

## Article

# Analysis of Electromagnetic Vibration in Permanent Magnet Motors Based on Random PWM Technology

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**Abstract:** High vibration noise limits the application of permanent magnet motors in electric locomotive traction. This paper focuses on the high-frequency electromagnetic vibration in traction permanent magnet motors introduced by inverters. It explores the impact of periodic and random switching frequency pulse-width modulation (PWM) schemes on the high-frequency electromagnetic vibration performance of permanent magnet motors. The studied works are as follows: (1) The sources of higher-order harmonic components in the stator current are analyzed, and the characteristics of electromagnetic forces generated by these higher-order harmonic currents are studied. (2) The principles for suppressing high-frequency electromagnetic vibrations through random PWM are introduced. (3) The impact of the random switching frequency on higher-order harmonic currents in permanent magnet motors is analyzed through simulations. (4) The comprehensive experimental validation and evaluation of the random PWM technique are conducted on a permanent magnet motor. The results show that the vibration near the carrier frequency can be effectively weakened, but the overall vibration level has not been effectively reduced.



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**Keywords:** PM motor; high frequency; electromagnetic vibration; random PWM

## 1. Introduction

Permanent magnet synchronous motors (PMSMs) are widely used in electric vehicles, such as BMW, Tesla, Toyota Prius, and BYD, due to their high power density. However, the vibroacoustics of vehicle motor systems powered by inverters have become a perceptible harm to the human ear in automobiles [1,2]. The harmonic voltages and currents generated by the inverter power supply can induce distortions in the magnetic field within the air gap of the motor. This, in turn, results in increased torque pulsation [3] and contributes to a rise in motor noise levels. In recent years, scholars have carried out a lot of research on the electromagnetic vibration noise of permanent magnet motors.

For the vibration noise of the motor powered by the inverter, some conclusions have been achieved and summarized [4–6]: The frequencies of the stator magnetic field when the inverter is supplied satisfy the relation  $f_k = k_1 f_s \pm k_2 f_0$ , where  $k_1$  and  $k_2$  are positive integers with a different parity, e.g.,  $f_s + 2f_0$ ,  $f_s - 4f_0$ ...  $f_s$  is the carrier frequency, and  $f_0$  is the motor fundamental frequency. The main increased excitation force when the inverter is supplied is generated by the interaction of the fundamental magnetic field and the  $k$ th harmonic current generating the fundamental magnetic field with the order of 0th or  $2p$ th

order at the frequency  $f_n = k_3 f_s \pm k_4 f_0$ , where  $k_3$  and  $k_4$  are positive integers with the same parity, e.g.,  $f_s + f_0$ ,  $f_s - 3f_0 \dots$

Scholars have made great efforts to solve motor vibration problems caused by converters. In summary, there are several solutions as follows:

#### (1) Adding filters

The simplest solution is to add a filter between the motor and the inverter [7], and the larger the value of the reactance, the better the high-frequency filtering effect, but the larger the reactance voltage drop on the reactor, the greater the increases in the capacity requirements of the inverter. For the problems caused by reactors, some scholars have proposed a passive filtering method using inductors, capacitors, and their combination devices [8]. This device is able to eliminate current harmonics of a certain bandwidth, thus reducing high-frequency excitation force and vibration, but this kind of inductor–capacitor requires a larger power rating. J.A. Ferreira et al. [9] proposed a structure with a variable filtering frequency, which can be adjusted between two limiting frequencies by switching the capacitance value. To address the problem of passive filters, many scholars have proposed an active filter structure [10], where the inverter and the motor are connected to each other by passive filter devices, and at the same time, another set of inverters using the high switching frequency MOSFET or SiC devices outputs harmonic components in an inverse phase with the harmonics in the passive filters on the main circuit.

#### (2) PWM control strategy

The PWM control strategy can significantly enhance the sinusoidal characteristics of the supply voltage and current, thereby reducing harmonic content [11]. Furthermore, vibration noise suppression research is mainly focused on the PWM control strategy, which is based on the idea of dispersing the voltage, current, vibration, and noise harmonic components near the carrier frequency band into a wider frequency band range.

A randomized carrier frequency is used to extend the originally concentrated sideband harmonic energy to a wider frequency range [12], and the results show that the noise suppression effect at the center frequency reaches 22 dB. In order to make the sampling frequency fixed, a random switching frequency pulse-width modulation method with a variable delay technique is proposed [13], and the experiments show that this method can effectively distribute the current spectrum uniformly in a wider range. A pseudo-random carrier modulation scheme is proposed [14], and its harmonic spreading effect is thoroughly examined. The results demonstrate that this scheme effectively disperses the harmonic spectra of both voltage and current across a broad range of frequency bands. In [15], a pseudo-random high-frequency square wave signal random switching frequency pulse-width modulation method is proposed, and the results show that the noise at the center frequency has been reduced by at least 10 dB. A specific harmonic elimination random switching frequency pulse-width modulation prevention is proposed in [16], and the simulation and experimental results validate the effectiveness of the proposed scheme. In order to reduce the high-frequency electromagnetic vibration noise of a dual three-phase permanent magnet motor, Prof. Y. Miyama et al. [17] proposed a carrier-phase shifted modulation method to eliminate the switching frequency harmonics in the inverter, which removes the dominant harmonic currents by optimizing the trigger angle of the power tubes.

A. Ruiz et al. [18] proposed a trapezoidal waveform as the modulating waveform PWM control scheme, in which the current harmonic quantity is greatly reduced. Jean L.B. et al. [19] explored the influence of the carrier frequency from a theoretical point of view and came to the conclusion that the optimal carrier frequency should be selected to avoid the 0th order and the 2nd order of the motor's intrinsic frequency.

The harmonic power propagation capabilities of Sinusoidal PWM (SPWM), Random Carrier PWM, and Random Pulse Position PWM are compared through simulation [20], and the simulation results are validated using a prototype of the inverter that was designed for this purpose. Notably, the RPPWM effectively disperses the acoustic switching noise spectrum of the induction motor drive. The periodic switching frequency modulation technique of the sawtooth wave is investigated in [21], and the results showed that the amplitude of the line voltage in the center frequency band was reduced by at least 30%. Subsequently, the scholar combined the sawtooth wave periodic switching frequency modulation technique with the asynchronous carrier modulation technique [22], which further reduced the amplitude of the high-frequency voltage. In [22], it was shown that the spread-spectrum modulation can effectively suppress the motor current harmonics and optimize the noise in the center band of the switching frequency band by more than 10 dBA.

In summary, the effects of random and periodic switching frequency PWM modulation strategies proposed by scholars are judged by the switching frequency and the nearby vibration noise amplitude. Admittedly, the amplitude can be used as a kind of evaluation index, but the vibration noise is also a matter of people's overall perception of the external objects; therefore, it should be necessary to add other means of judgement in the performance evaluation.

This paper comprehensively evaluates the high-frequency electromagnetic vibration characteristics of motors under the commonly used periodic switching frequency PWM modulation (PPWM) and random switching frequency PWM modulation (RPWM) strategies in terms of vibration performance. Firstly, the electromagnetic field and electromagnetic force of the motor under an inverter power supply are calculated and analyzed, and then the vibration suppression principles of PPWM and RPWM are briefly introduced. Next, the simulation verification of random PWM technology is carried out in the motor, and the influence of the scheme on the high-frequency harmonic current component in the motor is explored. Finally, the vibration of a permanent magnet synchronous motor is actually measured to analyze the suppression effect of the two schemes and explore the influencing factors of the two suppression schemes in detail.

## 2. Analysis of Electromagnetic Force

The source of electromagnetic vibration in the motor is the Maxwell stress tensor force, which has both radial and tangential components that primarily act on the inner surface of the stator. Among these, the radial component is the predominant contributor to electromagnetic vibration, and the radial pressure  $FF_n$  can be expressed as follows:

$$FF_n = B_n^2 / (2\mu_0) \quad (1)$$

where  $B_n$  is the normal component of the air gap magnetic field, and  $\mu_0$  is the air permeability. Disregarding the effect of the magnetic reluctance of the magnetic circuit of the iron core, the air gap magnetic field of the permanent magnet synchronous motor can be expressed as follows:

$$B_n(\theta, t) = f_a(\theta, t) \cdot \lambda(\theta, t) \quad (2)$$

where  $\theta$  is the spatial mechanical angle,  $f_a(\theta, t)$  is the air gap magnetomotive force, and  $\lambda(\theta, t)$  is the air gap specific permeance. For stator slotting, the air gap specific permeance can be approximated as follows:

$$\lambda(\theta) = \lambda_0 + \sum_{\substack{l \\ \text{②}}} \lambda_l \quad (3)$$

①

where  $\lambda_0$  is the average component of the specific permeance and  $\lambda_l$  is the specific permeance with harmonic order  $l$ .

When the inverter is supplied, the magnetomotive force can be divided into the following items: (1) the fundamental magnetomotive force generated by the stator fundamental current and rotor permanent magnet; (2) the harmonic magnetomotive force generated by the stator fundamental current; (3) the harmonic magnetomotive force generated by rotor permanent magnet; and (4) the fundamental magnetomotive force generated by the stator harmonic current.

When the inverter is supplied, the frequency of the motor current harmonics can be expressed as follows:

$$f_k = k_1 f_s \pm k_2 f_o \quad (4)$$

where  $f_s$  is the carrier frequency of inverter,  $f_o$  is the base wave frequency of motor,  $k_1$  and  $k_2$  are parity integers, and in three-phase motors,  $k_2$  cannot be taken as a multiple of three, such as  $f_s \pm 2f_o$ ,  $f_s \pm 4f_o$ ,  $2f_s \pm f_o$ ,  $2f_s \pm 5f_o$ , and so on.

Near the carrier frequency and its octave, the current harmonics introduced by the inverter cause high-frequency vibration of order 0 or order  $2p$ , and the frequency of the electromagnetic excitation force  $f_n$  is expressed as follows:

$$f_n = (k \pm 1)f_o = k_3 f_s \pm k_4 f_o$$

where  $k_3, k_4$  is a positive integer with the same parity, e.g.,  $f_s \pm f_o$ ,  $f_s \pm 3f_o$ ,  $2f_s \pm 2f_o$ ,  $2f_s \pm 4f_o$ , etc.

### 3. Basic Principles of Random PWM Technology

The high harmonic currents generate high-frequency electromagnetic vibration and noise. The variable carrier period scheme is used to solve the problem of the current switching frequency concentration without increasing the hardware cost. This section introduces common variable switching frequency PWM techniques, including periodic switching frequency pulse-width modulation (PPWM) and random switching frequency pulse-width modulation (RPWM).

The PPWM involves the addition of a period-varying component to the original fixed switching frequency, and the switching frequency is expressed in the following form:

$$f_s = f_{s0} + R(t)\Delta f \quad (5)$$

where  $f_{s0}$  is the center frequency,  $R(t)$  is a periodic function, and  $\Delta f$  is a fixed constant, representing the bandwidth of the switching frequency.

During a cycle, the switching frequency varies within  $[f_{s0} - \Delta f, f_{s0} + \Delta f]$ . The selection of  $f_{s0} = 10,000$  Hz,  $\Delta f = 500$  Hz,  $R(t)$  is an approximate triangular wave function. In the programming implementation, each time comes into the ePWM interrupt, so that  $f_s$  increased or decreased by 1 Hz, and the  $R(t)$  period is approximated as  $T = 4\Delta f / (f_{s0} \cdot \Delta f_0)$ , where  $\Delta f_0$  represents the size of each interrupt frequency change.

The RPWM is one of the most commonly used random PWM methods, which is similar to the periodic switching frequency, and it is expressed as the random switching frequency:

$$f_s = f_{s0} + \text{rand}(-1, 1)\Delta f \quad (6)$$

where  $f_{s0}$  is the center frequency,  $\Delta f$  is the range of variation in the switching frequency, and the function  $\text{rand}(-1, 1)$  is a random number generator that produces a value uniformly distributed within the interval  $(-1, 1)$ , and  $f_s$  is a randomly varying value in the range of  $[f_{s0} - \Delta f, f_{s0} + \Delta f]$ .

The RPWM with natural sampling is achieved by randomly setting the slope of the triangular wave signal, and in SVPWM, randomly changing the voltage vector angle can realize the RPWM.

## 4. Implementation of Random PWM Technology

### 4.1. Simulation Implementation

In order to verify the suppression effect of the random PWM technique on the high harmonic components in the motor current, a motor simulation model is built. In the simulation, the switching frequency in the control system is set at 10 kHz, 8 kHz, 6 kHz, and 4 kHz, respectively, and the currents are compared and analyzed with and without the addition of the random PWM technique; when the random PWM technique is added, the FFT analysis of the currents is carried out according to Equation (5), where  $\Delta f$  is set to 0.5 k, 1 k, and 1.5 k, respectively, for the simulation to determine the effect of this parameter on the suppression effect. The generation module of random frequency carrier built in Simulink is shown in Figure 1:

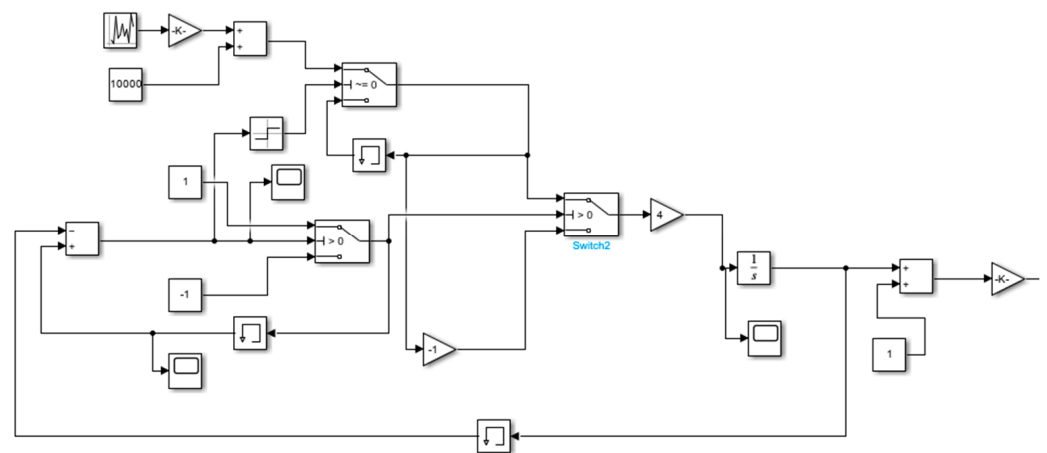


**Figure 1.** Simulation module for random PWM.

In Figure 1, a generator is used to randomly generate random numbers in the interval from  $-1$  to  $1$ . The switching frequency in the random PWM is obtained by multiplying it with the given  $\Delta f$  plus the value of the original fixed switching frequency. In the triangular waveform generation part, the rising edge part and the falling edge part of the triangular waveform can be obtained by giving two constants of  $1$  and  $-1$  and integrating them. By judging to switch its state when it reaches the peak value, and by changing the frequency of the triangular waveform every time it passes through the triangular waveform, a triangular carrier waveform whose frequency varies randomly within the given switching frequency can be obtained.

The control system of the permanent magnet synchronous motor is carried out by using the random PWM module to output a triangular wave with a random frequency instead of the original carrier wave. The parameters of the simulated PM motor are as follows:  $p = 4$ , rated speed is 1500 r/min, stator resistance  $1.1 \Omega$ ,  $L_d = L_q = 2.22$  mH, and the flux is 0.22 Wb.

Figures 2–5 show the results of the FFT analysis of the motor current for varying carrier frequency bandwidths when the switching frequency is 10 kHz. It should be emphasized that the fundamental current value is located at the top of Figures 2–9, and the unit is A. The amplitude of the fundamental current is considered as 100%. The  $y$ -axis represents the percentage of harmonic current relative to the fundamental current. THD means the total harmonic distortion of the current.

It can be found from Figures 2–5 that the addition of the random frequency carrier reduces the amplitude of the current harmonics at the switching frequency to some extent, as compared to the case without random PWM, and the best effect is achieved at  $\Delta f = 0.5$  kHz, where the harmonic amplitude as a percentage of the fundamental amplitude at the switching frequency is reduced from 0.55% to 0.44%. Meanwhile, the THD of the current increases as the range of the random frequency variation increases, but the THD of the current is approximated when  $\Delta f = 1$  kHz and  $\Delta f = 1.5$  kHz.

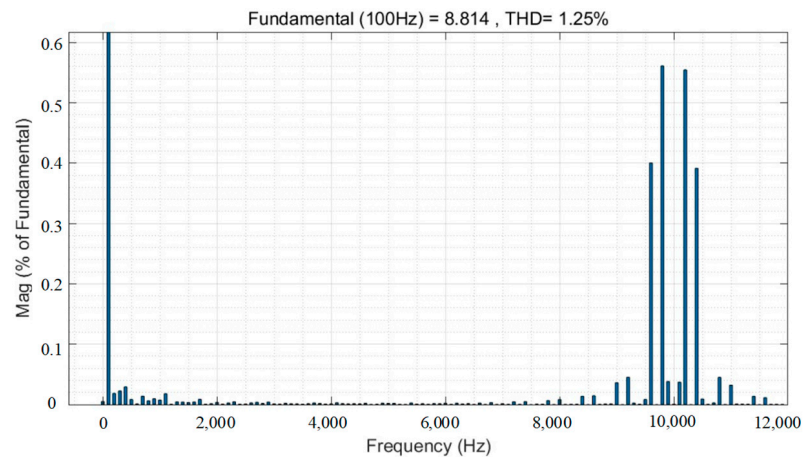


**Figure 2.** Switching frequency of 10 kHz,  $\Delta f = 0$ .

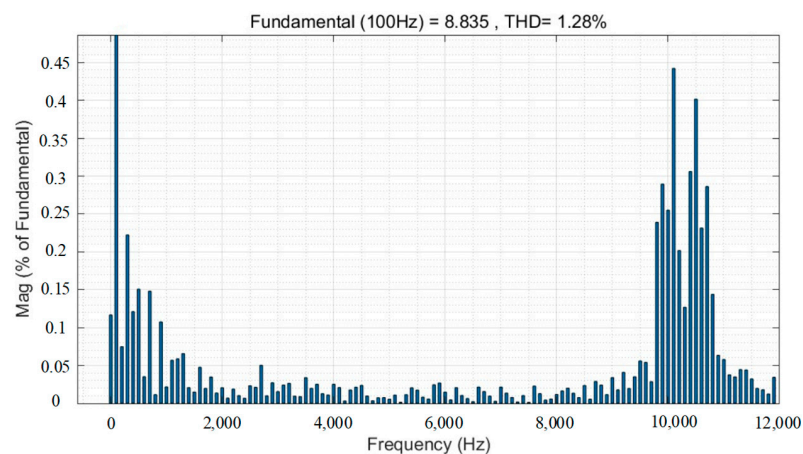


**Figure 3.** Switching frequency of 10 kHz,  $\Delta f = 0.5$  kHz.

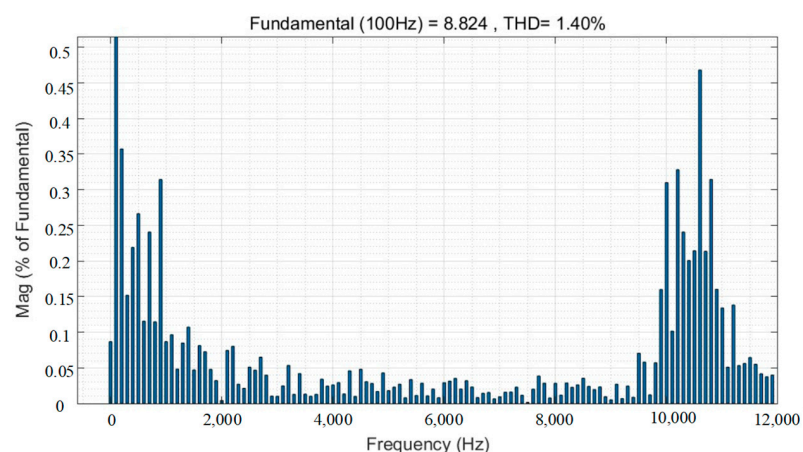


**Figure 4.** Switching frequency of 10 kHz,  $\Delta f = 1$  kHz.

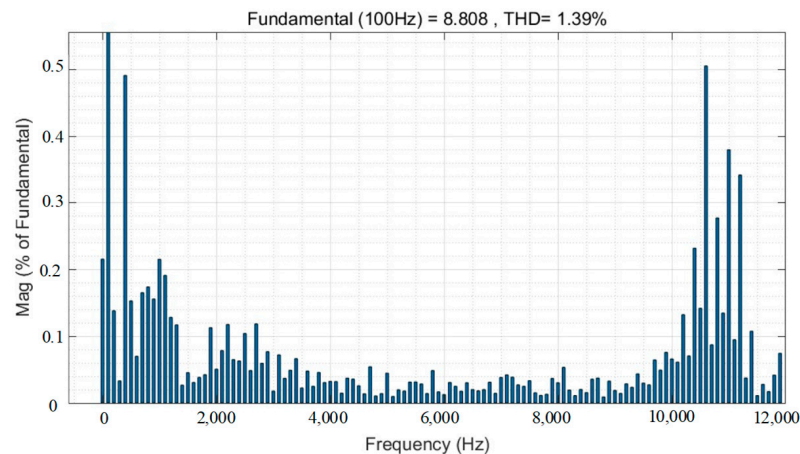


**Figure 5.** Switching frequency of 10 kHz,  $\Delta f = 1.5$  kHz.

Figures 6–9 show the results of the FFT analysis of the currents for a random quantity that changes the carrier frequency when the switching frequency is 8 kHz.

It can be seen from Figures 6–9 that the harmonic amplitudes at the switching frequency are suppressed with the addition of the random carrier, and the suppression is relatively the best at  $\Delta f = 1$  kHz, where the harmonic amplitude as a percentage of the fundamental amplitude is reduced from 0.7% to about 0.47%. Meanwhile, the THD of current harmonics changes very little from 1.57% to 1.59% at  $\Delta f$  from 0 to 1 kHz, and then THD becomes larger, reaching 1.70% at  $\Delta f = 1.5$  kHz.

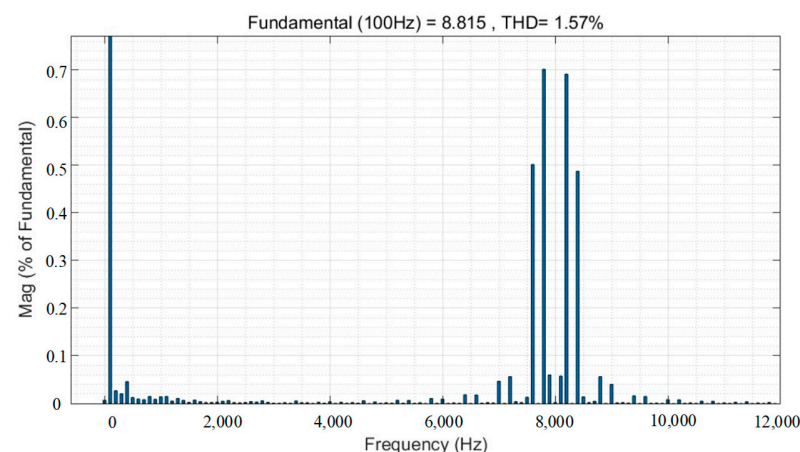


**Figure 6.** Switching frequency of 8 kHz,  $\Delta f = 0$ .

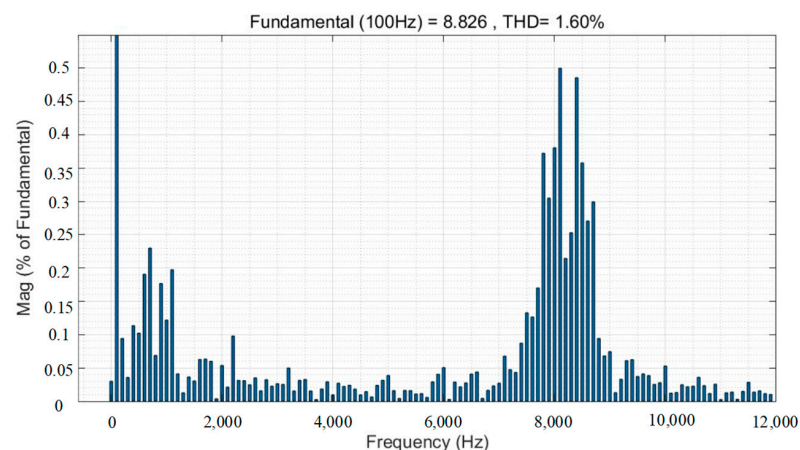


**Figure 7.** Switching frequency of 8 kHz,  $\Delta f = 0.5$  kHz.

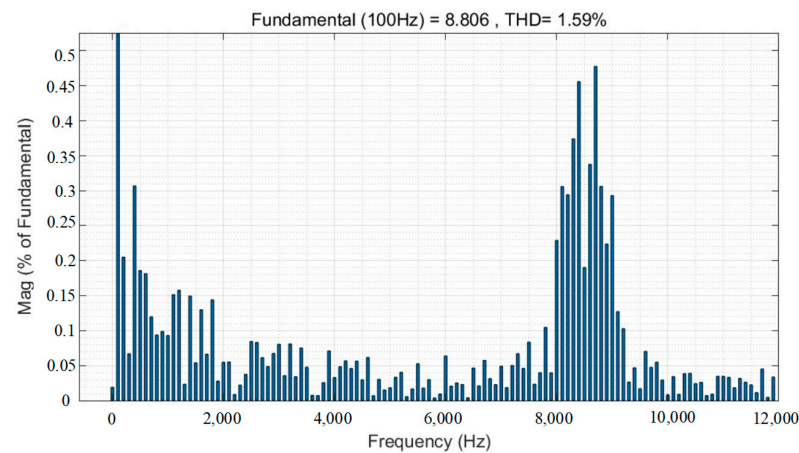


Figure 8. Switching frequency of 8 kHz,  $\Delta f = 1$  kHz.

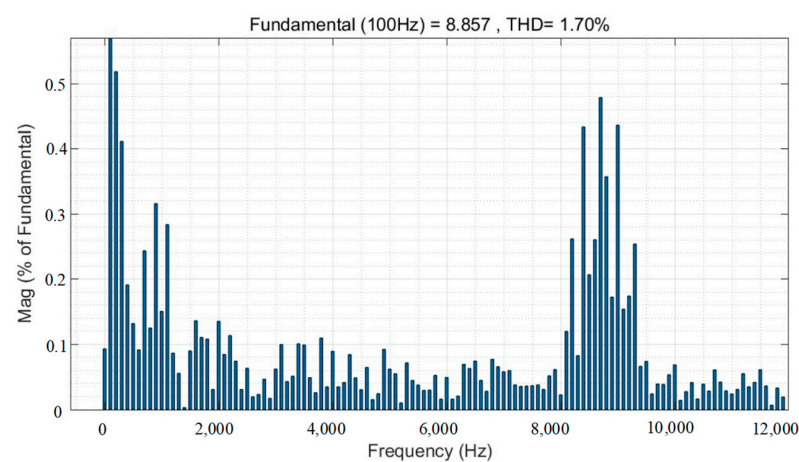


Figure 9. Switching frequency of 8 kHz,  $\Delta f = 1.5$  kHz.

Harmonic voltages and currents generated by the inverter supply can induce distortions in the magnetic field within the air gap of the motor. This distortion can result in increased torque pulsations, subsequently leading to heightened motor vibration. Consequently, it is essential to analyze the torque variations across different random frequency ranges. Figure 10 illustrates the torque variation over time for different  $\Delta f$  values.

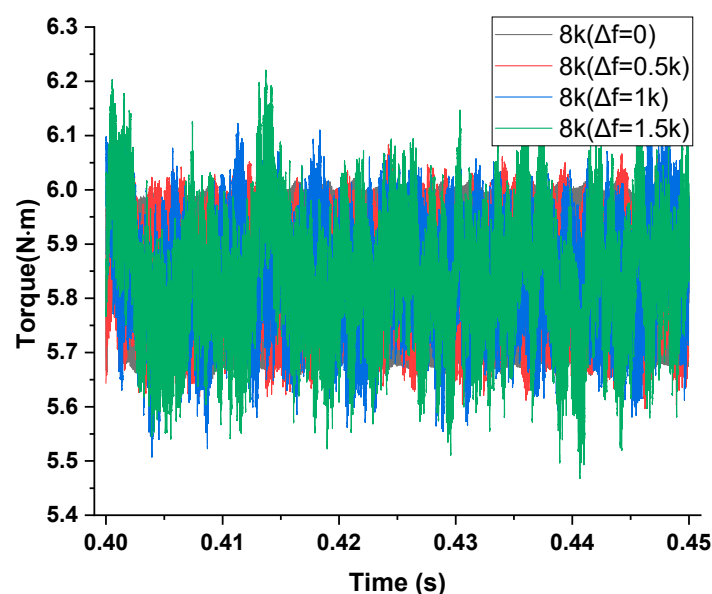


Figure 10. Torque time-domain diagram.

From Figure 10, it is evident that the torque pulsation gradually increases with the widening of the frequency  $\Delta f$ . To further analyze the impact of variations on the torque ripple, an FFT analysis is performed on the torque. The segments corresponding to the switching frequencies within the 6 kHz to 10 kHz range are extracted from the FFT results, as illustrated in Figure 11.

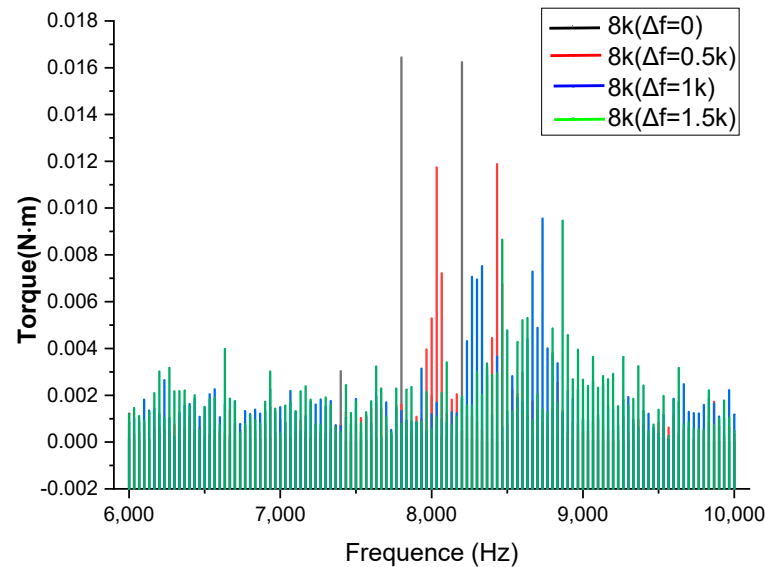


Figure 11. Switching frequency of 8 kHz,  $\Delta f = 0$  kHz.

From Figure 11, it is evident that the harmonic amplitudes near the switching frequency are reduced with the introduction of the  $\Delta f$ . Notably, the most effective suppression occurs at a frequency difference of  $\Delta f = 1.5$  kHz.

The simulation results are summarized to obtain the variation in current harmonic amplitude and THD at different switching frequencies, as shown in Table 1.

Table 1. Degree of variation of current harmonic amplitude and THD at different switching frequencies.

| Switching Frequency | Amplitude Reduction | Reduction Ratio in Amplitude | Elevated THD |
|---------------------|---------------------|------------------------------|--------------|
| 10 kHz              | 0.011 A             | 21.4%                        | 0.15         |
| 8 kHz               | 0.02 A              | 32.9%                        | 0.13         |
| 6 kHz               | 0.03 A              | 37%                          | 0.56         |
| 4 kHz (at 4 kHz)    | 0.035 A             | 28.6%                        | 0.7          |
| 4 kHz (at 8 kHz)    | 0.067 A             | 63.3%                        | 0.7          |

It can be seen from Table 1 that by using the random PWM technique, the magnitude of the higher harmonics can be suppressed, but the THD value of the current will be elevated. Table 2 demonstrates the simulation results of the magnitude of the fundamental current for different switching frequencies and bandwidths, and it shows that by using the technique of random PWM, there is almost no effect on the fundamental amplitude of the current.

Table 2. Current fundamental amplitude at different switching frequencies.

| Switching Frequency | Base Wave Current Amplitude $\Delta f = 0$ | Base Wave Current Amplitude $\Delta f = 0.5$ kHz | Base Wave Current Amplitude $\Delta f = 1$ kHz |
|---------------------|--------------------------------------------|--------------------------------------------------|------------------------------------------------|
| 10 kHz              | 8.814 A                                    | 8.835 A                                          | 8.824 A                                        |
| 8 kHz               | 8.815 A                                    | 8.826 A                                          | 8.806 A                                        |
| 6 kHz               | 8.816 A                                    | 8.817 A                                          | 8.831 A                                        |
| 4 kHz               | 8.817 A                                    | 8.841 A                                          | 8.847 A                                        |

#### 4.2. Test Validation

In order to investigate the effects of random PWM techniques on the current and vibration of the motor, detailed experiments are carried out on an eight-pole surface-mounted permanent magnet motor, and the experimental platform is shown in Figure 12. The experimental setup includes a permanent magnet motor, loader, controller, vibration sensor, and Jiangshu Union energy vibration test system.

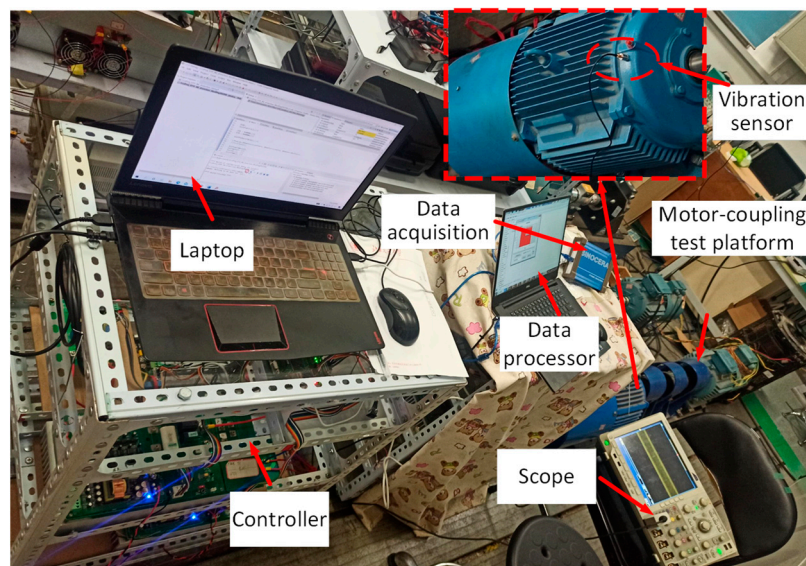


Figure 12. Experimental platform.

The experimental setup is as follows: in order to avoid the effect of the random PWM technology scheme on speed fluctuation, an encoder is used to measure the motor position and speed control. In the fixed switching frequency control scheme for motor speed control,  $i_d = 0$  is used, the speed runs at 1500 rpm, the load is 6 N·m, and the carrier frequency is 8 kHz. In the PPWM, the effect of four lengths of frequency change steps of 1 Hz, 5 Hz, 10 Hz, and 20 Hz on vibration is investigated, and the effect of the frequency band ranges on the vibration damping of PPWM and RPWM is also explored. During the experiments, the phase current and the vibration on the surface of the motor casing are measured, and the intercepted data processing segments are stable and non-fluctuating time-domain current and vibration signals, and the results are summarized as follows.

The vibration sensor is located at the upper part of the motor end cover. The vibration sensor, produced by SINOCERA PIEZOTRONICS, INC. in Yangzhou, China, is a piezoelectric acceleration sensor with model CA-YD-1182, the reference sensitivity is 10.15 mV/m·s<sup>2</sup>, and the frequency range is from 1 to 30,000 Hz.

##### 4.2.1. Current Waveforms

Figures 13–15 show the current waveforms under the conventional control scheme, PPWM, and RPWM, respectively. From Figure 13, it can be seen that in addition to the high-frequency harmonics brought by the inverter, the experimental motor itself also carries some low harmonics, and although the low current harmonics will cause low-frequency noise, the narrow bandwidth of high-frequency current harmonics near the carrier frequency and its integer multiples, caused by the vibration of the motor, will produce a narrow-band noise that is very sensitive to the human ear.

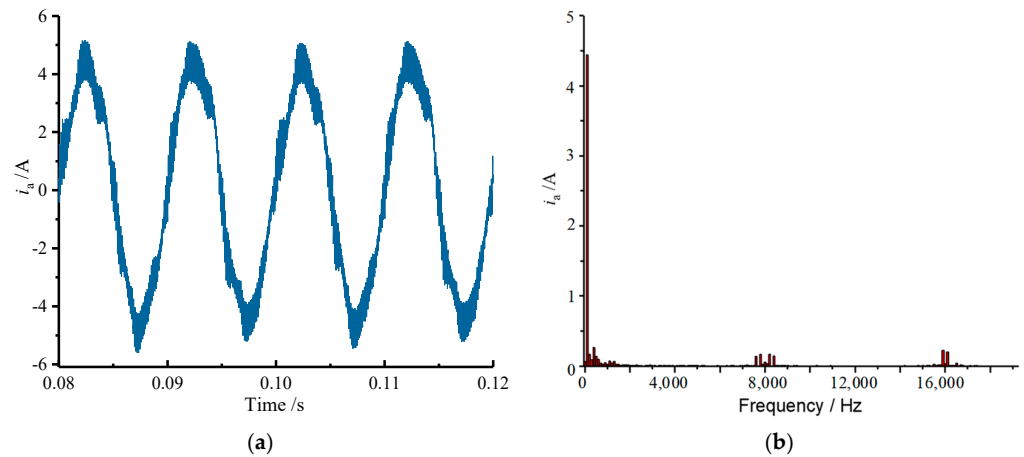


**Figure 13.** Current waveform under conventional control strategy. (a) Time domain. (b) Fourier decomposition.

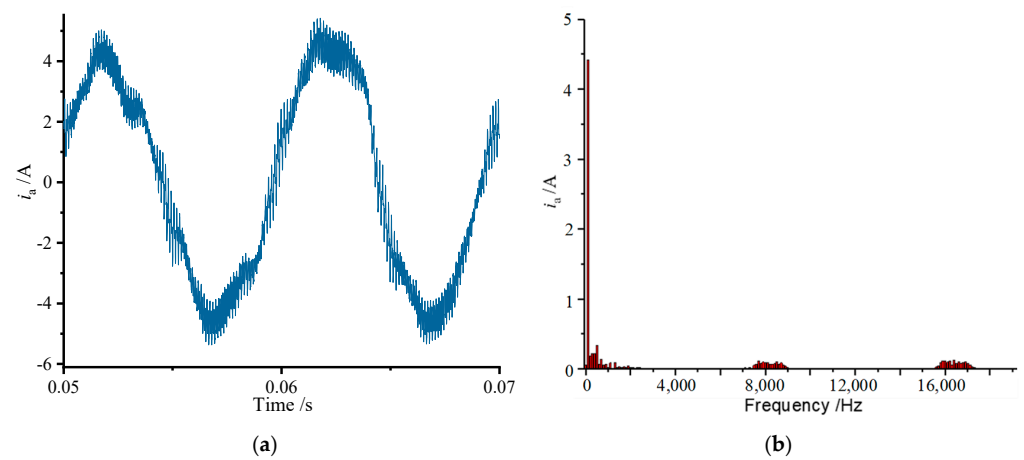


**Figure 14.** Current waveforms under the period-variable switching frequency strategy. (a) Time domain. (b) Fourier decomposition.

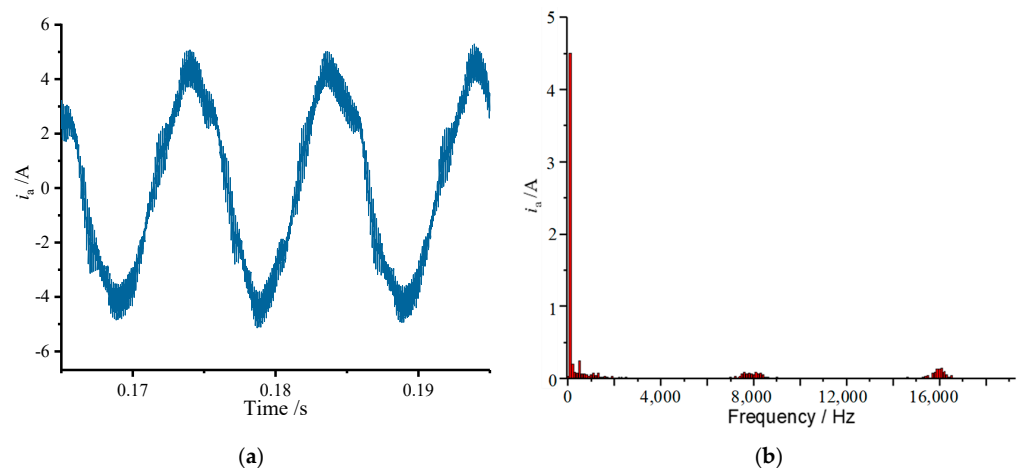


**Figure 15.** Current waveforms under random switching frequency strategy. (a) Time domain. (b) Fourier decomposition.

As can be seen in Figure 14, compared with the conventional scheme, the PPWM technique can significantly attenuate the amplitude of the narrow-bandwidth high-frequency harmonic currents near the carrier frequency  $f_c$  and its integer multiples of the frequency  $mf_c$  ( $m \geq 2$ ) at all operating conditions. For the narrow-bandwidth high-frequency harmonic components, the current amplitude of these components is attenuated by more than 50%

around the carrier frequency  $f_c$ , by more than 60% around the carrier frequency  $2f_c$  and  $3f_c$ , and by more than 70% around the carrier frequency  $4f_c$ , for all operating conditions.

It can be seen from Figure 15 that the RPWM technique can greatly weaken the amplitude of the high-frequency current harmonic components with a narrow bandwidth near the carrier frequency  $f_c$  and its integer multiples  $mf_c$  ( $m \geq 2$ ) under each working condition. For the narrow-bandwidth high-frequency harmonic components, the current amplitudes of these components are essentially attenuated by more than 50% around the carrier frequency  $f_c$ , and by more than 55% around the carrier frequency  $2f_c$ ,  $3f_c$ , and  $4f_c$  for all operating conditions.

#### 4.2.2. Electromagnetic Vibration Waveforms

Figure 16 shows the vibration histograms for the conventional control scheme, PPWM, and RPWM.

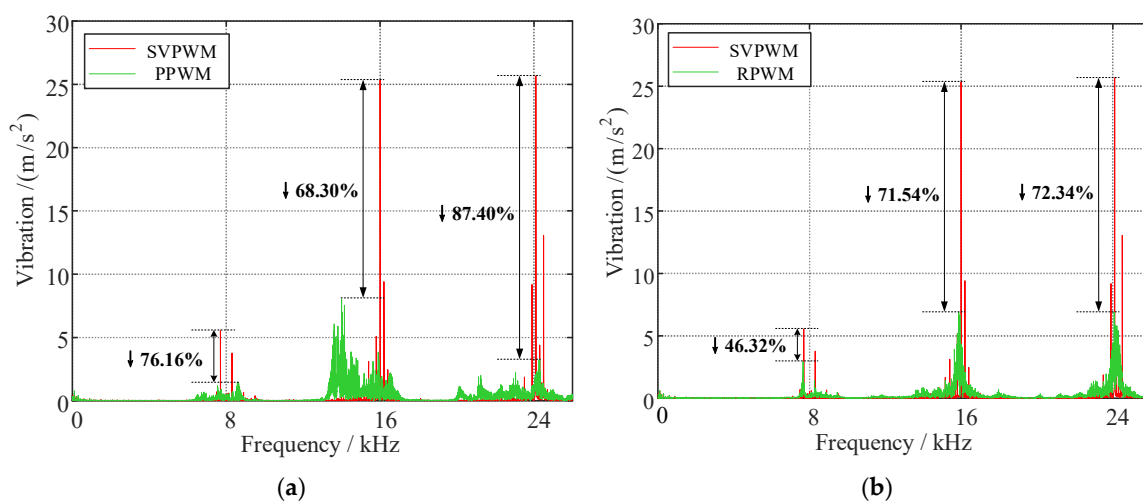


**Figure 16.** Waveforms in vibration. (a) With PPWM. (b) With RPWM.

It can be clearly seen from Figure 16a that the higher amplitude harmonic components of the radial vibration acceleration are still mainly concentrated near the carrier frequency  $f_c$  and its integer multiple frequency  $mf_c$  ( $m \geq 2$ ), which is very similar to the FFT results of the current. The high-frequency harmonics of the current give rise to the high-frequency harmonic components of the radial vibration acceleration, which will cause intolerable high-frequency noise. For the high-frequency harmonic components with a narrow bandwidth, the amplitude of the radial vibration acceleration of these components is attenuated by more than 75% around the carrier frequency  $f_c$ , by more than 65% around the carrier frequency  $2f_c$ , and by more than 85% around the carrier frequency  $3f_c$  for all the operating conditions using the PPWM technique.

Furthermore, it can be seen from Figure 16b that, because the RPWM technique greatly reduces the harmonic amplitude of high-frequency currents with a narrow bandwidth, the radial vibration acceleration amplitude will be reduced accordingly. For the high-frequency harmonic components with a narrow bandwidth, the amplitude of the vibration acceleration of these components is basically reduced by about 50% near the carrier frequency  $f_c$  under various operating conditions, and more than 70% near the carrier frequency  $2f_c$  and the frequency  $3f_c$ .

#### 4.2.3. Comparison of Reduction Method

As mentioned earlier, both the PPWM and RPWM can reduce the switching frequency vibration of the motor. Here, it will compare the effects of two suppression schemes. Figure 17 shows the total vibration acceleration level under different PWM control strategies.

in the carrier frequency range (7.8 kHz~8.2 kHz). As it can be seen from Figure 17, the total vibration level of the motor under three PWM strategies shows a tendency to increase and then slowly decrease as the rotational speed increases, and the total vibration level of the PPWM technique is higher, whereas that of the RPWM and the conventional PWM is almost the same.

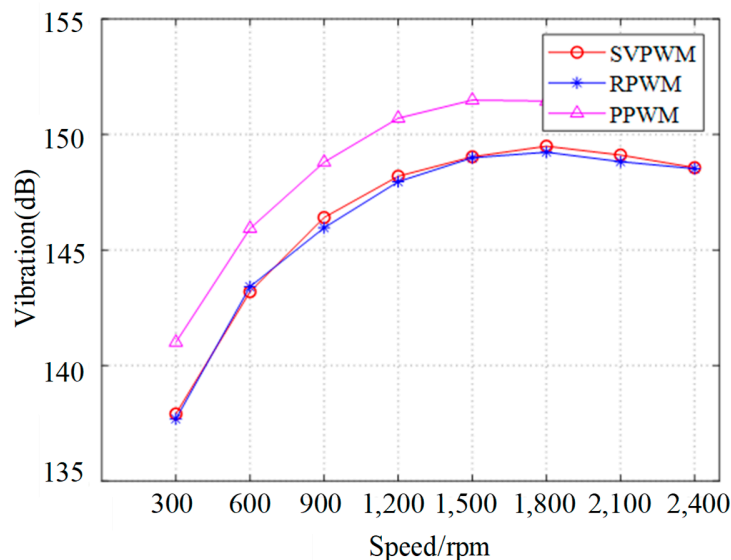


**Figure 17.** Total vibration acceleration level under different PWM control strategies in carrier frequency range (7.8 kHz~8.2 kHz).

When the carrier frequency range is expanded from 7.8 kHz~8.2 kHz to 7.5 kHz~8.5 kHz, the total vibration level can be shown in Figure 18. The total vibration levels of the motors under the three PWM strategies exhibit a trend of initially increasing and then gradually decreasing with rising rotational speed. Notably, the total vibration level is highest for the PPWM technique. In contrast to the carrier frequency range of 7.8 kHz to 8.2 kHz, the total vibration level of RPWM is slightly greater than that of SVPWM at speeds between 300 and 900 rpm. However, at speeds exceeding 900 rpm, the total vibration levels of both RPWM and SVPWM are nearly identical. This similarity at higher speeds suggests that the characteristics of total vibration levels in the two frequency ranges are compatible.

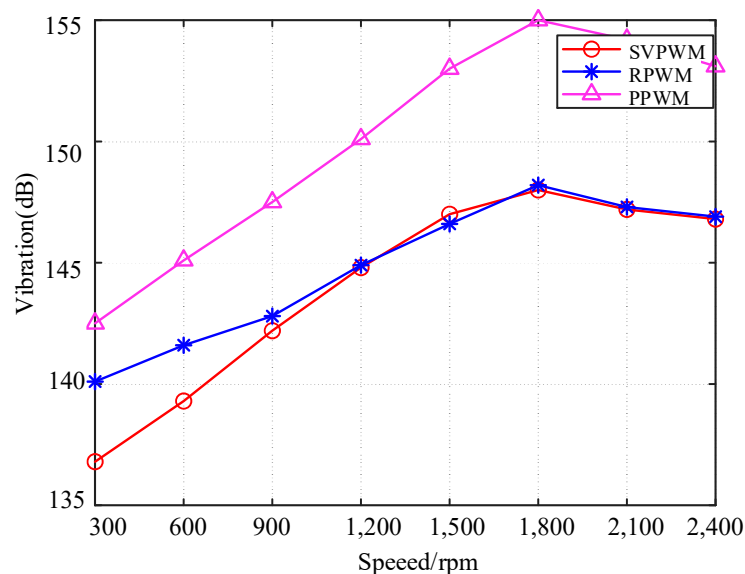


**Figure 18.** Total vibration acceleration level under different PWM control strategies in carrier frequency range (7.5 kHz~8.5 kHz).

Figures 19 and 20 show the total vibration level at the carrier frequency range, 7.2 kHz~8.8 kHz and 6.6 kHz~9.4 kHz, respectively. The total vibration level of RPWM technology has already surpassed conventional PWM at all speeds, and the total vibration level of PPWM technology is the largest. Furthermore, under the random PWM strategy, the total vibration level increases gradually with the carrier frequency range, and the total vibration level curve of the conventional SVPWM lies between the total vibration level curves of different carrier frequency ranges. This seems to contradict the analysis in Section 4.2.2. In fact, the essence of using random PWM technology is to disperse the concentrated frequencies near the carrier wave, and although the amplitude of the prominent frequency has decreased, the amplitude of the dispersed frequency has also increased. Therefore, for random PWM technology, although the vibration near the carrier frequency can be effectively weakened, the overall vibration level has not been effectively reduced.

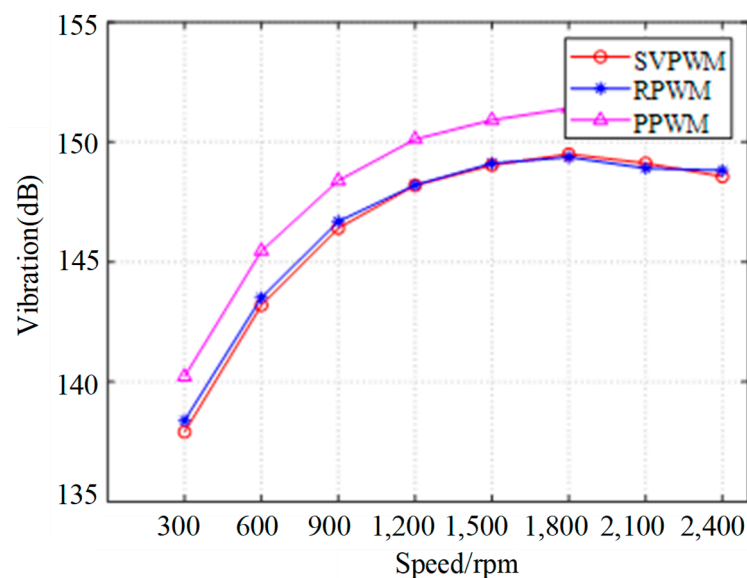


**Figure 19.** Total vibration acceleration level under different PWM control strategies in carrier frequency range (7.2 kHz~8.8 kHz).

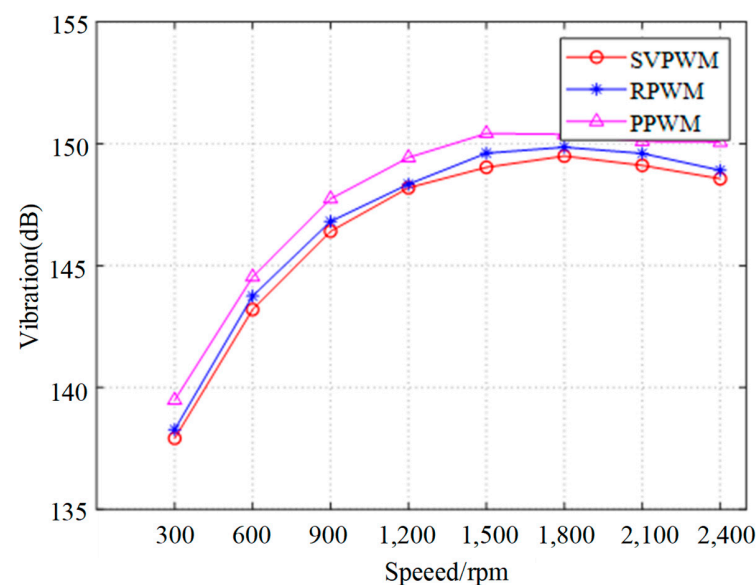


**Figure 20.** Carrier frequency range (6.6 kHz~9.4 kHz) and total vibration acceleration level with different PWM control strategies.

## 5. Conclusions

In this paper, the random PWM technique was introduced to reduce the high harmonic currents and vibrations, and the simulation and experimental results verify the effectiveness of the suppression scheme. The following conclusions can be obtained:

- (1) The high-frequency current harmonics generated by the rich high-frequency voltage harmonics embedded in the square wave output from the inverter are basically concentrated near the carrier frequency  $f_c$  and its integer multiple frequency  $mf_c$  ( $m \geq 2$ ), and these high-amplitude, narrow-bandwidth high-frequency current harmonics generate noise that is particularly sensitive to the human ear. Therefore, it is crucial to mitigate the high-frequency harmonic components of the current and the associated motor vibrations near the carrier frequency by employing a PWM modulation strategy.
- (2) Simulation results show that for switching frequencies of 4 k, 6 k, 8 k, and 10 kHz, the amplitude suppression of high current harmonics by the random PWM technique reaches more than 20%. However, it increases the total harmonic distortion of the current by a maximum of 12%.
- (3) The experimental results demonstrate that the control strategy employing both the PPWM and RPWM techniques significantly reduces the amplitude of high-frequency harmonic components in the current supplied by the inverter, as well as the motor vibrations induced by these harmonics. This attenuation occurs around the carrier frequency  $f_c$  and its integer multiples  $mf_c$  (where  $m \geq 2$ ), which in turn can greatly weaken the sharp high-frequency noise, reducing the amplitude by more than 60%. However, the random PWM techniques cannot reduce the total vibration levels compared with conventional PWM.

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