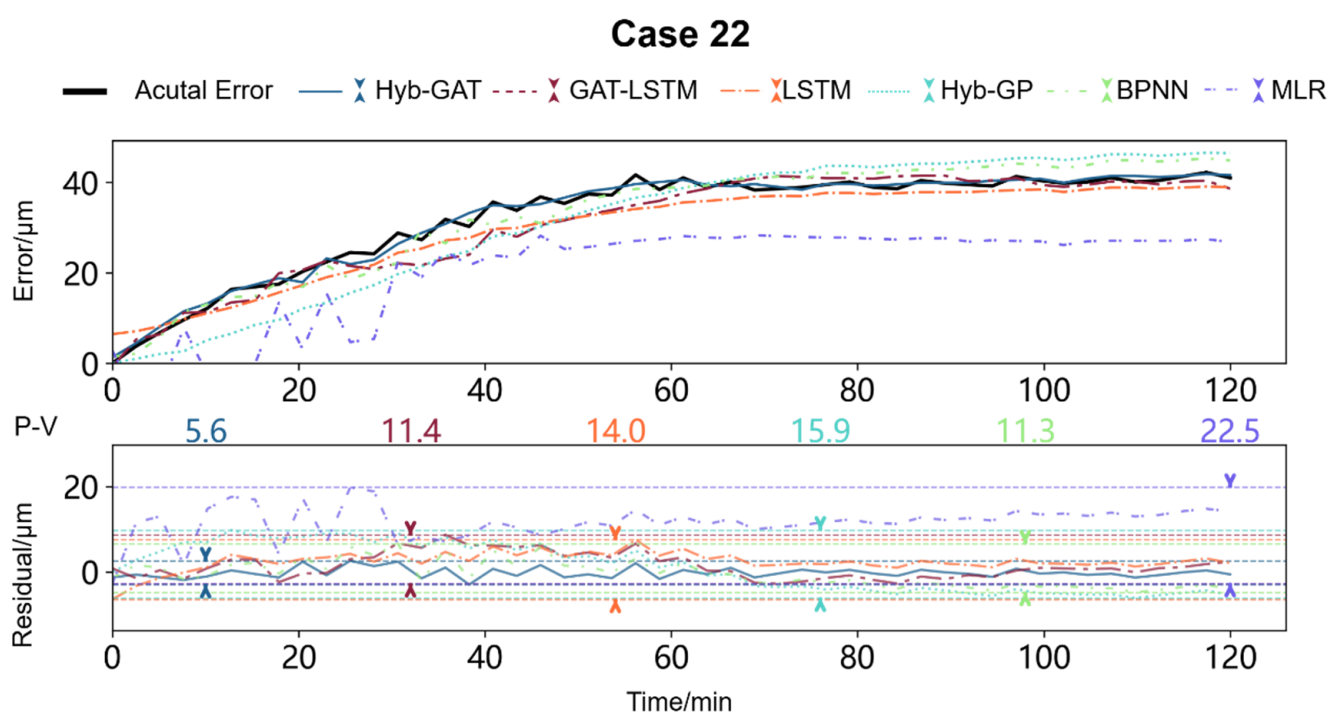


Figure 15. Fitting and residuals on the validation set.

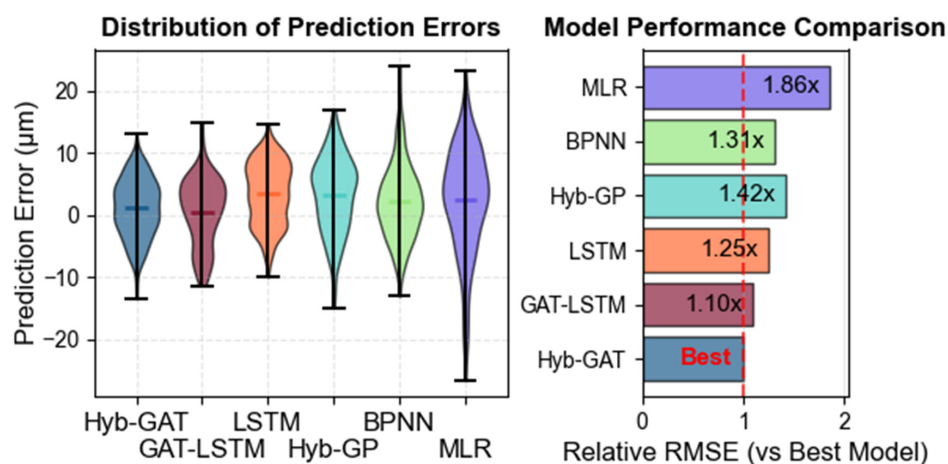


Figure 16. Statistical analysis results on validation set.

Table 7. Comparison of RMSE, MAE and P-V on the training set and validation set.

Dataset	Model	Average RMSE (μm)	Standard Deviation of RMSE (μm)	Average MAE (μm)	Average P-V (μm)
Training Set	Hyb-GAT	3.5	1.9	3.0	9.9
	GAT-LSTM	3.8	1.3	3.2	11.6
	LSTM	3.9	2.4	3.4	12.7
	Hyb-GP	5.9	2.0	5.1	16.4
	BPNN	5.2	2.6	4.5	14.7
	MLR	7.8	2.7	6.4	26.8
Validation Set	Hyb-GAT	4.6	1.7	4.0	10.3
	GAT-LSTM	5.1	1.5	4.5	12.7
	LSTM	5.5	2.6	5.1	15.0
	Hyb-GP	6.7	1.8	5.7	15.8
	BPNN	5.8	2.7	4.9	15.6
	MLR	8.4	3.5	7.3	24.2

On both the training and validation sets, the proposed hybrid model, Hyb-GAT, consistently demonstrates the best overall predictive performance. When trained on the full dataset (12 operating conditions), Hyb-GAT achieves an average RMSE of 4.6 μm and an MAE of 4.0 μm on the validation set, outperforming all baseline models. Relative to GAT-LSTM, the RMSE is reduced by 9.8%; compared with LSTM and BPNN, the improvements reach 16.4% and 20.7%, respectively. The Hyb-GP model, which combines the physical model with Gaussian process regression, achieves an RMSE of 6.7 μm on the validation set—exceeding MLR (8.4 μm) yet still falling short of Hyb-GAT and the purely data-driven GAT-LSTM (5.1 μm). This suggests that while incorporating a probabilistic residual model provides some benefit, it does not capture nonlinear dynamics as effectively as the attention-based GAT-LSTM architecture employed in Hyb-GAT. The MLR model exhibits the poorest performance, underscoring the limitations of purely linear approaches in modeling complex thermal behavior.

Statistical analysis confirms that the Hyb-GAT model achieves superior predictive accuracy relative to all baseline models. Paired *t*-tests reveal statistically significant reductions in prediction error: compared to GAT, Hyb-GAT reduces mean error by 0.5746 μm ($t = 4.0266$, $p = 6.58 \times 10^{-5}$); relative to LSTM, by 2.3710 μm ($t = -23.5254$, $p = 6.54 \times 10^{-82}$); and versus Hyb-GP, by 2.2229 μm ($t = -10.5415$, $p = 1.66 \times 10^{-23}$). Although Cohen's *d* effect sizes are modest (0.1144, -0.4894 , and -0.4102 , respectively), the consistency, magnitude, and extreme statistical significance (all $p < 10^{-23}$) of these improvements—coupled with directional uniformity (i.e., error reduction across all comparisons)—strongly support the model's reliability and practical advantage. Furthermore, Hyb-GAT maintains high accuracy across diverse operating conditions, demonstrating both exceptional point-wise precision and robust generalization capability—key indicators of methodological soundness and real-world applicability.

4.3. Few-Sample Training Performance

To evaluate robustness with limited data, models were trained on reduced sets: three-case (case 1, case 5, case 11) and single-case (case 11). Figure 17 illustrates the prediction performance of each model on the validation operating conditions under different numbers of training samples. Detailed performance for each specific condition can be found in Appendix B. Figure 18 presents a statistical summary of each model's performance on the validation set. Table 8 summarizes the overall performance in terms of RMSE, MAE, and P-V. Herein, the suffix of each model name indicates the number of training conditions.

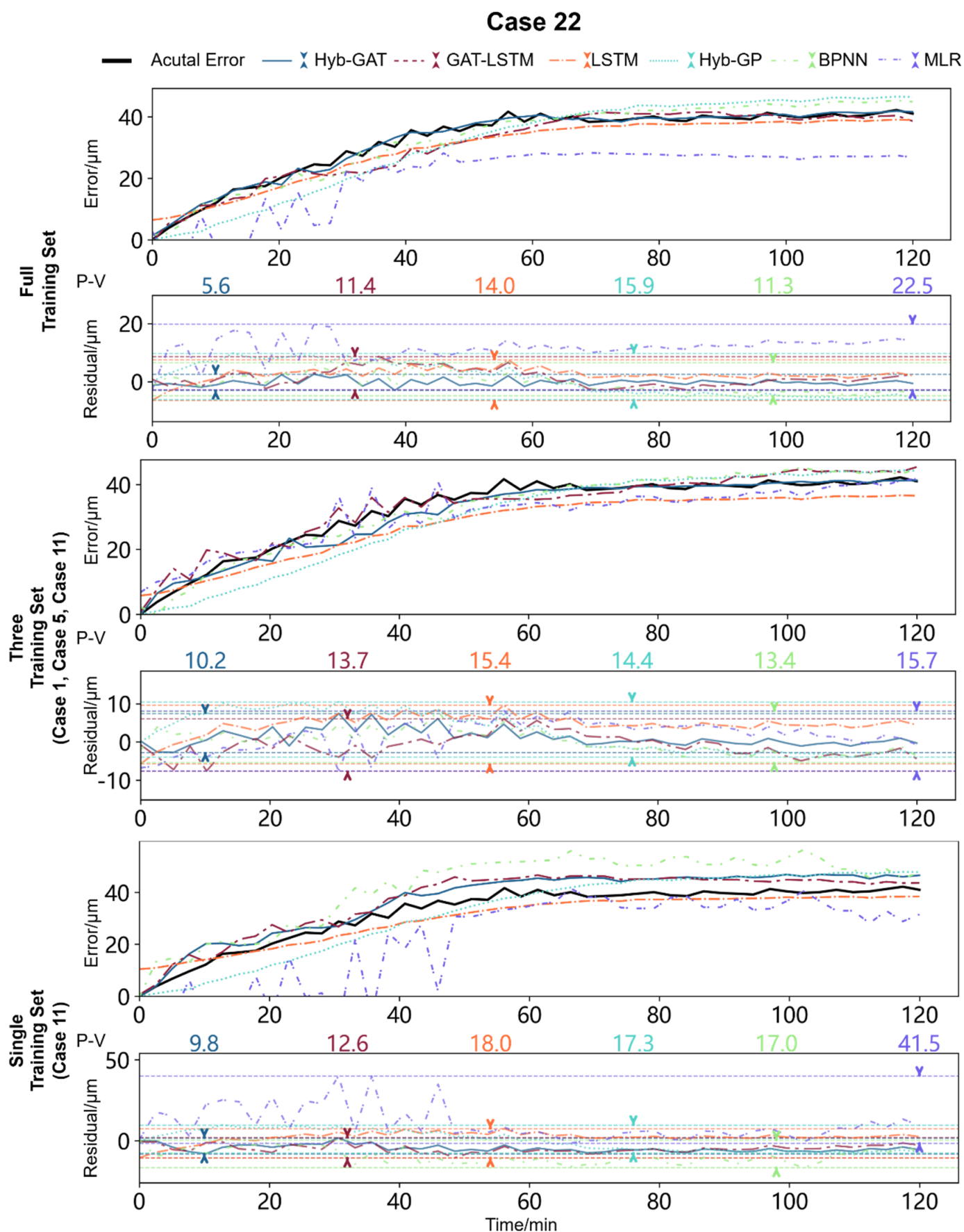


Figure 17. Fitting and residuals on three different sets.

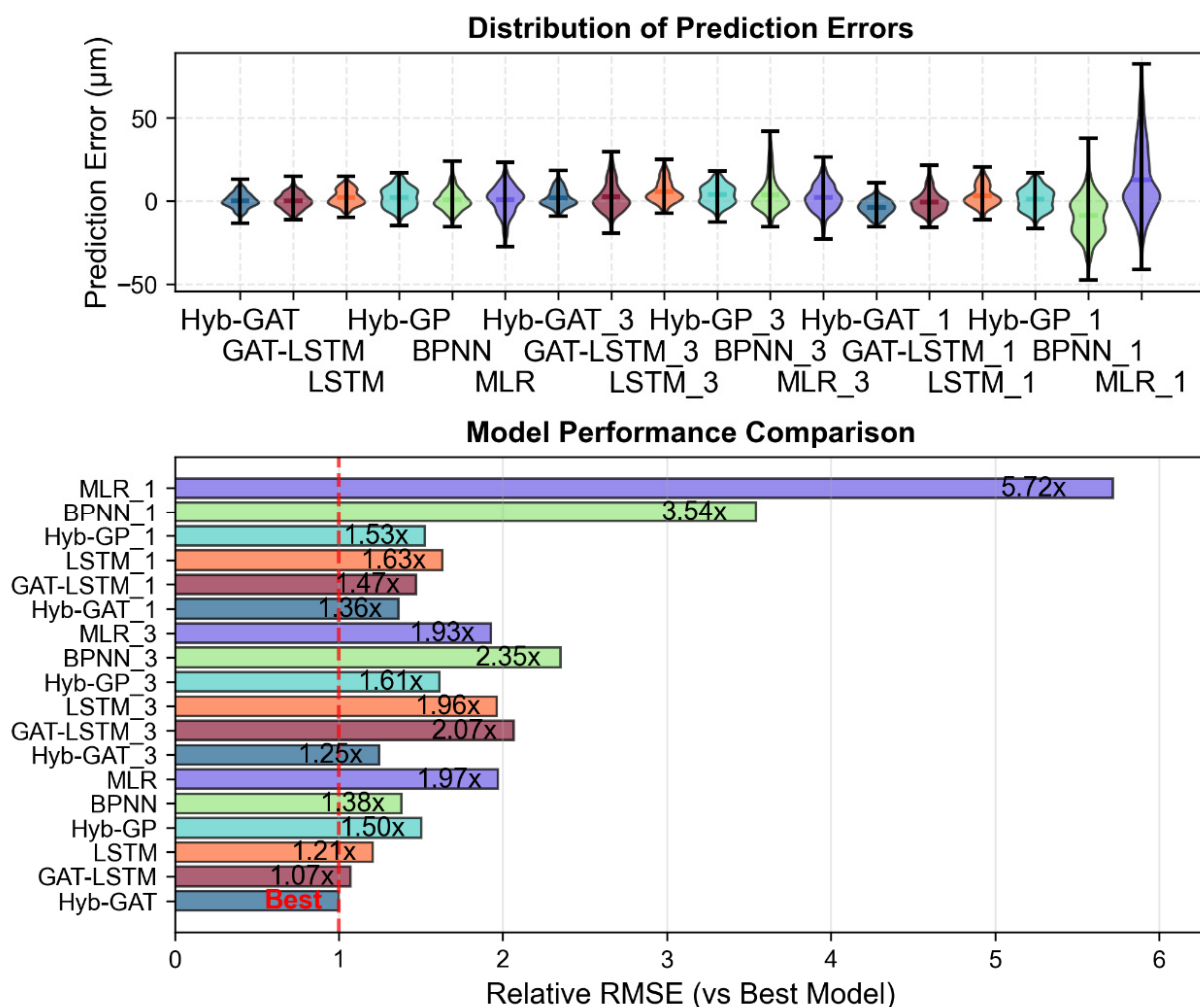


Figure 18. Statistical analysis results on three different sets.

Table 8. RMSE, MAE and P-V on three different sets.

Training Set		Total	Three	Single
Average RMSE (μm)	Hyb-GAT	4.6	5.5	4.8
	GAT-LSTM	5.1	8.4	6.3
	LSTM	5.5	8.5	7.1
	Hyb-GP	6.7	7.3	6.7
	BPNN	5.8	8.8	13.1
	MLR	8.4	9.3	24.3
Standard deviation of RMSE (μm)	Hyb-GAT	1.7	2.9	2.7
	GAT-LSTM	1.5	4.9	1.8
	LSTM	2.6	5.3	3.9
	Hyb-GP	1.8	2.5	1.4
	BPNN	2.7	6.1	6.7
	MLR	3.6	3.2	12.1
Average MAE (μm)	Hyb-GAT	4.0	4.9	4.3
	GAT-LSTM	4.5	7.3	5.1
	LSTM	5.1	7.8	6.4
	Hyb-GP	5.7	6.4	5.6
	BPNN	4.9	7.5	11.3
	MLR	7.3	7.8	19.8
Average P-V (μm)	Hyb-GAT	10.3	12.1	11.5
	GAT-LSTM	12.7	19.4	17.0
	LSTM	15.0	19.8	21.2
	Hyb-GP	15.8	15.2	16.8
	BPNN	15.6	19.9	30.1
	MLR	24.2	27.2	68.5

As expected, reducing the training data led to performance degradation across all models. Nevertheless, Hyb-GAT exhibited the smallest decline and consistently achieved the highest accuracy in both limited-data scenarios.

Under three-case training, Hyb-GAT attained an average RMSE of $5.5\ \mu\text{m}$, substantially lower than GAT-LSTM ($8.4\ \mu\text{m}$), LSTM ($8.5\ \mu\text{m}$), BPNN ($8.8\ \mu\text{m}$), and MLR ($9.3\ \mu\text{m}$). Hyb-GP recorded an RMSE of $7.3\ \mu\text{m}$ —outperforming the purely data-driven models yet still trailing Hyb-GAT by approximately 24.7%. This indicates that while Hyb-GP benefits from the physical prior, its residual modeling capacity remains constrained under scarce data.

In the single-case training scenario, Hyb-GAT maintained an RMSE of $4.8\ \mu\text{m}$, whereas Hyb-GP increased to $6.7\ \mu\text{m}$, and BPNN and MLR deteriorated sharply to $13.1\ \mu\text{m}$ and $24.3\ \mu\text{m}$, respectively. The relative improvement of Hyb-GAT over Hyb-GP reached 28.4%, and over BPNN 63.4%, underscoring its exceptional few-shot learning capability.

Statistical tests confirm the significance of these advantages. In the three-case setting, Hyb-GAT significantly outperformed all baseline models, including Hyb-GP ($p < 0.001$). In the single-case setting, the effect sizes were large (Cohen's $d = 0.97$ vs. BPNN, and -0.91 vs. MLR), further validating the robustness and generalizability of the proposed hybrid approach.

These results demonstrate that Hyb-GAT not only excels in data-rich environments but also maintains high predictive accuracy under extreme data scarcity, rendering it well-suited for real-world applications where extensive thermal data collection is impractical.

The above results directly address the core research problem: achieving accurate thermal error prediction with limited data. When training is reduced to a single condition, purely data-driven models (GAT-LSTM, BPNN) deteriorate sharply (RMSE $6.3\ \mu\text{m}$ and $13.1\ \mu\text{m}$), whereas Hyb-GAT maintains high accuracy (RMSE $4.8\ \mu\text{m}$). This confirms that the mechanistic baseline provides a strong inductive bias, preventing overfitting and enabling robust few-shot learning.

4.4. Model Compensation Architecture

The compensation architecture employed in this study is based on a surrogate model structure [20], which has been patented (CN118192419A), as illustrated in Figure 19. This architecture operates by deploying a high-order model on a high-performance computing device at the edge. This high-order model is used to fit a low-order model. Subsequently, the low-order model—which maintains the predictive accuracy of the high-order model within a defined operational interval—is embedded within the CNC system. This design enables real-time thermal error compensation during the machining process.

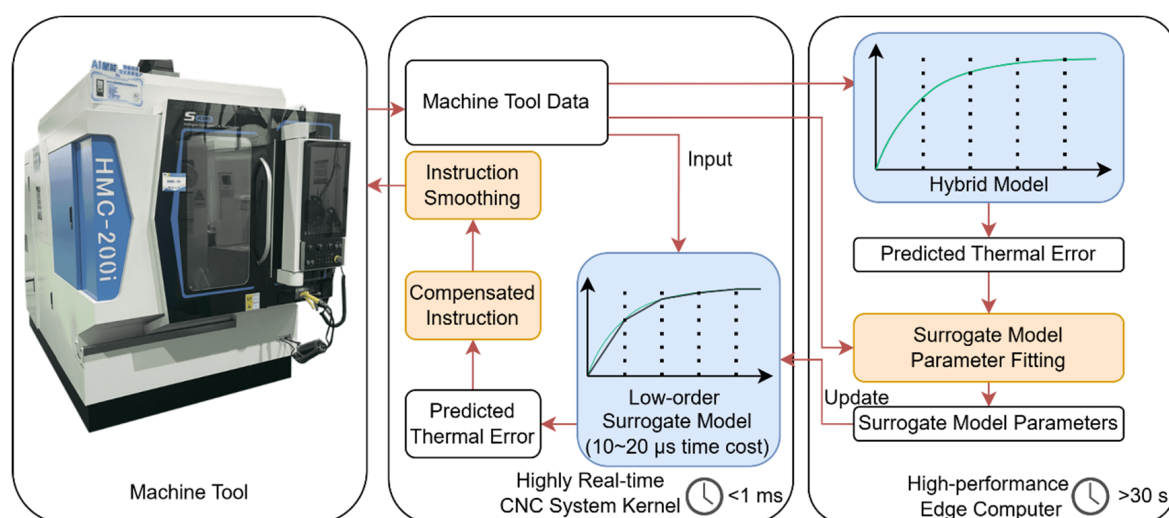


Figure 19. Surrogate compensation architecture.

This study conducted a compensation experiment using the same experimental configuration as shown in Figure 3. The compensation test was performed under a variable-speed condition with spindle speeds ranging from 12,000 rpm to 15,000 rpm, employing the full-training-set hybrid model for evaluation. The compensation results are presented in Figure 20.

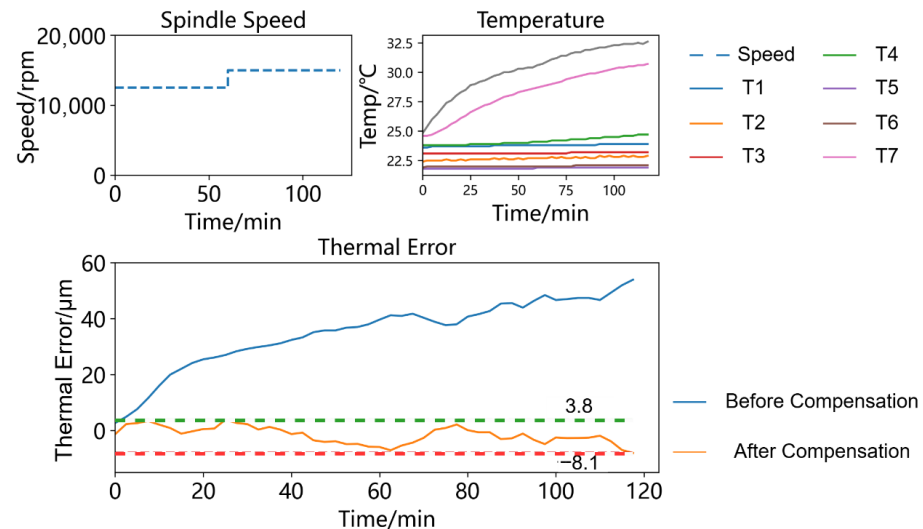
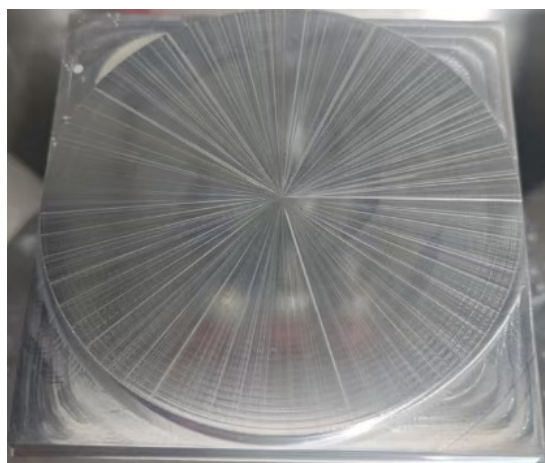


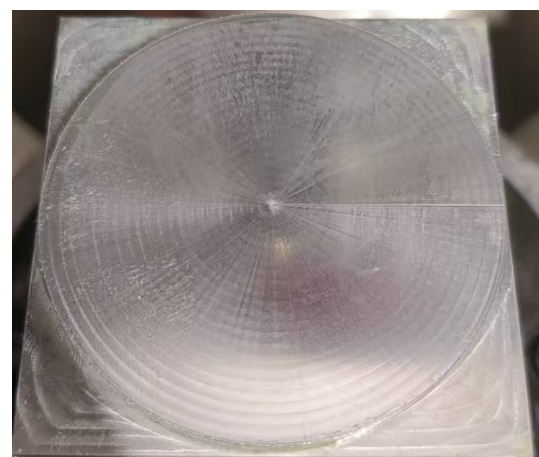
Figure 20. Compensation results.

The results indicate that although the spindle speed in this condition fell outside the training range of the model, the thermal error was reduced from 53.9 μm to 11.9 μm after compensation, corresponding to a 78% reduction in spindle thermal error. This demonstrates that the thermal error prediction model and compensation architecture employed in this study can not only effectively achieve high real-time compensation for machine tools but also exhibit strong robustness.

However, after actual cutting tests, it was observed that the surface polish of the workpiece did not meet the expected standards, as shown in Figure 21a. This issue stemmed from the insufficient engineering maturity of the proxy model compensation architecture software used during the writing of this paper. To enhance the practical effectiveness of the compensation software in real-world machining processes, we subsequently resolved the problem, as illustrated in Figure 21b, with the specifics to be published in the next paper.



(a) Experimental Prototype Results



(b) Product sample result

Figure 21. Machining results.

5. Conclusions

This study presents a hybrid-mechanism deep learning model for robust machine tool spindle thermal error prediction. Key findings are:

- (1) This study presents a residual-correction-based mechanism-data fusion modeling approach. The mechanism model serves as the primary model, with its predictions contributing approximately 70% to the final output. The data-driven model only corrects the residual error of the remaining 30%. This ‘mechanism as the main model, data as the auxiliary’ structure allows the fusion model to inherit the strong robustness of the mechanism model under physical constraints, preventing predictions that violate fundamental physical laws. Meanwhile, the data-driven model precisely fits the unmodeled dynamics in localized regions, significantly enhancing the overall prediction accuracy.
- (2) The proposed hybrid model not only achieves superior accuracy (RMSE 4.6 μm , MAE 4.0 μm) under full training, but also exhibits remarkable robustness under limited data—degrading only 20% (Hyb-GAT) and 9% (Hyb-GP) with 17% training data, compared to 65% and 52% for GAT-LSTM and BPNN—and reduces thermal error by 78% in untrained operating conditions, demonstrating its practical advantage in capturing complex spatiotemporal thermal dynamics.
- (3) The hybrid model proposed in this study approximates the complex thermal-deformation process via linear thermal expansion. Although a residual model is introduced to correct errors induced by this simplification, the approximation may still limit the prediction accuracy of the hybrid framework. Future work could incorporate higher-fidelity mechanistic representations to further improve predictive performance.
- (4) As the machine tools used in this study are housed in the constant-temperature workshop for actual production, cross-temperature-range testing was not feasible under the current production schedule. To address this, a temperature-controlled laboratory is being established, and future work will involve collecting multi-temperature-zone data to validate the model’s performance. In the meantime, cross-speed-range tests have been conducted and confirm the robustness of the model under the available conditions. It is recommended that subsequent studies incorporate cross-temperature-range experiments to fully evaluate the model under varying thermal environments.
- (5) In the compensation experiments, although the hybrid model demonstrates effective compensation results under the proposed architecture—confirming its validity—actual machining tests revealed that the current compensation method can produce uneven surface patterns on the workpiece. This issue is currently under investigation and will be resolved in future work.

Author Contributions: H.C.: data curation, software, validation, writing—original draft, writing—review and editing; Y.H.: conceptualization, methodology; C.Q.: formal analysis, writing—review and editing; X.N.: data curation, investigation, project administration, resources; P.W.: project administration, funding acquisition, writing—review and editing, resources; K.Y.: data curation, visualization. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by Wuhan Municipal Science and Technology Bureau. Author Yubin Huang has received research support from Huazhong University of Science and Technology.

Data Availability Statement: The data presented in this study are available on request from the corresponding author due to the data involving confidential performance information of industrial machine tools, thus requiring authorization from the corresponding author for data distribution.

Conflicts of Interest: The authors have no relevant financial or non-financial interests to disclose.

Appendix A

Appendix A.1

Training Set

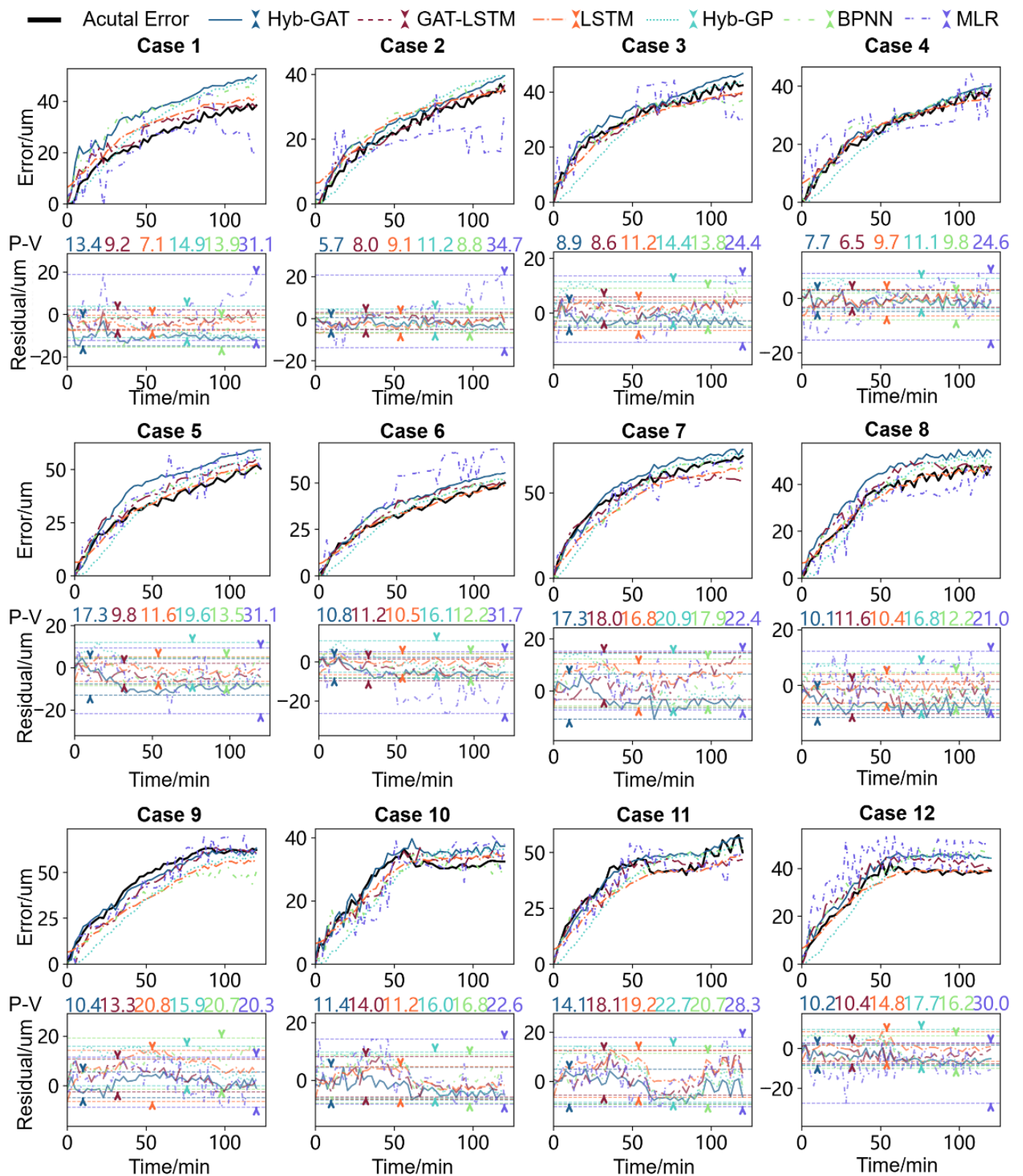


Figure A1. Fitting and residuals on the training set.

Appendix A.2

Validation Set

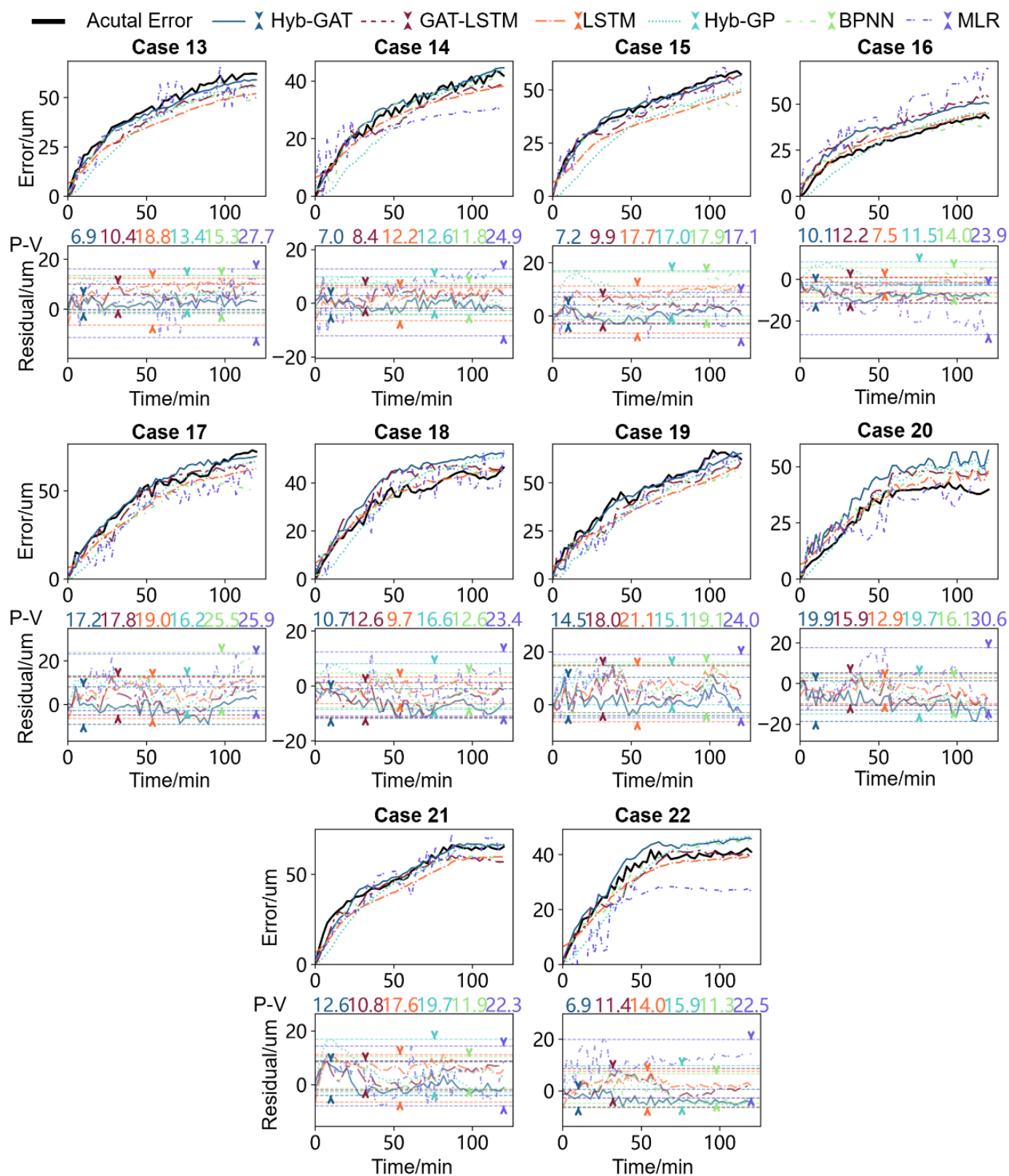


Figure A2. Fitting and residuals on the training set.

Appendix B

Appendix B.1

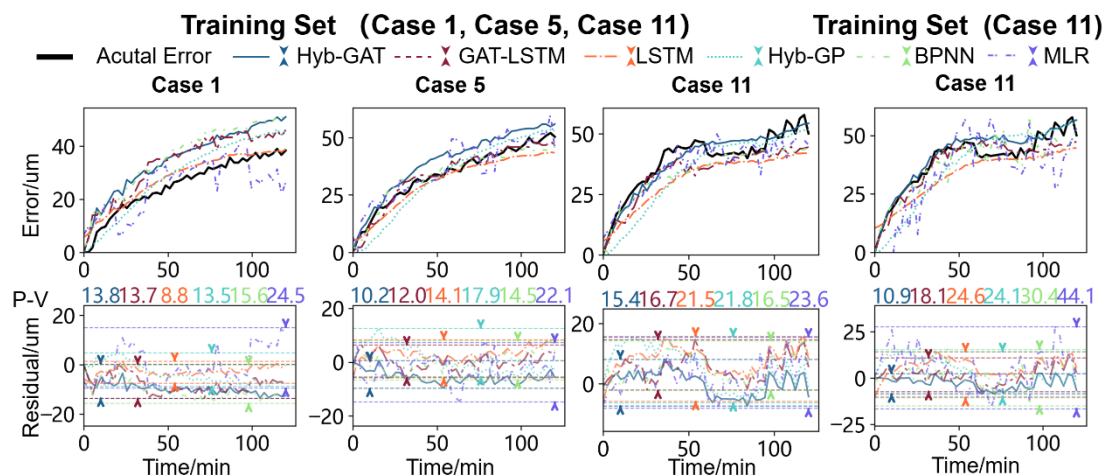


Figure A3. Fitting and residuals on the training set.

Appendix B.2

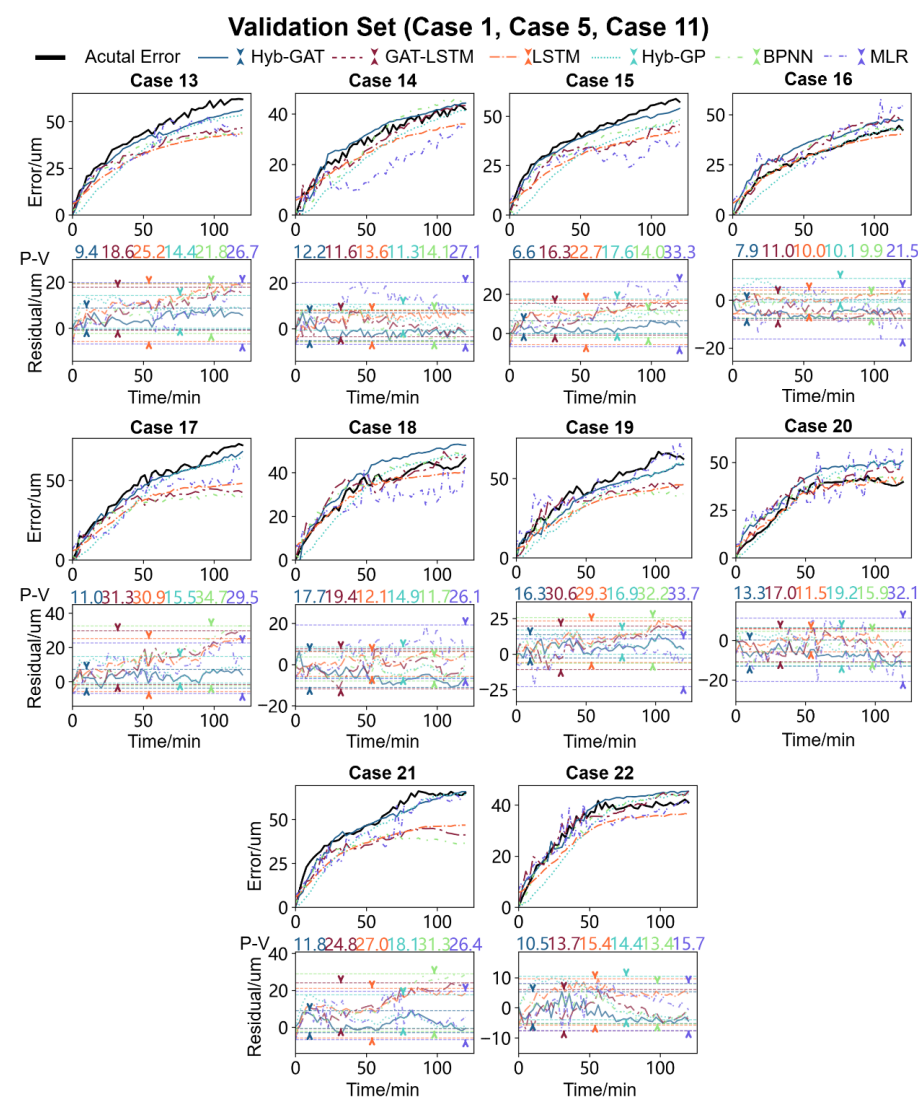


Figure A4. Fitting and residuals with the three-case training set on the validation set.

Appendix B.3

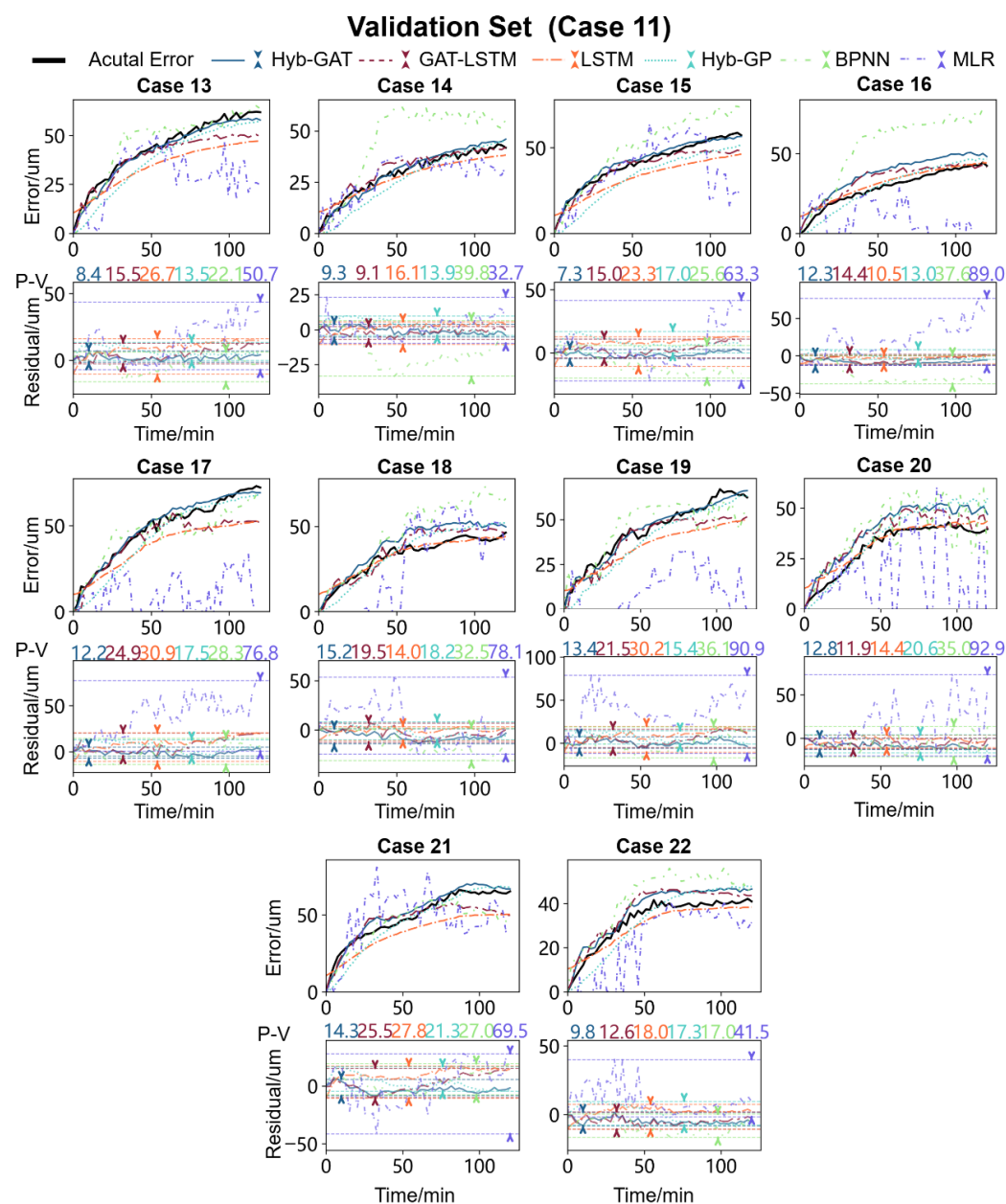


Figure A5. Fitting and residuals with the one-case training set on the validation set.

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