

Article

Multi-Objective Optimization of Battery Pack Mounting System for Construction Machinery

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Abstract

With the accelerated electrification of engineering machinery, the battery pack mounting system plays a critical role in enhancing the vehicle's structural safety and vibration-damping performance. This paper proposes an optimization framework for the multi-layer battery pack mounting systems used in such machinery. The framework integrates a multi-degree-of-freedom (MDOF) dynamic model, uncertainty analysis, and a multi-objective evolutionary algorithm (MOEA) to resolve the vibration suppression challenges associated with large-mass battery packs under harsh operating conditions. A parameter optimization method is introduced with the objectives of increasing natural frequencies, enhancing modal decoupling, and avoiding resonance. By identifying key influencing parameters and performing a comprehensive optimization of mount locations and stiffness, this approach achieves a highly efficient improvement in dynamic performance. Simulation and analysis results demonstrate that, compared to the initial design, the proposed method significantly elevates the system's first six natural frequencies (by 13.6%, 7.8%, 3.3%, 2.5%, 11.7%, and 9.4%, respectively). Furthermore, it enhances the energy decoupling between modes, with the decoupling rates for Y-direction translation and Z-axis rotation both increasing by 11.3%. This achieves a synergistic improvement in the system's vibration avoidance and decoupling performance. The methodology offers an effective means to optimize the safety and operational stability of battery systems in electric engineering machinery.



Academic Editor: Francesco Castellani

Received: 12 July 2025

Revised: 30 July 2025

Accepted: 7 August 2025

Published: 9 August 2025

Citation: Lin, D.; Sun, R.; Wei, H.; Wang, Y. Multi-Objective Optimization of Battery Pack Mounting System for Construction Machinery. *Machines* **2025**, *13*, 705. <https://doi.org/10.3390/machines13080705>

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Keywords: construction machinery; multi-objective optimization; mounting system; battery pack

1. Introduction

With the accelerating trend of electrification in construction machinery, the battery pack, as the core energy source, directly influences the machine's power, reliability, and durability [1–3]. Compared to traditional electric vehicles, construction machinery frequently operates in complex, variable environments with significant impact loads, thus imposing higher requirements on the battery pack's structural design and mounting system. Unlike the battery packs in electric cars, those in construction machinery are often stacked to form a multi-layer structure. To meet the power and endurance requirements of this machinery, the battery packs are designed for high energy storage capacity and high energy density, which results in a large volume and considerable mass. The mounting system is composed of elastic support elements (mounts) that serve critical functions under both static and dynamic conditions. In a static state, the mounts must bear the mass of the

battery pack, preventing excessive static displacement that could impair its operation [4]. Dynamically, their primary objective is to provide effective vibration isolation [5]. This function is twofold: on one hand, as the connecting elements between the chassis and the battery pack, they must attenuate the transmission of harmful vibrations from the chassis to protect the battery pack's mechanical structure. On the other hand, the mounting system must simultaneously control the displacement of the battery pack itself under various forces to prevent structural interference with surrounding components [6–8]. Therefore, the performance of the battery pack mounting system is crucial for the long-term, stable operation of construction machinery [9].

In recent years, the safety of electric vehicles has become closely linked to the stability of the battery pack. In the electrification of construction machinery, the safety of its battery pack is similarly a factor that cannot be overlooked. Wang et al. [10] investigated the effects of vibrations at different frequencies on the electrochemical performance of lithium-ion batteries, finding that vibrations exacerbate the loss of lithium inventory and active materials, thereby significantly reducing battery energy storage capacity and cycling performance. Brand et al. [11] conducted vibration and shock tests on pouch and cylindrical cells according to the UN 38.3 standard. They discovered that the mandrel in some cylindrical cells could become dislodged and strike internal components and that long-term vibration has additional adverse effects on the cells. With the advancement of finite element modeling, multibody dynamics simulation, and multi-objective evolutionary algorithms, multi-objective optimization research for mounting systems has gradually emerged, offering new approaches for enhancing the comprehensive performance of battery pack mounting systems. Li et al. [12] developed a finite element model of a battery pack to simulate and analyze its impact characteristics and fatigue life. By identifying critical components, they optimized the finite element model to achieve the goals of weight reduction and improved lifespan. Kang et al. [13] established a six-degree-of-freedom simulation model of a tractor cabin and optimized its rigid body modal frequencies and modal decoupling rate using the NSGA-II algorithm. Voparil et al. [14] designed two different mounting systems for an automotive powertrain. Their experimental results indicated that when vibration on the passive side is a key concern, different mounting point configurations have a significant impact on the vibration outcomes. N.H. and D.N. [15] integrated the SPEA/R and HNSGA-III algorithms, tested them on several typical benchmark problems, and applied them to optimize the stiffness parameters and achieve cost optimization for a powertrain mounting system, demonstrating the new algorithm's potential in solving parameter optimization problems for such systems. However, current research on mounting systems for the multi-layer battery packs used in construction machinery remains deficient in several areas. Specifically, there is a lack of in-depth studies on the dynamic modeling of the upper-layer battery packs and on the relationship between mount layout, stiffness characteristics, and the dynamic behavior of the battery pack. Concurrently, for systems with multiple mounting points, there is a scarcity of systematic theoretical analysis and exploration of engineering applications.

In response to the urgent demand for high-performance mounting systems for battery systems in modern construction machinery, this paper focuses on large-scale battery packs for such equipment. A multi-degree-of-freedom dynamic model of the upper-layer battery pack mounting system is constructed, taking into account the complex vibration and load environment encountered under typical operating conditions. This paper proposes a mount parameter optimization method with the core objectives of increasing natural frequencies, achieving modal decoupling, and avoiding resonance. This method aims to reduce the degree of dynamic coupling between different degrees of freedom, enabling orderly control and separation of the battery pack's vibration response, thereby enhancing structural

stability and operational safety. Furthermore, by introducing sensitivity analysis, this study identifies the key design parameters that significantly influence the system's dynamic response, specifically concerning the position and stiffness design of the battery pack's multiple mounting points. This approach effectively reduces the design dimensionality and improves optimization efficiency. Finally, a multi-objective evolutionary algorithm is employed to perform a system-level optimization of the screened parameters. This process seeks to achieve the best overall performance by balancing various performance indicators, thereby realizing the combined goals of vibration avoidance, decoupling, and the protection of decoupled modes.

2. Vibration Analysis of the Battery Pack

2.1. Modeling of the Battery Pack Mounting System

The design is based on a battery pack for a large-scale construction machine. Figure 1 shows a side view of the battery pack.



Figure 1. A specific construction machinery battery pack.

A dynamic model of the battery pack is established, as shown in Figure 2. The battery pack mounting system can be treated as a six-degree-of-freedom (6-DOF) rigid body. The length, width, and height of this rigid body are L , K , and G , respectively. A coordinate system for the battery pack, $OX_bY_bZ_b$, is defined by setting one corner of the rigid body's base as the origin. In this system, the positive X -axis points in the vehicle's direction of travel, the positive Y -axis points toward the left side of the driver's cabin, and the positive Z -axis is oriented vertically upward, perpendicular to the horizontal plane. Within this coordinate system, the generalized displacement vector is defined as $\mathbf{q}(t) = \{x, y, z, \theta_x, \theta_y, \theta_z\}$. Consequently, the corresponding velocity vector is $\dot{\mathbf{q}}(t) = \{\dot{x}, \dot{y}, \dot{z}, \dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z\}$, and the acceleration vector is $\ddot{\mathbf{q}}(t) = \{\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\theta}_x, \ddot{\theta}_y, \ddot{\theta}_z\}$.

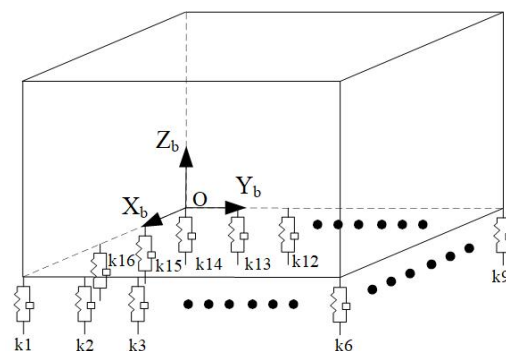


Figure 2. Schematic of the battery pack mounting system.

Without loss of generality, the coordinates of the center of mass in a static state are $\left(\frac{K}{2}, \frac{L}{2}, \frac{G}{2}\right)$. In the preliminary design of this battery pack mounting system, 16 mounts are uniformly arranged on the bottom of the battery pack. The mounts are regarded as elastic elements with tri-axial stiffness (in the X-, Y-, and Z-directions). The coordinate positions of each mounting point are listed in Table 1.

Table 1. Coordinates of the battery pack mounting points.

Mount No.	Coordinates		
	X	Y	Z
1	1000	0	0
2	1000	340	0
3	1000	680	0
4	1000	1020	0
5	1000	1360	0
6	1000	1700	0
7	666	1700	0
8	333	1700	0
9	0	1700	0
10	0	1360	0
11	0	1020	0
12	0	680	0
13	0	340	0
14	0	0	0
15	333	0	0
16	666	0	0

For the battery pack in a static state, its inertia tensor is expressed as follows:

$$J = \begin{bmatrix} \frac{m}{3}(K^2 + G^2) & -\frac{m}{4}LK & -\frac{m}{4}LG \\ -\frac{m}{4}LK & \frac{m}{3}(L^2 + G^2) & -\frac{m}{4}KG \\ -\frac{m}{4}LK & -\frac{m}{4}KG & \frac{m}{3}(L^2 + K^2) \end{bmatrix} \quad (1)$$

For simplicity, this is expressed as follows:

$$J = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{xy} & J_{yy} & J_{yz} \\ J_{xz} & J_{yz} & J_{zz} \end{bmatrix} \quad (2)$$

Since the vibration of a multi-layer battery pack tends to cause significant translational and rotational motion, the coordinates of its center of mass during vibration can be expressed in the time domain as $\left(\frac{L}{2} + \Delta x(t), \frac{K}{2} + \Delta y(t), \frac{G}{2} + \Delta z(t)\right)$, where $\Delta x(t)$, $\Delta y(t)$, and $\Delta z(t)$ represent the time-domain displacements of the battery pack's center of mass in the X-, Y-, and Z-directions, respectively.

After analyzing and superimposing the inertia tensors for the translational and rotational motions of the battery pack, the inertia tensor for when the battery pack undergoes translational motion is expressed as follows:

$$J_M = \begin{bmatrix} J_{M_{xx}} & J_{M_{xy}} & J_{M_{xz}} \\ J_{M_{xy}} & J_{M_{yy}} & J_{M_{yz}} \\ J_{M_{xz}} & J_{M_{yz}} & J_{M_{zz}} \end{bmatrix} \quad (3)$$

where

$$\begin{aligned}
 J_{M_{xx}} &= J_{xx} - m(\Delta y^2 + \Delta z^2) \\
 J_{M_{yy}} &= J_{yy} - m(\Delta x^2 + \Delta z^2) \\
 J_{M_{zz}} &= J_{zz} - m(\Delta y^2 + \Delta x^2) \\
 J_{M_{xy}} &= J_{xy} + m\Delta x\Delta y \\
 J_{M_{xz}} &= J_{xz} + m\Delta x\Delta z \\
 J_{M_{yz}} &= J_{yz} + m\Delta y\Delta z
 \end{aligned} \tag{4}$$

When the battery pack rotates by an angle of θ_z about the X-axis (parallel to the long side of the rigid body), the inertia tensor is expressed as follows:

$$J_R = \begin{bmatrix} J_{R_{xx}} & J_{R_{xy}} & J_{R_{xz}} \\ J_{R_{xy}} & J_{R_{yy}} & J_{R_{yz}} \\ J_{R_{xz}} & J_{R_{yz}} & J_{R_{zz}} \end{bmatrix} \tag{5}$$

where

$$\begin{aligned}
 J_{R_{xx}} &= J_{xx} \\
 J_{R_{yy}} &= J_{yy} \cos^2 \theta + J_{zz} \sin^2 \theta - 2J_{yz} \sin \theta \cos \theta \\
 J_{R_{zz}} &= J_{yy} \sin^2 \theta + J_{zz} \cos^2 \theta + 2J_{yz} \sin \theta \cos \theta \\
 J_{R_{xy}} &= J_{xy} \cos \theta - J_{xz} \sin \theta \\
 J_{R_{xz}} &= J_{xy} \sin \theta + J_{xz} \cos \theta \\
 J_{R_{yz}} &= (J_{zz} - J_{yy}) \sin \theta \cos \theta + J_{yz} (\cos^2 \theta - \sin^2 \theta)
 \end{aligned} \tag{6}$$

The inertia tensor from Equation (3), which accounts only for translation, is combined with the inertia tensor from Equation (5), which accounts only for rotation about the X-axis. The resulting expression is

$$J' = J_M + J_R - J \tag{7}$$

Based on the combined inertia tensor, the mass matrix can be expressed as follows:

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & J' & \\ 0 & 0 & 0 & & & \end{bmatrix} \tag{8}$$

By substituting the total kinetic energy and total potential energy of the system into Lagrange's equation [16], the system's equation of motion can be obtained:

$$\frac{d}{dt} \left(\frac{\partial E_T}{\partial \dot{q}} \right) - \frac{\partial E_T}{\partial q} + \frac{\partial E_V}{\partial q} + \frac{\partial E_D}{\partial \dot{q}} = Q \tag{9}$$

where E_T is the kinetic energy of the system, E_V is the potential energy of the system, E_D is the dissipative energy of the system, and Q is the generalized force applied to the system.

The kinetic energy of the system can be expressed as

$$E_T = \frac{1}{2} \dot{q}^T M \dot{q} \tag{10}$$

where M is the mass matrix of the system, and $\dot{q}$ is the generalized velocity vector.

The potential energy of the system can be expressed as

$$E_V = \frac{1}{2} q^T K q \tag{11}$$

where K is the stiffness matrix of the system, and q is the generalized displacement vector. The dissipative energy of the system can be expressed as

$$E_D = \frac{1}{2} \dot{q}^T C \dot{q} \quad (12)$$

where C is the damping matrix of the system.

Substituting Equations (10)–(12) into Equation (9) yields

$$M\ddot{q} + C\dot{q} + Kq = F \quad (13)$$

where F is the external excitation force applied to the battery pack system.

Based on road surface profile measurement reports, road conditions can be classified into grades (A–H), where Grade A represents superior, smooth roads, and Grade H represents poor, rough roads. The displacement and velocity power spectral densities (PSD) for various road conditions can be expressed as

$$G_d(n) = G_d(n_0) \left(\frac{n}{n_0} \right)^{-\omega} \quad (14)$$

$$G_v(n) = G_d(n) (2\pi n)^2 \quad (15)$$

where $G_d(n)$ is the displacement power spectral density, $G_v(n)$ is the velocity power spectral density, n is the spatial frequency, and n_0 is the reference spatial frequency.

When considering the vehicle speed, the velocity power spectral density in the time frequency domain can be expressed as

$$G_v(f) = \frac{G_v(n)}{u} \quad (16)$$

where $G_v(f)$ is the velocity power spectral density in the time frequency domain, and u is the speed of the construction machinery.

As shown in Table 2, three grades of displacement and velocity road roughness coefficients are presented. The working environment for construction machinery often includes construction sites, mines, and other areas with complex terrain and harsh conditions, corresponding to road grades F–H.

Table 2. Road coefficients for poor road conditions.

Road Grade	$G_d(n_0)/10^{-6}m^3$	$G_v(n_0)/10^{-6}m^3$
F	16,384	6468.1
G	65,536	25,872.6
H	262,144	103,490.3

Note: The reference spatial frequency n_0 is generally taken as 0.1 m^{-1} , and the fitting exponent ω is 2.

As shown in Figure 3, the lower the time frequency, the higher the corresponding road power spectral density, and the greater the risk and hazard of resonance with the road surface. For the mounting system, increasing the first-order natural frequency as much as possible can improve the overall distribution of natural frequencies, thereby reducing the risk of damage to the battery pack.

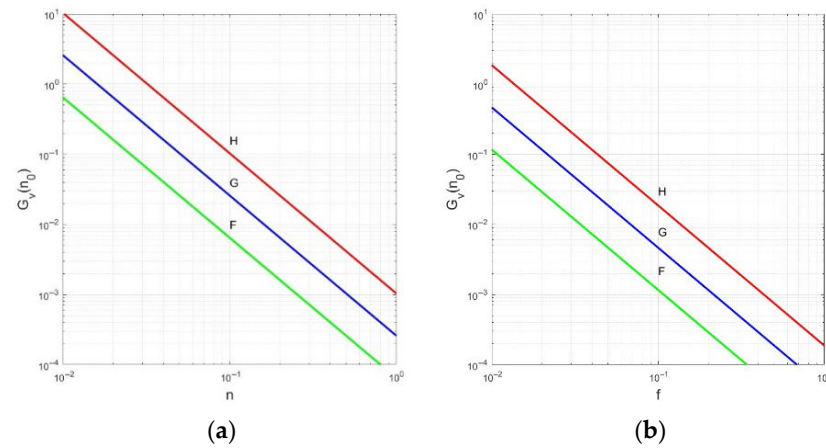


Figure 3. Power spectral density: (a) PSD in spatial frequency; (b) PSD in time frequency.

2.2. Calculation of the Battery Pack System's Natural Frequencies

To calculate the natural frequency characteristics of the complete system assembly, which includes the battery pack body and all of its mounting points, [17], the undamped equation can be written as

$$M\ddot{q} + Kq = 0 \quad (17)$$

The theoretical solution for displacement in any direction can be written as

$$q_i = \varphi_i \sin(\omega_i t + \phi_i) \quad (18)$$

Substituting this into Equation (17) yields the following characteristic equation:

$$(K - \omega^2 M)\varphi = 0 \quad (19)$$

This can be further expressed as

$$M^{-1}K\varphi = \omega^2\varphi \quad (20)$$

The natural frequency of the system is therefore

$$f = \frac{\omega}{2\pi} \quad (21)$$

2.3. Calculation of the Battery Pack System's Decoupling Rate

The decoupling rate of a mounting system is a key indicator of the independence of its vibration modes [18]. It represents the percentage of vibrational energy in a specific direction (e.g., vertical, lateral, longitudinal, or rotational) within a particular mode relative to the total vibrational energy. A higher decoupling rate in a specific direction for a given mode indicates that vibration in that direction is less likely to excite vibrations in other directions, meaning the coupling is weak.

The maximum kinetic energy for the i -th vibration mode can be expressed as

$$T_{i\max} = \frac{\omega_i^2 [\varphi_i] [M] [\varphi_i]}{2} \quad (22)$$

where φ_i and ω_i are the mode shape and natural frequency corresponding to the i -th vibration mode, respectively.

In the i -th vibration mode, the kinetic energy corresponding to the j -th direction can be expressed as

$$t_{ij} = \frac{\omega_i^2 \sum_{l=1}^6 M_{jl}(\varphi_{il})(\varphi_{ij})}{2} \quad (23)$$

where M_{jl} is the element in the j -th row and j -th column of the mass matrix, and φ_{il} and φ_{ij} are the elements in the l th row and j th row of the i th mode, respectively.

Therefore, in the i -th vibration mode of the battery pack mounting system, the modal decoupling rate corresponding to the j -th generalized coordinate can be expressed as

$$T_{ij} = \frac{t_{ij}}{T_{\text{imax}}} = \frac{\sum_{l=1}^6 M_{jl}(\varphi_{il})(\varphi_{ij})}{[\varphi_i]^T [\mathbf{M}] [\varphi_i]} \quad (24)$$

From Equations (21) and (24), it is evident that the key performance indicators of the battery pack mounting system—natural frequency and modal decoupling rate—are closely related to the system's inertial parameters, such as the moment of inertia, mass, and center of mass coordinates. They are also significantly influenced by the stiffness and spatial distribution of the mounting points. This mathematical relationship theoretically reveals the fundamental mechanism of the system's dynamic response, providing a parametric basis for subsequent performance tuning. To further clarify the influence of each parameter on the system's performance, this paper will quantitatively calculate and analyze the dynamic performance indicators of this battery pack model based on known structural and physical parameters, thereby supporting the theoretical foundation for the optimization design.

3. Parameter Calculation for a Case Study of a Battery Pack Mount

Based on the analysis of the vibration characteristics of the battery pack mounting system in the previous section, and combined with the actual mass and geometric dimension parameters of the battery pack, its inertia tensor and center of mass coordinates can be calculated. The specific values are shown in Tables 3 and 4. To ensure the symmetry of the structural layout and feasibility of installation, this study uniformly distributes 16 mounting points along the edge of the battery pack base. Their specific coordinates have been provided in Table 1.

Table 3. Inertia tensor parameters of the battery pack mounting system.

$J_{xx}/\text{kg} \times \text{mm}^2$	$J_{yy}/\text{kg} \times \text{mm}^2$	$J_{zz}/\text{kg} \times \text{mm}^2$	$J_{xy}/\text{kg} \times \text{mm}^2$	$J_{xz}/\text{kg} \times \text{mm}^2$	$J_{yz}/\text{kg} \times \text{mm}^2$
5.2×10^9	8.9×10^9	7.5×10^9	-2.4×10^9	-3.2×10^9	-1.9×10^9

Table 4. Mass and center of mass coordinates of the battery pack mounting system.

Battery Pack Mass/t	Center of Mass Coordinates/mm		
5.8	X	Y	Z
	500	850	650

The mount stiffness parameters are selected with reference to the typical configuration of the previous generation of construction machinery battery packs, as detailed in Table 5.

Table 5. Stiffness parameters of the battery pack mounts.

Stiffness Direction	X	Y	Z
Stiffness (N/mm)	1700	1700	2800

As shown in Table 6, based on the mathematical model established in the previous chapter and solved on the MATLAB R2024b platform, the natural frequencies and modal decoupling rates of the battery pack mounting system can be obtained. The calculation results show that the first-order natural frequency of the system is 4.4 Hz, which poses a risk. When operating in this frequency band, the mounting system is susceptible to resonance induced by road excitation, which poses a threat to the structural stability and service life of the battery pack. Therefore, it is necessary to appropriately increase the first-order natural frequency in the subsequent optimization to reduce the system's response to low-frequency external excitation, thereby effectively suppressing the resonance effect.

Table 6. Natural frequencies and energy-based modal decoupling rates of the battery pack mounting system.

Natural Frequency (Hz)	4.4	6.4	8.9	12.1	14.5	21.2
X	26.4%	0.0%	10.1%	61.3%	0.0%	2.2%
Y	0.3%	19.5%	59.5%	11.5%	0.0%	9.2%
Z	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%
RXX	2.2%	29.7%	−3.1%	5.0%	0.0%	66.2%
RYY	69.2%	0.0%	−1.9%	22.2%	0.0%	10.6%
RZZ	1.9%	50.8%	35.4%	0.1%	0.0%	11.8%

Furthermore, in terms of modal decoupling rates, the battery pack exhibits low decoupling rates in the Y-direction translation and Z-axis rotation degrees of freedom, at 59.5% and 50.8%, respectively, which do not meet the desired level of modal independence. This coupling characteristic may lead to dynamic energy coupling and mutual excitation of vibrations between these two degrees of freedom, reducing the system's vibration isolation effectiveness. Therefore, it is necessary to focus on improving the decoupling rate parameters in these two directions in the subsequent optimization design to enhance the system's stability and anti-interference capability under actual working conditions.

4. Multi-Objective Optimization of Battery Pack Mount Parameters

4.1. Sensitivity Analysis of Decision Variables

Integrating the formulas and results from the previous two chapters, it is clear that in the construction machinery battery pack mounting system, the position distribution and stiffness parameters of each mounting point have a decisive impact on the battery pack's natural frequencies and modal decoupling rates. This, in turn, significantly affects its vibration response level under different operating conditions, as well as the battery's service life and operational safety. The calculation results in Table 6 show that this model of the battery pack mounting system has issues such as a low first-order natural frequency and low decoupling rates in the X-direction translation and X-axis rotation. Therefore, the subsequent parameter optimization process should focus on improving these three performance indicators.

However, if a comprehensive optimization of the stiffness and spatial coordinates of all mounting points in the system were to be conducted, the total number of decision variables would reach 96. This would not only increase the dimensionality and complexity of the optimization problem but also lead to a significant decrease in computational

efficiency and a substantial increase in optimization cost [19]. Therefore, it is necessary to perform a sensitivity analysis of the decision variables of the mounting system before optimization [20,21]. The main objectives include the following:

- (1) Identifying the key variables among all variables that have a significant impact on the system's performance indicators.
- (2) Determining the variables that have an independent influence on the system's performance.
- (3) Identifying variables with significant interaction effects to be given special consideration in the subsequent optimization.

The Sobol method [22] is a typical global sensitivity analysis technique that can be used to assess the contribution of each input variable to the uncertainty of the output response in a mathematical model [23]. This allows for the effective reduction of model complexity and uncertainty while ensuring model accuracy, providing a theoretical basis and variable screening support for the optimization of the mounting system [24].

Any mathematical model with n input parameters can be simply written as $Y = f(\mathbf{X}) = f(X_1, X_2, X_3, \dots, X_n)$, $\mathbf{X} \in \Omega$, where Y is a single output variable. This model can be extended to multiple output variables without loss of generality.

The output function can be decomposed as

$$Y = f_0 + \sum_{i=1}^n f_i(X_i) + \sum_{i=1}^n \sum_{j=i+1}^n f_{ij}(X_i, X_j) + \dots + f_{1,2,3,\dots,n}(X_1, X_2, \dots, X_n) \quad (25)$$

When all the above terms are orthogonal, where

$$f_0 = E(Y) \quad (26)$$

$$f_i(X_i) = E(Y|X_i) - f_0 = \int_{\Omega} f_i(X_i) g_{y|x_i}(Y|X_i) dX_i - f_0 \quad (27)$$

$$\begin{aligned} f_{ij}(X_i, X_j) &= E(Y|X_i, X_j) - f_0 - f_i - f_j \\ &= \int_{\Omega} \int_{\Omega} f_{ij}(X_i, X_j) g_{y|x_i, x_j}(Y|X_i, X_j) dX_i dX_j - f_0 - f_i - f_j \end{aligned} \quad (28)$$

The variance of the output term Y can be decomposed as

$$Var(Y) = \sum_i^n V_i + \sum_i^n \sum_{j=i+1}^n V_{ij} + \dots + V_{12\dots n} \quad (29)$$

where

$$V_i = Var(E(Y|X_i)) \quad (30)$$

$$V_{ij} = Var(E(Y|X_i, X_j)) \quad (31)$$

The first-order sensitivity index is expressed as

$$S_i = \frac{V_i}{Var(Y)} \quad (32)$$

The total sensitivity index is expressed as

$$S_{Ti} = 1 - \frac{Var(E(Y|X_{\sim i}))}{Var(Y)} \quad (33)$$

During the sensitivity analysis, the decision variables in the battery pack mounting system need to be categorized and assigned reasonable value ranges. Specifically, the stiffness range for each mounting point in the X- and Y-directions is set to [750, 2000]

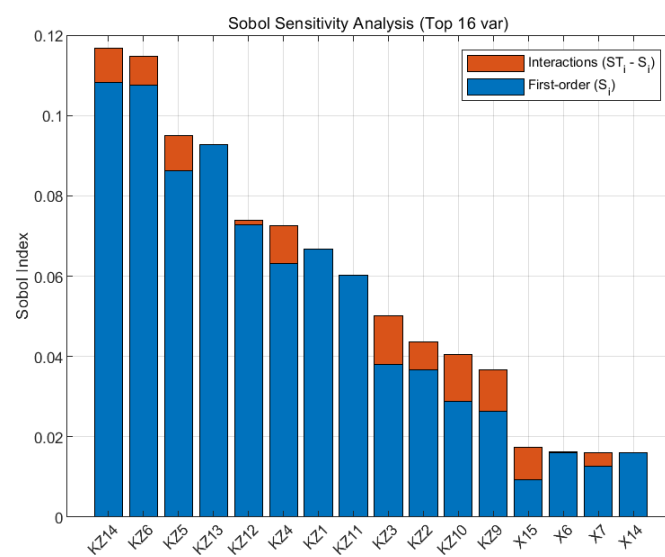
N/mm, and the Z-direction stiffness range is set to [2000, 8000] N/mm. For the position parameters, each mounting point is allowed to move within a range of ± 150 mm along the edge of the battery pack, while the height remains unchanged, located on the bottom plane of the battery pack. Furthermore, to ensure the statistical stability and comprehensive coverage of the sensitivity analysis results, the sample size is set to 600,000.

As shown in Figure 4, the horizontal axis of each graph represents the decision variables, and the vertical axis represents the sensitivity index. The blue part within the graph represents the first-order sensitivity index, while the orange part represents the interaction index (the difference between the total sensitivity index and the first-order sensitivity index). The figure presents a Sobol global sensitivity analysis for the first six natural frequencies and the maximum modal decoupling rates for the six corresponding generalized coordinates, showing the top 16 variables with the highest influence index (sensitivity) for each response indicator. The blue part measures the independent influence of a parameter on the evaluation metric, while the orange part measures the influence of interaction effects.

From the sensitivity analysis results in Figure 4, the key decision variables that significantly affect the system's performance for each response indicator can be identified, as well as the minor variables with less impact. After consolidation and comparative analysis, the main influencing factors are summarized in Table 7. Among them, the parameters with high first-order Sobol indices and significant interaction effects are mostly concentrated at the four corner mounting points of the battery pack mounting system, indicating that the parameters in this region have a stronger dominant effect on the system's natural frequencies and decoupling characteristics.

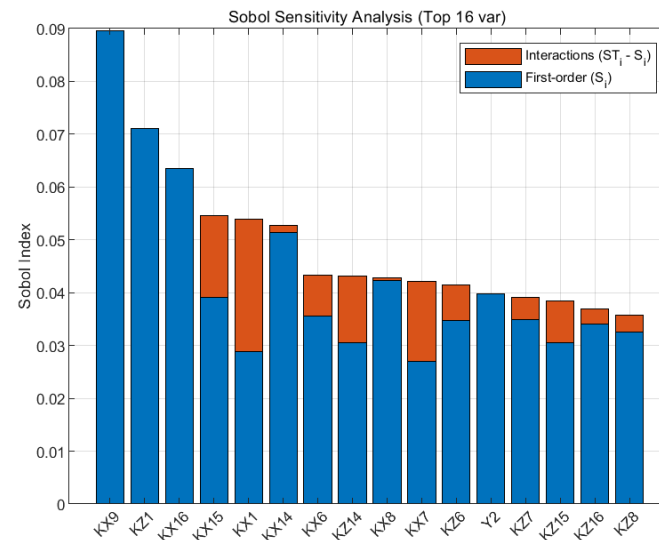
Table 7. Summary of sensitivity analysis results.

System Response	Parameters with Significant Individual Effects	Parameters with Significant Interaction Effects	Insignificant Parameter
natural frequency and decoupling ratio	KX1, KZ6, KX9, KZ9, KX16	KZ1, KZ14, KZ9	KX3, KX4, KX5.

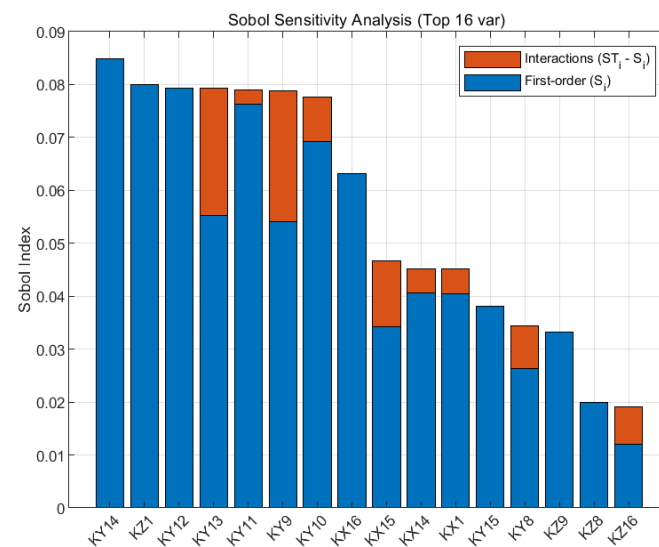


(a)

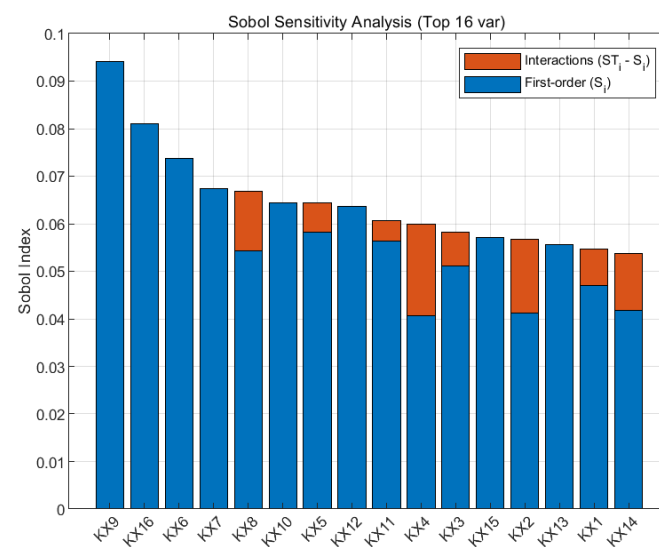
Figure 4. Cont.



(b)



(c)



(d)

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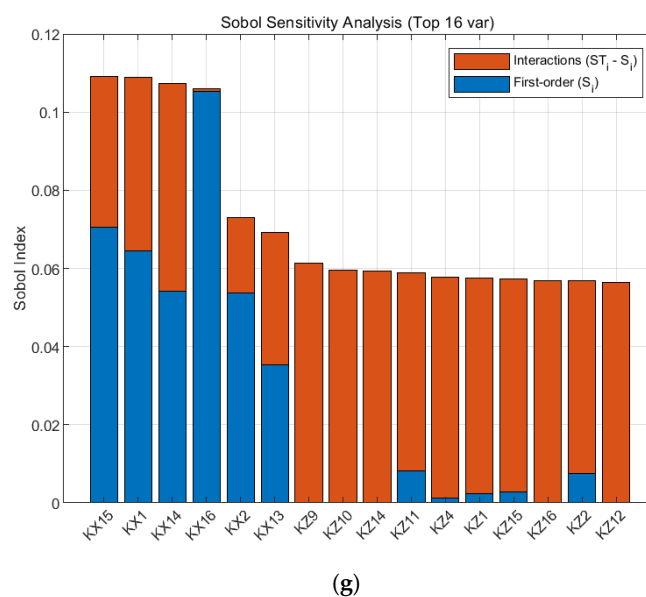
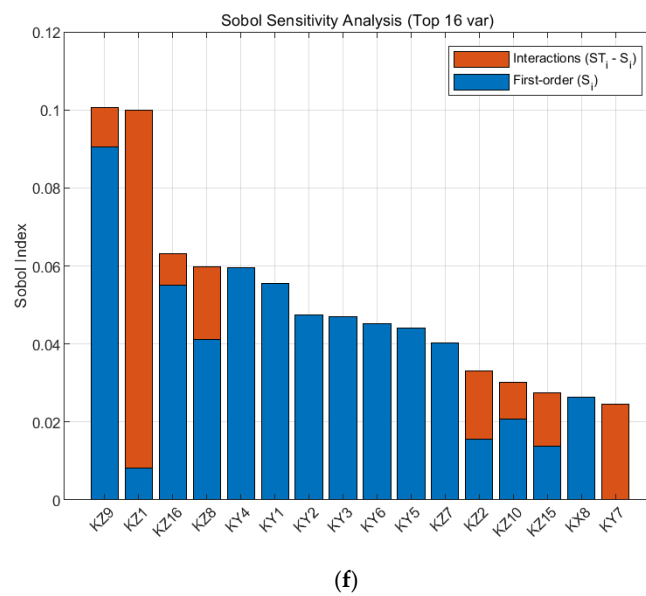
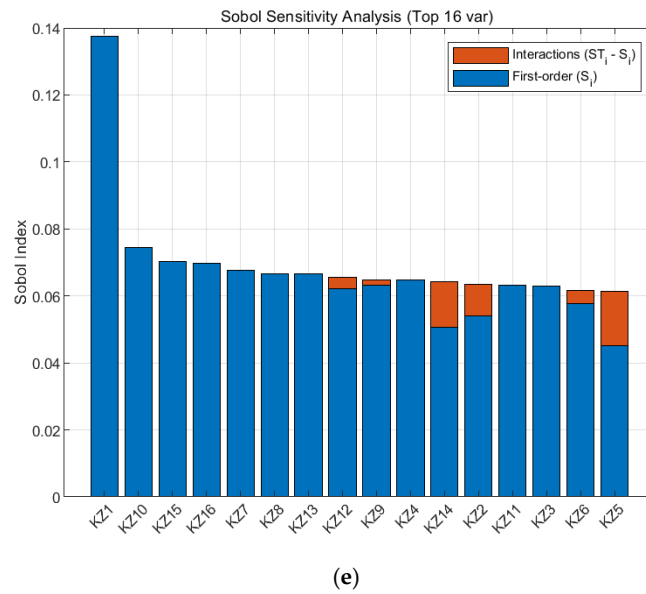
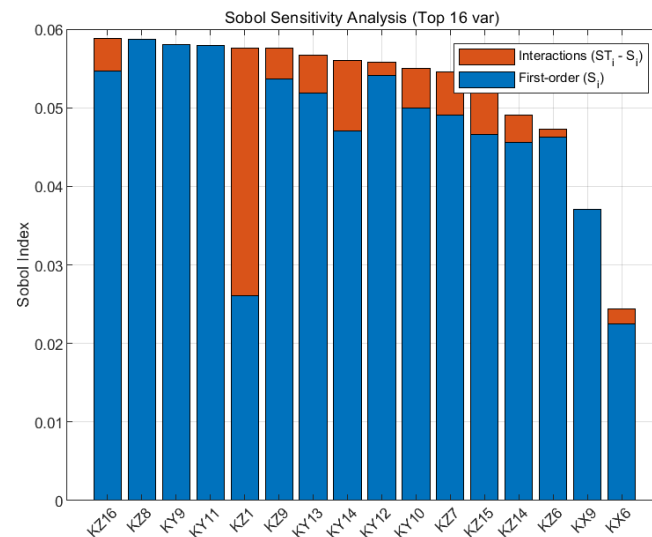
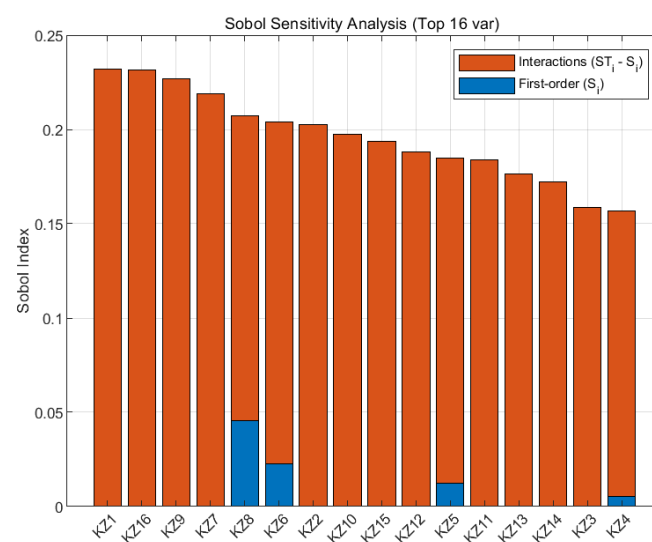


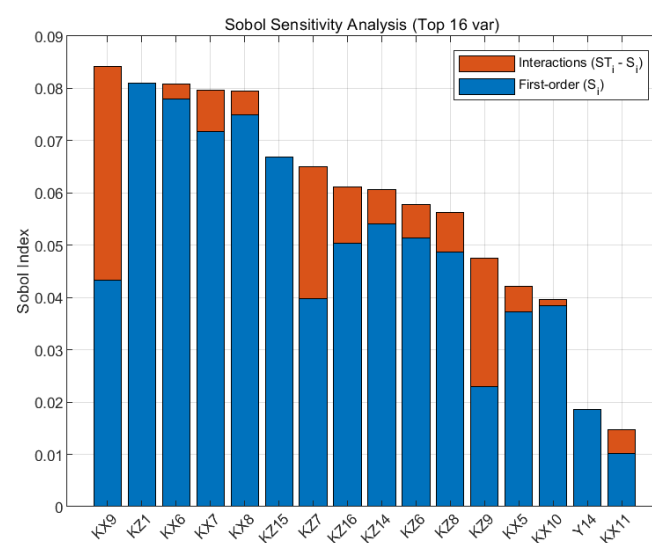
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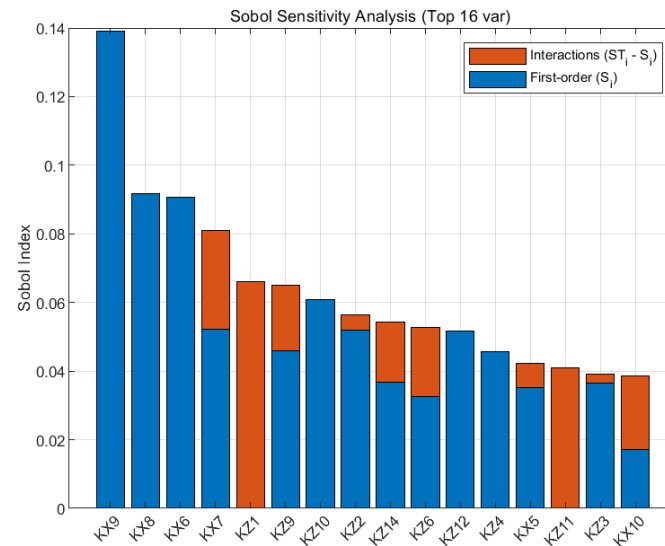


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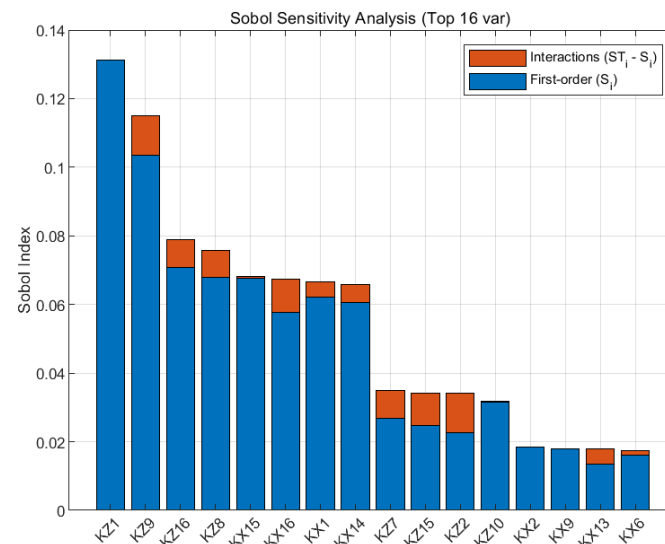


(j)

Figure 4. Cont.



(k)



(l)

Figure 4. Sensitivity analysis results for each evaluation metric: (a) first-order natural frequency; (b) second-order natural frequency; (c) third-order natural frequency; (d) fourth-order natural frequency; (e) fifth natural frequency; (f) sixth natural frequency; (g) X-direction decoupling rate; (h) Y-direction decoupling rate; (i) Z-direction decoupling rate; (j) RXX-direction decoupling rate; (k) RYY-direction decoupling rate; (l) RZZ-direction decoupling rate.

Further analysis reveals that compared to the spatial position parameters of the mounting points, the stiffness parameters have a more significant impact on the system's performance. Especially in the four corner regions, changes in their stiffness have a stronger sensitivity to the natural characteristics and the degree of modal decoupling. Therefore, in the subsequent optimization design process, priority should be given to the stiffness settings of the four corner mounting points to improve optimization efficiency, reduce computational dimensionality, and ensure effective improvement of the system's dynamic performance.

4.2. Sensitivity Variation Under Different Stiffness Ratios

After determining the decision variables for optimization, to further investigate the robustness of the system with respect to the ratio of axial to radial stiffness in the mounts, this study sets different ratios while keeping the X-direction to Y-direction stiffness ratio fixed. A sensitivity analysis is conducted on the stiffness ratio between the Z-direction (axial) and the X/Y-directions (radial). Specifically, the stiffness ratio is varied from 0.1 to 1.2, and the corresponding trends in the first-order and total sensitivity indices are analyzed. Figures 5 and 6 show the trends in the first-order and total sensitivity indices of each decision variable on the performance metrics under different stiffness ratios. The horizontal axis represents the stiffness ratio, the vertical axis represents the sensitivity index, and the legend on the right indicates the lines corresponding to each variable.

Figure 5 shows that the first-order Sobol sensitivity indices of the various stiffness parameters are strongly dependent on the stiffness ratio. The graphs exhibit clear sensitivity crossover phenomena, where the dominant influencing parameter changes as the stiffness ratio is altered, showing different characteristics depending on the evaluation metric. Additionally, some parameters show sharp sensitivity peaks in specific narrow regions. For example, for KX9 in the second and third natural frequencies, as the stiffness ratio changes from 0.1 to 0.3, the sensitivity rapidly drops from 0.14 to nearly 0. Therefore, the importance ranking of parameters is dynamic, and the system has highly sensitive regions to changes in certain parameters under specific configurations. These highly sensitive regions should be avoided in the design process.

Figure 6 shows that the total sensitivity indices of the parameters also exhibit a strong dynamic dependence on the stiffness ratio. This indicates that the interaction effects of the decision variables are not constant. The graphs also clearly show the transition of dominant influencing parameters (crossover phenomena) and sensitivity peaks in specific regions. Therefore, from a global perspective, the importance of parameters and the strength of their interactions are dynamically changing with the system state. Identifying these highly sensitive configurations is crucial for assessing the overall uncertainty and robustness of the model.

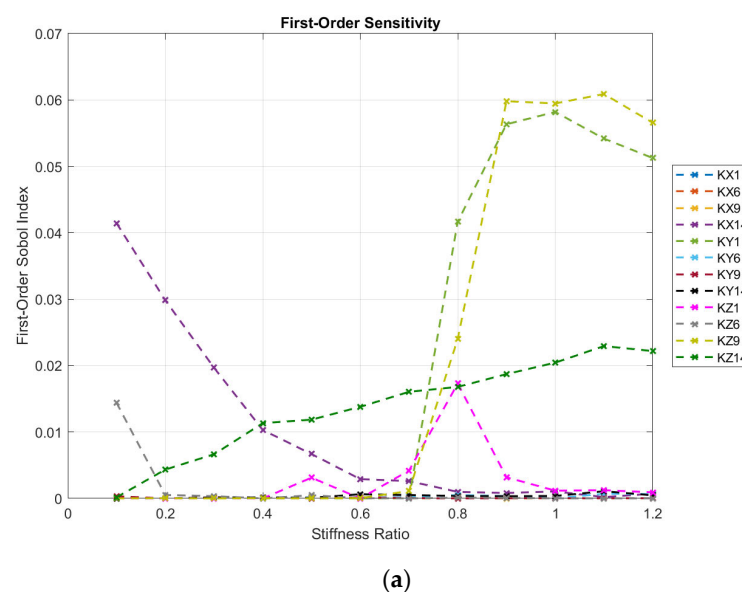
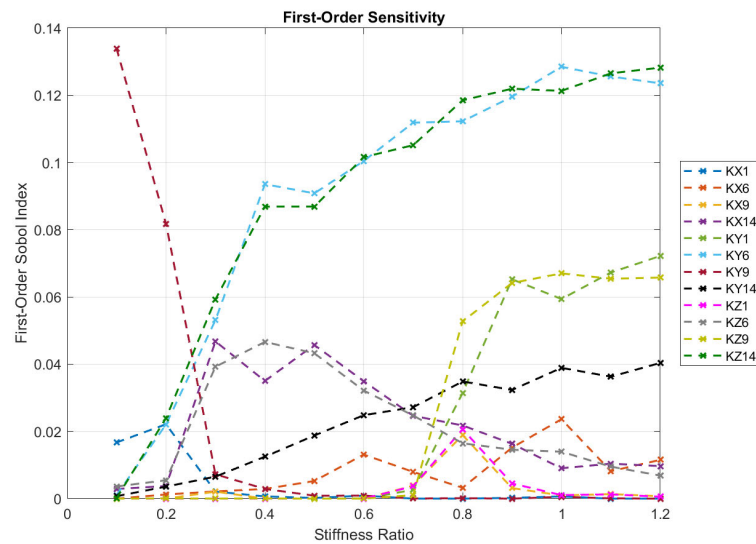
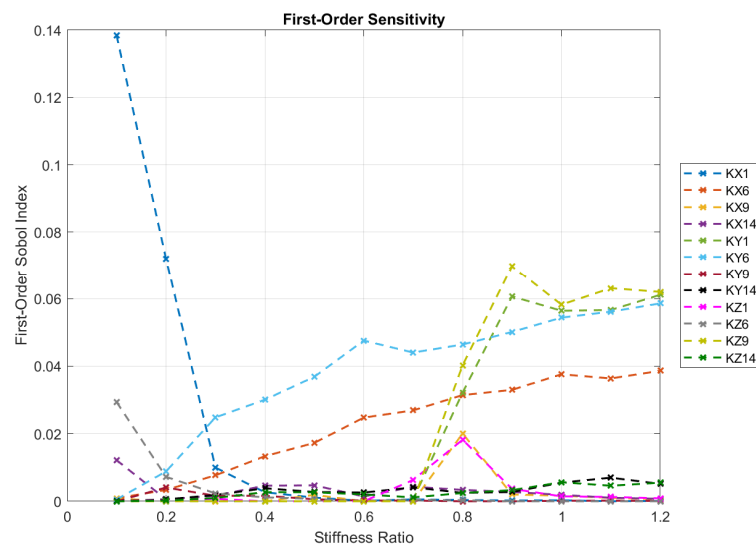


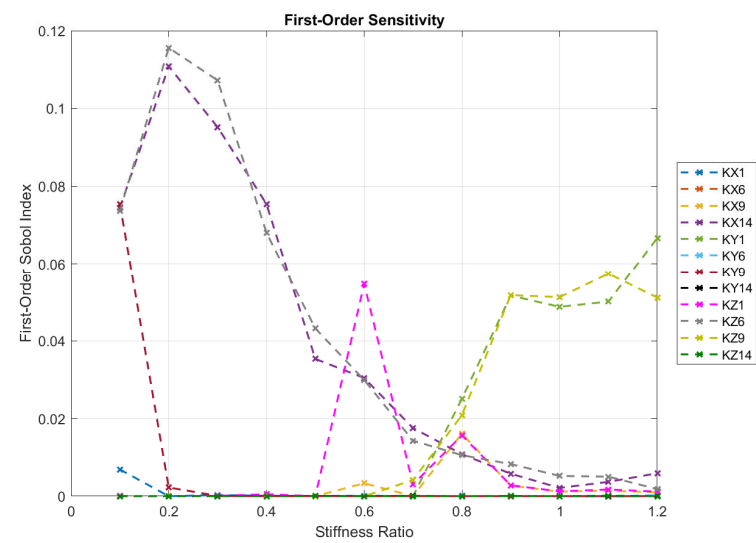
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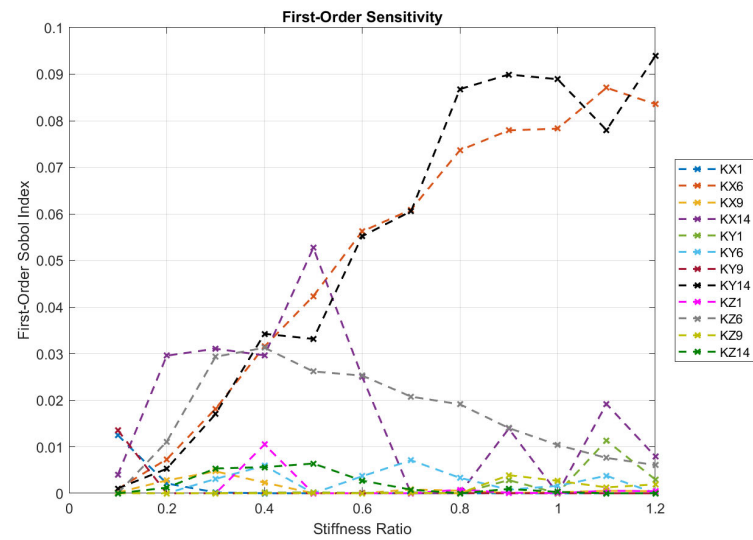


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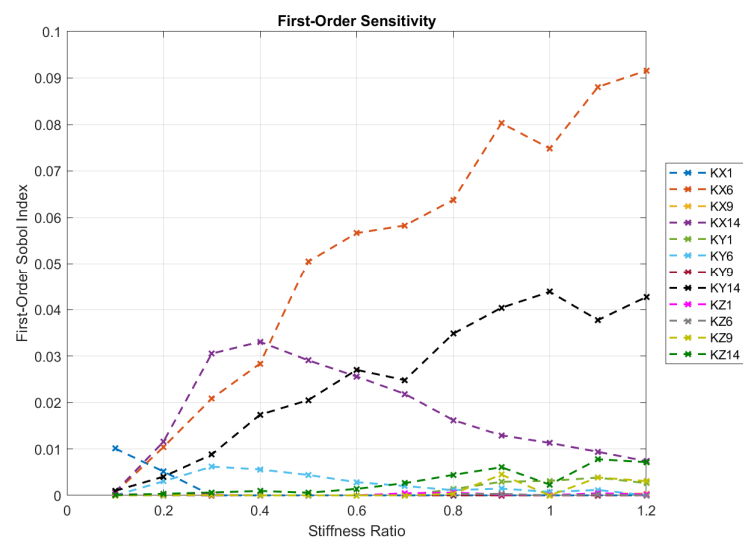


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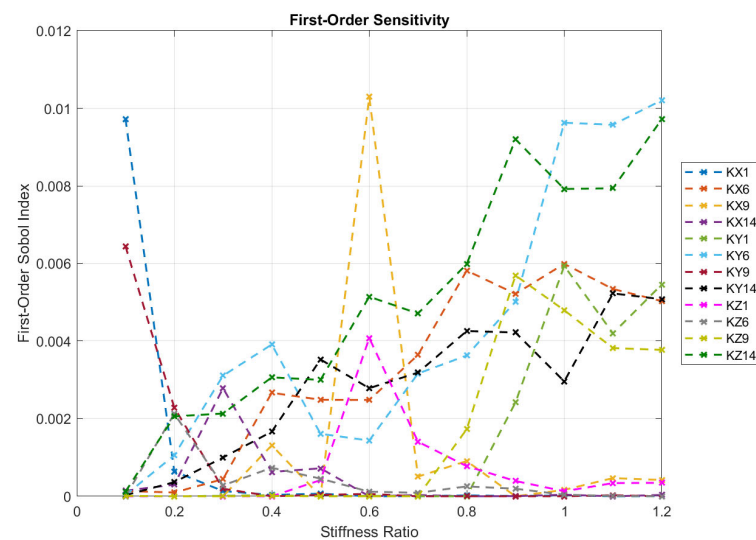
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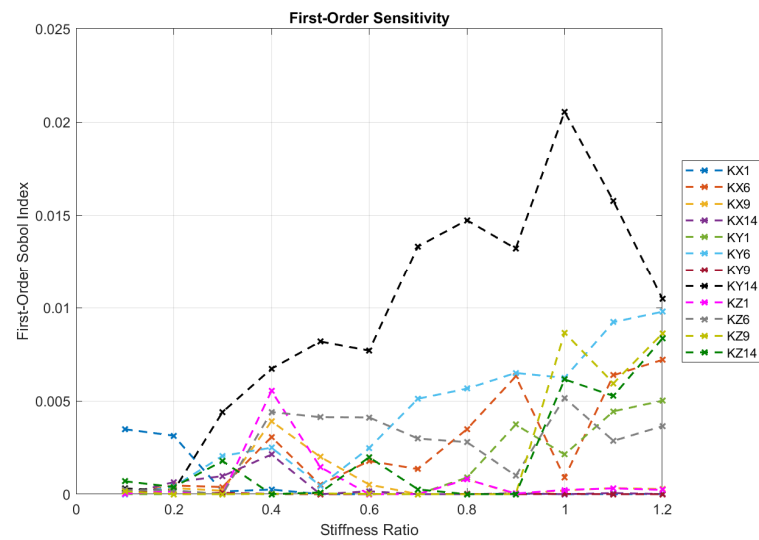


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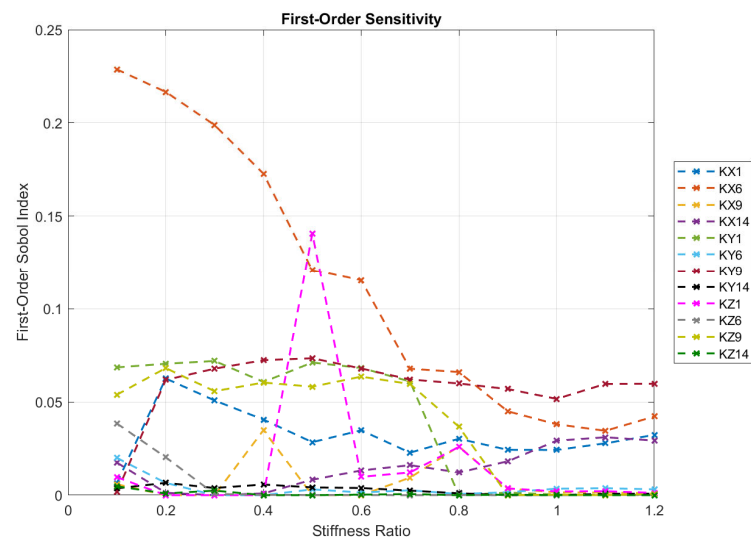


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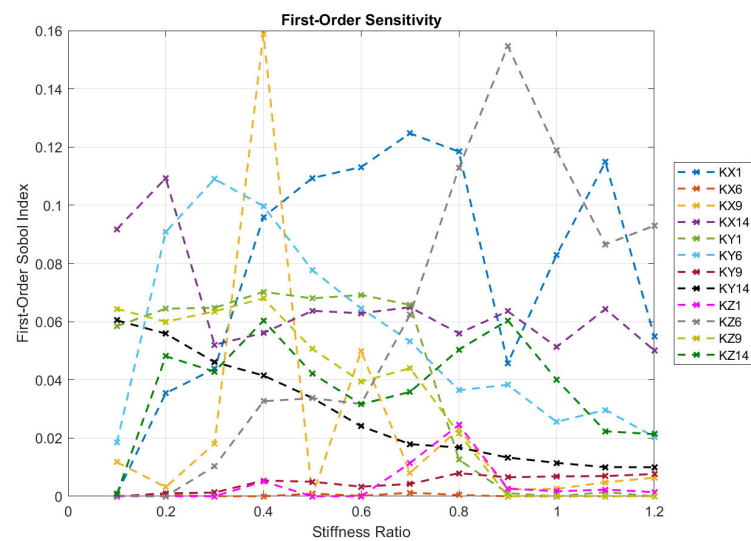
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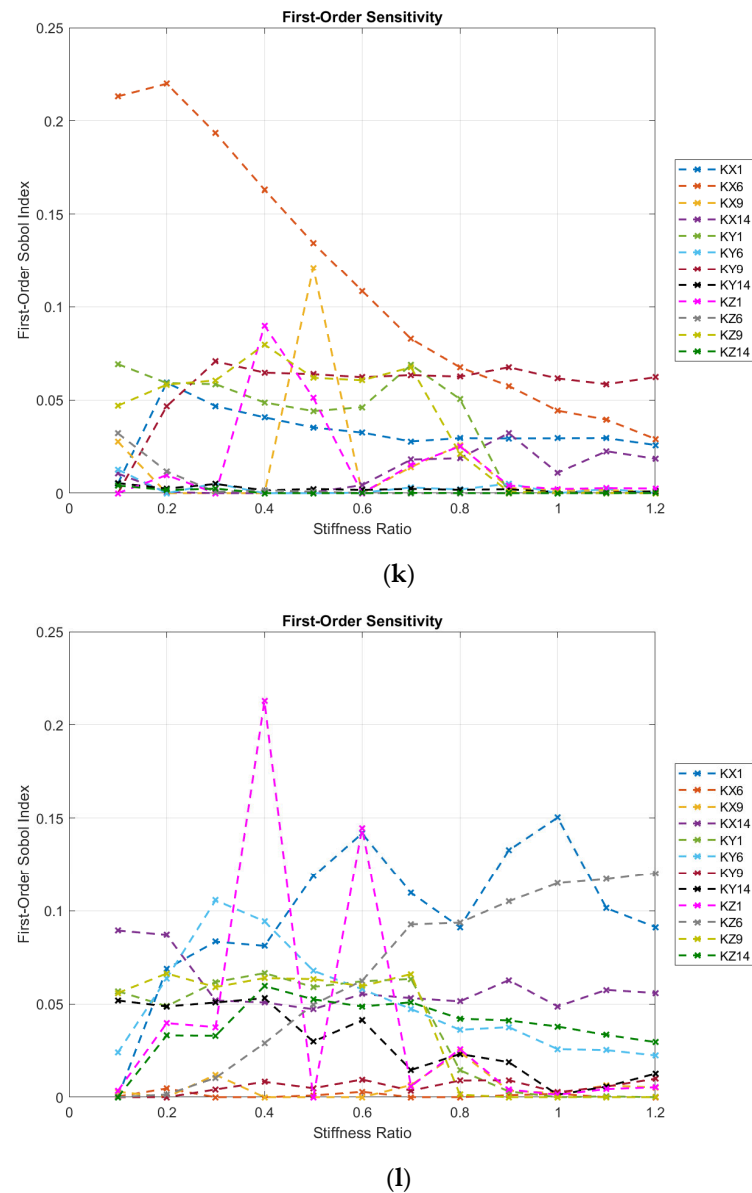


Figure 5. Variation of first-order sensitivity indices: (a) first-order natural frequency; (b) second-order natural frequency; (c) third-order natural frequency; (d) fourth-order natural frequency; (e) fifth natural frequency; (f) sixth natural frequency; (g) X-direction decoupling rate; (h) Y-direction decoupling rate; (i) Z-direction decoupling rate; (j) RXX-direction decoupling rate; (k) RYY-direction decoupling rate; (l) RZZ-direction decoupling rate.

4.3. Optimization Function for the Battery Pack Mounting System

Based on the data and results from this chapter and the previous one, the stiffnesses of the four mounting points are taken as the decision variables for optimization. The first-order natural frequency, the Y-direction decoupling rate T_Y , and the RZZ-direction decoupling

rate T_{RZZ} are set as the three optimization objectives. The established mathematical model for optimization can be expressed as follows:

$$\left\{ \begin{array}{l} \text{find} \quad \mathbf{x} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}] \\ \text{min} \quad J_1 = -f_1 \\ \text{min} \quad J_2 = -T_Y \\ \text{min} \quad J_3 = -T_{RZZ} \\ \text{s.t.} \quad 750 \leq x_i \leq 2000, i = 1, 2, 3, \dots, 8 \\ \quad 1500 \leq x_j \leq 8000, j = 9, 10, 11, 12 \\ \quad g_1(\mathbf{x}) = f_1 - 4.3 \geq 0 \\ \quad g_2(\mathbf{x}) = T_X - 60 \geq 0 \\ \quad g_3(\mathbf{x}) = T_Y - 60 \geq 0 \\ \quad g_4(\mathbf{x}) = T_Z - 60 \geq 0 \\ \quad g_5(\mathbf{x}) = T_{RXX} - 60 \geq 0 \\ \quad g_6(\mathbf{x}) = T_{RYY} - 60 \geq 0 \\ \quad g_7(\mathbf{x}) = T_{RZZ} - 60 \geq 0 \end{array} \right. \quad (34)$$

where the twelve decision variables $[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]$ are the stiffnesses of the four mounting points. The first eight decision variables are the X-direction and Y-direction stiffnesses, and the last four are the Z-direction stiffnesses, expanded as $[KX1, KX6, KX9, KX14, KY1, KY6, KY9, KY14, KZ1, KZ6, KZ9, KZ14]$. The constraints limit the X- and Y-direction stiffnesses to $[750, 2000]$ (unit: N/mm) and the Z-direction stiffnesses to $[1500, 8000]$ (unit: N/mm). Based on engineering practice, the maximum decoupling rate in all six directions should not be less than 60%.

4.4. Application of Multi-Objective Optimization Algorithms

This paper selects the MOEA/D-CMT (MOEA/D with competitive multitasking) multi-objective evolutionary algorithm to optimize the stiffness of the battery pack mounting system. According to Equation (34), the optimization model is a constrained multi-objective optimization problem with 12 decision variables, 3 objective functions, and 19 constraints. Therefore, the problem is very difficult to solve and places new demands on the solving capabilities of the algorithm.

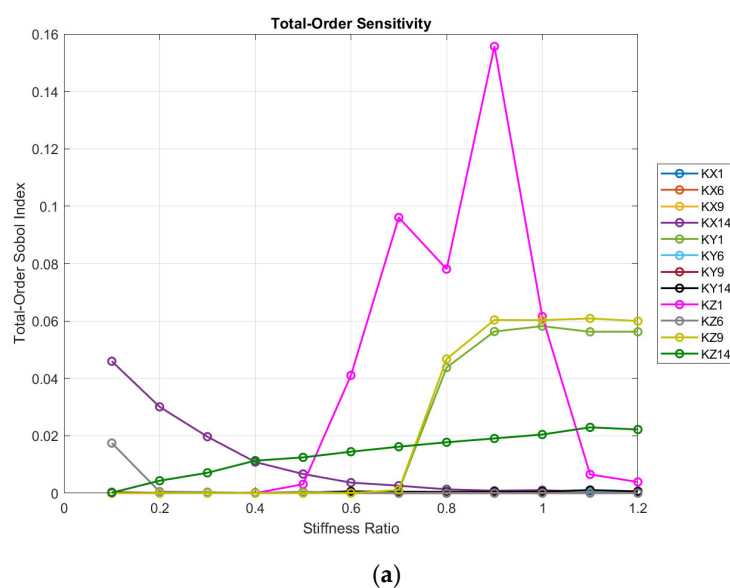
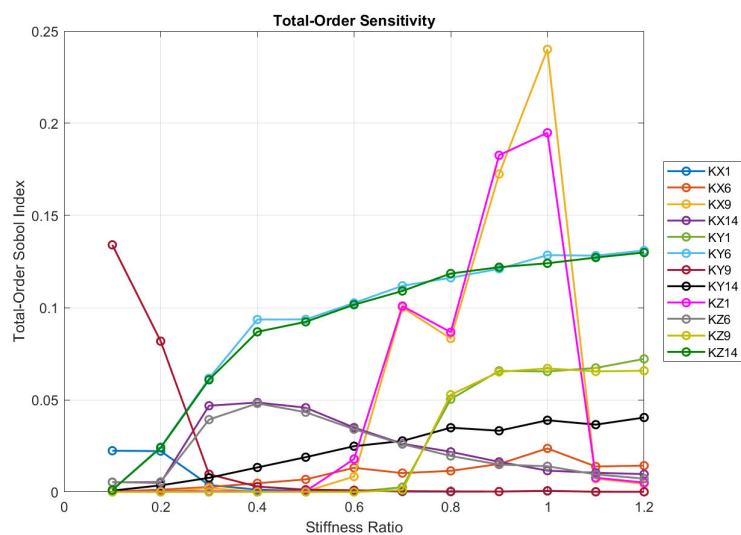
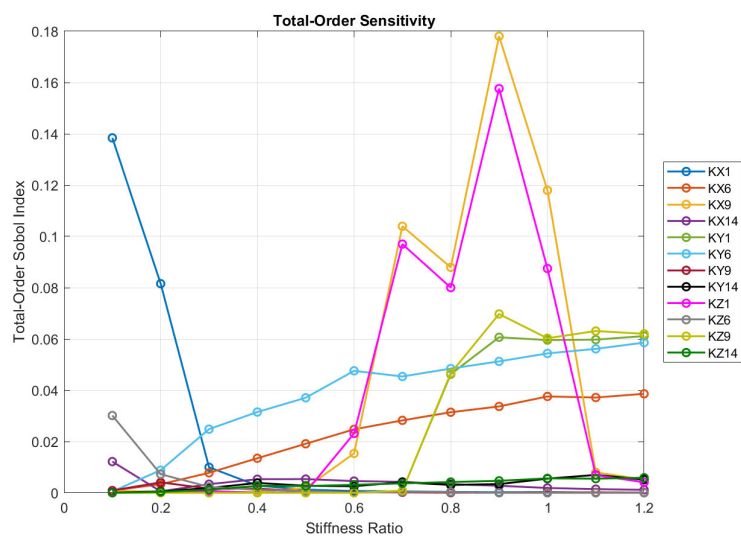


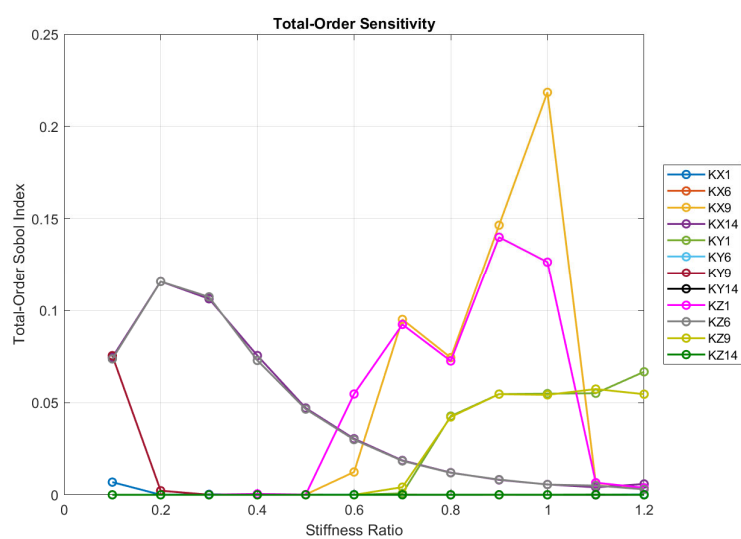
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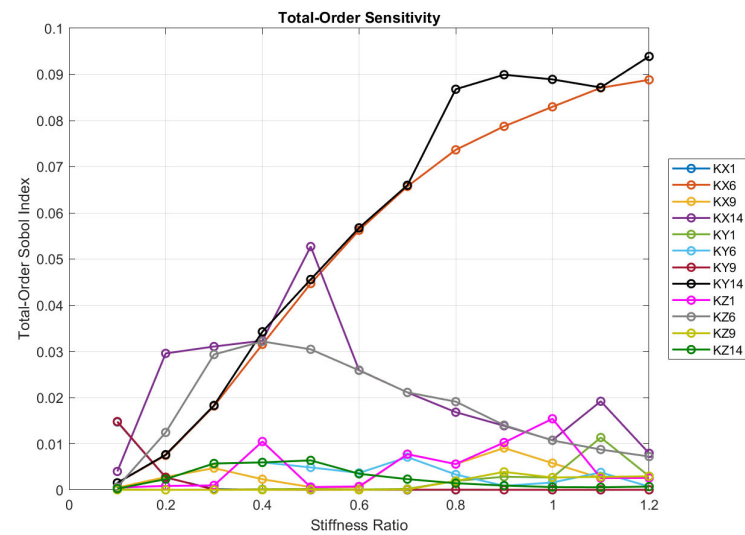


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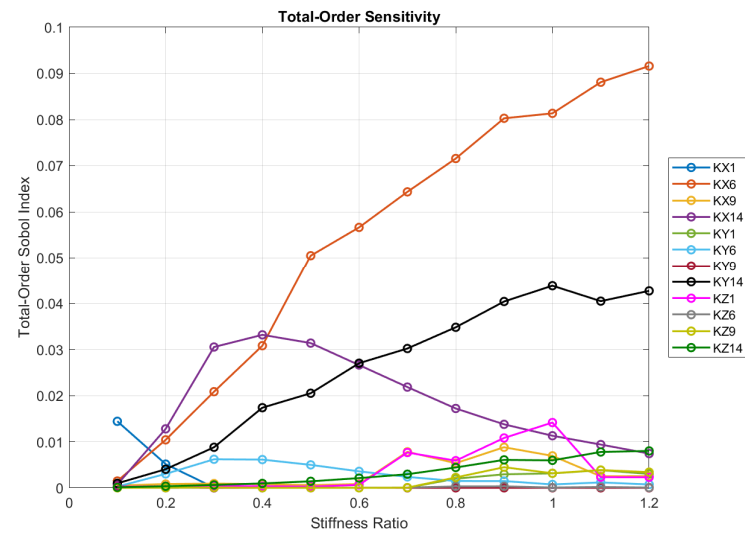


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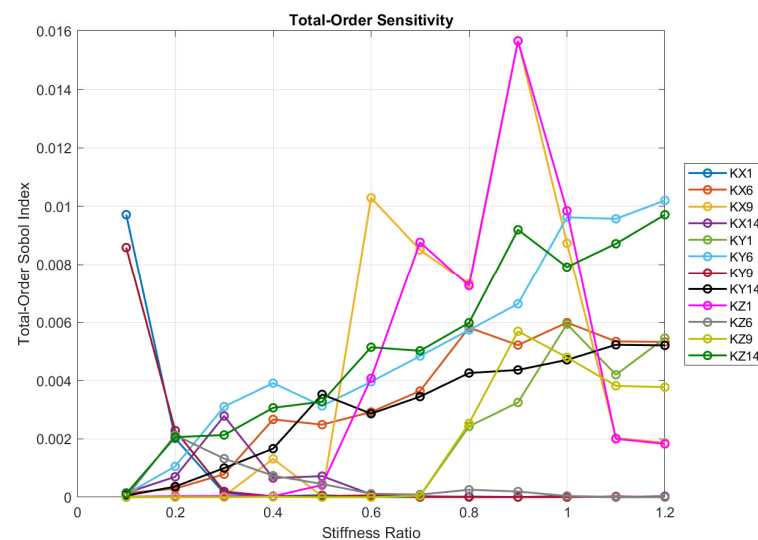
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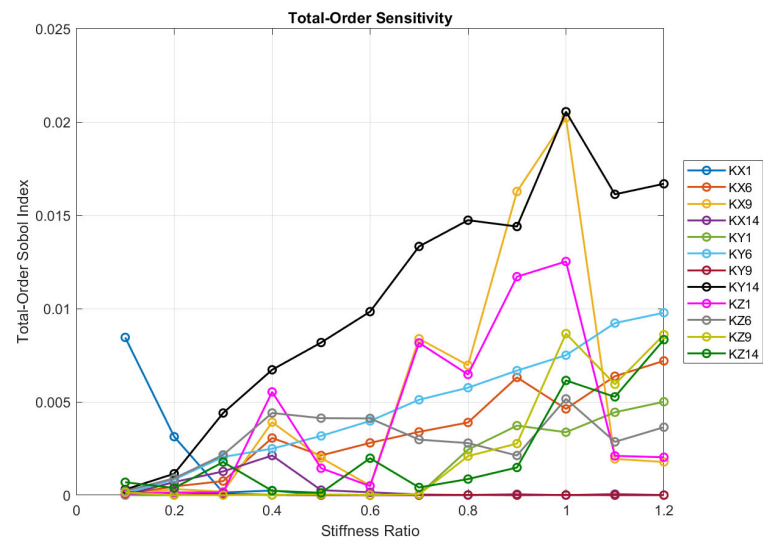


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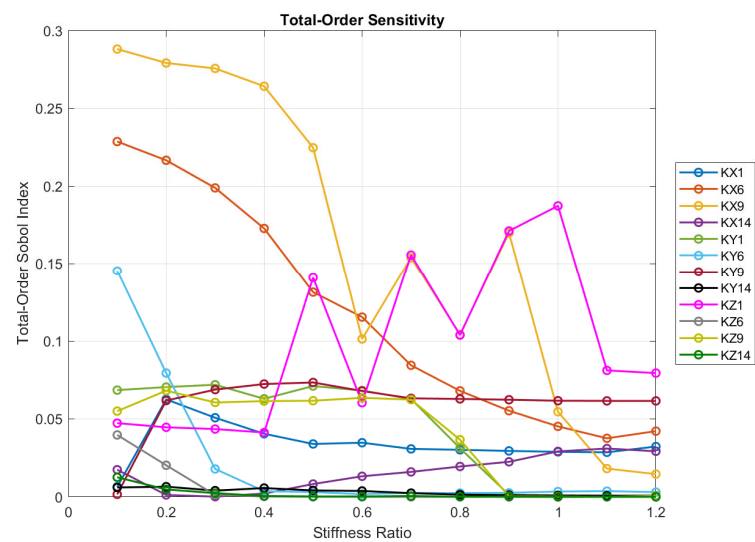


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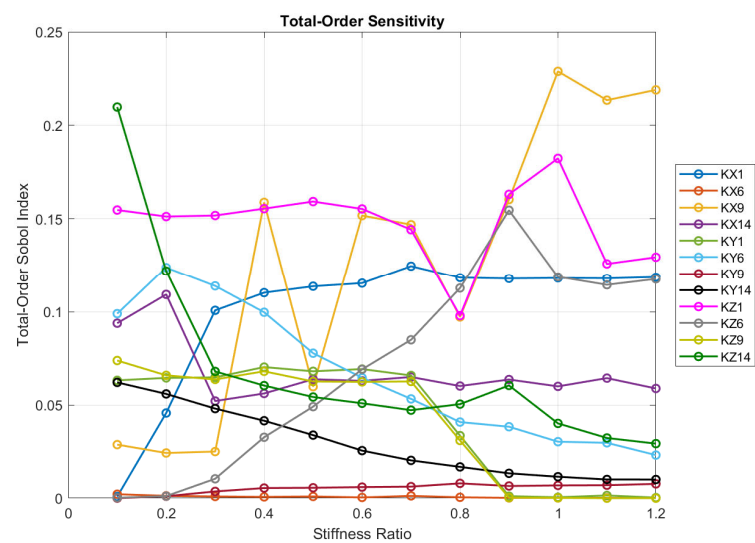
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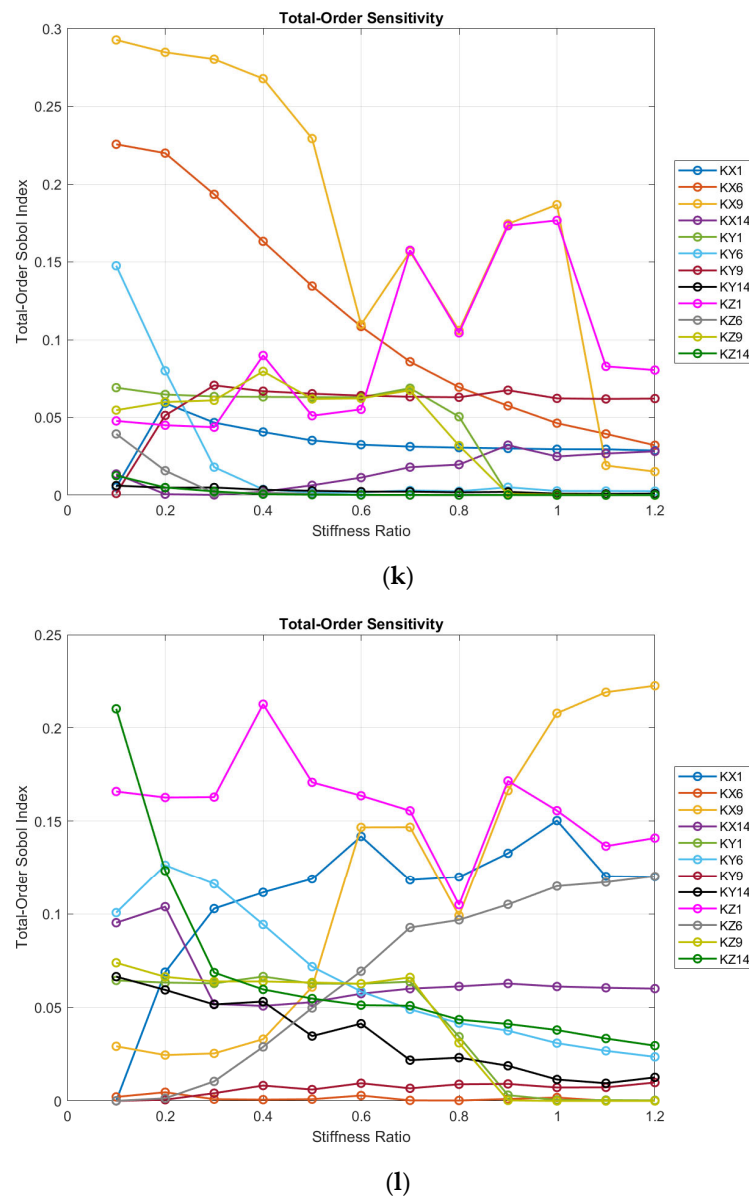


Figure 6. Variation of total sensitivity indices: (a) first-order natural frequency; (b) second-order natural frequency; (c) third-order natural frequency; (d) fourth-order natural frequency; (e) Fifth natural frequency; (f) sixth natural frequency; (g) X-direction decoupling rate; (h) Y-direction decoupling rate; (i) Z-direction decoupling rate; (j) RXX-direction decoupling rate; (k) RYY-direction decoupling rate; (l) RZZ-direction decoupling rate.

MOEA/D-CMT [25] is an algorithm specifically designed for constrained multi-objective optimization problems (CMOPs), based on a novel framework of competitive multitasking (CMT). The algorithm decomposes a complex constrained multi-objective optimization problem into multiple subtasks. Through a competitive mechanism, it dynamically selects a main task to which computational resources are centrally allocated, while other tasks serve as auxiliary tasks. This allows for efficient problem-solving while ensuring the diversity and convergence of the target solutions. Thus, the architectural advantages of MOEA/D-CMT are uniquely aligned with the challenges presented by our optimization task.

NSGA-II (Non-dominated Sorting Genetic Algorithm II), proposed by Deb et al. in 2002 [26], is a widely used multi-objective optimization evolutionary algorithm. The

algorithm simulates the principles of natural selection and genetics, aiming to find a set of solutions that can balance multiple conflicting objectives in a single run.

To verify the rationality and effectiveness of the chosen optimization algorithm, this paper selects two currently widely used and representative multi-objective evolutionary algorithms for comparative analysis: NSGA-II and the MOEA/D-CMT. Both algorithms were run under the same experimental conditions, with each algorithm independently repeated 30 times to ensure the statistical significance and robustness of the results.

In each run, the maximum number of evaluations was set to 50,000, meaning the algorithm performed 50,000 calls and evaluations of the objective functions during the search process. At the same time, to better capture the diversity and convergence characteristics of the solutions during the evolutionary process, 500 representative population individuals from each run were retained for analysis. The number of algorithm iterations was uniformly set to 100 to observe the performance differences between the two algorithms in terms of optimization effect, solution distribution, and decoupling characteristics within the same evolutionary period.

As shown in Figure 7, under the same optimization settings, the target solutions obtained by the MOEA/D-CMT algorithm are generally distributed closer to the origin of the objective space, indicating better convergence in multi-objective optimization and a more effective approximation of the theoretical Pareto front. In contrast, the solution set obtained by the NSGA-II algorithm generally has larger values in each objective dimension and is somewhat deviated from the origin, showing a deficiency in convergence accuracy. In terms of diversity, the MOEA/D-CMT algorithm demonstrates a broader coverage of the solution space. Its solution set has a larger and more uniform distribution in the objective space, reflecting good solution set extension and comprehensiveness, and more fully representing the trade-off relationships between multiple objectives. In comparison, the solution set of NSGA-II is more concentrated, with obvious clustering phenomena and limited coverage, making it difficult to fully characterize the entire Pareto front. This affects the diversity and practicality of the solutions.

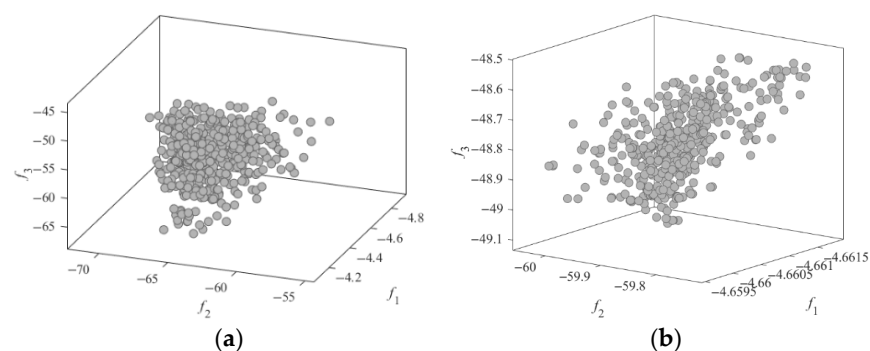


Figure 7. Pareto fronts under the two algorithms: (a) Pareto fronts under the MOEA/D-CMT; (b) Pareto fronts under the NSGA-II.

As shown in Figure 8, the MOEA/D-CMT algorithm shows a clear advantage in its search capability in the variable space. The solution set it obtains exhibits higher combinatorial diversity and distribution breadth in the variable dimensions, with a larger and more uniform range of variable values. This indicates that the algorithm has a stronger global exploration capability, can fully explore the decision space, and can effectively avoid getting trapped in local optima, thereby obtaining a more representative and feasible solution set. In contrast, the variable distribution of the solutions obtained by the NSGA-II algorithm is relatively concentrated, with limited coverage in some variable dimensions, indicating a weaker exploration capability in the variable space. This tendency toward local

distribution may lead to premature convergence, reducing the diversity and practicality of the solution set and limiting the possibility of providing multiple options for comparison and selection for decision-makers in practical problems.

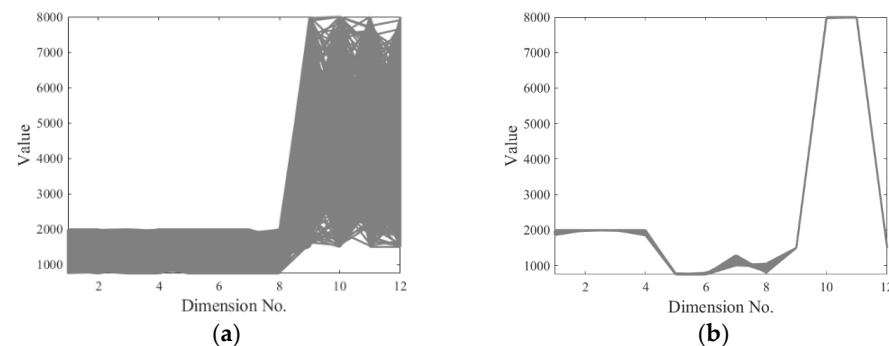


Figure 8. Variable distributions under the two algorithms: (a) variable distributions under MOEA/D-CMT; (b) variable distributions under NSGA-II.

Therefore, for the optimization problem of this battery pack mounting system, the MOEA/D-CMT algorithm demonstrates superior optimization performance over NSGA-II in both the objective space and the variable space. Not only is it closer to the theoretical Pareto front in terms of convergence, improving the accuracy and reliability of the solutions, but it also has broader and more balanced coverage in terms of diversity and variable distribution. It can generate a richer and more reasonably distributed solution set, thus providing more representative and practical solutions for multi-objective optimization problems, and it has stronger potential for engineering applications and promotion.

Based on engineering practice, after ensuring that the maximum decoupling rate in the six degrees of freedom reaches 60%, the further optimization objective should be to increase the first-order natural frequency of the system as much as possible. To this end, the original multi-objective optimization problem can be transformed into a single-objective optimization problem to obtain a set of decision variables with practical feasibility and application value. The mathematical expression for this single-objective optimization problem is as follows:

$$\min J_4 = \alpha J_1 + \beta J_2 + \gamma J_3 \quad (35)$$

where α , β , and γ are the weight coefficients for the three objectives—first-order natural frequency, Y-direction decoupling rate, and Z-axis rotation decoupling rate—set to 6, 1, and 1, respectively; J_4 is the single objective to be optimized.

As shown in Table 8, in the selection of decision variable parameters, a set of optimal input decision variables was obtained for each of the MOEA/D-CMT and NSGA-II methods. Based on the optimization results, the improvement effects of the two algorithms on the first six natural frequencies were further analyzed, with the comparison shown in Table 9.

Specifically, after optimization with the MOEA/D-CMT algorithm, the first to sixth natural frequencies of the system increased by 13.6%, 7.8%, 3.3%, 2.5%, 11.7%, and 9.4%, respectively. Each natural frequency achieved varying degrees of improvement, showing good optimization effects. In contrast, the optimization effect of the NSGA-II algorithm was relatively weaker, with corresponding improvement rates of 9.0%, 1.6%, −1.1%, 1.7%, 3.4%, and 1.5%. The third natural frequency even showed a slight decrease, indicating that it had insufficient convergence or was trapped in a local optimum during the optimization of some modes.

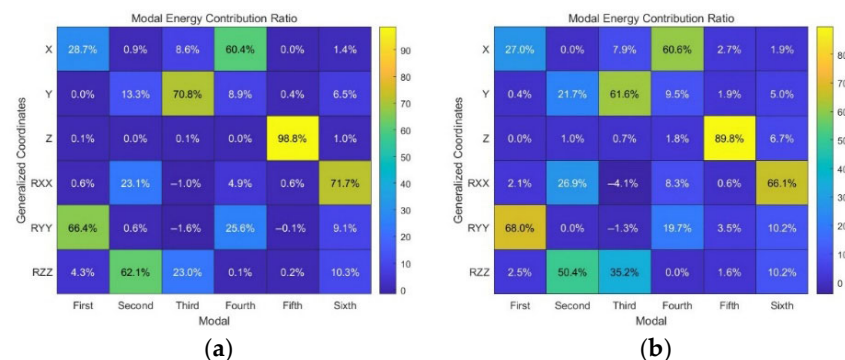
Table 8. Optimized decision variable parameters.

Variable(N/mm)	Initial State	MOEA/D-CMT	NSGA-II
KX1	1620	1647	1985
KX6	1620	1985	2000
KX9	1620	1551	1993
KX14	1620	1874	1992
KY1	1620	1990	750
KY6	1620	1810	750
KY9	1620	1971	1118
KY14	1620	1527	893
KZ1	3200	7832	1500
KZ6	3200	6959	8000
KZ9	3200	3518	7998
KZ14	3200	6493	1501

Table 9. Natural frequency optimization results.

Algorithm	First	Second	Third	Fourth	Fifth	Sixth
Initial State	4.4	6.4	8.9	12.1	14.5	21.2
MOEADCMT	5.0	6.9	9.2	12.4	16.2	23.2
NSGA-II	4.8	6.5	8.8	12.3	15.0	22.3

Figure 9 shows a comparison of the decoupling rates obtained by the MOEA/D-CMT and NSGA-II optimization algorithms. After optimization with MOEA/D-CMT, the X-direction decoupling rate slightly decreased by 0.7%, the Y-direction decoupling rate significantly increased by 11.3%, the Z-direction decoupling rate decreased by 3.2%, the RXX-direction decoupling rate increased by 5.5%, the RYY-direction decoupling rate decreased by 2.9%, and the RZZ-direction decoupling rate increased by 11.3%.

**Figure 9.** Decoupling rate optimization results: (a) decoupling rate optimized by MOEA/D-CMT; (b) decoupling rate optimized by NSGA-II.

In comparison, the optimization results of the NSGA-II algorithm were weaker in terms of decoupling rates. The X-direction decoupling rate decreased by 0.7%, the Y-direction decoupling rate increased by 2.1%, the Z-direction decoupling rate significantly decreased by 10.2%, the RXX-direction decoupling rate decreased by 0.1%, the RYY-direction decoupling rate decreased by 1.2%, and the RZZ-direction decoupling rate decreased by 0.3%. Except for the X-direction decoupling rate, where MOEA/D-CMT was 0.2% lower than NSGA-II, it was superior in all other aspects and met the strict condition that all six degrees of freedom reach 60%.

In comparison, except for the X-direction decoupling rate, the MOEA/D-CMT algorithm outperformed NSGA-II in the decoupling rates of the other five degrees of freedom,

with a significant difference. Among them, the improvement in decoupling performance for Y-direction translation and Z-axis rotation (RZZ) was particularly noticeable. Furthermore, the optimal solution obtained by MOEA/D-CMT has fully met the design requirement that the decoupling rates in all six degrees of freedom are not less than 60%, further verifying its reliability and effectiveness in the optimization of multi-degree-of-freedom dynamic systems.

5. Conclusions

This paper, based on a specific construction machinery battery pack, constructs a multi-degree-of-freedom dynamic model for an upper-layer battery pack. It comprehensively considers the coupling relationships between mount layout, stiffness directionality, and vibration excitation conditions, and it proposes a mounting system optimization method aimed at improving modal decoupling and avoiding resonance response. By introducing sensitivity analysis to evaluate the impact of the position and stiffness parameters of multiple mounting points, the key design variables that dominate the system's dynamic performance were identified. Combined with the MOEA/D-CMT algorithm, the vibration avoidance performance and decoupling characteristics were synergistically optimized. Importantly, the proposed framework is not limited to the specific model in this paper; its principles can be extended to optimize the mounting systems of other heavy machinery (e.g., agricultural tractors, mining vehicles) and to handle more complex, non-uniform mounting layouts. The analysis of the case study and the optimization results indicate the following:

- (a) In a battery pack mounting system distributed on the same plane, the sensitivity analysis results show that between the two categories of decision variables—spatial position and stiffness of the mounting points—the influence index of the stiffness parameters is generally higher, playing a more significant dominant role. Further analysis reveals that among all stiffness parameters, the stiffness of the four corner mounting points has the most prominent impact on the system's performance, indicating that they should be the primary focus of adjustment in the optimization design.
- (b) The sensitivity of the stiffness parameters is highly dependent on the stiffness ratio, exhibiting significant phenomena of parameter dominance transition and sensitivity peaks. This indicates that the importance ranking of parameters is dynamic. To ensure the stability of the system design, it is necessary to avoid falling into highly sensitive regions and to pay attention to the parameter interaction effects that are enhanced with configuration changes.
- (c) Through multi-objective optimization based on MOEA/D-CMT, the first six natural frequencies of the battery pack mounting system were increased by 13.6%, 7.8%, 3.3%, 2.5%, 11.7%, and 9.4%, respectively. Among them, the first-order natural frequency has been effectively shifted away from the main road excitation frequency band, with the optimized frequency increasing by more than 1 Hz, significantly reducing the risk of resonance. At the same time, the modal decoupling rates of the system in all six degrees of freedom exceed 60%, meeting the vibration isolation design requirements. The Y-direction translation and Z-axis rotation decoupling rates, which were the focus of the optimization, increased by 11.3% and 11.3%, respectively, demonstrating the effectiveness of the optimization scheme in improving the system's dynamic performance.

Author Contributions: Conceptualization, D.L. and Y.W.; Methodology, D.L.; Software, D.L., R.S. and H.W.; Resources, R.S., H.W. and Y.W.; Data curation, R.S. and H.W.; Writing—original draft, D.L.; Writing—review & editing, Y.W.; Project administration, Y.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by National Natural Science Foundations of China (22262005); Guangxi Science and Technology Major Program grant number 2023AB01210; and Guangxi Key Laboratory of Automobile Components and Vehicle Technology Independent Research Project grant number 2023GKLACVTZZ09.

Data Availability Statement: The original contributions presented in this study are included in the article material. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: Authors Run Sun and Hai Wei were employed by the company Guangxi Liugong Machinery Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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