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Review

A Review on Cutting Force and Thermal Modeling, Toolpath Planning, and Vibration Suppression for Advanced Manufacturing

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Abstract

Achieving precise prediction and intelligent control remains a pivotal challenge in cutting processes. This need is addressed through a comprehensive survey of three critical enabling technologies: cutting force/temperature modeling, tool path planning, and vibration suppression. First, the evolution of cutting force and temperature modeling is analyzed, tracing its progression from traditional analytical methods and finite-element numerical simulations to data-driven models such as machine learning (ML) and physics-informed neural networks. This analysis highlights multiphysics coupling and model–data fusion as key to enhancing prediction accuracy. Subsequently, the evolution of tool path planning is examined, showing its development from a geometric interpolation problem into a multi-objective optimization challenge incorporating dynamic constraints, involving computational geometry, graph theory, and meta-heuristic algorithms. Finally, stability analysis based on time-delay differential equations, state identification via signal processing and ML, and active control strategies for vibration suppression are discussed. In conclusion, mathematical methods are shown to be fundamentally integrated throughout the 'perception–prediction–decision–control' closed-loop of the cutting process. This integration provides a solid theoretical foundation and technical support for building high-performance manufacturing systems dedicated to complex curved critical components.

Keywords: cutting force; cutting temperature; tool path planning; vibration suppression; data-driven modeling

1. Introduction

As sectors, including aerospace, precision optics, and new energy vehicles, impose escalating performance demands on complex components, the corresponding requirements for manufacturing precision, efficiency, and consistency have accordingly reached a level of unprecedented stringency [1,2]. As the most widespread and fundamental subtractive manufacturing method, the precise prediction and intelligent control of machining processes have become critical factors determining the performance and reliability of high-end equipment [3,4]. As these sectors demand ever-greater part geometric complexity, machining accuracy, surface integrity, and production efficiency, traditional processing models reliant on trial-and-error methods and worker experience can no longer meet modern manufacturing needs [5]. Consequently, achieving a paradigm shift in cutting processes—from 'experience-driven' to 'science-driven' and ultimately 'intelligence-driven'—by precisely modeling, planning, and optimizing machining phenomena, serves as a critical prerequisite for digital and intelligent manufacturing [6,7].

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The majority of heat generated during cutting is conducted into the tool and workpiece, with only a small fraction removed by the chip from the cutting zone [8]. Temperature increases negatively impact surface quality and the performance of the cutting edge during the process [9,10]. Furthermore, high temperatures cause surface and subsurface damage, such as thermal stresses, microstructural changes, crack formation, dimensional and shape alterations, and even workpiece scorching [11]. Moreover, cutting is a complex, nonlinear, multi-parameter, coupled physical process inherently involving severe plastic deformation of materials, high-speed friction temperature modeling, tool interference, and chatter [12]. Cutting force and temperature modeling generated during machining directly influence machining accuracy, surface quality, tool life, and energy efficiency [13]. Consequently, the development of accurate cutting force and temperature models is fundamental, forming the basis for process parameter optimization, machining defect prediction, and overall production efficiency enhancement [14,15]. However, modeling remains challenging due to the process's multidisciplinary nature, involving complex interactions across mechanics, thermodynamics, and materials science [16].

On the other hand, tool path planning determines the spatial trajectory of the cutting tool, which not only affects machining efficiency (idle travel time) but also directly relates to fluctuations in cutting force, accumulation of cutting heat, and the dynamic stability of the process system [17,18]. Planning the optimal tool path in cutting operations to ensure geometric accuracy while balancing machining efficiency, cutting force stability, and surface quality remains a core challenge [18,19]. Currently, toolpath planning during cutting extends beyond traditional geometric interpolation to encompass multi-objective optimization. The goal is to simultaneously optimize machining efficiency, cutting force stability, and vibration suppression while meeting geometric accuracy requirements [20,21]. Furthermore, toolpath planning has extended from computational geometry (e.g., generating equidistant curves, curvature-adaptive step sizes) to digital twin optimization of idle paths to reduce machining time. Machine learning methods, such as genetic algorithms and particle swarm optimization, are employed to synergistically optimize feed rates, cutting parameters, and path morphology [22,23]. Furthermore, physically constrained path planning has emerged as a cutting-edge focus, where path generation simultaneously accounts for physical constraints, such as cutting force fluctuations, chatter stability, and tool wear [24].

The central challenge in vibration suppression lies in effectively controlling machining vibrations, particularly regenerative chatter, which can compromise efficiency, degrade surface quality, and even damage the tool or machine tool [25,26]. For instance, turbine blades used in the aerospace industry are typical thin-walled bending components manufactured via 5-axis ball-nose end milling. Unstable machining is a common issue during this process, leading to poor surface quality [27]. Vibration suppression constitutes a quintessential dynamic system control problem, forming a complete closed-loop "modeling–sensing–control" framework [28,29]. Dynamic system modeling based on time-delay differential equations and stability leaflet diagram theory provides theoretical foundations for selecting chatter-free cutting parameters [30]. Furthermore, modern control theories (adaptive control) are employed to design feedforward or feedback controllers, achieving active vibration suppression [31]. Currently, machine learning has also been introduced for the unsupervised learning of vibration features and the generation of intelligent vibration suppression strategies, driving the evolution of vibration suppression strategies toward intelligent approaches during the cutting process [32–34].

Recent advancements in computational mechanics, artificial intelligence, and experimental measurement techniques have enabled the continuous refinement of cutting force and temperature modeling methods [35,36]. These efforts primarily focus on analytical

modeling, numerical simulation, and finite-element simulation. However, challenges persist in the modeling process, including theoretical simplifications, complex multiphysics coupling calculations, and difficulties in experimental validation [37]. Secondly, tool path planning, as the core component of machining process optimization, has seen rapid advancements in intelligence and adaptability. The traditional constant residual height method, which relies on geometric constraints, is shifting toward multi-objective collaborative optimization [38]. This approach leverages integrated cutting force and temperature models to jointly optimize process parameters and tool paths, as seen in variable feed rate planning. Nevertheless, it faces challenges, including the solution of high-dimensional nonlinear objective functions and the need for real-time response to dynamic machining conditions [39,40]. Furthermore, vibration suppression is crucial for ensuring machining stability. Flutter suppression technology, in particular, has evolved from passive damping to active control, integrating digital twin technology to enable vibration prediction and online compensation [41]. However, challenges remain in identifying time-varying modal parameters under complex conditions and addressing control delay effects.

In summary, cutting force/temperature modeling, tool path planning, and vibration suppression are deeply interconnected within a tightly coupled closed-loop system. Path planning influences cutting force and temperature distributions, which then alter the dynamic state of the process system and directly affect vibration behavior [42]. Vibration, in turn, exerts a counterforce on the cutting, altering actual cutting force and surface finish quality. The primary challenge lies in systematically understanding, modeling, and controlling this coupled system. As illustrated in Figure 1, this paper provides a systematic review of research advances in cutting force and temperature modeling, tool path planning, and vibration suppression. It analyzes the strengths and limitations of different methodologies and explores future development trends.

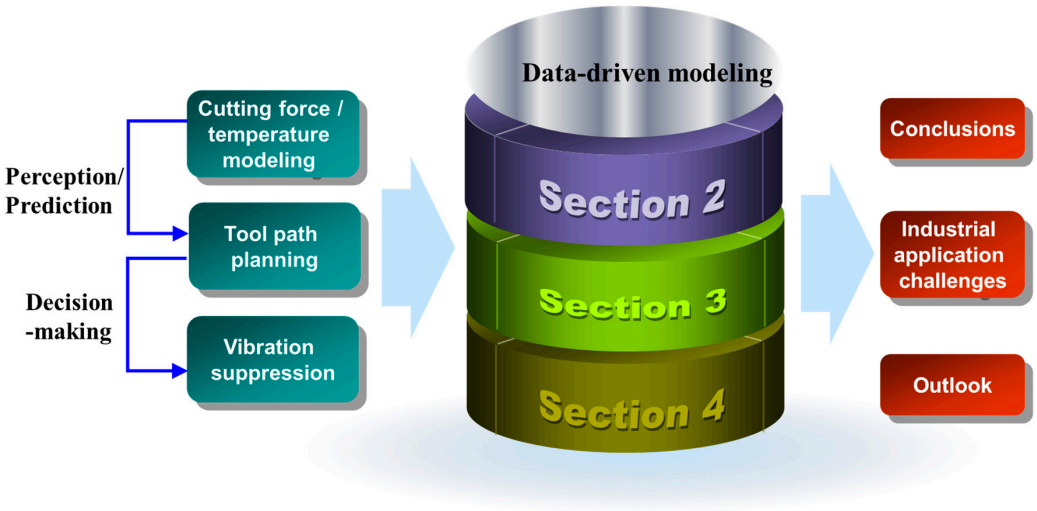


Figure 1. Narrative framework of this paper.

2. Cutting Force/Temperature Modeling

Cutting is the most widely used material forming method in mechanical manufacturing. The cutting force and heat generated during the process directly impact machining accuracy, surface quality, tool life, and energy efficiency. Therefore, establishing accurate models for cutting force and heat is crucial for optimizing process parameters, predicting machining defects, and enhancing production efficiency [43]. However, modeling remains challenging due to the complex interdisciplinary nature of the cutting process, involving mechanics, thermodynamics, materials science, and other fields. In recent years, advancements in computational mechanics, artificial intelligence, and advanced experimen-

tal measurement techniques have led to continuous improvements in methods for cutting force and temperature modeling. These efforts primarily focus on analytical modeling, numerical simulation, and finite-element simulation, as illustrated in Figure 2. Nevertheless, challenges persist in the modeling process, including theoretical simplifications, complex multiphysics coupling calculations, and difficulties in experimental validation. This section systematically reviews research progress in cutting force and temperature modeling, analyzing the strengths and limitations of different methodologies.

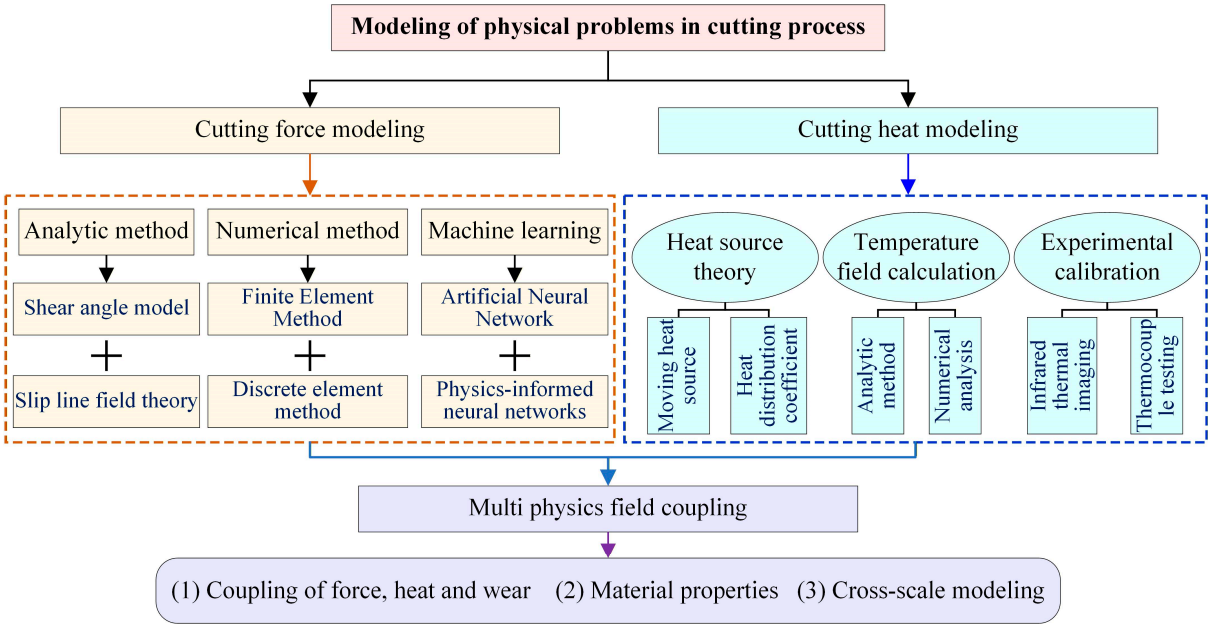


Figure 2. Schematic flowchart of cutting force and temperature modeling.

2.1. Analytical Model

The analytical modeling method for cutting force primarily focuses on analyzing the force acting on the tool cutting edge. It is a modeling approach that integrates tribology and materials science by comprehensively considering factors such as material removal mechanisms, cutting-edge geometry, and machining parameters. The analytical cutting force model provides a mathematical representation describing the variation in force magnitude throughout the entire cutting process [44]. Analytical modeling approaches originate from cutting mechanisms, requiring minimal data analysis and offering broad applicability [45]. Despite the complexity of considering multiple factors and the necessity of numerous assumptions during modeling, analytical modeling remains the most reliable method for deeply investigating machining mechanisms. Concurrently, cutting force serves as the primary form of input energy during material removal. Fluctuations in cutting force impact machining quality and tool wear by influencing deformation and vibration within the machining system, as well as stress conditions in the cutting zone. Cutting heat, a significant byproduct of this process, interacts with cutting force to affect workpiece surface quality, tool wear, and chip morphology [46]. Therefore, it is essential to investigate the development process and key influencing factors of cutting force and cutting heat modeling. The recent literature on cutting force and cutting temperature modeling, as shown in Figure 3 [47,48], indicates growing attention and substantial research investment in this field.

Specifically, Priest et al. [49] compared the accuracy of predicting cutting force for variable-pitch and helical bullnose-end mills using a mechanically optimized cutting force coefficient identification method based on rapid testing (requiring only single-angle cutting with increased radial engagement) against the traditional mechanical method involving multi-pass unidirectional cutting. Kang et al. [50] developed an instantaneous process material identifi-

cation method by synchronizing machine tool tip position data with cutting force measured by a dynamometer, employing optimization to minimize prediction errors. A cutting force calculation flow chart considering the wiper edge cutting effect is shown in Figure 4.

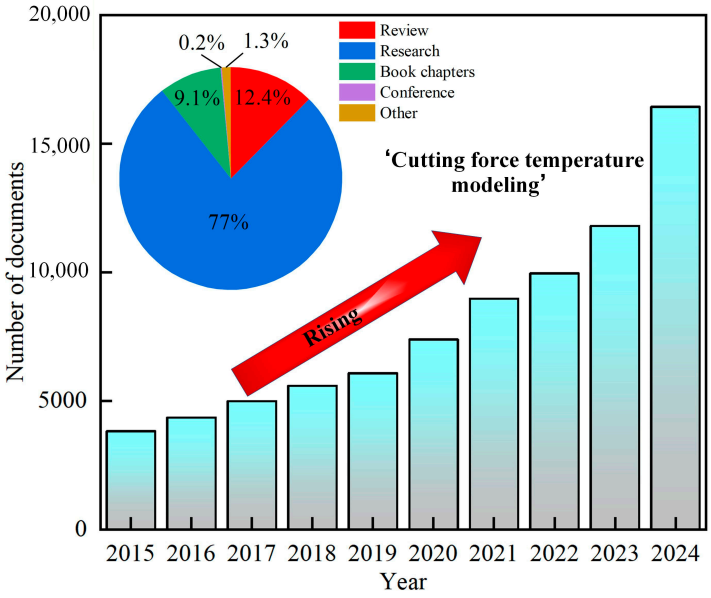


Figure 3. Literature reviews on cutting force and temperature modeling in recent years.

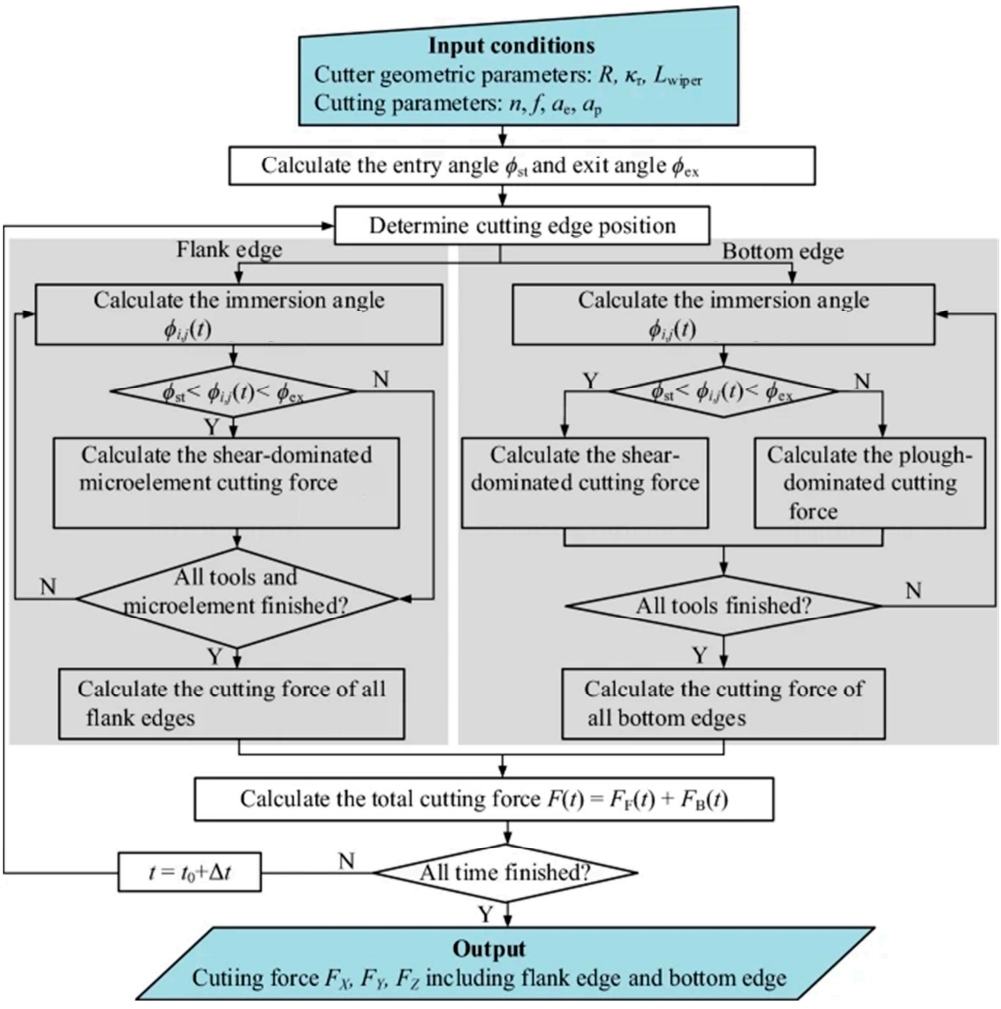


Figure 4. Cutting force calculation flow chart considering wiper edge cutting effect [51].

However, this approach is limited to simple cylindrical end mills with two grooves of consistent geometry. Liu et al. [52] predicted cutting force in screw whirling by combining mathematical modeling with finite-element analysis, accounting for tool inclination and relative tool–workpiece motion. They optimized process parameters to derive a suitable range for screw whirling operations. Li et al. [27] proposed a milling force model for longitudinal torsional ultrasonic vibration-assisted 5-axis ball-end milling, accounting for tool runout effects and ultrasonic vibration influences. They investigated the impact of various parameters on dynamic separation characteristics during machining via tool-workpiece contact rate analysis. Yao et al. [53] employed a stiffness averaging method to investigate the mechanical properties of 2.5D woven SiO_2/SiO_2 ceramic matrix composites. The establishment of these grinding force prediction models primarily comprises three steps: First, the motion trajectories of individual abrasive grains on the end face and side face of the grinding wheel were analyzed during both conventional and ultrasonic-assisted grinding processes. Second, the effective cutting depth and processing time of each grain were calculated. Finally, axial and lateral grinding force were predicted by considering the force contributions from end-face and side-face grains, with grinding force coefficients calibrated. In addition, Xie et al. [54] developed a bending-considering uncut chip thickness model based on classical assumptions. The new model posits that material flow over the tool contact area is described by a continuous vector field. The bending uncut chip thickness is measured along this vector field, while cutting forces are distributed along these paths. The proposed computational process extends cutting force prediction based solely on geometric parameters, orthogonal cutting parameters, and orthogonal-to-oblique transformations. Liao et al. [55] addressed the critical need to control cutting force and temperatures during orthopedic surgery, which can cause mechanical damage and necrosis of bone tissue. They proposed a novel cutting force and temperature mechanism model for bone milling. Based on orthogonal cutting parameters, a bone material cutting stress model considering anisotropic properties was established.

Additionally, Xu et al. [56] developed an innovative ultrasonic vibration-assisted electrical discharge milling (UV-EDAM) technique and established a three-dimensional cutting force model for UV-EDAM. Experimental results demonstrated that the finite volume-based model accurately predicted cutting force.

Furthermore, the surface heat source of the EDM workpiece was simplified by creating a longitudinal temperature distribution curve for the workpiece material. This approach eliminated the multi-pulse point heat source, resulting in a uniform surface heat source. The temperature distribution of the discharge thermal field was analyzed under different capacitance and feed rate conditions. The surface temperature of the workpiece material increased with increasing capacitance and then decreased. However, cutting force serves as the core characteristic of tool-workpiece interaction, directly influencing machine tool vibration, tool wear, and machining deformation, as shown in Figure 5. Analytical models based on mechanical equilibrium equations offer high computational efficiency but rely on strong assumptions, making it challenging to accurately describe nonlinear material behavior [57,58]. Dong et al. [59] used Arctic Parrotfish Optimization (APO) to determine the optimal milling machine coefficient between the ball-end mill and the workpiece, and conducted experimental verification of the proposed method, as shown in Figure 6. The results indicate that this method has high accuracy and excellent performance in instantaneous force prediction. This method establishes a relationship between the force coefficient and energy consumption signal through a model, and the predicted milling force can also be used for machining deformation analysis.

Furthermore, the precise determination of heat distribution ratios between the shear zone and friction zone in temperature modeling remains challenging. Significant varia-

tions exist in thermal distribution coefficients across different materials (metallic materials, composite materials) [60]. The heat source theory currently employed in temperature field solutions within analytical models is not suitable for complex geometric models under demanding operating conditions. Consequently, these analytical approaches necessitate integration with subsequent numerical simulations (finite-element simulations) and machine learning methodologies. An improved analytical model for the orthogonal cutting temperature of titanium alloys was developed, in which the temperatures of cutting tools, notches, and workpiece points were calculated using the moving heat source method. As displayed in Figure 7, the tool relief angle is introduced into the model, and a fictional mirror heat source of the shear surface heat source and friction heat source is applied to calculate the temperature rise in a semi-infinite medium [61]. The heat distribution ratio along the tool chip interface is determined by discretization methods.

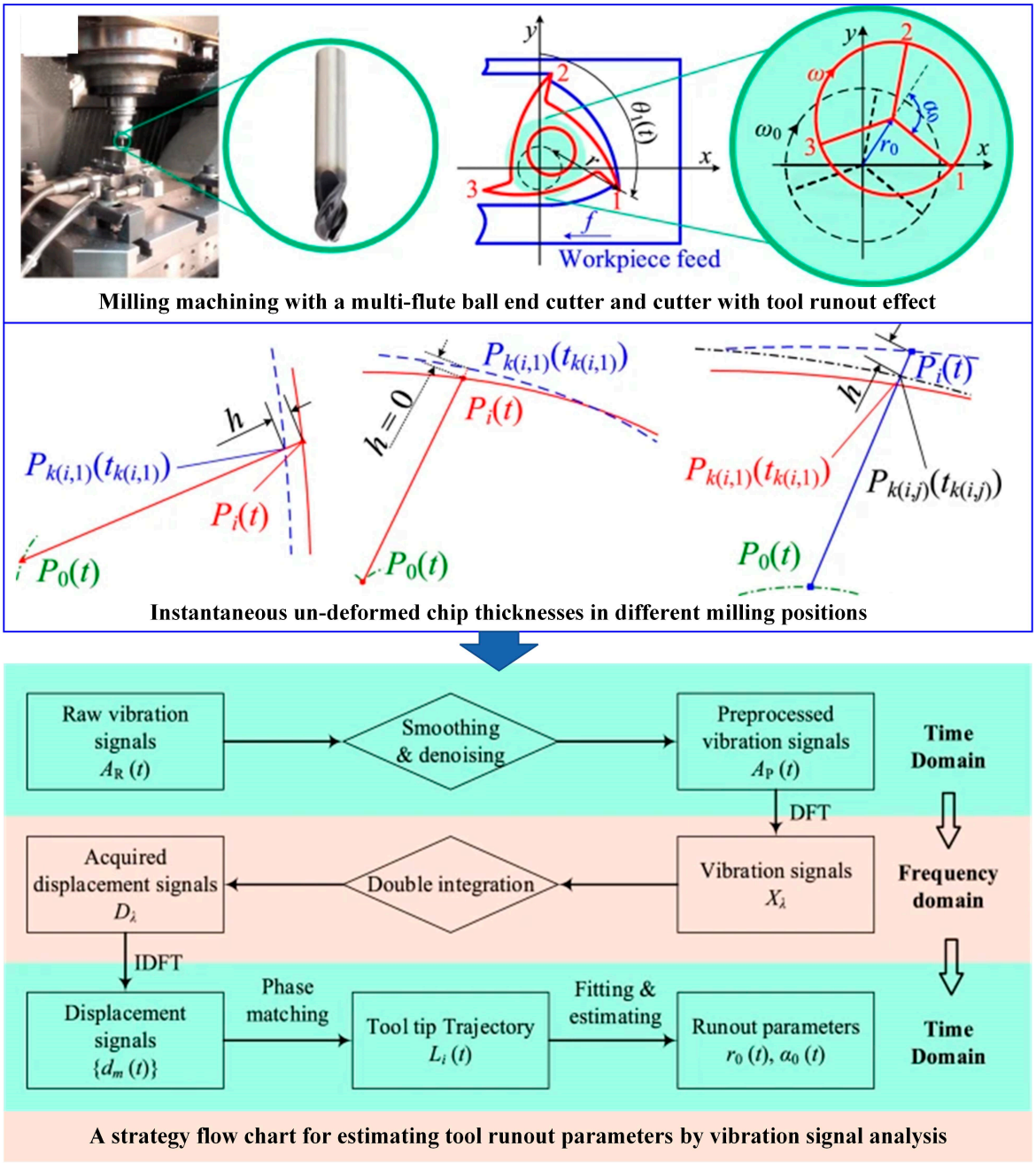


Figure 5. Modeling of cutting force and state identification in high-speed milling [57].

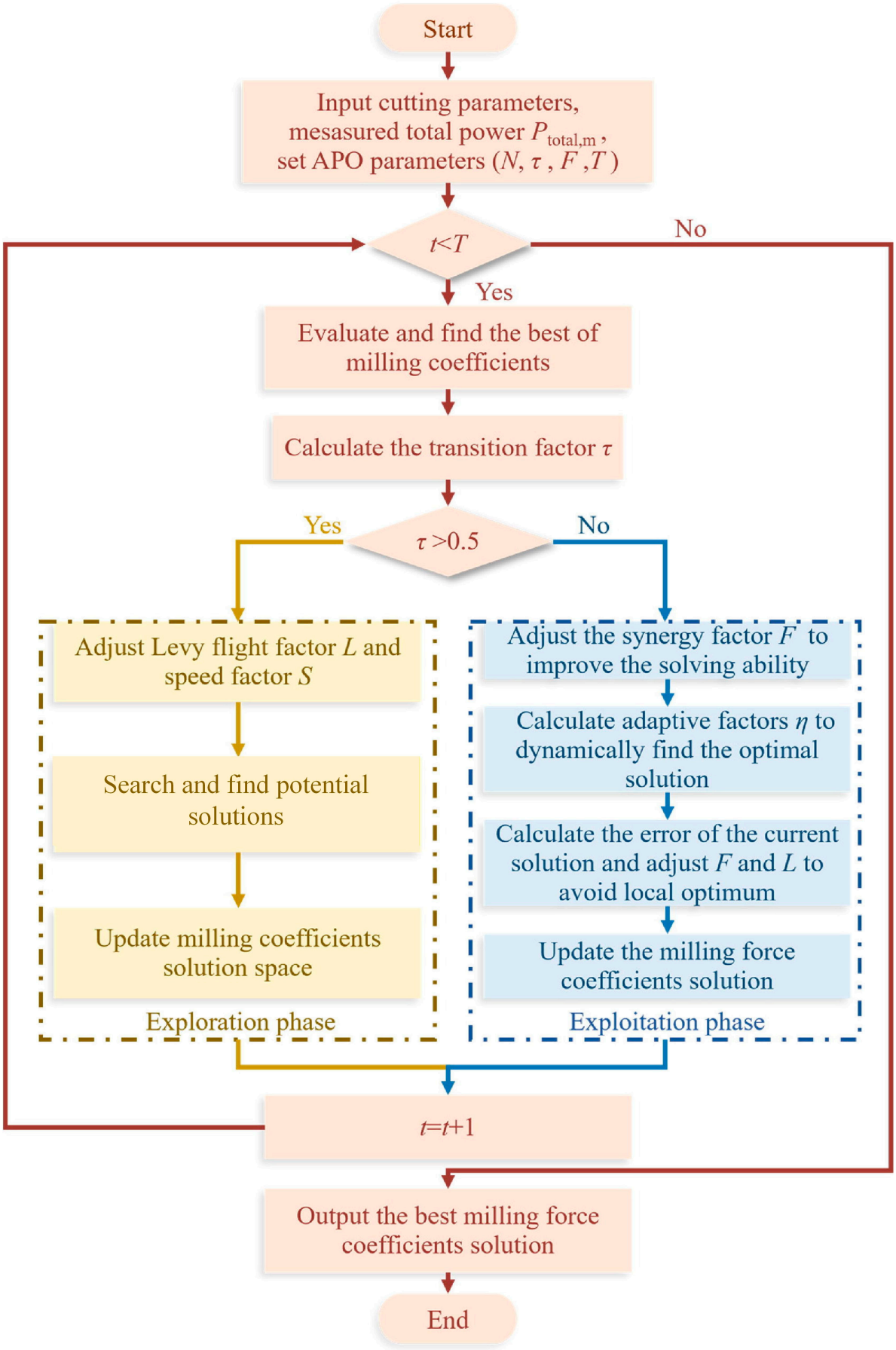
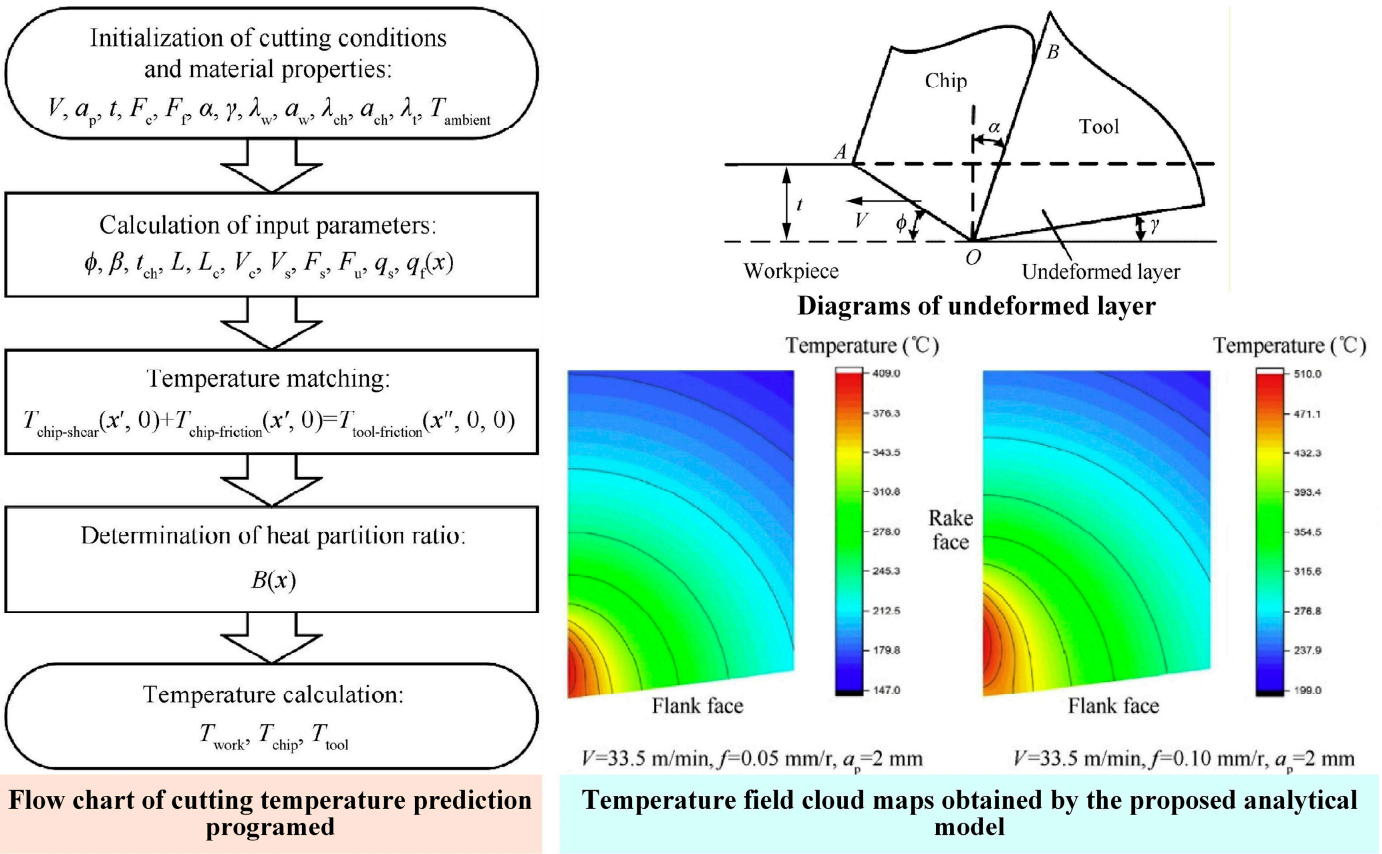


Figure 6. Flowchart of the proposed APO method for milling force coefficients [59].



Flow chart of cutting temperature prediction programed

Temperature field cloud maps obtained by the proposed analytical model

Figure 7. An improved analytical model of cutting temperature in orthogonal cutting of Ti6Al4V [61].

2.2. Numerical Model

Traditional methods for cutting force and temperature modeling in machining processes include analytical models as well as numerical simulations (such as the finite-element method and discrete element method), which serve as primary research approaches. Numerical simulations offer unique capabilities for investigating force–heat coupling phenomena during cutting operations [47,62]. The core approach involves solving a series of physical conservation equations (mass, momentum, energy) and material constitutive models governing the cutting process through discretization and iterative computation on computers. This enables the reproduction of complex phenomena such as chip formation, cutting force generation, and temperature modeling, while also capturing the complete distribution of field variables (stress field, temperature field) that are difficult to measure experimentally [63]. Second, while numerical simulation can model complex operating conditions, it incurs high computational costs. Early research primarily focused on simplified two-dimensional orthogonal cutting models, but has now been widely extended to simulate three-dimensional helical cutting, complex tool geometries, and multi-tooth milling, bringing it closer to real machining scenarios [64]. The application approach of numerical simulation in cutting force and temperature modeling is illustrated in Figure 8.

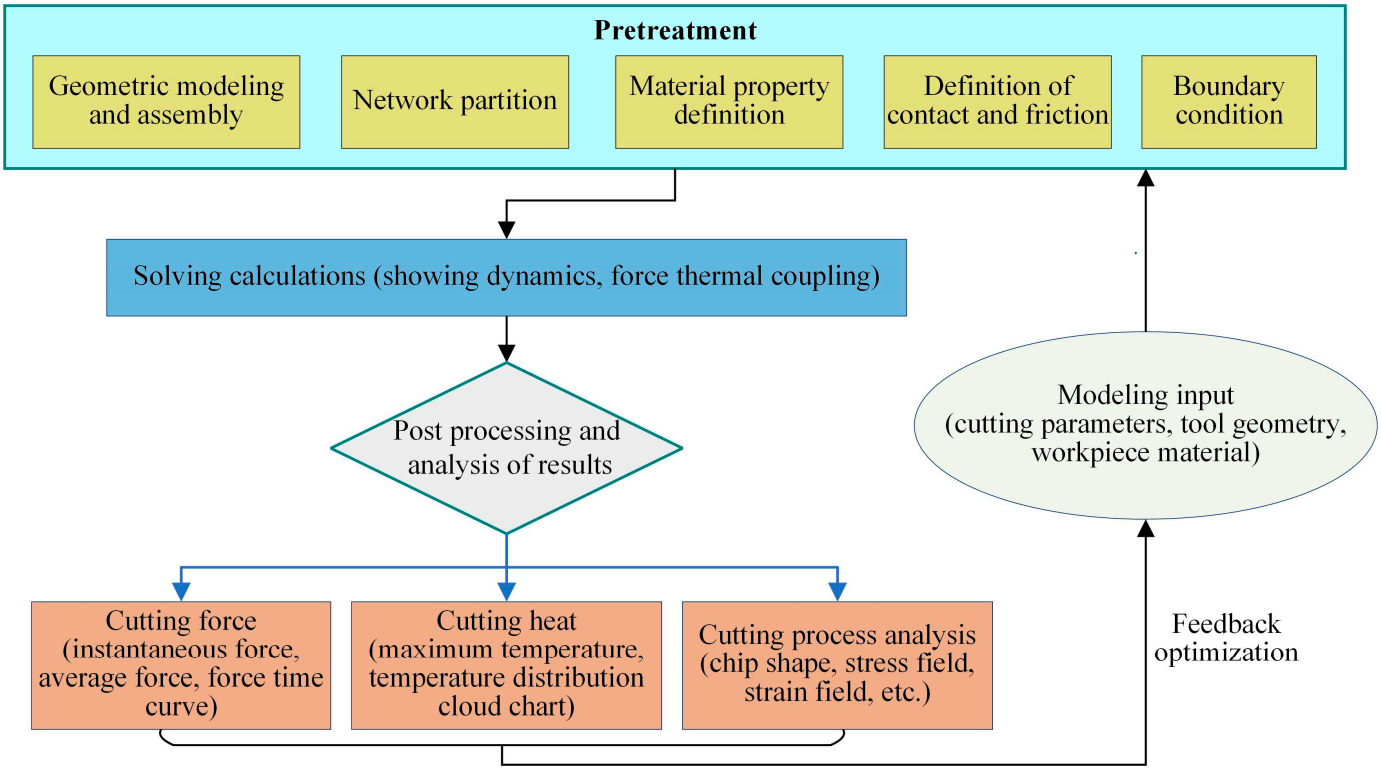


Figure 8. Application approach of numerical simulation in the modeling process of cutting force and temperature modeling.

Cutting force originates from the workpiece material undergoing plastic deformation under the tool's action and the work performed by friction between the tool's front and rear cutting edges. Numerical simulations directly obtain transient and steady-state cutting force by calculating the stress field distribution within the workpiece and integrating the stresses at the tool–workpiece contact interface [65,66]. Similarly, temperature modeling originates from plastic deformation energy (over 90%) and friction work. Numerical simulations calculate the spatiotemporal distribution of the temperature field by solving the energy conservation equation (heat conduction equation), accounting for internal heat generation (conversion of plastic work) and boundary conditions (workpiece heat dissipation and cooling) [67,68]. Hao et al. [69] developed a novel acoustic-plastic constitutive model for SiCp/Al by integrating dislocation density and crystal plasticity theory with stress superposition and acoustic softening. This model was applied to finite-element simulations and experimental validation of ultrasonic-assisted cutting. The study demonstrates that the new model improves the accuracy of cutting force prediction. The principal cutting force exhibits a monotonically decreasing trend with increasing amplitude, while higher frequencies result in a significant increase in the periodic density of the cutting force time-domain signal. As shown in Figure 9, Lu et al. [70] studied the influence of cutting paths on the machinability of SiCp/Al composite materials in multi-step ultra-precision diamond cutting by combining finite-element simulation with experimental observation and characterization. The simulation results revealed different particle tool interactions and failure modes in SiC particles, as well as the evolution of forces during the machining process.

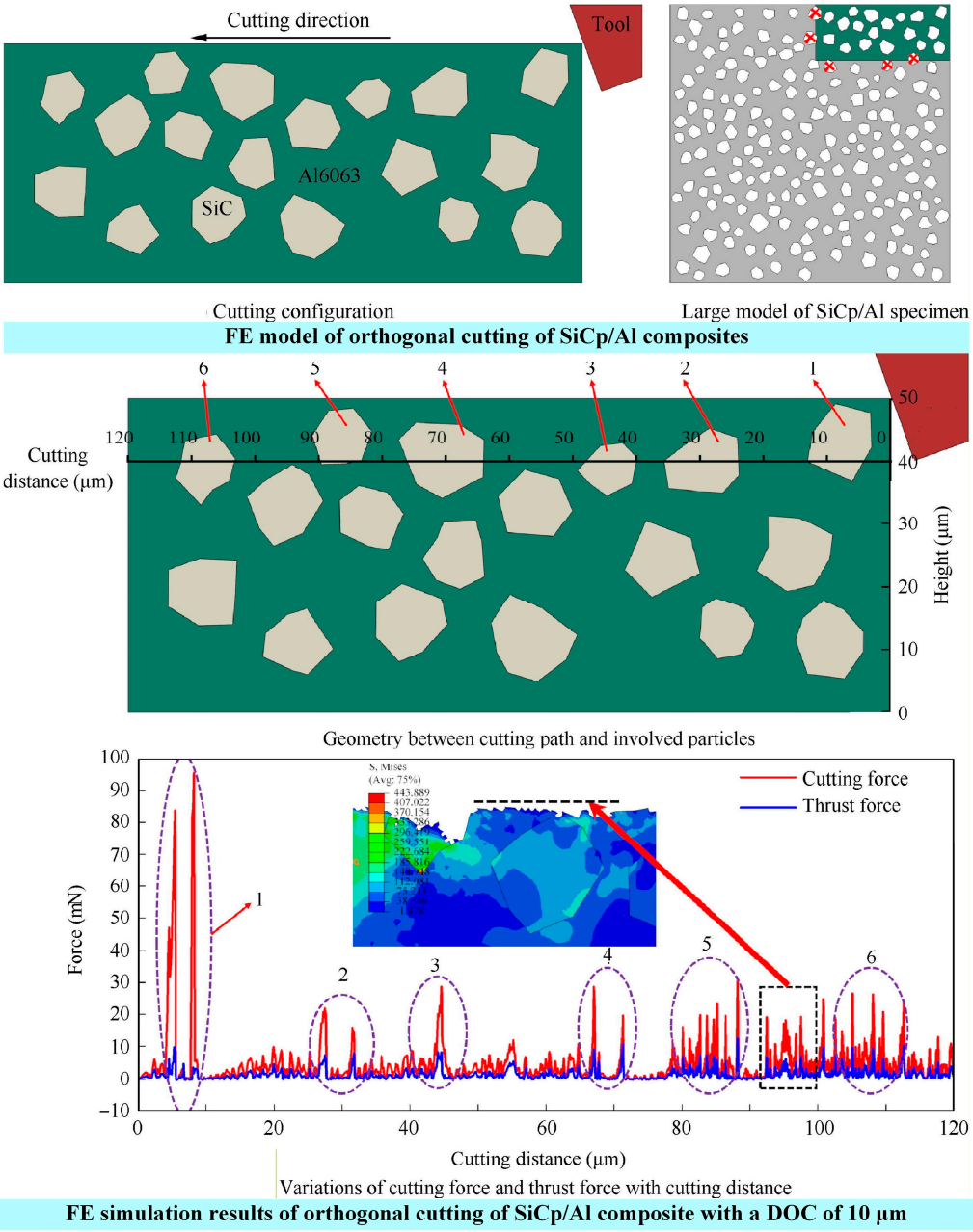


Figure 9. Cutting path-dependent machinability of SiCp/Al composite under multi-step ultra-precision diamond cutting [70].

Additionally, numerical simulation can simultaneously predict cutting force and cutting heat, employing thermo-mechanical coupling models to analyze plastic deformation of materials in the cutting zone and friction behavior at the tool-chip interface [44,71]. It can also investigate the influence patterns of process parameters on cutting force distribution, cutting temperature fields, and strain field distribution. Simulation also yields distributions of physical quantities such as stress fields, strain fields, and temperature fields, while predicting chip morphology and cutting temperature [72]. Wang et al. [73] established a three-dimensional macroscopic numerical model to simulate orthogonal cutting of CFRP at four typical fiber cutting angles, addressing the underrepresentation of strain rate effects in material modeling that often leads to inaccurate predictions of cutting and damage levels. As shown in Figure 10, the proposed model demonstrated good agreement between simulated machining processes and cutting force with experimental results for orthogonal CFRP cutting. Compared to models neglecting strain rate effects, prediction accuracy was

improved. Additionally, the influence of machining conditions on subsurface damage during CFRP processing at 135° was evaluated. Similarly, Mun et al. [74] employed the finite-element method to investigate the formation mechanism of interfacial damage in CFRP/Ti6Al4V laminates after machining, with particular focus on stress and temperature distributions. A micro-mechanical orthogonal cutting model and a three-dimensional drilling model for CFRP/Ti6Al4V laminates were developed. The study confirmed that the cutting sequence strategy significantly influences stress and temperature distributions during CFRP/Ti stack processing, which are primary causes of various interfacial damages. Li et al. [75] constructed two models for numerical simulation of continuous multi-cutting-edge rotary rock drilling. Comparisons with experimental data revealed that during multi-disk rotary rock breaking, the cutting tools exert lateral compression on the rock on both sides, thereby promoting crack propagation and penetration. Cheng et al. [76] used the simulation software to simulate finite-element cutting based on laser-assisted ultrasonic elliptical vibration turning, conventional turning, and laser-assisted turning. By analyzing the simulated cutting process, systematically analyzing the cutting mechanism, and establishing a cutting force model that combines the combined effect of composite tool temperature and SiC reinforcement, Cheng studied the influence that laws of force had on temperature characteristics under various machining conditions.

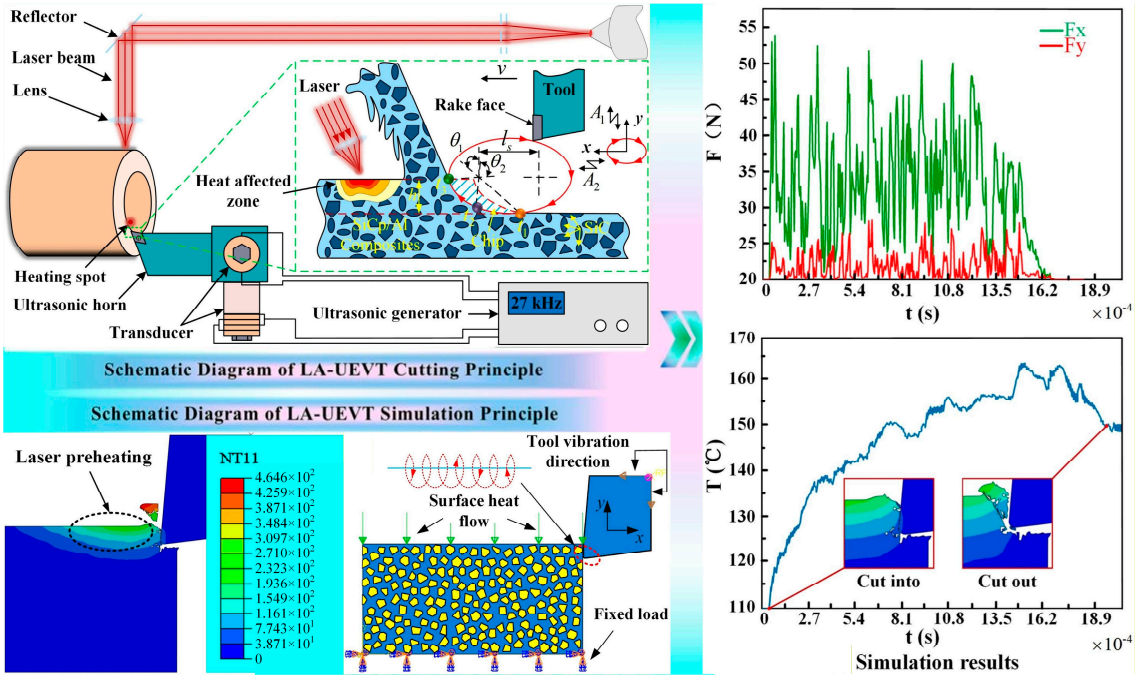


Figure 10. Laser-assisted ultrasonic elliptical vibration turning of high-volume fraction SiCp/Al force-thermal research [76].

Xu et al. [77] employed both numerical and experimental methods to investigate the cutting process during the machining of Inconel 718 with worn tools, encompassing chip formation, force variations, tool wear, and thermal distribution. The presence of tool wear induces intense friction force during tool–surface contact, generating substantial heat. This alters stress conditions and temperature distribution gradients, resulting in the formation of a distinct white layer beneath the surface. Additionally, some researchers [78] employed Lagrangian and coupled Eulerian–Lagrangian methods to investigate the orthogonal cutting of Ti6Al4V titanium alloy using different tool edge microgeometries. The developed cutting model incorporates a constitutive model (plasticity and damage) that accounts for the effects of strain, strain rate, temperature, and stress state. As shown in Figure 11, a

comparison was made between numerical simulation and experimental measurement of cutting force on SiCp/Al composite materials at a cutting depth of 0.05 mm and a cutting speed of 785 mm per second. The relative error predicted by the simulation was 10.47%. To simulate the cutting temperature field, the tool surface is divided into three regions: inclined surface, cutting edge, and side surface. The temperature at the front corner and cutting edge is significantly higher, mainly due to the strong friction between the chip substrate and the front corner [79].

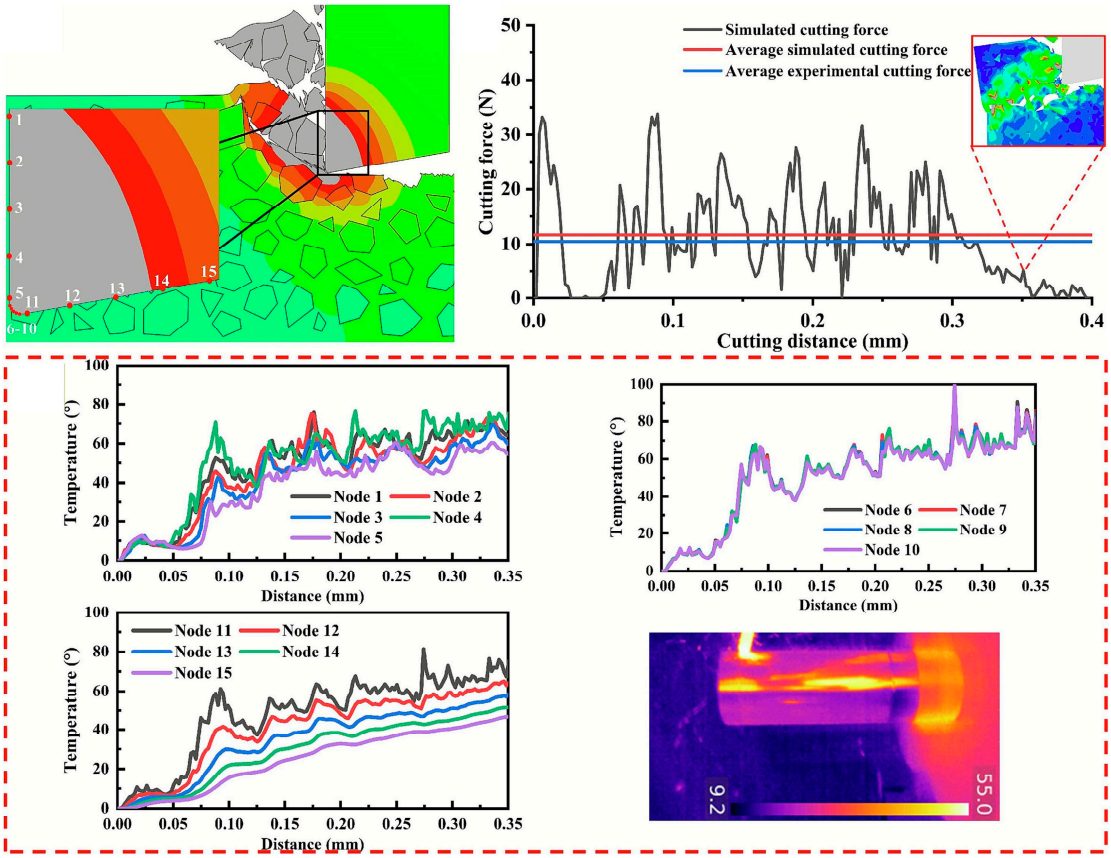


Figure 11. Comparison of simulation and experimental results for cutting force and temperature [79].

On the other hand, early cutting simulation models primarily focused on orthogonal two-dimensional cutting due to their relatively lower computational demands, facilitating mechanism studies. With advancements in computational capabilities, three-dimensional models have gained widespread application. These models can more accurately reflect the cutting processes of multi-tooth tools such as end mills and multi-abrasive grinding wheels, analyzing the periodic variations in cutting force and temperature modeling during composite energy field-assisted machining [80,81]. Tobias Wolf et al. [82] extended numerical models to account for thermal flows from chip formation to tools and fluids. They employed finite-element chip formation simulations to discuss modulation and validated results through thermal and mechanical load experiments. The derived optimized number of interruptions facilitates temperature-monitoring-based thermal tool wear management. Han et al. [83] focused on the heterogeneous deformation behavior of carbon fiber-reinforced polymer composites (CFRP) under mechanical loading. They proposed a novel cutting strategy involving epoxy coating application on pre-machined surfaces, observing that the measured cutting force exhibited less sensitivity to time compared to the simulated cutting force. Some researchers established force–heat coupled numerical simulation models for laser-assisted machining and composite drilling models. They ana-

lyzed material removal mechanisms and damage evaluation methods during machining, obtaining cutting force and temperatures through numerical models to investigate the mapping relationship between process parameters and machining quality [84,85].

In summary, future efforts should focus on enhancing the predictive capability and reliability of simulation models while developing more efficient computational methods to reduce the computational cost of three-dimensional simulations. Additionally, models should be selected and refined based on actual physical processes, such as adopting more advanced physical failure criteria or incorporating friction models that account for temperature effects. Constitutive parameters for materials under high strain rates should be obtained through tests like the SHPB, with calibration of hard-to-measure parameters achieved via methods such as inverse engineering. Finally, simulation results can be quantitatively compared with experimental data on cutting force, cutting temperatures, chip morphology, and residual stresses from multiple perspectives to validate the reliability of numerical models.

2.3. Machine Learning

Traditional cutting force models are predominantly based on analytical methods (Merchant formula) or mechanism-based modeling approaches. These methods heavily rely on physical experiments and simplifying assumptions, often proving complex and limited in accuracy when handling nonlinear, high-dimensional real-world machining conditions [86,87]. In recent years, ML has provided a powerful, new data-driven paradigm for cutting force modeling. The fundamental approach of ML in cutting force and thermal modeling is to treat the cutting process as a complex nonlinear system. ML algorithms learn the mapping relationship between input parameters (cutting parameters, tool geometry, material properties) and output responses (cutting force and temperature) from historical data, thereby constructing highly accurate predictive models [88]. In essence, the process involves defining the cutting force or temperature modeling problem, establishing an objective function, and acquiring experimental data, which may include cutting force in various directions measured by force transducers, or average temperature, peak temperatures, and thermal field distributions obtained via thermocouples or infrared thermal imaging [89]. The complete workflow for ML modeling is illustrated in Figure 12 [90].

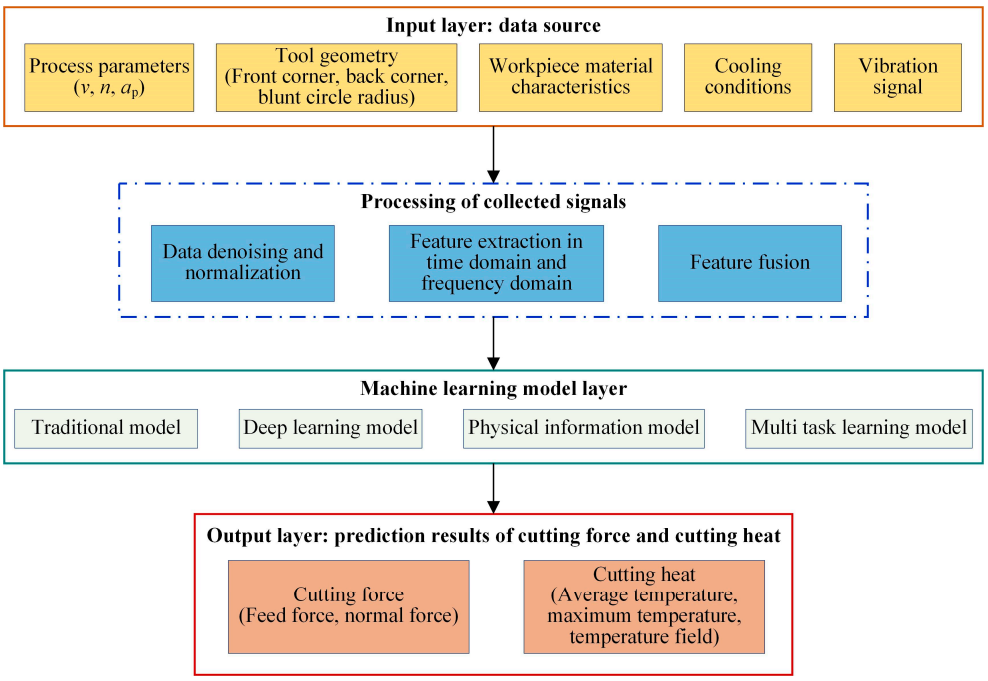


Figure 12. Application approach of ML in cutting force/temperature modeling.

The complete process of ML modeling generally consists of four parts. First is the input layer, which aggregates multi-source data, including easily controllable process parameters, constant tool parameters, workpiece attributes, and real-time sensor signals [91]. Next is the processing layer, which performs preprocessing and feature engineering on raw data to prepare it for model training. This is followed by the core model layer, where an appropriate ML algorithm is selected based on the objective (predicting cutting force or temperature modeling), data volume, and required accuracy [92]. Finally, the output layer delivers high-precision predictions of cutting force or temperature modeling, providing critical data support for process optimization, condition monitoring, and digital twin applications.

In establishing ML models for predicting cutting force, Amir Hossein Rabiee et al. [67] employed support vector regression to model and forecast cutting force and temperatures during cortical bone micro-milling processes. A large dataset was generated based on the derived model, and extended Fourier amplitude sensitivity testing was utilized to determine the validity of output variables relative to input parameters. These research findings enable surgeons to predict temperatures and force under different machining parameters prior to surgery, thereby selecting optimized machining conditions to reduce temperatures and cutting force. Lena Geißel et al. [93] proposed a Bayesian Markov-chain Monte Carlo method to predict cutting force during turning processes. The parameters of this cutting force model are conditioned on only a small number of measurements from turning experiments, yielding accurate simulations of cutting force during machining. This approach avoids the need for extensive, time-consuming, and costly experiments required for calibrating cutting force prediction models [94]. Employing three distinct ML models, Arash Ebrahimi et al. [95] proposed a hybrid modeling approach integrating physical simulation with ML for high-precision milling force prediction. This method trains ML models using a combination of limited experimental data and finite-element simulation results, significantly enhancing prediction accuracy under multi-material and multi-tool conditions. As shown in Figure 13, a novel and accurate method was introduced to characterize the characteristics of specially prepared samples at the intersection of full-field optical strain, thermal measurement, and machine learning (ML), simultaneously obtaining strain and thermal data at multiple temperatures in a single test [96].

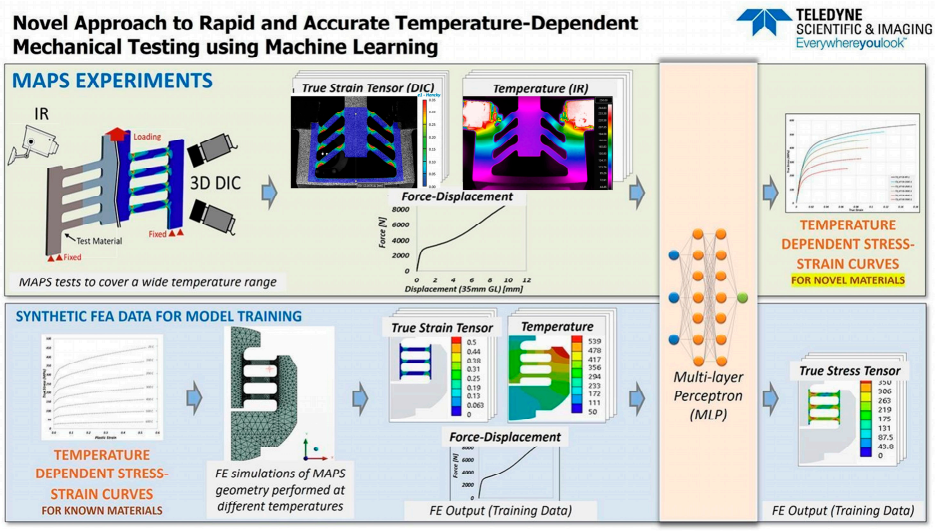


Figure 13. Novel approach to rapid and accurate temperature-dependent mechanical testing using machine learning [96].

Additionally, Xie et al. [54] proposed a novel deep learning-based instantaneous cutting force prediction model aimed at addressing the overly simplified model inputs and frame-

works inherent in traditional cutting force prediction models. They creatively represented the integrated geometric and machining information during the machining process as a multi-channel digital image (ICGPI). A deep learning network was then designed to take ICGPI as the input and output three-dimensional instantaneous cutting force. Experimental validation demonstrated the method's superior modeling accuracy. Furthermore, the ML-based identifier comprises three multi-layer perceptrons (MLPs) organized into five layers: one input layer, three hidden layers, and one output layer. Scholars developed a method that integrates finite-element modeling, ML-based predictive analysis, and multi-objective genetic algorithm (GA) optimization for the design of microgroove tools, which is illustrated in Figure 14 [97]. Machine learning algorithms were used to develop predictive surrogate models, which subsequently drove the general algorithm optimization of key geometric parameters.

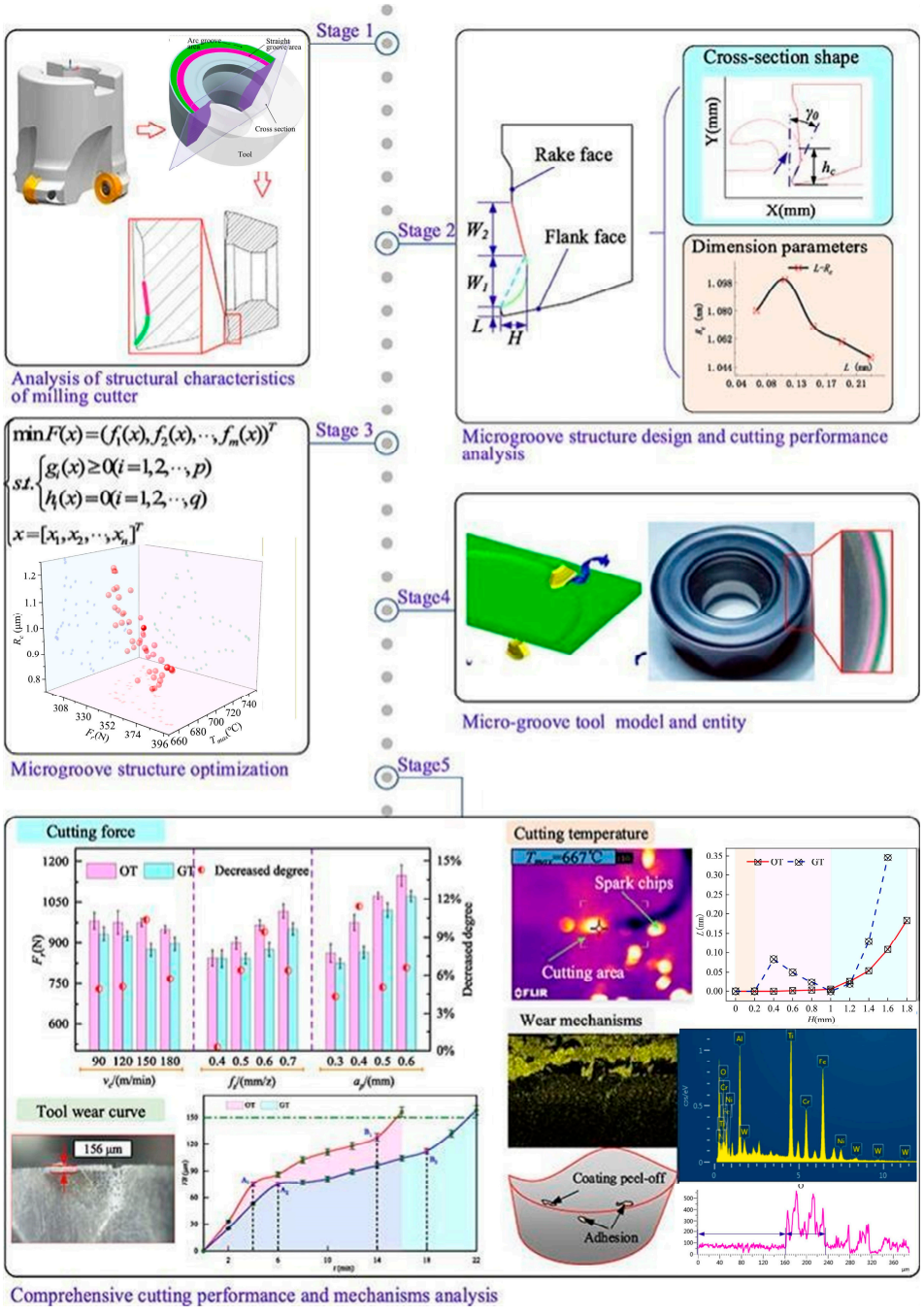


Figure 14. Design and performance analysis of wear-resistant micro-groove cutting tool; numerical simulation and experimental study [97].

Gregory W. Vogl et al. [98] developed a novel method for measuring cutting force, enabling real-time estimation based on accelerometer readings from machine tools. One approach employs ML to generate a model that estimates cutting force from machine acceleration, validating the potential for real-time monitoring of cutting force via accelerometer measurements and providing intelligence for optimizing part production. Wajdi Rajhi et al. [99] developed an optimized artificial intelligence model to predict milling surface characteristics of CFRP. In addition, Yeh et al. [100] proposed a machine learning-based method that integrates a cutting force model calibrated by a genetic algorithm (GA) with vibration analysis to optimize the feed rate during machining, which generates theoretical cutting force datasets under different processing conditions, and then uses fast Fourier transform (FFT) for frequency domain analysis to determine the feed rate that minimizes vibration, as shown in Figure 15.

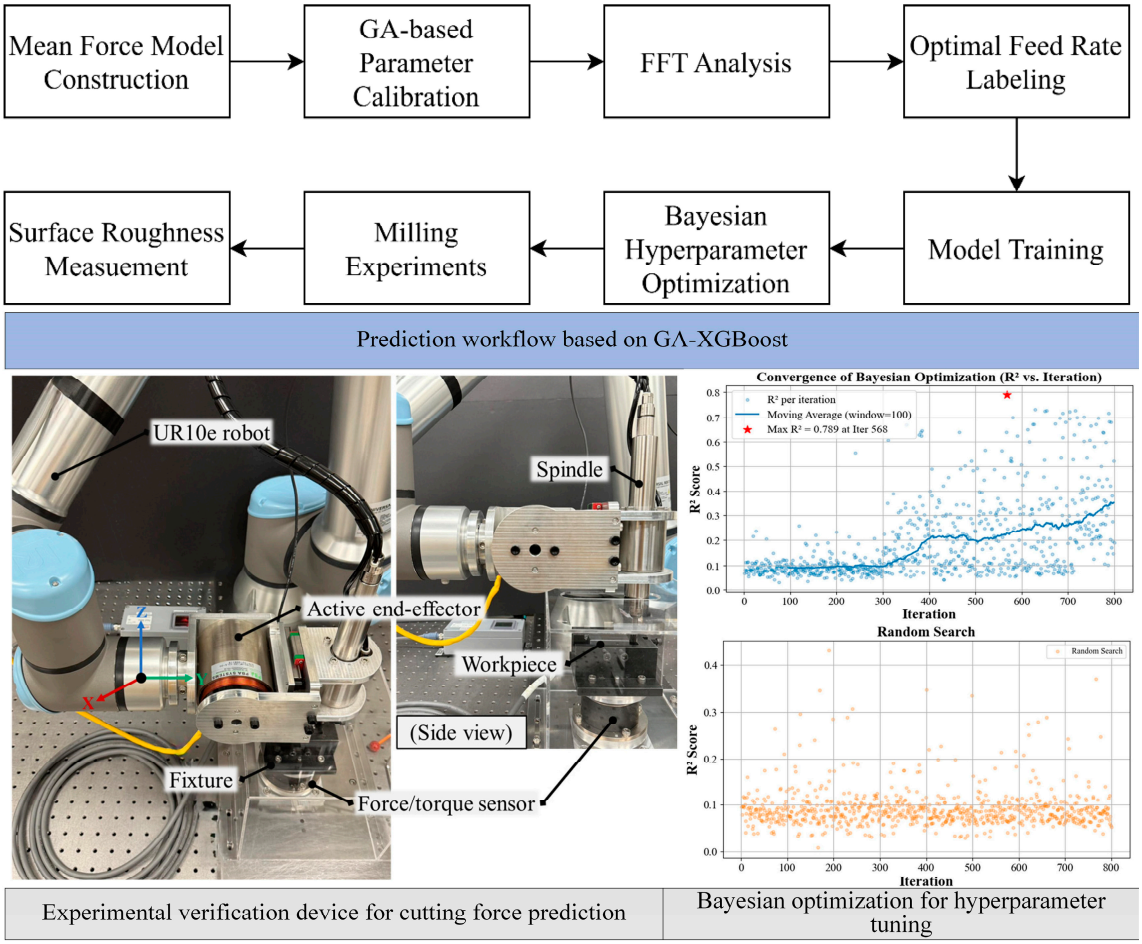


Figure 15. Cutting force modeling and vibration analysis based on machine learning [100].

This research achieved high-precision monitoring of milling force by installing accelerometers and flange force sensors on the spindle, combined with Kalman filter compensation for dynamic delays. The measured force signals showed good agreement with the reference force, achieving 95.27% accuracy in the time domain, effectively resolving the challenge of real-time cutting force monitoring in robotic machining processes.

Digital twin technology, as a core means for achieving deep integration of cyber-physical systems, has demonstrated significant advantages in modeling and measurement of cutting processes in recent years. By constructing virtual models synchronized in real time with physical cutting systems, this technology enables high-fidelity mapping and precise prediction of dynamic cutting force characteristics. Taking the research by Dai

et al. [101] as an example, to overcome the limitations of traditional cutting force models in identifying tool axis offset, they innovatively proposed a twin-data-driven modeling approach focused on tool axis offset identification in 5-axis ball-nose milling. This method first utilizes geometric modeling techniques to extract characteristic parameters of the tool axis offset from measured cutting force signals. It then constructs twin data pairs with mapping relationships between theoretically calculated critical cutting positions and measured critical cutting positions. By minimizing the discrepancy between twin data through intelligent optimization algorithms, it achieves high-precision, high-efficiency identification of tool-axis offset parameters. This research not only demonstrates the adaptability of digital twin technology in complex manufacturing scenarios but also showcases its significant potential for enhancing the modeling and measurement accuracy of cutting processes. As shown in Figure 16, adopting multi-objective response surface methodology (MORSM) to optimize cutting speed, feed rate, and axial cutting depth, with the aim of improving machining efficiency and product quality [102].

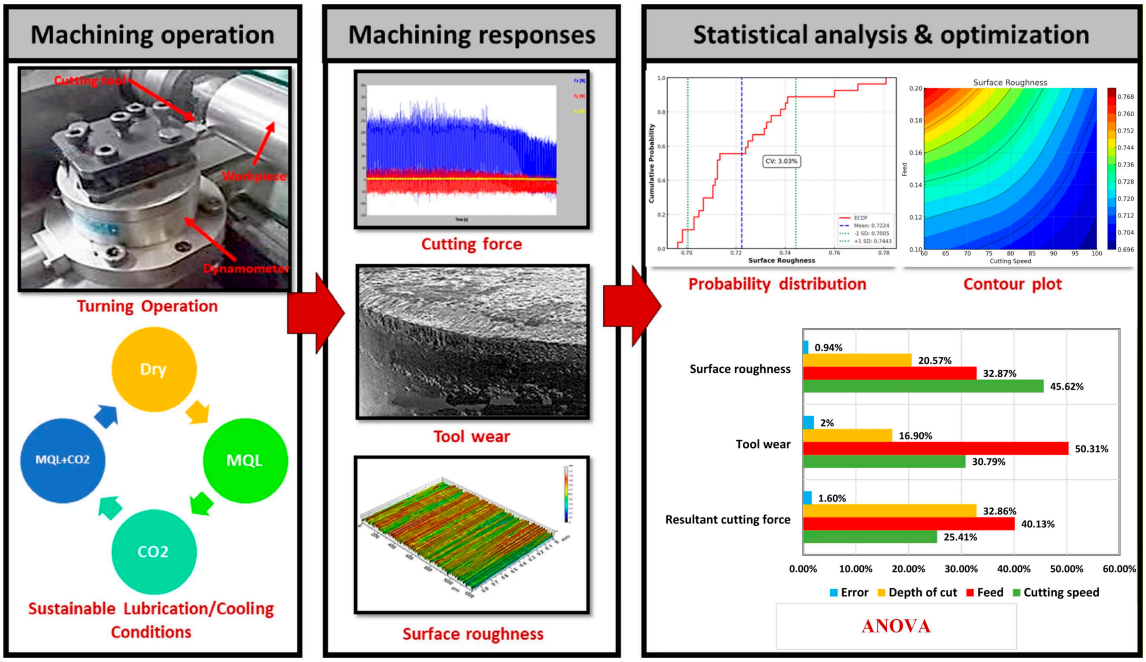


Figure 16. Multi-objective optimization framework for sustainable machining in cutting force [102].

Therefore, ML methods have significantly reduced the difficulty of constructing high-precision cutting force models under complex operating conditions, decreased reliance on traditional massive experimental data, and demonstrated outstanding predictive performance. However, their development still faces challenges: the interpretability of these models is weaker than that of models established based on cutting mechanisms. Furthermore, the predictive performance of the models is highly dependent on the acquisition of high-quality, large-scale labeled datasets, and engineering difficulties exist. Future research will increasingly focus on small-sample learning, transfer learning, and the deep integration of physical models to develop more reliable, versatile, and efficient intelligent cutting force models [103]. ML is also applied to temperature modeling. Utilizing ML for temperature modeling has emerged as a crucial research direction in recent years to address the limitations of traditional methods. The advantage of ML lies in bypassing complex and uncertain physical mechanisms (constitutive relations, friction models, and boundary conditions) by establishing data-driven nonlinear mapping relationships from machining input parameters to thermal outputs (temperature, thermal distribution). Some researchers

have explored using ML to directly predict temperatures from machining parameters and sensor data, or incorporating heat transfer laws into neural networks to compensate for data deficiencies. Chen et al. [104] investigated ablation mechanisms in CFRP laser cutting, identifying two processing mechanisms: thermal ablation caused by material vaporization from accumulated laser energy, and mechanical deposition resulting from material detachment from the substrate due to plasma plume impact. Zhou et al. [105] discovered that holes for displaced embedded thermocouples compromise cutting-edge strength, and chips obstruct the line-of-sight between the tool tip and thermal imager. They proposed two feedforward multilayer perceptron artificial neural network (ANN) models using the Levenberg–Marquardt backpropagation training algorithm to determine steady-state tool tip temperatures during dry turning operations. Additionally, Wang et al. [106] proposed a process optimization framework integrating molecular dynamics simulation, ML, and high-throughput optimization algorithms. Physical insights obtained through molecular dynamics enriched the dataset for training ML models, thereby reducing the required data volume. ML rapidly and accurately established regression models between laser parameters and target machining performance, while high-throughput optimization algorithms determined optimal processes within the process space. For example, based on the SSA algorithm, an alternative model of BP neural network has been improved to achieve rapid prediction of residual stress and cutting force, as shown in Figure 17 [107].

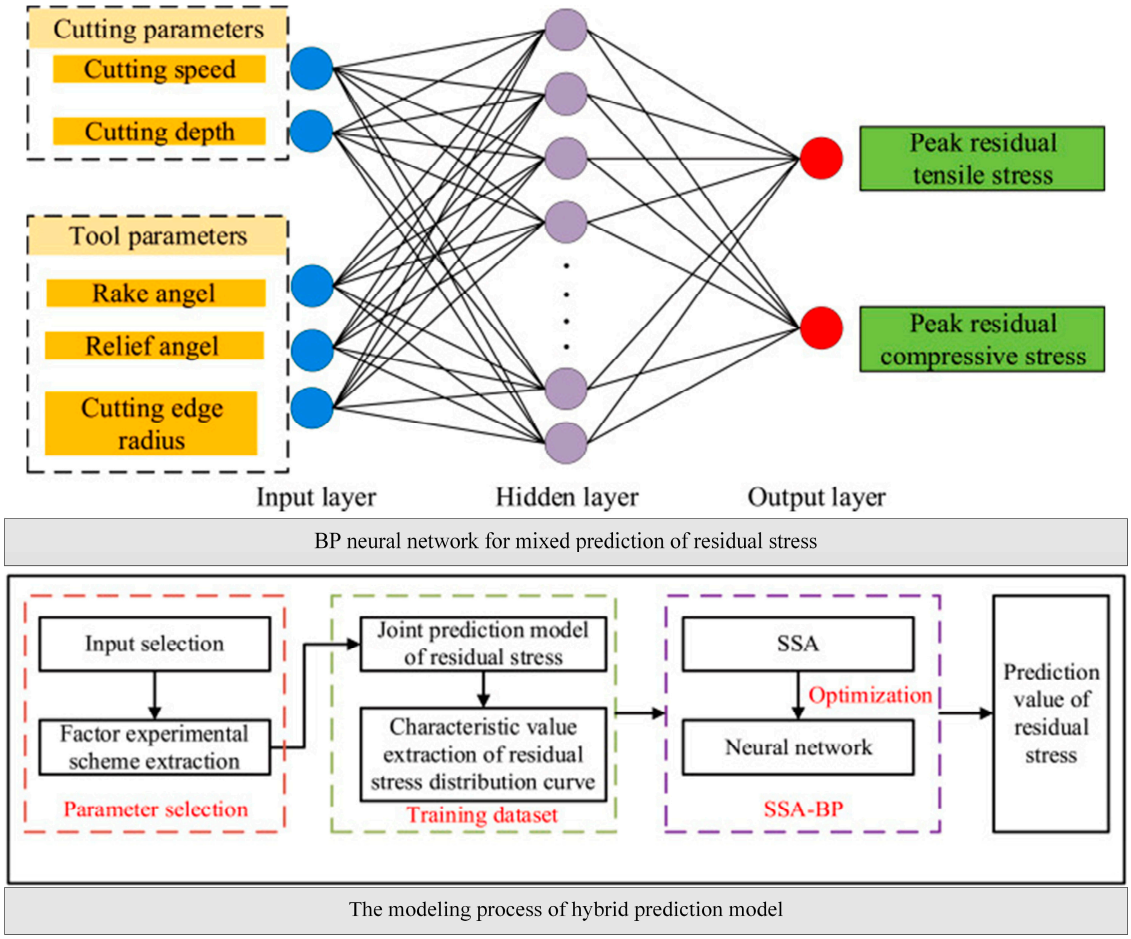


Figure 17. Using finite-element analysis neural network for hybrid modeling to predict residual stress [107].

In summary, the application of ML in cutting force/temperature modeling has evolved from simple regression fitting to complex intelligent perception paradigms. Its core value

lies in establishing end-to-end nonlinear mapping relationships from machining parameters and sensor signals to target outputs (force/temperature), effectively circumventing the complex challenges of constitutive relationships and boundary condition settings inherent in traditional physical modeling. First, algorithmic diversity has expanded from early traditional methods like support vector machines and random forests to today's dominant deep learning models. These ML algorithms efficiently predict cutting force with high precision directly from cutting parameters (spindle speed, feed rate, cutting depth), tool geometry, and sensor signals (current, vibration, acoustic emission). Second, multi-source data fusion has expanded research beyond process parameter inputs. By integrating multi-sensor data (force, vibration, acoustic emission) and employing deep learning for feature extraction, end-to-end cutting force/temperature prediction models have been developed, significantly enhancing model generalization and robustness. Finally, the integration of physical mechanisms with data emerges as a key advancement. Emerging Physical Information Neural Networks attempt to embed physical conservation laws (energy conservation) as constraints within neural network training. This ensures predictions remain physically consistent even in regions with sparse training data, representing a crucial direction for next-generation intelligent modeling.

2.4. Integrated Discussion on Cutting Force/Temperature Modeling

The predictive accuracy of cutting force and temperature modeling forms the foundation for enabling perception and prediction capabilities across the entire intelligent machining system. Comparison of modeling methods for cutting force/temperature is shown in Table 1. Its output data serves as critical inputs driving downstream optimization and control decisions. This synergistic relationship manifests in two key aspects: First, it supports toolpath planning, where dynamic force/heat prediction models provide the core basis for path planning. Based on predicted cutting force distributions, adaptive feed rate optimization can be performed. Feed rates are reduced in high-load zones to protect tools and workpieces, while increased in low-load zones to enhance efficiency, achieving a comprehensive optimization of machining time, tool wear, and surface quality. Additionally, predicted temperature fields aid in planning cooling strategies or adjusting tool trajectories to prevent localized thermal damage to the workpiece. On the other hand, chatter occurrence is closely related to dynamic forces during the cutting process. Physics-based or data-driven force models form the core for constructing time-varying delayed differential equations and plotting stability phasor diagrams. Accurate models predict stable regions across varying spindle speeds and depth-of-cut combinations, providing direct guidance for selecting chatter-free process parameters during the decision phase. In active control, force models serve as the basis for feedforward system design, enabling the control system to preemptively compensate for periodic cutting force disturbances and enhance vibration suppression performance.

Table 1. Comparison of modeling methods for cutting force/temperature.

Model Type	Principle	Parameter Source	Applicability Stage
Empirical/analytical models	Empirical formulas	Small number of experiments	Process design, online rough prediction
Mechanistic Models	Mechanics of shear zone, friction zone	Calibration tests	Process analysis, optimization
Finite-Element Models	Numerical solution of governing equations	Fine mesh and model	Offline research, mechanism verification
ML	Learning mapping relations from data	Large experimental datasets	Online monitoring, prediction for specific systems

3. Tool Path Planning

The core task of toolpath planning is to generate the optimal spatial motion trajectory of the tool while satisfying constraints such as geometric accuracy, surface quality, and machining efficiency [108].

3.1. Mathematical Methods in Tool Path Planning

Toolpath planning deeply integrates various mathematical methods, including computational geometry and differential geometry, matrix and linear algebra, mappings and geometric transformations, among others [109,110]. Specifically, non-uniform rational B-splines serve as the primary tool for describing complex curved surfaces, enabling precise representation of workpiece geometry and generation of smooth tool paths. Furthermore, complex 3D surfaces are mapped onto a 2D parametric domain, where planning is performed before being inverse-mapped back to 3D space, simplifying the planning problem. Matrix and linear algebra are applied for least-squares plane fitting, coordinate transformations (calculating tool positions from tool contact points), and other operations [111]. Tool path planning is also a critical technology in aerospace and gear manufacturing, where triangular meshes commonly model these complex surfaces. Huang et al. [112] proposed a novel toolpath planning method for machining triangular meshed surfaces, obtaining toolpaths based on an offset heat geodesic field. Compared to sawtooth paths, the proposed method achieved a 24.28% increase in machining efficiency while maintaining the same scallop height constraint. Furthermore, a complete discrete contour path design was established based on a discrete non-iterative semi-geodesic algorithm by directly solving the first-order differential equation of tangent vectors on triangular meshes, particularly suitable for complex mandrel contour path design [113]. Wang et al. [114] proposed a novel method to compute a smooth preferred feed direction field with a high material removal rate for constructing CNC tool paths in 5-axis machining with ball-end mills. As shown in Figure 18, this method integrates harmonic functions and geometric dimensionality reduction [115]. By focusing on the accurate prediction of interpolation points between adjacent CC points, the accuracy of nonlinear error prediction and compensation within each interpolation cycle has been improved.

In 5-axis CNC machining of complex curved surface parts, the constant residual height method is widely applied to ensure surface quality by controlling residual height. Simultaneously, smoothing algorithms can effectively reduce machine tool vibration, enhancing machining stability and precision. Li et al. [116] proposed a hybrid robot non-singular toolpath generation method based on the post-processing stage. They developed an S-curve weighting algorithm for C-axis path optimization, which can be integrated into CNC systems. Furthermore, an efficient feed rate scheduling method with drive limits was proposed to achieve rapid and stable motion within singularity neighborhoods. The effectiveness of this approach was validated through simulations and experiments on the hybrid robot. Yan et al. [117] addressed cutting force causing blade deformation and vibration during 5-axis milling of turbine blades, which ultimately affect milling accuracy and surface quality. Firstly, NURBS theory is employed to obtain surfaces before and after finishing. Next, milling information, such as coordinate systems, is extracted from the NC code. Subsequently, the boundary is extracted from the NURBS surfaces, and information is derived. The proposed model combines discrete and analytical methods. In addition, the concepts of size reduction and mapping were introduced. By discretizing the tool path and extracting new base variables, an abstract two-dimensional space was constructed, which has a mapping relationship between tool position points in surface 5-axis machining [118], as illustrated in Figure 19.

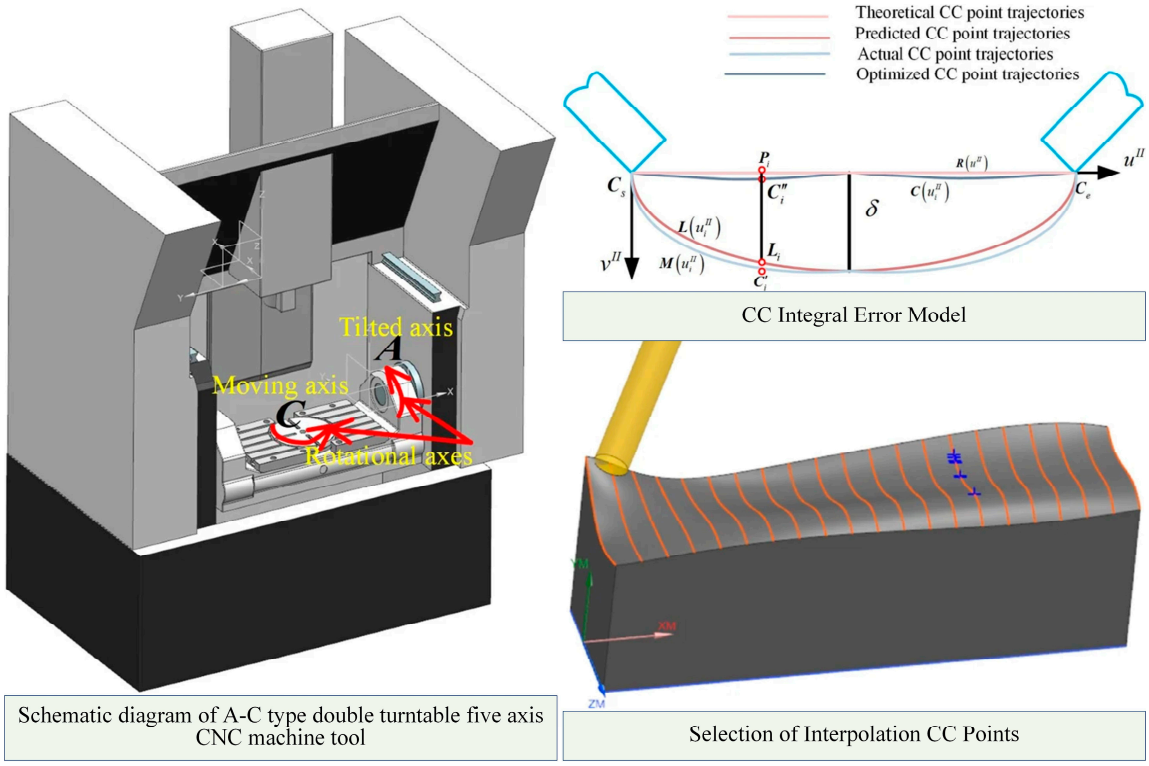


Figure 18. A tool contact point trajectory prediction method combining harmonic functions and dimensionality reduction methods [115].

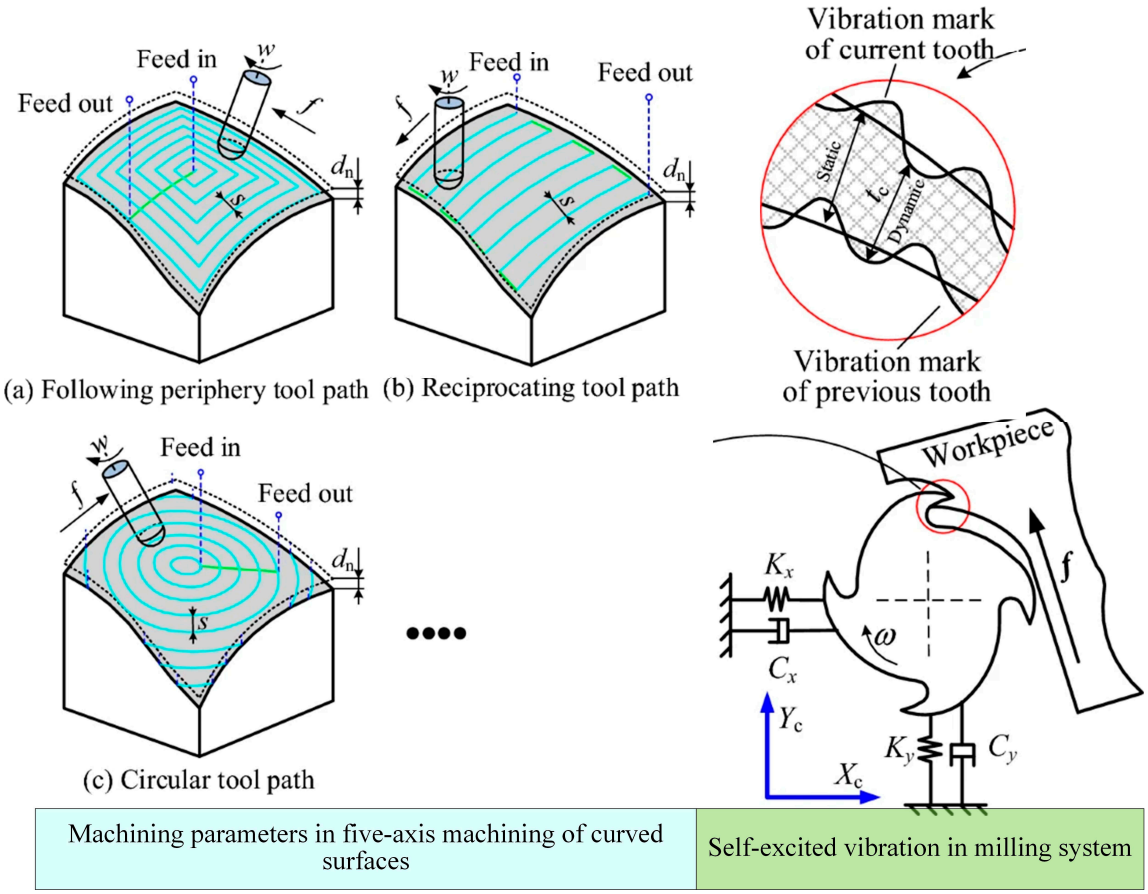


Figure 19. Dynamic analysis model for 5-axis machining of curved surfaces based on size reduction and mapping [118].

Beyond this, NURBS interpolation technology has become a standard feature in CNC systems, enabling high-precision, high-speed 'flexible' machining [119]. Tool path planning has also evolved from purely geometric considerations to physically constrained path generation. During path planning, efforts are made to maintain stable cutting force, minimize fluctuations, and extend tool life. Second, regarding dynamic constraints, the servo systems and joint dynamics of machine tools must be considered to generate paths with continuous acceleration and deceleration. This maximizes machine performance while preventing overshoot and vibration. Tool path planning is no longer an isolated post-processing step but is deeply integrated with process design, fixture layout, and error compensation, becoming a vital component of digital twins and intelligent manufacturing systems [120]. A systematic summary of mathematical descriptions and modeling approaches in tool path planning is shown in Figure 20.

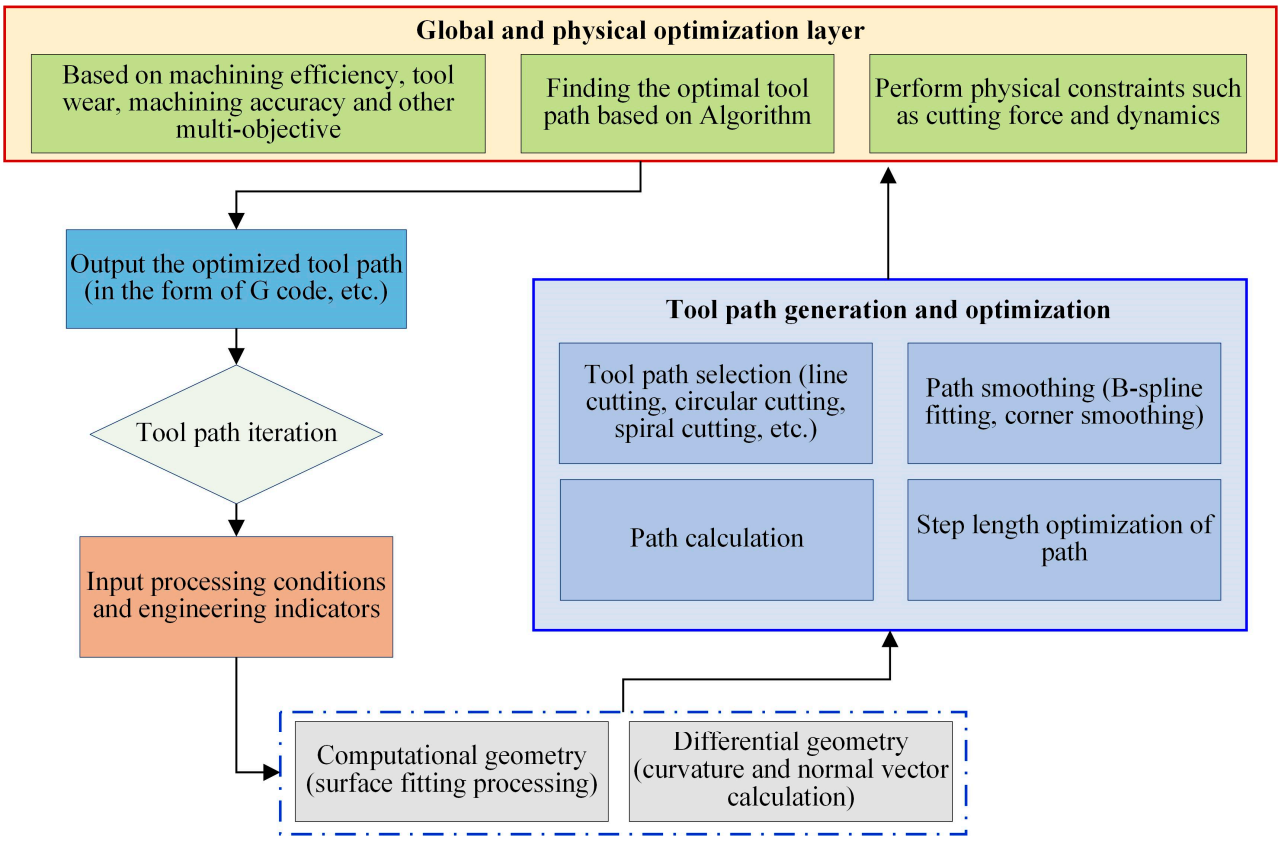


Figure 20. Mathematical description and modeling ideas in tool path planning.

Tool path planning for 5-axis single-point diamond turning is a complex decision-making process. While ensuring a reasonable distribution of contact points (CC points), geometric constraints must also be considered to avoid interference caused by adjusting the tool-axis orientation [121]. Wang et al. [122] proposed a tool-path planning method for 5-axis turning, encompassing both driving point planning and tool axis orientation planning. Research findings established a mathematical model for tool contact points by integrating the mapping relationship between scallop height and path parameters. Furthermore, driving points can be generated on complex free-form surfaces, facilitating the analytical determination of tool contact points while accounting for scallop height. Secondly, based on workpiece information obtained from initial machining, a secondary planning method for tool contact points compensates for machining errors. This approach effectively enhances contour accuracy, achieving approximately 66.7% improvement. As shown in Figure 21, the sudden increase in radial depth is a unique phenomenon in milling,

usually characterized by a sharp increase in cutting force at the end of the tool's axial feed, accompanied by harsh machine vibration sounds, which can have a negative impact on the reliability of submerged milling. Shen et al. [123] proposed an efficient path planning algorithm tailored for porous structures, employing an adaptive equal-shell-height method to create a parametric mesh. Subsequently, a traveling-cell algorithm constructs a cell mesh capable of preserving arbitrary boundaries. Based on the graph structure naturally induced by traveling cells, a minimum spanning tree is established using a specified weighting method, and paths with singular start and end points are generated on this tree.

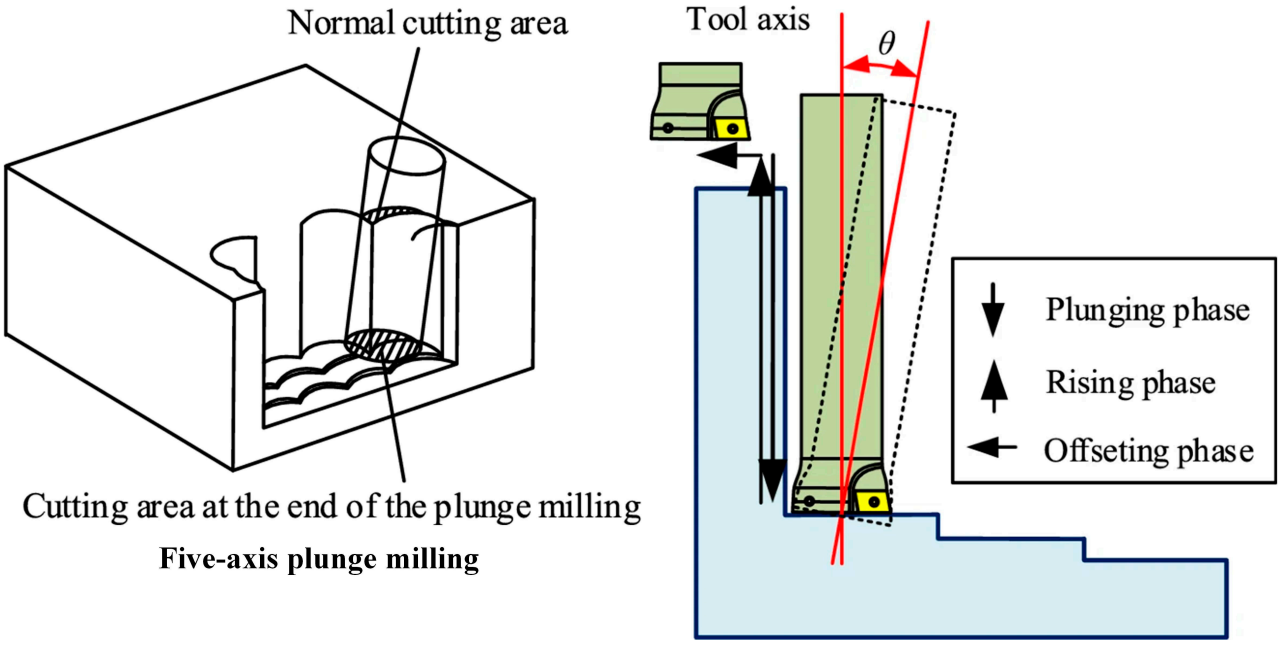


Figure 21. Optimization method for tool path in 5-axis milling, considering radial depth [124].

For triangular mesh surfaces, harmonic mapping can be employed to project the surface onto the unit disk domain. Subsequently, toolpaths satisfying line spacing requirements can be generated based on the shortest boundary path map [125]. For point-cloud data, toolpaths can be directly generated from the point cloud. For instance, by segmenting the machining area, partitioning based on geometric complexity, and assigning different tools to distinct regions, efficiency can be enhanced. Facing the significantly increased complexity of robot path planning in industrial manufacturing, researchers [126] parameterized the hard and soft constraints of the objective function and underlying optimization problems to find optimal robot joint space paths, thereby addressing complex continuous manufacturing path planning tasks. Furthermore, 5-axis milling is widely employed for manufacturing parts with straight-grain surface designs, such as centrifugal impellers, turbine blades, and fan blades. However, toolpath generation for fairing end-face milling remains challenging due to compact structures prone to interference and difficult-to-control machining errors. To date, iterative toolpath adjustments to enhance accuracy and avoid interference remain common practice. As shown in Figure 22, Sun et al. [127] proposed a toolpath generation method based on a divide-and-conquer strategy to handle multiple geometric constraints in 5-axis end-face milling. By optimizing a concise model using meta-heuristic algorithms, they obtained a tool orientation pool with acceptable tool surface deviations.

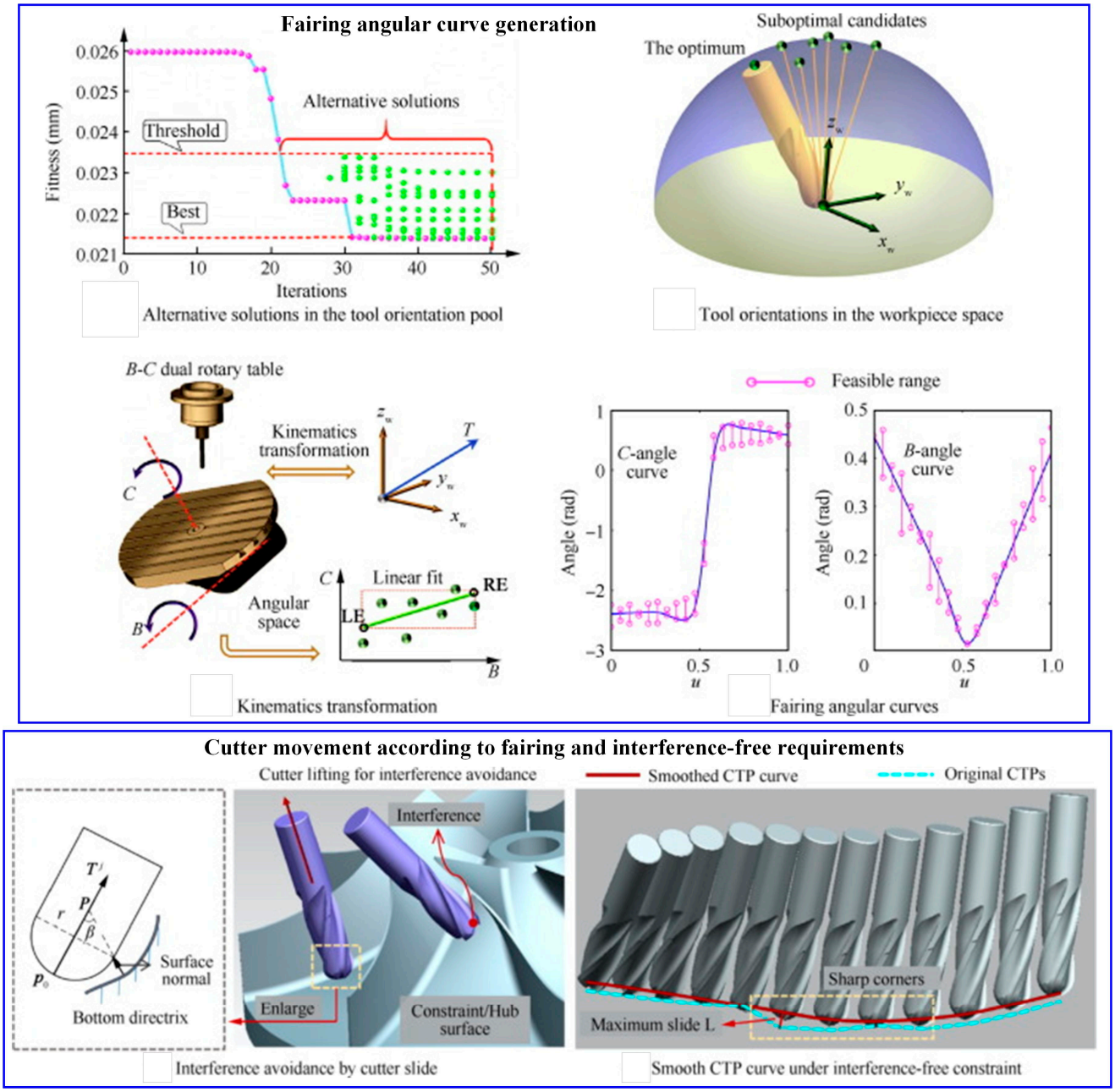


Figure 22. Tool path generation method for 5-axis end milling of ruled surface considering multiple constraints [127].

Tool path planning is a key research focus for machining complex and free-form surfaces, while enhancing machining efficiency remains crucial. To optimize efficiency, large tools can be used for flat areas and small tools for intricate regions, reducing idle tool movement and frequent tool changes [128]. Concurrently, applying ML techniques to path planning in complex dynamic environments improves decision-making robustness and efficiency. Simultaneously, research focuses on the combined optimization of various interpolation methods (linear, circular, spline) to generate more efficient composite toolpaths [129]. Besides, comparison of tool path planning strategies with different optimization objectives is shown in Table 2.

Table 2. Comparison of tool path planning strategies with different optimization objectives.

Planning Strategy	Primary Optimization Objective	Algorithm Complexity
Geometric shortest path	Reduce non-cutting travel time	Low
Constant material removal rate	Maintain stable cutting load	Medium
Dynamics-constrained optimization	Avoid chatter, smooth motion	High

Tool path planning deeply integrates multiple mathematical tools. To more intuitively demonstrate how mathematical methods are applied across different stages and scenarios of tool path planning, the mathematical methods designed for tool path planning are summarized in Table 3 [130–132].

Table 3. Application of some mathematical methods in tool path planning.

Application Objectives	Mathematical Methods Involved	Notes
Geometric description and interpolation	Computational geometry	Parametric curves/surfaces (NURBS, Bézier) describe tool paths, isometric curves generate circular cutting paths, etc.
Path smoothing	Differential geometry (B-spline curve, least square fitting, etc.)	Curvature calculation guidance point encryption (high curvature area), normal vector control tool attitude, Gaussian spherical mapping optimization tool-axis vector
Global path optimization	Numerical calculation	Particle swarm optimization (PSO) for curvature optimization and simulated annealing for NURBS control point weight optimization
Segmentation and optimization of processing area	Optimization algorithm, normal vector calculation	Geometric division of point cloud region
Accuracy compensation Tool location and tool-axis planning	Numerical calculation and interpolation/coordinate transformation, spherical mapping	Spline interpolation smoothing sensor signal, adaptive iterative adjustment of interpolation points, etc.
Dynamic path planning	Deepening learning and other ML	-

Future research directions in tool path planning are shifting from geometry-driven approaches toward performance-driven and intelligence-driven methodologies, pursuing high precision, high efficiency, high smoothness, and strong adaptability. Key developments can be pursued in three primary areas:

(1) Precision and error control. Optimized algorithms can compensate for robotic absolute positioning errors, significantly enhancing path accuracy. Combining techniques such as the constant residual height method and smoothing algorithms improves machining smoothness and precision [133].

(2) Full-process optimization. Tool path planning should integrate more closely with process planning, cutting parameter optimization, error compensation, and online inspection to form a closed-loop intelligent machining system [134].

(3) Big data cloud-based collaborative control. Complex tool path planning algorithms for subsequent machining stages can be executed in the cloud, while lightweight models or instructions are downloaded to the machine tool for execution, addressing computational resource bottlenecks.

3.2. Integrated Discussion on Tool Path Planning

Tool path planning essentially serves as the decision-making hub of the machining process. The tool trajectories and feed rate schemes generated by planning algorithms (such as optimization based on constant material removal rate) constitute the input boundary conditions for the force/thermal model. A complex decision alters the cutting geometry and conditions, thereby driving the model to perform new predictive calculations. Simultaneously, tool vibration states can be directly avoided through optimized tool decisions (such as altering tool entry/exit angles), employing variable-speed milling, or scheduling machining idle strokes. Conversely, improper tool path planning may introduce significant force impacts or periodic excitation. Thus, tool path planning serves as the bridge connecting predictive models to actual tool processing states, forming the cornerstone of achieving global optimization.

4. Vibration Suppression

Vibration suppression in cutting is a deeply interdisciplinary field involving mechanical dynamics, control theory, signal processing, computer science, and other disciplines [135]. Vibration in the cutting process, due to the irregular change in cutting thickness caused by regenerative chatter, the amplitude of the cutting force increases and fluctuates violently at the place where chatter occurs [136]. Vibration suppression has always been the core challenge in the field of precision and ultra-precision cutting [137]. In aerospace, some thin-walled parts and precision actuators are widely required, such as engine blades, frames, impellers, bladed disks, bulkheads, and fuselage skins. Generally, the lack of protection during cutting will lead to vibration, resulting in poor surface quality, or flutter in some cases [138].

4.1. Mathematical Methods in Vibration Suppression

Aiming to address structural vibration suppression during sandwich structure machining, researchers designed and fabricated a novel fully composite double-layer tube-core sandwich structure to achieve vibration suppression and enhanced mechanical properties [139]. Additionally, poly (methyl methacrylate) foam was filled within the sandwich structure to improve structural vibration characteristics (i.e., natural frequency and mode shapes). Beyond structural design to suppress chatter during machining, tool wear during cutting also induces changes in dynamic performances, namely, process damping. Researchers proposed a novel method for monitoring tool wear through the online identification of cutting vibration-damping behavior [140]. To accurately identify time-varying process damping during increased tool wear, vibration components near the dominant mode of the cutting system were extracted from cutting vibration signals. It was found that the local average damping ratio of milling vibrations increased with tool wear progression. Yan et al. [141] proposed a flexible vibration-damping device for slender turning tools during internal hole turning, capable of enhancing damping capacity or adjusting tool frequency. Based on this, an optimization strategy for tool dynamics adjustment related to cutting conditions was introduced. Experimental results validated the effectiveness of the flexible device and corresponding adjustment strategy in preventing tool chatter and improving surface quality. Zhuang et al. [142] addressed the impact of machine tool vibration on surface roughness during metal mirror finishing. They employed transfer matrices and predictive correction methods to establish a dynamic model identifying different-order vibration frequencies of Z-axis components. Through extensive random simulations, a probability density distribution of vibration frequency bifurcation with workpiece geometry variation was established. Based on this, an adaptive optimization algorithm for interpolation point positioning was proposed. Experimental validation demonstrated

that the model's prediction exhibited a relative error of less than 10% compared to actual measurements, and vibration amplitude was reduced by over 50% after optimization via the interpolation algorithm. As shown in Figure 23, scholars have proposed an adaptive vibration control method for wind tunnel models and designed a split-type adaptive vibration-reduction structure. Secondly, they derived a multidimensional vibration characteristic characterization method and designed a vibration characteristic recognition method for the system [143].

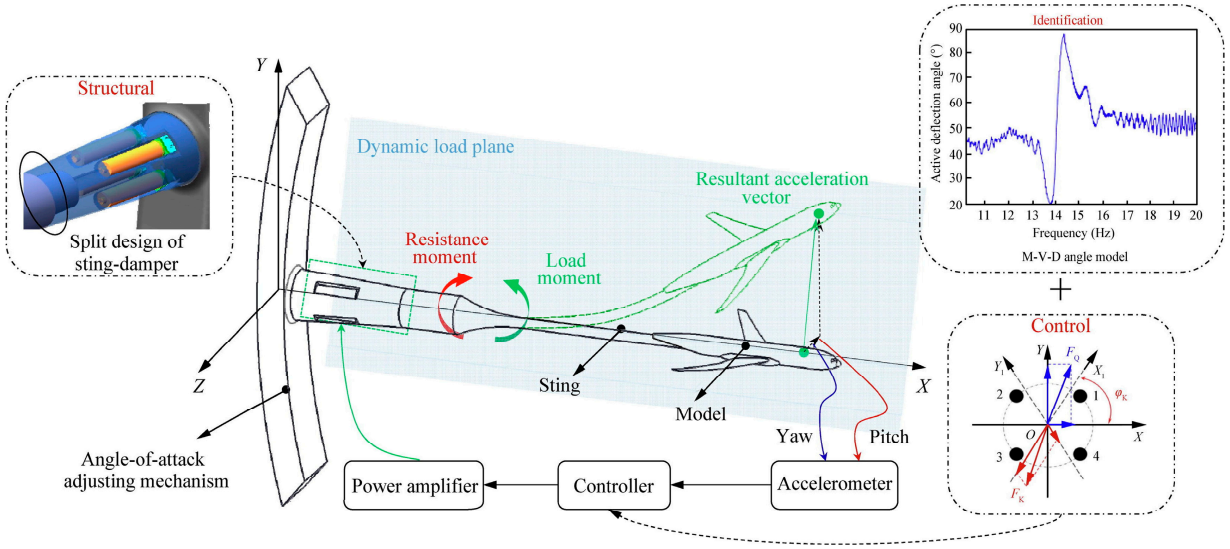


Figure 23. Adaptive vibration control method for wind tunnel model [143].

In addition to the above methods employing passive vibration suppression (tuned mass dampers) and semi-active vibration suppression (with adjustable damping characteristics) to increase system damping and dissipate vibration energy, a cutting vibration suppression device has been developed [144]. Second, due to the high structural stiffness of machine tool bodies, researchers in milling dynamics studies (such as chatter vibration trace analysis) often neglect low-frequency modes dominated by the machine tool structure [145,146]. In robotic milling, the end-effector's low-frequency modes, dominated by the robot body structure, exhibit strong dependence on position and orientation. These modes form a crucial basis for studying robotic milling vibrations, including regenerative chatter, mode-coupled chatter, forced vibration, and vibration control [147,148]. The end-effector dynamic characteristics dominated by the milling robot's body structure play a crucial role in vibration control and chatter avoidance during robotic milling. Wu et al. [149] demonstrated the directionality of end-effector modal vibrations based on the body structure mode shapes of milling robots. They modeled the distribution of end-effector dynamic compliance and excitation direction in robotic modes, proposing a convenient method for obtaining dual-sphere dynamic compliance. Through experimental validation, they introduced a feed direction selection method that reduces milling vibrations without requiring exhaustive computational searches.

Additionally, cutting vibration suppression can establish stability blade diagrams to select cutting parameter combinations that avoid chatter, enabling precise acquisition of time-varying dynamic characteristics [150]. By installing sensors on machine tools to collect real-time signals and determine whether chatter is occurring or imminent, unsupervised or semi-supervised learning methods are employed for online monitoring and intelligent recognition. This reduces reliance on large amounts of labeled data while enhancing the monitoring system's generalization capability and adaptability [151]. Zhang et al. [152] addressed low-frequency chatter observed in the friction braking of novel high-speed

train brake blocks. They designed a series of parallel micro-groove textures with varying densities on the friction block surface and conducted stick-slip vibration experiments using a self-developed vehicle braking performance simulation test bench. Velocity signals from brake blocks with varying numbers of parallel microgroove textures were captured using a laser vibrometer, followed by a comprehensive analysis to evaluate stick-slip vibration behavior. Totis G et al. [153] developed a novel nonparametric advanced optimal inverse filter for processing long transients and generic signals, analyzing its potential for effective and nearly fully automated dynamic cutting force compensation during thin-walled structure milling. Novel actuators can provide a controllable damping force to absorb and suppress vibrations. Liu et al. [154] proposed a novel elastic origami structure with quasi-zero stiffness characteristics to deliver effective low-frequency vibration isolation performance. Through the integration of an elastic joint and a compression spring, the mechanical model was established, and the dynamic behavior of the origami starter was obtained by a multi-scale method. The research on the influence of parameters verified that the vibration isolation system had strong design flexibility and superior vibration suppression ability. When exploring the optimal operating conditions for plant factory transplantation (PFT) based on the digital twin method, the mechanical vibration analysis model is mainly used to evaluate the performance of the transplantation equipment [155]. The hardware configuration of the PFT mechanical vibration analysis based on a digital twin is shown in Figure 24. The system includes physical and virtual entities of the transplant machine, a data-twin service system, 3-axis acceleration sensors, industrial cameras, position sensors, and so on.

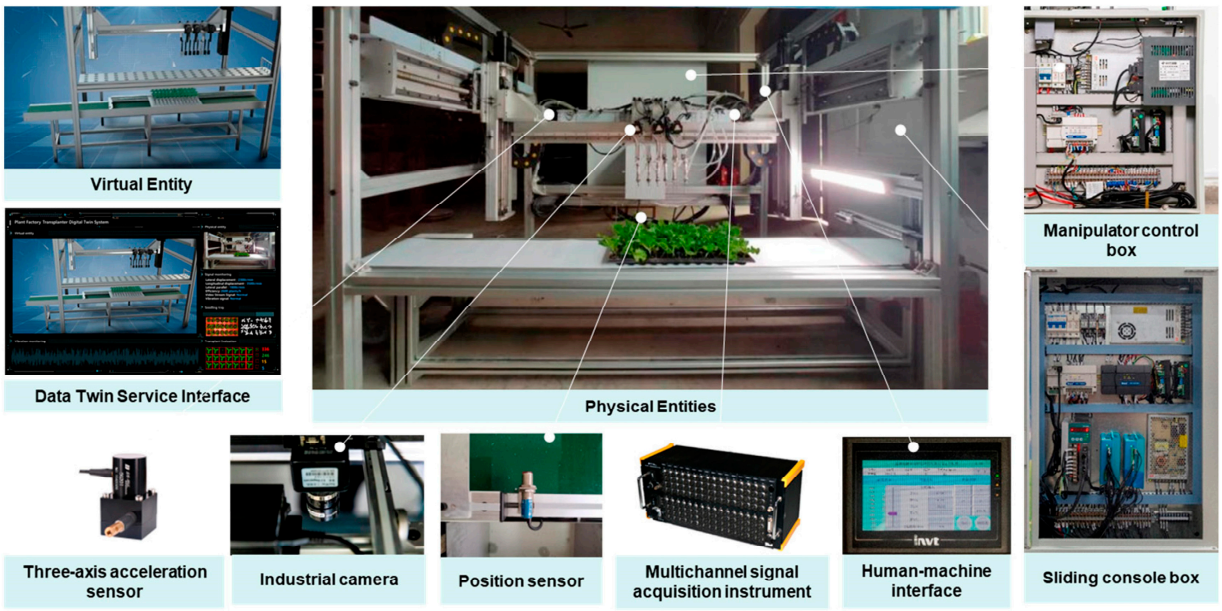


Figure 24. Multi-domain PFT model based on digital twin [155].

Analysis of chatter mechanisms during cutting and chatter detection form the foundation for chatter suppression. While conservative selection of cutting parameters can prevent chatter occurrence, industrial production demands not only machining quality but also efficiency and profitability. Vibration suppression strategies fall into two categories: active chatter suppression and passive chatter suppression [156]. Active chatter suppression compensates for chatter by driving actuators based on chatter detection results. Passive chatter suppression expands the stable cutting boundary by reducing cutting force and altering the dynamic characteristics of the micro-milling system. From a time-domain discretization perspective, methods primarily include semi-discretization, full discretization,

and time-domain finite-element analysis for predicting the stability of time-delay systems during periodic milling processes [157].

Among these, discretization methods exhibit reduced computational efficiency but improved accuracy. However, the computational efficiency of numerical integration methods is significantly influenced by the order of interpolation. Notably, the frequency-domain method offers the highest computational efficiency, requiring minimal calculations for stability prediction and analysis. Sun et al. [158] validated the milling stability and reliability of high-speed milling of superalloys using self-developed ceramic milling cutters. They conducted comparative analyses of chatter failure for self-developed ceramic cutters and commercial cemented carbide cutters, examining both tool material types and cutter structures. The vibration signal data required for constructing the frequency response function curves and chatter stability waveflap diagrams of each cutter originated from actual experiments. Secondly, accurately acquiring cutting process data is crucial for detecting and suppressing chatter, necessitating the selection of signals that precisely reflect the cutting process. Therefore, cutting force is commonly used as a characteristic signal for optimizing cutting parameters and detecting chatter. An intelligent manufacturing system for cutting processes must possess real-time, accurate, and reliable characteristics. Further research is needed on the application of digital twin methods for chatter suppression during cutting. A digital twin model for vibration suppression strategies in cutting is illustrated in Figure 25 [159].

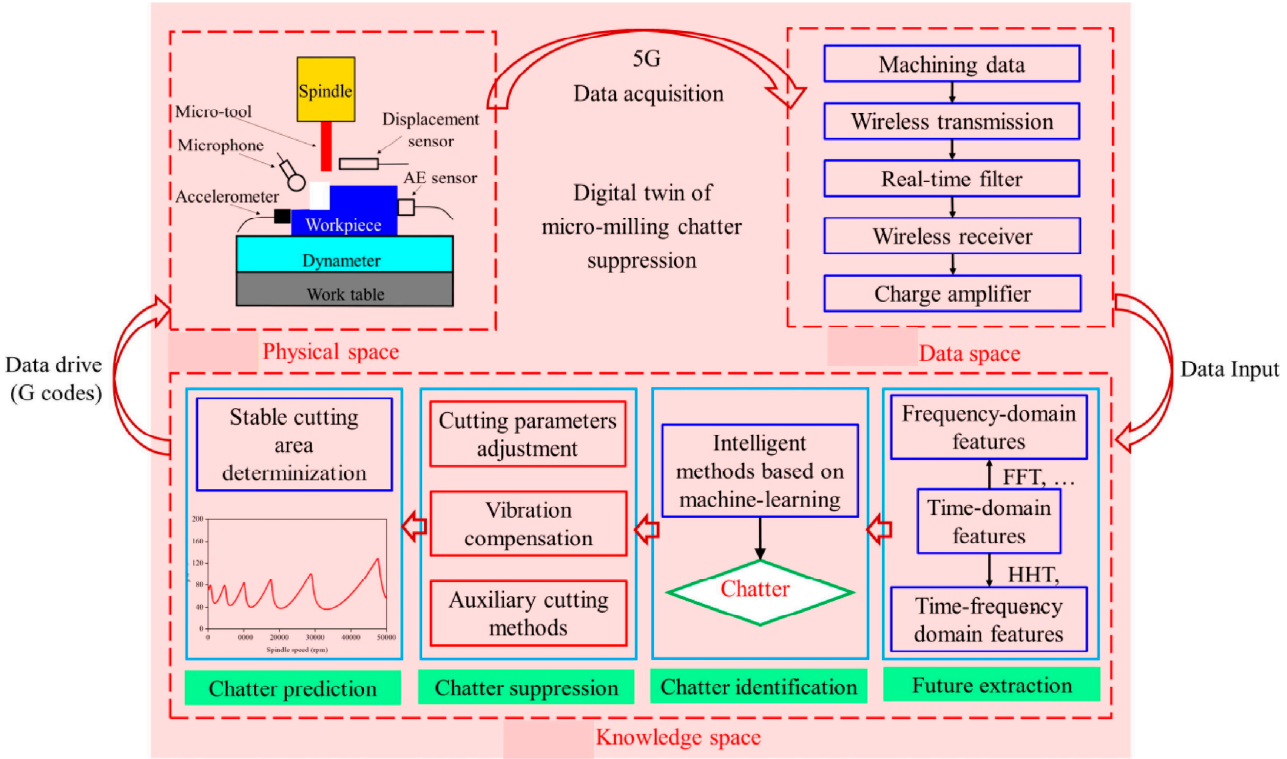


Figure 25. Digital twin model for vibration suppression strategy in cutting [159].

Figure 26 illustrates a knowledge and data-driven digital twin approach for suppressing chatter in machining. This method integrates the physical, digital, and knowledge domains, aligning with the development trends of intelligent and green manufacturing. Research on cutting vibration suppression has evolved from offline passive vibration avoidance to online active vibration suppression, exhibiting characteristics of multidisciplinary convergence. In addition, the performance and applicability comparison of major chatter suppression strategies is shown in Table 4.

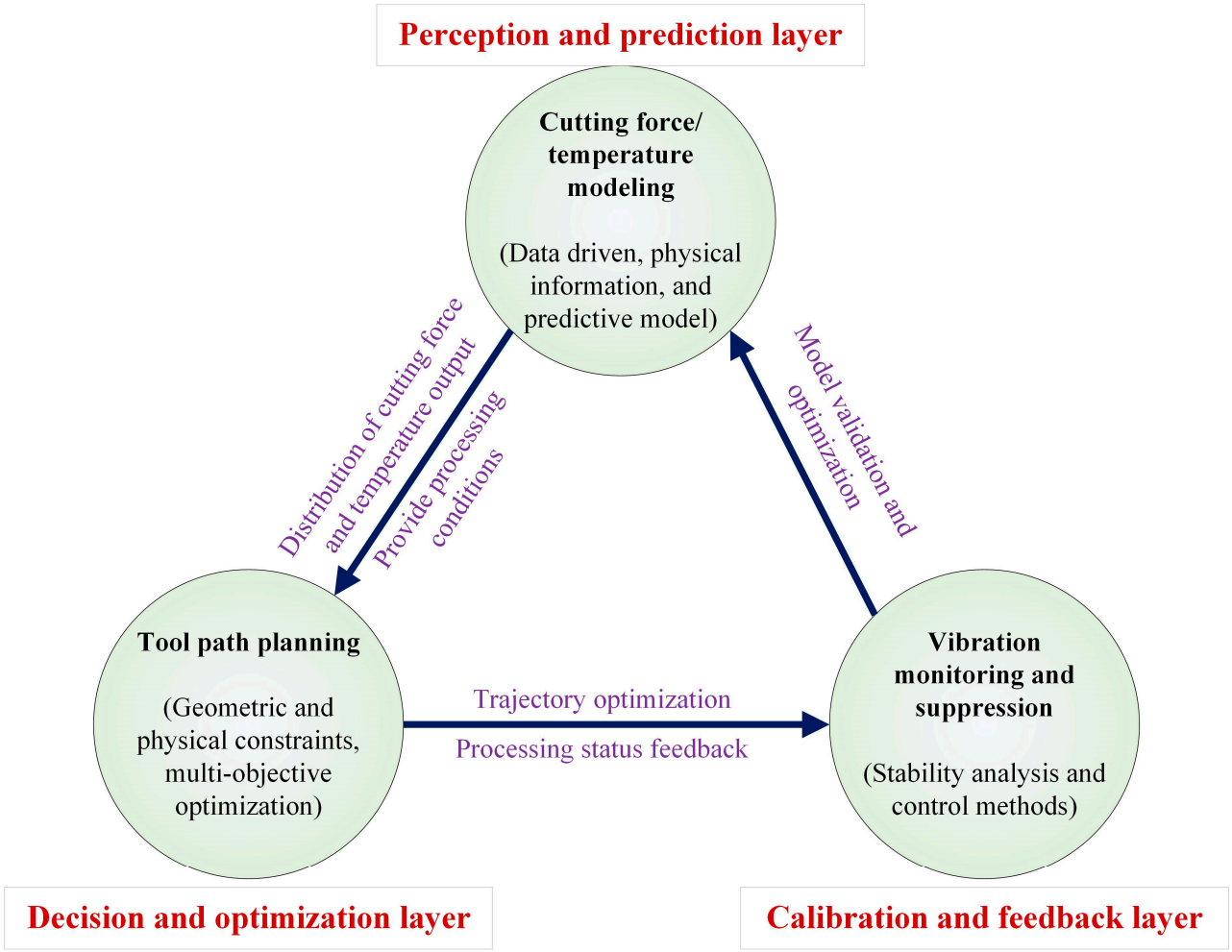


Figure 26. Dynamic adaptive closed-loop framework diagram for 'perception–prediction-decision–control'.

Table 4. Performance and applicability comparison of major chatter suppression strategies.

Suppression Strategy	Operating Principle	Implementation Stage
Parameter optimization	Operate outside unstable lobes in stability diagram	Process planning
Variable spindle speed	Disrupt regenerative chatter period	Online
Passive dampers	Add mass/damping to absorb energy	Machine design/retrofitting
Active control	Apply counter-phase control force to cancel vibration	Online, real-time

In the future, advancements in artificial intelligence and intelligent material algorithm optimization will enable the construction of high-fidelity virtual machine tool systems. These systems will predict, simulate, and optimize cutting processes within the digital space while achieving real-time interaction with physical machine tools. Consequently, intelligence-integrated vibration suppression systems will emerge as the developmental trend, ultimately realizing autonomous intelligent optimization of the machining process.

4.2. Integrated Discussion on Vibration Suppression

Whether for offline stability optimization or online active control, vibration suppression strategies form a tightly coupled closed-loop with preceding stages. Effective vibration suppression heavily relies on accurate predictive models. After vibration suppression

actions are executed, actual cutting conditions have changed. This requires the system to perform online calibration or rapid re-prediction of cutting force/temperature modeling based on sensor feedback, thereby achieving real-time closed-loop control-sensing-prediction to ensure models remain synchronized with physical processes. Furthermore, vibration states monitored online can directly serve as feedback signals to trigger real-time micro-adjustments of tool paths, enabling highly adaptive intelligent machining.

5. Conclusions and Outlook

5.1. Conclusions

The digitalization and intelligentization of cutting processes represent the core direction for the advancement of manufacturing, with mathematical methods playing a pivotal role in transitioning from 'description' to 'prediction' and ultimately to 'control'. This paper systematically reviews the current application status of mathematical methods across three core domains: cutting force/heat modeling, tool path planning, and vibration suppression. The following conclusions can be drawn:

(1) The methodology for cutting force and temperature modeling has become increasingly sophisticated. From analytical models based on shear-slip theory and finite-element numerical simulations capable of revealing intricate details in complex physical fields, to data-driven ML models that do not require explicit physical equations, each approach possesses distinct advantages and applicable scenarios. Among these, hybrid modeling that integrates physical laws with data science demonstrates significant potential to overcome the limitations of traditional models, positioning it as a frontier for future research.

(2) Tool path planning has progressed from a purely geometric task to a multi-objective optimization problem. The mathematical toolkit has expanded from computational geometry (NURBS curves, iso-scallop tool paths) to graph theory (shortest-path search), metaheuristic algorithms (genetic algorithms and particle swarm optimization for machining sequence planning), and real-time control theory. Today, physically constrained path planning is mainstream, requiring paths to satisfy not only geometric accuracy but also physical objectives such as stable cutting forces, vibration suppression, and thermal management.

(3) Vibration suppression relies on a mathematically grounded 'modeling–sensing–control' framework. This spans stability lobe theory rooted in time-delay differential equations, state identification using Fourier transforms, wavelet analysis, and machine learning, to active control strategies based on modern control theory (such as adaptive control). Together, these mathematical tools provide a systematic methodology for analyzing, predicting, and mitigating vibrations during machining.

In summary, mathematical methods underpin every facet of cutting machining, bridging physical mechanisms, process data, and intelligent control. They are the fundamental catalyst driving the technology toward greater precision, efficiency, and reliability.

5.2. Industrial Application Challenges

Firstly, the complexity of system integration is relatively high. It is necessary to seamlessly integrate independent predictive models, planning algorithms, or vibration suppression controllers with existing computer-aided manufacturing software, numerical control systems, and manufacturing execution systems at the workshop level. This leads to the dependence of planning algorithms on the interpolation function of CNC systems and the possible instruction lag problem when generating complex trajectories. Secondly, it is necessary to analyze the dependence of data-driven methods on high-quality, multi-dimensional sensor data (such as force, vibration, sound emission), as well as the additional

cost and reliability of data acquisition systems. At the same time, it is important to discuss the requirements of advanced control algorithms (such as active vibration suppression) for actuator performance and computing hardware computing power. At the same time, it is necessary to objectively evaluate the comprehensive cost of technology implementation and quantify the benefits brought by technology (such as extended tool life, reduced scrap rate, and improved processing efficiency). In addition, it is necessary to address the dependency of planning algorithms on the interpolation function of CNC systems when generating complex trajectories, as well as the potential issue of instruction lag.

5.3. Outlook

Although mathematical methods have yielded significant achievements in cutting machining, numerous challenges and development opportunities remain. Future research should focus on breakthroughs in the following areas:

(1) Deep integration and systemization: Future research can seamlessly integrate multiple modules, including cutting force/temperature models, path planning, and vibration suppression, into a comprehensive 'digital twin' framework through mathematical methods. Developing efficient multiphysics coupling algorithms enables real-time interaction and bidirectional optimization between virtual spaces and physical machine tools, forming an autonomous closed-loop system of 'perception-decision-control'. Additionally, exploring how to more effectively embed prior knowledge, such as physics conservation laws and boundary conditions, into deep learning architectures addresses the weak generalization capabilities and poor interpretability of pure data models.

(2) Specialized solutions for complex machining scenarios: Existing general-purpose models may prove inadequate for new processes and materials such as composite materials, superhard material machining, micro/nano machining, and additive–subtractive hybrid manufacturing. There is an urgent need to develop specialized constitutive models, damage criteria, and path generation algorithms tailored to these applications, alongside establishing toolpath optimization databases for high-performance manufacturing of complex curved components. For example, for carbon fiber composite materials, one can establish a specialized constitutive model that can characterize anisotropic cutting forces and interlayer delamination damage; thus, developing multi-scale simulation methods that consider size effects and grain orientation for micro milling. Secondly, by utilizing limited experimental data on new materials through transfer learning, a reliable set of tool path optimization parameters can be quickly generated.

(3) Application of autonomous intelligence and reinforcement learning: Current optimization heavily relies on offline algorithms. Autonomous intelligence based on deep reinforcement learning enables agents to interact with the machining environment through trial and error. This approach facilitates the development of higher-level autonomous optimization systems for machining strategies. Autonomous learning algorithms acquire optimal vibration suppression strategies, tool path parameters, or cooling strategies, ultimately forming an "adaptive machining decision center" capable of handling unknown conditions and self-evolving and adjusting. Examples of this include deploying ML in the digital twin of the machining process, designing a high-fidelity virtual environment that can comprehensively reflect vibration, force/heat, tool wear, and other states, as well as a multi-objective function that considers efficiency, quality, and stability.

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