

Article

Electromagnetic Vibration Analysis and Mitigation of FSCW PM Machines with Auxiliary Teeth

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Abstract

Auxiliary teeth are usually used in fractional-slot concentrated winding (FSCW) machines for fault tolerance. However, the influence of auxiliary teeth on torque and electromagnetic vibration performance differs with different slot–pole configurations. Thus, this paper investigates electromagnetic vibration and mitigation methods in FSCW permanent magnet (PM) machines with auxiliary teeth. The relationship between yoke forces and tooth parameters of two dual three-phase (DTP) FSCW-PM machines with 12-slot/14-pole configuration and 12-slot/10-pole configuration is studied and compared. Results reveal that (1) the $2p$ -order airgap electromagnetic force reduces second-order yoke force in the 12-slot/14-pole machine but increases it in the 12-slot/10-pole machine. (2) Through optimized tooth width, slot harmonics can be mitigated, but the fundamental winding magnetic field in the 12-slot/10-pole machine is also weakened, whereas the 12-slot/14-pole machine achieves fundamental field preservation or enhancement. Based on these findings, auxiliary tooth optimization and rotor pole profile shaping are proposed for vibration reduction in 12-slot/14-pole machine. Electromagnetic–mechanical coupled simulations conducted in ANSYS Maxwell/Workbench 2023 demonstrate that the optimized design reduces the cogging torque peak from 11.4 mN·m to 2.9 mN·m (74.6% reduction), suppresses housing surface vibration acceleration by 21%, and maintains the average output torque without reduction.

Keywords: electromagnetic vibration mitigation; permanent magnet machine; auxiliary teeth; tooth modulation effect; rotor pole profile shaping



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1. Introduction

With the widespread application of motors in electric vehicles and electric vertical take-off and landing (eVTOL), there has been increasing attention on motors with high power density, high reliability, and low electromagnetic vibration [1,2]. Compared to three-phase integer-slot distributed winding motors, multiphase fractional-slot concentrated winding (FSCW) motors exhibit higher efficiency and reliability due to their shorter end-windings and winding redundancy [3]. However, the FSCW motor inherently contains abundant spatial harmonic components in the magnetic field, leading to significant vibration. Consequently, vibration analysis and optimization for FSCW permanent magnet (PM) motors have become a popular topic of research [4–6].

The excitation of low-order electromagnetic vibrations in FSCW-PM motors is mainly generated by the interaction between the fundamental magnetic field and slot harmonics. Therefore, suppressing the slot harmonics can effectively mitigate electromagnetic vibrations [7,8]. In [9], a 20-slot/18-pole V-type interior permanent magnet motor used an eccentric pole design with auxiliary slots on the stator and rotor, suppressing the 11th-order slot harmonic and significantly reducing second-order vibration. In [10], a 12-slot/10-pole (12s10p) single-layer concentrated winding motor suppressed the 7th-order slot harmonic and associated second-order electromagnetic vibration through unequal tooth-width optimization. However, this method increased cogging torque. In [11], an 18-slot/16-pole double-layer winding motor introduced inter-phase auxiliary teeth, which suppressed 10th-order slot harmonic-induced second-order vibration without sacrificing output torque but doubled torque ripple. Reference [12] proposed intra-slot auxiliary teeth to suppress winding short-circuit fault currents and slot harmonics, but reduced output torque by 33%.

Furthermore, high order airgap electromagnetic forces contribute to low-order vibrations through the tooth modulation effect [13]. Reference [14] compared a 12s10p surface-mounted PM motor with bread-loaf magnets versus conventional magnets. The results indicate that the bread-loaf design produced stronger $2p$ -order (where p is pole pairs) forces, causing higher second-order vibration. Reference [15] suppressed the $2p$ -order airgap electromagnetic force by enhancing the $3p$, $5p$, and $7p$ harmonics in PM magnetic field, thereby reducing the second-order vibration. The above references show that the $2p$ -order airgap electromagnetic force enhances the second-order vibration due to the tooth modulation effect. However, high-order airgap forces can also suppress the low-order vibration due to a different tooth modulation effect [16]. Reference [17] reveals that the tooth modulation effect varies with slot opening width, and an analytical model of the tooth modulation effect was proposed in [18] by mapping the tooth-surface radial airgap force to the yoke. However, neglecting the slot-opening force leads to significant errors in tangential force analysis [19].

In summary, the auxiliary teeth (unequal teeth) can reduce slot harmonics and enhance fault tolerance, but they introduce drawbacks such as increased cogging torque [10], higher torque ripple [11], and reduced output torque [12]. Furthermore, existing studies focus on the tooth modulation effect in the equal teeth stator [16–18], but analysis with unequal teeth remains scarce.

This paper aims to clarify the impact of auxiliary teeth on electromagnetic vibration in two types of FSCW-PM machines (12s14p and 12s10p) whose slot and pole combinations satisfy $2p = Q_s \pm 2$ (where Q_s denotes the number of slots). The equivalent yoke force model is developed based on the rigid-body theory detailed in Section 2. Then, in Section 3, the impact of auxiliary teeth on winding magnetomotive force (MMF), airgap permeance, and tooth modulation effects is analyzed. Specifically, tooth modulation coefficients incorporating auxiliary teeth are further proposed based on the equivalent yoke force model, which quantifies the transfer mechanism from high-order airgap forces to low-order yoke forces. Based on this analysis, a synergistic approach combining auxiliary tooth-width optimization and rotor shaping is proposed in Section 4 to mitigate electromagnetic vibrations in $2p = Q_s + 2$ machines. Finally, in Section 5, Electromagnetic–mechanical coupled simulations conducted in ANSYS Maxwell/Workbench to verify the effectiveness of the optimal design.

2. Equivalent Yoke Force Model

It is well known that electromagnetic forces are mainly acting on the airgap and tooth interface. Forces are transferred by teeth, acting on the yoke, and then cause radial vibration. For convenience of analysis, some simplifications are made as follows [20,21]:

1. The motor's stator core and housing undergo small elastic deformation;
2. The teeth are considered rigid bodies; under this assumption, forces acting on the teeth are transferred to the interface of teeth and yoke (no deformation-induced force loss);
3. Bending moments can be equivalent to a pair of forces that have the same amplitude, opposite directions, and parallel non-collinear action lines.

According to the assumption 1 to 3, the force equivalence relationships are illustrated in Figure 1. Radial forces are directly mapped to the yoke, while tangential forces are transferred with additional bending moments and then converted into equivalent radial forces. Since the accuracy of the equivalent forces depends on the stiffness of the teeth and yoke, and the stiffness of the teeth is related to the width, length and elastic modulus, the results of the calculated yoke forces based on the rigid-body assumption may be higher than actual yoke force values under the condition of lower stiffness of the teeth.

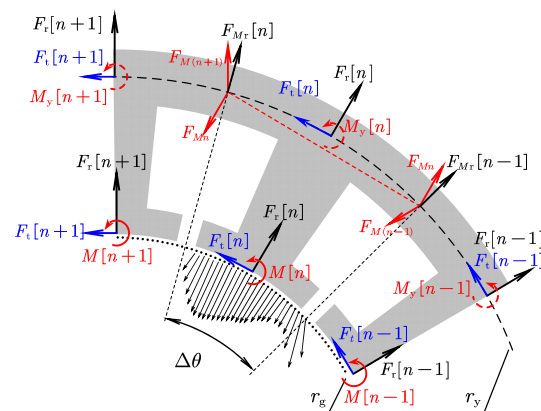


Figure 1. Airgap force mapping model.

The equivalent concentrated forces $F_r[n]$, $F_t[n]$ and bending moments $M[n]$ on the tooth shoulders are derived by [16,22]:

$$\begin{cases} F_r[n] = r_g \int_{\theta_n - \Delta\theta/2}^{\theta_n + \Delta\theta/2} f_r \cos(\theta - \theta_n) - f_t \sin(\theta - \theta_n) d\theta, \\ F_t[n] = r_g \int_{\theta_n - \Delta\theta/2}^{\theta_n + \Delta\theta/2} f_r \sin(\theta - \theta_n) + f_t \cos(\theta - \theta_n) d\theta, \\ M[n] = r_g^2 \int_{\theta_n - \Delta\theta/2}^{\theta_n + \Delta\theta/2} f_t d\theta - r_g F_t[n]. \end{cases} \quad (1)$$

where f_r and f_t are the radial and tangential airgap force, respectively, θ_n denotes the angular position of the n th tooth, $\Delta\theta$ denotes the slot pitch of the tooth, r_g and r_y denote the radii of the airgap and yoke centerline, respectively. Furthermore, based on the rigid-body theory, the concentrated forces and bending moments in (1) are transferred to the yoke centerline, with bending moments expressed as

$$\begin{aligned} M_y[n] &= M[n] - (r_y - r_g)F_t[n] \\ &= r_g^2 \int_{\theta_n - \Delta\theta/2}^{\theta_n + \Delta\theta/2} f_t d\theta - r_y F_t[n]. \end{aligned} \quad (2)$$

The bending moment $M_y[n]$ can be equaled to forces F_{Mn} , acting on the yoke centerline [21], and the following relationship is established:

$$M_y[n] = 2F_{Mn}r_y \sin(\Delta\theta/2). \quad (3)$$

Then, the equivalent radial forces of the bending moment on the yoke can be expressed as

$$\begin{aligned} F_{Mr}[n] &= (F_{M(n+1)} - F_{Mn}) \cos(\Delta\theta/2) \\ &= \frac{M_y[n+1] - M_y[n]}{2r_y} \frac{\cos(\Delta\theta/2)}{\sin(\Delta\theta/2)}. \end{aligned} \quad (4)$$

Finally, the airgap forces can be equivalently represented as radial force pulse trains with π/Q_s spacing acting on the yoke:

$$\bar{f}_{yr}(\theta, t) = \sum_{n=0}^{Q_s-1} F_r[n] \delta\left(\theta - \frac{2n\pi}{Q_s}\right) + \sum_{n=0}^{Q_s-1} F_{Mr}[n] \delta\left(\theta - \frac{(2n+1)\pi}{Q_s}\right). \quad (5)$$

Obviously, after the airgap forces are transmitted through the teeth, the distribution and harmonic content of the forces will differ. This phenomenon is called the tooth modulation effect. The harmonic order relationship between airgap forces and yoke forces is detailed in the following sections.

3. Electromagnetic Force Harmonics Analysis

3.1. Airgap Forces and Their Harmonics

The magnetomotive force (MMF) generated by PM and the fundamental component of the winding current across the slotless airgap can be expressed as

$$\begin{cases} F_{pm}(\theta, t) = \sum_{\mu=2k+1} F_{\mu} \cos(\mu p \theta - \mu \omega_e t), & k \in \mathbb{Z}. \\ F_{wd}(\theta, t) = \sum_v F_v \cos(v \theta - \omega_e t - \varphi_0). \end{cases} \quad (6)$$

where μp and v denote the space orders, p is the pole pairs of the machine, and μ is the odd integer owing to the symmetry in the PM MMF distribution. The electrical angular velocity of the rotor satisfies $\omega_e = 2\pi f_e$, where f_e represents the electrical frequency. φ_0 is the initial phase and $\varphi_0 = \pi/2$ with the control strategy that $i_d = 0$. The winding MMF spatial orders v are determined by

$$\begin{cases} v = \frac{2mp}{d}k + p, & k \in \mathbb{Z}. \\ q = \frac{Q_s}{2mp} = \frac{N}{d}. \end{cases} \quad (7)$$

where m denotes the number of winding phases. The numerator N and the denominator d in the slot per pole per phase q are coprime. The neglective value of v denotes the reverse rotating direction.

The harmonic amplitudes of the PM MMF and winding MMF satisfy the relationship:

$$\begin{cases} F_{\mu} \propto \frac{1}{\mu} \sin\left(\mu \frac{\alpha_p \pi}{2}\right), \\ F_v \propto \sin(v\tau)/v. \end{cases} \quad (8)$$

where α_p denotes the pole arc coefficient, $\sin(v\tau)$ is the pitch coefficient, 2τ is the coil pitch.

The greatest common divisor (GCD) of poles and slots satisfies $\text{GCD}(2p, Q_s) = 2p/d$, details are provided in Appendix A. For $2p = Q_s \pm 2$ FSCW-PM machines, $\text{GCD}(2p, Q_s) = 2$. Thus, the winding MMF spatial orders in (7) can be represented as

$$v = \frac{kQ_s}{N} + p = 2mk + p, \quad k \in \mathbb{Z}. \quad (9)$$

According to the MMF-permeance method and Maxwell stress tensor method, the radial airgap force is calculated by

$$f_r(\theta, t) \approx \frac{B_r^2}{2\mu_0} = \frac{(F_{pm} + F_{wd})^2 \lambda^2(\theta, t)}{2\mu_0} = \frac{\lambda^2(\theta, t)}{2\mu_0} \left\{ \begin{array}{l} \underbrace{\sum_{\mu_1} \sum_{\mu_2} F_{\mu_1 \mu_2} \cos[(\mu_1 \pm \mu_2)p\theta - (\mu_1 \pm \mu_2)\omega_e t]}_{\text{Pm-Pm}} \\ + \underbrace{\sum_{\mu} \sum_v F_{\mu v} \cos[(\mu p \pm v)\theta - (\mu \pm 1)\omega_e t \mp \varphi_0] d}_{\text{Pm-Wd}} \\ + \underbrace{\sum_{v_1} \sum_{v_2} F_{v_1 v_2} \cos[(v_1 \pm v_2)\theta - (1 \pm 1)\omega_e t - (\varphi_0 \pm \varphi_0)]}_{\text{Wd-Wd}} \end{array} \right\}. \quad (10)$$

where the airgap permeance $\lambda(\theta, t)$ is represented as

$$\begin{aligned} \lambda(\theta, t) &= \lambda_s(\theta, t) \cdot \lambda_r(\theta, t) \\ &= \lambda_{s0}\lambda_{r0} + \sum_k \lambda_{r0}\lambda_{sk} \cos(kQ_s\theta) + \sum_n \lambda_{s0}\lambda_{rn} \cos(2np\theta - 2n\omega_e t) \\ &\quad + \sum_n \sum_k \lambda_{rn}\lambda_{sk} \cos((2np + kQ_s)\theta - 2n\omega_e t) \end{aligned} \quad (11)$$

For DTP FSCW PM machines with 12s10p configuration and 12s14p configuration, the winding MMF spatial orders v are given by $v = p + kQ_s$. For concise expression, the notation (x, y) is used to represent the harmonic components with spatial order x and temporal harmonic y . According to (10) and (11), the airgap force components follow the $(2np + kQ_s, 2nf_e)$ order–frequency relationship. Table 1 shows all the deduced airgap force components.

Table 1. The airgap forces of 12s10p and 12s14p DTP-PM machines.

$\lambda^2(\theta, t)$	Pm-Pm	Pm-Wd	Wd-Wd	
	$(2np, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(kQ_s, 0)$	$(2np + kQ_s, 2f_e)$
$(0, 0)$	$(2np, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(kQ_s, 0)$	$(2np + kQ_s, 2f_e)$
$(kQ_s, 0)$	$(2np + kQ_s, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(kQ_s, 0)$	$(2np + kQ_s, 2f_e)$
$(2np, 2nf_e)$	$(2np, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(2np + kQ_s, 2nf_e)$
$(2np + kQ_s, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(2np + kQ_s, 2nf_e)$	$(2np + kQ_s, 2nf_e)$
Initial phase	0 or π	$\pm\pi/2$	0 or π	

Components independently generated by PM and armature fields exhibit initial phases of 0 or π , corresponding to the real components of the airgap force complex Fourier series (Figure 2a), while components generated by the interaction of PM and armature fields show $\pm\pi/2$ phases, corresponding to the imaginary components (Figure 2b). The second order $(2p - Q_s)$ components (red highlighted in Figure 2) are considered to be the most significant contributors to vibration and mainly come from the following.

1. The interaction between PM's p th-order field and armature's slot harmonic component $(p - Q_s)$.
2. Stator permeance modulation of $2p$ -order forces (from PM/winding self/interaction effects) generating lower-order $2p - Q_s$ components.

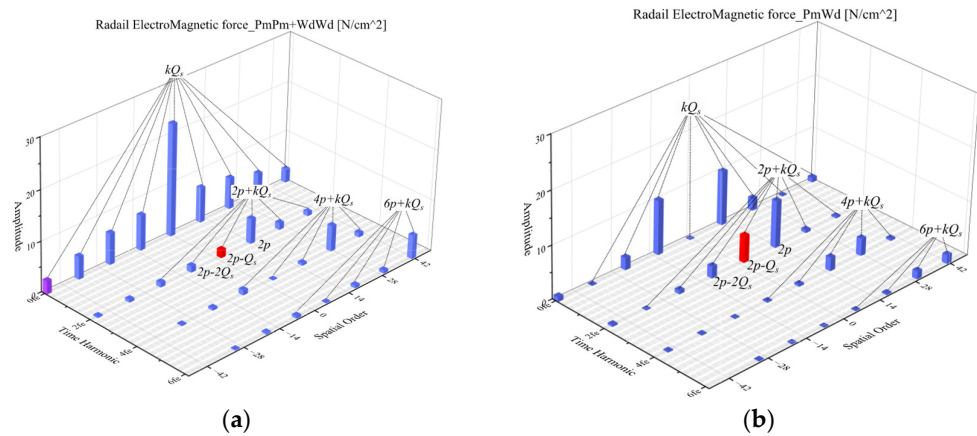


Figure 2. Radial airgap forces of 12s14p: (a) PmPm + WdWd; (b) PmWd.

3.2. Yoke Forces and Tooth Modulation Coefficients

As expressed in Section 2, distributed forces can be equivalently represented as concentrated forces [20,21]. For convenience of harmonic analytical analysis, taking a v th order radial airgap force with $2f_e$ electrical frequency ($v, 2f_e$) as an example and neglecting the curvature effects of the tooth structure, the resulting radial tooth concentrated forces (1) become

$$F_v[n] = r_g \int_{\theta_n - \tau}^{\theta_n + \tau} f_v \cos(v\theta - 2\omega_e t - \phi) d\theta = r_g \frac{2\sin(v\tau)}{v} f_v \cos(v\theta_n - 2\omega_e t - \phi), \quad (12)$$

where 2τ denotes the integration region of the tooth, ϕ is the initial phase.

For the stator with auxiliary teeth shown in Figure 3, the stator tooth axis positions satisfy $\theta_n = \pi n / Q_s$, where Q_s stands for the number of wound teeth (auxiliary teeth excluded). Slot opening width is depicted by τ_s , and τ_a, τ_t are the widths of wound teeth and auxiliary teeth, respectively. Two integration regions exist for concentrated forces: $2\tau_1$ for wound teeth and $2\tau_2$ for auxiliary teeth, with the following relationship:

$$2\tau_1 + 2\tau_2 = 2\pi / Q_s. \quad (13)$$

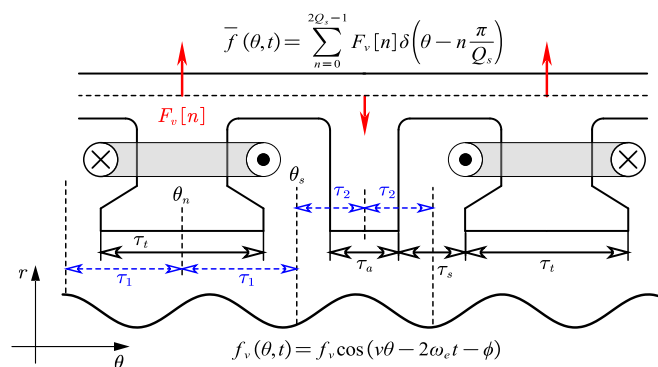


Figure 3. Airgap force transformation in the stator with auxiliary teeth.

As described in (5) in Section 2, the concentrated forces are pulse trains with π / Q_s spacing acting on the yoke:

$$\bar{f}(\theta, t) = \sum_{n=0}^{2Q_s-1} F_v[n] \delta\left(\theta - n \frac{\pi}{Q_s}\right). \quad (14)$$

The Fourier series of (14) are

$$\begin{aligned} Y_i &= \frac{1}{2\pi} \int_0^{2\pi} \bar{f}(\theta, t) e^{-ji\theta} d\theta = \frac{1}{2\pi} \sum_{n=0}^{2Q_s-1} F_v[n] e^{-jin \frac{\pi}{Q_s}} \\ &= \begin{cases} \frac{r_g Q_s f_v}{2\pi v} [\sin(v\tau_1) + e^{jk\pi} \sin(v\tau_2)] e^{-j2\omega_e t - j\phi}, & i = v - kQ_s, \\ \frac{r_g Q_s f_v}{2\pi v} [\sin(v\tau_1) + e^{-jk\pi} \sin(v\tau_2)] e^{j2\omega_e t + j\phi}, & i = kQ_s - v. \end{cases} \end{aligned} \quad (15)$$

With the transmission through stator teeth, the v th-order airgap force not only induces its inherent spatial order but also generates additional spatial orders in the stator yoke. These additional forces are called tooth modulation forces, whose spatial order i satisfies the relationship $i = v - kQ_s$ (where $k \in \mathbb{Z}$, an arbitrary integer). Therefore, the force density form of the i th-order yoke force can be expressed as

$$\begin{aligned} \bar{f}_i(\theta, t) &= (Y_i e^{ji\theta} + Y_{-i} e^{-ji\theta}) \\ &= \frac{r_g Q_s [\sin(v\tau_1) + (-1)^k \sin(v\tau_2)]}{\pi v} f_v \cos(i\theta - 2\omega_e t - \phi). \end{aligned} \quad (16)$$

Compared with the i th-order modulated force (16) and the original airgap force, the tooth modulation effect can be expressed by the modulation coefficient $K_{i,v}$:

$$K_{i,v} = r_g \frac{Q_s}{\pi} \frac{[\sin(v\tau_1) + (-1)^k \sin(v\tau_2)]}{v}, \quad i = v - kQ_s. \quad (17)$$

The tooth modulation coefficient represents the amplitude and phase relationship between the v th-order airgap force $f_v(\theta, t)$ and the modulated i th-order yoke force $\bar{f}_i(\theta, t)$. Under a specific airgap force order v , the tooth modulation coefficient varies with the slot pitch $2\tau_1$ of wound teeth in a trigonometric pattern. For given slot pitches, $2\tau_1$ and $2\tau_2$, the tooth modulation coefficient changes with airgap force order v for a sinc function profile. In the special case that $\tau_1 = \pi/Q_s$, $\tau_2 = 0$, the stator becomes the traditional equal-teeth and double layer winding machine. The tooth modulation coefficient of the traditional equal-teeth geometry is

$$K_{i,v} = r_g \frac{\sin(v\pi/Q_s)}{v\pi/Q_s}, \quad i = v - kQ_s. \quad (18)$$

4. Auxiliary Teeth Impact Analysis and Vibration Optimization

According to the analysis in Section 3, the second-order yoke force in $2p = Q_s \pm 2$ FSCW-PM machines can be concluded as

$$\bar{f}_{2nd} \approx \underbrace{K_{2,2} F_{2p-Q_s} \lambda_0}_{\text{Caused by } p_{pm} \text{ and } (p-Q_s)_{wd}} + \underbrace{K_{2,2} F_{2p} \lambda_{s1}}_{\text{Permeance modulated}} + \underbrace{K_{2,2p} F_{2p} \lambda_0}_{\text{Tooth modulated}}. \quad (19)$$

where the slot harmonic force, permeance-modulated force and tooth modulated force are related to auxiliary teeth structural parameters.

4.1. Influence of Auxiliary Teeth on Winding MMF and Stator Permeance

The pitch factor $\sin(v\tau)$ in the winding MMF (8) directly related to the coil span 2τ . For FSCW-PM machines with auxiliary teeth, the coil span is the slot pitch $2\tau_1$ of the wound-tooth and varies simultaneously with the auxiliary teeth slot pitch $2\tau_2$. Their variation satisfies the relationship in (13). The normalized amplitude variations in fundamental and low-order slot harmonics in winding MMF for 12s10p and 12s14p DTP machines

are presented in Figure 4a. As the slot pitch $2\tau_1$ decreases, the 7th-order winding MMF initially increases, reaches maximum at $2\tau_1 = \pi/p$ (25.7°), then decreases, while the 5th-order amplitude exhibits a monotonically decreasing trend. For 12s10p machine, the fundamental winding MMF corresponds to $v = 5$, with the low-order slot harmonics at $|p + kQ_s| = 7, 17, 19$; for the 12s14p machine, the fundamental is $v = 7$, and low-order slot harmonics are $|p + kQ_s| = 5, 17, 19$. Therefore, an optimized auxiliary teeth width can suppress slot harmonics (5th, 17th, and 19th orders) of the 12s14p machines, and thus the slot harmonic force in (19), while maintaining or enhancing the fundamental MMF.

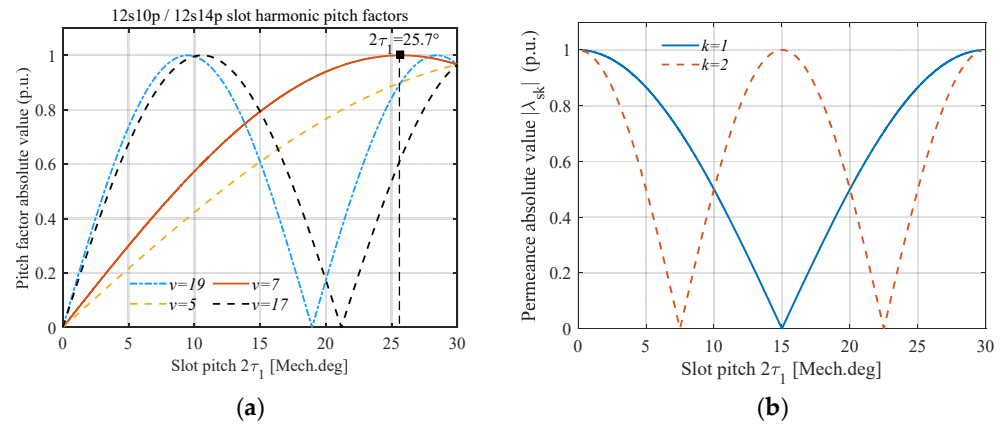


Figure 4. Influence of auxiliary teeth: (a) Winding MMF; (b) Stator permeance.

The permeance of the stator with auxiliary teeth satisfies the following relationship:

$$\lambda_{sk} \propto \frac{Q_s}{2\pi} \left(\int_{-\tau_t/2}^{\tau_t/2} e^{-jkQ_s\theta} d\theta + \int_{\pi/Q_s - \tau_a/2}^{\pi/Q_s + \tau_a/2} e^{-jkQ_s\theta} d\theta \right) = -\frac{2}{k\pi} \cos(kQ_s\tau_1) \sin\left(\frac{kQ_s\tau_s}{2}\right). \quad (20)$$

By fixing the slot opening τ_s , the variations in permeance harmonics (tooth-number order and double-tooth-number order) versus the wound tooth slot pitch $2\tau_1$ are shown in Figure 4b. As $2\tau_1$ decreases from $\pi/Q_s = 30^\circ$, the tooth-order permeance harmonics exhibit a decreasing trend and become zero at $2\tau_1 = 15^\circ$, where the widths of auxiliary teeth and wound teeth are equal.

4.2. Influence of Auxiliary Teeth on Tooth Modulation Effect

In actual machines, the wound tooth is wider than the auxiliary tooth ($2\tau_1 > 2\tau_2$). For the $2p$ -order airgap force, the sign of the tooth modulation coefficient $K_{2,2p}$ ($v = 2p, i = 2$ and $k = 1$ in (17)) satisfies the following relationship:

$$\begin{cases} K_{2,2p} < 0, & 2p = Q_s + 2. \\ K_{2,2p} > 0, & 2p = Q_s - 2. \end{cases} \quad (21)$$

Detailed proof of (21) is provided in Appendix A. The phase of the modulated second-order force remains consistent with that of the original $2p$ airgap force when the tooth modulation coefficient $K_{2,2p} > 0$, but exhibits a 180-degree phase shift when $K_{2,2p} < 0$. The variation trends of the tooth modulation coefficient $K_{2,2p}$ with slot pitch $2\tau_1$ in 12s10p and 12s14p machines are shown in Figure 5. A few values of $K_{2,2p}$ in (17) are shown in Table 2. Results show that the tooth modulation coefficient $K_{2,10} > 0$ for 12s10p machine when $2\tau_1 > 2\tau_2$ ($2\tau_1 > 15^\circ$), and $K_{2,14} < 0$ for 12s14p machine. These trends are consistent with the relationship in (21).

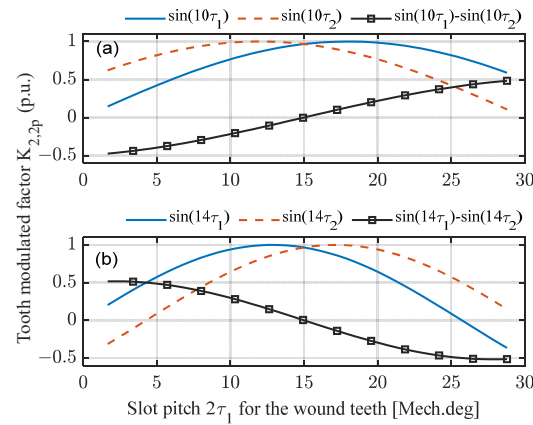


Figure 5. Variation in tooth modulation coefficient $K_{2,2p}$ with slot pitch: (a) 12s10p; (b) 12s14p.

Table 2. Values of $K_{2,2p}$ of the 12s10p and 12s14p spoke-type PM motors.

$2\tau_1$	5°	10	15	20	25
12s10p	−0.006	−0.0033	0	0.0033	0.006
Sign	$K_{2,10} < 0$		$K_{2,10} = 0$		$K_{2,10} > 0$
12s14p	0.0052	0.0032	0	−0.0032	−0.0052
Sign	$K_{2,14} > 0$		$K_{2,14} = 0$		$K_{2,14} < 0$

According to (6), the $2p$ and $(2p - Q_s)$ order airgap forces from the interaction between the p th-order PM field and $(p, p - Q_s)$ th order armature field are phase-synchronized. In addition, according to (20), the tooth number permeance $\lambda_{s1} > 0$, which means that the permeance-modulated force $F_{2p}\lambda_{s1}$ aligns with the $2p$ -order force $F_{2p}\lambda_0$ as shown in Figure 6. For $2p = Q_s + 2$ machines, the tooth modulation coefficient $K_{2,2p} < 0$ results in the tooth-modulated force $K_{2,2p}F_{2p}\lambda_0$ suppressing both permeance-modulated force $K_{2,2}F_{2p}\lambda_{s1}$ and slot harmonic force $K_{2,2}F_{2p-Q_s}\lambda_0$. Conversely, $2p = Q_s - 2$ machines with $K_{2,2p} > 0$ exhibit force enhancement.

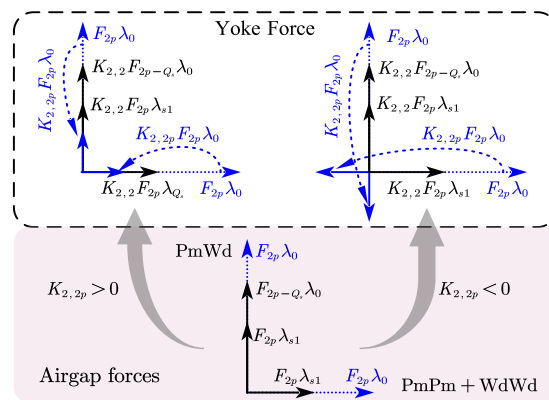


Figure 6. Second-order phase relationship of $2p = Q_s \pm 2$ machines.

4.3. Vibration Mitigation Method

This section therefore focuses on vibration optimization for 12s14p machines. As discussed in Section 4.2, optimal auxiliary teeth width and enhancement of $2p$ -order airgap force can effectively suppress second-order electromagnetic vibrations. For spoke-type PM machines, adjusting the rotor magnetic pole shape modifies the p , $3p$, and $5p$ airgap

magnetic harmonics, thereby adjusting the $2p$ -order electromagnetic force. The thickness of the magnetic pole is defined as

$$d = \text{abs}(L(A_1 \cos p\theta + A_3 \cos 3p\theta + A_5 \cos 5p\theta)). \quad (22)$$

where L denotes the max thickness of the magnetic pole, A_1, A_3, A_5 represent the amplitudes of the injected 1st, 3rd, and 5th harmonics, respectively, as shown in Figure 7.

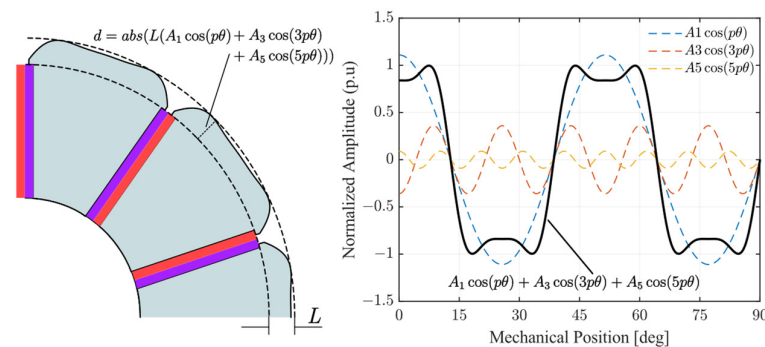


Figure 7. Harmonic injection of magnetic pole shape.

Structural parameters (auxiliary teeth width At , slot opening So , injected harmonic amplitudes A_1, A_3, A_5) are difficult to optimize theoretically. Thus, a multi-objective NSGA-III algorithm is employed to maximize torque, minimize 2nd- and 4th-order forces, and determine the optimal parameter combination. To satisfy the winding coil placement requirements and ensure the structural rigidity of auxiliary teeth, the optimization objectives and constraints are formulated as follows:

$$\left\{ \begin{array}{l} \text{Function : } [\text{Min } f_1(f_{2nd}), \text{Min } f_2(f_{4th}), \text{Max } f_3(T_{out})], \\ \text{Constraints :} \\ \quad So > 2\text{mm}, At > 2\text{mm}; \\ \quad A_1 \in [1, 1.5], A_2 \in [0, 0.8], A_5 \in [-0.1, 0.15]; \\ \quad \max(d) \leq L. \end{array} \right. \quad (23)$$

Since genetic algorithm optimization explores the Pareto front through population-based crossover/mutation operations, requiring intensive magnetic field computations. Thus, this study employs the semi-analytical dimension-reduced harmonic modeling (DRHM) method in [23] for rapid magnetic field calculations. The simplified machine topology is shown in Figure 8, and the non-optimized structural parameters detailed in Table 3. The number of coil turns of the winding is designed to be 15, and the rated operating current is 12 Arms. The motor is driven by the $i_d = 0$ control strategy and the rotating velocity is 1200 Rpm.

Table 3. Parameters of the 12s14p spoke-type PM motor.

Parameters	Values	Parameters	Values
r_{yo} (mm)	62	Tooth Height h_s	16 mm
r_{sy} (mm)	56	Tooth body width	$(12 - At)$ mm
r_{is} (mm)	43	Rotor Opening	2 mm
r_{at} (mm)	40	Rotor Insert (L)	1.5 mm
r_{pa} (mm)	39	PM Insert	1.5 mm
r_{po} (mm)	37.5	PM Length	15 mm
r_{pi} (mm)	22.5	PM Thickness	2.5 mm
B_{re} (T)	1.395	Axial Length	80 mm

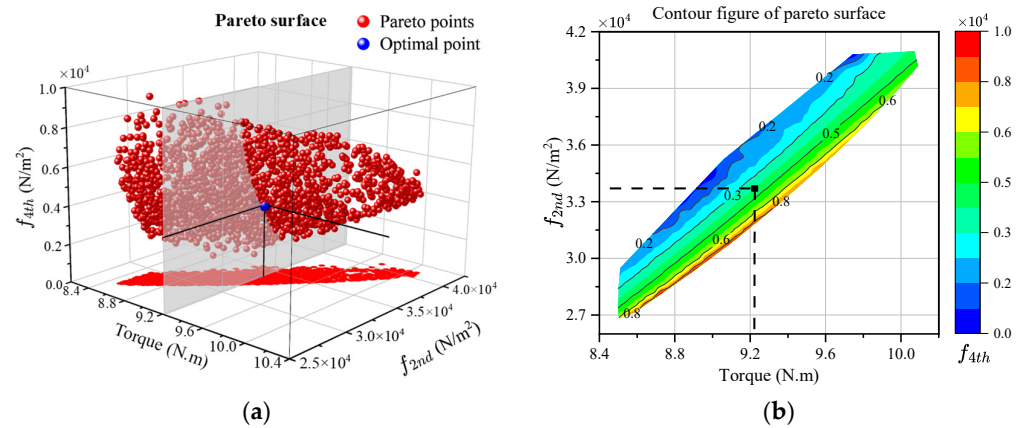


Figure 10. Results of multi-objective optimization: (a) Pareto surface; (b) Contour figure of pareto surface.

5. Optimal Design Verification Via Finite Element Analysis

To validate the effects of auxiliary tooth and rotor pole shape optimization on both output performance and electromagnetic vibration in $2p = Q_s + 2$ type machines as discussed in Section 4, this section established 2D electromagnetic and structural vibration simulation models for a 12s14p spoke-type DTP-PM motor on the ANSYS, as shown in Figure 11. In Maxwell 2D simulations, the airgap forces are calculated as concentrated forces and bending moments that act on the center of each tooth surface. Subsequently, the temporal harmonics of the concentrated forces and bending moments are transferred to ANSYS Mechanical as vibrational excitation for structural vibration response calculation.

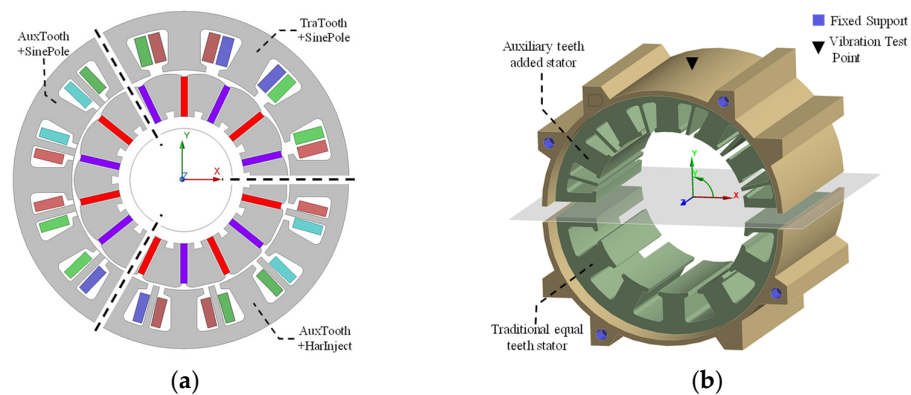


Figure 11. ANSYS simulation model: (a) Maxwell2D models; (b) Structural vibration simulation models for the traditional equal-teeth stator and the auxiliary teeth added stator.

The simulation models include (a) Maxwell2D models which include three configurations: stator with traditional equal teeth and rotor pole sine-shaped (TraTooth + SinePole), stator with auxiliary teeth and rotor pole sine-shaped (AuxTooth + SinePole), and stator with auxiliary teeth and rotor pole profile harmonic injection (AuxTooth + HarInject); (b) Structural vibration models include stator with traditional equal teeth and stator with auxiliary teeth. The differences in parameter values among the three configurations are listed in Table 4.

Table 4. Design parameters of the three types of motors.

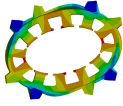
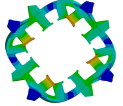
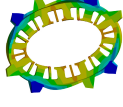
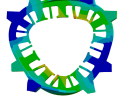
Parameters		TraTooth + SinePole	AuxTooth + SinePole	AuxTooth + HarInject
Slot open width So (mm)		5	2	2
Auxiliary teeth width At (mm)		-	2.5	2.5
Magnet poles inset L (mm)		1.5	1.5	1.5
Magnet pole shape parameters	A_1	-	-	1.079
	A_3	-	-	−0.212
	A_5	-	-	0.036

The structural vibration simulation model comprises the housing and stator. Fixed constraints are applied to the blue positioning holes at both housing ends, and a tied connection is established between housing and stator. Windings are modeled as additional mass on stator teeth, and the stator laminations could be replaced by a solid with orthotropic elastic parameters [24]. The material parameters are listed in Table 5, where ρ is the mass density; E_r , E_t , and E_z are Young's modulus in radial, tangential and axial directions, respectively. ν_{rt} , ν_{rz} , and ν_{tz} are Poisson's ratios in the direction of rt , rz , and tz , respectively. As shown in Table 6, nature frequencies of stators with auxiliary teeth are lower than those of traditional stators. These frequency shifts are caused by the increased mass due to the auxiliary teeth and may result in a higher vibration response under the same force.

Table 5. Material parameters.

Name	Parameters	Equivalent Stator	Housing
Mass density	ρ (kg/m ³)	7372	2770
Young's modulus	E_r, E_t (GPa)	194	71
	E_z (GPa)	67.3	71
Shear modulus	G_{rt} (GPa)	74.65	26.7
	G_{rz}, G_{tz} (GPa)	25.9	26.7
Poisson's ratio	$\nu_{rt}, \nu_{rz}, \nu_{tz}$	0.3	0.33

Table 6. Modal shapes and nature frequencies.

Modal Order	2	3	4
Traditional teeth			
Nature frequency	2091 Hz	3056 Hz	4854 Hz
Auxiliary teeth			
Nature frequency	2067 Hz	3012 Hz	4777 Hz

5.1. Comparison of Electromagnetic Performance

The airgap flux density harmonics of the three types of motors under open load and rated load are shown in Figure 12. As shown in Figure 12a, the application of the auxiliary teeth reduced the permeance of tooth-number and double tooth-number orders, leading to 75%, 89% and 70% reductions in the 5th-order ($|p - Q_s|$), 17th-order ($|p - 2Q_s|$), and 19th-order ($p + Q_s$) harmonics, respectively. The increase in p th order airgap flux density is

caused by the increase in zero-order permeance and p th-order winding MMF. Following the injection of 3rd and 5th spatial harmonics into the magnetic pole profile, the $3p$ -order airgap magnetic flux density harmonics exhibit approximately threefold amplification. Compared to the open load condition, the 5th-order harmonic amplitude in Figure 12b demonstrates a significant increase under the rated load due to the presence of $(p - Q_s)$ winding slot harmonic.

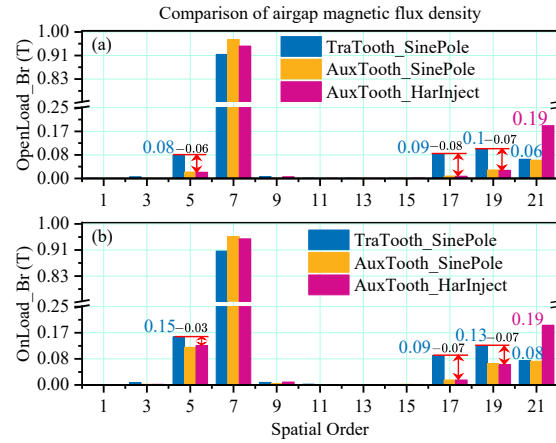


Figure 12. Airgap radial flux density comparison: (a) Open load; (b) On load.

The influence of the auxiliary teeth and rotor pole profile harmonic injection on both motor output torque and cogging torque is illustrated in Figure 13. The standalone auxiliary teeth configuration improved the average output torque due to the enhanced fundamental magnetic flux density. However, the application of rotor pole profile harmonic injection led to an increase in $3p$ -order harmonic magnetic flux density, which reduced the average output torque.

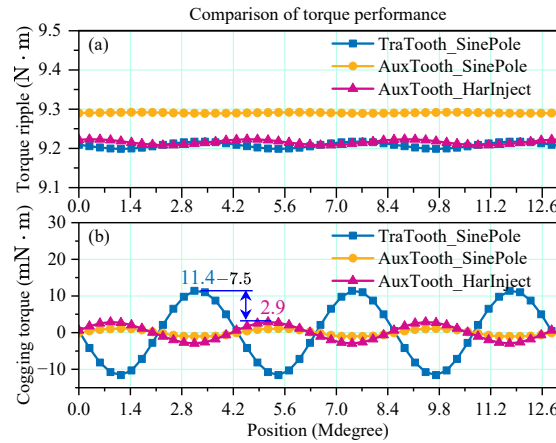


Figure 13. Torque performance comparison: (a) Output torque; (b) Cogging torque.

The cogging torque satisfies the following relationship:

$$T_c = \frac{R^2 L_{core}}{\mu_0} \int_0^{2\pi} B_r B_t d\theta \propto \lambda_0 \lambda_{sk} \sin(2n\omega_e t), \text{ when } 2np - kQ_s = 0. \quad (24)$$

The cogging torque is related to the stator permeance, and the primary part is obtained when $k = 7$ and $n = 6$. According to (20), $\lambda_{sk} = 0$ when $\text{mod}(kQ_s\tau_1, \pi) = \pi/2$, indicating that the cogging torque becomes zero. In the optimal design, $So = 2$ mm, $At = 2.5$ mm, $2\tau_1 = 23.55^\circ$, and $\lambda_{s7} \approx 0$. Thus, as shown in Figure 13b, after applying auxiliary teeth,

the cogging torque amplitude decreased from 11.4 mN·m to 2.9 mN·m, achieving a 74.6% amplitude reduction.

5.2. Comparison of Electromagnetic Force and Vibration Response

The harmonic components of equivalent yoke force for three configurations under both open-load and rated-load conditions are comparatively presented in Figure 14. Through the synergistic application of auxiliary teeth and rotor pole profile harmonic injection, the second-order electromagnetic force components are both reduced, specifically, from 4.14×10^4 N/m² to 3.38×10^4 N/m² under rate load (phase current 12 Arms and load angle 90°, i.e., $i_d = 0$), representing an approximate 18% reduction. Although the 4th-, 6th-, and 8th-order components exhibit increases in magnitude, their higher spatial orders result in a limited amplitude growth of electromagnetic vibration response.

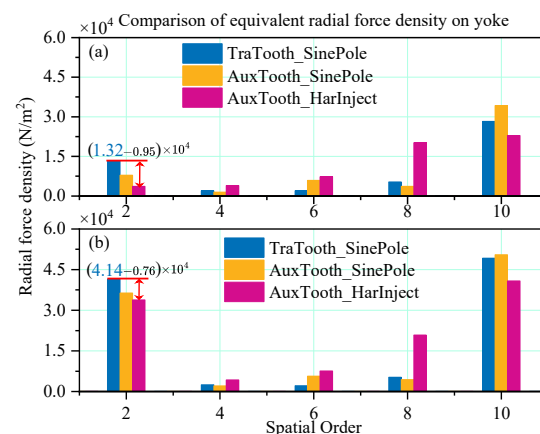


Figure 14. Comparison of equivalent yoke force density: (a) Open load; (b) On load.

The vibration response amplitudes at different positions on the motor surface differ. Thus, a fixed measurement point (black triangle in Figure 11b) is selected for comparing the trends of the three configurations. Radial vibration acceleration responses under open-load and rated-load conditions are shown in Figure 15. At a mechanical speed of 1200 rpm, the second-order electromagnetic force rotating at twice the electrical frequency ($2f_e = 280$ Hz) dominates the vibration spectrum. The vibration response trends of the three configurations match the electromagnetic force magnitude variations as shown in Figure 14. Compared to the conventional structure, the synergistic application of auxiliary teeth and rotor profile harmonic injection reduces the second-order electromagnetic vibration acceleration under a rated load (from 3.01 m/s² to 2.39 m/s²), with a 21% amplitude reduction.

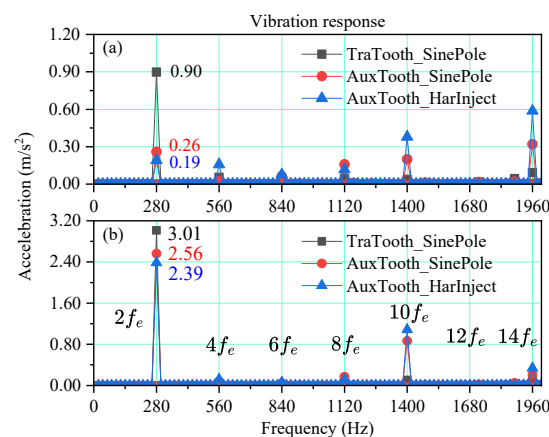


Figure 15. Comparison of vibration response: (a) Open load; (b) On load.

The amplitude of vibration responses under multi-speed and Campbell diagram of the equivalent radiated power levels from 900 to 1500 rpm are shown in Figure 16. Except for the main vibrations at frequencies $2f_e$, $4f_e$, $6f_e$, and $10f_e$, resonance appears around 2067 Hz (elliptical marked) which is the 2nd-order natural frequency of the structure.

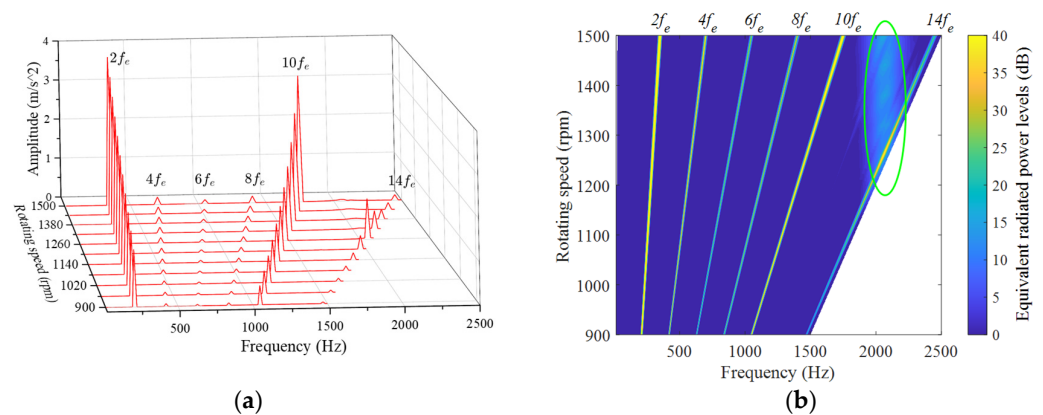


Figure 16. Vibrations under multi-speed: (a) vibration amplitude; (b) Campbell diagram.

6. Conclusions

This study has analyzed the effects of auxiliary teeth on FSCW-PM machines with $2p = Q_s \pm 2$ slot-pole combinations through three key aspects: winding MMF, stator permeance, and tooth force modulation effect. The auxiliary teeth have been shown to effectively reduce slot-order permeance harmonics and suppress electromagnetic forces caused by permeance modulation. For $2p = Q_s \pm 2$ configurations, optimized auxiliary teeth width has demonstrated the ability to mitigate slot harmonics without sacrificing fundamental MMF amplitude.

Furthermore, based on the equivalent yoke force model, tooth modulation coefficients for airgap electromagnetic forces have been established. Comparative analysis reveals distinct modulation effects in $2p = Q_s \pm 2$ machines:

1. For $2p = Q_s - 2$ configurations, the tooth modulation coefficient $K_{2,2p} > 0$, where $2p$ -order airgap forces enhance second-order electromagnetic vibrations;
2. In $2p = Q_s + 2$ machines, $K_{2,2p} < 0$, resulting in $2p$ -order forces suppressing second-order vibrations.

Based on these characteristics, this study proposes a synergistic approach utilizing optimal auxiliary teeth width and rotor pole profile harmonic injection to further optimize electromagnetic vibrations in $2p = Q_s + 2$ spoke-type machines. The NSGA-III genetic algorithm was employed to obtain the optimal Pareto front for three key parameters: auxiliary tooth width, slot opening width, and rotor pole harmonic injection coefficients. With optimized parameters, the cogging torque and second-order electromagnetic vibrations were reduced by 74.6% and 21%, respectively, while maintaining uncompromised output torque.

The core of this method lies in utilizing the characteristic that the slot pitch of the motor with $2p = Q_s + 2$ is larger than the pole pitch, as well as the suppression effect of the $2p$ -order airgap electromagnetic force on the 2nd-order yoke force through the tooth modulation effect. Therefore, this method is valid for other $2p = Q_s + 2$ FSCW tooth coil motors. The method proposed in this paper verifies the trends in vibration variation before and after optimization through simulations. However, the auxiliary tooth may also reduce the slot area and increase the slot filling, which may, in turn, impact mechanical performance. Moreover, there are still some differences between the simulation model and actual motor, such as the neglected endcap and the simplified connection between housing

and stator core. Therefore, further experimental verification is still necessary and will be one of the follow-up research directions from this paper.

Author Contributions: H.Z. wrote this manuscript; W.W. proposed the methodology and conceptualization; X.L. reviewed and edited this manuscript; Z.W. was responsible for the funding acquisition. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement: The original contributions presented in this study are included in the article. Additionally, the NSGA-III software is openly available in <https://github.com/lfarizav/NSGA-III> (accessed on 15 May 2024). Any questions about the DRHM method or further inquiries can be directed to the corresponding author.

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Nomenclature

f_r	Radial airgap force density [N/m ²].	p	Pole pairs
f_t	Tangential airgap force density [N/m ²].	Q_s	Slot number
$\bar{f}_{yr}$	Radial force density on yoke [N/m ²].	θ_n	Position of the n th tooth.
r_g	Radius of airgap central circle [m].	$\Delta\theta$	Slot pitch of the tooth.
r_y	Radius of yoke central circle [m].	τ_1	Slot pitch of wound teeth.
$F_r[n]$	Radial concentrated forces [N/m].	τ_2	Slot pitch of auxiliary teeth.
$F_t[n]$	Tangential concentrated forces [N/m].	τ_a	Auxiliary teeth width.
$M[n]$	Bending moments on tooth shoulder.	τ_t	Wound teeth width.
$M_y[n]$	Bending moments on yoke.	τ_s	Slot opening width.
F_{Mn}	Equivalent forces by $M_y[n]$.	m	Winding phases number.
$F_{Mr}[n]$	Equivalent radial forces by $M_y[n]$.	λ	Airgap permeance.
λ_{sk}	kQ_s -th order stator permeance.	ω_e	Electrical angular velocity.
λ_{rn}	$2np$ -th order rotor permeance.	f_e	Electrical frequency.

Appendix A

Appendix A.1 Proof of Equation $Q_s / \text{GCD}(2p, Q_s) = m \cdot k, k \in \mathbb{Z}$

The greatest common divisor of slot and pole numbers $\text{GCD}(2p, Q_s)$ can be calculated by:

$$\frac{Q_s}{2p} = \frac{Q_0 \cdot N_t}{2p_0 \cdot N_t}. \quad (\text{A1})$$

where $N_t = \text{GCD}(p, Q_s)$ is the number of unit motors, Q_0, p_0 are slots and pole-pairs number of the unit motor, respectively. Then, $\text{GCD}(2p, Q_s)$ satisfies:

$$\text{GCD}(2p, Q_s) = \begin{cases} N_t, & Q_0 \text{ is odd,} \\ 2N_t, & Q_0 \text{ is even.} \end{cases} \quad (\text{A2})$$

Obviously, when Q_0 is odd,

$$\frac{Q_s}{\text{GCD}(2p, Q_s)} = Q_0 = m \cdot k, \quad (\text{A3})$$

where k is an integer. When Q_0 is even, for the symmetrical winding, $Q_0 = 2mk$ is existing, thus:

$$\frac{Q_s}{\text{GCD}(2p, Q_s)} = \frac{Q_0}{2} = m \cdot k. \quad (\text{A4})$$

According to (A3) and (A4), the relationship $Q_s / \text{GCD}(2p, Q_s) = m \cdot k$ is established.

Appendix A.2 Proof of Equation $\text{GCD}(2p, Q_s) = 2p/d$

The number of slots per pole per phase q can be expressed by:

$$q = \frac{Q_s}{2mp} = \frac{Q_s/m}{2p} = \frac{N \cdot t}{d \cdot t'}, \quad (\text{A5})$$

where t is calculated by:

$$t = \frac{2p}{d} = \text{GCD}\left(2p, \frac{Q_s}{m}\right). \quad (\text{A6})$$

According to the proof in Appendix A.1, for a machine with symmetrical winding, it holds that:

$$\begin{aligned} \text{GCD}\left(2p, \frac{Q_s}{m}\right) &= \text{GCD}(2p, k \cdot \text{GCD}(2p, Q_s)) \\ &\geq \text{GCD}(2p, Q_s). \end{aligned} \quad (\text{A7})$$

And due to,

$$\text{GCD}\left(2p, \frac{Q_s}{m}\right) \leq \text{GCD}(2p, Q_s). \quad (\text{A8})$$

Thus, according to (A5)–(A8),

$$\frac{2p}{d} = \text{GCD}\left(2p, \frac{Q_s}{m}\right) = \text{GCD}(2p, Q_s). \quad (\text{A9})$$

Appendix A.3 Sign Derivation of Tooth Modulation Coefficient $K_{2,2p}$

According to (13) and (17), tooth modulation coefficient $K_{2,2p}$ satisfies the following relationship:

$$\begin{aligned} K_{2,2p} &\propto \sin(2p\tau_1) - \sin(2p\tau_2) \\ &\propto 2 \cos\left(\frac{p\pi}{Q_s}\right) \sin\left(2p\tau_1 - \frac{p\pi}{Q_s}\right) \end{aligned} \quad (\text{A10})$$

Since the relationship of poles and slots $2p = Q_s \pm 2$, (A10) can be rewritten as

$$K_{2,2p} \propto \begin{cases} \sin\left(\frac{\pi}{Q_s}\right) \cos\left(2p\tau_1 - \frac{\pi}{Q_s}\right), & 2p = Q_s + 2 \\ -\sin\left(\frac{\pi}{Q_s}\right) \cos\left(2p\tau_1 + \frac{\pi}{Q_s}\right), & 2p = Q_s - 2 \end{cases} \quad (\text{A11})$$

Obviously, $\sin(\pi/Q_s) > 0$ is true. In general, the slot pitch $2\tau_1$ of wound-tooth satisfies the following relationship:

$$\frac{\pi}{Q_s} < 2\tau_1 \text{ and } 2\tau_1 < \frac{2\pi}{Q_s}. \quad (\text{A12})$$

Thus,

$$\begin{cases} \frac{\pi}{2} < 2p\tau_1 - \frac{\pi}{Q_s} < \pi + \frac{2\pi}{Q_s}, & 2p = Q_s + 2 \\ \frac{\pi}{2} < 2p\tau_1 + \frac{\pi}{Q_s} < \pi - \frac{2\pi}{Q_s}, & 2p = Q_s - 2 \end{cases} \quad (\text{A13})$$

The slot number of three-phase $2p = Q_s \pm 2$ FSCW-PM machines satisfy $Q_s \geq 6$, then, $\pi - 2\pi/Q_s$ lies in the second quadrant and $\pi + 2\pi/Q_s$ lies in the third quadrant. Therefore,

$$\begin{cases} \cos(2p\tau_1 - \pi/Q_s) < 0, & 2p = Q_s + 2 \\ \cos(2p\tau_1 + \pi/Q_s) < 0, & 2p = Q_s - 2 \end{cases} \quad (\text{A14})$$

According to (A11)–(A14), the following relationship is valid:

$$\begin{cases} K_{2,2p} < 0, & 2p = Q_s + 2 \\ K_{2,2p} > 0, & 2p = Q_s - 2 \end{cases} \quad (\text{A15})$$

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