

## Article

# A Cutting Force Prediction Model for Corner Radius End Mills Based on the Separate-Edge-Forecast Method and BP Neural Network

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## Abstract

Corner radius end mills (CREMs) are widely used in machining due to their unique tool geometry, which improves surface quality. Variations in cutting force during machining significantly impact machining quality. Therefore, precisely predicting cutting forces is critical for controlling machining chatter and enhancing accuracy. Traditional element force models have complex formulas and high computational demands when considering tool runout. This paper proposes a hybrid prediction model for CREMs that integrates the separate-edge-forecast method and the BP neural network. The integration approach incorporates runout effects into cutting force coefficients and addresses nonlinear effects from runout. The accuracy of the cutting force prediction model was validated through side milling on 7075 aluminum alloy. The results indicate that the maximum error between the predicted and measured forces is 9.43%, demonstrating that this model ensures high prediction accuracy while reducing computation cost.

**Keywords:** corner radius end mills; separate-edge-forecast method; BP neural network; cutting force prediction model; tool runout



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## 1. Introduction

As a crucial physical parameter in the machining process, cutting force plays a significant role in areas such as chatter identification, tool wear detection, and machining accuracy prediction. Therefore, establishing reasonable cutting force models to accurately predict cutting forces represents an important research direction in the field of machining. Currently, commonly used cutting force prediction models can be categorized into empirical formula models, element force models, numerical models, and artificial intelligence models.

Empirical formula models establish the functional relationship between cutting forces and cutting parameters through methods such as regression analysis, relying on extensive machining experimental data. While simple to construct, this model suffers from high time and economic costs, difficulty in predicting transient forces during machining, and significant limitations in practical applications. Ning et al. [1] established an empirical formula for the segmented specific cutting energy of CFRP and used it to predict the cutting forces in CFRP multidirectional laminates. To reduce the influence of machine tool vibration on the empirical model, Azeem et al. [2] fitted the acquired noisy signals to a polynomial function and subsequently employed an iterative two-stage numerical procedure to solve

for the model coefficients, thereby developing two distinct solution approaches. Lin et al. [3] developed a cutting force model for ball-end mills under various cutting conditions using quadratic regression equations, where the corresponding regression coefficients were determined using experimental cutting data.

The core concept of the element force models lies in the axial superposition of forces analyzed on discretized cutting edge elements. Its modeling process requires comprehensive consideration of factors such as tool geometry and workpiece material properties. A well-formulated differential force model can accurately predict transient cutting forces and finds wide application in precision machining; however, the model development process is relatively complex. Wang et al. [4] proposed a differential force prediction model incorporating the influence of tool wear, conducting a comprehensive analysis of the relationship between cutting force and tool wear, and experimentally validating the accuracy of the derived model. Zhu et al. [5] developed a cutting force prediction model for Ti-6Al-4V, considering the Taylor factor, acquiring cutting forces during machining experiments using a dynamometer, and observing microstructural evolution on the machined surfaces before and after cutting via Electron Backscatter Diffraction. Comparative analysis of the experimental results and model predictions verified the reliability of the predictive model. Yang et al. [6] established an element force model incorporating tool wear, with its accuracy validated through slot milling experiments on 304 stainless steel. Wei et al. [7] discretized the curved surface into a series of infinitesimal inclined planes corresponding to cutter location points, parametrically defined the geometric relationships between the tool axis, feed direction, and inclined planes, and proposed a novel cutting force prediction model for ball-end milling of curved surfaces. Sela et al. [8] presented a mechanistic cutting force prediction model for three distinct tool edge radii, based on two cutting speeds and a wide range of feed rates. Su et al. [9] developed a differential cutting force model suitable for complex profile milling cutters by analyzing the working state of each section on the complex milling tool. Srinivasa et al. [10] proposed a differential cutting force model considering edge radius and material strengthening effects, specifically addressing the overlapping tooth engagement phenomenon in micro-milling. Matsumura et al. [11] presented a differential force model integrating tool axis tilt and chip flow direction. Liu et al. [12] proposed a differential force prediction model based on fracture mechanisms and the evolution of material mechanical properties. This model achieves accurate prediction of cutting forces during machining by introducing a fracture coefficient, a slip angle coefficient, and a compression coefficient. Lu et al. [13] developed an accurate and reliable differential force prediction model by comprehensively considering the influence of tool runout and workpiece deformation on instantaneous chip thickness. Sun et al. [14] formulated a high-precision differential force prediction model by transforming the tool-workpiece contact relationship into the relationship between a helical line and the cutting region within a two-dimensional plane.

Numerical models predict cutting forces by constructing 3D simulation models to simulate material deformation and stress distribution. A key advantage of this approach is its ability to visualize machining details intuitively, making it suitable for analyzing complex conditions. However, these simulations are computationally expensive and demand significant hardware resources and precise parameter settings. Wang et al. [15] proposed a multi-scale crystal plasticity finite element model to investigate subsurface generation during ultra-precision grating cutting of FCC alloys, specifically accounting for micro-scale chip thickness variation and the size effect in ultra-precision machining. Qiao et al. [16] utilized finite element analysis to examine the initial stage of orthogonal cutting with negative rake angle tools. By combining slip-line theory and a shear band model, they established a cutting force calculation model for negative rake angle tools

machining workpieces under varying depths of cut. Timothy et al. [17] determined the uncertainty in Johnson–Cook model coefficients via orthogonal cutting FE simulations. Using the tool’s rake and clearance profiles along with random samples from the Johnson–Cook parameter distributions, they determined cutting force model coefficients related to factors such as the cutting area and cutting width, and subsequently predicted cutting forces. Pal et al. [18] performed finite element simulations using the ABAQUS software and the Johnson–Cook model, focusing on investigating the relationship between the lead angle and cutting forces, thereby achieving accurate analysis and prediction of machining process dynamics. Hu. et al. [19] investigated the relationship between drilling forces, torque, and microstructure parameters by establishing a three-dimensional model of the drilling process, and developed predictive models for drilling force and torque based on microstructure parameters. Jagadesh et al. [20] employed a strain gradient plasticity-modified Johnson–Cook material model to describe material flow stress and validated the mechanistic model’s predicted cutting forces using micro-turning experiments.

Artificial intelligence models predict cutting forces by training algorithms on experimental data. These models offer strong adaptability to nonlinear problems without requiring explicit mechanistic interpretation of the cutting process. Nevertheless, their performance depends heavily on large volumes of high-quality training data, and a significant drawback is their poor interpretability. Dai et al. [21] developed an improved Radial Basis Function neural network model based on extensive sample simulations and utilized it to achieve high-precision prediction of cutting forces during the semi-finishing machining of thin-walled parts. Wang et al. [22] leveraged simulation-derived data and, integrating theories and methods from the field of transfer learning, established a transfer network that demonstrates enhanced error control capability. Liu et al. [23] acquired experimental data through cutting tests, embedded the gradient descent algorithm into a standard particle swarm optimization algorithm, and proposed an improved particle swarm optimization-optimized fuzzy system method for cutting force prediction. Wu et al. [24] developed a cutting force prediction model incorporating an extreme learning machine model enhanced by the Grey Wolf Optimizer algorithm. They determined the number of hidden layer nodes using a second-order multivariate regression model for analysis. Hu et al. [25] employed the genetic algorithm and multivariate regression analysis to establish a cutting force prediction model for multi-diamond rapid milling of brittle-hard materials. Bian et al. [26] proposed a prediction model based on a Chimpanzee Optimization Algorithm–Random Forest hybrid algorithm driven by finite element data. Peng et al. [27], considering the non-linear mapping relationship between spindle current and cutting force signals, proposed a method for predicting instantaneous cutting forces based on a neural network model trained on current signals.

CREMs, characterized by their corner cutting edges, offer relatively stable cutting forces and superior machining quality during the machining process [28–30]. Consequently, they exhibit broad application potential in areas such as contour machining of curved surfaces, profile milling, and corner rounding operations.

In the research on cutting forces for CREMs, Qi et al. [31] developed a differential cutting force prediction model for variable-helix CREMs incorporating the effect of tool runout. They provided an algorithm based on the linear least squares method and oblique cutting theory to solve for the initial values of the model parameters, and verified the model’s prediction accuracy by milling 1Cr18Ni4Mo3N stainless steel alloy. Zhang et al. [32] calculated the flow stress in the shear zone of the cutting edge element using an oblique cutting model. They established a differential cutting force prediction model for CREMs by modifying the calculation process according to the helix angle parameter of the cutting edge element, and verified the model’s prediction accuracy by milling 6061 aluminum alloy.

Duan et al. [33], accounting for the influence of tool inclination angle on the instantaneous undeformed chip thickness, proposed an online identification method for cutting force model coefficients considering tool posture, thereby achieving accurate prediction of cutting forces for CREMs. The predictive accuracy of the model was demonstrated through milling tests on 2024-T6 aluminum alloy.

In summary, due to the inherent geometric complexity of CREMs, constructing corresponding differential cutting force models involves a cumbersome process, significant computational demands, and often results in limited prediction accuracy.

The cutting force model for CREMs proposed in this paper enhances prediction accuracy by incorporating tool runout. Specifically, the separate-edge-forecast method combined with a BP neural network integrates the influence of tool runout directly into the cutting force coefficients. This method significantly reduces the complexity of the force model and improves computational efficiency. These advantages make the model highly valuable for practical machining applications.

## 2. Materials and Methods

### 2.1. Geometric Parameters of CREM

The three-edge CREM is illustrated in Figure 1; the pitch angles are  $\varphi_1 = \varphi_2 = \varphi_3$ , and the helix angles are  $\beta_1 = \beta_2 = \beta_3$ . The identical helix angles result in the constant pitch angles at axial height along the tool.

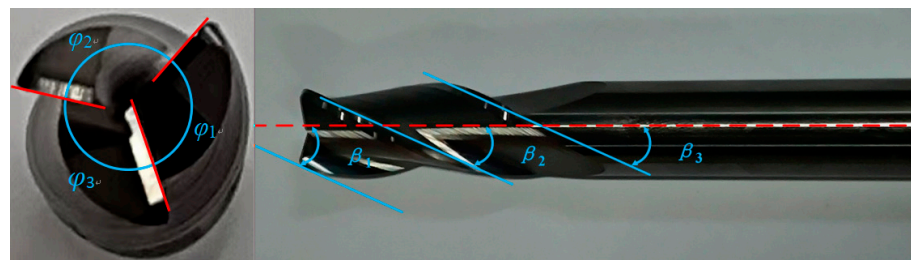


**Figure 1.** Geometric parameters of a three-edge CREM.

Geometric structure of CREMs is illustrated in Figure 2; the primary distinction between CREMs and flat end mills lies in their cutting edges: the former feature a cutting edge composed of a corner cutting edge segment and a cylindrical cutting edge segment, which meet at the vertex of the round cutting edge section. The radial lag angle  $\phi(z)$  of the cutting edge at axial height  $z$  is given by the following:

$$\phi(z) = \frac{z \tan \beta}{R} \quad (1)$$

where  $R$  is the cutter radius.

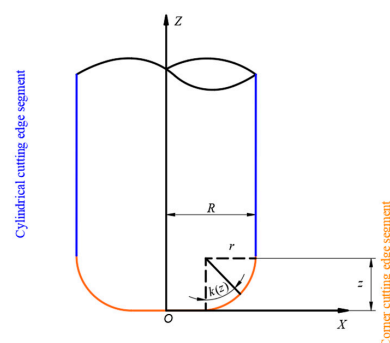


**Figure 2.** Geometric structure of CREMs.

The axial immersion angle  $\kappa(z)$  of the cutting edge at axial height  $z$  is given by the following:

$$\kappa(z) = \begin{cases} \arccos\left(\frac{r-z}{r}\right) & z \leq r \\ \frac{\pi}{2} & z > r \end{cases} \quad (2)$$

where  $r$  is the corner radius.

## 2.2. Establishment of Cutting Force Prediction Model

Tool runout, inevitably caused by manufacturing and clamping errors, significantly compromises the accuracy of milling force prediction models. Most researchers characterize tool runout using two unknown parameters: eccentricity and eccentric angle. In element force modeling, runout primarily alters the instantaneous undeformed chip thickness and secondarily modifies the tool entry/exit angles. Consequently, the element force models considering tool runout often incorporate eccentricity and eccentric angle into chip thickness calculations. This requirement leads to mathematically cumbersome formulations and high computational costs. The separate-edge-forecast method effectively circumvents this complexity by integrating errors caused by tool runout into cutting force coefficients. Tool runout causes cutting force coefficients to differ among edges with identical geometric parameters, and the coefficient curves exhibiting this discrepancy as illustrated in Figure 3.

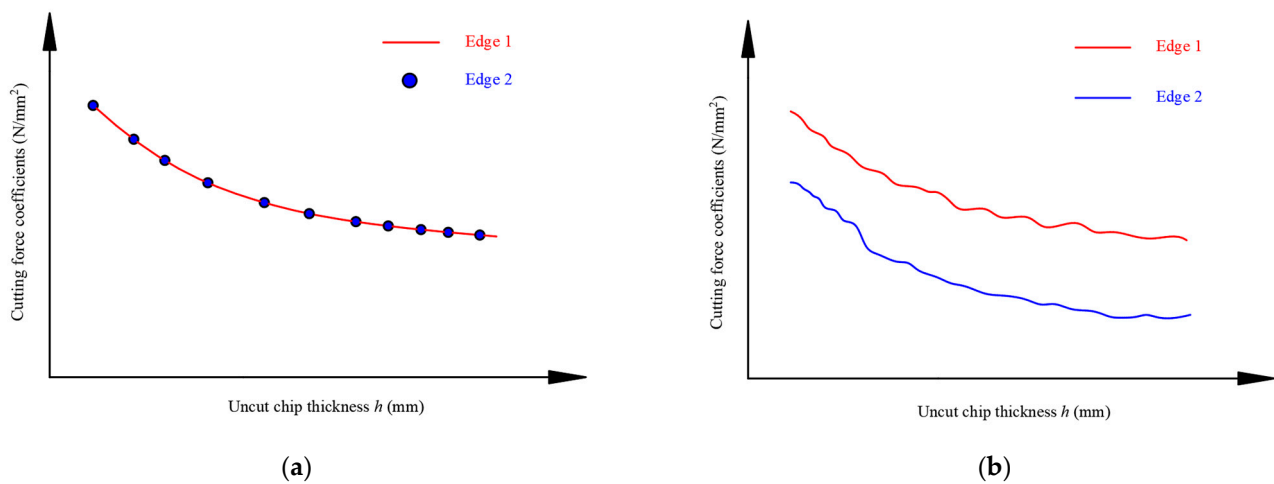


**Figure 3.** The cutting force coefficient curves (a) without considering runout; (b) considering runout.

Extensive research has demonstrated significant disparities in cutting force coefficients between the corner cutting edge segment and cylindrical cutting edge segment of CREMs, necessitating separate computational treatment rather than consolidated modeling. When applying the separate-edge-forecast method to establish cutting force prediction models for end mills, the cutting forces are solely governed by the cutting force coefficients and instantaneous undeformed chip thickness. The tangential, radial, and axial element cutting forces are given by Equation (3):

$$\begin{aligned} dF_{j,t} &= \begin{cases} A[\theta_j(z)]K'_{j,t} h[\theta_j(z)]dz & z \leq r \\ A[\theta_j(z)]K_{j,t} h[\theta_j(z)]dz & z > r \end{cases} \\ dF_{j,r} &= \begin{cases} A[\theta_j(z)]K'_{j,r} h[\theta_j(z)]dz & z \leq r \\ A[\theta_j(z)]K_{j,r} h[\theta_j(z)]dz & z > r \end{cases} \\ dF_{j,a} &= \begin{cases} A[\theta_j(z)]K'_{j,a} h[\theta_j(z)]dz & z \leq r \\ A[\theta_j(z)]K_{j,a} h[\theta_j(z)]dz & z > r \end{cases} \end{aligned} \quad (3)$$

where  $\theta_j(z)$  is the positional angle of the  $j$ -th cutting edge at axial position  $z$ .  $K'_{j,t}$ ,  $K'_{j,r}$ ,  $K'_{j,a}$  are tangential, radial, and axial cutting force coefficients for the corner cutting edge segment of the  $j$ -th edge.  $K_{j,t}$ ,  $K_{j,r}$ , and  $K_{j,a}$  are tangential, radial, and axial cutting force coefficients for the cylindrical cutting edge segment of the  $j$ -th edge.  $h[\theta_j(z)]$  is the instantaneous undeformed chip thickness for the  $j$ -th cutting edge at positional angle  $\theta_j(z)$ , which can be calculated by Equation (4) [34].  $A[\theta_j(z)]$  is the window function, which is used to judge the cutting state of the  $j$ -th edge.

$$h[\theta_j(z)] = f_z \sin \theta_j(z) \sin \kappa(z) \quad (4)$$

$$A[\theta_j(z)] = \begin{cases} 1 & \theta_{st} < \theta_j(z) < \theta_{ex} \\ 0 & \text{other} \end{cases} \quad (5)$$

where  $\theta_{st}$  and  $\theta_{ex}$  are the tool entry and exit angles, which could be calculated in reference [35].

Through coordinate transformation, the calculation formulas for triaxial forces in the machine-tool Cartesian coordinate system are derived as Equation (6), and the force analysis is illustrated in Figure 4.

$$\begin{bmatrix} dF_x(\theta_j(z)) \\ dF_y(\theta_j(z)) \\ dF_z(\theta_j(z)) \end{bmatrix} = \begin{bmatrix} \cos \theta_j(z) & \sin \theta_j(z) & 0 \\ \sin \theta_j(z) & -\cos \theta_j(z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dF_t(\theta_j(z)) \\ dF_r(\theta_j(z)) \\ dF_a(\theta_j(z)) \end{bmatrix} \quad (6)$$

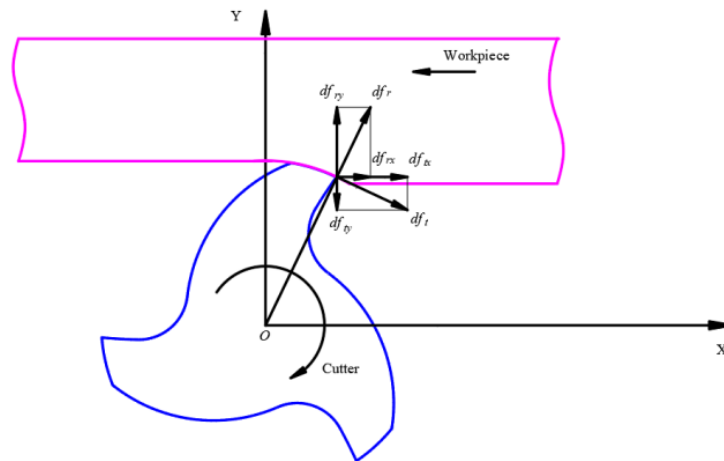


**Figure 4.** The force analysis.

The triaxial forces on the  $j$ -th cutting edge can be calculated as in Equation (7):

$$F_{j,q} = \int_0^{a_p} dF_{j,q}(q = x, y, z \vee q = t, r, a) \quad (7)$$

The total milling forces acting on the cutter can be calculated as in Equation (8):

$$F_q = \sum_{j=1}^N F_{j,q}(q = x, y, z \vee q = t, r, a) \quad (8)$$

### 2.3. Condition of Single-Tooth Cutting

To determine the cutting force coefficients for the  $j$ -th edge using the separate-edge-forecast method, it is required that all other edges do not engage in the cutting process. This ensures the accuracy of derived coefficients. Consequently, the acquired cutting force data must satisfy the specific condition that at any time, the measured cutting forces along

three directions must belong to only one edge. The cutting force curve under the single-tooth cutting condition is illustrated in Figure 5.

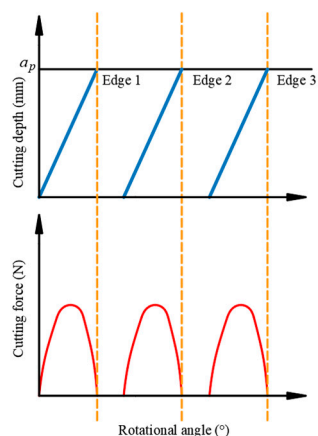


**Figure 5.** Cutting force curve under single-tooth cutting condition.

This condition can be satisfied solely by adjusting the cutting depth value without altering the tool or machining method. The calculation formula for the critical single-tooth cutting condition of flat-end mills is given in Equation (9) [36].

$$\begin{cases} a_p \leq R(\phi - \frac{\pi}{2} + \arcsin(1 - \frac{a_r}{R})) / \tan \beta & a_e \leq R \\ a_p \leq R(\phi - \frac{\pi}{2} - \arcsin(\frac{a_r}{R} - 1)) / \tan \beta & a_e > R \end{cases} \quad (9)$$

where  $a_p$  is axial cutting depth and  $a_e$  is radial cutting depth.

For CREMs, when the cylindrical cutting edge segment is under single-tooth cutting, the corner cutting edge segment also operates in single-tooth cutting since the cutter is an integrated entity. It follows that the critical single-tooth cutting condition for CREMs is given by the following:

$$\begin{cases} a_p \leq R(\theta - \frac{\pi}{2} + \arcsin(1 - \frac{a_r}{R})) / \tan \beta + r & a_e \leq R \\ a_p \leq R(\theta - \frac{\pi}{2} - \arcsin(\frac{a_r}{R} - 1)) / \tan \beta + r & a_e > R \end{cases} \quad (10)$$

The limit axial cutting depth can be calculated using Equation (10) when the radial cutting depth has been given. The relationship between radial and axial cutting depths under the current experimental conditions is illustrated in Figure 6.

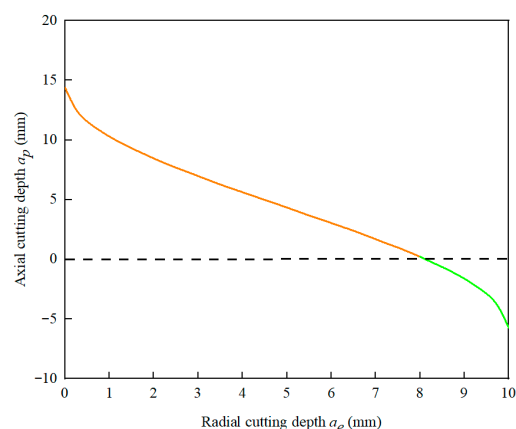


**Figure 6.** The relationship between radial and axial cutting depths.

Figure 6 indicates that when the selected depth parameters lie below the curve and  $a_p > 0$ , the cutting process operates under single-tooth cutting conditions. The green segment of the curve denotes that when the radial depth is sufficiently large, the single-tooth cutting state cannot be achieved regardless of the axial depth value.

#### 2.4. Calibration of Cutting Force Coefficients

When tool runout is considered, the relationship between uncut chip thickness and cutting force coefficients is not a simple linear function. Consequently, fitting a high-precision nonlinear relationship curve is critical for ensuring the accuracy of predictive models. Given its strong adaptability to nonlinear problems, a classical BP neural network is employed in this paper for curve fitting. The computational workflow for this process is illustrated in Figure 7.

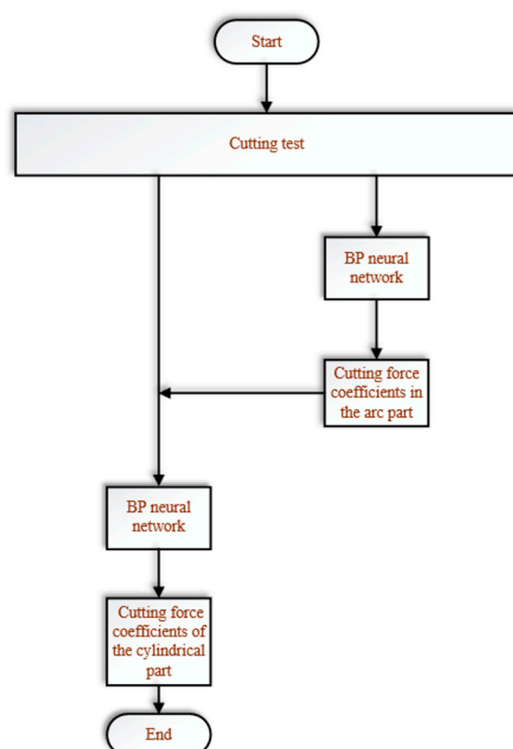


Figure 7. Flowchart of calibration process.

The cutting force coefficients for the corner cutting edge segment at different uncut chip thicknesses are calculated by Equations (3)–(5), and the resulting coefficients were then fitted by the BP neural network.

The cutting force coefficients for the cylindrical cutting edge segment are calculated using Equation (11).

$$F_{j,q}^m - \int_0^1 K'_{j,q} h[\theta_j(z)] dz = \int_1^{a_p} K_{j,q} h[\theta_j(z)] dz \quad (q = t, r, a) \quad (11)$$

where  $F_{j,q}^m$  is the measured cutting forces.

#### 2.5. Experimental Verification

This paper employed a carbide CREM (ALG-3R-D10.0R1.0) manufactured by ZCCCT to perform side milling experiments on 7075 aluminum alloy. The tool parameters are listed in Table 1. A Kistler 9123C1011 rotary dynamometer was directly mounted on the tool to enhance measurement accuracy (experimental setup is illustrated in Figure 8). The cutting parameters determined from Figure 6 are listed in Table 2. Radial cutting depth

was maintained constant at 1 mm. The experimental equipments and their manufacturers are listed in Table 3.

**Table 1.** The Tool parameters.

| Diameter (mm) | Number of Edges | Helix Angle (°) | Corner Radius (mm) |
|---------------|-----------------|-----------------|--------------------|
| 10            | 3               | 30              | 1                  |

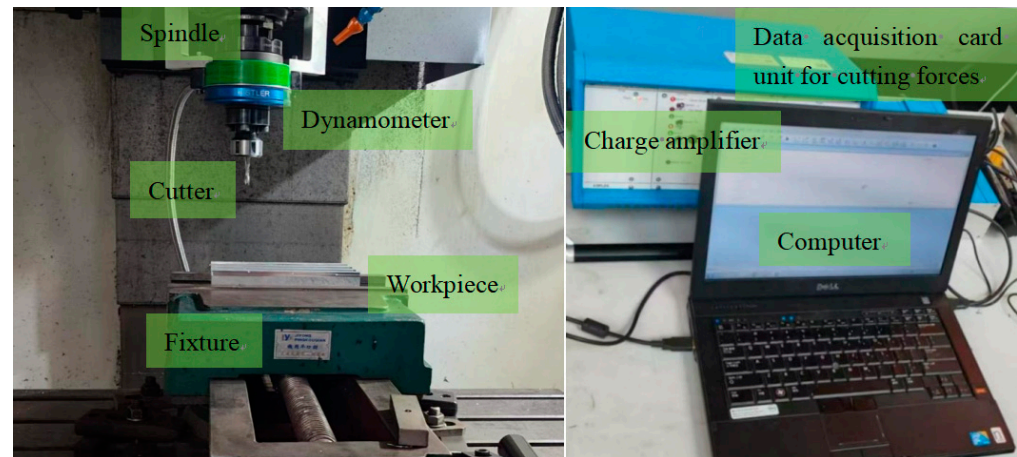


**Figure 8.** Experimental setup.

**Table 2.** The cutting parameters.

| Test Number | Type         | Spindle Speed (r/min) | Feed Per Tooth (mm/z) | Axial Cutting Depth (mm) |
|-------------|--------------|-----------------------|-----------------------|--------------------------|
| 1           | Side milling | 1910                  | 0.07                  | 1                        |
| 2           |              | 1910                  | 0.07                  | 2                        |
| 3           |              | 1910                  | 0.03                  | 4                        |
| 4           |              | 3185                  | 0.03                  | 2                        |
| 5           |              | 1910                  | 0.05                  | 2                        |
| 6           |              | 1910                  | 0.03                  | 2                        |

**Table 3.** The experimental equipments and their manufacturers.

| Equipment             | Manufacturers                                    |
|-----------------------|--------------------------------------------------|
| CREM                  | ZCCCT Company Limited, Zhuzhou, China            |
| Rotary dynamometer    | Kistler Company Limited, Winterthur, Switzerland |
| Charge amplifier      | Kistler Company Limited, Winterthur, Switzerland |
| Data acquisition card | Kistler Company Limited, Winterthur, Switzerland |

### 3. Results

The cutting force parameters for the corner cutting edge segment could be obtained from the first experiment, where only the corner cutting edge participated in the cutting process; the cutting force coefficient curve is illustrated in Figure 9.

Here,  $h$  is the instantaneous average chip thickness, which is calculated by the following:

$$h = \frac{\int_0^{a_p} h[\theta_f(z)] dz}{a_p} \quad (12)$$

A cutting test was conducted using the second parameter; the cutting force coefficient curve fitted by the BP neural network is illustrated in Figure 10.

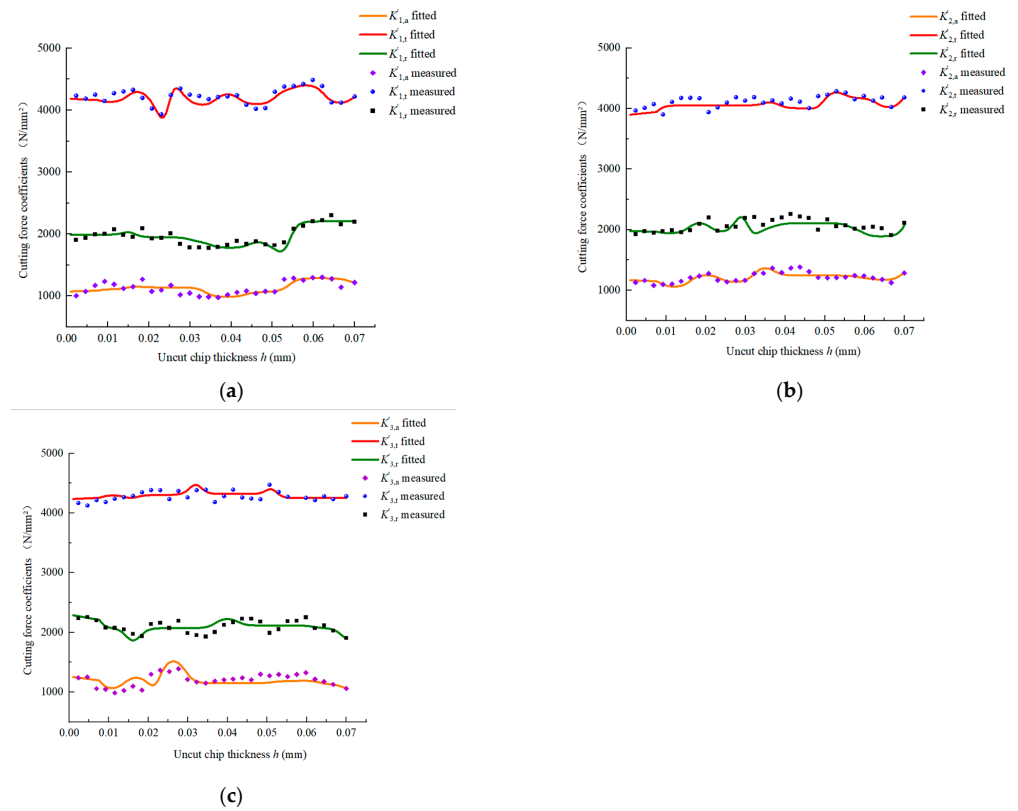


**Figure 9.** The cutting force coefficient curves of the corner cutting edge segment: (a) Edge 1; (b) Edge 2; (c) Edge 3.

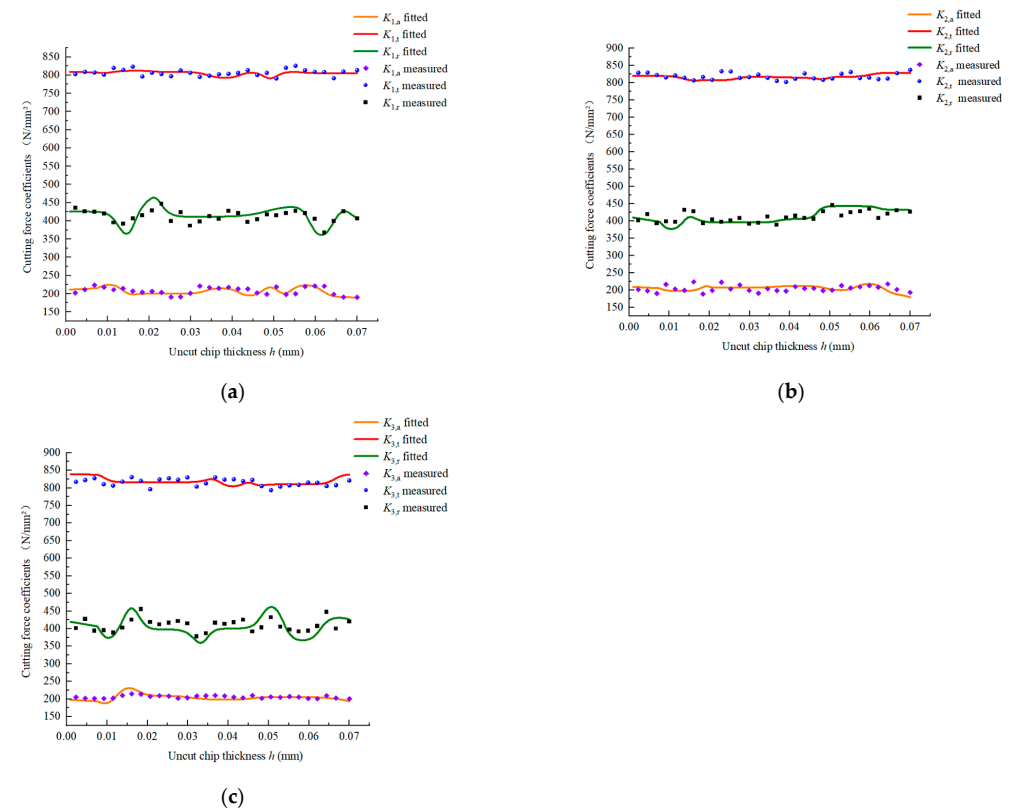


**Figure 10.** The cutting force coefficient curves of cylindrical cutting edge segment: (a) Edge 1; (b) Edge 2; (c) Edge 3.

The latter four sets of test parameters from Table 1 were selected to validate the prediction model. The cutting tool, experimental methodology, and workpiece remained identical. The comparison between the measured and predicted milling forces is presented in Figure 11.

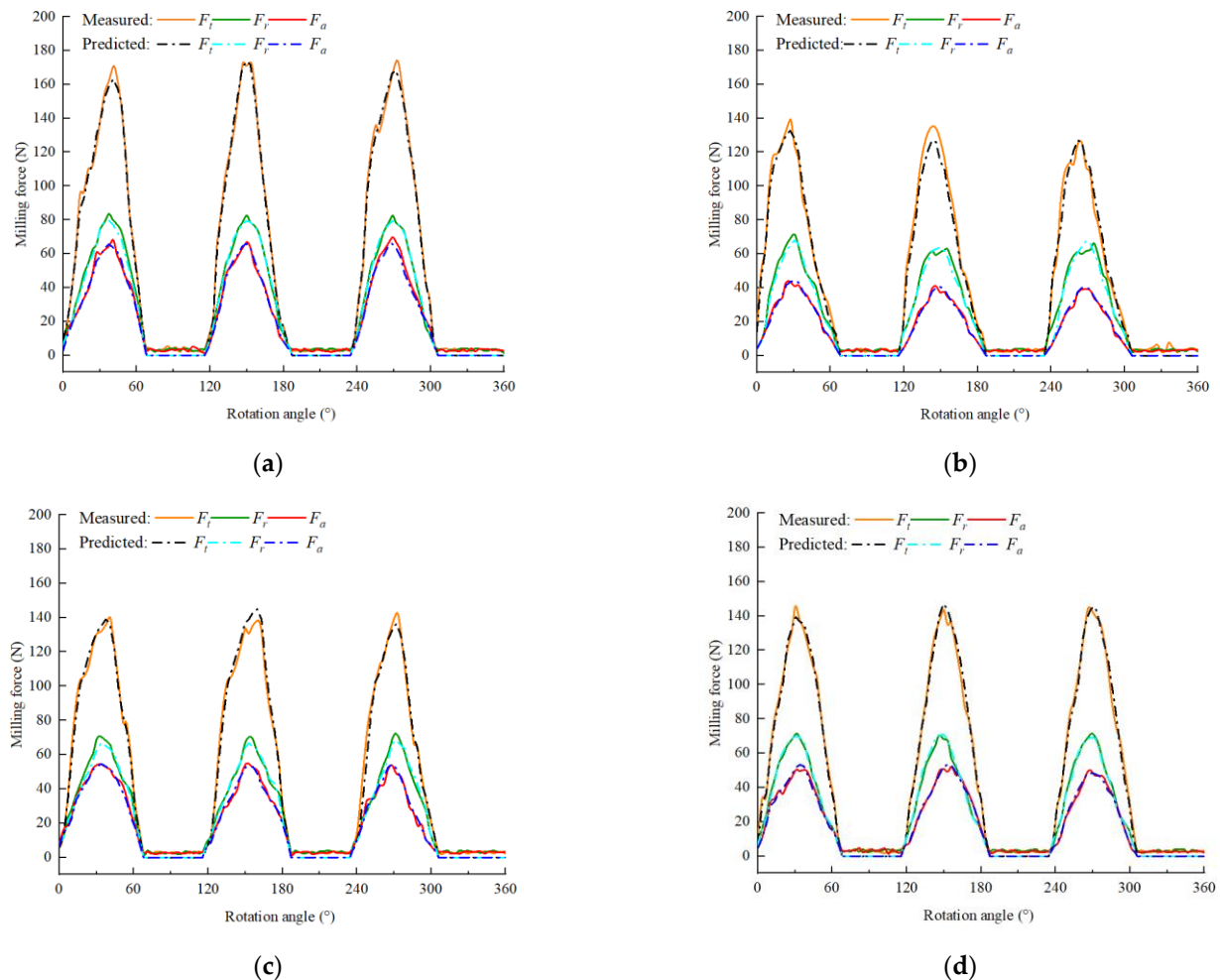


**Figure 11.** Comparison between measured and predicted milling forces: (a) Test 3; (b) Test 4; (c) Test 5; (d) Test 6.

#### 4. Discussion

Figure 9 reveals that the corner cutting edge segments' cutting force coefficients for different edges in the same force direction show minimal variations. For example, the tangential force coefficients— $K_{1,t}'$ ,  $K_{2,t}'$ , and  $K_{3,t}'$ —all remain around  $4000 \text{ N/mm}^2$ . However, the corner cutting edge segments' cutting force coefficients for one edge in different force directions show significant differences: the tangential force coefficients are the highest, followed by the radial force coefficients, while the axial force coefficients are the lowest. Taking the first edge as an example, the following relationships are observed:  $K_{1,t}' > K_{1,r}' > K_{1,a}'$ . This phenomenon occurs because the tangential force measured under the first set of experimental conditions exceeds the radial force, which in turn exceeds the axial force.

Figure 10 reveals that the cylindrical cutting edge segment cutting force coefficients for different edges in the same force direction show minimal variations. For example, the tangential force coefficients— $K_{1,t}$ ,  $K_{2,t}$ , and  $K_{3,t}$ —all remain around  $850 \text{ N/mm}^2$ . The cylindrical cutting edge segments' cutting force coefficients for one edge in different force

directions show significant differences: the tangential force coefficients are the highest, followed by the radial force coefficients, and the axial force coefficients are the lowest. Taking the first edge as an example, the following relationships are observed:  $K_{1,t} > K_{1,r} > K_{1,a}$ . This phenomenon occurs because the tangential force measured under the second set of experimental conditions exceeds the radial force, which in turn exceeds the axial force.

Figures 9 and 10 reveal that the corner cutting edge segment cutting force coefficients for one edge in the same force directions are significantly higher than the cylindrical cutting edge segment cutting force coefficients. Taking the first edge as an example, the following relationships are observed:  $K_{1,t}' > K_{1,t}$ ,  $K_{1,r}' > K_{1,r}$ , and  $K_{1,a}' > K_{1,a}$ . This phenomenon indicates that the corner cutting edge segment of a CREM generates a higher force per unit area during side milling. Ref. [31] also substantiates this conclusion. Under the experimental conditions selected in this paper, the force per unit area generated by the corner cutting edge segment of the CREM is approximately five times higher than that generated by the cylindrical cutting edge segment.

Figure 11 reveals that the peak values of the curves represent the maximum instantaneous cutting forces attainable by each cutting edge of this CREM. Variations in these maximum instantaneous cutting forces across edges arise due to errors caused by tool runout and other factors. The force magnitude was evaluated by calculating the average of peak values in the force signal [37]: Test 3—tangential: 172.438 N, radial: 81.363 N, axial: 68.132 N; Test 4—tangential: 133.238 N, radial: 66.451 N, axial: 41.214 N; Test 5—tangential: 140.331 N, radial: 70.869 N, axial: 54.207 N; Test 6—tangential: 144.648 N, radial: 74.235 N, axial: 50.688 N; Test 4—tangential: 133.238 N. These results clearly demonstrate that in all four sets of side milling tests, the magnitudes of the forces were as follows: the tangential force was the largest, followed by the radial force, and the axial force was the smallest.

Calculations reveal that this prediction model yields a maximum error of 9.43% in tangential force prediction for Test 5; 9.41% in radial force prediction for Test 4; and 6.12% in axial force prediction for Test 6. These results demonstrate the model's high prediction accuracy.

The milling force curves from the four tests exhibit shapes consistent with the single-tooth cutting condition curves in Figure 6. However, non-zero values persist during non-cutting phases due to machine tool structural vibration. This confirms that the cutter operated under single-tooth cutting conditions in all four tests, demonstrating that single-tooth cutting states can be achieved by adjusting the cutting depth.

A limitation of this study is that the proposed predictive model is applicable to cutting tests with small parameters. During the cutting tests for calibrating the corner cutting edge coefficients, axial cutting depth should be constrained not to exceed the corner radius to ensure exclusive engagement of the corner cutting edges. While theoretically viable, this approach carries a risk of edge tipping during practical machining. Achieving exclusive corner cutting edges engagement necessitates reducing cutting parameters such as feed per tooth, consequently restricting the predictive force model's applicability to small parameter cutting tests.

## 5. Conclusions

A cutting force prediction model for CREMs based on the separate-edge-forecast method and BP neural network is established. This model characterizes the variations induced by tool runout through fitted cutting force coefficient curves. This approach simplifies the model's formulation and reduces its computational costs.

The critical conditions for achieving single-tooth cutting states in CREMs were investigated. Based on these conditions, machining parameters were selected for cutting tests.

Analysis of the experimental force signals demonstrated that adjusting the cutting depth enables the tool to attain single-tooth cutting states under specific conditions.

The prediction model was validated through four cutting tests. The results indicate that under the selected parameters, the tangential force is the largest, followed by the radial force, and the axial force is the smallest. The maximum error of the milling force prediction model is 9.43%, demonstrating that this model maintains high prediction accuracy while reducing computational costs.

This predictive force model's applicability to small parameter cutting tests. To address this situation, calibrating the corner cutting edge coefficients under large cutting parameters via milling workpiece at inclination angles will be the focus of our team's future research.

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## Abbreviations

The following abbreviations are used in this manuscript:

|       |                                        |
|-------|----------------------------------------|
| CREM  | Corner radius end mill                 |
| BP    | Back Propagation                       |
| AI    | Artificial Intelligence                |
| CFRP  | Carbon Fiber-Reinforced Polymer        |
| ZCCCT | ZHUZHOU CEMENTED CARBIDE CUTTING TOOLS |

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