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Study of the Influence of the Two-Drive-Axle Bogie Parameters on the Three-Axle Vehicle Handling

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Abstract: The presence of a bogie in three-axle vehicles when moving along a curved trajectory leads to deterioration in its handling and maneuverability. The paper developed a mathematical model of the elastic bogie wheel while moving along a curvilinear trajectory, according to which the bogie wheel simultaneously participates in curvilinear and plane-parallel motion with a slip angle. Such movement of the bogie wheels develops significant lateral and longitudinal forces on the steered wheels, which leads to the movement of the steered wheels with slip, redistribution of the load on them, tire twisting, and a decrease in the steering angle of the outside steered wheel due to the elasticity of the steering trapezoid. Based on the mathematical model of the bogie wheel, an analytical dependence was obtained to determine the minimum turning radius of a three-axle vehicle. The reliability of the analytical dependencies characterizing the movement of the bogie wheel along a curvilinear trajectory was determined by comparing the minimum turning radii of a three-axle vehicle with the intermediate axle lowered and raised. It has been established that the minimum turning radius of a vehicle with a bogie increases compared to a two-axle vehicle and depends on the cornering stiffnesses of the tires of the bogie and steered wheels, the bogie and vehicle wheelbases, the kinematic and elastic parameters of the steering trapezoid, the direction of turning of the steered wheels, and the load on the steered and the bogie wheels.

Keywords: three-axle vehicle; bogie; minimum turning radius; slip angle; lateral and longitudinal force on the steered wheels; steering trapezoid

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Academic Editor: Ning Sun

Received: 31 March 2025
Revised: 27 April 2025
Accepted: 7 May 2025
Published: 9 May 2025

Citation: Mateichyk, V.; Soltus, A.; Klimov, E.; Smieszek, M.; Kostian, N.; Tarandushka, L.; Marchuk, N. Study of the Influence of the Two-Drive-Axle Bogie Parameters on the Three-Axle Vehicle Handling. *Machines* **2025**, *13*, 394. https://doi.org/10.3390/machines13050394

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1. Introduction

With the growth in motorization, the load on the road network increases. Heavy traffic requires truck drivers to be more attentive and precise in driving. The maneuverability and handling of trucks are critically important factors, as they affect road safety and transportation efficiency [1]. Trucks are large in size and weight, so reducing their maneuverability can lead to accidents [2]. The maneuverability of a vehicle saves time and fuel, optimizing the transportation process, while its handling ensures a quick response to emergencies and reduces the risk of damage to the cargo. The presence of a bogie at three-axle vehicles allows

increasing their load capacity, but at the same time increases the resistance to curvilinear movement and worsens their fuel economy, maneuverability, and handling [3]. At the same time, unlike a two-axle vehicle, the bogie affects the size of the wheelbase of a three-axle vehicle and develops significant lateral and longitudinal forces on the steering axle wheels while moving along a curved trajectory, which leads to an increase in the minimum turning radius [4]. The significant minimum turning radius of three-axle vehicles, caused by the presence of a bogie, makes it impossible to operate them on mountain roads with small radii of curvature and in the absence of asphalt concrete surface [5]. Thus, the author of this paper conducted studies on the possibility of exploiting three-axle KrAZ–256B (JSC "AutoKrAZ", Kremenchuk, Ukraine) dump trucks with a 6 × 4 wheel formula on the roads of a high-mountain quarry of the Comibol company (Bolivia) with an altitude of up to 4800 m above sea level have shown that their operation on such roads is dangerous since during movement the loaded three-axle dump truck does not turn stably.

Table 1 shows the following technical characteristics of three-axle general-purpose vehicles: wheel formula, distance from the steering axle to the intermediate axle of the vehicle, ratio of bogie and vehicle wheelbases, bogie wheelbase, and presence of single or dual tires on the bogie axles. At the same time, the vehicle wheelbase L is understood as the distance between the steering axle and the equalizer axis. In this case, the vehicle wheelbase consists of the distance between the steering axle and the intermediate axle and half of the bogie wheelbase.

Table 1. Technical characteristics of the three-axle vehicles.

Vehicle	Wheel Formula	Distance Between the Steering Axle and the Intermediate Axle, mm	Wheel-Base Ratio l_b/L	Bogie Characteristics	
				Wheel-Base l_b, mm	Single/Dual Tires
KrAZ–6322–02 (JSC "AutoKrAZ", Kremenchuk, Ukraine)	6 × 6	4600	0.2642	1400	single
KrAZ–65055–040 (JSC "AutoKrAZ", Kremenchuk, Ukraine)	6 × 4	4080	0.2929	1400	dual
MAZMAN–752559 (MAN Truck & Bus SE, Munich, Germany)	6 × 4	3300	0.35	1400	dual
MAZ–6501 (OJSC "MAZ", Minsk, Belarus)	6 × 4	3875	0.306	1400	dual
JAC N350 (JAC Motors, Hefei, China)	6 × 4	4100	0.2827	1350	dual
RENAULT KERAX chassis (Renault Trucks, Bourg-en-Bresse, France)	6 × 4	3845	0.3024	1370	dual
Foton Auman GTL BJ 3259 (Foton Motor, Beijing, China)	6 × 4	3950	0.2919	1350	dual
MAN–TGS 26.400 6X2 4 BL Rechtslenker (MAN Truck & Bus SE, Munich, Germany)	6 × 4	4200	0.2857	1400	dual
KAMAZ–5350 (PJSC "KAMAZ", Naberezhnye Chelny, Russia)	6 × 6	3690	0.3034	1320	single
KAMAZ 6520 (PJSC "KAMAZ", Naberezhnye Chelny, Russia)	6 × 4	3600	0.3333	1440	dual
KAMAZ 65115 (PJSC "KAMAZ", Naberezhnye Chelny, Russia)	6 × 4	3190	0.3428	1320	dual
Ford Trucks 2533 DC chassis (Ford Otosan, Inonu, Turkey)	6 × 4	4250	0.2688	1320	dual
Ford Trucks 3333 DC chassis (Ford Otosan, Inonu, Turkey)	6 × 4	4500	0.2609	1350	dual
Chassis Iveco T-WAY AD380T43H (Iveco S.p.A., Piacenza, Italy)	6 × 4	4209	0.2817	1380	single
Chassis Iveco T-WAY AD380T48H (Iveco S.p.A., Piacenza, Italy)	6 × 4	4479	0.267	1380	dual

From the analysis of Table 1, it can be seen that the bogie wheelbases of general-purpose trucks are in the range from 1.32 to 1.44 m. With such values of the wheelbases, the traction/braking and lateral forces that develop in the contact of the bogie wheels with the supporting surface negatively affect the characteristics of passing curves and cause significant tire wear [6]. During the movement of such vehicles along the trajectory of the minimum turning radius, traces of tire wear of the truck wheels remain on the supporting surface with a high coefficient of adhesion. Intensive operation of three-axle vehicles in such conditions has an important effect on traffic safety and stability of their operation [7].

As for the three-axle truck wheelbase, on the one hand, its reduction to a certain value will lead to a decrease in the minimum turning radius on roads with a high coefficient of adhesion, and on the other hand, to a loss of handling on roads with a reduced coefficient of adhesion. Taking into account the existing bogie wheelbases of three-axle vehicles in the range of 1.32–1.44 m, the analysis showed that the vehicle wheelbases, defined here as the distance between the steering axle and the bogie axis, will be in the range of 3.850–5.175 m, and the ratio of the bogie wheelbase to vehicle one is in the range of 0.2609–0.3428.

The analysis of [8] on the handling of three-axle trucks with a bogie testified that the bogie increases the radius of the curved trajectory and movement resistance, which is caused by significant slip angles of the bogie wheels when moving along a curved trajectory. Since the minimum turning radius of the vehicle will depend on the kinematics of the steered wheels and the forces acting on them, the design and elastic parameters of the steering trapezoid and the direction of turning of the steered wheels will affect the value of the minimum turning radius. Handling parameters deteriorate as the load on the vehicle increases [9].

At the same time, the minimum turning radius is dependent on the bogie and the vehicle wheelbase, the elastic characteristics of tires and dimensions their contact patches, the load on the wheels, and the maximum steering angles.

As for the maximum steering angles, their values depend on the design of the kingpin unit of the steering axle [10]. If the steering axle is driving, then its design includes CV joints, the designs of which limit the maximum steering angles. Thus, an analysis of the designs of all-wheel drive vehicles KamAZ (PJSC "KAMAZ", Naberezhnye Chelny, Russia), MAZ (OJSC "MAZ", Minsk, Belarus), and KrAZ (JSC "AutoKrAZ", Kremenchuk, Ukraine) showed that disk CV joints are installed on these vehicles, and their design provides a maximum steering angle of up to 32°.

On non-all-wheel drive three-axle vehicles, the maximum steering angle is limited by special stops on the steering axle, which prevent the steered wheel tires from contacting the suspension parts at their maximum steering angles in extreme conditions. This design of the steering axle provides maximum steering angles of up to 42°.

Note that the designs of the steering axles ensure the turning of the right steering wheel by transmitting the force through the steering trapezoid. To do this, it is necessary to create by the power steering a moment about the left steered wheel kingpin axis, sufficient to overcome the steering resistance of the right steered wheel. The value of this moment is proportional to the power transmission ratio of the steering trapezoid. Analysis of the results of the conducted studies of the transmission ratios of the steering trapezoid showed the following: in the KrAZ–6322–02 truck, at the maximum steering angle, when steering to the left, the transmission ratio is 0.55, and when steering to the right it is 2.15; in the KamAZ–5511 truck, under such conditions, when steering to the left, the transmission ratio is 0.62, and when steering to the right it is 1.67; in the KamAZ–4310 truck, when steering to the left, the transmission ratio is 0.66, and when steering to the right it is 1.52.

From the analysis of the above, it follows that the transmission ratio of the steering trapezoid when steering the wheels to maximum angles to the left (0.55–0.66) is significantly

less than when steering to the right (1.52–2.15). It is obvious that the moment that must be applied about the left steered wheel kingpin axis to steer the right wheel to the right will be 2.3–3.91 times greater than for its steer to the left, which must be taken into account when conducting experimental studies of the minimum turning radius to the right.

J. R. Ellis considers the features of the movement of a two non-steering-axle bogie along a curvilinear trajectory in his work [11]. The center of the bogie moves with a linear velocity U, and with an angular velocity r, the bogie turns relative to this center and moves with a lateral slip angle α, which is determined by external factors.

The author notes that under these conditions, the following slip angles will develop on each axle of the bogie:

$$\alpha_f = \alpha + \frac{e \times r}{U}$$

$$\alpha_r = \alpha - \frac{e \times r}{U}$$

where α_f and α_r are the slip angles of the front and rear axles of the bogie, respectively; e is the distance from the axis to the center of the bogie.

Assuming that the vertical load between the axles is evenly distributed, the cornering stiffness of the tires of the front and rear axles are the same, and the slip angles of the tires do not exceed 5°, the author obtains a dependence for determining the moment created by lateral forces:

$$M = 2Ce^2 \frac{r}{U} \tag{1}$$

where M is the moment generated by lateral forces; C is the cornering stiffness of the axle tires.

Considering the movement of the center of the bogie along a curvilinear trajectory with radius R, the author defines by the relation $\frac{r}{U} = \frac{1}{R}$ that expression (1) will take shape:

$$M = 2Ce^2 \frac{1}{R} \tag{2}$$

Based on the results of the research, the author concludes that when driving on a curve with a constant radius, the reaction to lateral slip is greater for a vehicle equipped with a two-axle bogie than for a vehicle with a single rear axle. The steering angle required to ensure the vehicle moves along a trajectory of a given radius increases if a two-axle bogie is installed on the vehicle, and replacing a multi-non-steering-axle bogie with a single axle located in the center of the bogie leads to errors in the assessment of handling.

In [12], the authors present a linear equation for the steady-state turning mode of a three-axle vehicle equipped with a bogie with two non-steering axles and dual tires:

$$\delta = \frac{l}{R}\left[1 + \left(\frac{\Delta^2 + D^2 \frac{C_{sR}}{C_{\alpha R}}}{l^2}\right)\left(1 + \frac{2C_{\alpha R}}{C_{\alpha F}}\right)\right] + M\frac{V^2}{R}\left[\frac{b/l(2C_{\alpha R}) - a/lC_{\alpha F}}{C_{\alpha F}(2C_{\alpha R})}\right] \tag{3}$$

where δ is the steering angle; a is the distance between the center of gravity of the vehicle and the front axle; b is the distance between the center of gravity of the vehicle and the midpoint between the rear axles; C_{sR} is the summation of the longitudinal stiffnesses of the four tires of a rear axle and is the same for each rear axle; $C_{\alpha F}$ is the summation of the cornering stiffnesses of the two tires on the front axle; $C_{\alpha R}$ is the summation of the cornering stiffnesses of the four tires of a rear axle and is the same for each rear axle; D is one-half of the spacing between dual tires and is the same for each set of duals; l is the vehicle wheelbase; M is the mass of the vehicle; R is the turn radius; V is the rgw velocity of the vehicle; Δ is one half of the bogie wheelbase.

When traveling on a circle at low speeds, replacing dual tires with a single tire [13], from the analysis of Equation (3), we determine the radius of the trajectory of the vehicle's center of gravity:

$$R = \frac{l}{\delta}\left[1 + \frac{\Delta^2}{l^2}\left(1 + \frac{2C_{\alpha R}}{C_{\alpha F}}\right)\right] \quad (4)$$

Comparing expression (4) with the expression for determining the radius of the trajectory of a single rear axle vehicle $R = l/\delta$, it can be noted that during movement in a circle at low speeds with the same steering angle, a two-non-steering-axle bogie vehicle will move in a circle with a larger radius than a single-rear-axle vehicle. That is, at a fixed value of the steering angle, the presence of a bogie with non-steering axles in a vehicle will lead to an increase in the radius of the trajectory of movement.

In [14], it is noted that a vehicle with several non-steering axles can be represented as an equivalent two-axle vehicle, which, with the same steering angle, will have the same radius and approximately the same turn center as a vehicle with several non-steering axles. The physical interpretation of the equivalent wheelbase for such vehicles and the method of its measurement are given. The equation for determining the equivalent wheelbase of a two-axle vehicle has the form:

$$l_e = l\left[1 + \frac{\Delta^2}{l^2}\left(1 + \frac{2C_{\alpha R}}{C_{\alpha F}}\right)\right] \quad (5)$$

where l_e is the equivalent wheelbase; l is the vehicle wheelbase.

From the analysis of works [11–14], it follows that in work [11], a bogie is considered and during the study, the radius of the trajectory of the bogie center is taken, while in works [12–14], a vehicle with a bogie is considered and during the study, the radius of the trajectory of the vehicle center of gravity is taken.

In works [15], when considering a three-axle vehicle, the author adds a third non-steering axle to the bicycle model of a two-axle vehicle. At the same time, the equivalent wheelbase of a three-axle vehicle is defined as follows:

$$l_e = (l - t) + t\frac{C_1C_3l + C_2C_3t}{C_1C_2(l - t) + C_1C_3l} \quad (6)$$

where l is the distance between the steering axle and the rear axle of the bogie; t is the bogie wheelbase; C_1, C_2, and C_3 are the cornering stiffnesses of the tires of the steering axle, intermediate, and rear axle of the vehicle, respectively.

If we substitute $l = L + \Delta$, $t = 2\Delta$ in dependence (6), where L is the vehicle wheelbase and Δ is one-half the bogie wheelbase, then with equal values of cornering stiffnesses of the intermediate and rear axle $C_2 = C_3 = C_{\alpha R}$ after elementary transformations, we obtain dependence (5).

In [16], a generalized system of equations is presented that describes the dynamics of vehicle handling with an arbitrary number of axles. Two- and three-axle models are considered as special cases of the generalized model. Paper [17] investigates the effect of rear axle steering of a three-axle vehicle on stationary handling in terms of equivalent wheelbase. It is noted that the handling of a three-axle vehicle at low speed is improved by steering the third axle due to a decrease in the effective wheelbase.

In [18], the equivalent wheelbase of a multi-axle vehicle is proposed to be defined as the sum of the distances from the center of gravity of the vehicle to the front axle and from the center of gravity of the vehicle to the equivalent rear axle. It is assumed that the steering axle wheels cornering stiffnesses, the distance between the center of gravity of the vehicle and the steering axle, the forces and moments at the center of gravity of the multi-axle model and its equivalent two-axle model are the same. The equivalent cornering

stiffness and the distance from the center of gravity to the equivalent axle are determined by the optimization method.

The work [19] proposed a control algorithm that determines the optimal steering angles of a multi-axel crane in order to minimize the turning radius and increase the efficiency of driver control. When creating a simplified model of a crane, a bicycle model was used.

From the analysis of the above, it follows that the presence of a bogie increases the turning radius of a three-axle vehicle relative to a two-axle vehicle with the same wheelbase.

The authors of all the above-mentioned works used a bicycle model, which is limited to two degrees of freedom and assumes the absence of wheel tracks and the location of the center of gravity at the road level, to predict the handling and stability characteristics of three-axle vehicles [20]. However, the prediction of the behavior of a real vehicle can be significantly improved if the analysis takes into account the dimensions of the vehicle, the compliance of the suspension, and the steering system [21].

Taking into account the above, the purpose of the study is to evaluate the influence of bogie parameters on the minimum turning radius of a three-axle vehicle. To do this, the following must be carried out:

- Develop a mathematical model of the movement of the bogie wheel along a curvilinear trajectory;
- Determine the kinematics of the steered wheels taking into account the bogie and vehicle wheelbases, kinematic and elastic parameters of the steering trapezoid, tire characteristics, and direction of turning;
- Conduct research and obtain an analytical dependence for calculating the minimum turning radius of a three-axle vehicle;
- Experimentally determine the reliability of the obtained analytical dependencies that characterize the movement of the bogie wheels along a curvilinear trajectory and determine the minimum turning radius.

The article consists of five sections. Section 1 analyzes the influence of the design and kinematic parameters of the bogie and the vehicle on its handling. Section 2 develops a mathematical model of the movement of the bogie wheel along a curvilinear trajectory, investigates the kinematics of the steered wheels taking into account the bogie and vehicle wheelbases, the parameters of the steering trapezoid, and the direction of turning, and obtains an analytical dependence for determining the minimum turning radius. Section 3 presents the results of experimental and analytical studies of the minimum turning radii of three-axle vehicles, which confirm the reliability of the obtained dependencies. Section 4 discusses the results of the conducted studies. Section 5 presents the conclusions of the research results.

2. Materials and Methods

2.1. Mathematical Model of a Bogie Wheel

For the movement of a wheeled vehicle, it is necessary to overcome the wheels' moving resistance, which depends on the design and kinematic, weight, and elastic parameters of the tires and their installation location on the vehicle.

The design parameters of the tire are characterized by the cord, tread, presence of tube, contact patch, and type of tire. The cord is determined by the presence of a metal mesh, material, and number of layers design (radial, bias). The tire tread is characterized by the type (summer, winter, presence of metal spikes), pattern, saturation, and height of the spikes. Depending on the presence of the tube, tires are divided into tube-type and tubeless. As for the tire contact patch, it is characterized by the area and shape (oval, ellipse,

approaching a circle or rectangle, etc.). By type, tires are divided into tires with regulated and non-regulated pressure.

The kinematic parameters of wheel movement are characterized by the trajectory of movement (straight line, curved with a constant radius of curvature, curved with a variable radius of curvature), speed, and acceleration.

The conditions under which the elastic wheel will move depend on its installation location on the vehicle. Thus, the non-steered wheel rotates only about the spin axis, the steered wheel has two degrees of freedom—it rotates about the spin axis and the kingpin axis—and the presence of the bogie causes the non-steered wheel to move with an additional slip angle when moving along a curved trajectory.

To determine the features of the movement of the bogie wheels along a curved trajectory, the following assumptions were made:

- The elastic bogie wheel is considered as a complex mechanism that includes a hard disk, an elastic tire body, and a tire contact patch [22];
- The support surface is a horizontal road with an asphalt concrete surface;
- The tire contact patch is reduced to an equal-sized rectangle [10];
- The movement of the bogie is considered at a low constant speed without taking into account inertial forces;
- The power cylinder of the hydraulic power-assisted steering system provides a stable maximum steering angle of the inside steered wheel while moving along a trajectory of minimum radius.

Taking into account the accepted assumptions, the scheme for determining the parameters of the bogie dynamics while moving along a curved trajectory is shown in Figure 1.

From the analysis of Figure 1, it is seen that the vehicle moves along a curvilinear trajectory relative to the center of transport motion, point O, with angular velocity $\overline{\omega}_{tr}$. The bogie has a wheelbase l_b, the longitudinal axis of the tire contact patch of the bogie wheel a. The center of the inner side of the bogie, point A, moves along a curvilinear trajectory of radius R, and the center of the outer side, point F, moves along a trajectory of radius $R + K_b$, where K_b is the bogie wheel track. The centers of the bogie wheels, points B, C, D, and E, move along trajectories of radii OB, OC, OD, and OE, respectively. Perpendiculars to these radii, drawn from the centers of the wheels, points B, C, D, and E, will determine the directions of the velocity vectors of the centers of each of the bogie wheels, respectively, $\overline{V}_B$, $\overline{V}_C$, $\overline{V}_D$, and $\overline{V}_E$.

From the viewpoint of classical mechanics, the velocity vector of the center of each of the bogie wheels can be written as:

$$\begin{aligned} \overline{V}_B &= \overline{V}_A + \overline{V}_{BA} \\ \overline{V}_C &= \overline{V}_A + \overline{V}_{CA} \\ \overline{V}_D &= \overline{V}_F + \overline{V}_{DF} \\ \overline{V}_E &= \overline{V}_F + \overline{V}_{EF} \end{aligned} \tag{7}$$

where $\overline{V}_B$, $\overline{V}_C$, $\overline{V}_D$, and $\overline{V}_E$ are the velocity vectors of the centers of the wheels B, C, D, and E during the movement of the bogie along a curvilinear trajectory relative to the center of transport motion, point O; $\overline{V}_A$ and $\overline{V}_F$ are the velocity vectors of the centers of the inner bogie side, point A, and the outer bogie side, point F, respectively, during the movement along trajectories of radii R and $R + K_b$; $\overline{V}_{BA}$, $\overline{V}_{CA}$, $\overline{V}_{DF}$, and $\overline{V}_{EF}$ are the relative velocity vectors of the centers of the wheels B, C, D, and E relative to the centers of the bogie sides, point A and point F, respectively.

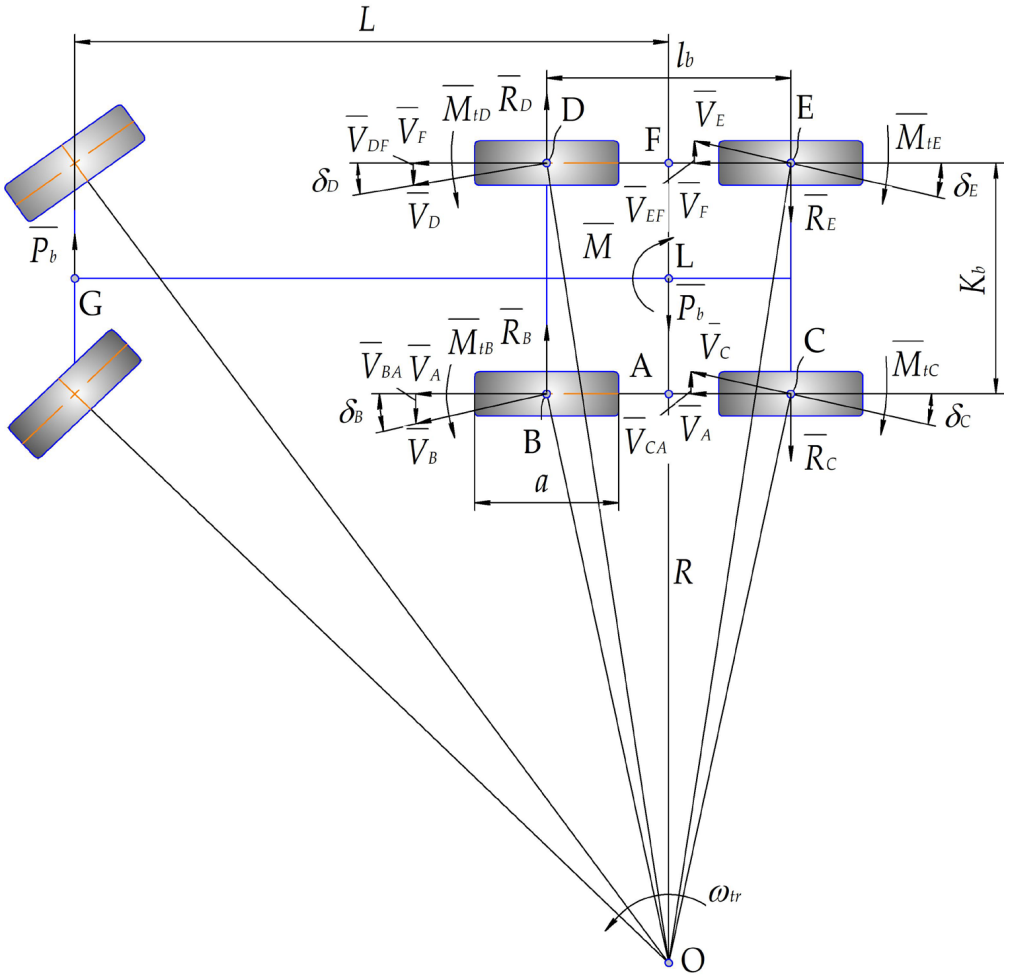


Figure 1. Scheme for determining the parameters of the bogie dynamics while moving along a curved trajectory.

The vectors of the centers of the inner, point A, and the outer, point F, bogie sides, according to Figure 1, are written as follows:

$$\overline{V}_A = R\overline{\omega}_{tr}$$
$$\overline{V}_F = (R + K_b)\overline{\omega}_{tr}$$

At the same time, the vectors of relative velocity of the wheel centers are determined by the product of the angular velocity of rotation of the bogie $\overline{\omega}_{tr}$ by the distance from the center of the bogie side to the center of the wheel. Since the bogie rotates with the same angular velocity as the vehicle, in this case, these vectors can be written as follows:

$$\begin{aligned} \overline{V}_{BA} &= \mathrm{AB}\overline{\omega}_{tr} \\ \overline{V}_{CA} &= \mathrm{AC}\overline{\omega}_{tr} \\ \overline{V}_{FD} &= \mathrm{FD}\overline{\omega}_{tr} \\ \overline{V}_{FE} &= \mathrm{FE}\overline{\omega}_{tr} \end{aligned} \tag{8}$$

where AB, AC, FD, and FE are the distances from the centers of the wheels B, C, D, and E to the centers of the bogie sides A and F, respectively, which, as an absolute value, is equal to half the bogie wheelbase $l_b/2$.

Note that the vectors $\overline{V}_{BA}$ and $\overline{V}_{FD}$ are perpendicular to the longitudinal axis of the vehicle and directed toward the center of transport motion of the vehicle, point O, and the vectors $\overline{V}_{CA}$ and $\overline{V}_{FE}$ are opposite to them.

Thus, the motion of the centers of wheels B and C of the inner bogie side consists of motion along a curvilinear trajectory of radius R and their motion relative to the center of point A along a radius of $l_b/2$. Similarly, the motion of the centers of the wheels D and E of the outer bogie side consists of motion along a curvilinear trajectory of radius $R + K_b$ and their motion relative to the center of point F along a radius of $l_b/2$.

Above, the movement of the centers of the bogie wheels along a curvilinear trajectory was considered without taking into account the tires. At the same time, according to [23], in the presence of an elastic body of the tire, the angle between the velocity vector of the center of each of the bogie wheels and the disk will be the slip angle with which the bogie wheel moves along the curvilinear trajectory. In this case, the bogie wheels with centers B, C, D, and E will move along a curvilinear trajectory with slip angles δ_B, δ_C, δ_D, and δ_E, respectively. From the analysis of Figure 1, it can be seen that the slip angle of the bogie wheel with center B will be determined from the analysis of ΔAOB. The angle AOB = δ_B as angles with mutually perpendicular sides. Considering that the slip angles do not exceed 10°, we can, with sufficient accuracy for practice, equate tgδ_B = δ_B (rad). In this case, δ_B = AB/AO (rad). Taking into account that AB = $l_b/2$ and AO = R, the slip angle at which the bogie wheel with center B will move, we determine by the expression:

$$\delta_B = \frac{57.3 l_b}{2R} \tag{9}$$

Considering that the radius of the trajectory of the bogie wheel with center C is OC = OB, then the slip angle at which the wheel with center C moves is determined as follows: $\delta_C = \delta_B = l_b/(2R)$ (rad).

The bogie wheels with centers D and E move along the trajectories of radii OD and OE, which, according to Figure 1, are equal to each other, i.e., OD = OE. Taking into account the above, the slip angles of these wheels are determined as follows:

$$\delta_D = \delta_E = \frac{57.3 l_b}{2(R + K_b)} \tag{10}$$

where δ_D and δ_E are the slip angles of the bogie wheels with the centers D and E, respectively, deg.

From the above, it follows that the movement of the bogie wheel along a curvilinear trajectory consists of movement along the trajectory of the center of the bogie side and additional plane-parallel movement with a slip angle, which is a function of half the bogie wheelbase and the radius of the trajectory of the center of the bogie side.

At the same time during the plane-parallel movement of an elastic wheel with slip, a lateral force and a self-aligning torque of the tire develop, which are functions of the slip angle. Provided that the slip angles do not exceed 5°, and only adhesion zones are present in the tire contact patch, the lateral force and the self-aligning torque of the tire, according to [10], are defined as follows:

$$P_s = k_s \delta \tag{11}$$

$$M_t = \frac{K_t C_\theta \theta_A}{3} \left[1 - \left(\frac{\delta}{\theta_A} - 1 \right)^2 \right] \tag{12}$$

where P_s is the lateral force acting on the bogie wheel during plane-parallel motion with slip, N; δ is the slip angle, which is determined by the expression (9) or (10) depending on the radius of the wheel motion trajectory, deg; k_s is the cornering stiffness of the bogie wheel, N/deg. Direct experimental determination of the cornering stiffness is associated with great difficulties, and therefore the studies presented in [22] showed that k_s, with sufficient accuracy for practice, is determined by the expression:

$$k_s = \frac{2C_\theta}{a} \tag{13}$$

where C_θ is the angular stiffness of the tire relative to the vertical axis, which is determined experimentally, N·m/deg. In the absence of experimental data, it is recommended to calculate the angular stiffness of the tire using the expression [24]:

$$C_\theta = (9 \ldots 11) \times 10^{-3} \times G_w \tag{14}$$

where G_w is the wheel load, N; a is the longitudinal axis of the tire contact patch, reduced to an equal-sized rectangle, m; K_t is the proportionality coefficient of the tire self-aligning torque, which takes into account the influence of longitudinal reactions and is determined experimentally. Its value is in the range of 1.05–1.32; θ_A is the largest steering angle in static, at which there is a conditionally linear relationship between the tire steering resistance torque in static and the steering angle.

If the slip angles determined by expressions (9) or (10) are within $5^\circ < \delta \leq 13^\circ$, and at the same time there are zones of adhesion and sliding in the tire contact patch, then the lateral force and self-aligning torque of the tire are determined as follows:

$$P_s = G_w\varphi - (G_w\varphi - k_s\theta_\mathrm{A})\left(\frac{\theta_\mathrm{B} - \delta}{\theta_B - \theta_A}\right)^2 \tag{15}$$

$$M_t = \frac{K_t C_\theta \theta_A}{3}\left[1 - \left(\frac{\delta - \theta_A}{\theta_B - \theta_A}\right)^2\right] \tag{16}$$

where θ_A and θ_B are angles characterizing the state of the tire contact patch (the presence of adhesion and sliding zones). On dry asphalt concrete surfaces, for truck tires, their values do not exceed $\theta_A \leq 5^\circ$ and $\theta_B \leq 13^\circ$. For car tires, their values on dry asphalt concrete are as follows: $\theta_A \leq 4^\circ$, $\theta_B \leq 10^\circ$; φ is the friction coefficient between the tire and the support surface.

At the slip angle $\delta \geq 13^\circ$ the lateral force reaches the maximum value limited by the adhesion forces $P_s = \varphi G_w$. As for the self-aligning torque of the tire, at the slip angle $\delta = 13^\circ$, it tends to zero, and at slip angles $\delta > 13^\circ$, it acquires negative values. There are currently no dependencies for its determination.

At the same time, during the movement of three-axle vehicles, for the bogie wheelbases in the range of 1.32–1.44 m, even when traveling along the minimum radius of the trajectory, as the analysis has shown, the slip angles do not exceed $\delta < 5^\circ$. Therefore, the lateral forces and self-aligning torques of the tires will be determined by expressions (11) and (12).

The lateral forces that develop during the movement of the bogie wheels along a curved trajectory generate reactions from the support surface in the contact patches of the tires in opposite direction to the vectors of lateral velocities $\overline{V}_{BA}$, $\overline{V}_{CA}$, $\overline{V}_{DF}$, and $\overline{V}_{EF}$. On the bogie wheels with centers B, C, D, and E, lateral reactions, respectively, $\overline{R}_B$, $\overline{R}_C$, $\overline{R}_D$, and $\overline{R}_E$ and the self-aligning torques of the tires generated by their displacement relative to the centers of the contact patches, respectively, $\overline{M}_{tB}$, $\overline{M}_{tC}$, $\overline{M}_{tD}$, and $\overline{M}_{tE}$ are developed. The vectors of lateral reactions $\overline{R}_B$, $\overline{R}_C$, $\overline{R}_D$, and $\overline{R}_E$ are directed opposite to the corresponding vectors of relative velocities $\overline{V}_{BA}$, $\overline{V}_{CA}$, $\overline{V}_{DF}$, and $\overline{V}_{EF}$, which will affect the direction of action of the self-aligning torques of the tires. Thus, the self-aligning torques of the tires of the bogie wheels with centers B and D are directed counterclockwise, and those of the wheels with centers C and E are directed clockwise. Since the slip angles of the wheels $\delta_B = \delta_C$, and $\delta_D = \delta_E$, then the self-aligning torques of the tires $M_{tB} = M_{tC}$ and $M_{tD} = M_{tE}$, respectively. In this case, the resulting self-aligning torque of the tires during the movement of the bogie along a curved trajectory, taking into account the above, is zero.

At the same time, the lateral reactions acting on the bogie wheels R_B and R_D are directed oppositely to the reactions R_C and R_E, are parallel to each other, and act at a

distance of the bogie wheelbase, generating bogie turning resistance moment while moving along a curved trajectory, which we will determine by the expression:

$$M = (R_B + R_D)l_b \tag{17}$$

Considering that the slip angles of three-axle vehicles do not exceed 5°, the reactions R_B and R_D are defined as follows:

$$R_B = \frac{57.3}{2} k_s l_b \left(\frac{1}{R}\right) \tag{18}$$

$$R_D = \frac{57.3}{2} k_s l_b \left(\frac{1}{R + K_b}\right) \tag{19}$$

After the appropriate substitution of the values of the reactions R_B and R_D into expression (17), we obtain the value of the bogie turning resistance moment:

$$M = \frac{57.3 k_s l_b^2}{2} \left(\frac{1}{R} + \frac{1}{R + K_b}\right) \tag{20}$$

From the analysis of dependence (20), it follows that when the bogie moves along a curved trajectory, a moment about the vertical axis occurs due to the lateral reactions of the support surface, proportional to the cornering stiffness of the bogie wheels, the square of the bogie wheelbase, and the curvature of the trajectory of the bogie wheels.

This moment generates a force on the steering axle perpendicular to the vehicle frame. Provided that the slip angles of the bogie wheels while moving along the curved trajectory of the vehicle do not exceed 5°, this force is determined by the expression:

$$P_b = \frac{57.3 k_s l_b^2}{2L} \left(\frac{1}{R} + \frac{1}{R + K_b}\right) \tag{21}$$

where P_b is the force on the steering axle generated by the movement of the bogie along a curved trajectory, N.

If the slip angles δ_B and δ_D, calculated according to expressions (9) and (10), exceed 5° and are in the range from 5 to 13°, then the reactions R_B and R_D, taking into account dependence (15), are determined as follows:

$$R_B = G_w \varphi - (G_w \varphi - k_s \theta_A) \left(\frac{\theta_B - \delta_B}{\theta_B - \theta_A}\right)^2 \tag{22}$$

$$R_D = G_w \varphi - (G_w \varphi - k_s \theta_A) \left(\frac{\theta_B - \delta_D}{\theta_B - \theta_A}\right)^2 \tag{23}$$

Taking into account the above bogie, the turning resistance moment is determined by the expression:

$$M = \left[2(G_w \varphi - (G_w \varphi - k_s \theta_A)) \left[\left(\frac{\theta_B - \delta_B}{\theta_B - \theta_A}\right)^2 + \left(\frac{\theta_B - \delta_D}{\theta_B - \theta_A}\right)^2\right]\right] l_b \tag{24}$$

At the same time, a force acts on the steering axle, which is determined by the expression:

$$P_b = \frac{M}{L} = \left[2(G_w \varphi - (G_w \varphi - k_s \theta_A)) \left[\left(\frac{\theta_B - \delta_B}{\theta_B - \theta_A}\right)^2 + \left(\frac{\theta_B - \delta_D}{\theta_B - \theta_A}\right)^2\right]\right] \frac{l_b}{L} \tag{25}$$

As for the cornering stiffness of the bogie wheels, its value significantly depends on the design features of the tire and the load on it. Increasing the load on the bogie wheels will cause an increase in the cornering stiffness and, accordingly, the force on the steering axle. At the same time, increasing the vehicle wheelbase will lead to a decrease in this force, but will cause an increase in the minimum turning radius of the vehicle.

2.2. Study of Steering Wheel Angles

In general, the force generated by the movement of the bogie along the curved trajectory P_b acts on the steering axle, causing a change in the dynamic and kinematic parameters of the steered wheels. Such a change in the kinematic parameters of the steered wheels will affect the value of the minimum radius of the vehicle's trajectory. Since the kinematics of the steered wheels depends not only on the design and stiffness parameters of the steering trapezoid, but also on the direction of turning of the steered wheels, we will consider its influence on the steering angles.

Figure 2 shows a scheme for studying the steering angles of the steered wheels of an all-wheel drive vehicle in the case of a left-hand placement of the steering wheel while turning to the left.

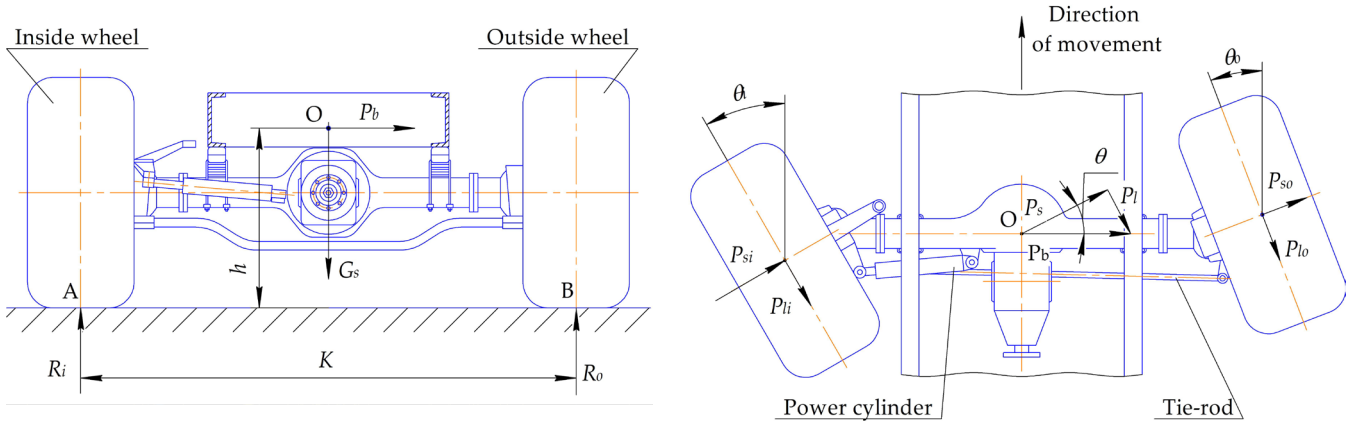


Figure 2. Scheme of kinematics and dynamics of steered wheels when turning to the left: (**a**) Rear view; (**b**) Top view.

From the analysis of Figure 2, it is seen that the left steered wheel turned inside to the center is turned directly by the power cylinder to the maximum angle θ_i and limited by the stop of the wheel disk in the limiter. This position of the left wheel hard disk does not change during movement. The steering angle of the right θ_o, outside to the turn center steered wheel, will be formed by the kinematics of the steering trapezoid and the influence of the bogie. The force P_b is applied to the center of the frame above the steering axle at point O, generated by the movement of the bogie wheels along a curvilinear trajectory, which is perpendicular to the longitudinal axis of the frame. If the average steering angle $\theta = (\theta_i + \theta_o)/2$, then the force P_b is decomposed into two forces that directly act on the steered wheels. The lateral force P_s and the longitudinal force P_l (Figure 2b) are defined as follows:

$$P_s = P_b \cos\theta \tag{26}$$

$$P_l = P_b \sin\theta \tag{27}$$

The lateral force P_s causes the steered wheels to move with slip and leads to the redistribution of the load on the steered wheels. If we do not take into account the effect of the normal stiffness of the suspension and tires, assuming with sufficient accuracy for practice that their effect is not significant, then, according to Figure 2a, the normal reactions on the steered wheels are defined as follows:

$$R_o = \frac{G_s}{2} + P_s \frac{h}{K} \tag{28}$$

$$R_i = \frac{G_s}{2} - P_s \frac{h}{K} \tag{29}$$

where R_o and R_i are the normal reactions, respectively, on the outside and inside steered wheel, m; G_s is the load on the steering axle, N; h is the distance from the support surface to the center of the frame above the steering axle, m; K is the track of the steered wheels, m.

The lateral force on the steered wheels P_s will cause them to move with a slip angle, which we define as follows:

$$\delta_s = \frac{P_s}{k_{si} + k_{so}} \tag{30}$$

where δ_s is the slip angle of the steered wheels developed by the lateral force during the movement of the bogie along a curved trajectory, deg; k_{si} and k_{so} are the cornering stiffness of the inside and outside steered wheels, respectively, N/deg.

Therefore, the lateral force generated by the movement of the bogie along a curved trajectory causes the steered wheels to move with slip and increase the load on the outside wheel.

As for the longitudinal force P_l, it will generate a twisting moment about the kingpin axis, which is determined by the expression:

$$M_{tw} = \frac{P_l}{2} l_k \cos(\alpha_k + \gamma_{k0}) \tag{31}$$

where M_{tw} is the twisting moment on the steered wheel tire about the kingpin axis, generated by the longitudinal force, N·m; l_k is the spindle length, m; α_k is the kingpin inclination angle, deg; γ_{k0} is the camber angle when moving in a straight line ($\theta = 0$), deg.

The moment M_{tw} is applied to the outside wheel about the kingpin axis will cause, due to the deformation of the steering trapezoid and the twisting of the tire relative to the hard disk, a decrease in the steering angle of the outside wheel θ_o. At the same time, the value of the tire twisting angle, which will lead to a decrease in the steering angle of the outer wheel, is determined as follows:

$$\Delta\theta_t = \frac{M_{tw}}{C_\theta} \tag{32}$$

where $\Delta\theta_t$ is the angle developed by the tire twisting relative to the disk, deg; C_θ is the angular stiffness of the tire of the outside steered wheel relative to the vertical axis, N·m/deg.

If the steering trapezoid is deformed by the moment M_{tw}, the steering angle of the outside wheel will decrease by the value:

$$\Delta\theta_{tr} = \frac{M_{tw}}{C_{tr}} \tag{33}$$

where $\Delta\theta_{tr}$ is the reduction in the steering angle of the outside wheel due to the deformation of the steering trapezoid; C_{tr} is the stiffness of the steering trapezoid, N·m/deg, which is determined by the experimental graph $\Delta\theta_{tr} = f(M_{tw})$.

So, in the case of a left turn, the actual steering angle of the right outside wheel while moving along a curved trajectory is determined by the expression:

$$\theta_o = \theta_{omax} - (\delta_s + \Delta\theta_t + \Delta\theta_{tr}) \tag{34}$$

where θ_{omax} is the maximum steering angle of the outside steered wheel when turning left, which is determined experimentally for a vehicle when installing steered wheels on self-centering platforms, or from the vehicle's technical specifications.

The moment M_{tw} on the inside steered wheel will only cause the tire to twist relative to the disk, the position of which is fixed by the stop in the limiter. In general, the steering angle to the left of the inside wheel θ_i, according to Figure 2, is determined by the expression:

$$\theta_i = \theta_{imax} - (\delta_s - \Delta\theta_t) \tag{35}$$

where θ_{imax} is the maximum steering angle of the inside steered wheel when turning left, which is determined experimentally for a vehicle when installing steered wheels on self-centering platforms or from the vehicle's technical specifications.

According to Figure 1, the force P_b acts on the bogie wheels, directed toward the turn center, point O, which will cause the bogie wheels to move with slip angle:

$$\delta_b = \frac{P_b}{4k_s} \tag{36}$$

where δ_b is the slip angle of the bogie wheels caused by the force P_b, deg; k_s is the cornering stiffness of the bogie wheel, N/deg.

Therefore, when turning left, the steering angle of the outside wheel θ_o will significantly decrease, and the inside wheel will remain almost unchanged, and therefore, with sufficient accuracy for practice, the minimum turning radius when turning left will be determined by the steering angle of the outside wheel θ_o and the slip angle of the bogie wheels δ_b.

2.3. Minimum Turning Radius

Taking into account the above, the scheme for calculating the minimum radius of a left turn is shown in Figure 3.

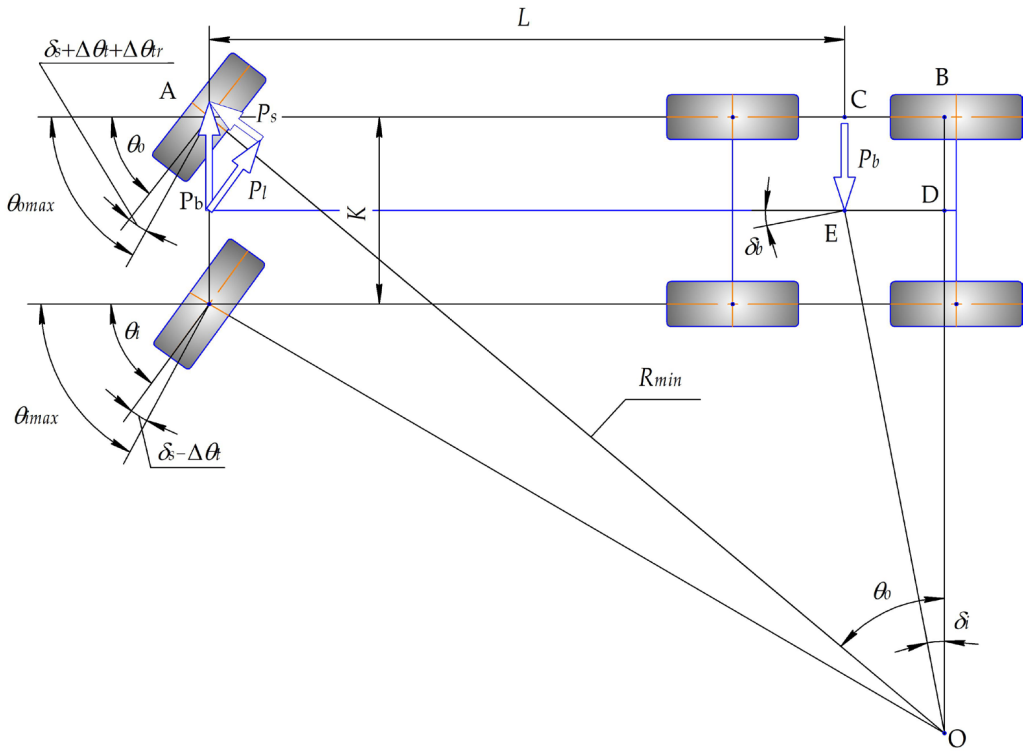


Figure 3. Scheme for calculating R_{min} when turning left.

From the analysis of ΔABO, we determine the minimum turning radius, taking into account AO = R_{min}, as follows:

$$\mathrm{AO} = \frac{\mathrm{AB}}{\sin\theta_o} \tag{37}$$

According to Figure 3, the value of AB is written as follows: AB = AC + BC. In this case, AC = L, and the value of BC = DE is determined from the analysis of ΔODE by the expression:

$$\mathrm{DE} = \mathrm{OD}\mathrm{tg}\delta_b$$

According to Figure 3 OD = OB − BD =OB − $K/2$, where K is the track of the steered wheels.

From the analysis of ΔOAB, taking into account AB = L + ODtgδ_b, we can write:

$$\text{OB} = \text{ABctg}\theta_o \tag{38}$$

Substituting OB = OD + $K/2$, AB = L + ODtgδ_b into Equation (38), we obtain:

$$\text{OD} + \frac{K}{2} = (L + \text{ODtg}\delta_b)\text{ctg}\theta_o \tag{39}$$

The unknown value OD is determined by analyzing Equation (39) using the expression:

$$\text{OD} = \frac{L\text{ctg}\theta_o - \frac{K}{2}}{1 - \text{ctg}\theta_o\text{tg}\delta_b} \tag{40}$$

After elementary substitutions in expression (37), we obtain the dependence for determining the minimum radius of a vehicle's left turn:

$$R_{min} = \left[L + \left(\frac{L\text{ctg}\theta_o - \frac{K}{2}}{1 - \text{ctg}\theta_o\text{tg}\delta_b}\right)\text{tg}\delta_b\right]\frac{1}{\sin\theta_o} \tag{41}$$

where θ_o is the steering angle of the outside steered wheel, taking into account the effect of the bogie on its value, which we will determine by expression (34); δ_b is the slip angle of the bogie wheels, determined by expression (36).

The scheme for determining the kinematics of the steered wheels when turning to the right is shown in Figure 4.

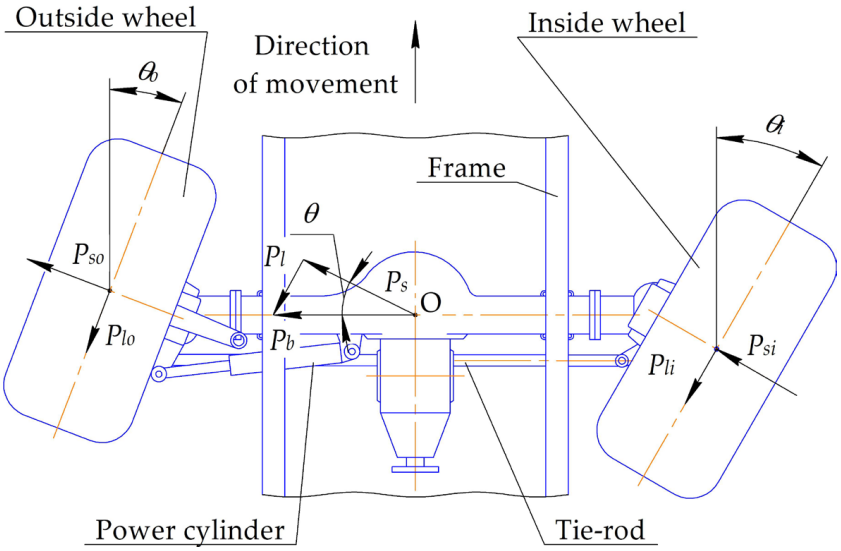


Figure 4. Scheme of kinematics and dynamics of steered wheels when turning right.

From the analysis of Figure 4 it follows that when turning right, the hard disk of the right (inside) wheel is turned by the power cylinder through the steering trapezoid until it stops in the limiter. While driving, this steering angle is provided by overturning the left (outside) steered wheel.

The right (inside) fully turned steered wheel is subjected to forces similar to those of the left wheel when turning left, which cause the tire to roll with slip and twist relative to the disk. In this case, the slip angle and the twisting angle of the tire relative to the disk have opposite directions, and their resulting value is equal to the difference in these angles.

The movement of the left (outside) steered wheel during a right turn due to the action of lateral and longitudinal forces on it generated by the movement of the bogie along a

curved trajectory occurs with a slip angle and twisting of the tire relative to the disk, which has the same direction and reduces its steering angle. To ensure the turning of the wheel disk during a right turn to the stop in the limiter, the power cylinder, acting on the steering trapezoid, must compensate for this angle by turning the left wheel.

Therefore, when turning to the right, the steering angle of the outside steered wheel will increase. As a result, increasing the steering angle of the outside wheel when turning to the right will result in a smaller minimum turning radius for the vehicle to turn right relative to turning to the left.

From the analysis of the above, it the following can be concluded:

- During the movement of a three-axle vehicle along a curved trajectory, a force perpendicular to the vehicle frame acts from the bogie on the steering axle, which we will decompose into lateral and longitudinal.
- The lateral force causes the steered wheels to move with a slip, increases the load on the outside steered wheel, and decreases it on the inside one.
- The longitudinal force on each steered wheel generates a twisting moment relative to the kingpin axis, the effect of which on the kinematics of the steered wheels depends on their direction of turning, stiffness, and kinematics of the steering trapezoid as a spatial four-link mechanism.
- When turning left, the moment M_{tw} applied to the outside (right) wheel relative to the kingpin axis will cause, due to the deformation of the steering trapezoid and the elasticity of the tire, a decrease in the steering angle of the outside wheel. At the same time, the steering angle of the inside steered wheel is limited by the stop, and the twisting moment deforms the tire in the direction of increasing its steering angle.
- When turning right, the inside (right) steered wheel is subjected to a force through the steering trapezoid from the outside (left) steered wheel, which provides its maximum steering angle, limited by the stop. At the same time, the twisting moment due to the elasticity of the tire increases the steering angle. The outside (left) steered wheel is subjected to a twisting moment, which due to the elasticity of the tire reduces the steering angle. However, to ensure that the inside wheel turns to a stop, the steering angle of the outside wheel will be increased by the power cylinder of the power assist.

Thus, when turning left, the lateral and longitudinal forces generated by the movement of the bogie along a curved trajectory will cause the steered wheels to move with a slip and change the steering angle and load on the outside steered wheel, which will lead to an increase in the minimum turning radius of the vehicle. When turning right, the steering angles will not change significantly and the minimum turning radius to the right will be smaller compared to turning left.

At the same time, the front and rear axles of the bogie will move, according to Figure 1, with different slip angles, since the slip angle of the wheels of the front bogie axle is the sum of the angles from the plane-parallel movement with slip and the slip angle from the action of the force P_b. As for the slip angle of the wheels of the rear axle, it is equal to the difference in these angles. Based on the above, the slip angles of the bogie wheels during movement along a curved trajectory are determined by the expressions:

$$\sum \delta_f = \frac{(\delta_B + \delta_D)}{2} + \delta_b \tag{42}$$

$$\sum \delta_r = \frac{(\delta_C + \delta_E)}{2} - \delta_b \tag{43}$$

where $\Sigma\delta_f$ and $\Sigma\delta_r$ are the slip angles of the wheels of the front and rear axles of the bogie, respectively; δ_B, δ_D, δ_C, and δ_E are the slip angles of the bogie wheels with centers B, D, C,

and E, which are calculated according to expressions (9) and (10); δ_b is the slip angle of the bogie wheels developed by the force P_b, which is calculated according to expression (36).

From the above, it follows that the wheels of the front bogie axle move with a larger slip angle relative to the wheels of the rear bogie axle.

Based on the results of the research, a mathematical model of the movement of a bogie wheel along a curvilinear trajectory and a dependence for calculating the minimum turning radius of a three-axle vehicle were obtained.

3. Results

3.1. Initial Data of Research Objects

From the analysis of the above, it follows that when a three-axle vehicle moves along a curved trajectory, the steering angle of the outside steered wheel is formed by the structural and elastic parameters of the steering trapezoid, the slip angle, and the twisting of the tire relative to the disk. All these will lead to a decrease in the steering angle of the outside steered wheel when turning left. At the same time, the load on this wheel is increased relative to the inside one, and therefore its influence on the minimum turning radius of the vehicle will be decisive. As for turning right, the steering angle of the outside steered wheel will increase to ensure stable support of the disk of the right in-side wheel in the limiter.

To confirm the reliability of analytical dependencies that determine the features of the movement of the elastic bogie wheel along a curvilinear trajectory and the minimum turning radius, experimental studies of the minimum turning radius of three-axle vehicles with a raised and lowered intermediate axle were carried out. The all-wheel-drive serial three-axle vehicle KrAZ–6322–02 truck (JSC "AutoKrAZ", Kremenchuk, Ukraine) and the experimental non-all-wheel-drive three-axle vehicle KrAZ truck (JSC "AutoKrAZ", Kremenchuk, Ukraine) with a wheel formula of 6 × 4 (hereinafter referred to as KrAZ–6 × 4) were chosen as the object of research. The vehicles are equipped with identical steering axles with a power cylinder on the axle beam and with CV joints, the design of which ensures stable maximum steering angles during experimental studies of the minimum turning radius. The vehicles differ in tires: the all-wheel-drive vehicle KrAZ–6322–02 truck has tires with a wide cross-section and regulated pressure with single tires on the bogie wheels, and the non-all-wheel-drive KrAZ–6 × 4 truck has high-pressured tires with dual tires on the bogie wheels. The vehicles differ in their wheelbases: the KrAZ–6322–02 truck has a wheelbase of 5.3 m and the KrAZ–6 × 4 truck has a wheelbase of 5.7 m. The vehicles differ in their transmission: on the KrAZ–6322–02 truck, an interaxial asymmetric differential is installed between the steering axle and the bogie wheels, on the KrAZ–6 × 4 truck, the steering axle is non-driven, and the interaxial symmetric differential is installed between the intermediate and rear axles.

Experimental studies were conducted on a flat, dry, horizontal surface with asphalt concrete pavement at an ambient temperature of +18–22 °C with the steered wheels turned to the left and right for three states of each vehicle. The first state is a two-axle vehicle without a load, in which the intermediate axle is raised using spacers between the rear axle beam and the frame spars. The second state is a three-axle vehicle without a load, and the third state is a three-axle vehicle with a load.

Figure 5 shows a scheme of lifting the intermediate axle using spacers installed between the rear axle beam and the frame spars.

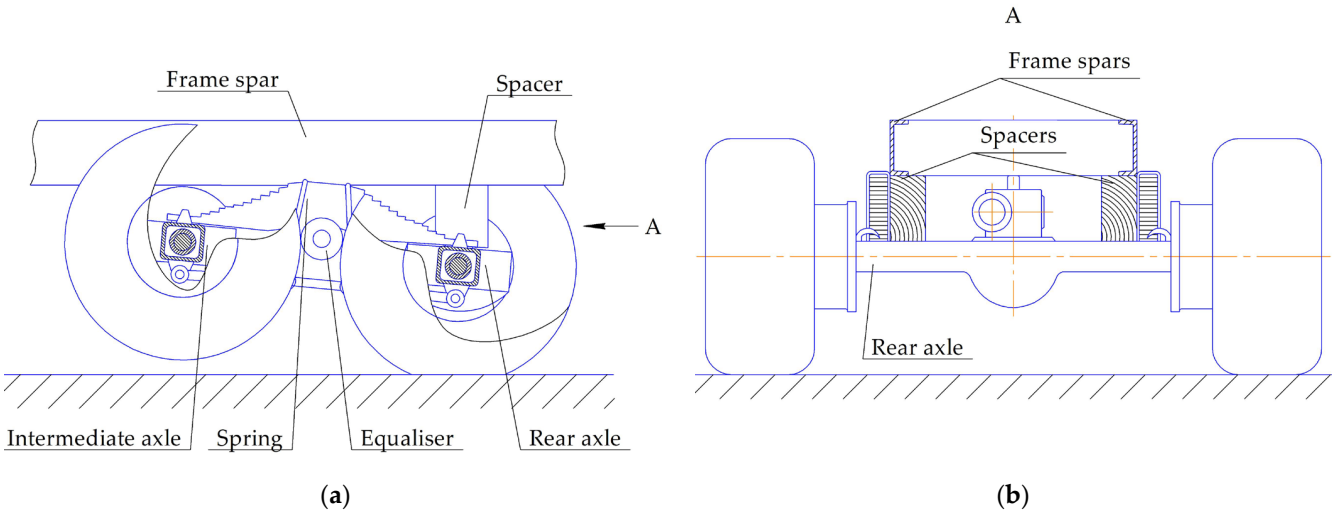


Figure 5. Scheme of lifting the middle axle: (**a**) side view; (**b**) rear view.

From the analysis of Figure 5, it can be seen that two spacers are installed between the rear axle beam and the frame spars, which, in the presence of a leaf spring of the equalizer suspension, when the rear axle is lowered onto the support surface, ensure the lifting of the intermediate axle. In this case, the spring together with the equalizer rotates relative to the equalizer axis and raises the intermediate axle above the support surface. In this case, a three-axle vehicle with a raised intermediate axle becomes a two-axle vehicle. For a KrAZ–6322–02 truck, the wheelbase increases by 0.7 m (half the bogie wheelbase) and is 5.3 m + 0.7 m = 6.0 m. For a KrAZ–6 × 4 truck with a base of 5.7 m in a two-axle state, the wheelbase is 5.7 m + 0.7 m = 6.4 m.

Since the structural and elastic parameters of the steering trapezoid affect the steering angles, and the minimum turning radius of the vehicle is determined when the inside steered wheel is turned to the maximum angle, the maximum steering angles were experimentally determined.

During experimental studies of maximum steering angles, the steered wheels were jacked up, which minimized the deformation of the steering trapezoid due to the tire steering resistance torque and the weight aligning torque caused by the kingpin inclinations. Analysis of experimental studies showed that when turning left, the maximum steering angle of the left wheel was 32.2° and of the right wheel was 26.6°. When turning right, the maximum steering angle of the right wheel was 32.1° and of the left wheel was 26.7°.

Figure 6 shows a KrAZ–6322–02 truck with the intermediate axle raised during experimental studies of the minimum turning radius.



Figure 6. Experimental studies of KrAZ–6322–02 truck with raised intermediate axle.

The presence of the twisting moment M_{tw} on the outside steered wheel when turning left with elastic deformation of the steering trapezoid will lead to a decrease in the steering angle of the outside wheel, and when turning right to an increase in the steering angle of the outside wheel. In this case, the value of this angle will depend on the nature of the change in the steering trapezoid elasticity curve depending on the value of the twisting moment. Since the minimum turning radius of the vehicle was determined at the maximum steering angle of the inside wheel, experimental studies of the trapezoid stiffness were carried out at this steering angle. The outside wheel was installed on a self-centering platform, and the inside one rested on a stand with a high coefficient of adhesion and was fixed in the extreme position with a special bracket. A force was applied to the outside steered wheel through a lever with a length of 3.05 m and a dynamometer attached to it. For each value of the applied moment M_{tw}, the change in the steering angle of the wheel $\Delta\theta_{tr}$ was determined, which is given in Table 2.

Table 2. Results of studies of steering trapezoid stiffness.

M_{tw}, N×m	$\Delta\theta_{tr}$, deg	M_{tw}, N×m	$\Delta\theta_{tr}$, deg
0	0	610	0.82
152.5	0.27	915	1.03
305	0.5	1220	1.15

According to the data in Table 2, Figure 7 shows the experimental dependence $\Delta\theta_{tr} = f(M_{tw})$.

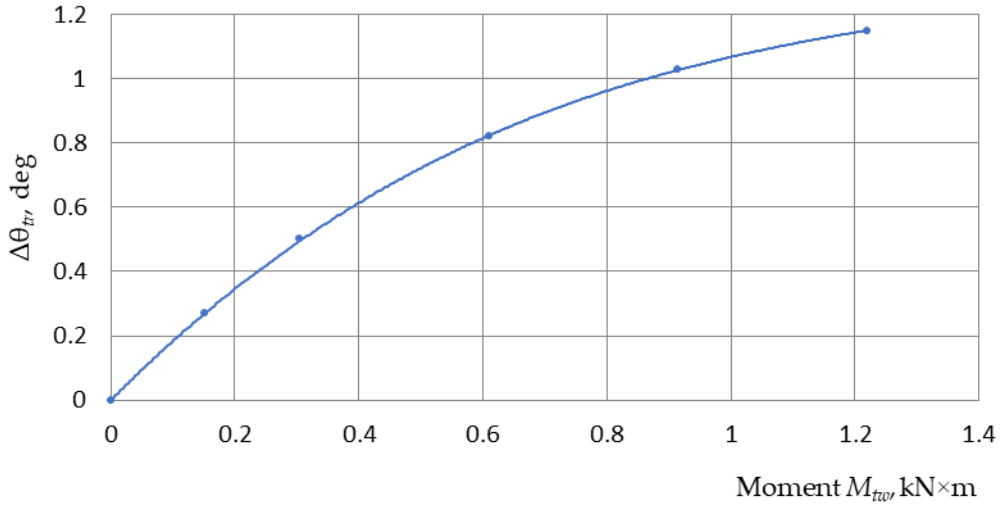


Figure 7. Dependence of $\Delta\theta_{tr} = f(M_{tw})$.

From the analysis of Figure 7, it can be seen that depending on the moment applied to the wheel during the deformation of the steering trapezoid, the steering angle of the outside wheel smoothly changes, which was taken into account in further studies of the minimum turning radius of the vehicle.

Taking into account the research of the parameters of the kinematics of the steered wheels, Table 3 shows the input data of the KrAZ–6322–02 truck with the intermediate axle raised and lowered, taking into account the load.

Table 3. Input data of the KrAZ–6322–02 truck.

Truck Characteristics	Truck State		
	Two-Axle Without Load	**Three-Axle Without Load**	**Three-Axle With Load**
Truck weight, kg	12,220	12,220	23,870
Load on the steering axle, N	62,800	62,800	68,500
Load on the bogie (rear axle), N	59,400	59,400	170,200
Truck wheelbase, m	6.0	5.3	5.3
Bogie wheelbase, m	–	1.4	1.4
Wheel track: bogie (rear axle), m steering axle, m	 2.16 2.16	 2.16 2.16	 2.16 2.16
Maximum steering angles: outside wheel θ_{omax}, deg inside wheel θ_{imax}, deg	 26.6 32.2	 26.6 32.2	 26.6 32.2
Tires: model inflation pressure, MPa bogie wheel tire	 Dneproshina 21.5/75R21 model ID-370 (Dneproshina Ltd., Dnipro, Ukraine) 0.545 single		
Dimensions of tire contact patches, m: steered wheel non-steered wheel	 a = 0.217; b = 0.393 a = 0.212; b = 0.397	 a = 0.217; b = 0.393 a = 0.18; b = 0.335	 a = 0.232; b = 0.405 a = 0.25; b = 0.414
Angular stiffness of tires C_θ, N·m/deg (14): steered wheel non-steered wheel	 345.4 326.7	 345.4 163.4	 376.8 468.0
Cornering stiffness k_s, N/deg (13): steered wheel, non-steered wheel	 3183 3082	 3183 1816	 3248.3 3744
The distance from the support surface to the center of the frame h, m	0.86	0.86	0.86

At the same time, the cornering stiffness and the stiffness of the tires were determined by empirical dependencies (13) and (14) taking into account the actual load on them. The tire contact patches were determined experimentally and reduced to equal-sized rectangles.

The input data of the KrAZ–6 × 4 truck are given in Table 4.

Table 4. Input data of the KrAZ–6 × 4 truck.

Truck Characteristics	Truck State		
	Two-Axle Without a Load	**Three-Axle Without a Load**	**Three-Axle With Load**
Truck weight, kg	13,780	13,780	25,800
Load on the steering axle, N	59,400	59,400	59,600
Load on the bogie (rear axle), N	78,400	78,400	198,400
Truck wheelbase, m	6.4	5.7	5.7
Bogie wheelbase, m	–	1.4	1.4

Table 4. *Cont.*

Truck Characteristics	Truck State		
	Two-Axle Without a Load	**Three-Axle Without a Load**	**Three-Axle With Load**
Wheel track:			
bogie (rear axle), m	1838	1838	1838
steering axle, m	2070	2070	2070
Maximum steering angles:			
outside wheel θ_{omax}, deg	26.6	26.6	26.6
inside wheel θ_{imax}, deg	32.2	32.2	32.2
Tires:			
model	KAMA-201 12.00R20 (KAMA TYRES Ltd., Nizhnekamsk, Russia)		
inflation pressure, MPa	0.58 MPa		
bogie wheel wheel	dual		
Dimensions of tire contact patches, m:			
steered wheel,	a = 0.264; b = 0.232	a = 0.264; b = 0.232	a = 0.265, b = 0.233
non-steered dual wheel	a = 0.219; b = 0.541	a = 0.153; b = 0.536	a = 0.246; b = 0.546
Angular stiffness of tires C_θ, N·m/deg (14):			
steered wheel	297	297	298
non-steered dual wheel	392	196	496
Cornering stiffness k_s, N/deg (13):			
steered wheel;	2249	2249	2250
non-steered dual wheel.	3580	2562	4032.5
Distance from the support surface to the center of the frame h, m	0.85	0.85	0.85

3.2. Analysis of Experimental Research Results

Experimentally, the minimum radius of the vehicle's trajectory was determined by the trajectory of the outside steered wheel on dry asphalt concrete along the track left by the steered wheel during movement. The power cylinder on the beam of the steering axle provided the maximum steering angles by turning the inside wheel all the way to the stop in the limiter, the functions of which were performed by a disk CV joint. The steering angle of the outside steered wheel was formed by the kinematic and elastic parameters of the steering trapezoid.

Knowing the minimum turning radius of a two-axle vehicle and its wheelbase, this turning radius was subsequently calculated for the wheelbase of a three-axle vehicle, assuming that the steering angles do not change. This allowed us to experimentally determine the influence of the bogie on the minimum turning radius for identical bases of two-axle and three-axle vehicles.

The results of experimental and calculated values of the minimum turning radii for two states of the KrAZ–6322–02 truck when turning left are given in Table 5. At the same time, for a two-axle vehicle with a base of 5.3 m, the calculated values of the minimum radius are given, based on the experimental data of this vehicle with a base of 6.0 m.

The turning radii of the sides of the bogie R and $R + K_b$ and the steering angles θo and θi were determined according to well-known dependencies, knowing the turning radius of the outside steered wheel, the vehicle wheelbase, and the track of the steered wheels and bogie wheels.

Table 5. Experimental and calculated data of the KrAZ–6322–02 truck when turning left.

Experimental and Calculated Data	Truck State			
	Two-Axle Without Load		Three-Axle Without Load	Three-Axle With Load
	L = 6.0 m	L = 5.3 m	L = 5.3 m	L = 5.3 m
Minimum turning radius (experiment) R_{min}, m	13.4	11.84	12.5	13.15
Turning radii of the bogie sides, m:				
inner side R	9.82	8.427	9.16	9.96
outer side $R + K_b$	11.98	10.587	11.32	12.03
Bogie turning resistance moment M, N·m (20):	–	–	20,136	38,579
Force on the steering axle P_b, N (21)			3799.2	7279
Calculated values of steering angles $\theta = f(R_{min})$, deg:				
outside wheel θ_o	26.6	26.58	25.09	23.76
inside wheel θ_i	31.4	32.16	30.05	28.21
average value θ	29.0	29.37	27.57	25.98
Lateral force on steered wheels P_s, N (26)	–	–	3368	6440
Slip angle of steered wheels δ_s, deg (30)	–	–	0.529	1.007
Load on the steered wheel, N:				
outside (28)	31,400	31,400	32,907	36,846
inside (29)	31,400	31,400	29,895	31,654
Longitudinal force on steered wheels P_l, N (27)	–	–	1759	3188.6
Twisting moment on the steered wheel M_{tw}, N·m (31)	–	–	195.4	351.5
Tire twisting angle $\Delta\theta_t$, deg (32)	–	–	0.56	0.933
Reduction in the steering angle of the outside wheel due to the deformation of the steering trapezoid $\Delta\theta_{tr}$, deg	–	–	0.3	0.53
The total decrease in the steering angle of the outside wheel $\Sigma\theta = \delta_s + \Delta\theta_t + \Delta\theta_{tr}$, deg	–	–	1.389	2.47
Steering angle of the outside wheel $\theta_o = \theta_{omax} - \Sigma\theta$, deg	26.6	26.58	25.183	24.1
Slip angle of the bogie wheels δ_b, deg (36)	–	–	0.52	0.48
Minimum turning radius R_{min}, m (41)	13.4	11.84	12.67	13.02
Relative error, %	0	0	1.36	0.98

From the analysis of Table 5, it can be seen that the steering angle of the outside wheel is formed by the parameters of the steering trapezoid, the slip angle, the deformation of the steering trapezoid, and the twisting of the tire from the moment generated by the longitudinal component. As for the left (inside) wheel when turning left, the slip angle and the twisting angle of the tire are directed opposite to each other, and the difference between them tends to zero. At the same time, the outside (right) steered wheel is turned by a smaller angle relative to the theoretically required one, the load on this wheel is greater relative to the inside one, and therefore, it will have a decisive influence on the value of the minimum turning radius.

Analysis of the results of the calculated and experimental data showed their coincidence, which confirms the reliability of the obtained results in determining the features of the movement of the bogie wheels along a curvilinear trajectory.

From the analysis of Table 5, it can be seen that the bogie when turning left increases the minimum turning radius of the loaded KrAZ–6322–02 truck by 1.31 m compared to a two-axle vehicle with equal wheelbases.

Table 6 shows the experimental values of the minimum turning radii of the KrAZ–6322–02 truck on dry asphalt concrete to the left and right.

Table 6. Minimum turning radii of the KrAZ–6322–02 truck.

Truck State	Truck Weight, kg	Direction of Turning	Minimum Turning Radius R_{min}, m
Three-axle without load	12,220	Left	12.5
		Right	11.95
Three-axle with load	23,870	Left	13.15
		Right	12.35

From the analysis of Table 6, it follows that when turning right, the minimum turning radius decreases for an empty vehicle to 11.95 m, and for a loaded vehicle to 12.35 m compared to turning left, which was 12.5 m and 13.15 m, respectively.

Table 7 presents experimental and calculated data for the KrAZ–6 × 4 truck, which has dual wheels installed on the bogie, and the truck wheelbase is 5.7 m.

From the analysis of Table 7, it can be seen that the bogie when turning left increases the minimum turning radius of the loaded vehicle by 1.61 m compared to a two-axle vehicle with equal wheelbases. At the same time, the load on the bogie leads to an increase in the minimum radius from 13.7 m to 14.35 m.

Table 7. Experimental and calculated data of the KrAZ–6 × 4 truck when turning left.

Experimental and Calculated Data	Truck State			
	Two-Axle Without Load		Three-Axle Without Load	Three-Axle With Load
	L = 6.4 m	L = 5.7 m	L = 5.7 m	L = 5.7 m
Minimum turning radius (experiment) R_{min}, m	14.3	12.74	13.7	14.35
Turning radii of the bogie sides, m:				
inner side R	10.834	9.556	10.16	11.22
outer side $R + K_b$	12.672	11.39	12.0	13.05
Bogie turning resistance moment M, N·m (20)	–	–	26,145	36,990
Force on the steering axle R_b, N (21)	–	–	4586	6489
Calculated values of steering angles $\theta = f(R_{min})$, deg:				
outside θ_o	26.6	26.58	25.5	23.4
inside θ_i	30.9	30.5	28.6	26.7
average value θ	28.7	28.54	26.7	25.05
Lateral force on steered wheels P_s, N (26)	–	–	4097	5878
Slip angle of steered wheels δ_s, deg (30)	–	–	0.91	1.306
Load on the steered wheel, N:				
outside (28)	29,700	29,700	31,402	32,142
inside (29)	29,700	29,700	27,998	27,260
Longitudinal force on steered wheels P_l, N (27)	–	–	2060	2747
Twisting moment on the steered wheel M_{tw}, N·m (31)	–	–	227	302.85
Tire twisting angle $\Delta\theta_t$, deg (32)	–	–	0.764	1.02

Table 7. *Cont.*

Experimental and Calculated Data	Truck State			
	Two-Axle Without Load		Three-Axle Without Load	Three-Axle With Load
	L = 6.4 m	L = 5.7 m	L = 5.7 m	L = 5.7 m
Reduction in the steering angle of the outside wheel due to the deformation of the steering trapezoid $\Delta\theta_{tr}$, deg			0.27	0.5
The total decrease in the steering angle of the outside wheel $\Sigma\theta = \delta_s + \Delta\theta_t + \Delta\theta_{tr}$, deg	–	–	1.94	2.826
Steering angle of the outside wheel $\theta_o = \theta_{omax} - \Sigma\theta$, deg	26.6	26.58	24.63	23.754
Slip angle of the bogie wheels δ_b, deg (36)	–	–	0.4475	0.4023
Minimum turning radius R_{min}, m (41)	14.3	12.74	13.89	14.36
Relative error, %	0	0	1.37	0.07

Table 8 shows the experimental values of the minimum turning radii of a KrAZ–6 × 4 truck on dry asphalt concrete to the left and right.

Table 8. Minimum turning radii of a KrAZ–6 × 4 truck.

Truck State	Truck Weight, kg	Direction of Turning	Minimum Turning Radius R_{min}, m
Three-axle without load	13,780	Left	13.7
		Right	12.65
Three-axle with load	25,780	Left	14.35
		Right	12.95

The analysis of the research results presented in Tables 5–8 showed the following:

- The elastic bogie wheel while moving along a curvilinear trajectory simultaneously participates in two motions: curvilinear motion with the radius of the trajectory of the center of the bogie side and movement with a slip angle $\delta = 57.3 l_b/(2R)$ (deg);
- Increasing the load and the cornering stiffness of the steered wheel tires and reducing the load and the cornering stiffness of the bogie tires reduce the value of the minimum turning radius of the vehicle;
- The minimum turning radius of a three-axle vehicle reaches its maximum value when turning left when the vehicle is fully loaded. This must be taken into account when operating it;
- The coincidence of calculated and experimental data confirms the reliability of the mathematical model of the movement of the elastic bogie wheel along a curvilinear trajectory and the dependence for calculating the minimum turning radius, which allows determining the influence of the tire characteristics, the wheelbases of the bogie and the vehicle, the kinematic and elastic parameters of the steering trapezoid, the distribution of the load on the wheels, and the direction of turning.

4. Discussion

It has been established that during the movement of a three-axle vehicle along a curved trajectory in the presence of a bogie, each bogie wheel participates in two movements: movement along a curved trajectory with the radius of movement of the center of the bogie side and plane-parallel movement with a slip angle, which is a function of the bogie

wheelbase and the radius of the trajectory of movement of the center of the corresponding bogie side. The movement of the bogie wheels with slip generates the lateral forces and self-aligning torques of the tires. The lateral forces on the bogie axles are oppositely directed and generate a moment about the bogie center. Analysis of the dependence (20) showed that this moment is a function of the cornering stiffness of the bogie wheels, the bogie wheelbase and the radius of the trajectory of movement of the centers of the bogie sides. As for the self-aligning torques of the tires of the bogie wheels, they have opposite directions on the axles, and their resulting value tends to zero.

The moment generated by the bogie is transmitted to the steering axle, developing a force perpendicular to the vehicle frame, and to the bogie, generating a force equal in magnitude and opposite in direction to the force acting on the steering axle. The dependence (21) determines this force. Relative to the bogie, it is a lateral force that causes the bogie wheels to move with the slip angle, which is determined by expression (36).

At the same time, the force acting on the steering axle develops lateral and longitudinal forces on each steered wheel, which are determined by expressions (26) and (27). The lateral force causes the movement of the steered wheels with a slip angle, which is determined by expression (30), and the redistribution of normal reactions on the steered wheels: the outside wheel to the turn center is loaded, and the normal load on the inside wheel decreases. As for the longitudinal force on the steered wheel, it will generate a twisting moment about the kingpin axis, which is determined by expression (31). This moment, if there is a pneumatic tire, will develop a twisting angle of the tire, which is determined by expression (32). At the same time, on the outside steered wheel, it will reduce the steering angle, and on the inside one, it will increase the steering angle.

The analysis of the conducted studies showed that the minimum turning radii of the vehicle to the left and to the right differ from each other. In this case, the minimum turning radius to the left is greater than when turning to the right. This is explained by the difference between the steering angles due to the peculiarities of the transmission of forces from the left steered wheel to the right by the steering trapezoid. In the case of a left turn, the left inside wheel turns to the maximum angle until it stops in the limiter. The right wheel turns due to the transmission of forces from the left wheel by the steering trapezoid. In this case, if the steering angle of the left wheel disk remains constant, then the steering angle of the right wheel decreases due to rolling with slip, the elasticity of the steering trapezoid, twisting of the tire, and is determined by expression (34). Such a decrease in the steering angle will lead to an increase in the minimum radius of the vehicle when the vehicle turns to the left.

In the case of a right turn, the right inside steered wheel is turned by force from the left steered wheel through the steering trapezoid to the maximum steering angle until it stops at the limiter. In this case, the left steered wheel is the outside one and its steering angle will decrease similarly to the outside wheel when turning left. To ensure the maximum steering angle of the right wheel until it stops at the limiter, the left one will be turned to compensate for this decrease. Therefore, when turning right, the steering angle of the left steered wheel will increase, which will cause a decrease in the minimum turning radius relative to turning left.

It is established that to calculate the minimum turning radius of a vehicle, it is necessary to determine the steering angle of the outside steered wheel and the slip angle of the bogie wheels. The paper presents the relations (34) and (36) for their determination. Based on the obtained values of these angles, taking into account the vehicle wheelbase and the track of the steered wheels, the dependence (41) was obtained to determine the minimum turning radius of a three-axle vehicle with a bogie.

The results of theoretical studies were confirmed by experimental data from studies conducted on three-axle vehicles, which differed in wheelbase, type of tires and their number on the axle. At the same time, the results of the studies show that the minimum turning radius differs when turning left and right, and the results confirmed the existence of this difference.

Reducing the wheelbase of a three-axle vehicle to a certain limit when driving on roads with a low coefficient of adhesion can lead to loss of vehicle handling, therefore, when determining the minimum value of the vehicle's wheelbase, it is necessary to take into account the operating conditions of the vehicle.

5. Conclusions

1. The analysis of literary sources confirmed the influence of the bogie parameters on the handling, maneuverability, and safety of the movement of three-axle vehicles.

2. A mathematical model of the bogie elastic wheel while moving along a curvilinear trajectory was obtained from the viewpoint of classical mechanics, which allowed us to assess the influence of the bogie parameters on the steering angles of a three-axle vehicle. It was established that during such movement lateral forces develop, one of which acts on the steering axle, and the other on the bogie wheels. Their values are proportional to the cornering stiffnesses of the bogie tires, the square of the bogie wheelbase, and the curvature of the trajectory of movement of the centers of the bogie sides, and are inversely proportional to the vehicle wheelbase.

3. It was found that the forces acting on the steered wheels during the movement of the bogie along a curvilinear trajectory increase the minimum turning radius of a three-axle vehicle due to the change in the steering angles and the movement of the bogie wheels with a slip compared to a two-axle vehicle with the same wheelbases. At the same time, the value of the minimum turning radius depends on the direction of the turning. The larger steering angle of the outside left wheel compensates for the decrease in the steering angle of this wheel from rolling with a slip, deformation of the steering trapezoid, and the moment from the longitudinal force when turning to the right. This leads to a decrease in the minimum turning radius to the right compared to turning to the left. For the given three-axle vehicles with a full load with wheelbases of 5.3 and 5.7 m, this decrease was 5.5% and 9.7%, respectively.

4. An analytical dependence of the minimum turning radius of a three-axle vehicle with a bogie on the steering angle of the outside steered wheel, the slip angles of the bogie wheels, the vehicle wheelbase, and the track of the steered wheels was obtained. This dependence made it possible to determine that the minimum radius of the vehicle's trajectory increases with an increase in the bogie wheelbase and the load on the bogie, and decreases when tires with increased cornering stiffnesses are installed on the steered wheels and the load on them is increased. Experimental studies have confirmed the reliability of the obtained dependence with a relative error of up to 1.37%.

5. The bogie parameters and operating conditions must be taken into account at the design stage of three-axle vehicles to predict and improve their handling and maneuverability indicators. Further studies will be aimed at determining the influence of the bogie parameters on the wear of the vehicle's tires.

Author Contributions: Conceptualization, V.M. and A.S.; methodology, A.S., M.S. and V.M.; software, M.S., E.K. and N.K.; validation, A.S., E.K. and N.K.; formal analysis, A.S., E.K., N.K. and N.M.; investigation, A.S. and E.K.; resources, E.K.; data curation, A.S.; writing—original draft preparation, A.S. and E.K.; writing—review and editing, N.K., L.T. and N.M.; visualization, E.K., L.T. and N.M.; supervision, V.M. and M.S.; project administration, V.M. and A.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

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