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Modified Dynamic Movement Primitive-Based Closed Ankle Reduction Technique Learning and Variable Impedance Control for a Redundant Parallel Bone-Setting Robot

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Abstract: Traditional fracture reduction relies heavily on the surgeon's experience, which hinders the transmission of skills. This specialization bottleneck, coupled with the high demands on physical strength, significantly limits the efficiency of daily treatments in trauma orthopedics. Currently, most fracture surgery robots focus on open or minimally invasive reduction techniques, which inherently carry the risk of iatrogenic damage due to surgical incisions or bone pin insertions. However, research in closed reduction-oriented robotic systems is remarkably limited. Addressing this gap, our study introduces a novel bone-setting robot for the closed reduction of ankle fractures designed with a redundant parallel platform. The parallel robot's design incorporates three sliding redundancy actuators that enhance its tilt flexibility while maintaining load performance. Moreover, a singularity-free redundant kinematic solver has been developed, optimizing the robot's operational efficacy. Building upon the demonstrations from professional closed reduction techniques, we propose the use of a multivariate Student-t process as a multi-output regression model within dynamic movement primitive for accurately learning stable reduction maneuvers. Additionally, we develop an anthropomorphic variable impedance controller based on inverse dynamics. The simulation results demonstrate convincingly that the developed ankle bone-setting robot is proficient in effectively replicating and learning the nuanced closed reduction techniques.

Keywords: bone-setting robot; ankle; redundant parallel platform; dynamic movement primitive; multivariate Student-t processes; variable impedance control



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1. Introduction

The ankle–foot complex plays a crucial role in lower limb functionality, serving as the foundation for balance, support, and locomotion. Ankle fractures, however, represent the most common type of intra-articular fractures, accounting for approximately 3.9% of all fractures [1,2]. Treatment options are guided by the fracture type, ranging from open reduction and internal fixation to non-invasive reduction and external fixation. The latter approach mitigates iatrogenic trauma associated with surgical interventions and the potential for cutting-type injuries due to pin insertion or loosening. However, traditional closed fracture reduction techniques rely heavily on the surgeon's experiential methods, making

the expertise of master-level practitioners scarce and difficult to disseminate. Moreover, the daily diagnostic and treatment efficiency in trauma orthopedics is limited by the physical strain on surgeons. Medical robots represent a significant frontier in the evolution of health-care technology. In the domain of fracture reduction, surgical robots offer the promise of precise and safe ankle fracture reduction while also holding the potential to significantly reduce radiation exposure and physical fatigue among medical personnel [3–7]. Despite advances in orthopedic surgical robots, most research has focused on open or minimally invasive procedures, with limited international progress in developing an intelligent robotic platform for the closed manipulation of ankle fractures. Therefore, addressing the challenge of developing a new intelligent ankle bone-setting robot system that integrates the key technologies of closed fracture reduction techniques with clinical application requirements represents a meaningful and challenging endeavor.

Orthopedic surgical robots represent a pivotal technology driving the development of personalized, precise, and minimally invasive orthopedic surgeries. Given the diversity of fracture types, the existing robotic systems still struggle to meet the complex demands of actual surgical procedures. The development of open surgery robots in the field of fracture reduction has been a focal point in recent years [8–12]. The team led by Thomas developed a serial reduction robot based on the RX90 industrial robot, which achieves high-precision reduction by integrating teleoperation and intraoperative navigation technologies [13]. The FRAC-Robo, which was developed by the University of Tokyo, represents a serial system designed to assist in the reduction of long bone fractures [14]. Liang et al. [15] conducted a kinematic analysis and performance comparison on three redundant parallel mechanisms used for fracture reduction surgeries and identified a redundant parallel fracture reduction robot structure with great tilting dexterity. However, this research did not further carry out work on trajectory planning and compliant control for fracture reduction. Jamwal et al. [16] proposed a parallel robot driven by six intrinsically flexible actuators, which can be used for the automated reduction of femoral fractures. Lin et al. [17] propose a new reduction strategy using the X-ray morphology of femoral neck fractures for preoperative planning. Lu et al. [18] proposed a computer-assisted automatic preoperative path planning method for pelvic fracture reduction surgery based on an enlarged RRT* algorithm, which is difficult to integrate into the surgeon's bone-setting experience. Gebremeskel [19] addresses the challenge of achieving precise alignment and reducing joint space in surgeries for distal tibiofibular syndesmotic injuries by developing an image-guided surgical robot to enhance restoration accuracy. However, most of the above studies fall under the category of open or minimally invasive surgical robots. In the field of ankle fracture management, considerable research has focused on rehabilitation robots [20–23], while the surgical reduction of ankle fractures has received comparatively little attention. It is essential to distinguish between fracture reduction and rehabilitation. Rehabilitation robots typically operate within the physiological range of three degrees of freedom, whereas fracture reduction involves the more complex movements of six degrees of freedom, often requiring significant load-bearing manipulations. In fact, some ankle fractures, such as minor supination fractures, require only closed reduction. Traditional closed reduction involves a variety of complex manual maneuvers; that is, highly skilled experts often have detailed skills for different situations. Such techniques largely rely on subjective experience, necessitating years of learning, understanding, and practice, and have yet to be precisely quantified or theorized through mathematical models. For distal radius fractures, Zha et al. [24] developed a robot-assisted closed reduction and rehabilitation system. However, this study does not involve the learning of closed reduction skills and compliant control of robots. According to the authors' limited investigation, it has been found that there is no work on developing

an intelligent orthopedic robot platform from the perspective of closed reduction for ankle fractures.

Current surgical robots, which are predominantly controlled remotely by surgeons, exhibit limited autonomy during operations. To enhance the autonomy and safety of surgical robots, it is crucial to transfer operational skills from humans to robots by imitating expert maneuvers. Learning from Demonstration (LfD), a significant branch within the field of robotic skill learning, has seen substantial advancements in recent years [25–30]. For instance, Insert and colleagues [31] developed dynamical movement primitive (DMP), which generalizes point-to-point and periodic actions through learning from a single demonstration trajectory. Fanger et al. [32] replaced Local Weighted Projection Regression (LWPR) with Gaussian Process Regression to learn nonlinear forcing terms in DMP. Additionally, Khansari-Zadeh's team [33] introduced the Stable Estimator of Dynamical Systems (SEDS), optimizing Gaussian Mixture Model (GMM) parameters based on multiple samples to ensure stability in Gaussian Mixture Regression (GMR). Paraschos and others [34] presented the concept of Probabilistic Movement primitive (ProMP), estimating trajectory parameters probabilistically and adjusting trajectories through conditional Gaussian probabilities. Calinog et al. [35] developed Task-Parameterized GMM (TP-GMM), projecting trained trajectories into local coordinate systems for probabilistic modeling, addressing the generalization issue of GMM in robotic task spaces. Huang et al. [36–39] proposed Kernelized Movement Primitive (KMP), achieving non-parametric skill learning by minimizing the Kullback–Leibler divergence between parametric and sample trajectories using kernel techniques. With the ability to transfer human skills relying on minimal samples without the need for prior knowledge or data, LfD has been widely applied in various fields such as medical robots [40–42]. Yang et al. [40] developed a human–robot impedance adaptive skill transfer interface within a human–machine communication framework. Osa et al. [41] developed an automatic knot-tying system that learns to tie knots from a single surgeon's demonstration. However, a single demonstration lacks consistency and is insufficient for modeling a robust skill library. Thus, learning from multiple demonstrations is essential for extracting operational skills and training skill models. Hang et al. [42] transferred motion skills from multiple human demonstrations in open surgery to robotic manipulators in robot-assisted minimally invasive surgery (RA-MIS) by using LfD. Pan et al. [43] mapped a surgeon's closed reduction trajectories for pelvic fracture by using discrete dynamic movement primitive (DMP). However, this study only involves the collection and analysis of the closed reduction action and does not adopt the closed reduction scheme based on the robot. Zhang et al. [44] proposed a framework for autonomously performing appendectomies by using DMP, which is generated from learned surgical movements. In [45], Liu et al. proposed the PSKMP by kernelizing the postural synergistic subspaces with the kernel treatment to preserve human natural movement characteristics. Notably, most existing studies focus solely on applying LfD, such as DMP, to surgical or rehabilitation robots. However, they often overlook how the precision of imitation learning algorithms influences the outcomes of surgical or rehabilitation processes.

Safety is a paramount concern in the control of surgical robots. Typically, robots utilize impedance–admittance control technology to achieve compliance, thereby ensuring safe control within human–robot collaborative systems [46–49]. Constant impedance parameters often fall short of meeting the diverse requirements of different work tasks, contact environments, and collaboration partners. Variable impedance controllers, superior to constant impedance controllers, exhibit enhanced performance in human–robot interaction tasks [50–52]. In the field of rehabilitation robots, extensive research has been conducted on variable impedance control. Liu and colleagues [53] introduced a novel application of characteristic impedance control for assistive/rehabilitation robots based on Variable Stiff-

ness Actuators (VSAs) utilizing a cascaded position-torque control loop. Brahmi et al. [54] proposed a new adaptive impedance, incorporating backstepping control, delay estimation, and disturbance observers for executing passive assistive rehabilitation movements. Fereydooni et al. [55] designed an adaptive impedance control for rehabilitation robots using surface electromyography (sEMG) and a dual closed-loop control strategy based on human force. Prashant et al. [56] developed a parallel ankle rehabilitation robot powered by Pneumatic Muscle Actuators (PMAs) and introduced an impedance control-based interactive training paradigm. Wang et al. [57] combined PI-2 reinforcement learning optimization with impedance control to propose a new layered adaptive impedance robot control strategy. In the surgical robot domain, Zhang et al. [58] proposed a fuzzy adaptive sliding mode impedance control method for reduction robots to compensate for various uncertainties and nonlinearities in the fracture reduction parallel robot system. Xiao and colleagues [59] introduced a robotic control system for intraoperative ultrasound imaging, employing variable impedance control to adjust the contact force of the ultrasound transducer, thus optimizing the visual quality of ultrasound examinations. Considering Remote Center of Motion (RCM) constraints in the operational space formulation for a 7 DoFs redundant robot, Su et al. [60] proposed a hierarchical task impedance control scheme for minimally invasive surgery (MIS). Bian [61] proposed a new probabilistic model approach to learning the skills of minimally invasive surgery and to solving the inconsistencies between the RCM constraint space and the surgical task space. However, impedance control for the redundant parallel mechanisms is rarely discussed at present.

This study proposes an integrated system framework that uses a variable impedance control strategy based on modified DMP, aiming to safely and effectively transfer the surgeon's closed reduction techniques to a bone-setting robot while minimizing patient harm. The overall framework diagram of the bone-setting robot system is depicted in Figure 1. The framework consisted of three key parts: First, professional surgeons demonstrated the closed reduction techniques for ankle fractures multiple times, while an Optical Tracking System captured several 6D reduction trajectories. The reduction trajectories underwent coordinate system transformation and data preprocessing to create a comprehensive expert demonstration dataset. Second, the proposed MV-TP-DMP algorithm was employed to perform imitation learning on both position and orientation trajectories from the expert demonstration, generating the desired reduction trajectories for the control strategy. Lastly, a novel variable impedance controller was designed based on the desired reduction trajectories and inverse dynamic analysis, mimicking the closed reduction techniques and ensuring operational flexibility and effectiveness. The contributions of this study are as follows:

- (1) We designed a novel redundant parallel bone-setting robot for closed ankle reduction incorporating a singularity-free redundant kinematic solver. The proposed redundant parallel platform has the advantages of great dexterity and load, which can meet the needs of the closed ankle reduction process.
- (2) We proposed a modified DMP algorithm, termed MV-TP-DMP, which improves the accuracy of 6D reduction trajectory reproduction while enhancing generalization capabilities. Considering the correlation of multidimensional trajectories, the proposed algorithm can show better trajectory reproduction accuracy and generalization performance than the classic DMP algorithm.
- (3) We performed a dynamic analysis for the redundant parallel bone-setting robot and introduced a position-based variable impedance controller featuring a novel variable stiffness strategy. Compared with the constant impedance controller, the proposed variable impedance controller can better adapt to different types of disturbances and exhibit more human-like compliance characteristics.

of the ankle joint. Medically, ankle fractures are commonly classified using the Lauge-Hansen system, which categorizes fractures into four main types: supination–adduction, supination–external rotation, pronation–adduction, and pronation–external rotation [63]. Depending on the type of ankle fracture, treatment may involve closed reduction or open reduction. Moreover, studies have shown that up to one-third of ankle fractures and dislocations present as open injuries, with a higher incidence of dislocations without fractures. The prompt closed reduction of identifiable ankle fracture-dislocations is essential to reduce the risk of neurovascular and skin complications, relieve soft-tissue tension, minimize cartilaginous impingement, and avoid delays in surgery [64,65]. Therefore, the development of a robotic system capable of timely closed reduction aligns significantly with medical needs. Figure 3a,b illustrates the classic closed reduction maneuvers.

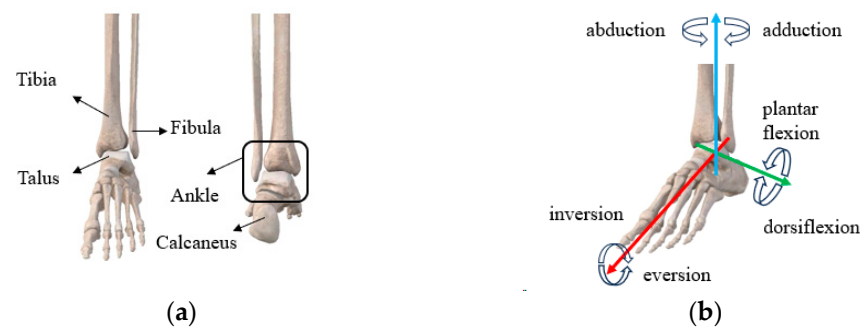


Figure 2. Ankle illustration. (a) The anatomical and physiological structure of the ankle complex. (b) The movements of the ankle.

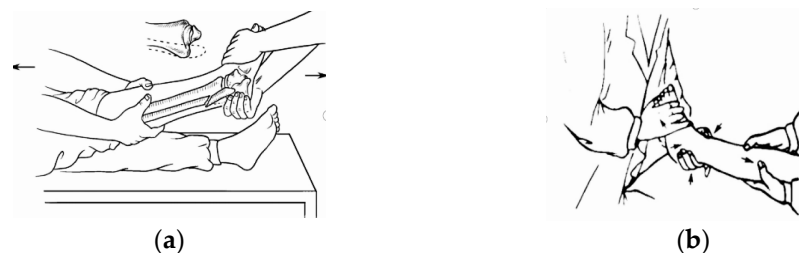


Figure 3. Classic closed reduction maneuvers. (a) Traction. (b) Inversion and lift.

2.1.2. Mechanical Structure Design of Bone-Setting Robot

Ankle fracture reduction typically requires extension, traction with significant force, plantar flexion with large rotation angles, and so on. To address these demands, we propose a redundant parallel bone-setting robot capable of performing autonomous closed reduction, eliminating the need for manual intervention by surgeons. The system includes a redundant parallel reduction platform, fixation boot, adjustable calf support, and so on.

The redundant parallel reduction robot proposed in this study is an enhancement of the classic Stewart–Gough platform [66]. The Stewart–Gough platform consists of a moving platform connected to a base through six branches, each comprising a passive universal joint, a driven prismatic joint, and a spherical joint attached to the moving platform. This parallel mechanism, known for its excellent load capacity and high control precision, has been widely applied in clinical settings [66–68]. Therefore, it can be shown that similar parallel robot solutions are clinically feasible and safe. Considering the intricate and diverse anatomical presentations of severe ankle fracture-dislocations and talar avulsion fractures with significant angular deformities, the reduction mechanism requires a high demand for tilt dexterity. Therefore, this study introduces a new redundant parallel platform structure based on the Stewart–Gough platform. The CAD model of the proposed redundancy mechanism is shown in Figure 4. This mechanism features six prismatic actuators and three

sliding actuators on the base box, controlling the end-effector with six degrees of freedom. It should be pointed out that the two actuators here are essentially single-degree-of-freedom moving mechanisms, but the driving principle is slightly different. Moreover, for the six prismatic actuators, one end is connected to the moving platform via Hooke joints and the other ends are connected in pairs to the sliding blocks on the sliding actuators through Hooke joints, which are the classical universal joint and offer a wide angular range and a high load capacity. Similar redundant parallel mechanisms have been verified in the literature [15]. Compared to the Stewart–Gough platform, the redundant parallel platform proposed can demonstrate enhanced motion range and dexterity, making it a promising solution for ankle fracture reduction robots.

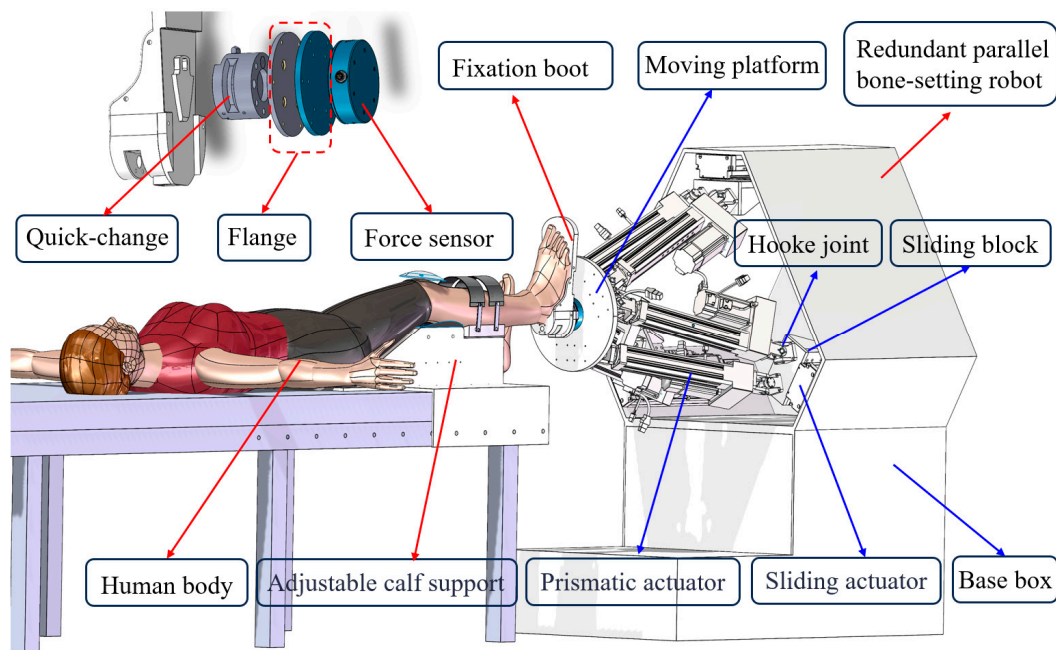


Figure 4. Schematic diagram of the bone-setting robot system. The red arrow points to the core component of the bone-setting robot system, while the blue arrow indicates the core component of the redundant parallel bone-setting robot.

In addition, to achieve the purpose of non-invasive and stable fixation for the patient's foot, this study has designed a specialized fixation boot featuring quick-release straps and cushioning materials by referring to non-invasive traction structures that have already been used in the clinic. The base of the fixation boot is connected to the parallel platform through a quick-change device and a six-axis force sensor. It should be noted that fixation boots can be manufactured using 3D printing, which not only reduces costs but also allows for integration with fixation devices to create a comprehensive orthotic boot for post-surgical stabilization of ankle fractures. Additionally, considering that patients often need to maintain a bent knee posture during closed reduction, the design includes a calf support adjustable to the leg length of different patients, aimed at providing comfortable and stable support. A detailed illustration of the overall design scheme is presented in Figure 4.

Figure 5 presents the process of robot bone-setting. It should be noted that the focus of this paper is on imitation learning trajectory planning and control for the redundant parallel bone-setting robot. Specifically, the operation procedures related to the system proposed in this paper are as follows: The patient is positioned supine on the operating table, with the affected limb placed on an adjustable calf support and the knee flexed. The calf is secured using straps, and a fixation boot is applied, achieving non-invasive fixation with convenient

straps. The robot is then switched to drag-teaching mode, and the redundant parallel bone-reduction robot, equipped with a six-dimensional force sensor, is securely connected to the fixation boot via a quick-change mechanism. Coordinate transformation and registration are performed using an Optical Tracking System in conjunction with the reconstructed model. With the aid of interactive software, the surgeon defines the end-effector pose for bone reduction. The robot utilizes a modified dynamic movement primitive (DMP) algorithm to plan the reduction trajectory and executes bone-setting by using variable impedance control. Finally, postoperative bone fixation is completed. Compared with the existing open or minimally invasive orthopedic robots [43,58], the scheme proposed in this paper has the characteristics of non-invasive, high load, high dexterity, convenient operation, and low cost.

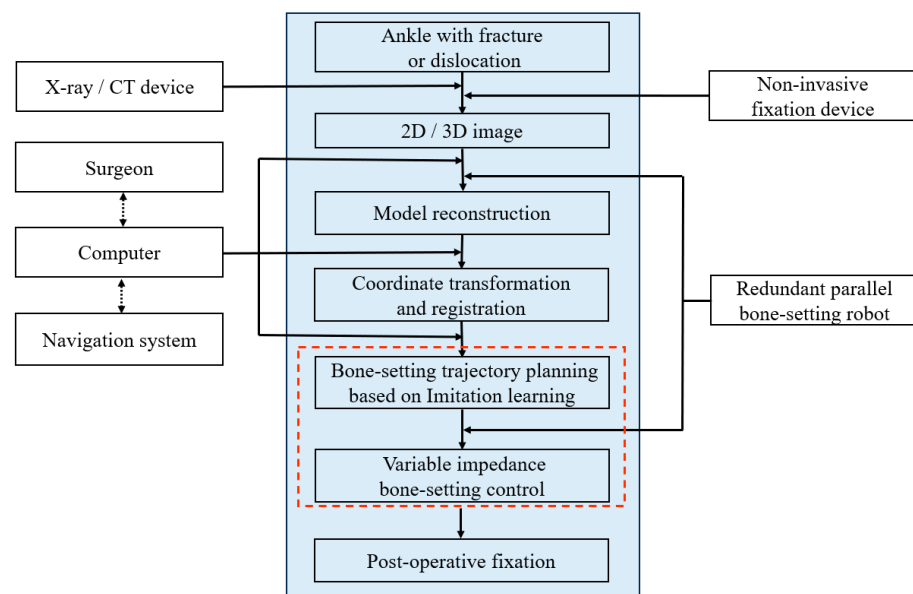


Figure 5. Process of robot bone-setting.

2.2. Kinematics Analysis of Redundant Parallel Platform

In this section, to gain a comprehensive understanding of the structural and kinematic characteristics of the redundant parallel robot platform, we have developed a foundational framework model, as illustrated in Figure 6a. The origin of this framework is defined at the intersection point of the projections of three linear sliding rails on the base, with the detailed layout presented in Figure 6b,c. The X-axis is aligned with the projection of one of the linear rails, while the Z-axis is perpendicular to the base of the platform. The angles between the projections of the three linear rails are equal, set at $2\pi/3$. The origin of the moving platform's coordinate system is located at its center. The X'-axis is perpendicular to the line connecting points b_1 and b_2 , passing through their midpoint, and the Z'-axis remaining consistently perpendicular to the moving platform.

In the context of describing the spatial positioning of an arbitrary point on an end-effector, it can be delineated through the position vector of the origin of the body-fixed frame P with respect to the reference frame B. Additionally, the orientation of the end-effector within frame B is characterized by a trio of Euler angles $(\psi \ \theta \ \varphi)$ following a Z-Y-X sequence, which defines the orientation through successive rotations about the Z, Y, and X axes, respectively. The transformation matrix facilitating the transition from P to B encapsulates this spatial relationship.

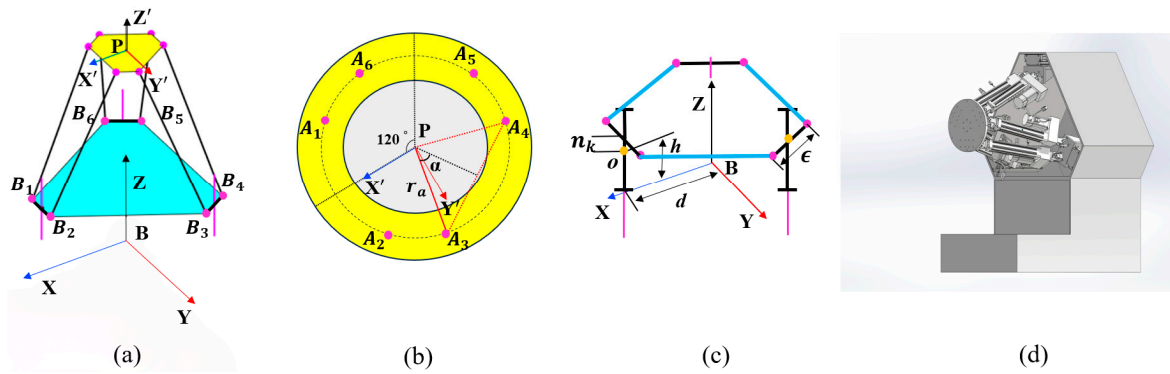


Figure 6. Schematic diagram of redundant parallel platform. (a) Diagram of overall. (b) Moving platform. (c) Base. (d) Three-dimensional model.

$$R = R_z(\psi)R_y(\theta)R_x(\varphi) = \begin{bmatrix} c\psi c\theta - s\psi c\varphi + c\psi s\theta s\varphi & s\psi s\varphi + c\psi s\theta c\varphi \\ s\psi c\theta & c\psi c\varphi + s\psi s\theta s\varphi & -c\psi s\varphi + s\psi s\theta c\varphi \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix} \quad (1)$$

The pose vector of a_i in the base coordinate system can be expressed as follows:

$$a_i = p + R\hat{s}_i \quad (2)$$

where $p = [p_x \ p_y \ p_z]^T$ represents the position vector of P in the base coordinate. a_i represents the position vector of A_i in moving platform coordinate with $i = 1, 2, 3, 4, 5, 6$. Then, $\hat{s}_i$ is the position vector of A_i relative to moving platform coordinate $PX'Y'Z'$, which can be obtained as follows:

$$\hat{s}_i = \vartheta_i \begin{bmatrix} r_a \cos \alpha & (-1)^i r_a \sin \alpha & 0 \end{bmatrix}^T \quad (3)$$

where r_a represents the radius of the circle which can be determined by the point A_i , and α denotes half of the angle between PA_1 and PA_2 , PA_3 and PA_4 , and PA_5 and PA_6 . $\vartheta_i = Rot(Z, \sigma_i)$, and the three sets of points are evenly distributed on the moving platform at an interval of 120 degrees. Then, we have $\sigma_1 = \sigma_2 = 0$, $\sigma_3 = \sigma_4 = 2\pi/3$, and $\sigma_5 = \sigma_6 = 4\pi/3$.

According to the geometry of the mechanism, we can obtain the expression of the position vector of B_i ,

$$b_i = \vartheta_i \begin{bmatrix} d(-1)^i \epsilon / 2 h + n_k \end{bmatrix}^T \quad (4)$$

where d is defined as the distance from the X -axis projection of the linear rail's midpoint to the point o . The variable n_k represents the distance from the sliding block to the midpoint of the k -th linear rail, where k is assigned values of 1, 2, or 3. The direction in which the sliding block moves downward along the linear rail is designated as positive, as illustrated by the red arrows in Figure 6b. Accordingly, n_k is assigned a positive value when the sliding block descends past the linear rail's midpoint. In contrast, it is deemed negative when the movement is in the opposite direction. The symbol ϵ denotes the distance between the bottom ends of two adjacent legs. The variable h quantifies the vertical distance between the linear rail's midpoint and its projection. The relationship between k and h is defined as follows: k equals 1 when i is either 1 or 2; k is 2 when i is 3 or 4; and k is set to 3 when i is either 5 or 6.

Then, by combining Equations (2) and (3), we can obtain the position vector for each leg,

$$L_i = a_i - b_i = p + R \begin{bmatrix} r_a \cos \alpha & (-1)^i r_a \sin \alpha & 0 \end{bmatrix}^T - \vartheta_i \begin{bmatrix} d(-1)^i \epsilon / 2 h + n_k \end{bmatrix}^T \quad (5)$$

The length of each legchain can be expressed as

$$l_i = \|a_i - b_i\| \quad (6)$$

For velocity analysis, by defining $\Omega = [\psi \ \theta \ \varphi]^T$ as the orientation of the moving platform, and ω represents the angular velocity of the moving platform with respect to the base frame, and $\omega = [\omega_x \ \omega_y \ \omega_z]^T$, the relation of ω and the derivative of the moving platform orientation can be expressed as

$$\omega = R_a \dot{\Omega}, \quad (7)$$

$$\dot{\omega} = R_a \ddot{\Omega} + \dot{R}_a \dot{\Omega} \quad (8)$$

where

$$R_a = \begin{bmatrix} 0 & -s\psi & c\theta c\psi \\ 0 & c\psi & c\theta s\psi \\ 1 & 0 & -s\theta \end{bmatrix}, \quad (9)$$

$$R_b = \begin{bmatrix} 0 & -c\psi\dot{\psi} & -s\theta c\psi\dot{\theta} - c\theta s\psi\dot{\psi} \\ 0 & -s\psi\dot{\psi} & -s\theta s\psi\dot{\theta} + c\theta c\psi\dot{\psi} \\ 0 & 0 & -c\theta\dot{\theta} \end{bmatrix} \quad (10)$$

The velocity vector of a_i can be computed by the time derivative of Equation (2),

$$\dot{a}_i = \dot{p} + \omega \times s_i \quad (11)$$

where $s_i = R\hat{s}_i$. The above relation can be represented as

$$\dot{a}_i = \left[I_{3 \times 3} - \tilde{s}_i R_a \right] \begin{bmatrix} \dot{p} \\ \dot{\Omega} \end{bmatrix} \quad (12)$$

where $\tilde{s}_i$ is the skew-symmetric matrix of s_i , defined as a square matrix S that satisfies the condition $S^T = -S$.

The derivation of Equation (4) can be written as

$$\dot{b}_i = \mathfrak{H} \dot{n}_k \quad (13)$$

where $\mathfrak{H} = [0 \ 0 \ 1]$.

The vector a_i on the moving platform can also be expressed as

$$a_i = b_i + l_i \vec{u}_i \quad (14)$$

where $\vec{u}_i$ denotes the unit vector corresponding to the leg of the robot. The derivative of Equation (14) can be written as

$$\dot{a}_i = \dot{b}_i + \dot{l}_i \vec{u}_i + l_i \mathfrak{N}_i \times \vec{u}_i \quad (15)$$

where $\mathfrak{N}_i$ is the angular velocity of the leg, which can be obtained as

$$\mathfrak{N}_i = \frac{\vec{u}_i \times (\dot{a}_i - \dot{b}_i)}{l_i} \quad (16)$$

According to the literature [69], we can obtain that

$$\dot{l}_i = \vec{u}_i^T \left[I_{3 \times 3} - \tilde{s}_i R_a \right] \begin{bmatrix} \dot{p} \\ \dot{\Omega} \end{bmatrix} - \vec{u}_i^T \mathfrak{H} \dot{n}_k \quad (17)$$

Defining the pose vector of the center point of the moving platform as $x = [p \ \Omega]^T$, and letting the length of the actuated joints be $q = [q_l \ q_n]^T$ with $q_l = [l_1 \ l_2 \ l_3 \ l_4 \ l_5 \ l_6]^T$ and $q_n = [n_1 \ n_2 \ n_3]^T$, Equation (17) can be rewritten as

$$J_x \dot{x} - J_q \dot{q} = 0 \quad (18)$$

where x represents the pose of the end-effector, and J_x and J_q can be denoted as

$$J_x = \begin{bmatrix} \vec{u}_1^T [I_{3 \times 3} - \tilde{s}_1 R_a] \\ \vec{u}_2^T [I_{3 \times 3} - \tilde{s}_2 R_a] \\ \vec{u}_3^T [I_{3 \times 3} - \tilde{s}_3 R_a] \\ \vec{u}_4^T [I_{3 \times 3} - \tilde{s}_4 R_a] \\ \vec{u}_5^T [I_{3 \times 3} - \tilde{s}_5 R_a] \\ \vec{u}_6^T [I_{3 \times 3} - \tilde{s}_6 R_a] \end{bmatrix} \quad (19)$$

$$J_q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \vec{u}_1^T \mathfrak{H} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \vec{u}_2^T \mathfrak{H} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \vec{u}_3^T \mathfrak{H} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \vec{u}_4^T \mathfrak{H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \vec{u}_5^T \mathfrak{H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \vec{u}_6^T \mathfrak{H} & 0 & 0 \end{bmatrix} \quad (20)$$

It can be seen from the equation that the elements of the direct Jacobian matrix are non-homogeneous in dimension. In order to obtain a Jacobian matrix with homogeneous dimensions, we directly divide the elements in the fourth, fifth, and sixth columns of the Jacobian matrix by the characteristic length $l = (a \cos \alpha + b \sin \alpha)/2$.

For acceleration analysis, by the time derivative of Equation (11), we can obtain the acceleration of a_i , which can be represented as

$$\ddot{a}_i = \ddot{p} + \omega \times (\omega \times s_i) + \dot{\omega} \times s_i \quad (21)$$

In combination with Equation (7), Equation (21) can be rewritten as

$$\ddot{a}_i = [I_{3 \times 3} - \tilde{s}_i R_a] \begin{bmatrix} \ddot{p} \\ \ddot{\Omega} \end{bmatrix} + (\tilde{\omega})^2 s_i - \tilde{s}_i R_b \dot{\Omega} \quad (22)$$

where $\tilde{\omega}$ is the skew-symmetric matrix of ω .

The acceleration vector $\ddot{b}_i$ can be obtained by the derivation of Equation (13),

$$\ddot{b}_i = \mathfrak{H} \ddot{n}_k \quad (23)$$

Similarly, the acceleration of the lateral actuators can be deduced by the time derivation of Equation (17),

$$\ddot{l}_i = \vec{u}_i^T \left[I_{3 \times 3} - \tilde{s}_i R_a \right] \begin{bmatrix} \ddot{p} \\ \ddot{\Omega} \end{bmatrix} + \vec{u}_i^T \left(\left(\tilde{\omega} \right)^2 s_i - \tilde{s}_i R_b \dot{\Omega} \right) - \vec{u}_i^T \mathfrak{H} \ddot{n}_k - \left(\mathfrak{n}_i \times \vec{u}_i \right)^T \mathfrak{H} \dot{n}_k + \left(\mathfrak{n}_i \times \vec{u}_i \right)^T \left[I_{3 \times 3} - \tilde{s}_i R_a \right] \begin{bmatrix} \dot{p} \\ \dot{\Omega} \end{bmatrix} \quad (24)$$

In this study, the sliding blocks of the redundant actuators are locked at the midpoint of each linear rail. The geometric parameters of the redundant parallel robot are listed in Table 1.

Table 1. Geometric parameters.

Parameters	Values or Bounds	Unit
α	42.54	deg
r_a	0.1	m
d	0.26	m
l_i	(0.42, 0.57)	m
n_k	(−0.05, 0.05)	m
ϵ	0.09	m
h	0.05	m

2.3. Singularity Free Redundant Kinematics Solver

When robots enter singular configurations, their performance can significantly degrade or become uncontrollable. It is a critical issue that must be avoided, especially in surgical applications where safety is paramount. Therefore, considering the imperative of safety, the redundant parallel bone-setting robot should steer clear of working in singular configurations. In this study, we use the condition number as a criterion to assess the singularity performance of redundant parallel robots. The singular values of the Jacobian matrix J_x can be expressed as follows:

$$\tau_i = \sqrt{\gamma_i(J_x^T J_x)} \quad (25)$$

where γ_i is the eigenvalues of $J_x^T J_x$. Consequently, the condition number of J_x can be calculated as follows:

$$\text{Cond}(J_x) = \frac{\tau_{\max}}{\tau_{\min}} \quad (26)$$

where $1 \leq C \leq +\infty$. A lower condition number reflects improved singularity performance, ensuring more stable and predictable robot operation. When the condition number approaches infinity, the robot is considered to be in a singular configuration.

Due to the system's three degrees of redundancy, multiple solutions exist for the end-effector's pose within the workspace. To enhance surgical safety, it is crucial to optimize the reduction trajectory to avoid singular points. This is achieved by minimizing the Jacobian matrix's condition number as the objective function. Parallel mechanisms encounter three types of motion singularities [69,70], with the most problematic one arising when the direct Jacobian matrix determinant J_x equals zero. In such cases, the platform can still move locally even with locked joints. The objective function for a singularity-free kinematic solver is defined as follows:

$$\begin{aligned} \argmin_{q_{n+1}} & \text{Cond}(J_x(t_{n+1})) + \delta_l \|q_l(t_{n+1}) - q_l(t_n)\| + \delta_n \|q_n(t_{n+1}) - q_n(t_n)\| \\ \text{s.t. } & q_{\min} < q < q_{\max} \quad \dot{q}_{\min} < \dot{q} < \dot{q}_{\max} \quad \ddot{q}_{\min} < \ddot{q} < \ddot{q}_{\max} \end{aligned} \quad (27)$$

where δ_l and δ_n are the adjustable coefficients, which can affect the solution performance of the inverse kinematics. In this study, the optimization problem was addressed using the FMINCON function, such as SQP, a component of the nonlinear optimization toolbox in MATLAB.

3. Closed Reduction Techniques Learning Based on Modified Dynamic Movement Primitive

3.1. Dynamic Movement Primitive

Dynamic movement primitive (DMP) constitutes a framework for modeling movements using dynamic systems defined by nonlinear differential equations [31]. It is adept at capturing the intricacies of one or a limited number of demonstration trajectories, facilitating the replication and generalization of proficiently performed movements. A standard one-dimensional DMP can be expressed as follows:

$$\begin{cases} \tau_s \dot{z} = \alpha_z (\beta_z (y_g - y) - v) + F(s) \\ \tau_s \dot{y} = z \\ \tau_s \dot{s} = -\gamma s \end{cases} \quad (28)$$

where y is the current position, y_g is the desired position, z is the scaled velocity, τ_s is the temporal scaling parameter, s is the scalar phase variable, and α_z and β_z are the design parameters. Generally, $\alpha_z = 4\beta_z$, so that gets a critically damped system. $F(s)$ is the nonlinear forcing term, which allows DMP to generate any smooth trajectory. It is the product of the synthetic center point of the force field and the phase variable obtained by the linear combination of N force fields,

$$F(s) = \frac{\sum_{i=1}^N \Psi_i(s) \omega_i}{\sum_{i=1}^N \Psi_i(s)} s \quad (29)$$

$$\Psi_i(s) = \exp(-h_i(s - c_i)^2) \quad (30)$$

where $\Psi_i(s)$ is radial basis function, ω_i represents the weight coefficient, and N is the number of the basis functions. $F(s)$ does not depend directly on the time variable, but on the phase variable s , so the expression of s can be obtained by the integral regular system,

$$s = \exp\left(-\frac{\gamma}{\tau_s} t\right) \quad (31)$$

For DMP learning of closed ankle fracture reduction trajectories, we first collected multiple sets of expert reduction data using an Optical Tracking System. These trajectories were then preprocessed to create a dataset of expert reduction techniques,

$$G = \left\{ \left\{ t_n, x_{h,n} \right\}_{h=1}^H \right\}_{n=1}^N \quad (32)$$

where t_n represents the n -th time step, and $x_{h,n}$ represents the h -th I-dimensional input and O-dimensional output vectors of the n -th demonstration in the database. H and N denote the number of demonstrations and the length of each demonstration in the database, respectively. The details of the technique acquisition process are described in Section 5.1.

For the robotic imitation learning of reduction trajectories, the system must not only adapt to translational adjustments, such as traction, but also accommodate complex variations in the orientation space. Hence, we designed two multidimensional DMPs to learn the position trajectories and the orientation trajectories, respectively. Furthermore, in order to ensure the synchronization between the multidimensional motions, the whole process is

driven by the same regular system. The corresponding multidimensional position DMP and multidimensional orientation DMP can be uniformly defined as

$$\begin{cases} \tau_s \dot{v} = \alpha_p (\beta_p (p_g - p) - v) + F_p(s) \\ \tau_s \dot{\omega} = \alpha_\Omega (\beta_\Omega (\Omega_g - \Omega) - \omega) + F_\Omega(s) \\ \tau_s \dot{p} = v \\ \tau_s \dot{\Omega} = \omega \\ \tau_s \dot{s} = -\gamma s \end{cases} \quad (33)$$

where p and Ω are the current position and current orientation; p_g and Ω_g are the desired position and orientation; v and ω represent the scaled velocity and angular velocity; τ_s is the temporal scaling parameter; and α_p , β_p , α_Ω and β_Ω are the design parameters. $F_p(s)$ is the nonlinear forcing term for position DMP and $F_\Omega(s)$ is the nonlinear forcing term for orientation DMP. The uniform system of specifications eliminates the explicit time dependence of forced terms and therefore allows for online reactions. $F_p(s)$ and $F_\Omega(s)$ are usually modeled using Locally Weighted Projection Regression (LWPR) [71], which can be approximated by a set of radial basic functions,

$$F_p(s) = \frac{\sum_{i=1}^{N_p} \Psi_i(s) \omega_i^p}{\sum_i^N \Psi_i(s)} s \quad (34)$$

$$F_\Omega(s) = \frac{\sum_{i=1}^{N_\Omega} \Psi_i(s) \omega_i^\Omega}{\sum_i^N \Psi_i(s)} s \quad (35)$$

where ω_i^p and ω_i^Ω are represent the weight coefficient.

3.2. MV-TP-DMP

DMP serves as a framework for representing goal-directed or periodic movements acquired through observation or demonstration [31]. It uses flexible function approximators to capture various movements while ensuring goal convergence in point-to-point motion. In earlier studies on dynamic movement primitive (DMP), Local Weighted Projection Regression (LWPR) was employed to approximate the nonlinear forcing terms F_p and F_Ω from Equations (34) and (35) based on demonstration data. This approach incorporates dimensionality reduction and applies a finite number of linear models to fit the data. A notable advantage of LWPR is its low computational complexity, $O(m)$, relative to the number of training samples, which makes it particularly well suited for real-time learning applications. Nevertheless, LWPR requires the meticulous manual adjustment of hyperparameters and the deployment of multiple local models, as its linear assumptions hold true only within small regions. Gaussian Process Regression (GPR) is particularly favored for its superior regression performance, offering more precise predictions compared to LWPR and Gaussian Mixture Regression. Notably, one study [32] demonstrated the applicability of GPR in learning the nonlinear forcing terms within DMP by replacing LWPR, also providing a quantifiable measure of motion uncertainty, thus proving GPR's effectiveness in ensuring motion stability.

Traditional Gaussian Regression models often treat each output dimension independently, disregarding potential interdependencies. To overcome this limitation, a recent study [72] proposed the Multivariate Gaussian Process Regression (MV-GPR) algorithm. MV-GPR not only outperforms LWPR in fitting accuracy but also provides a more robust framework for assessing uncertainties in demonstration data. The main advantage of MV-GPR is its ability to capture both data point correlations and relationships between different responses. Additionally, MV-GPR handles large outputs efficiently without re-

quiring matrix vectorization, making it ideal for tasks like Multivariate Student-t Process Regression (MV-TPR).

In this study, we propose the adoption of MV-TPR in place of LWPR for learning the nonlinear forcing terms F_p and F_Ω in DMP, aiming to emulate and learn multiple closed reduction trajectories. This approach, named MV-TP-DMP, is expected to enhance both the precision and efficiency of DMP in multidimensional output prediction while also improving the quantification of motion uncertainty. The MV-TPR algorithm is briefly described next.

The multivariate Gaussian process $\mathcal{G}$ on the dataset is defined as follows: $\mathcal{G}$ is a multivariate Student-t process on χ with parameter $\hat{\nu} > 2$, vector-valued mean function $\mu : \chi \rightarrow \mathbb{R}^d$, covariance function (also called kernel) $\hat{k} : \chi \times \chi \rightarrow \mathbb{R}$, and positive semi-definite parameter matrix $\hat{\Omega} \in \mathbb{R}^{d \times d}$, then any finite collection of vector-valued variables have a joint matrix-variate Student-t distribution:

$$\left[\mathcal{F}(\hat{x}_1)^T, \dots, \mathcal{F}(\hat{x}_n)^T \right]^T \sim MT\left(\hat{\nu}, \hat{M}, \hat{\Sigma}, \hat{\Omega}\right), n \in N, \quad (36)$$

where $\mathcal{G}, \hat{u} \in \mathbb{R}^d$ are row vectors whose components are the functions $\{\mathcal{F}_i\}_{i=1}^d$ and $\{\mu_i\}_{i=1}^d$, respectively. Furthermore, $\hat{M} \in \mathbb{R}^{n \times d}$ with $\hat{M}_{ij} = \mu_j(\hat{x}_i)$ and $\hat{\Sigma} \in \mathbb{R}^{n \times n}$ with $\hat{\Sigma}_{ij} = \hat{k}(\hat{x}_i, \hat{x}_j)$. So we can denote $\mathcal{F} \sim MTP(\hat{\nu}, \hat{u}, \hat{k}, \hat{\Omega})$.

Given n pairs of observations $\{(\hat{x}_i, \hat{y}_i)\}_{i=1}^n, \hat{x}_i \in \mathbb{R}^p$, we assume

$$\mathcal{F} \sim MTP\left(\hat{\nu}, \hat{u}, \hat{k}', \hat{\Omega}\right), \hat{\nu} > 2, \hat{y}_i = \mathcal{F}(\hat{x}_i), \text{ for } i = 1, \dots, n, \quad (37)$$

where m is the degree of freedom of the Student-t process and the remaining parameters have the same meaning as the MV-GPR model. Consequently, the predictive distribution is obtained as follows:

$$p\left(\mathcal{F}_* \middle| \tilde{X}, \tilde{Y}, X_*\right) = MT\left(\tilde{\nu}, \tilde{M}, \tilde{\Sigma}, \tilde{\Omega}\right), \quad (38)$$

where

$$\tilde{\nu} = \hat{\nu} + n, \quad (39)$$

$$\tilde{M} = K'\left(X_*, \tilde{X}\right)^T K'\left(\tilde{X}, \tilde{X}\right)^{-1} \tilde{Y}, \quad (40)$$

$$\tilde{\Sigma} = K'(X_*, X_*) - K'\left(X_*, \tilde{X}\right)^T K'\left(\tilde{X}, \tilde{X}\right)^{-1} K'\left(X_*, \tilde{X}\right) \quad (41)$$

$$\tilde{\Omega} = \hat{\Omega} + \tilde{Y}^T K'\left(\tilde{X}, \tilde{X}\right)^{-1} \quad (42)$$

According to the expectation and the covariance of matrix-variate Student-t distribution, the predictive mean and covariance are given by

$$E[\mathcal{F}_*] = \tilde{M} = K'\left(X_*, \tilde{X}\right)^T K'\left(\tilde{X}, \tilde{X}\right)^{-1} \tilde{Y} \quad (43)$$

$$\text{cov}\left(\text{vec}\left(\mathcal{F}_*^T\right)\right) = \frac{1}{\hat{\nu} + n - 2} \tilde{\Sigma} \otimes \tilde{\Omega} \quad (44)$$

In summary, the implementation process of the modified DMP algorithm proposed in this paper can be seen in Figure 7. The left side shows the learning and reproduction process of expert samples, while the right side shows the skill generalization process of coping with new tasks based on the learned model.

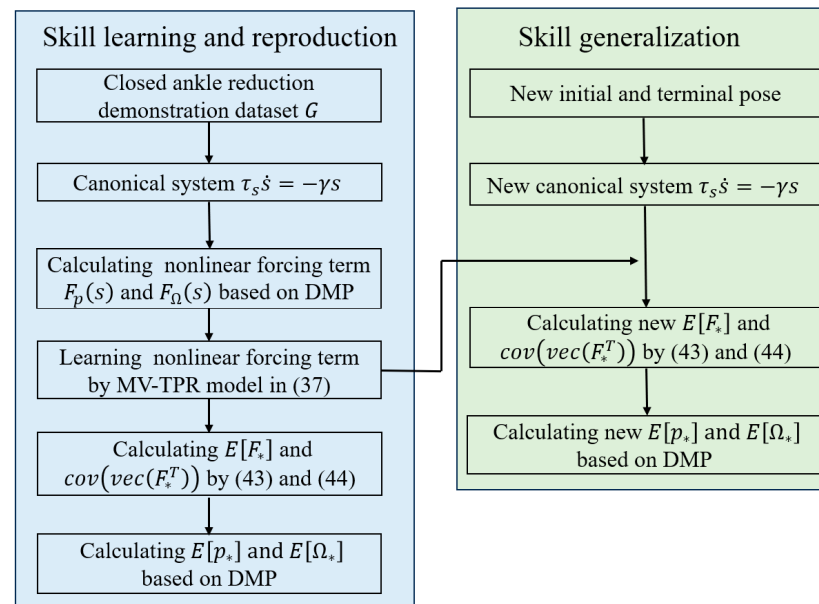


Figure 7. Flow chart of MV-TP-DMP.

4. Variable Impedance Control Based on Inverse Dynamic

4.1. Inverse Dynamics Analysis

Building upon the kinematic analysis presented in Section 2, this section extends the investigation to the dynamics of a redundant robotic platform using the principle of d'Alembert. In this analysis, the flexibility of the limbs, as well as joint friction and backlash, are neglected. The forces in the prismatic and sliding actuators are denoted as f_i and f_k , respectively. For the current mechanism under consideration, we have,

$$m_a \ddot{p} \cdot \nabla p + \sum_k^3 m_b \ddot{n}_k \nabla n_k + (J_m \dot{\omega} + \omega \times J_m \omega) \cdot \nabla \omega_k + m_a g \cdot \nabla p + \sum_k^3 f_k \nabla l_k + \sum_i^6 f_i \nabla l_i = 0 \quad (45)$$

The mass of the moving platform and the concentrated mass at three sliding actuators are represented by m_a and m_b , respectively. The inertia tensor of the moving platform relative to the reference frame is derived as $J_m = R^T \tilde{J} R$, in which $\tilde{J}$ is the inertia matrix about point P in the moving coordinate frame. The virtual change in the length of prismatic actuators is designated by ∇l_i . An expression for ∇l_i is obtained using the kinematic approach in terms of ∇p and $\nabla \Omega$, given by

$$\nabla l_i = \vec{u}_i^T [I_{3 \times 3} - \tilde{s}_i] \begin{bmatrix} \nabla p \\ \nabla \Omega \end{bmatrix} - \vec{u}_i^T \mathfrak{H} \nabla l_k \quad (46)$$

Combining Equations (45) and (46), we can obtain,

$$m_a \ddot{p} \cdot \nabla p + \sum_k^3 m_b \ddot{n}_k \nabla n_k + (J_m \dot{\omega} + \omega \times J_m \omega) \cdot \nabla \omega_k + m_a g \cdot \nabla p + \sum_k^3 f_k \nabla l_k + \sum_i^6 f_i \vec{u}_i^T [I_{3 \times 3} - \tilde{s}_i] \begin{bmatrix} \dot{p} \\ \dot{\Omega} \end{bmatrix} - \sum_i^6 f_i \vec{u}_i^T \mathfrak{H} \nabla l_k = 0 \quad (47)$$

By simplifying, we can obtain the following dynamic equation,

$$\begin{bmatrix} M_u & 0 \\ 0 & M_d \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{q}_n \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{q}_n \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} = \begin{bmatrix} -G_u & 0 \\ G_d & -I_{3 \times 3} \end{bmatrix} \begin{bmatrix} f_l \\ f_n \end{bmatrix} \quad (48)$$

where matrices M_u , M_d , C , and E are

$$M_u = \begin{bmatrix} m_a I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & R_a^T R^T \tilde{J} R R_a \end{bmatrix}, \quad (49)$$

$$M_d = \begin{bmatrix} m_b & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_b \end{bmatrix}, \quad (50)$$

$$C = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & R_a^T \tilde{\omega}^T \tilde{J} R R_a + R_a^T R^T \tilde{J} R R_b \end{bmatrix}, \quad (51)$$

$$E = \begin{bmatrix} m_a \\ 0_{3 \times 3} \end{bmatrix} \quad (52)$$

And matrices G_u , G_d , f_l , and f_n are as follows:

$$G_u = \begin{bmatrix} \begin{bmatrix} \vec{u}_1 \\ -R_a^T \tilde{s}_1 \vec{u}_1 \end{bmatrix} & \begin{bmatrix} \vec{u}_1 \\ -R_a^T \tilde{s}_2 \vec{u}_2 \end{bmatrix} & \begin{bmatrix} \vec{u}_1 \\ -R_a^T \tilde{s}_3 \vec{u}_3 \end{bmatrix} & \begin{bmatrix} \vec{u}_1 \\ -R_a^T \tilde{s}_4 \vec{u}_4 \end{bmatrix} & \begin{bmatrix} \vec{u}_1 \\ -R_a^T \tilde{s}_5 \vec{u}_5 \end{bmatrix} & \begin{bmatrix} \vec{u}_1 \\ -R_a^T \tilde{s}_6 \vec{u}_6 \end{bmatrix} \end{bmatrix}, \quad (53)$$

$$G_d = \begin{bmatrix} \vec{u}_1^T \tilde{s} & \vec{u}_1^T \tilde{s} & 0 & 0 & 0 & 0 \\ 0 & 0 & \vec{u}_1^T \tilde{s} & \vec{u}_1^T \tilde{s} & 0 & 0 \\ 0 & 0 & 0 & 0 & \vec{u}_1^T \tilde{s} & \vec{u}_1^T \tilde{s} \end{bmatrix}, \quad (54)$$

$$f_l = [f_{l1} \ f_{l2} \ f_{l3} \ f_{l4} \ f_{l5} \ f_{l6}]^T, \quad (55)$$

$$f_n = [f_{n1} \ f_{n2} \ f_{n3}]^T. \quad (56)$$

4.2. Position-Based Variable Impedance Control

During the interactive process of fracture reduction, the interaction between the bone-setting robot and the patient's limb may induce significant muscular forces, including spasms and other sudden loads caused by unforeseen circumstances. These forces risk damaging skeletal muscle tissues. To mitigate this risk, this study introduces an impedance control strategy to ensure that the robot can maintain appropriate compliance throughout the whole reduction process.

Based on the practical requirements for closed ankle reduction, the impedance control algorithm must account for variations in the robot's position and force. Additionally, it must consider the impact of changes in the orientation of the end-effector torque during task execution. Consequently, impedance control for redundant robots necessitates establishing a dynamic relationship in Cartesian space among three-dimensional position trajectories, orientation trajectories, force profiles, and torque profiles. According to the principles outlined in [46–48] and the fundamentals of impedance control, the second-order differential equation governing Cartesian space impedance control can be expressed as follows:

$$M_d \ddot{\tilde{x}} + C_d \dot{\tilde{x}} + K_d \tilde{x} = -\tilde{F} \quad (57)$$

where M_d , C_d , and K_d represent the inertia, damping, and stiffness matrices of the desired impedance control model. $\tilde{x}$, $\dot{\tilde{x}}$, and $\ddot{\tilde{x}}$ denote the error between the actual and the desired position, velocity, and acceleration of the end-effector, respectively. $\tilde{F}$ is the force error. In

this study, we still consider position and orientation separately. Therefore, the impedance model can be expressed as follows:

$$M_{dp}(\ddot{p}_d - \ddot{p}) + C_{dp}(\dot{p}_d - \dot{p}) + K_{dp}(p_d - p) = -F_I \quad (58)$$

$$M_{d\Omega}(\ddot{\Omega}_d - \ddot{\Omega}) + C_{d\Omega}(\dot{\Omega}_d - \dot{\Omega}) + K_{d\Omega}(\Omega_d - \Omega) = -M_I \quad (59)$$

In the equations above, the first equation represents the position impedance model, while the second equation represents the orientation impedance model. Here, M_{dp} and $M_{d\Omega}$, forming M_d , are the desired inertia matrices for the position and orientation impedance models, respectively. Likewise, C_{dp} and $C_{d\Omega}$ denote the desired damping matrices, and K_{dp} and $K_{d\Omega}$ are the desired stiffness matrices for the position and orientation impedance models, respectively. And C_{dp} and $C_{d\Omega}$ can constitute C_{dp} , while K_{dp} and $K_{d\Omega}$ can form K_d . To simplify computations, these desired parameter matrices are typically chosen as symmetric positive definite matrices. The vectors p_d , $\dot{p}_d$, and $\ddot{p}_d$ represent the position, velocity, and acceleration of the reference trajectory, respectively, while p , $\dot{p}$, and $\ddot{p}$ denote the position, velocity, and acceleration of the actual trajectory. Ω_d and Ω indicate the reference and actual orientations, respectively. The vectors $\dot{\Omega}_d$, $\ddot{\Omega}_d$, $\dot{\Omega}$, and $\ddot{\Omega}$ correspond to the angular velocities and angular accelerations of the reference and actual trajectories, respectively. The force F_I represents the actual contact force matrices. Similarly, M_I denotes the actual contact torque matrices. Both can be measured using a six-dimensional force sensor.

Let $p_e = p_d - p$ and $\Omega_e = \Omega_d - \Omega$. According to the second-order differential equation for position-based Cartesian impedance control, the corrected accelerations for the desired end-effector displacement and orientation can be expressed as follows:

$$\ddot{p}_e = M_{dp}^{-1} [F_r^B - C_{dp}\dot{p}_e + K_{dp}p_e] \quad (60)$$

$$\ddot{\Omega}_e = M_{d\Omega}^{-1} [M_r^B - C_{d\Omega}\dot{\Omega}_e + K_{d\Omega}\Omega_e] \quad (61)$$

Impedance control enables the reduction robot to maintain flexibility during the bone-setting process. The selection of impedance parameters directly influences the contact characteristics between the robot and the patient's limb [58]. Since variations in the inertial parameter M_d can easily induce system oscillations, adaptive adjustment of the damping parameter C_d or stiffness parameter K_d is commonly employed. The damping matrix C_d and stiffness matrix K_d , respectively, modulate the damping and stiffness characteristics when the robot interacts with the limb. It should be stressed that this study aims to achieve the following objectives: during the normal reduction process, the controller should ensure high tracking accuracy. However, in the event of sudden disturbances such as high-speed or heavy-load impacts, the controller must demonstrate sufficient compliance to ensure the safety of the reduction procedure. To address this, we defined the following variable stiffness strategy:

$$\mathcal{T} = \exp(-|c_a \langle \dot{D}_I \rangle_\rho|^{c_b}) \quad (62)$$

where $\mathcal{T}$ represents a variable stiffness function. D_I is the interaction force in a dimension that is made of F_I and M_I . $\dot{D}_I$ is the rate of change of the interacting force in a dimension. c_a and c_b represent the fine and coarse adjustment coefficients, respectively. $\langle \cdot \rangle_\rho$ represents the moving average calculation, and ρ is a constant that represents the size of the sliding window. It should be pointed out that the sliding average of the interaction force change rate is used to mitigate instability in the impedance system caused by rapid changes in

stiffness. Figure 8a,b show the change characteristics of the variable stiffness coefficient under different parameters.

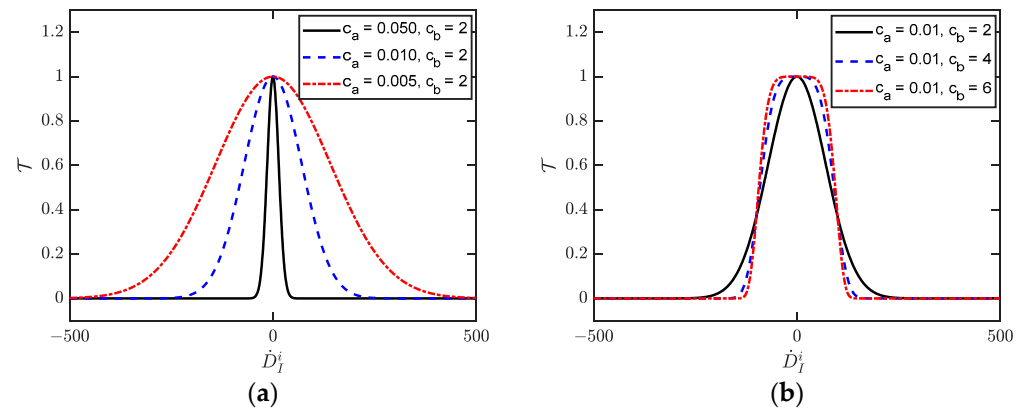


Figure 8. Effects of different coefficients on the variable stiffness function. (a) c_a . (b) c_b .

Therefore, the actual stiffness change expression can be expressed as follows:

$$K_d^i = (1 - \hbar)K_{cd}^i + \hbar \tau K_c^i \quad (63)$$

where $\hbar$ is the constant, constrained by $0 < \hbar < 1$. K_d^i and K_{cd}^i represent the desired stiffness and the defined reference stiffness in a given dimension, where K_{cd}^i is derived from both K_{cdp} and $K_{cd\Omega}$. To achieve the desired impedance model for interaction between the bone-setting robot and the limb, a PD controller based on inverse dynamics is employed, enabling the precise tracking of the desired position, velocity, and acceleration.

4.3. PD Control Strategy Base on Inverse Dynamic

Building on the dynamic analysis outlined previously, we have developed an inverse dynamics Proportional–Derivative (PD) control strategy for a redundant parallel rehabilitation robot. The control equation incorporates six spatial degrees of freedom, x , along with the redundant joint variables q_n , as the state variables, denoted by $\zeta = [x \ q_n]$. The inputs to the controller are represented by $f = [f_l \ f_n]$, where f_n and f_l correspond to the forces in the lateral and normal directions, respectively. To achieve precise motion control in the task space, the desired motion variables, ζ_d , are specified. Utilizing a singularity-free redundancy resolution algorithm discussed earlier, the desired motion variables in the joint space, q_d , are computed. Motion control of the redundant mechanism is then executed by employing the error vector $e = \zeta_d - \zeta$, which measures the deviation from the desired trajectory. According to Equation (48), the control expression is designed as follows:

$$f = \begin{bmatrix} -G_u & 0 \\ G_d & -I_{3 \times 3} \end{bmatrix}^{-1} \left(\begin{bmatrix} M_u & 0 \\ 0 & M_d \end{bmatrix} u + \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{q}_n \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} - \begin{bmatrix} D_l \\ 0 \end{bmatrix} \right) \quad (64)$$

in which

$$u = \ddot{\zeta}_d + \mathcal{K}_d \dot{e} + \mathcal{K}_p e \quad (65)$$

where $\mathcal{K}_d$ and $\mathcal{K}_p$ are the proportional and derivative control gain matrices.

5. Virtual Prototype Simulation

5.1. Demonstration Collection

To evaluate the proposed method, we conducted tests using expert demonstration data collected in real-world settings, aiming to validate its feasibility. Expert demonstrations

were collected by using a specialized acquisition device and the AIMOOE OP-M620 Optical Tracking System, which communicates via ethernet with a sampling frequency of 50 Hz. Prior to data collection, a set of marker balls with four markers was attached to a specially designed optical positioning tool to track position and orientation.

Figure 9 presents the sequence of closed reduction maneuvers for a simulated ankle fracture with dislocation. It highlights the key steps involved in the bone-setting process, including “Pre-reduction”, “Traction”, “Inversion”, “Lift” and “End of reduction”. Correspondingly, we imported the bone-setting trajectory data collected by the Optical Tracking System into the specially built Preoperative Simulation Platform for bone-setting verification simulation. That is, the five figures in Figure 10 correspond to the five bone-setting steps in Figure 9. Note that the Preoperative Simulation Platform used here is constructed using the OpenSim 4.1 software, while the feasibility of this scheme has been successfully verified in multiple studies of fracture reduction robots [18,73]. As shown in Figure 10a, prior to reduction, the talus exhibits displacement, resulting in an abnormal and uneven joint space (JS), with a direct collision between the talus and tibia. The joint space (JS) refers to the distance between the tibia and talus observed in the anteroposterior X-ray of the ankle joint [74]. This metric directly reflects the alignment of the ankle joint surface and indirectly indicates the health of the cartilage and ligamentous structures. When the reduction is complete, the value of JS in Figure 10e is clearly uniform and within the normal range.

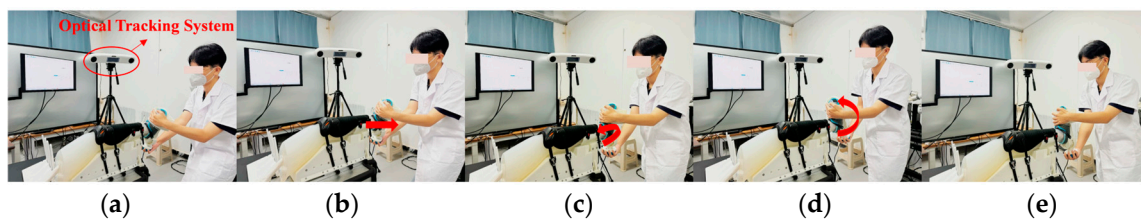


Figure 9. Process of the bone-setting. (a) Pre-reduction. (b) Traction. (c) Inversion. (d) Lift. (e) End of reduction.

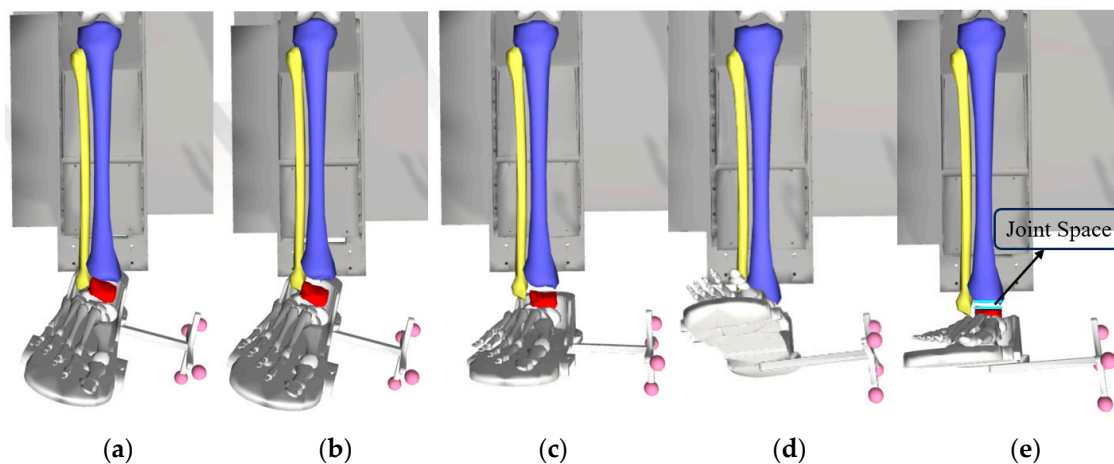


Figure 10. Preoperative simulation for the process of bone-setting. (a) Pre-reduction. (b) Traction. (c) Inversion. (d) Lift. (e) End of reduction.

It is worth noting that the entire demonstration was performed five times to emphasize the reproducibility of the procedure. However, significant variability in trajectory duration occurred due to the challenge for physicians to maintain precision during repeated complex tasks. Therefore, to effectively model these operational skills, it is crucial to align the trajectories of the operational curves. This study employed Dynamic Time Warping

(DTW) [75] to align the five temporal trajectories across all the axes, ensuring consistent trajectory lengths for input data and enabling simultaneous modeling at equivalent time steps. Following alignment, the dynamic movement primitive (DMP) of the operational curves was modeled using Multivariate Gaussian Processes (MV-GPs) and multivariate Student-t processes (MV-TPs). From these trained DMP, a trajectory planning model mimicking closed fracture reduction was subsequently derived and computed.

5.2. Comparison Metrics

5.2.1. Similarity with Original Trajectory

The average distance between samples from the learned distribution and the demonstrated trajectories is computed [76]. This metric is defined mathematically as follows:

$$DTW_{demo} = \frac{1}{m \ n} \sum_{j=1}^m \sum_{i=1}^n DTW(g_{learn}^i, g_{demo}^j) \quad (66)$$

where m and n represent the number of demonstrated and learned trajectories, respectively. The DTW distance between two trajectories is calculated using the distance metric in the Lie group, which can be computed as $d_{norm}(g_1, g_2) = \|g_1 - g_2\|_2$, and $\|\cdot\|_2$ represent the 2-norm.

5.2.2. Similarity with Reference Trajectory

To assess reproducibility, we evaluate the similarity between the learned trajectory and the reference. The error metric is defined as the mean square error (MSE) between the learned trajectory g_{learn} and the reference end-effector trajectories g_{ref} , i.e.,

$$MSE_{ref} = \frac{1}{T} \sum_{t=1}^T \|g_{learn} - g_{ref}\|_2 \quad (67)$$

5.2.3. Adaptability to Target Pose

The distance between the end-pose of the learned trajectory and the desired target pose μ_t^* , i.e.,

$$D_{tar} = \frac{1}{n} \sum_{i=1}^n d_{norm}(g_{learn,t}^i, \mu_t^*) \quad (68)$$

5.3. Reproducing and Generalizing for Reduction Trajectory

This section validates the effectiveness of the proposed modified DMP algorithm through trajectory learning tests involving both reproduction and generalization. Five expert demonstrations were utilized, with the first four datasets used for training and the fifth dataset reserved for validation. The demonstrated trajectories, which are represented by the gray lines in Figure 11, served as input for DMP training. Under the guidance of experienced surgeons, all the demonstrations were categorized into three distinct phases: “Traction” (0–4 s), “Inversion” (4–7 s), and “Lift” (7–10 s). These phases align with the corresponding actions shown in Figure 9b–d and are visually delineated by three color blocks in Figure 11, highlighting the sequence of the closed reduction process. Comparative evaluations were conducted for standard dynamic movement primitives (DMPs) and the proposed methods, including Multivariate Gaussian Process-based DMP (MV-GP-DMP) and multivariate Student-t process-based DMP (MV-TP-DMP), to assess their ability to replicate the demonstrated trajectories accurately. The parameters used for DMP are as follows: $\alpha_p = 500$, $\beta_p = 125$, and $N_p = 50$ for position and $\alpha_\Omega = 300$, $\beta_\Omega = 88$, and $N_\Omega = 50$ for orientation.

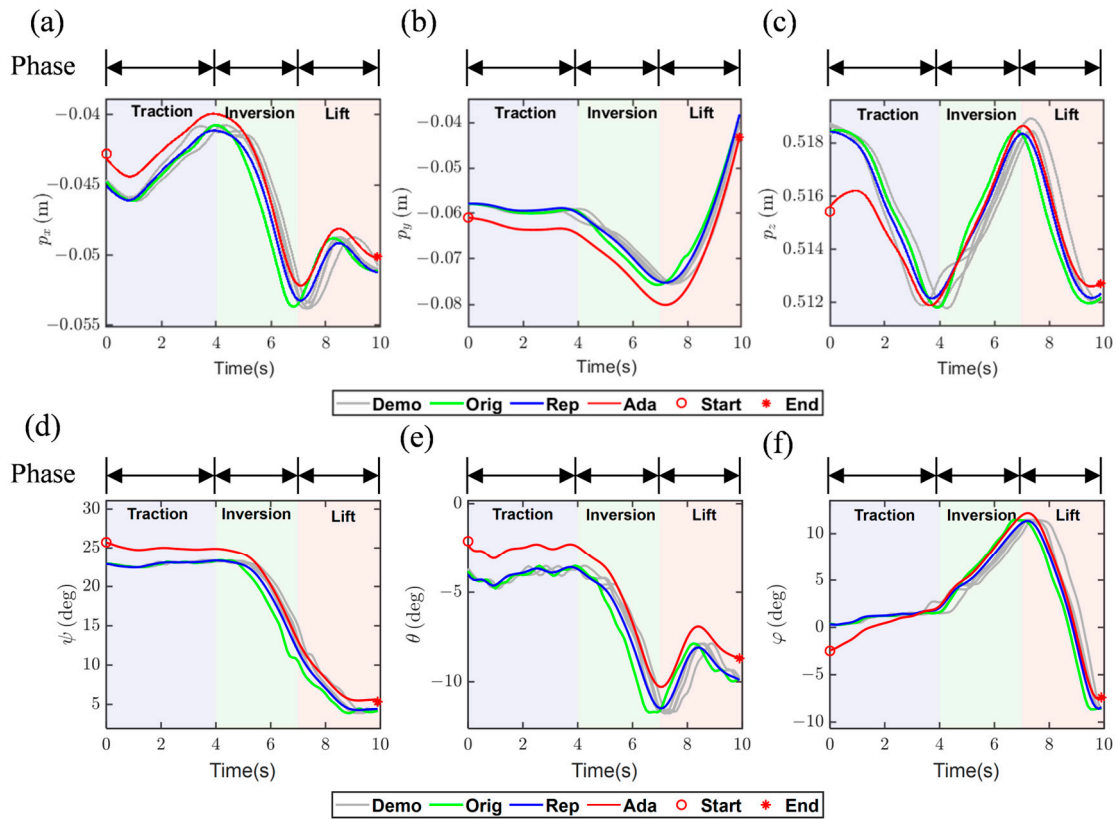


Figure 11. Reproducing and generalizing tests of position and orientation trajectory for MV-TP-DMP. The gray lines represent the demo trajectories used for training, which are processed by DTW and denoised. The green lines denote the reference trajectories used for the reproduction test. The blue lines indicate the trajectories generated by the MV-TP-DMP algorithm during the reproduction test, while the red lines represent the trajectories produced by the algorithm in the generalization test. The reduction process is divided by using three phases, which correspond to “Traction”, “Inversion”, and “Lift”, respectively. (a–c) represent the position along the X, Y and Z axes, respectively. (d–f) are respectively three Euler angles.

During the reproduction tests, the fifth demonstrated trajectories are used as the reference, which are highlighted by the green lines in Figure 11. The position and orientation of this reference trajectory were input into both the original and modified DMP models for the reproduction test. Subsequently, the algorithms were tasked with reproducing the trajectory, starting from a fixed initial point, $p_s = [-0.0448, -0.0579, 0.5184]$, $\Omega_s = [22.877, -3.8319, 0.3820]$ and terminating at $p_e = [-0.0511, -0.0383, 0.5122]$, $\Omega_e = [4.2005, -9.8356, -8.5427]$. Both the original and modified DMP algorithms successfully reproduced the reference trajectory, but the modified algorithms, particularly MV-TP-DMP, demonstrated superior accuracy in preserving finer trajectory details. The intuitive image results from the MV-TP-DMP algorithm are shown in Figure 11, which are highlighted by the blue lines. Moreover, the improvement in the MV-TP-DMP is also reflected in the Dynamic Time Warping (DTW) similarity measure. Specifically, for position trajectory reproduction, the DTW values for DMP, MV-GP-DMP, and MV-TP-DMP were 0.1756, 0.1186, and 0.0490, respectively. For orientation trajectory reproduction, the DTW values were 772.609, 53.486, and 26.248, respectively. Additionally, the modified DMP algorithms exhibited lower endpoint errors compared to the original DMP, with detailed comparisons provided in Table 2.

Table 2. Comparison of trajectory reproduction tests.

	Position			Orientation		
	D_{tar}	MSE_{ref}	DTW_{demo}	D_{tar}	MSE_{ref}	DTW_{demo}
DMP	4.6×10^{-4}	2.8×10^{-6}	0.1756	0.2268	0.7651	72.609
MV-GP-DMP	3.1×10^{-4}	7.8×10^{-7}	0.1186	0.1196	0.6425	53.486
MV-TP-DMP	1.2×10^{-4}	6.7×10^{-7}	0.0490	0.1287	0.5907	26.248

It is important to note that in clinical practice, patients with ankle fractures present a wide variety of conditions. Even fractures of the same type can exhibit significant differences in initial joint angles. However, the reduction techniques used for these fractures are often quite similar, highlighting the importance of generalization in trajectory planning algorithms. Therefore, in the generalization tests, we altered the initial and terminal pose coordinates to assess the DMP algorithms' ability to generalize to different fracture conditions. The newly defined starting coordinates were $p_s = [-0.0428, -0.0609, 0.5154]$, with orientation $\Omega_s = [25.742, -2.1130, -2.4828]$, and the terminal coordinates were $p_e = [-0.0501, -0.0433, 0.5127]$, with orientation $\Omega_e = [5.3464, -8.6897, -7.3968]$. A detailed comparison of generalization test results is provided in Table 3. All three algorithms demonstrated good generalization capability under new pose conditions, but the modified DMP algorithms, which take into account the correlations between multiple variables, outperformed the original DMP in terms of accuracy at the initial and terminal points. As a result, the generalized reduction trajectories closely resembled the demonstration trajectories. Specifically, the DTW values for orientation trajectory generalization using the MV-TP-DMP algorithm were 0.3691 and 221.46, which is lower than DMP and MV-GP-DMP. The results of the MV-TP-DMP algorithm in the generalization test are displayed in Figure 11, which are indicated by the red line. As seen in the figure, the algorithm effectively handles extrapolated scenarios, with both the start and end points extending beyond the demonstrated range. When the initial pose changes, the entire trajectory shifts accordingly, but the intrinsic relationship between the pose and the trajectory's shape remains consistent.

Table 3. Comparison of trajectory generalization tests.

	Position		Orientation	
	D_{tar}	DTW_{demo}	D_{tar}	DTW_{demo}
DMP	4.5×10^{-4}	0.4811	0.2091	242.85
MV-GP-DMP	3.2×10^{-4}	0.3709	0.1012	231.43
MV-TP-DMP	1.0×10^{-4}	0.3691	0.1115	221.46

Both the reproduction and generalization tests show that all three dynamic movement primitive (DMP) algorithms effectively replicate and generalize closed reduction trajectories. However, the MV-TP-DMP outperforms the others in both accuracy and generalization due to the consideration of the correlations among multiple dimensions. This enables MV-TP-DMP to achieve effectively lower MSE and DTW values, particularly in orientation trajectory reproduction. These improvements allow for more accurate mimicry of expert techniques, making MV-TP-DMP ideal for robotic applications requiring high precision. Additionally, MV-TP-DMP excels in generalization, adapting well to variations in the initial and terminal poses. It maintains improved via-point accuracy under altered fracture conditions, demonstrating its ability to transfer professional closed reduction skills to new scenarios. This adaptability is critical for clinical applications, where patient-specific conditions can vary. In summary, MV-TP-DMP offers superior accuracy, better adaptability, and improved generalization over existing methods, making it a promising solution for

achieving robotic autonomy in orthopedic surgery, where precise and flexible trajectory planning is essential for successful outcomes.

5.4. Results of Variable Impedance Control

To validate the effectiveness of the proposed impedance control algorithm for ankle bone-setting in a redundant parallel robot, we conducted a detailed simulation analysis in the MATLAB environment. The simulation model incorporates both the complete dynamics and kinematics of the parallel robot. The dynamic model accounts for gravity, inertial forces, and unknown external disturbances, while the kinematic model integrates key modules such as a singularity-free inverse kinematics solver, a motion interference detector (accounting for actuator rod interference, Hooke joint constraint detection, and singularity detection), and so on. In this study, the low limb was simplified to a variable stiffness linear model. Furthermore, to assess the robot's response to sudden disturbances, we introduced a sudden lower limb force disturbance in the simulation. Specifically, between 2 and 3 s, an additional disturbance force of -120 N was applied along the Z-axis, as depicted in Figure 12. These disturbances are designed to mimic potential external factors, such as muscle spasms or other unforeseen interferences during the reduction process. Moreover, Table 4 provides detailed parameters for the variable impedance control algorithm.

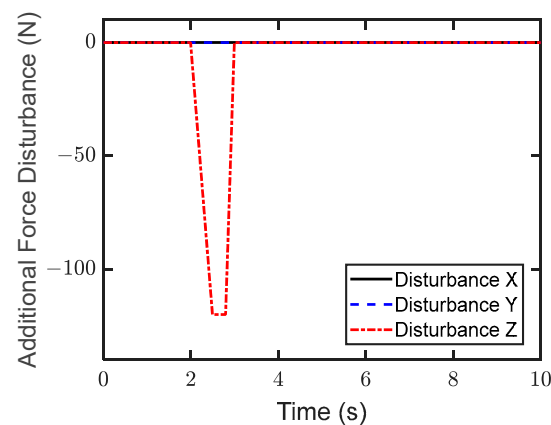


Figure 12. Additional disturbance forces.

Table 4. Parameters of the variable impedance control algorithm.

Parameters	Values or Bounds
M_{dp}	30
$M_{d\Omega}$	30
C_{dp}	200
$C_{d\Omega}$	50
K_{cdp}	500,000
$K_{cd\Omega}$	100
h	0.4
c_a	0.01
c_b	4
ρ	20

Figure 13 compares the reduction trajectory tracking results under constant and variable impedance control. By employing the MV-TP-DMP algorithm to generate the desired reduction trajectory, both controllers achieved high-precision trajectory tracking and demonstrated a certain degree of flexibility. Under the influence of human–robot interaction forces, the impedance control algorithm continuously updated the reduction trajectory in real time, introducing a degree of compliance that enhanced the safety of the reduction procedure.

Moreover, when there is a large muscle disturbance, that is, 2–3 s, the variable impedance control can reduce the human–machine interaction force by reducing the stiffness, which can be seen in Figure 14a. From the enlarged view in Figure 13c and the tracking error shown in Figure 14b, it can be observed that variable impedance control results in greater displacement deviation compared to constant impedance control. This effectively reduces the human–machine interaction force, enhancing both compliance and safety. Additionally, at other times, variable impedance control offers relatively higher trajectory tracking accuracy than constant impedance control. Specifically, for variable impedance control, the root-mean-square (RMS) tracking errors for the position trajectories were 8.5×10^{-10} m, 6.7×10^{-9} m, and 2.7×10^{-11} m, with maximum errors of 6.8×10^{-5} m, 2.3×10^{-4} m, and 8.5×10^{-4} m, respectively. For the orientation trajectories, the RMS tracking errors were 0.0012° , 0.0020° , and 0.0506° , with maximum errors of 0.0945° , 0.1041° , and 0.6803° , respectively. The experimental results show that the proposed variable impedance control algorithm has better tracking accuracy and compliance than constant impedance control, and exhibits anthropomorphic closed reduction characteristics.

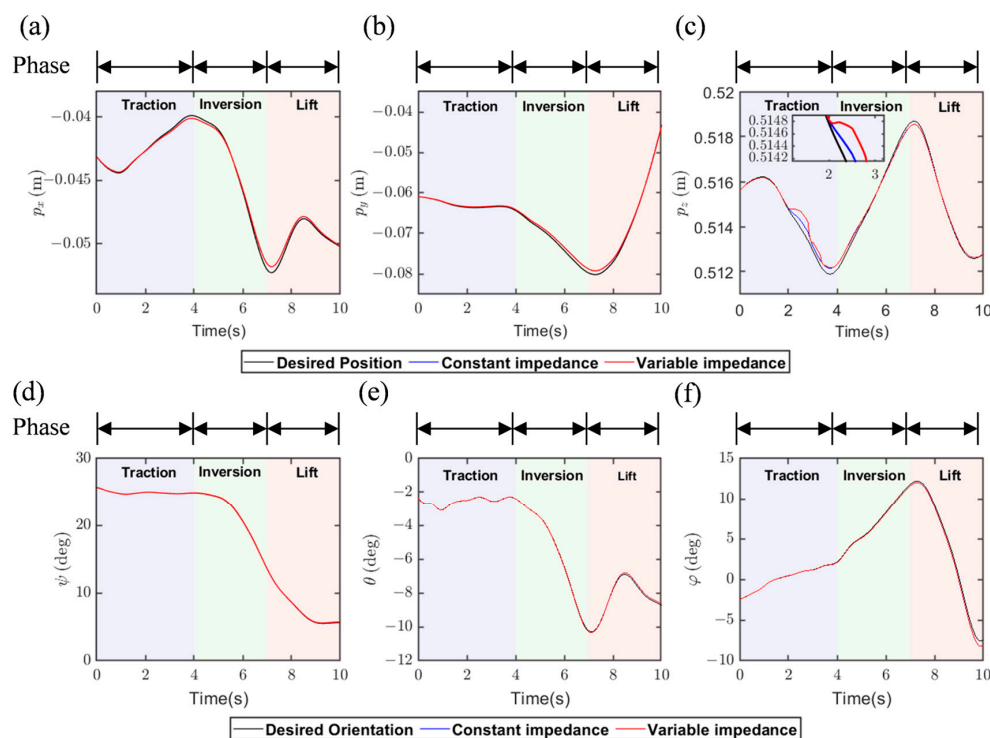


Figure 13. End-effector position and orientation trajectory under the control of constant impedance controller and variable impedance controller. The closed reduction process is divided using three phases, which correspond to “Traction”, “Inversion”, and “Lift”, respectively. (a–c) represent the position along the X, Y and Z axes, respectively. (d–f) are respectively three Euler angles.

Figure 15 shows the interaction forces between humans and robots during bone-setting. It can be seen that during the 0–4 s stretching process, the traction interaction force in the Z-axis direction continues to increase. When there is a sudden disturbing force in 2–3 s, the impedance controller shows a compliant action by correcting the trajectory, which is similar to a professional surgeon. Table 5 presents a comparison of the mean square error (MSE) of trajectory tracking, as well as the mean absolute value (MAV) and peak value (PV) of human–robot interaction forces F_I across the three axes under two impedance control schemes. The results demonstrate that along the undisturbed axial direction, the proposed variable impedance control achieves superior trajectory tracking accuracy. In the presence of disturbances along the Z-axis, the proposed algorithm effectively reduces

both the MAV and PV of interaction forces, thereby improving the compliance and safety of the reduction process. These findings highlight the practical significance of the proposed control algorithm in enhancing the accuracy, safety, and adaptability of robotic systems for ankle fracture reduction.

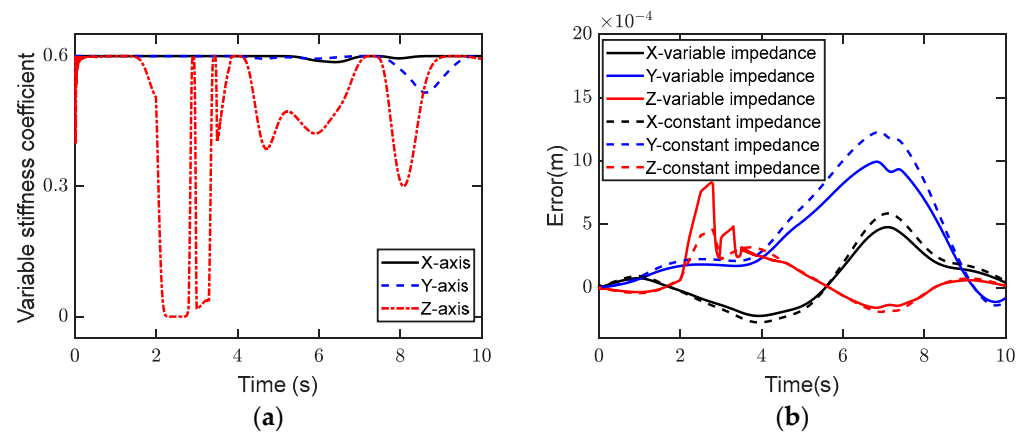


Figure 14. Variable impedance control test under disturbance. (a) Diagram of variable stiffness coefficient. (b) Tracking error diagram of position trajectory.

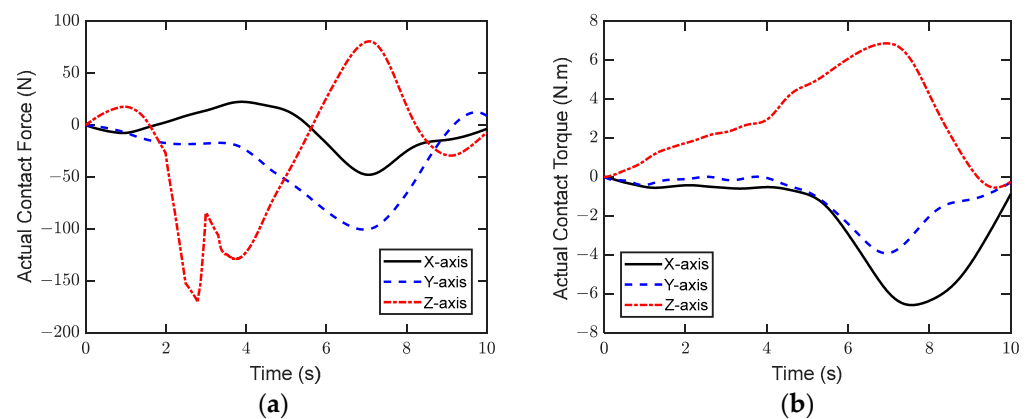


Figure 15. Actual contact force F_I and torque M_I between humans and robots. (a) Actual contact force. (b) Actual contact torque.

Table 5. Performance comparison of impedance control.

	Constant Impedance Control			Variable Impedance Control		
	X-axis	Y-axis	Z-axis	X-axis	Y-axis	Z-axis
MSE (10^{-10} m)	8.550	67.784	200.08	8.249	65.731	523.81
MAV (N)	6.222	36.973	25.147	6.010	35.758	22.833
PV (N)	47.824	100.59	187.15	47.601	100.26	168.52

To solve the optimization problem at each time step, we employed the FMINCON function from the MATLAB optimization toolbox. We used the sequential quadratic programming (SQP) method to handle large-scale sparse problems while ensuring adherence to actuator boundary conditions throughout the iterative process. In the inverse kinematics calculation for the reduction trajectory, the time step was set to 0.1 s, with each optimization step completed in less than 0.08 s. It highlights the method's computational efficiency and suitability for practical clinical applications. Furthermore, by using a singularity-free redundancy solver, we plotted the time-varying lengths of the six prismatic actuators

and three sliding actuators, as shown in Figure 16. From this figure, it is evident that the motion trajectories of all nine actuators remain within the designated range and exhibit smooth characteristics. Our tests also demonstrated that under the same structural parameters, the classical non-redundant parallel platform failed to solve the inverse kinematics, as the solution exceeded the predefined motion range. Under the inverse dynamics PD control, all the actuators exhibited high tracking precision, with the RMS tracking errors for the six prismatic actuators at 5.6×10^{-8} m, and for the three sliding actuators at 9.9×10^{-8} m. Figure 17a,b show the force profiles of the six prismatic actuators and three sliding actuators, respectively.

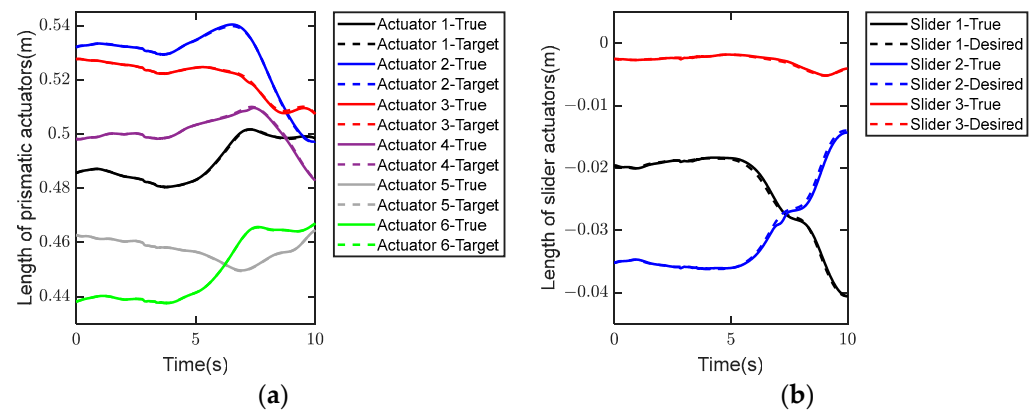


Figure 16. Trajectories of the actuators. (a) Prismatic actuators. (b) Sliding actuators.

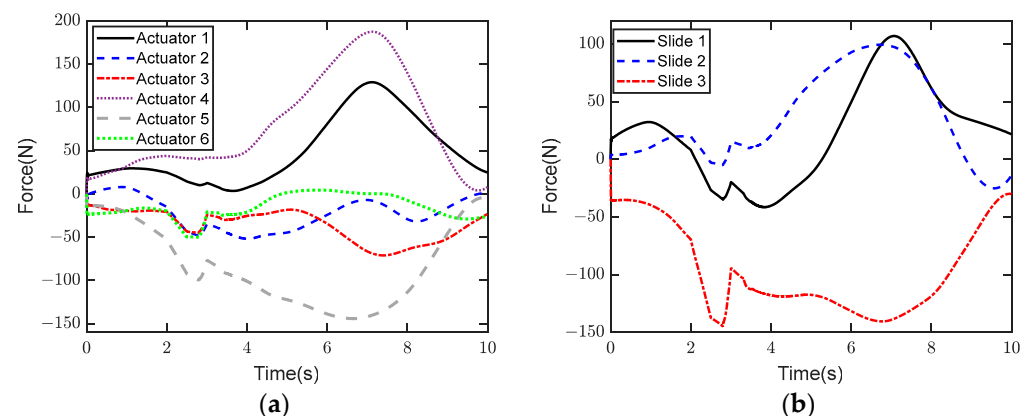


Figure 17. Forces in the actuators. (a) Forces of the prismatic actuators. (b) Forces of the sliding actuators.

6. Discussion

6.1. Comparison

The proposed ankle fracture closed reduction robotic system is inspired by the existing non-invasive traction devices and incorporates clinically validated principles. The system features a human–robot coupling design and a redundant tilt-degree-of-freedom parallel platform, effectively meeting the clinical demands for load capacity, dexterity, and non-invasiveness in closed reduction procedures. Moreover, using an improved dynamic movement primitive method, the system transfers the reduction techniques of experienced orthopedic surgeons to the robotic platform, achieving a human-like reduction process through variable stiffness compliance control. This design aligns closely with clinical requirements, demonstrating its feasibility and safety. Table 6 compares this system with the existing solutions, highlighting its superior performance in closed, load capacity, tilt dexterity, imitation learning, and compliance. Given the lack of directly comparable systems, approximate case studies are provided to validate its advantages.

Table 6. Comparison of our work with the state of the art.

	Zha [24]	Bian [43]	Jamwal [16]	Lei [58]	Liang [15]	Ours
Structure type	Series	Series	Parallel	Parallel	Parallel	Parallel
Closed	✓	×	×	×	×	✓
High load capacity	×	×	✓	✓	✓	✓
High tilt dexterity	✓	✓	×	×	✓	✓
Imitation learning planning	×	✓	×	×	×	✓
Compliance and safety	×	×	✓	✓	×	✓

6.2. Medical Application Prospect

The proposed bone-setting robotic system, while primarily designed for ankle fractures, demonstrates broader medical application potential due to its advantages in load capacity and dexterity. This system can be easily adapted for the postoperative rehabilitation of ankle fractures and, through the targeted design of a novel human–robot coupling mechanism, can be extended to closed bone-setting procedures for other fractures, including those of flexural joints and the tibia–fibula. Additionally, the improved imitation learning algorithm and variable impedance control method presented in this work can be rapidly scaled to other medical applications. Hence, this approach holds promise for advancing fracture treatment and care, especially in resource-limited regions where medical resources are scarce, skilled surgeons are in short supply, and access to quality treatment is limited. The robots, which have the ability to replicate expert reduction techniques, can improve surgical efficiency, reduce errors, and provide support for less experienced clinicians, ultimately benefiting more patients. Additionally, by decreasing physical demands on surgeons, the system increases procedural consistency and stability, contributing to more equitable healthcare distribution. However, several challenges remain, including human–robot coupling technologies, ethical considerations in medical robotics, and robot operation and maintenance.

6.3. Limitation

Although the proposed ankle closed reduction robot demonstrates significant potential, it still faces several technical limitations that must be addressed for broader clinical applicability. One key challenge is the inability of the current human–robot coupling mechanism to adapt to the varying anatomical shapes of different body parts. This limitation restricts the system’s versatility in treating fractures across multiple sites. Additionally, while the robot can replicate expert techniques, it does not yet incorporate tactile imitation learning, which is essential for accurately mimicking the touch and precision of human surgeons during the fracture reduction process. Furthermore, the system currently lacks integration with musculoskeletal biomechanics, which could optimize the bone-setting procedure by considering the physical properties and interactions of bones, muscles, and surrounding tissues. Addressing these gaps will require the development of more adaptable human–robot coupling mechanisms, the integration of advanced multidimensional tactile feedback, and the incorporation of musculoskeletal biomechanics to refine the robotic system and improve its performance in complex fracture reduction scenarios.

6.4. Ethical and Legal Concerns

The ethical concerns surrounding medical robotics, particularly in life-critical applications, remain significant. While robots excel in handling high loads and delivering precise operations, they cannot fully replace clinical judgment, especially in unexpected and complex scenarios. To address this challenge, the proposed ankle closed reduction robot can be positioned as a surgical “assistant” rather than a substitute. It offers preoperative planning based on expert simulations, but the physician must retain ultimate authority—conducting

patient evaluations, managing care, and responding to complications. By refining the interaction between the robot and surgeon, we ensure that the doctor maintains control throughout the procedure, intervening as needed to enhance safety and treatment quality. Instead of supplanting the physician's role, the technology serves as a decision-support tool. Furthermore, the system can integrate robust safety features, such as real-time data from six-dimensional force sensors and strict safety thresholds tailored to clinical demands. Incorporating temporal machine learning models to predict unsafe conditions further bolsters the system's safety and reliability.

6.5. Robot Operation and Maintenance

The effective application of the proposed ankle closed reduction robotic system hinges on the operational expertise and maintenance capabilities of medical personnel. Surgeons must undergo comprehensive preoperative training to master robotic operation and emergency response, particularly for complex cases requiring rapid intervention. Training programs should cover operating procedures, troubleshooting, and clinical response strategies. In addition, healthcare institutions need to establish systematic training protocols that ensure that medical staff receive regular updates on the latest technological advancements, maintaining a high level of operational proficiency. Maintenance teams must also possess sufficient technical support skills to guarantee the robot's continued reliability in clinical use. Simulated environments and virtual reality training platforms can be leveraged to allow medical staff to practice high-risk scenarios without involving real patients, thereby enhancing their readiness to handle unforeseen situations.

6.6. Clinical Test and Regulatory Approval

To ensure clinical viability and regulatory approval, several key steps are required for the proposed redundant parallel bone-setting robot system for ankle fractures. Initially, pre-clinical testing using cadaveric models is necessary to evaluate the robot's precision in replicating expert manual closed reduction techniques. This phase focuses on verifying force application, trajectory alignment, and overall performance. Subsequently, human pilot studies must be conducted to assess the robot's functionality in real-world settings. This phase will also help identify potential complications and refine the robot's design. Concurrently, regulatory approval from agencies such as the FDA or EMA is crucial. This process involves submitting detailed documentation on safety, risk analysis, clinical trial data, and system design. Compliance with medical device standards is necessary to ensure that the robot meets safety and quality requirements for clinical use.

7. Conclusions and Future Work

This study introduces a novel redundant parallel bone-setting robot system designed to inherit the closed reduction techniques for ankle dislocations and fractures. Compared to the Stewart–Gough platform, the robot's redundant structure, which incorporates redundant kinematics and singularity-free solvers, offers better reachable workspace and enhanced dexterity. Moreover, to address the limitations of traditional DMP in accurately imitating trajectories, we propose the MV-TP-DMP algorithm. It leverages the MV-TPR algorithm as a multi-output regression model for DMP, leading to improved trajectory imitation precision and generalization capability. Comparative tests based on demonstrated reduction trajectories from surgeons show that MV-TP-DMP outperforms both DMP and MV-GP-DMP. Furthermore, we developed a position-based variable impedance controller featuring a novel variable stiffness strategy. The simulation results demonstrate that the proposed controller can effectively improve trajectory tracking accuracy in the absence of disturbances. Under sudden perturbations, it exhibits enhanced compliance,

thereby improving safety during reduction procedures. Compared to constant impedance control, our variable impedance controller demonstrates more human-like interaction characteristics. In conclusion, this study integrates advanced imitation learning and variable impedance control strategy to develop an intelligent robotic system that precisely replicates surgeons' closed reduction techniques, providing a safer and more accurate solution for fracture reduction.

Future research will focus on the following areas: First, addressing the increased computational cost of MV-GPR and MV-TPR with larger datasets, potentially through Sparse Gaussian Processes. Second, developing multi-functional human-machine coupling mechanisms that integrate advanced multidimensional tactile feedback to accommodate scenarios involving multi-site fractures, including radial joints. Third, enhancing imitation learning trajectory planning by incorporating tactile feedback from the bone-setting process and musculoskeletal biomechanics, thereby improving the reliability and safety of surgical robots. Finally, a prototype based on the proposed model will be developed, and cadaveric experiments will be conducted to validate the system's effectiveness.

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