

Article

Dynamic Characteristics of a Transmission System Based on Tooth Surface Modification of Double-Pressure-Angle Helical Face Gears

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Abstract

To investigate the dynamic characteristics of a double-pressure-angle helical face gear transmission system, a digital tooth surface of a double-pressure-angle helical face gear, considering tooth surface modification, was established based on the generation principle of a disk grinding wheel. The time-varying mesh stiffness (TVMS) of the gear pair was calculated using the loaded tooth contact analysis (LTCA) method. A dynamic model of the double-pressure-angle helical face gear transmission system was then developed using the lumped-mass method, and the corresponding dynamic equations were derived. The dynamic characteristics of the system, as well as the effects of input speed, drive-side pressure angle, and tooth surface modification, were analyzed. The results show that increasing the drive-side pressure angle reduces the peak displacement vibration along the line of action, with a reduction of 31% at 32° compared with 24°. The input speed has a significant effect on the system dynamic response; as the speed increases, the vibration response first increases and then decreases, reaching its maximum at 9600 rpm. Bidirectional tooth surface modification on the drive side of the helical face gear changes the geometric contact pattern of the gear pair from localized concentration to a more uniform distribution. Meanwhile, the scatter points on the Poincaré section change from dispersed to concentrated, and the main acceleration peak along the line of action is reduced by 45.8%. These results provide theoretical support for the dynamic design of helical face gear transmission systems.

Keywords: helical face gear transmission; double pressure angle; dynamic characteristics; tooth surface modification



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1. Introduction

Face gear transmissions are characterized by compact structure, high load-carrying capacity, and ease of lightweight design, and therefore exhibit considerable application potential in aerospace and other engineering fields [1]. However, during practical operation, they are susceptible to external loads, transmission errors, and other factors, which may intensify system vibration and reduce operational stability [2,3]. Therefore, investigation of the dynamic characteristics of face gear transmissions is of great significance for improving the stability of gear transmission systems and mitigating vibration and noise.

Research on the dynamic characteristics of face gear transmissions has mainly focused on the development of dynamic models and the investigation of the effects of excitation factors and structural parameters on system dynamics. Regarding excitation factors, Qian et al. [4] established a dynamic model of a non-orthogonal face gear transmission and investigated the influence of backlash on the vibration characteristics of the system. Dong et al. [5], while considering backlash, proposed a dynamic model of a non-orthogonal face gear transmission incorporating tooth surface friction. The validity of the theoretical model was verified experimentally, and the influence of the harmonic amplitude of mesh stiffness on vibration was analyzed. Gao et al. [6], Lin et al. [7], and Mo et al. [8] investigated the effects of different excitation factors on the dynamic characteristics of face gear systems from the perspectives of grazing impact, eccentricity, and load fluctuation, respectively. Fu et al. [9] further analyzed the effects of multiple parameters on the load-sharing characteristics of an offset non-orthogonal helical face gear two-branch transmission system. Research on face gear dynamics has gradually progressed from single-factor analysis to multi-parameter studies under complex operating conditions.

Regarding the effects of structural parameters, Inoue et al. [10] investigated the combined effects of assembly and manufacturing errors on the transmission error of face gear drives. Their results indicated that the pressure-angle deviation of the pinion and the tooth-position angular error of the face gear are major influencing factors, while insufficient tooth surface modification may lead to edge contact and increased fluctuations in transmission error. Jia et al. [11] proposed a double-crowned tooth surface modification method for non-orthogonal helical face gears. Through tooth contact analysis (TCA) and loaded tooth contact analysis (LTCA), the effectiveness of the proposed method in reducing contact stress and tooth-root bending stress and improving tolerance to misalignment errors was demonstrated. Peng et al. [12] and Jia et al. [13] further investigated the effects of tooth surface modification parameters on system vibration and peak dynamic response. Related studies have also examined the effects of helix angle, center offset, eccentricity, mesh damping, and pinion arrangement on the contact characteristics and dynamic behavior of face gear transmissions [14–17]. These studies indicate that structural parameters are closely related to the contact and dynamic characteristics of face gear transmissions, and also provide a theoretical basis for further investigation of the dynamic characteristics of non-conventional face gears.

Compared with conventional single-pressure-angle face gears, double-pressure-angle face gears allow different pressure angles to be adopted on the drive and non-drive sides, thereby increasing the flexibility of tooth-profile parameter configuration. A proper pressure-angle configuration can improve tooth geometry and tooth surface contact conditions, and is beneficial for enhancing load-carrying capacity and meshing performance [18]. Meanwhile, different pressure-angle configurations may also provide potential for reducing system vibration by regulating meshing characteristics and dynamic excitation. However, the specific influence of pressure-angle configuration on the system vibration response has not yet been systematically clarified. Therefore, it is necessary to further investigate the relationship between pressure-angle parameters and dynamic response in double-pressure-angle face gear transmissions.

In summary, existing studies have mainly focused on face gear transmissions with a single pressure angle [19–22], whereas relatively few studies have addressed the dynamic characteristics of double-pressure-angle helical face gear transmissions. In this study, a double-pressure-angle helical face gear transmission system is investigated. Based on the generation principle of a disk grinding wheel, the tooth surface of a double-pressure-angle helical face gear with tooth surface modification is generated. Considering factors such as time-varying mesh stiffness (TVMS) and backlash, a dynamic model of the double-pressure-

angle helical face gear transmission system is established using the lumped-mass method. The dynamic characteristics of the system are analyzed, and the effects of input speed, drive-side pressure angle, and tooth surface modification on system vibration are revealed.

2. Digital Tooth Surface of a Double-Pressure-Angle Helical Face Gear

According to the tooth surface generation method for straight face gears in Ref. [23], the tooth surface of the double-pressure-angle helical face gear considered in this study is generated by a double-pressure-angle disk grinding wheel, whose tooth surface is a surface of revolution formed by rotating the end-section profile of a virtual shaper cutter. The mating double-pressure-angle helical gear is generated by a double-pressure-angle rack cutter. The end-section tooth profile of the rack cutter used to generate the double-pressure-angle helical gear and the virtual double-pressure-angle shaper cutter is shown in Figure 1, where c and d denote the coast side and the drive side, respectively; s_t and s_b denote the tooth width and tooth thickness of the double-pressure-angle rack cutter, respectively; O_t is the midpoint of s_t ; Q_i ($i = c, d$) denotes the end point of the tooth profile on the coast side and drive side; α_i ($i = c, d$) denotes the normal pressure angle on the coast side and drive side; u_{si} ($i = c, d$) denotes the tooth profile direction parameter on the coast side and drive side; and l_i ($i = c, d$) denotes the displacement parameter along the tooth trace direction of the rack cutter on the coast side and drive side. In addition, a_{si} ($i = c, d$) denotes the parabolic modification coefficient of the tooth profile on the coast side and drive side, respectively, and u_{s0} denotes the zero-point coefficient of the parabolic tooth profile modification, which is used to describe the tooth profile modification curve formed by the envelope of the rack cutter.

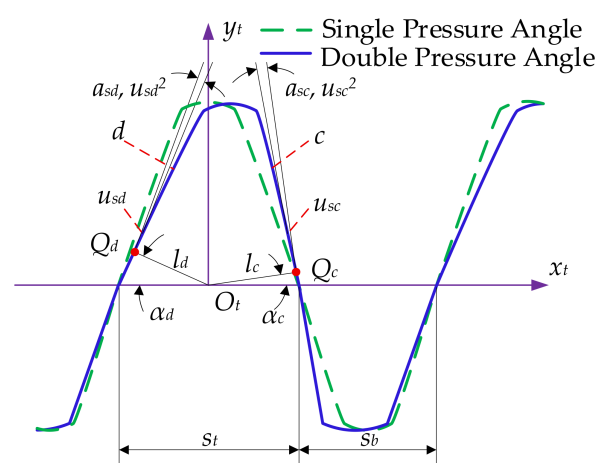


Figure 1. Transverse tooth profile of the rack cutter.

The generation method using a double-pressure-angle disk grinding wheel for grinding the double-pressure-angle helical face gear is illustrated in Figure 2. S_g , S_2 , and S_s denote the moving coordinate systems rigidly attached to the double-pressure-angle disk grinding wheel, the double-pressure-angle helical face gear, and the double-pressure-angle shaper cutter, respectively; O_g , O_2 , and O_s are the origins of S_g , S_2 , and S_s , respectively. Σ_g denotes the tooth surface of the double-pressure-angle disk grinding wheel, Σ_2 denotes the tooth surface of the double-pressure-angle helical face gear, and Σ_s denotes the tooth surface of the double-pressure-angle shaper cutter. ω_g is the angular velocity of the double-pressure-angle disk grinding wheel rotating about the x_g -axis, ω_s is the angular velocity of the double-pressure-angle disk grinding wheel rotating about the z_s -axis, and v_s is the velocity of the double-pressure-angle disk grinding wheel along the z_g -axis. L_g denotes the displacement of the double-pressure-angle disk grinding wheel along the z_s -axis, φ_s

denotes the rotation angle of the double-pressure-angle disk grinding wheel, and $\Delta\varphi_s$ denotes the additional rotation angle of the double-pressure-angle disk grinding wheel corresponding to the variation in L_g . ω_2 is the angular velocity of the double-pressure-angle helical face gear rotating about the z_2 -axis, and E_g denotes the distance between the z_g - and z_s -axes.

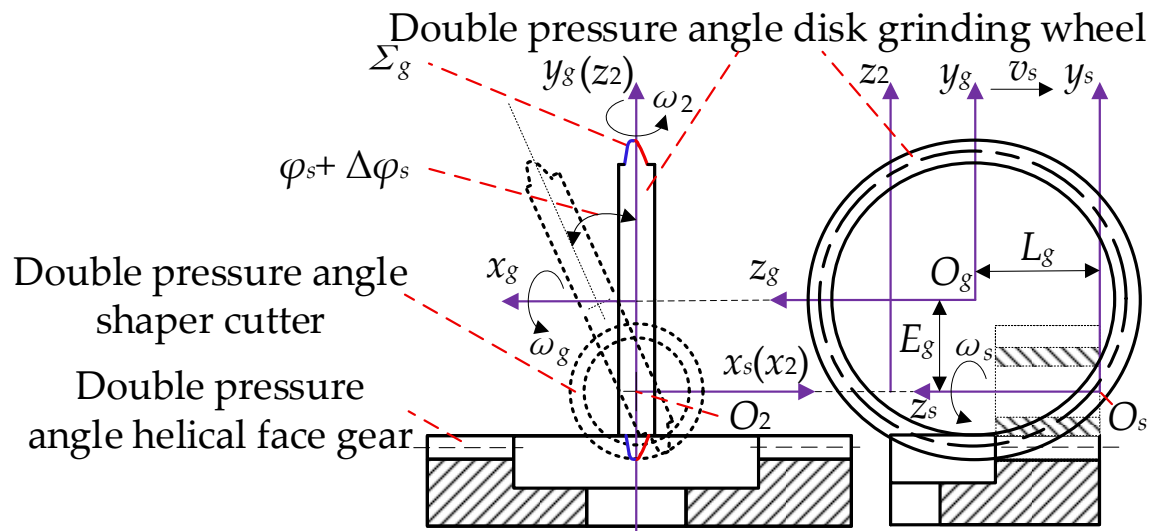


Figure 2. Schematic of the grinding principle for a double-pressure-angle helical face gear.

The key motions involved in the tooth surface machining process of the double-pressure-angle helical face gear are as follows: the double-pressure-angle disk grinding wheel rotates about the x_g -axis at an angular velocity of ω_g , performs a swinging motion about the z_s -axis at an angular velocity of ω_s , and undergoes a linear feed motion along the z_g -axis at a velocity of v_s ; meanwhile, the double-pressure-angle helical face gear rotates about the z_2 -axis at an angular velocity of ω_2 .

The root transition surface of the double-pressure-angle helical face gear is generated by enveloping the arcuate surface of the double-pressure-angle disk grinding wheel. The tip arc of the double-pressure-angle disk grinding wheel is shown in Figure 3, where $R_s(u_s)$ denotes the vector coordinate of the end-section tooth profile of the double-pressure-angle helical shaper cutter in the coordinate system S_s ; $n_s(u_s)$ denotes the unit normal vector of the tooth profile; ρ_g denotes the radius of the cutter tip arc; M_1 is the tangent point between the cutter tip arc and the tooth profile, and M_2 is the tangent point between the cutter tip arc and the addendum circle; θ_1 and θ_2 are the corresponding angular parameters of M_1 and M_2 , respectively; and O and O' are the centers of the arcs on the two sides of the cutter tip, respectively. In addition, a_{fi} ($i = c, d$) denotes the parabolic modification coefficient in the tooth trace direction on the coast side and the drive side, respectively, and L_{f0} denotes the zero-point coefficient of the parabolic tooth trace modification, which is used to describe the tooth trace modification curve formed by the envelope of the cutter tip arc of the double-pressure-angle disk grinding wheel.

Based on the tooth surface equation of the disk grinding wheel and the corresponding machining kinematics, a family of moving surfaces of the double-pressure-angle disk grinding wheel is established in the coordinate system of the double-pressure-angle helical face gear. The digital tooth surface of the double-pressure-angle helical face gear is then obtained as the envelope of this surface family. The digital tooth surface of the double-pressure-angle helical face gear is shown in Figure 4, where Σ_c and Σ_d denote the tooth surfaces on the coast side and drive side, respectively, with the relevant parameters listed in Table 1.

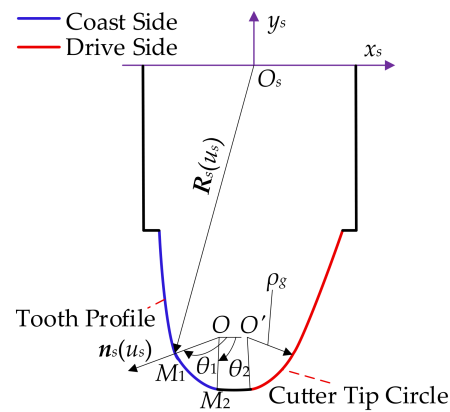


Figure 3. Cutter tip arc of the double-pressure-angle disk grinding wheel.

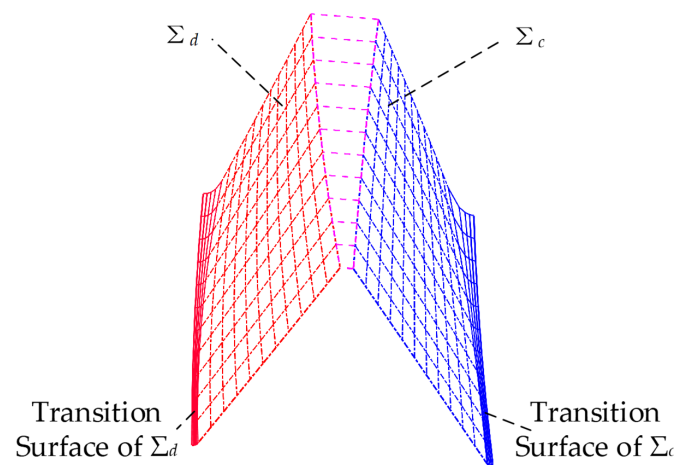


Figure 4. Digital tooth surface of the double-pressure-angle helical face gear.

Table 1. System parameters.

Parameter	Value
Number of teeth of the driving pinion N_1	21
Number of teeth of the driven face gear N_2	111
Normal module m_n/mm	4
Drive-side pressure angle $\alpha_d/^\circ$	24
Coast-side pressure angle $\alpha_c/^\circ$	20
Helix angle $\beta/^\circ$	10
Inner diameter of the driven face gear L_1/mm	225
Outer diameter of the driven face gear L_2/mm	250
Driving pinion support stiffness in the x -direction $K_{x1}/(\text{N}\cdot\text{m}^{-1})$	2×10^8
Driving pinion support stiffness in the y -direction $K_{y1}/(\text{N}\cdot\text{m}^{-1})$	2×10^8
Driving pinion support stiffness in the z -direction $K_{z1}/(\text{N}\cdot\text{m}^{-1})$	2×10^8
Driven face gear support stiffness in the x -direction $K_{x2}/(\text{N}\cdot\text{m}^{-1})$	8×10^8
Driven face gear support stiffness in the y -direction $K_{y2}/(\text{N}\cdot\text{m}^{-1})$	8×10^8
Driven face gear support stiffness in the z -direction $K_{z2}/(\text{N}\cdot\text{m}^{-1})$	8×10^8
Overall TVMS $k_m/(\text{N}\cdot\text{m}^{-1})$	2.75×10^8
Half backlash $b_c/\mu\text{m}$	10
Poisson's ratio μ	0.3
Young's modulus E/GPa	206
Density $\rho/(\text{kg}\cdot\text{m}^{-3})$	7850
Input speed n/rpm	20,000
Output torque $T_2/(\text{N}\cdot\text{m})$	600

3. Dynamic Model of the Double-Pressure-Angle Helical Face Gear Pair

3.1. Time-Varying Mesh Stiffness

The geometric meshing characteristics under no-load conditions are calculated using the TCA method. Subsequently, a flexibility coefficient matrix between discrete contact points is established through LTCA, and the load distribution and overall elastic deformation under load are solved to obtain the TVMS of the double-pressure-angle helical face gear pair. Further details are provided in Ref. [24].

At a given instantaneous meshing position, the total normal deformation δ_T of the teeth of the double-pressure-angle helical face gear pair is expressed as

$$\delta_T = \frac{\pi |T_G - T_L| r_2}{3600 \times 180} \quad (1)$$

where T_G is the geometric transmission error, T_L is the loaded transmission error, and r_2 is the base circle radius of the double-pressure-angle helical face gear.

A single meshing period of one tooth pair is divided into eight equal intervals, and the corresponding overall TVMS k_m of the double-pressure-angle helical face gear pair is expressed as

$$k_{m(j)} = \frac{F_m}{\delta_{T(j)}} \quad (j = 1, 2, \dots, 8) \quad (2)$$

where F_m is the normal meshing force obtained from LTCA.

The discrete values in Equation (2) are expanded into a periodic meshing function of the face gear transmission using a Fourier series, and the corresponding TVMS $k_n(t)$ is expressed as

$$k_n(t) = k_m + \sum_{i=1}^N A_i \cos(i\Omega_h t + \phi_i) \quad (3)$$

where A_i is the amplitude of the i th harmonic, Ω_h is the meshing frequency of the gear pair, t is time, and ϕ_i is the phase angle corresponding to the i th harmonic. Since the contribution of higher-order terms in the Fourier expansion is relatively small, the first ten harmonics are retained in this study [25].

3.2. Mesh Damping

The mesh damping c_n of the double-pressure-angle helical face gear pair is expressed as

$$c_n = 2\zeta_g \sqrt{\frac{k_m I_1 I_2}{I_1 r_1^2 + I_2 r_2^2}} \quad (4)$$

where I_1 and I_2 are the moments of inertia of the double-pressure-angle helical gear and the double-pressure-angle face gear, respectively; r_1 is the base circle radius of the double-pressure-angle helical gear; and ζ_g is the mesh damping ratio, which is taken as 0.1 in this study [26].

3.3. Dynamic Equations of the System

Using the lumped parameter method, the dynamic model of the double-pressure-angle helical face gear transmission system is established, as shown in Figure 5, with the coordinate system O -xyz introduced. The driving gear is the double-pressure-angle helical gear, and the driven gear is the double-pressure-angle helical face gear. M_1 and M_2 are the masses of the driving and driven gears, respectively; T_1 and T_2 are the input and output torques, respectively; K_{ij} ($i = x, y, z; j = 1, 2$) denotes the support stiffness of the gears in the x -, y -, and z -directions; C_{ij} ($i = x, y, z; j = 1, 2$) denotes the corresponding support damping, which is calculated as $C_{ij} = 2\zeta_s \sqrt{M_j K_{ij}}$, where ζ_s is the support damping ratio and is set to

0.1 according to Ref. [27]; φ_1 and φ_2 are the rotation angles about the rotational axes of the driving and driven gears, respectively; x_1 , y_1 , and z_1 are the vibration displacements of the driving gear in the x -, y -, and z -directions, respectively; and x_2 , y_2 , and z_2 are the vibration displacements of the driven gear in the x -, y -, and z -directions, respectively.

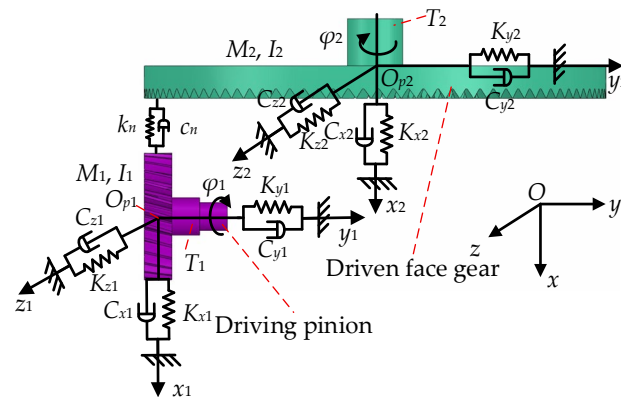


Figure 5. Dynamic model of the double-pressure-angle helical face gear transmission system.

The relative displacement δ_n of the double-pressure-angle helical face gear transmission system along the line of action is expressed as

$$\delta_n = (x_1 - x_2)\sin\alpha_d + (y_1 - y_2)\sin\beta\cos\alpha_d + (z_1 - z_2)\cos\alpha_d\cos\beta + (\varphi_1 r_1 - \varphi_2 r_2)\cos\beta\cos\alpha_d \quad (5)$$

where β is the helix angle at the reference circle of the driving pinion.

The normal meshing force F_n between the driving pinion and the driven face gear is expressed as

$$F_n = k_n(t)f(\delta_n) + c_n\dot{\delta}_n \quad (6)$$

where $\dot{\delta}_n$ is the relative velocity of the double-pressure-angle helical face gear transmission system along the line of action; $f(\delta_n)$ is the dynamic transmission error function [28], in which b_c denotes the half backlash.

Based on Newton's second law, the dynamic equations of the double-pressure-angle helical face gear pair system are established as follows:

$$\begin{cases} M_1\ddot{x}_1 + C_{x1}\dot{x}_1 + K_{x1}x_1 = -F_n \sin\alpha_d \\ M_1\ddot{y}_1 + C_{y1}\dot{y}_1 + K_{y1}y_1 = -F_n \sin\beta\cos\alpha_d \\ M_1\ddot{z}_1 + C_{z1}\dot{z}_1 + K_{z1}z_1 = -F_n \cos\beta\cos\alpha_d \\ I_1\ddot{\varphi}_1 = T_1 - F_n \cos\beta\cos\alpha_d r_1 \\ M_2\ddot{x}_2 + C_{x2}\dot{x}_2 + K_{x2}x_2 = F_n \sin\alpha_d \\ M_2\ddot{y}_2 + C_{y2}\dot{y}_2 + K_{y2}y_2 = F_n \sin\beta\cos\alpha_d \\ M_2\ddot{z}_2 + C_{z2}\dot{z}_2 + K_{z2}z_2 = F_n \cos\beta\cos\alpha_d \\ I_2\ddot{\varphi}_2 = -T_2 + F_n \cos\beta\cos\alpha_d r_2 \end{cases} \quad (7)$$

where $\dot{x}_1$, $\dot{y}_1$, and $\dot{z}_1$ are the velocities of the driving pinion in the x -, y -, and z -directions, respectively; $\dot{x}_2$, $\dot{y}_2$ and $\dot{z}_2$ are the velocities of the driven face gear in the x -, y -, and z -directions, respectively; $\ddot{x}_1$, $\ddot{y}_1$ and $\ddot{z}_1$ are the accelerations of the driving pinion in the x -, y -, and z -directions, respectively; $\ddot{x}_2$, $\ddot{y}_2$ and $\ddot{z}_2$ are the accelerations of the driven face gear in the x -, y -, and z -directions, respectively; $\ddot{\varphi}_1$ is the angular acceleration of the driving pinion about its rotational axis; and $\ddot{\varphi}_2$ is the angular acceleration of the driven face gear about its rotational axis.

3.4. Nondimensionalization of the Dynamic Equations

Since the system of dynamic equations is non-positive definite, the two torsional equations in Equation (7) are substituted into the second derivative of Equation (5), yielding

$$\begin{cases} m_e \ddot{\delta}_n - m_e (\ddot{y}_1 - \ddot{y}_2) \sin \beta \cos \alpha_d - m_e (\ddot{z}_1 - \ddot{z}_2) \cos \alpha_d \cos \beta \\ - m_e (\ddot{x}_1 - \ddot{x}_2) \sin \alpha_d + F_n \cos \alpha_d^2 \cos \beta^2 = F_1 + F_2 \\ m_e = I_1 I_2 / (I_1 r_2^2 + I_2 r_1^2) \\ F_1 = m_e T_1 r_1 \cos \beta \cos \alpha_d / I_1 \\ F_2 = m_e T_2 r_2 \cos \beta \cos \alpha_d / I_2 \end{cases} \quad (8)$$

where $\dot{\delta}_n$ and $\ddot{\delta}_n$ are the relative velocity and relative acceleration, respectively, of the double-pressure-angle helical face gear transmission system along the line of action; m_e is the equivalent mass of the gear pair.

Substituting Equation (8) into Equation (7) and nondimensionalizing the dynamic equations, the dimensionless equations of the system are obtained as

$$\begin{cases} \ddot{X}_1 + \zeta_{X1}^C \dot{X}_1 + \zeta_{X1}^K X_1 + (c_{h1} \dot{\delta} + k_{h1} f(\delta)) \sin \alpha_d = 0 \\ \ddot{Y}_1 + \zeta_{Y1}^C \dot{Y}_1 + \zeta_{Y1}^K Y_1 + (c_{h1} \dot{\delta} + k_{h1} f(\delta)) \cos \alpha_d \sin \beta = 0 \\ \ddot{Z}_1 + \zeta_{Z1}^C \dot{Z}_1 + \zeta_{Z1}^K Z_1 + (c_{h1} \dot{\delta} + k_{h1} f(\delta)) \cos \alpha_d \cos \beta = 0 \\ \ddot{X}_2 + \zeta_{X2}^C \dot{X}_2 + \zeta_{X2}^K X_2 - (c_{h2} \dot{\delta} + k_{h2} f(\delta)) \sin \alpha_d = 0 \\ \ddot{Y}_2 + \zeta_{Y2}^C \dot{Y}_2 + \zeta_{Y2}^K Y_2 - (c_{h2} \dot{\delta} + k_{h2} f(\delta)) \cos \alpha_d \sin \beta = 0 \\ \ddot{Z}_2 + \zeta_{Z2}^C \dot{Z}_2 + \zeta_{Z2}^K Z_2 - (c_{h2} \dot{\delta} + k_{h2} f(\delta)) \cos \alpha_d \cos \beta = 0 \\ \ddot{\lambda} - (\ddot{X}_1 - \ddot{X}_2) \sin \alpha_d - (\ddot{Y}_1 - \ddot{Y}_2) \cos \alpha_d \sin \beta \\ - (\ddot{Z}_1 - \ddot{Z}_2) \cos \alpha_d \cos \beta + \cos \alpha_d^2 \cos \beta^2 c_h \dot{\delta} \\ + \cos \alpha_d^2 \cos \beta^2 k_h f(\delta) = f_1 + f_2 \end{cases} \quad (9)$$

$$\omega_n = \sqrt{k_m / m_e} = 2\pi f_n, \tau = \omega_n t \quad (10)$$

$$f_m = N_1 n / 60 \quad (11)$$

where X_1 , Y_1 , and Z_1 are the dimensionless displacements of the driving pinion in the x -, y -, and z -directions, respectively; X_2 , Y_2 , and Z_2 are the dimensionless displacements of the driven face gear in the x -, y -, and z -directions, respectively; δ is the dimensionless displacement along the line of action; $\dot{X}_1$, $\dot{Y}_1$, and $\dot{Z}_1$ are the dimensionless velocities of the driving pinion in the x -, y -, and z -directions, respectively; $\dot{X}_2$, $\dot{Y}_2$, and $\dot{Z}_2$ are the dimensionless velocities of the driven face gear in the x -, y -, and z -directions, respectively; $\dot{\delta}$ is the dimensionless velocity along the line of action; $\ddot{X}_1$, $\ddot{Y}_1$, and $\ddot{Z}_1$ are the dimensionless accelerations of the driving pinion in the x -, y -, and z -directions, respectively; $\ddot{X}_2$, $\ddot{Y}_2$, and $\ddot{Z}_2$ are the dimensionless accelerations of the driven face gear in the x -, y -, and z -directions, respectively; $\ddot{\delta}$ is the dimensionless acceleration along the line of action; ζ_{X1}^C , ζ_{Y1}^C , and ζ_{Z1}^C are the dimensionless damping coefficients of the driving pinion in the x -, y -, and z -directions, respectively; ζ_{X2}^C , ζ_{Y2}^C , and ζ_{Z2}^C are the dimensionless damping coefficients of the driven face gear in the x -, y -, and z -directions, respectively; ζ_{X1}^K , ζ_{Y1}^K , and ζ_{Z1}^K are the dimensionless stiffness coefficients of the driving pinion in the x -, y -, and z -directions, respectively; ζ_{X2}^K , ζ_{Y2}^K , and ζ_{Z2}^K are the dimensionless stiffness coefficients of the driven face gear in the x -, y -, and z -directions, respectively; $f(\delta)$ is the dimensionless dynamic transmission error function; ω_n is the natural angular frequency, f_n is the natural frequency, and τ is the dimensionless time; and f_m is the gear pair mesh frequency, N_1 is number of teeth of the driving pinion, and n is the input speed.

4. Numerical Example and Analysis

The main system parameters of the double-pressure-angle helical face gear transmission system are assumed to be as listed in Table 1. To highlight the influence of tooth surface modification on the dynamic response of the system, the support characteristics of each gear in different directions are represented by the same equivalent support stiffness.

4.1. Analysis of Geometric Contact Characteristics

To investigate the influence of tooth surface modification on the subsequent dynamic characteristics, bidirectional tooth surface modification is applied to the drive side of the driven face gear. The bidirectional modification parameters are set as follows: $a_{sd} = -0.003$, $u_{s0} = 2$, $a_{fd} = -0.0002$, and $L_{f0} = 1.5$.

Under the same geometric parameters and operating conditions, the geometric contact patterns of the unmodified gear pair and the bidirectionally modified gear pair are compared, as shown in Figure 6. In the figure, 1 denotes the inner end; 2 denotes the theoretical contact point, and the curve formed by connecting a series of theoretical contact points represents the contact path; 3 denotes the major axis of the contact ellipse; 4 denotes the tooth tip; 5 denotes the outer end; 6 denotes the common tangent line between the usable tooth surface and the transition surface; and 7 denotes the tooth root. As shown in the figure, compared with the unmodified tooth surface, the geometric contact path of the double-pressure-angle helical face gear pair after bidirectional tooth surface modification exhibits a significantly wider distribution range, and the contact region changes from localized concentration to a broader distribution over the tooth surface. These results indicate that the selected bidirectional modification parameters can effectively adjust the spatial distribution of tooth surface contact and, to a certain extent, alleviate the tendency toward localized contact concentration [29]. Based on the modified tooth surface geometry, LTCA is recalculated to obtain the corresponding TVMS $k_n(t)$. The resulting TVMS is then nondimensionalized and substituted into Equation (9) for the dynamic analysis of the modified system.

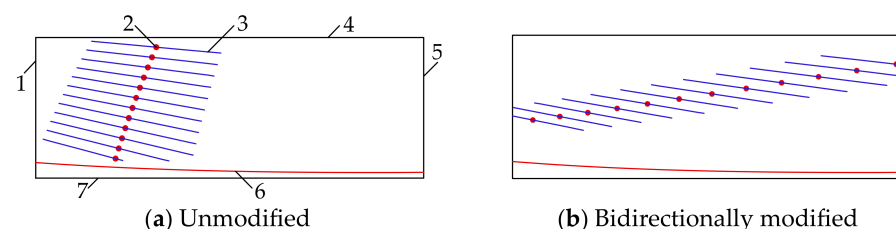


Figure 6. Geometric contact pattern of the gear pair.

4.2. Analysis of Dynamic Characteristics

The fourth- to fifth-order Runge–Kutta integration method is employed to numerically solve the dynamic equations. To ensure that the analysis results reflect the steady-state characteristics of the system, the initial transient responses are discarded.

The time-domain responses of the double-pressure-angle helical face gear transmission system are shown in Figure 7. Figure 7a–c show the displacement responses of the driving pinion in the x -, y -, and z -directions, respectively. The peak-to-peak value of the displacement response in the z -direction is relatively large, whereas that in the y -direction is relatively small. The displacement responses in all directions exhibit stable periodic fluctuations. The response periods remain essentially unchanged before and after modification, while the peak-to-peak values of the displacement responses in all directions are reduced. Figure 7d–f show the velocity responses of the driving pinion in the x -, y -, and z -directions, respectively. The relative magnitudes of the peak-to-peak values of the velocity responses

in different directions are consistent with those of the displacement responses. Compared with the unmodified tooth surfaces, the peak-to-peak values of the velocity responses in all directions are significantly reduced after modification. Figure 7g–i show the displacement responses of the driven face gear in the x -, y -, and z -directions, respectively. The peak-to-peak value of the displacement response in the z -direction is relatively large, whereas that in the y -direction is relatively small. The displacement responses in all directions also exhibit stable periodic fluctuations, and their peak-to-peak values are reduced after modification. Figure 7j–l show the velocity responses of the driven face gear in the x -, y -, and z -directions, respectively. The relative magnitudes of the peak-to-peak values of the velocity responses in different directions are consistent with those of the displacement responses, and the peak-to-peak values of the velocity responses in all directions are significantly reduced after modification. Figure 7m shows the displacement response δ_n along the line of action. After modification, its mean value increases, whereas its peak-to-peak value decreases significantly. Figure 7n shows the velocity response $\dot{\delta}_n$ along the line of action, whose peak-to-peak value is also significantly reduced after modification. Overall, bidirectional tooth surface modification has little influence on the vibration response period of the system, but effectively reduces the peak-to-peak values of the responses in each degree of freedom and along the line of action, with a more pronounced reduction in the peak-to-peak values of the velocity responses.

The variation in the RMS value of $\ddot{\delta}_n$ with input speed n before and after tooth surface modification is shown in Figure 8. As can be seen, the RMS value of $\ddot{\delta}_n$ varies nonlinearly with n , and the curve exhibits relatively complex variations in the range of 0–6000 rpm. These local peaks are mainly associated with the higher-order harmonics of the mesh frequency approaching the system natural frequency, which can induce additional resonance responses [30]. The RMS value reaches its peak at 9600 rpm. At approximately this speed, the gear pair mesh frequency f_m approaches the system natural frequency f_n , resulting in resonance amplification and the observed critical response. Compared with the unmodified tooth surfaces, bidirectional tooth surface modification effectively reduces the RMS value of $\ddot{\delta}_n$, with a reduction of 45.8% at the main peak. In the high-speed range, the RMS value of $\ddot{\delta}_n$ after modification remains at a relatively low level, indicating that bidirectional tooth surface modification can effectively suppress vibration over the speed range up to 20,000 rpm.

Under the unmodified condition, the dynamic characteristics of the system at different input speeds n are shown in Figure 9. As can be seen from the time-domain responses, the peak value of δ first increases and then decreases with increasing n . The maximum peak occurs at $n = 10,000$ rpm, followed by that at 5000 rpm. When n exceeds 10,000 rpm, the peak value gradually decreases with increasing n . The phase portraits show that the enclosed area first increases and then decreases as n increases. At 5000 rpm, a pronounced alternating variation in the response peaks appears in the time-domain response, while the phase portrait changes from a single closed trajectory to a double-loop structure with inner and outer loops, indicating that the system exhibits more complex motion under this operating condition. A comparison of Figures 8 and 9 shows that the variation trends of the peak value of δ and the area enclosed by the phase portrait are consistent with that of the RMS value of $\ddot{\delta}_n$ in Figure 8. Variations in input speed do not alter the boundedness of the system response, but significantly affect the vibration response intensity and the stability of system motion. In particular, at 10,000 rpm, the influence of n on the dynamic response of the system is more pronounced.

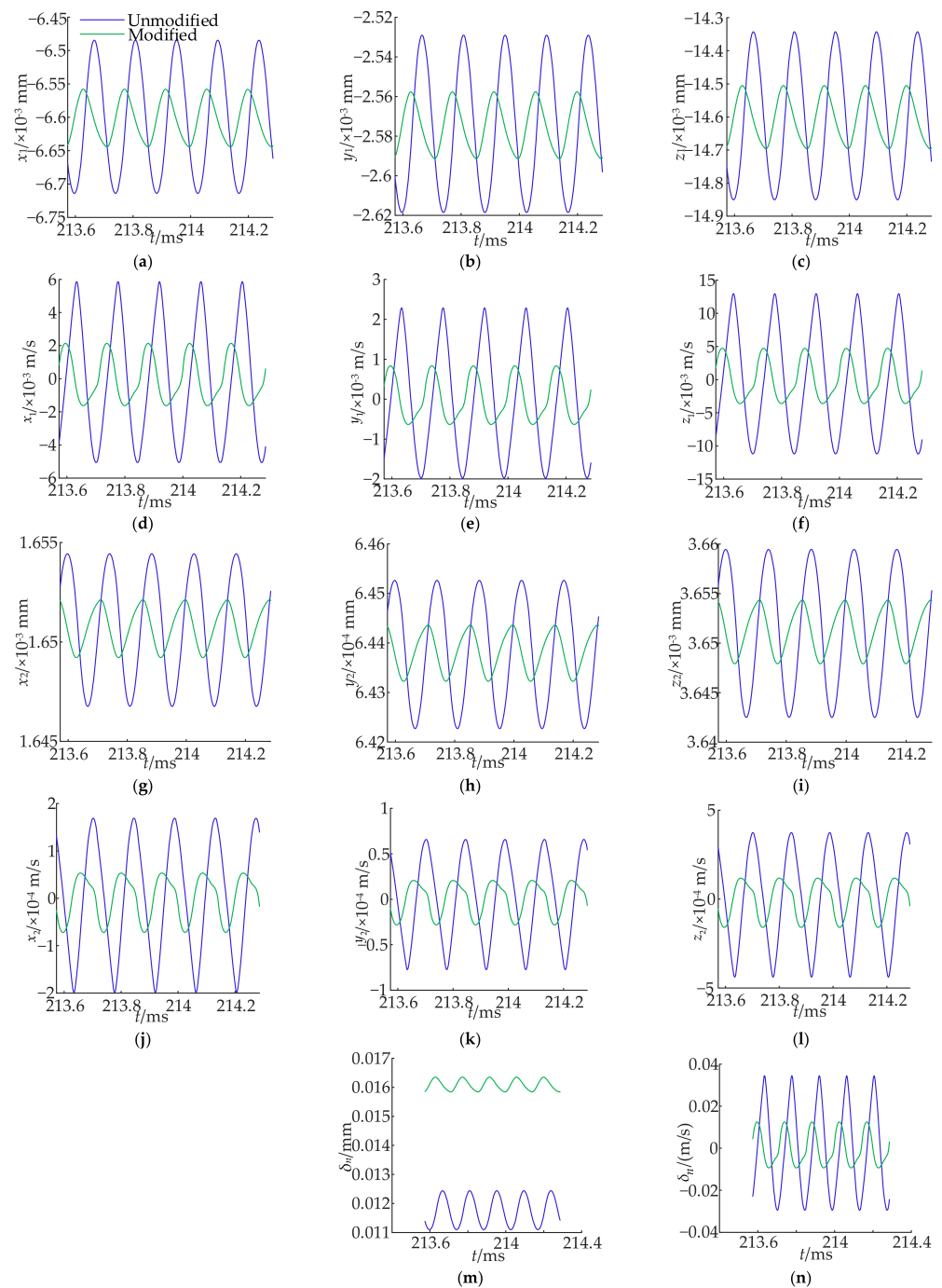


Figure 7. Time-domain responses of the double-pressure-angle helical face gear transmission system. (a) x -direction displacement of the driving pinion; (b) y -direction displacement of the driving pinion; (c) z -direction displacement of the driving pinion; (d) x -direction velocity of the driving pinion; (e) y -direction velocity of the driving pinion; (f) z -direction velocity of the driving pinion; (g) x -direction displacement of the driven face gear; (h) y -direction displacement of the driven face gear; (i) z -direction displacement of the driven face gear; (j) x -direction velocity of the driven face gear; (k) y -direction velocity of the driven face gear; (l) z -direction velocity of the driven face gear; (m) displacement along the line of action; (n) velocity along the line of action.

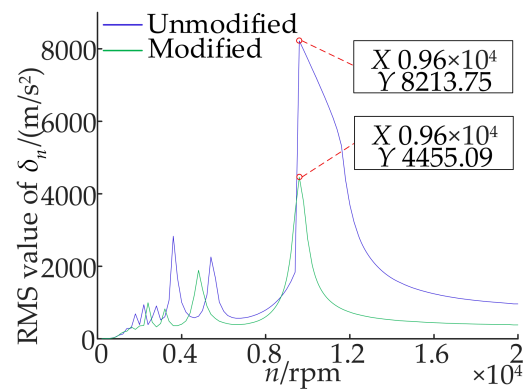


Figure 8. RMS value of δ_n versus input speed n .

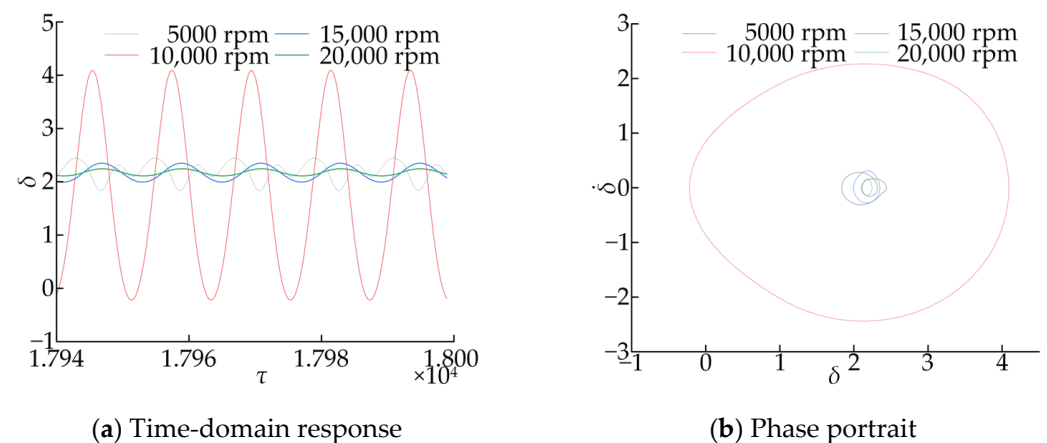


Figure 9. Dynamic characteristics of the system at different input speeds n .

The drive-side pressure angle α_d is varied from 24° to 32° , over which a continuous geometric contact pattern is maintained. For each α_d , the corresponding tooth surface geometry and TVMS are recalculated through TCA and LTCA before the dynamic analysis. Under the unmodified condition, the dynamic characteristics of the system at different drive-side pressure angles α_d are shown in Figure 10. As can be seen from the time-domain responses, with increasing α_d , the mean value of δ increases, while the vibration peak decreases. The vibration peak is the largest at $\alpha_d = 24^\circ$, whereas at $\alpha_d = 32^\circ$, the vibration peak is the smallest and the mean value is the highest. The phase portraits show that, as α_d increases, the envelope shifts toward the region of larger displacement, while its enclosed area gradually decreases, and the trajectory evolves from a regular closed curve to a progressively distorted shape. Compared with that at $\alpha_d = 24^\circ$, the vibration peak at $\alpha_d = 32^\circ$ is reduced by 31%. The vibration peak and phase portrait exhibit relatively small changes when α_d increases from 24° to 28° , whereas more pronounced changes in the vibration peak, the position of the phase portrait, and the enclosed area are observed when α_d increases from 28° to 32° . Overall, α_d mainly affects the peak value of δ and the trajectory characteristics of the phase portrait, while its influence on the periodic characteristics of the time-domain response is relatively small.

The dynamic responses of the modified and unmodified double-pressure-angle helical face gear transmission systems are compared in Figure 11. As shown in the figure, the system responses remain bounded both before and after tooth surface modification. The time-domain responses exhibit periodic fluctuations, the phase portraits form closed trajectories, and the frequency spectra are dominated by the main frequency component, indicating pronounced periodic motion of the system in both cases. Compared with

the unmodified case, bidirectional tooth surface modification reduces the peak value of the time-domain response δ , decreases the area enclosed by the phase portrait, and lowers the amplitudes of both the main spectral peak and the other frequency components, thereby effectively suppressing system vibration. Meanwhile, the scatter points on the Poincaré section change from a relatively dispersed distribution to a more concentrated distribution, indicating that the motion state of the system becomes more stable after tooth surface modification.

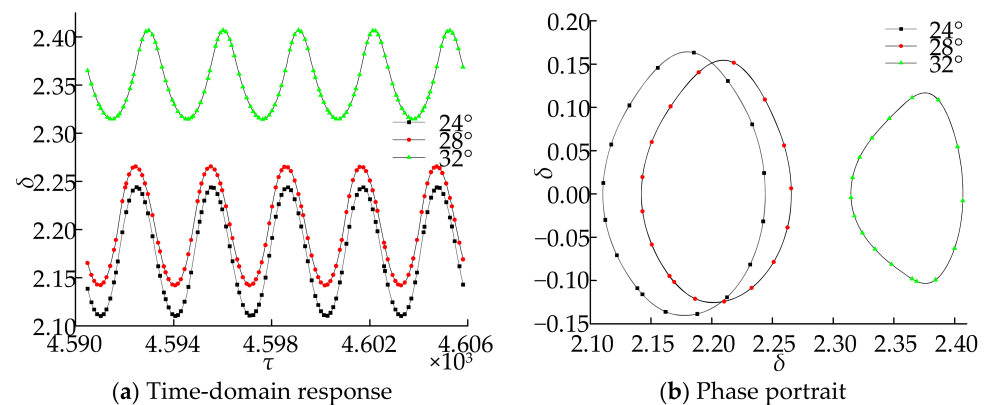


Figure 10. Dynamic characteristics of the system at different drive-side pressure angles α_d .

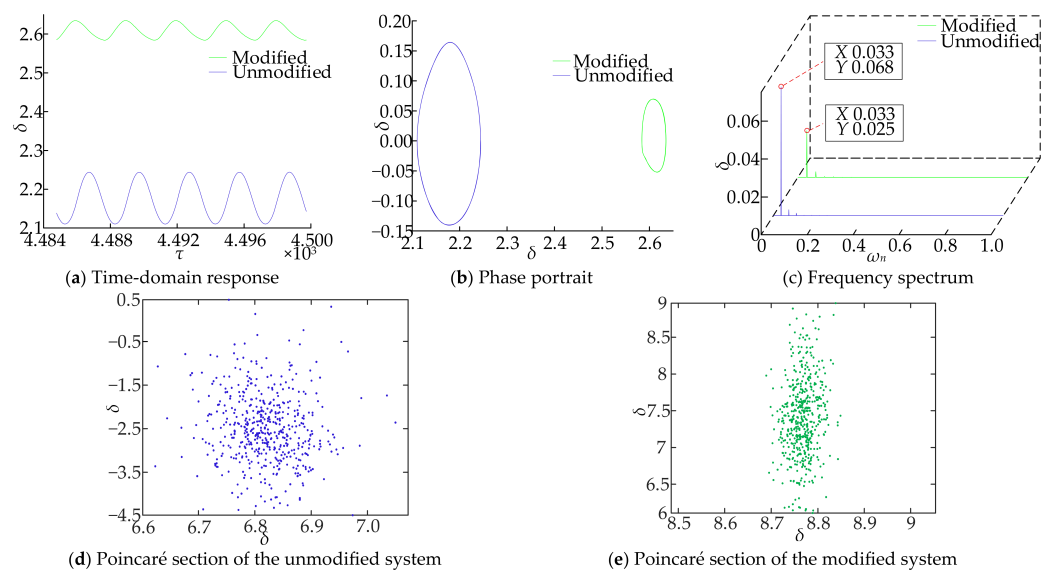


Figure 11. Dynamic responses of the gear system before and after tooth surface modification.

5. Conclusions

The digital tooth surface of the double-pressure-angle helical face gear was established using the disk grinding wheel generation method, and the TVMS before and after tooth surface modification was calculated using LTCA. The dynamic equations of the double-pressure-angle helical face gear transmission system were then established using the lumped-mass method. Furthermore, the dynamic characteristics of the system before and after bidirectional tooth surface modification on the drive side of the double-pressure-angle helical face gear were investigated. The main conclusions are as follows:

- (1) Bidirectional tooth surface modification reduces the peak-to-peak values of the displacement and velocity responses of the system in all degrees of freedom and significantly decreases the peak RMS value of the acceleration $\ddot{\delta}_n$ along the line of action, demonstrating its effectiveness in suppressing system vibration.

- (2) The input speed n significantly affects the dynamic characteristics of the system. As n increases, both the vibration peak and the area enclosed by the phase portrait first increase and then decrease. A strong dynamic response occurs near 9600 rpm. Therefore, long-term operation near this speed should be avoided.
- (3) As the drive-side pressure angle α_d increases, the mean value of the dimensionless displacement δ along the line of action increases, while its peak value decreases. Meanwhile, the phase portrait shifts toward the region of larger displacement, and its enclosed area decreases. Appropriately increasing α_d is conducive to reducing system vibration.
- (4) Bidirectional tooth surface modification on the drive side of the double-pressure-angle helical face gear can effectively reduce system vibration. After modification, the peak value of the time-domain response decreases, the phase portrait contracts, and the scatter points on the Poincaré section become more concentrated, indicating that bidirectional tooth surface modification is beneficial to the operational stability of the system.

This work is theoretical and numerical in nature, and independent experimental or finite element validation has not yet been performed but will be considered in future work.

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