

Review

Advances in Computer Numerical Control Geometric Error Compensation: Integrating AI and On-Machine Technologies for Ultra-Precision Manufacturing

Yassmin Seid Ahmed ^{1,2,*}  and Fred Lacerda Amorim ³

¹ Mechanical Engineering Department, King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia

² Interdisciplinary Research Center for Advanced Materials, King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia

³ Mechanical Engineering Graduate Program-PPGEM, Pontifícia Universidade Católica do Paraná-PUCPR, Av. Imaculada Conceição, 1155, Prado Velho, Curitiba 80215-901, PR, Brazil; fred.amorim@pucpr.br

* Correspondence: yassmin.seidahmed@kfupm.edu.sa

Abstract: Geometric inaccuracies in machine configuration and part specifications are a major source of errors in CNC machining. These discrepancies have long affected the quality of manufactured components and continue to be a key research area in academia and industry. Over the years, significant efforts have been made to minimize these errors and enhance machining precision. Researchers have explored various methodologies to identify, measure, and compensate for spatial inaccuracies, improving accuracy in modern machining systems. This paper comprehensively reviews recent advancements in geometric error measurement and compensation techniques, particularly in five-axis machine tools. It examines the latest methods for detecting errors and explores volumetric error modeling approaches designed to enhance machining precision. This review highlights the growing role of emerging technologies, including on-machine measurement systems, machine learning algorithms, and digital twin frameworks, in improving real-time error detection and compensation strategies. Furthermore, advanced tools such as laser interferometry and hybrid software–hardware approaches are discussed for their potential to drive innovation in ultra-precision machining. This paper also addresses key challenges in achieving high volumetric accuracy and outlines future opportunities for improving CNC machining performance. Future research can enhance precision and reliability in modern manufacturing by integrating intelligent systems and advanced measurement techniques.

Keywords: geometric discrepancies; error measurement techniques; error modeling; error adjustment; multi-axis machining; future trends



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1. Introduction

Over the last twenty years, machining technologies in manufacturing have advanced significantly, fueled by market competition and ongoing innovation [1]. CNC machines have transitioned from simple punch-driven mechanisms to advanced, computer-controlled systems. Modern five-axis technology now enables precise programming and machining of complex 3D surfaces, transforming the manufacturing landscape.

Figure 1a provides a comprehensive network visualization, offering valuable insights into the interconnected research areas within CNC geometric error compensation and ultra-precision manufacturing. Prominent clusters in the visualization highlight themes such as ‘AI-driven optimization’, ‘Forecasting’, and ‘industry 4.0’, reflecting the field’s

focus on enhancing precision and efficiency. The figure also emphasizes emerging concepts like ‘thermal error compensation’, ‘geometric error’, and ‘condition monitoring’, showcasing innovative approaches to error reduction. By illustrating the collaboration between advanced technologies and precision manufacturing techniques, this visual representation highlights the transformative impact of modern CNC systems, supporting the observed trends in high-precision manufacturing advancement. In addition, recent data from the World Machine Tool Survey indicate a notable increase in machine tool consumption, rising from USD 60 billion in 2019 to USD 110 billion in 2024 [2]. This upward trend reflects the growing demand for advanced machining systems, as shown in Figure 1b. The continuous improvement in machining precision and efficiency has been enabled by fast software, information technologies, hardware, and control system advancements. However, despite the mentioned technological improvements, imprecision still heavily restricts machine tool performance, especially during metal cutting processes.

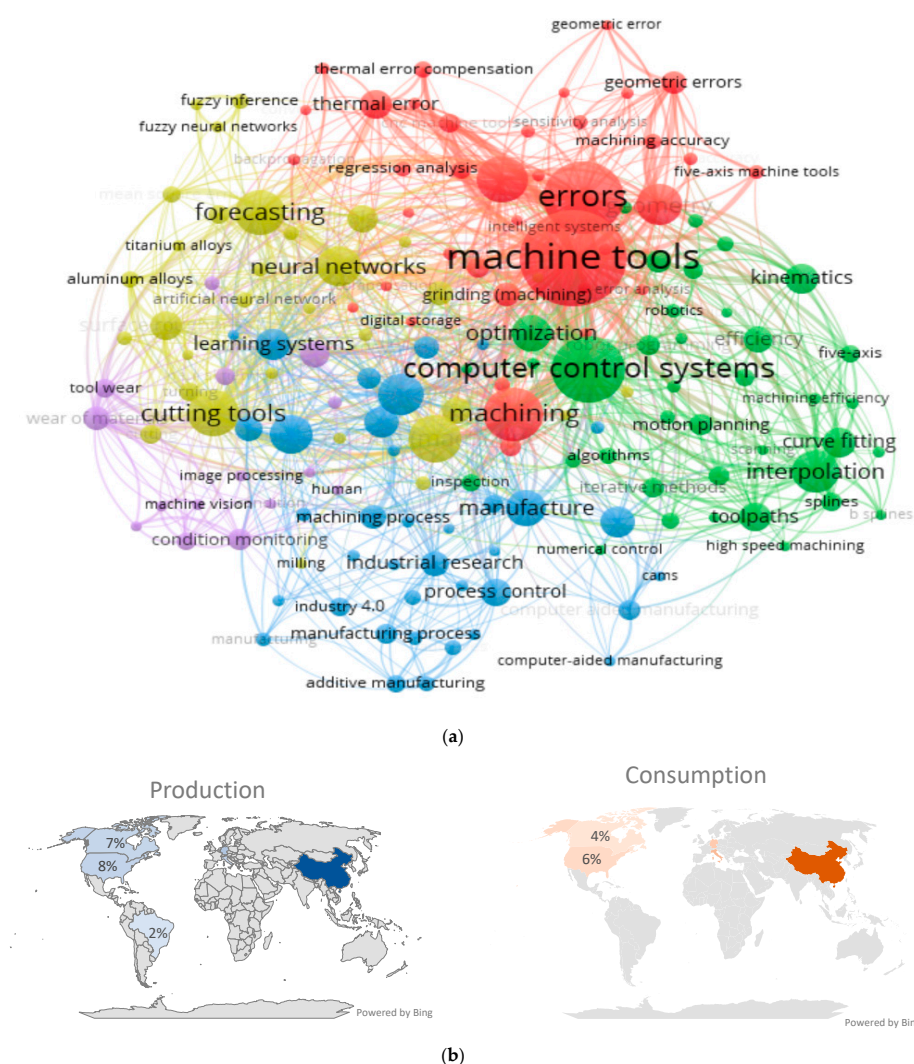


Figure 1. (a) Network visualization of research themes in CNC geometric error compensation and ultra-precision manufacturing; (b) world production and compensation countries for machines [2].

One of the main drivers for the development of ultra-precision machine tools [3] (UPMTs) was the need in modern manufacturing for greater precision and higher quality in machining. Unlike conventional CNC machines, UPMTs operate with sub-micron or nanometric accuracies, allowing intricate and high-quality components to be produced for the aerospace, optics, and semiconductor manufacturing industries. The very design of these

tools creates features such as advanced control systems that provide sub-nanometric position feedback and systems to control underlying temperature variations within ± 0.05 °C. However, the growing complexity of UPMTs introduces unique challenges for calibration, error measurement, and compensation.

Geometrical errors, thermal distortions, and dynamic disturbances are the most significant factors affecting UPMT performance. Unlike traditional CNC machines, where geometric and thermal errors dominate, UPMTs face motion control errors due to vibration, servo dynamics, and environmental factors such as air turbulence. Moreover, the high accuracies required from UPMT machines mean they very often need to work in controlled environments with small temperature fluctuations and vibration isolation systems to ensure system stability [4].

A proper calibration of UPMTs will be instrumental in securing and maintaining the requisite accuracy throughout the operational life of the tool. Regular calibration routines such as error measurement and compensation are unavoidable to obtain an overall optimum performance. In addition, iterative interaction in ultra-precision machining creates further requirements for systematic calibration approaches due to the fact that even minute irregularities in the correctness of a machine tool can sufficiently affect the final quality of the product [5].

Machining discrepancies is a technical term that refers to deviations between the real tool path and the intended workpiece contour. They have an impact on the quality and functionality of the final product. For instance, discrepancies in machining aero-engine blades can drastically affect their aerodynamic properties. This led the manufacturers to pay more attention to the problem of eliminating or reducing such error sources in machining devices. Extensive research [6–8] has significantly improved the precision of machining devices. For instance, modern lathes typically achieve precision within 3 micrometers under stable temperature conditions. In contrast, precision machines can attain one-micrometer tolerances or lower. Despite these improvements, it is impossible to eliminate discrepancies fully due to their inherent nature. No matter how much effort is put into improving machine tool parts and assembly, precision has its limits. Typically, the sources of discrepancies in CNC machining devices [9,10] are classified into spatial inaccuracies, thermal effects, cutting forces, etc., as shown in Figure 2. Geometric discrepancies are particularly impactful, affecting both the motion precision of machining devices and the dimensional precision of the machined workpiece [4,11]. As a result, spatial error adjustment has become a critical area of research in improving machining precision.

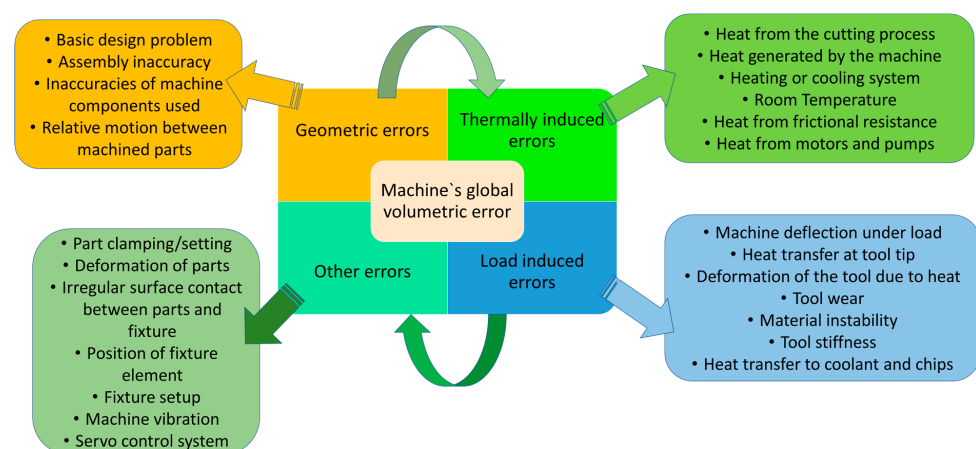


Figure 2. Machine error sources.

1.1. Research Gaps and Objectives

Geometric discrepancies remain a significant limitation in CNC machining despite decades of research and technological advancements [12]. The development of UPMTs has further highlighted the inadequacies of traditional approaches to error identification and compensation, particularly in dynamic and high-precision environments. This section elucidates the key research gaps and outlines objectives to address these limitations, providing a comparative analysis of the proposed work with the existing literature.

1.1.1. Objectives

This article reviews the latest advancements in identifying and compensating for spatial errors in CNC machining devices. Correcting geometric discrepancies also follows some procedures: using appropriate sensors and correction strategies to specify some forms of spatial inaccuracies; modeling the errors leads to the compensation methods that can be undertaken to help fix the repeatability of machining improvements. This review discusses significant developments achieved in volumetric accuracy through improved modeling, measurement, and compensation techniques. This review also discusses the application of modern technologies such as machine learning, digital twin frameworks, and advanced metrology systems to redefine error detection and compensation. Such innovations have yielded marked improvement in volumetric accuracy. This article presents some key observations and puts forward recommendations for future avenues of research toward further enhancing the precision of CNC manufacturing systems.

1.1.2. Research Gaps

1. Many existing studies have focused on offline compensation techniques, which rely on pre-measured errors and post-process adjustments [13–15]. While effective to some extent, these methods fail to address real-time variations in machining conditions, such as thermal drift, tool wear, and dynamic vibrations.
2. Limited integration of advanced emerging technologies such as AI, digital twins, and machine learning has shown potential in predictive maintenance and fault diagnosis. However, their application in geometric error compensation is still nascent [16–19].
3. While homogeneous transformation matrices and screw theory have been extensively utilized for error modeling, these approaches often focus on isolated errors or single-axis systems. Coupled interactions in multi-axis systems remain under-explored. Yanag et al. [20] provided insights into error propagation in multi-axis systems but did not comprehensively integrate thermal and dynamic factors. Fan et al. [21] emphasized the importance of volumetric geometric error modeling and proposed enhancements in five-axis machining systems. However, their focus on static conditions highlights a gap in dynamic compensation strategies.
4. On-machine measurement systems (OMMSs) promise reduced downtime and higher efficiency, yet challenges hinder their implementation in sensor integration and data processing. Studies by Zhao et al. [22] and Zhu et al. highlight these limitations, showing the need for more sophisticated hybrid measurement systems that combine multiple sensors and algorithms.
5. Existing compensation methods often lack scalability across different machine configurations and operating conditions. Models tailored for specific machines or environments fail to generalize, as evidenced by Ge et al. [23], Li et al. [24], and Zhao et al. [25].

1.2. Paper Structure

This review paper is arranged in the following way: Section 2 addresses the various classifications of error in CNC machining, including geometric, thermal, cutting force, and tool wear-induced inaccuracy. Section 3 discusses the general routine for geometric error calibration, including modeling, measurement, and compensation strategies. Section 4 analyses techniques for measuring geometric errors in linear and rotary axes, focusing on direct methods such as laser interferometry and indirect techniques like machining tests. Section 5 presents error modeling techniques that include homogeneous transformation matrices, screw theory, and differential motion matrices. Section 6 discusses compensation techniques using principles from software-based to real-time hardware adjustments. Section 7 discusses cutting-edge tools and technologies, such as machine learning, digital twin systems, and advanced metrology devices, expanding the landscape for error detection and compensation. Finally, Section 8 summarizes important conclusions and outlines further research directions to boost CNC machining performance.

2. Comprehensive Overview of CNC Errors

Machining is a complex field dictated by many aspects, with dimensional accuracy dependent on the interaction among unlike sources of errors. However, these errors complicate the situation, providing opportunities to deal with machining inaccuracies from many perspectives. This section will highlight the primary sources of errors affecting dimensional accuracy in machined parts and introduce methods of compensating for these errors.

2.1. Geometric Errors

Geometric errors are a considerable mistake source for CNC machining that influence the machine's precision and the quality of finished workpieces. These errors can arise from multiple sources: imperfections in the machine structure, misalignments during assembly, or performance deterioration due to wear and tear over time [4,11,26–28]. Geometric errors go through two main categories:

1. Position-Dependent Geometric Errors (PDGEs)

Position-Dependent Geometric Errors (PDGEs) are errors that vary based on the location of machine components. They arise from relative motion deviations of one machine part with respect to another during their operation [29,30]. PDGEs usually comprise the following:

- **Displacement Errors:** These are deviations in the positioning along the X, Y, and Z axes, which vary as the machine moves through its working space.
- **Rotational Errors:** They are angular in nature—pitch, yaw, and roll—and modify the orientation of the tool with respect to the workpiece. These errors depend on the position of the machine.

To keep a CNC machine with more than one axis accurate, measuring and correcting PDGEs is vital, particularly while working through complex geometries.

2. Position-Independent Geometric Errors (PIGEs)

Position-Independent Geometric Errors are errors in a machine that remain unchanged no matter its position in space. They usually arise due to initial assembly failures or imperfections in the construction of rotary axes or structural members of the machine [27]. Examples of the most common Position-Independent Geometric Errors are the following:

- **Angular Misalignments:** These occur when the axes responsible for rotation are not perfectly aligned, causing errors that persist throughout the machine's operation.

- **Axis Offsets:** These are fixed deviations in the origin of the machine's coordinate system, which remain constant during operation but lead to consistent inaccuracies in machining.

Because they are constant, PIGEs are much harder to manage, and, therefore, sophisticated measurement and correction techniques are required to compensate them effectively [26].

2.2. Thermal Errors

Thermal discrepancies arise from the deformation or distortion of the structure due to thermal variance and gradients. These are among the most significant contributors causing inaccuracies observed in machined parts. The heat generated by the cutting process, friction between moving parts, and thermal effects from cooling systems [31–33] all contribute to this. Changes in ambient temperature can also lead to a degradation in the stability of the machine [7,9,34,35]. Mitigation methods for thermal discrepancies usually fall into three major options:

- **Avoidance:** Systems designed with a view to minimize heat generation.
- **Control:** Temperature is actively controlled in the critical parts.
- **Compensation:** Predicting thermal distortions through models and then adjusting the tool path to counteract the disturbances.

In particular, thermal compensation is one of the most efficient and economical methods of addressing this aspect. This generally consists of temperature distribution analysis and machine structure influences, the development of predictive models, and the implementation of corresponding adjustments in real time.

Accurate temperature measurement is essential for understanding and mitigating thermal errors in CNC machining [36]. Various sensor technologies are employed to monitor temperature variations in machine components, each offering distinct advantages and limitations. Thermocouples are widely used due to their high sensitivity and fast response time but require careful calibration to ensure accuracy [37]. Resistance temperature detectors (RTDs) provide stability and precision, making them suitable for long-term temperature monitoring [38]. Infrared sensors offer non-contact temperature measurement and are particularly useful for high-speed spindle applications [39]. Embedded temperature sensors integrated within machine structures provide real-time thermal data but require advanced data acquisition systems [40,41]. The effectiveness of these sensors depends on their placement, measurement resolution, and response time, all of which influence the accuracy of thermal compensation strategies.

The accuracy of thermal compensation models is significantly affected by the positioning of temperature sensors [36]. Improper sensor placement can lead to inaccurate temperature mapping, resulting in suboptimal error corrections [42]. Sensors positioned at the spindle and tool interface help predict thermal expansion effects on machining accuracy, while those placed at the machine bed and frame track ambient and internal thermal influences. The positioning of sensors within coolant and lubrication systems also plays a critical role in understanding temperature variations affecting thermal drift [43].

Accurate temperature measurement is critical for validating thermal error simulation models. Computational techniques, such as finite element analysis (FEA) and data-driven machine learning models, rely on precise temperature inputs to predict machine deformations [44]. Direct measurement methods, such as thermocouples and RTDs, provide high precision but are limited in spatial coverage. Indirect methods, including infrared imaging and thermal cameras, offer broader insights but require calibration against direct measurements [45]. Multi-sensor data fusion, which integrates readings from different sensor types, enhances the reliability of thermal compensation models by reducing uncer-

tainties. Recent advancements in real-time adaptive compensation integrate temperature sensors with CNC controllers, allowing machines to dynamically adjust tool paths based on thermal feedback [46]. The integration of advanced temperature measurement techniques with simulation-driven error prediction significantly improves machining precision and reliability [47]. Future research should focus on enhancing the integration of multi-sensor data fusion, real-time adaptive compensation, and AI-based thermal error prediction to further improve machining accuracy. While earlier studies have focused extensively on thermal error compensation, the current review excludes this topic from its scope [48–50].

2.3. Cutting Force Errors

Accuracies may also be achieved in cutting forces through elastic deformation of a tool or workpiece. Some examples are that undercuts could result in under- or overcutting during the machining of thin-walled structures. These forces in dynamics make adjustment in real time quite challenging owing to hardware and software restrictions [9,10,51,52].

The most used methods to handle these discrepancies are offline adjustment methods. These methods involve an iterative computation based on the theory of elasticity, so as to adjust the tool paths for machining deformation. It is time-consuming and inaccurate. An emerging solution consists of auxiliary fixtures to offer stabilization to thin-walled workpieces during machining, which itself is a very active research area nowadays [49].

2.4. Tool Wear and Fixture Inaccuracies

Tool Wear: As tools undergo shape changes during machining, deviations in tool geometry affect workpiece precision. Predicting and monitoring tool wear [53,54], which is nonlinear and time-dependent, remains challenging [54].

Fixture Deformation: Clamping forces can cause elastic deformation in the fixture and workpiece. After removing the fixture, stress relief can also lead to distortions, requiring additional modeling and adjustment strategies [55].

3. General Routine of Geometric Error Calibration

Ultra-precision machine tools demand components of exceptional accuracy and advanced control systems to achieve sub-nanometer positioning precision during cutting processes [56,57]. For the machine base, natural or synthetic composite granite is a more advantageous material compared to cast iron or welded steel due to its superior vibration-damping characteristics, long-term stability, and lower assembly costs. To counteract low-frequency vibrations exceeding 2 kHz, an anti-vibration system integrated with an automatic levelling mechanism is essential.

The ISO standards ISO 230-1:2012 [58] provide a structured framework for defining geometric errors, encompassing measurement devices and data evaluation techniques. In addition to ISO 230-1:2012, the ISO 230-2 [59] standard specifically defines the measurement procedures for positioning accuracy and repeatability in CNC and ultra-precision machine tools. While ISO 230-1 focuses on static geometric accuracy, ISO 230-2 evaluates motion-related deviations, including backlash, hysteresis, and thermal drift effects on positioning.

However, these standards are primarily tailored for conventional precision machines or multipurpose machine tools rather than UPMTs (ultra-precision machine tools) with nanometer-level accuracy. Calibrating geometric errors in UPMTs diverges significantly from traditional CNC machines regarding measurement device capabilities, accuracy benchmarks for reference setups, error identification, and correction approaches. This process involves a systematic protocol encompassing error modeling, measurement, and compensation techniques [12,13].

3.1. Error Modeling

The error transfer function model examines how motion deviations in machine components quantitatively affect the static alignment and positioning of the cutting tool relative to the workpiece. This analysis utilizes principles from rigid body kinematics and multiplexing theory [60]. Motion errors are generally divided into two categories: those arising from movement and those independent of movement. The former involves positional defects that vary with displacement, while the latter relates to constant misalignments, often attributed to joint inaccuracies. Many mathematical frameworks represent the system as a collection of interconnected bodies and apply methods such as the HTM, differential motion matrix, and screw theory. Among these, the HTM is the most commonly applied due to its intuitive interpretation and systematic modeling technique [61].

Each rotation or translation axis is represented as revolving around the movement of distinct rigid bodies, with their respective local coordinate systems defined. To manage the complexity of large matrix dimensions, the coordinate systems of mobile components are converted into the fixed reference frame of the machine's base. For a standard three-axis turning machine, two kinematic chains are established: one connecting the machine base to the end of the workpiece and the other linking it to the tip of the cutting tool. Ideally, under normal operating conditions, these two endpoints should perfectly align [62].

The transition from coordinate system i to coordinate system j is represented by a transformation matrix, as outlined in Table 1. The abbreviations used are defined as follows: $[T_{ideal}]$ denotes the matrix for the ideal position; $[T_{pe}]$ represents the position error matrix; $[T_m]$ corresponds to the ideal transformation matrix for motion, whether rotational or translational; and $[T_{me}]$ accounts for the transformation error matrix related to motion. For a tool tip defined as $[Q_{tx}, Q_{ty}, Q_{tz}]$ in the cutting tool's coordinate system, the volumetric error vector influencing its calculated movement within the workpiece's coordinate system across the x , y , and z axes can be determined as follows:

$$[E_x E_y E_z 1]^T = {}^3_5 T_{actual} [Q_{tx} Q_{ty} Q_{tz} 1]^T - {}^3_5 T_{ideal} [Q_{tx} Q_{ty} Q_{tz} 1]^T$$

$${}^3_5 T_{actual} = {}^2_1 T_{pe} {}^2_1 T_m {}^2_1 T_{me} {}^1_0 T_m {}^1_0 T_{me} \left({}^4_0 T_{pe} {}^4_0 T_m {}^4_0 T_{me} \right)^{-1}$$

$${}^3_5 T_{ideal} = {}^3_2 T_m {}^2_1 T_m {}^1_0 T_m \left({}^5_4 T_m {}^4_0 T_m \right)^{-1}$$

Table 1. Error transformation matrices.

Adjacent Body	T_p	T_{pe}	T_m	T_{me}
${}^4_5 T$	$I_{4 \times 4}$	$I_{4 \times 4}$	$I_{4 \times 4}$	$I_{4 \times 4}$
${}^4_0 T$	$I_{4 \times 4}$	$\begin{bmatrix} 1 & 0 & E_{BOZ} & 0 \\ 0 & 1 & 0 & 0 \\ -E_{BOZ} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -E_{cz} & E_{BZ} & E_{XZ} \\ E_{CZ} & 1 & -E_{AZ} & E_{YZ} \\ -E_{BZ} & E_{AZ} & 1 & E_{ZZ} \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^1_0 T$	$I_{4 \times 4}$	$I_{4 \times 4}$	$\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -E_{cz} & E_{BX} & E_{XX} \\ E_{CX} & 1 & -E_{AX} & E_{YX} \\ -E_{BX} & E_{AX} & 1 & E_{ZX} \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^2_1 T$	$I_{4 \times 4}$	$\begin{bmatrix} 1 & 0 & E_{BOZ} & 0 \\ 0 & 1 & -E_{AOZ} & 0 \\ -E_{BOZ} & E_{AOZ} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos C & -\sin C & 0 & 0 \\ \sin C & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -E_{CC} & E_{BC} & E_{XC} \\ E_{CX} & 1 & -E_{AC} & E_{YC} \\ -E_{BX} & E_{AC} & 1 & E_{ZC} \\ 0 & 0 & 0 & 1 \end{bmatrix}$
${}^3_2 T$	$I_{4 \times 4}$	$I_{4 \times 4}$	$I_{4 \times 4}$	$I_{4 \times 4}$

Furthermore, modeling methods like the differential motion matrix (DMM) and screw theory involve complicated matrix calculations and analysis of motion errors in the context of integrated representations [20]. The DMM sees motion in an absolute sense and describes the geometric errors concerning each axis [63]. The transformation matrix of each moving part defining the movement from its initial position to the cutting point can be expressed using the product of the ideal motion matrix and the error differential matrix. The final error vector for the tool tip is then obtained by summing all differential matrices. While the DMM does give a clearer representation of immediate motion and aids in the definition of how geometric errors in each axis contribute to the total volume error of the machine, it creates a challenge in defining a suitable Jacobian matrix [64]. This challenge has been faced in implementing a compensation routine, mainly while using pseudo-inversion theory to calculate the compensation value.

In comparison, screw theory modeling represents error displacements for both longitudinal and rotary axes as the error twists corresponding to each driving axis in motion. The general error model is built by summing up all error twists along the kinematic chain. Unlike the HTM method, where individual local coordinate systems for each drive module are assigned, screw theory uses a single global coordinate system for the whole modeling process, which is more efficient since all components can be unified in one coordinate frame. The building of geometric error models, whether they involve the HTM, DMM, or screw theory, is a time-consuming process, especially since, for accurate modeling of a three-coordinate machine, one is supposed to consider over 21 errors, and 41 errors are relevant for a five-coordinate machine. Hence, some errors are virtually undetectable or they might overlap, thus complicating the efforts in effectively isolating and treating them [65].

3.2. Error Measurement

Error estimation aims to quantify the magnitude of errors. Various ISO standards have been established to guide the measurement and analysis of geometric and thermal inaccuracies to aid in this process. For instance, ISO 230-12 [66] focuses on assessing geometric errors in linear and rotary axes, while ISO 230-7 [67] is specific to the geometric evaluation of rotary axes. Guidance on testing procedures, including recommendations for equipment such as laser interferometers, alignment telescopes, and reference scales, is outlined in ISO 230-11 [68]. These standards provide a framework for evaluating machine tool performance effectively.

The geometric errors of ultra-precision machine tools rest between the submicron- to nano-level range, calling for measurement sensors displaying nanometer-level resolution and repeatability. For UPMT machines, positioning errors typically fall within the range of 50 nm to 200 nm, while repeatability errors are within ± 20 nm to ± 100 nm, depending on machine structure, environmental control, and sensor precision. Measurement uncertainty in ISO 230-2 evaluations depends on several factors, including the resolution of the measurement device, vibration isolation, and thermal stability. High-resolution laser interferometers provide uncertainty levels as low as ± 0.01 μm to ± 0.1 μm , while capacitive and inductive displacement sensors can achieve sub-nanometer precision. However, real-world measurement uncertainty is influenced by environmental factors such as temperature fluctuations, requiring compensation techniques to maintain high accuracy.

Capacitive sensors and laser interferometers stand amongst the most common instruments used for geometric error analysis [69]. Capacitive sensors are often integrated into commercially available metrological systems and can measure with nanometric resolution over a submillimeter range. This renders them capable of accepting an attractive application domain, which has been indicated in this paper. On the other hand, laser

interferometers and optical mirrors are also used to identify various linear and rotational errors along different axes. For instance, the latest such system includes the Renishaw XL-80 laser interferometers capable of measuring positioning errors down to 1 nm, an angular error detection of $0.1 \mu\text{m}/\text{m}$, and measuring straightness in the range of 10 nm [70]. In addition to diagonal displacement measurements described in ISO 230-6 [71], this modern technology has the potential for advanced five-axis CNC machines where axes are programmed following complex paths such as body or face diagonals. This allows for evaluating interconnected errors, such as linearity, straightness, perpendicularity, and angular deviations. Other devices used for precision measurements are double ball bars, grating interferometers, and R-tests, thus providing further options for satisfying specific error evaluations for CNC machines and test systems [72].

3.3. Error Compensation

Methods for error compensation are those which modify the path taken by the machine tool for efficiency in machining. These approaches are divided into two classes, offline and online [12,72]. Offline compensation employs a least-squares algorithm or similar curve-fitting techniques to fit measured error values at specific discrete points into a continuous function that is subsequently used to update the machining program without adjusting hardware settings. Online compensation employs an external PC controller capable of calculating and feeding the compensation values directly into the CNC machine controller, requiring real-time access to machine signals. Compensation can be carried out using changes to the NC code or a preconfigured lookup table within an open CNC controller. For instance, straightness error pre-compensation was implemented, leading to roughness improvements, namely, 86%, 25%, and 58% in front turning, taper turning, and spherical surface machining, respectively. In addition, various iterative and differential algorithms have been introduced to better compensate for the error provoked by rotary axes. These methodologies are based on inverting the matrix for mapping the actual position and orientation from the workpiece's coordinate system to the controlled axes [73]. The working procedure of error compensation and its function in machining and measurement is highlighted in Figure 3.

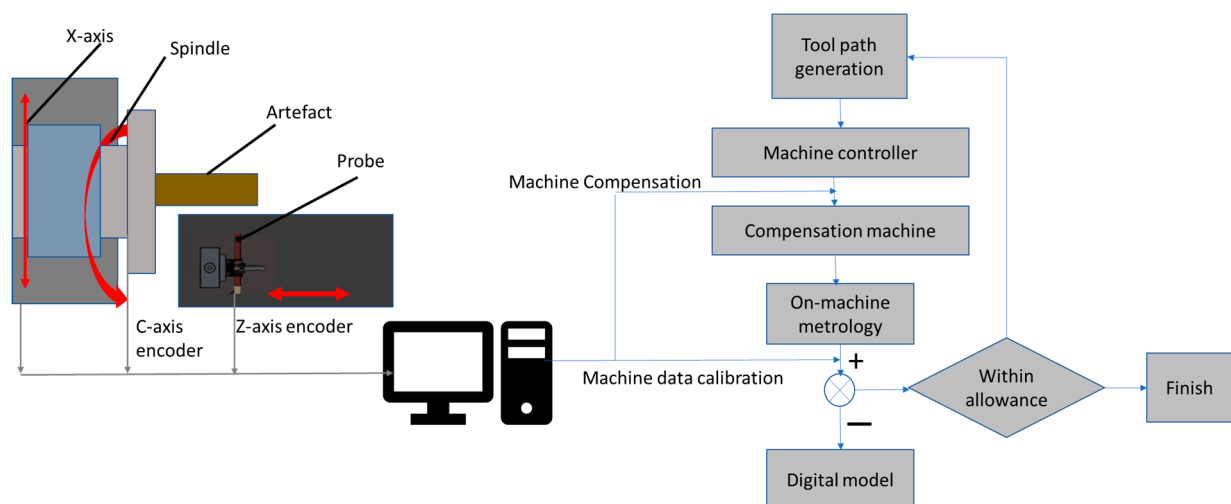


Figure 3. Understanding UPMT errors for improved machining accuracy.

4. Techniques for Measuring Geometric Errors

4.1. Linear Axis Error Measurement

Linear axes are essential parts of CNC machining devices [74]. Still, their motion is inherently subject to spatial inaccuracies arising from their six degrees of freedom (DOFs): three translational discrepancies (positioning deviations in X, Y, and Z directions) and three rotational discrepancies (roll, pitch, and yaw). These discrepancies are collectively referred to as position-dependent spatial discrepancies (PDGEs) [29,30] because they vary with the position of the axis, as illustrated in Figure 4. These spatial inaccuracies impact machining precision and require effective detection and correction strategies to maintain accuracy [75]. The measurement and identification of these discrepancies can be broadly categorized into direct and indirect methods. Both approaches aim to characterize discrepancies, support error adjustment, and enhance machining performance.

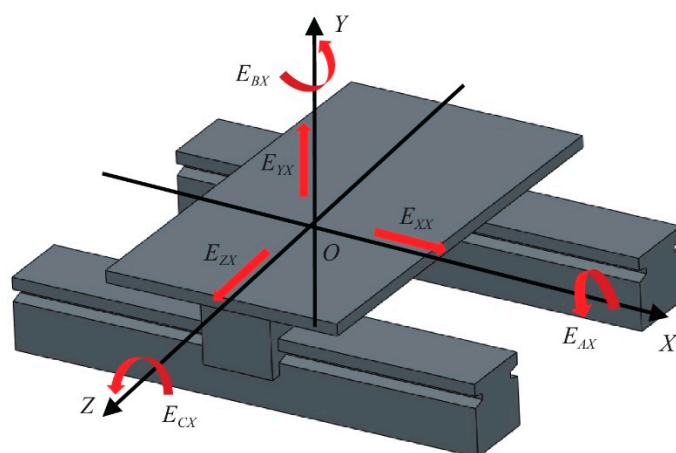


Figure 4. ISO 230-1:2012 geometric error descriptions [65].

4.1.1. Direct Measurement Methods

Direct measurement techniques directly capture spatial discrepancies using high-precision instruments without requiring intermediate modeling. Some common instruments employed for this purpose are laser interferometers and straightedges [76].

Single-Parameter

Single-parameter methods focus on individual error parts and utilize devices like laser interferometers for their high precision and versatility. Laser interferometers can measure linear positioning deviations and angular discrepancies by tracking changes in optical path length or angle between a reference beam and a target [77,78].

In ISO 230 [58], straightness error is determined by comparing it to a defined reference, such as a physical artifact or a laser beam generated by an optical system [79]. Figure 5 demonstrates the operating principle of a laser interferometer, where measurements of angular deviations or relative displacements allow error quantification. Recent advancements, such as heterodyne interferometry, enable simultaneous measurement of straightness magnitudes and positions, further improving precision and utility. Laser alignment techniques have also been studied for identifying straightness and angular errors, utilizing the uniform directional properties of collimated light [77]. While cost-effective and compact, these methods are sensitive to environmental conditions like atmospheric turbulence, particularly over long distances. Shielding techniques, such as those proposed by Liu [80], mitigate these effects and enhance measurement reliability.

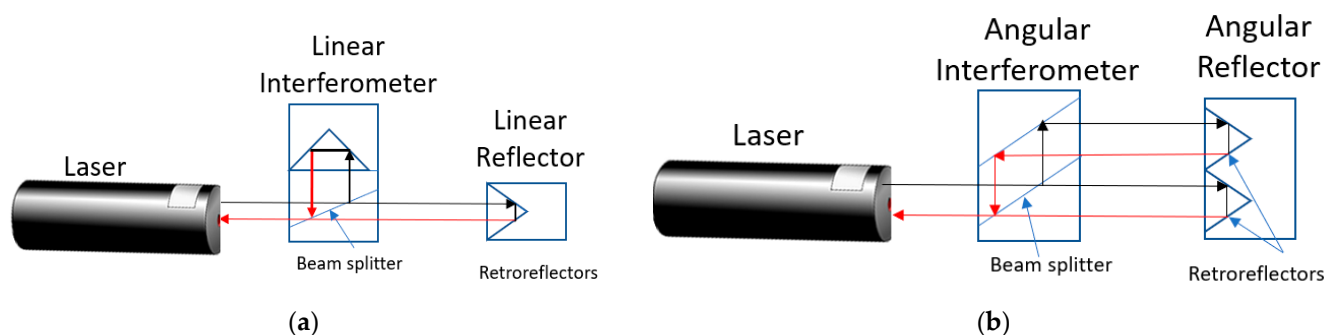


Figure 5. Laser interferometer measurement method for (a) position error and (b) rotational error [81].

Multi-DOF

The increasing complexity of CNC machining systems necessitates the simultaneous measurement of multiple spatial discrepancies. Multi-DOF (degree of freedom) measurement systems address this need by capturing several error parts concurrently. Commercial systems like the Renishaw XL-80 [81] use beam-splitting techniques to achieve high precision across multiple axes. However, these systems can be time-consuming to set up and sensitive to environmental disturbances over prolonged measurement periods. Figure 6 illustrates a setup utilizing diffraction gratings and interferometry to capture multiple real-time error parts.

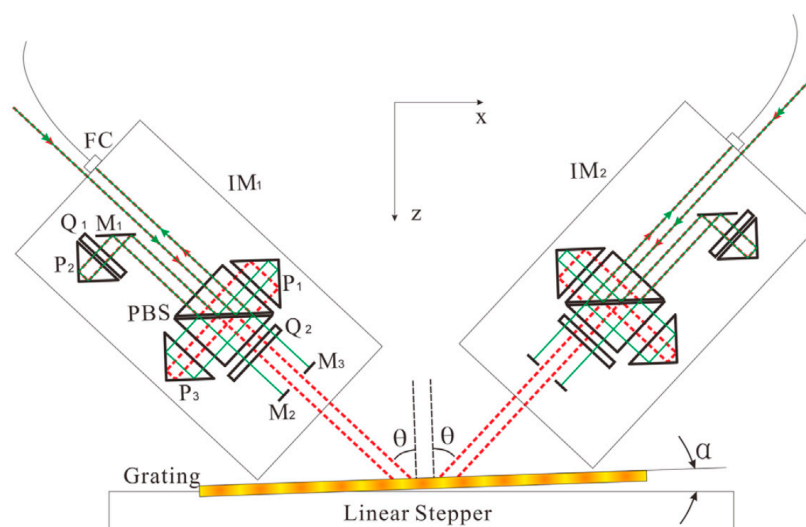


Figure 6. Measuring based on interferometry * [72]. * FC: Fiber Coupler, PBS: Polarizing Beam Splitter, IM₁, IM₂: Interferometer Modules, M₁, M₂, M₃: Mirrors, P₁, P₂, P₃: Polarizers, Q₁, Q₂: Quarter-Wave Plates, θ : Angle of Incidence, α : Grating Inclination Angle.

Collimated laser systems, shown in Figure 7, are also used for multi-DOF error identification. These systems divide laser beams into multiple paths, reflecting them off mirrors and retroreflectors to determine error variations [12]. This approach enables efficient multi-axis error detection but requires careful alignment and environmental control. Table 2 summarizes more information about the multi-DOF measurement critiques.

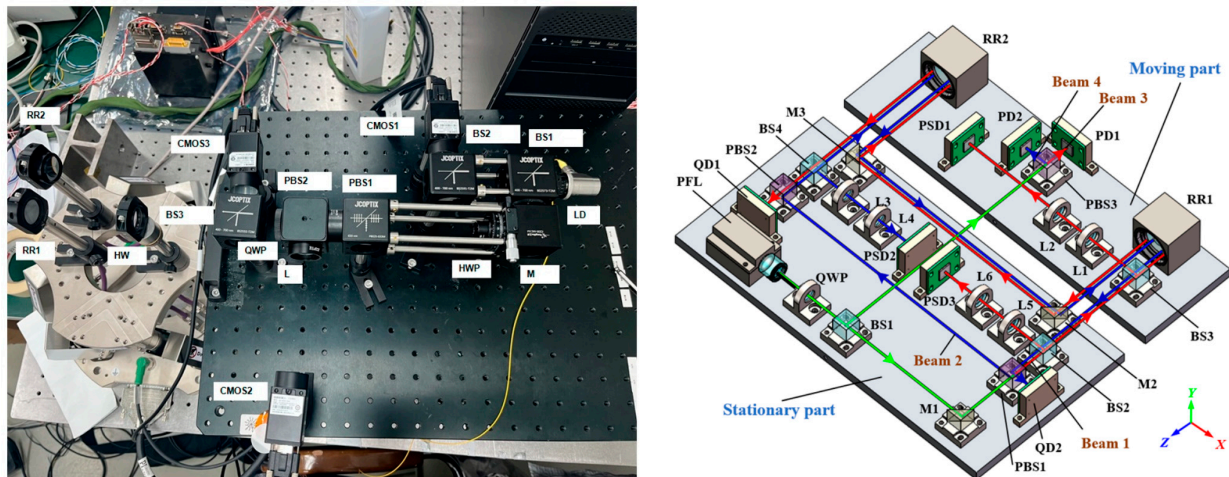


Figure 7. Measuring based on collimation [82].

Table 2. Multi-degree-of-freedom geometric error measurement methods.

Error Type	Description	Instrument	Key Features	Resolution/Accuracy
Positioning [55,83]	Involves errors in the location or displacement of the CNC machine tool along the axes, impacting the accuracy of tool movements.	Laser interferometer	Precise linear and rotational measurements are used for positioning errors.	1 μm resolution, 0.01 arcsec accuracy
Horizontal Straightness [79]	Refers to the deviation of a machine's horizontal motion along the X or Y axis, impacting flatness and overall geometry.	2D grating, heterodyne light source	High system stability for measuring horizontal straightness.	0.00141 μm resolution
Vertical Straightness [79]	Measures errors related to the vertical axis of the machine, such as any upward or downward deviation from a straight path.	Heterodyne light source, beam splitter	Measures both horizontal and vertical straightness, pitch, yaw, and roll.	0.002 μm resolution, 0.0103 arcsec accuracy
Multi-Axis Motion [80,84]	Involves combined errors across multiple axes, such as yaw, pitch, and roll, affecting the overall motion of the tool in space.	Linear grating, heterodyne source	Accurate multi-axis motion tracking, including vertical straightness.	$\pm 0.3 \mu\text{m}$ accuracy, ± 0.25 arcsec
Vertical Straightness [79,85,86]	Focused on vertical deviations in the system, often seen when measuring the linearity of vertical motion components.	Reflective grating, optical encoder	Enhanced resolution, simplified setup for measuring vertical straightness.	0.000137 μm resolution
Horizontal/Vertical Straightness [87]	Detects errors in both horizontal and vertical straightness, often using multiple sensors to ensure the precision of CNC machining along multiple axes.	Diode laser, autocollimator	Compact design, suitable for horizontal and vertical straightness.	$\pm 0.3 \mu\text{m}$ accuracy, $\pm 100 \mu\text{m}$, ± 0.6 arcsec
Pitch, Yaw, Roll [88,89]	These are angular errors related to the tool's orientation in space, affecting the angles of tool motion along the axes.	He-Ne laser, beam splitter	Versatile system for horizontal and vertical straightness, as well as angular measurements (pitch, yaw, and roll).	$\pm 1.5 \mu\text{m}$ accuracy, ± 1.9 arcsec

Table 2. Cont.

Error Type	Description	Instrument	Key Features	Resolution/Accuracy
Positioning and Straightness [87]	Deals with both the positioning of the machine tool and its straightness along the axis, particularly useful in simpler setups or compact designs.	Laser diode, position sensing detector	Simple, compact system for positioning and straightness measurement.	0.5 μm resolution, 0.4125 arcsec accuracy
Pitch, Yaw, Roll [27,90]	Similar to angular measurements, focused on ensuring that the CNC machine tool's orientation remains within tolerance, preventing defects in the final product.	He-Ne laser, position detector	Provides low-cost, simplified assembly for pitch, yaw, roll, and straightness.	$\pm 1 \mu\text{m}$ accuracy, $\pm 100 \text{ mm}$, $\pm 0.2 \text{ arcsec}$
Beam Stability [91]	Refers to maintaining a stable light beam in optical systems, reducing error during measurements of straightness and angular motion.	Pigtailed diode laser, beam splitter	Stable beam alignment, suitable for accurate angular and straightness measurements.	$\pm 0.25 \mu\text{m}$ accuracy, $\pm 160 \text{ arcsec}$
Horizontal/Vertical Measurement [92]	Measures horizontal and vertical straightness, ensuring that tool paths are as intended without distortion.	Fiber-coupled laser diode, reflector	Reliable and precise measurement for horizontal and vertical straightness.	$\pm 0.9 \mu\text{m}$ accuracy, $\pm 200 \text{ mm}$

4.1.2. Indirect Measurement Methods

Due to their cost-effectiveness and adaptability, indirect measurement methods have become indispensable for identifying spatial discrepancies in machining devices. Unlike direct methods, these approaches rely on analyzing deviations in machine movements and tool paths without requiring specialized equipment for each error type. Instead, measurement results are combined with computational models to extract error parameters. Two primary indirect methods include diagonal/step-diagonal tests and machining-based techniques.

Diagonal Tests

As ISO 230-6:2002 [71] specified, the diagonal test is a widely adopted method for evaluating volumetric precision in three-axis machining devices. In this test, the machine tool's axes are moved along its body diagonals, and deviations are measured using a laser interferometer, as illustrated in Figure 8a. While this method provides a quick overview of volumetric discrepancies, it cannot effectively separate individual error parts due to the combined effects of multiple error sources. To address this limitation, the step-diagonal test was introduced. This method segments the diagonal path into stepwise motions along each axis, as shown in Figure 8b, allowing for a more precise identification of error parts [93–95].

Table 3 summarizes the various diagonal and step-diagonal approaches, highlighting their strengths and limitations. For example, discrepancies introduced by misalignment in laser setups or the neglect of angular influences are mitigated by incorporating independent linear error measurements or using enhanced formulations of step-diagonal paths.

Previous studies have primarily focused on identifying spatial discrepancies, with limited attention given to discrepancies arising from the measurement system, which can significantly influence results. Peng et al. investigated how installation inaccuracies in the measurement setup could affect error identification. They introduced an optimization technique to determine the most influential ways of placing a laser interferometer. Similarly, another study [99] highlighted the impact of neglecting Abbe's principle, where angular discrepancies could be misinterpreted as linear or positioning discrepancies during

measurements with a laser interferometer. To address this, they analyzed the interaction between angular discrepancies and kinematic inaccuracies, developing a model that mitigates the interference between these error types [90].

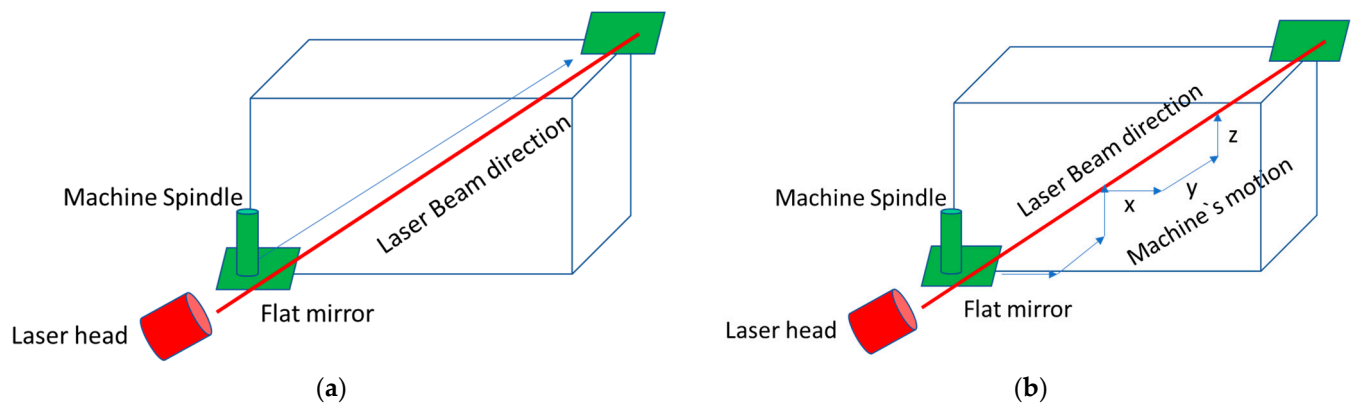


Figure 8. (a) Diagonal measurement and (b) step-diagonal measurement representation.

Table 3. Multi-step path analysis.

Reference	Approach	Axes	Observed Errors (PDGEs)	Observed Errors (PIGEs)
Ibaraki et al. [93]	2D Step-Diagonal	X,YX, YX,Y	Positional inaccuracies, horizontal and vertical deviations	Errors in Squareness
Ibaraki et al. [96]	3D Step-Diagonal	X,Y,ZX, Y, ZX,Y,Z	Positional inaccuracies, horizontal and vertical deviations	Errors in Squareness
Li et al. [75]	13-Line Measurement Approach	X,Y,ZX, Y, ZX,Y,Z	Positional inaccuracies, pitch, yaw, and roll errors	Errors in Squareness
Li et al. [97]	Sequential Step-Diagonal Approach	X,Y,ZX, Y, ZX,Y,Z	Positional inaccuracies, horizontal and vertical deviations, and pitch, yaw, roll errors (only in X,YX, YX,Y)	Errors in Squareness
Bui et al. [98]	Three-Face Step-Diagonal	X,Y,ZX, Y, ZX,Y,Z	Positional inaccuracies, horizontal and vertical deviations	Errors in Squareness

Laser Trackers

Laser trackers have become vital technologies in industrial measurement, demonstrating remarkable progress over the past two decades. Today, they are widely regarded as one of the most versatile tools for portable measurement in various industrial fields, including robotics metrology and the assembly of large-scale products [100].

The working principle of a laser tracker (illustrated in Figure 9) relies on projecting a laser beam onto a retroreflective optical target attached to the object being measured. The tracker captures the reflected beam, which computes the target's position based on the measured distance, azimuth, and zenith angles. These calculations enable determining the target's location. Extensive research has been conducted on the methodologies and practical applications of laser trackers [100].

Laser trackers have become a preferred tool for volumetric error assessments [95,101] due to their extended range and user-friendly operation. However, their performance may be compromised by systematic and random errors arising from design flaws, assembly inconsistencies, test setup variability, and environmental disturbances. A notable challenge is angular measurement errors, which tend to amplify with increasing measurement distances. Multiple laser trackers are frequently used to address these limitations, employing

a multilateration approach. This technique calculates target positions based solely on distance measurements, enhancing precision. For instance, Aguado et al. [102] demonstrated that optimal spatial positioning of multiple trackers can improve accuracy by 75% to 90%. Originally implemented in coordinate measuring machines (CMMs), multilateration has since been adapted for CNC machining tools.

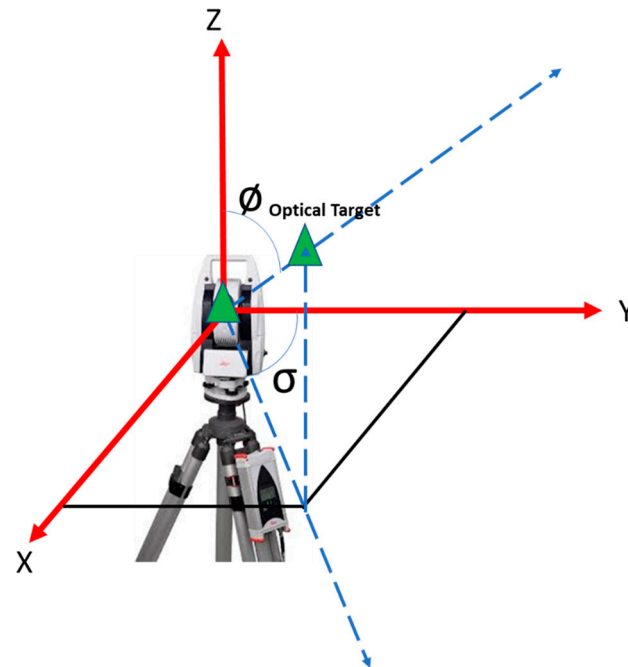


Figure 9. Laser tracker.

A viable alternative to multiple trackers is the pseudo-multilateration method, where a single tracker is repositioned systematically to optimize measurement geometry and reduce uncertainty [78]. This approach involves programming the machine tool to follow a predefined 3D motion path while the laser tracker records the motion at various base positions, as shown in Figure 10. In solving spatial equations, discrepancies in the trajectory can be mapped to specific errors. Zhang et al. [100] refined this technique by integrating different objects. These targets move incrementally along each axis, simplifying error identification through step-by-step multilateration.

The effectiveness of multilateration relies on the positioning and number of trackers in relation to the machine's operational area. Achieving precise alignment between the laser tracker's reference frame and the machining equipment is crucial, typically achieved through self-calibration processes. This alignment involves minimizing registration errors by matching reference points using least-squares optimization. Wan et al. [103] improved alignment precision by introducing directional weighting in their calculations and developing an iterative algorithm to optimize the registration process. For large-scale machining systems, Wang et al. designed an optimization model to determine ideal tracker placement, reducing the error magnification factor across measurement points. They found that spatial configuration often yields optimal results.

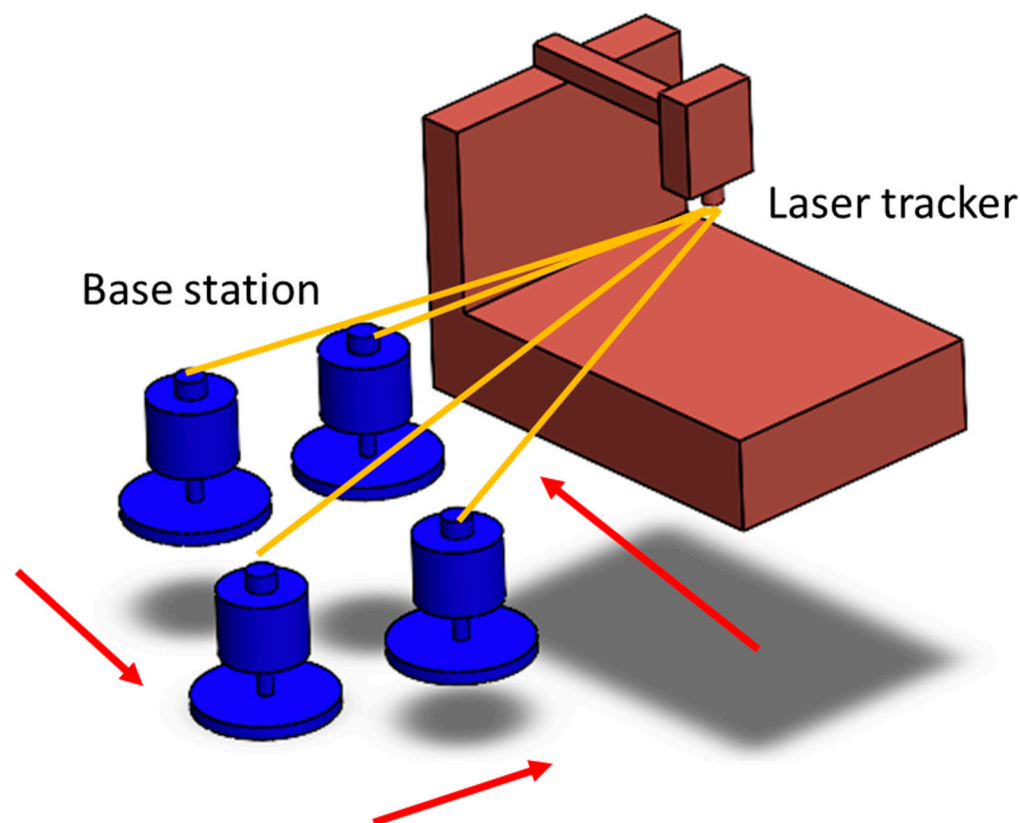


Figure 10. Sequential multilateration representation.

Researchers [104] developed laser tracers to enhance precision, which are particularly effective for detecting rotary axis errors. Laser tracers are designed by known institutions to achieve superior repeatability and reduced radial discrepancies compared to conventional laser trackers. In the research [104], the advantages of laser tracers for volumetric verification were demonstrated by using multilateration principles. Leveraging multi-body system theory and homogeneous transformation matrices, Ibaraki et al. proposed optimized iterative methods to isolate and identify individual discrepancies more accurately.

Cutting Experiments

Cutting experiments and analyzing manufactured parts are the common dependable methods for evaluating machine tool performance in manufacturing settings. A notable example is composite cutting, which assesses the precision of three-axis CNC machines by machining different shapes [3,6]. This test, called the ‘circle–diamond–square test’, later became part of the Agency’s NSA 979 standard [105]. Variants of this test were subsequently adopted in international and national standards, including ISO 10791-7:2014 and ASME B5.54:2005 [106]. Deviations [58,71] in straightness, parallelism, and perpendicularity for these profiles are evaluated using a coordinate measuring machine (CMM) [104,107], offering detailed insights into the accuracy of machined surfaces, particularly the flat or sloped edges of the test piece.

Despite its effectiveness in measuring spatial accuracy, this test does not differentiate between various kinematic errors that may impact machine tool performance. Pezeshki and Arezoo [106] developed a kinematic error model based on data derived from these test pieces to overcome this limitation. They created a specialized series of test components designed to detect such discrepancies in three-axis CNC machines, as shown in Figure 11.

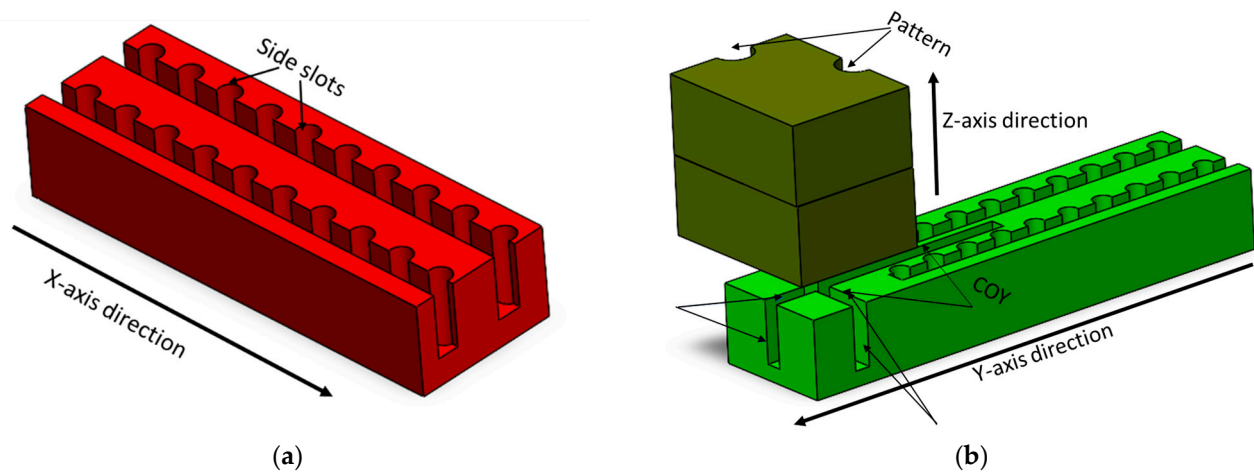


Figure 11. Kinematic errors of milling machine, related to (a) X-axis and (b) related to Y- and Z-axes.

It is important to note that multiple factors influence the overall precision of machined workpieces. These include machining parameters, the condition of measurement tools, tool wear, and spatial errors caused by the machine [6]. To mitigate dynamic effects introduced during machining processes, optimizing and simplifying tool path designs becomes critical for enhancing precision.

Double Ball Bar Tests

The double ball bar (DBB) is an established tool for assessing the performance of three-axis machining equipment, having evolved into a reliable and widely adopted measurement instrument [29,108]. The DBB includes a distance measurement system with two calibrated spheres at both extremes. Both linear axes are activated simultaneously during testing, causing the measuring ball to trace a circular trajectory around a central ball. A displacement sensor records the variations in distance between the two balls, which indicate dynamic discrepancies between the linear axes, as described in Figure 12.

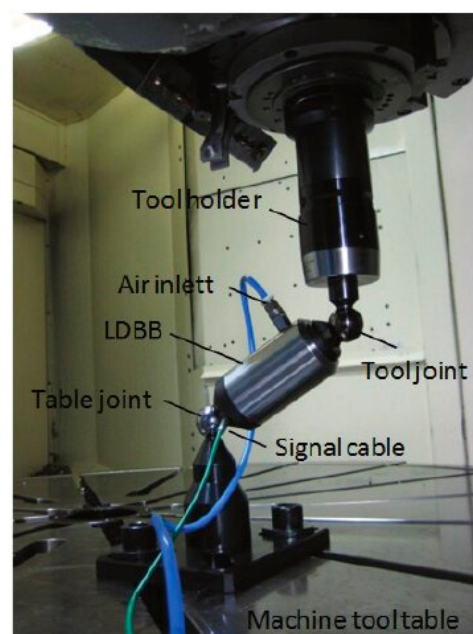


Figure 12. Double ball bar [109]. Reproduced with permission from Archenti et al. [109].

The typical DBB evaluation process for tri-axial machines includes a full rotational test in the XY plane and two segmental arc assessments in the YZ and XZ planes [29,108].

These tests help translate observed changes in length into spatial discrepancies, making the method ideal for assessing machine precision, mainly through circular planar tests. However, the complexity of error models and their nonlinear mappings can complicate the identification process, sometimes resulting in poorly posed problems. To enhance precision, Tian et al. [110] proposed representing the error components with polynomials, applying regularization techniques to determine the polynomial coefficients while overcoming the ill-posed nature of the problem. Conversely, Xu et al. [29] highlighted that linear dependencies in the polynomial error components can complicate the calibration process. To address this, they developed a method using non-integer exponents to model Position-Dependent Geometric Errors (PDGEs), which reduces the challenges introduced by linear correlations by implementing a pre-fitted model.

In traditional DBB measurements, the data are adjusted to the center of the optimal fit circle to compensate for installation discrepancies. However, when the eccentricity of the circle is substantial, the distortion can be significant, leading to inaccurate results. Lee and Yang investigated the effect of eccentricity on spatial errors, showing that measured data should be examined without any mathematical adjustments to maintain accuracy [29,111]. Furthermore, as circular deviations usually evaluate the accuracy of only two of the three translational axes, a new spherical deviation measure was proposed. This measure, which defines the maximum radial distance of deviations from a best-fit sphere (refer to Figure 13), is used to assess the performance of three-dimensional machines in unloaded states.

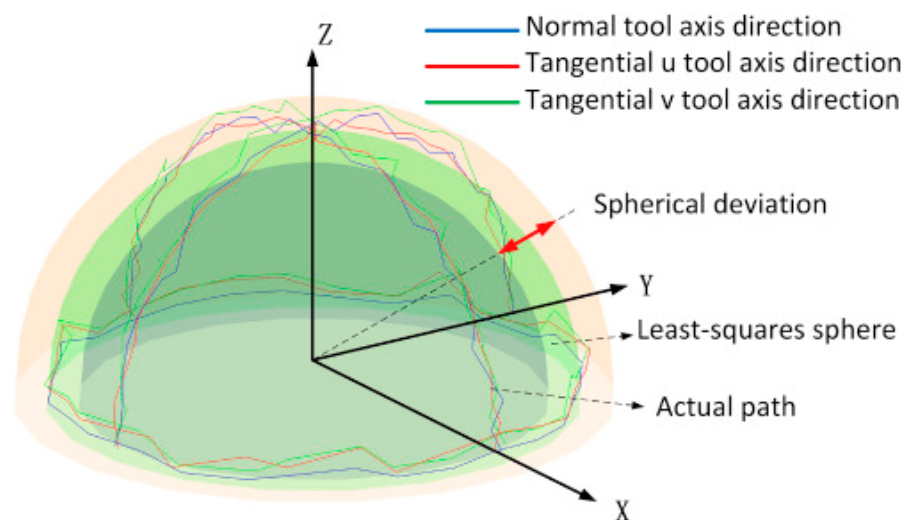


Figure 13. Spherical deviation [101]. Reproduced with permission from Zhong et al. [112].

Loaded DBB [113] is a new variation of the DBB, which was developed to assess machining performance under loaded conditions. Researchers have examined the combined impact of spatial errors and load-induced variations, using the LDBB to assess the static rigidity properties of the machinery and analyze how these factors affect part geometry.

4.1.3. Measurement Techniques for Specific Linear Axis Errors

- **Straightness Measurement [85,86]:** Straightness deviations can occur along horizontal and vertical planes. To measure the straightness of the X-, Y-, and Z-axes—especially for long travel axes—tools such as laser interferometers, dial indicators, and high-precision linear scales are commonly employed. These tools help detect any misalignment or distortion in the path of the axes. Straightness errors often arise from structural imperfections in the machine bed or thermal effects that alter the shape of the components during operation [79].

- Flatness Measurement [86]: Flatness errors arise as a result of straightness deviations along several axes. These are measured by moving the linear axis across its workspace and continually checking for any deviations from a flat reference surface. Instruments such as laser systems, straightedges, and electronic levels accurately give these measurements. This process helps identify compound errors that affect the overall geometry of the machine's movements.
- Squareness Measurement [77]: Squareness defines the angular error between two axes that are supposed to be at right angles. It is crucial to ensure proper squareness between the linear axes to avoid unexpected movements when one axis is moved. Typically, squareness errors are determined as the measurement of the angles between various moving machine members with accurate angular measuring devices. The correction of squareness errors involves ensuring that motions in one axis do not result in unwanted displacement in the direction of its perpendicular axis [77].

4.1.4. Error Detection in Complex Linear Movements

In multi-axis CNC machines, the linear motions are susceptible to direct errors in the individual axes and also to coupled errors arising out of interaction with rotary axes. Detection of more complicated errors requires measurement not only of the motion of each axis but also assessment of the behavior of the overall system.

- Multi-axis error detection: This is considered in machines with more than one linear axis. It concerns the dependency of errors against one another along several axes. A good example is an error in the X-axis, which would consequently affect the motion at Y- and Z-axes because of the mechanical coupling between these two axes. Multi-degree-of-freedom measurement systems usually keep track of several axes errors simultaneously. These systems offer a holistic view of machine performance and support the location of complex errors that result in a general decrease in machine accuracy [35,114].
- Machine Calibration and Compensation: In the event of finding error in the linear axes, it can be compensated by a machine's control system. First, the software is recalibrated according to the found discrepancies of the machine. The errors identified are then compensated by developing techniques for compensation, which mainly include geometric error models and adjusting the tool path. This allows it to continue to have a high level of accuracy during the machining process since this system is naturally filled with so many inaccuracies [5].

4.1.5. Advanced Calibration and On-Machine Measurements

1. Advanced Calibration and On-Machine Measurements

The precision and accuracy of modern CNC machines have been constantly improved by advances in calibration techniques. Among them, on-machine measurement and calibration systems represent a quantum leap in the field, enabling real-time detection and correction of machining errors directly during manufacturing. Unlike conventional calibration methods, which often require machine downtime and external equipment, on-machine systems integrate measurement tools within the machine, ensuring a seamless and efficient workflow [115].

The selection of measuring probes is crucial in determining the precision and effectiveness of OMM systems; probes are among the most precise tools available, offering exceptional sensitivity that enables the detection of minute forces. This capability makes them ideal for achieving nanometer-level accuracy in ultra-precision machining processes. These probes are particularly suited for inspecting complex geometries and maintaining consistent measurements in scenarios requiring sub-micron tolerances [116,117].

On the other hand, while durable and commonly utilized, kinematic resistive probes offer lower sensitivity and resolution than strain gauge probes. Their strengths lie in tasks where extreme precision is less critical, such as preliminary setups or measurements with relaxed accuracy requirements. This contrast underscores the necessity of selecting probes that align with the specific demands of the machining application [118,119]. For instance, strain gauge probes are often preferred when integrating OMM systems into ultra-precision workflows. Their high precision and resistance to variations caused by thermal or dynamic factors make them indispensable for reliable real-time monitoring and adaptive machining adjustments [120,121].

2. On-Machine Surface Measurements (OMSMs)

On-Machine Surface Measurements (OMSMs) use embedded sensors and measurement tools to measure and correct geometric errors without removing the workpiece or interrupting the machining process. These systems are critical in ultra-precision machining, where nanometric tolerances are critical. OMSMs can capture deviations in real time and provide immediate feedback to the control system to adjust tool paths or correct positioning errors [122–124]. The main differences between on-machine and conventional calibration are shown in Table 4.

3. Advantages of OMSMs

1. **Reduced Downtime:** OMSMs reduce machine idle time and increase overall productivity since there is no need to dismantle the setup for calibration outside.
2. **Real-Time Error Detection:** with continuous monitoring, errors can be identified and corrected immediately to ensure higher machining accuracy.
3. **Closed-Loop Precision Control:** the integrations with CNC controllers automatically compensate for errors to create a closed loop that further enhances reliability and repeatability [125].

4. Challenges in Implementing OMSMs

While OMSMs offer remarkable benefits, their implementation presents specific challenges:

- **Motion-Induced Measurement Errors:** embedded sensors must account for vibrations and dynamic motions of the machine, which can interfere with measurement accuracy.
- **Sensor Calibration:** the sensors require precise calibration to ensure their measurements align with the machine's actual motion characteristics.
- **Data Processing Requirements:** real-time measurement generates large volumes of data that must be quickly processed and analyzed to maintain machining speed and precision [124,126].

5. Calibration Strategies for OMSMs

Effective calibration of on-machine systems involves systematic error identification and compensation. The process typically includes the following:

- **Source of Error Identification:** distinguishing motion errors from environmental influences like temperature variations or vibrations through detailed analysis and modeling.
- **Integration of Advanced Sensors:** application of high-resolution tools like capacitive sensors, laser interferometers, or optical encoders for error measurements up to nanometric resolution.
- **Compensating Algorithms:** developing algorithms to forecast and counteract identified errors in real time guarantees constant performance for machining.

Table 4. On-machine vs. conventional calibration comparison.

Aspect	On-Machine Calibration	Conventional Calibration
Downtime	Minimal	Significant
Accuracy	Real-time, adaptive	Dependent on external conditions
Flexibility	High (adapts to changing errors)	Limited (static calibration settings)
Complexity	High (requires integration of sensors and algorithms)	Moderate (requires skilled operators)

6. Future Potential of OMSMs

The integration of OMSMs will play a revolutionary role in precision manufacturing. Advances in sensor technology [50,127], data analytics, and machine learning are expected to enhance their capabilities further. For instance, real-time data-driven predictive models can predict tool wear or thermal deformation and thus proactively adapt the machining process. Additionally, OMSMs can make autonomous machining systems feasible, where the machine continuously learns and improves accuracy over time. In fact, by leveraging OMSMs, manufacturers can achieve unprecedented levels of precision while reducing operational costs and sustaining high productivity. These systems represent a critical step forward in the journey toward fully automated and intelligent manufacturing processes [128–130].

4.1.6. Hybrid Measurement Systems

The hybrid indirect measurement method is an advanced approach to identifying and quantifying geometric errors in CNC machining systems. By combining the strengths of multiple indirect techniques, this method provides a comprehensive, efficient, and cost-effective solution for measuring errors without direct physical access to all machine components. It is particularly useful for complex multi-axis machining systems, where individual errors interact in intricate ways.

Combination of Measurement Techniques

Hybrid indirect methods capitalize on the strengths of various measurement approaches to detect and analyze errors comprehensively. For instance, the double ball bar (DBB) test is a well-established method for identifying planar errors in circular paths, especially in the XY, YZ, and XZ planes [131]. It excels in capturing deviations caused by misalignment or backlash in linear axes. Diagonal tests, on the other hand, provide volumetric accuracy insights by analyzing deviations along the machine's body diagonals. Integrating these two methods ensures that localized and systemic errors are identified effectively. Studies have shown that combining DBB and diagonal tests can reveal error patterns that might be missed when using either method alone [12]. Laser trackers and step-diagonal tests offer another complementary pairing. Laser trackers are renowned for their ability to measure spatial positioning over extensive ranges, making them ideal for large-scale or multi-axis machining systems. Step-diagonal tests isolate linear and angular errors along specific axes, allowing for detailed calibration and error separation [132]. These methods ensure precise error mapping across all axes, as Ibaraki et al. [133] demonstrated. Cutting experiments and coordinate measuring machines (CMMs) also play a crucial role in hybrid methods. Cutting experiments assess machining performance by analyzing deviations in manufactured parts, while CMMs provide high-precision validation of part geometries. The synergy between these techniques allows manufacturers to correlate machine tool behavior with actual workpiece accuracy, as shown in studies by Jodar et al. [134].

Error Decoupling Algorithms

Advanced mathematical models are essential for hybrid indirect methods to effectively separate and quantify overlapping error components. Regression analysis is commonly employed to identify linear relationships among errors [135], while Fourier transforms analyze periodic or cyclical error patterns [136]. Neural networks and machine learning algorithms enhance error decoupling by identifying nonlinear relationships and complex interactions between error sources. For example, Feng et al. [137] utilized a neural network-based model to isolate thermal distortions from geometric inaccuracies, achieving a 15% improvement in error compensation accuracy. The iterative refinement of these models ensures that compounded errors, such as those caused by tool deflection or machine misalignments, are addressed comprehensively.

Sensor Fusion

Sensor fusion is a cornerstone of hybrid indirect measurement methods. Multiple sensors, including laser interferometers, capacitive probes, and optical encoders, are integrated to collect high-resolution data [138]. Sensor fusion algorithms synthesize these inputs into a unified error map, leveraging the strengths of each sensor type. Laser interferometers, for instance, provide unparalleled accuracy for linear positioning measurements, while capacitive probes excel in detecting minute surface deviations [69]. Optical encoders add rotational accuracy, ensuring comprehensive error detection. Sensor fusion minimizes measurement uncertainties and enhances system robustness, as Guo et al. [139] demonstrated in their study on multi-axis machining systems.

Working Principle of Hybrid Indirect Measurement

The operation of hybrid methods begins with data collection, where the machine executes predefined movements or cutting paths, such as circular, diagonal, or stepped trajectories. Sensors record deviations from ideal trajectories, generating raw error data. These data are then processed using mathematical models to interpret and map deviations to the machine's coordinate system. For instance, transformation matrices convert positional deviations into kinematic relationships, while finite element analysis (FEA) predicts deformation-induced errors based on measured forces [137,140].

The next step involves error analysis and decoupling, combining data from various techniques to isolate overlapping errors. Thermal and geometric errors can be separated by analyzing patterns in positional deviations over time. Finally, the hybrid model undergoes validation and refinement using precision tools like CMMs or laser trackers. Discrepancies between predicted and actual measurements are addressed iteratively, ensuring the model's reliability [137].

Advantages of Hybrid Indirect Methods

Hybrid indirect methods offer comprehensive error capture by integrating multiple techniques, enabling the detection of position-dependent, position-independent, and dynamic errors. Their combination of diverse data sources reduces uncertainties, providing higher accuracy and precision. For example, laser trackers' broad-range capabilities complement the DBB's high-resolution localized measurements, resulting in a more holistic error analysis [131].

These methods are also cost-effective, reducing reliance on expensive direct measurement tools and maximizing the utility of existing devices. Furthermore, hybrid methods are non-invasive, making them ideal for in-process measurements that do not disrupt machining operations. Their flexibility allows them to adapt to various machine config-

urations, including three-axis, five-axis, and hybrid systems, ensuring their applicability across various manufacturing environments [139,140].

4.2. Rotary Axis Error Measurement

As CNC technology advances, multi-axis machining systems, particularly five-axis machines, have become essential in the aerospace, marine engineering, and precision manufacturing industries [55,85]. These systems significantly enhance machining flexibility and efficiency for complex geometries by integrating two rotary axes with three traditional linear axes. However, rotary axes also introduce additional geometric errors, which must be accurately measured and addressed to ensure high-precision machining [11].

Geometric errors in rotary axes can be broadly categorized into the following:

- Position-Dependent Geometric Errors (PDGEs): errors vary based on the rotary axis position or angular movement [30].
- Position-Independent Geometric Errors (PIGEs): constant errors resulting from assembly imperfections, such as misalignments or structural defects [27,141].

Each rotary axis contributes up to ten unique spatial error components (as shown in Figure 14), including radial deviations, tilt errors, and positional inaccuracies [104,142]. Due to the combined motion of its axes, a five-axis machine can exhibit more than forty spatial discrepancies. Therefore, effective identification and measurement techniques for rotary axes are crucial to improving machining accuracy. The methods for measuring geometric errors in rotary axes are generally divided into direct and indirect approaches [4]. Each method has distinct advantages and challenges, explored below.

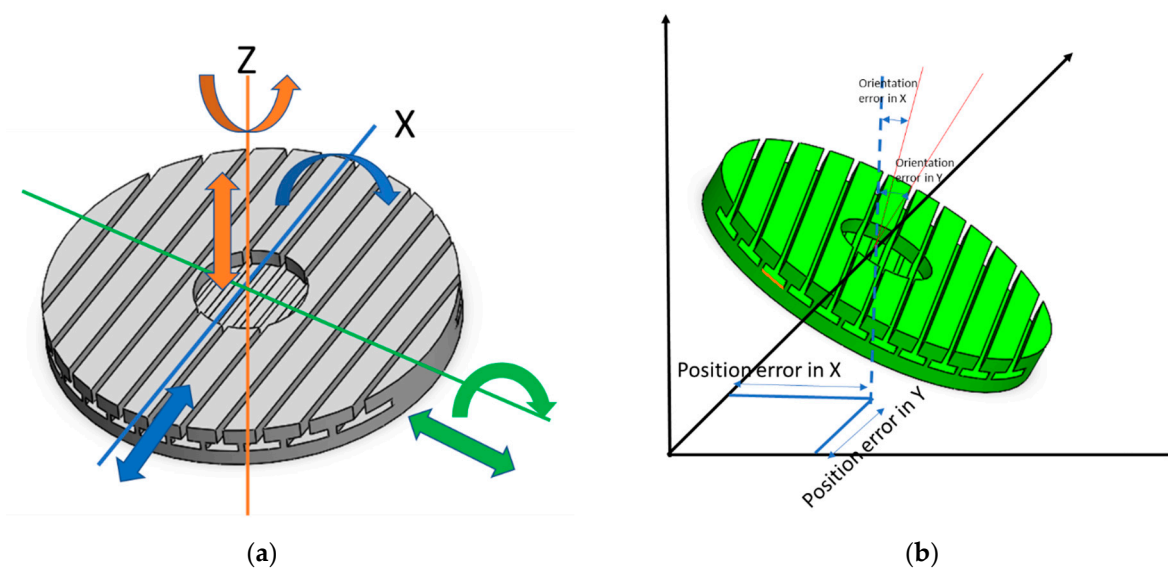


Figure 14. (a) PDGEs and (b) PIGEs.

4.2.1. Direct Measurement Techniques

Direct measurement techniques involve using precision instruments [143] to capture specific geometric errors of rotary axes. These methods measure individual error components, such as tilt, axial deviation, and run out, without requiring complex computational models.

1. Dial Gauges and Autocollimators

Dial gauges are used to measure run-out deviations of rotary tables or spindles, as the ISO 230-1 [58] standard describes. Autocollimators combined with polygon mirrors can evaluate angular positioning errors, including tilt and radial motion errors. For example,

optical systems, such as laser Doppler instruments, provide highly accurate measurements of angular and positional errors by analyzing reflected beams [144].

2. Multi-DOF Optical Measurement Systems

Liu et al. [80] developed a six-degree-of-freedom (6-DOF) measurement system using pyramid polygon mirrors and laser diodes paired with position-sensitive detectors (PSDs). This setup captures the rotary axis motion errors in multiple directions simultaneously. Another approach involves integrating conical lenses and optical sensors to enhance rotary axis error analysis precision. Despite their accuracy, direct measurement systems face challenges such as setup complexity, environmental sensitivity, and difficulty in measuring coupled motion errors. For these reasons, indirect techniques are often preferred for comprehensive error identification [145].

4.2.2. Indirect Measurement Methods

Indirect methods infer geometric errors by analyzing deviations from predefined tool paths or measurement results. These approaches are widely adopted due to their adaptability and lower equipment costs than direct techniques.

Laser Trackers and Tracers

Laser trackers have become increasingly preferred for rotary axis calibration because of their extensive measurement range and exceptional accuracy. The working principle involves projecting a laser beam onto a retroreflector mounted on the spindle or table and calculating positional deviations based on spherical coordinates [100]. Sequential multilateration, a modified method where a single laser tracker is repositioned to optimize measurement geometry, reduces errors and enhances precision. Additionally, combining data from multiple laser trackers mitigates angular error magnification at longer distances, improving overall accuracy.

Unlike conventional laser trackers, a laser tracer device employs a sequential multilateration approach with a fixed high-precision reference sphere. This configuration eliminates the need for continuous line-of-sight tracking, significantly improving accuracy and reducing setup complexity. The use of a stable reference sphere minimizes systematic errors compared to conventional laser trackers, ensuring higher measurement accuracy. Furthermore, a laser tracer provides greater stability for large-scale measurements, as it does not suffer from alignment errors at long distances, making it particularly well suited for precision calibration of both linear and rotary axes in machine tools and coordinate measuring machines (CMMs).

Figure 15 shows a schematic of a laser tracker system for rotary axis error identification. Recent studies, such as those by Zhang et al. [146], have demonstrated that optimized spatial positioning of trackers can improve accuracy by up to 90%, particularly for large-scale machining systems. Research on laser tracer systems further supports their effectiveness in achieving superior calibration performance. Schwenke et al. [147] demonstrated that a single tracking interferometer can effectively map errors in CMMs and machine tools, improving measurement precision through detailed volumetric error analysis. Their work emphasizes the importance of precise error mapping for achieving higher machining accuracy. Similarly, Zha et al. [148] explored volumetric error compensation using a laser tracer system combined with machining verification. Their study highlights how integrating real-time error compensation techniques can significantly enhance the accuracy of machine tools by minimizing geometric deviations, leading to improved manufacturing quality. Additionally, a study by Holub et al. [149] evaluates the application of volumetric compensation in a small three-axis vertical machining center, demonstrating its effectiveness in enhancing machining accuracy. While large machining centers often exhibit significant volumetric

deviations, their research highlights that even small machines can benefit from precise compensation techniques. Moreover, the study emphasizes the necessity of verifying compensation effectiveness through detailed pre- and post-compensation measurements to ensure the anticipated accuracy improvements are achieved. These findings align with our discussion on the importance of advanced calibration and compensation methods in optimizing precision across machining systems.

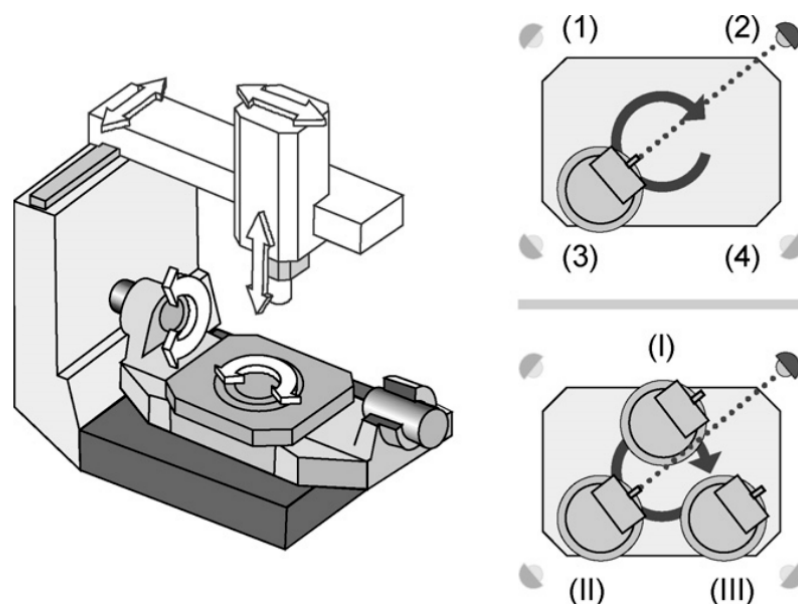


Figure 15. A diagram illustrating a laser tracer system for identifying rotary axis errors [150]. Reproduced with permission from Schwenke et al. [107]. (I–III) are positions and (1–4) are reflector locations.

Double Ball Bar (DBB) Tests

The DBB is a widely used tool for assessing rotary axis performance. By measuring deviations in a circular path traced by the tool center, the DBB identifies spatial discrepancies in rotary axes. Early methods focused on isolating PIGEs using radial, tangential, and axial motion patterns. Jiang et al. [151] introduced a streamlined two-step DBB test, reducing machine downtime while accurately identifying location discrepancies. For PDGEs, researchers like Ding et al. proposed multi-length DBB setups and optimized measurement patterns to decouple linear and rotary errors. Figure 16 illustrates the principle of a DBB test applied to rotary axes. While effective, the DBB is inherently limited to one-dimensional measurements, requiring multiple setups for comprehensive error identification [112,113].

The error arising from the ball bar setup is known to have significant effects on the measurement results due to the difference between the ball's install position and the reference coordinate system origin. The ball bar error in installation could thus be modeled as a constant added to the designed position of the ball. In addition to the above, they designed a fixture with flexure hinges at the tool nose to hold the ball in place while measurement was conducted. Chen et al. [152] optimized ball bar installation to reduce errors and suggested adjusting the mounting position to minimize inaccuracies. They used a standard clamp and circular path method to correct master ball installation errors and applied circular fitting to identify and correct linkage errors in tilt tables. This process also allowed for volumetric error analysis using a pseudo-inverse matrix, as shown in Figure 17.

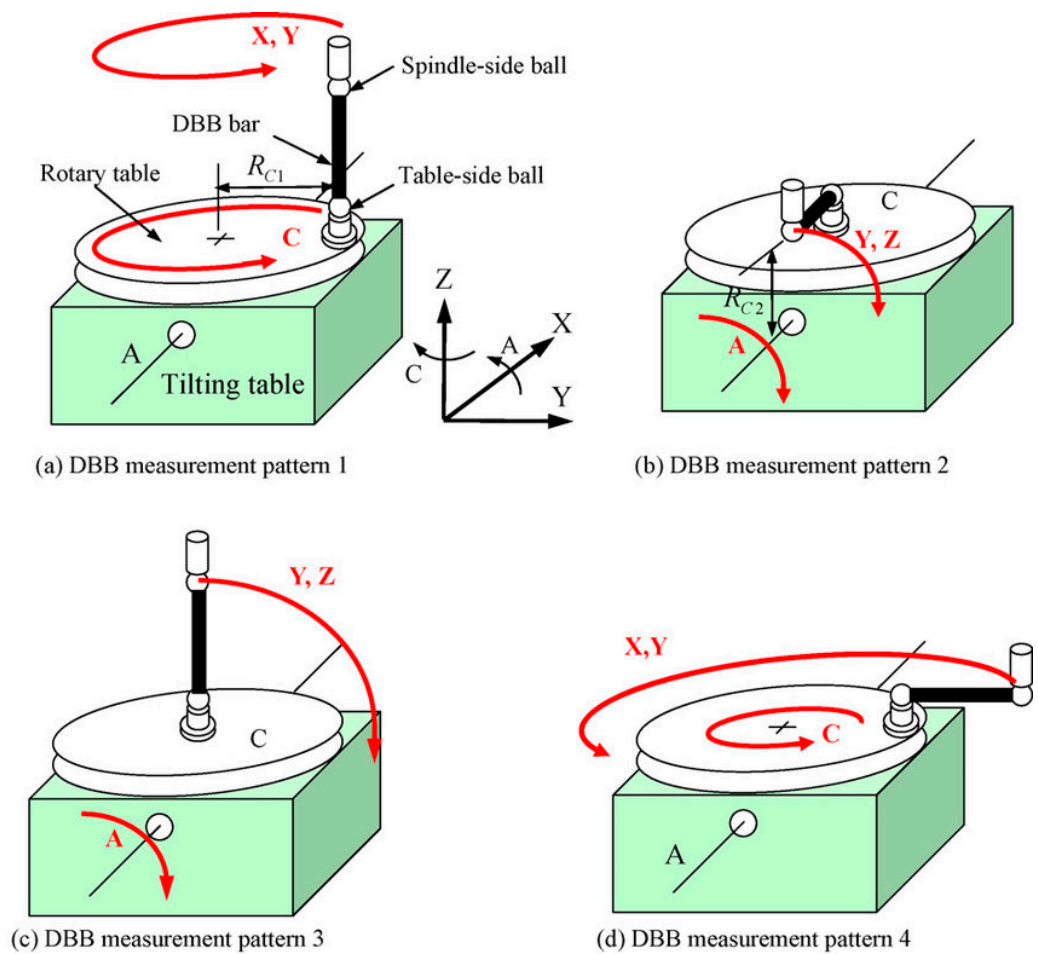


Figure 16. DBB measurement patterns used to identify kinematic error [90]. Reproduced with permission from Uddin et al. [90].

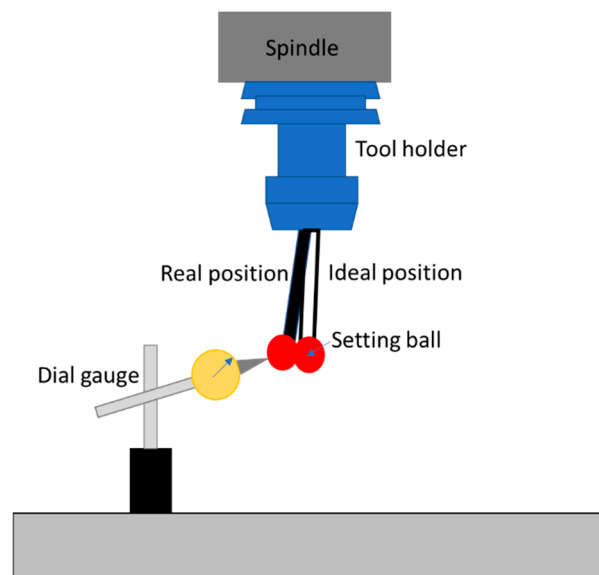


Figure 17. Correction of installation errors in the tool cup.

R-Test

The R-Test, developed to address DBB limitations, captures three-dimensional displacements using an array of linear displacement sensors. A sphere fixed to the rotary axis is monitored as it moves, enabling real-time error trajectory measurements [5,153].

- Advantages: faster data acquisition, higher efficiency, and automated testing cycles.
- Non-Contact Systems: recent innovations integrate laser displacement sensors to eliminate friction-induced errors during dynamic measurements.
- Applications: Ibaraki et al. extended R-Test algorithms to separate B- and C-axis error motions, constructing comprehensive error maps.

On-Machine Measurement (OMM)

OMM systems utilize touch-trigger probes and reference artifacts, such as precision spheres or blocks, to calibrate rotary axes directly on the machine. These systems automate routine precision checks without requiring external devices.

- Touch-Trigger Probing: Ibaraki et al. [93] calibrated rotary axes using a square-column artifact, achieving minimal influence from setup errors.
- Master Ball Artifacts: Precision spheres mounted on machine tables facilitate efficient identification of PIGEs and PDGEs.
- Integration: OMM systems provide accurate real-time calibration by incorporating homogeneous transformation matrices (HTMs) and least-squares optimization. Figure 18 demonstrates an OMM system set up with a master ball artifact.

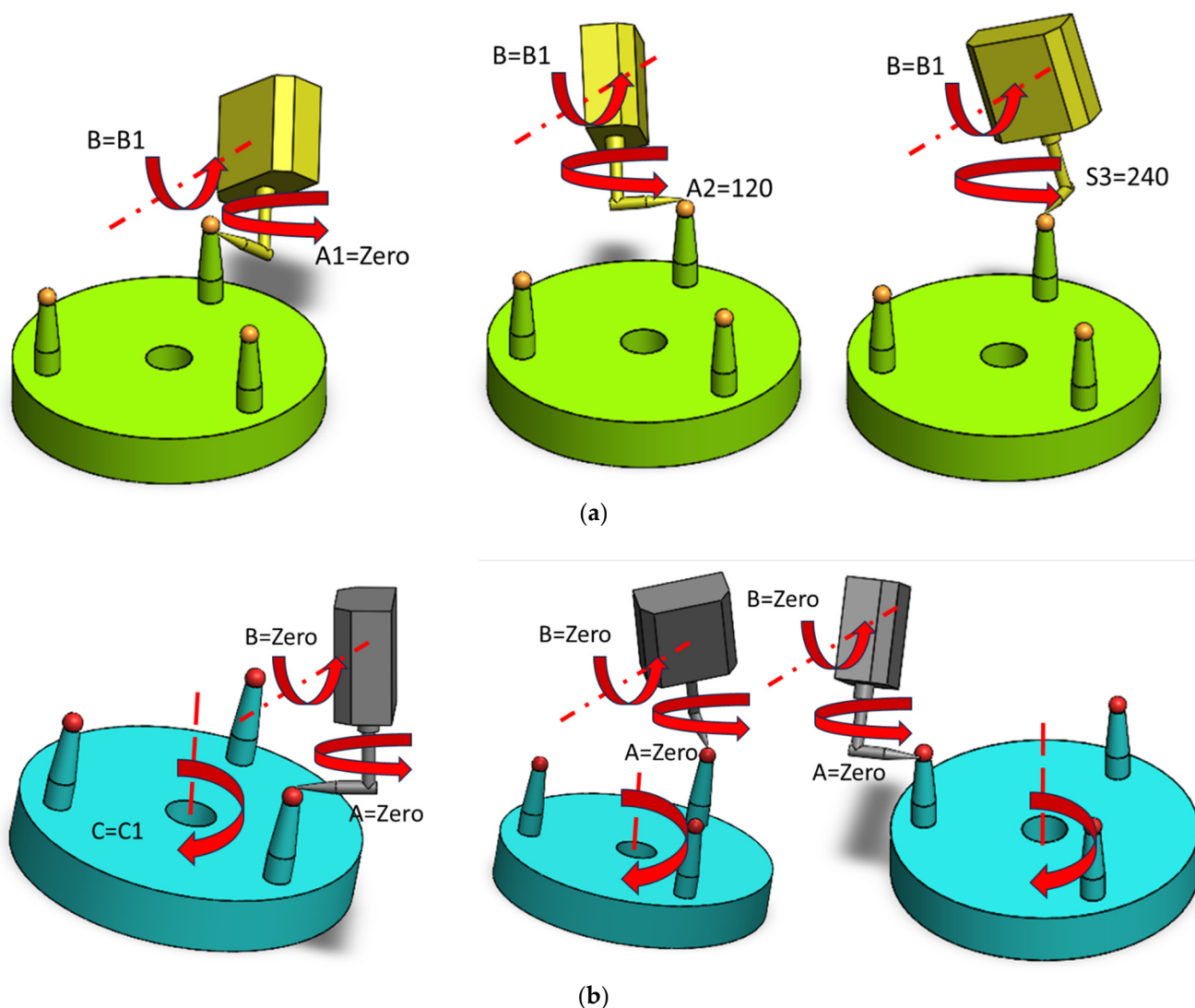


Figure 18. OMM system setup at each index angle, (a) B-axis and (b) C-axis.

When selecting probes for OMM systems, the choice between strain gauge and kinematic resistive probes plays a crucial role in determining the effectiveness of geometric error detection [154]. Strain gauge probes are highly sensitive and capable of detecting minute forces, offering nanometric accuracy essential for ultra-precision machining. They are particularly effective in scenarios requiring sub-micron tolerances and are less influenced by dynamic and thermal factors. This makes them ideal for applications where the highest levels of precision are required [116,155].

In contrast, kinematic resistive probes are more robust and better suited for tasks where extreme precision is not critical. They are commonly used for preliminary setups or measurements with relaxed accuracy requirements. While they offer lower resolution compared to strain gauge probes, their durability and ease of integration make them a practical choice for general-purpose applications [156].

The selection of the appropriate probe should be guided by the specific demands of the machining process. For example, ultra-precision workflows benefit significantly from strain gauge probes, as their high accuracy ensures reliable real-time monitoring and adaptive machining adjustments. Conversely, kinematic resistive probes may be preferred in environments where robustness and long-term stability are prioritized over nanometric resolution. By aligning the probe choice with process requirements, manufacturers can optimize on-machine measurement systems to effectively address geometric errors and enhance machining performance [116,133].

Machining Experiments

Indirect measurement techniques are commonly used to identify geometric errors in rotary axes of five-axis machine tools, especially under real machining conditions where errors differ significantly from static ‘cold’ states. Machining tests have become a practical solution to evaluate rotary axis errors by analyzing specific cutting patterns and the resulting workpiece geometry. For example, a square pyramid test, which involves straight side cuts using an end mill, can effectively reveal kinematic errors [90]. By examining the relationships between machining deviations and the machine’s kinematic model, errors can be identified with high accuracy, as shown in Figure 19. Additionally, thermal effects, such as deformation caused by spindle rotation, further influence rotary axis performance and must be factored into the analysis [7].

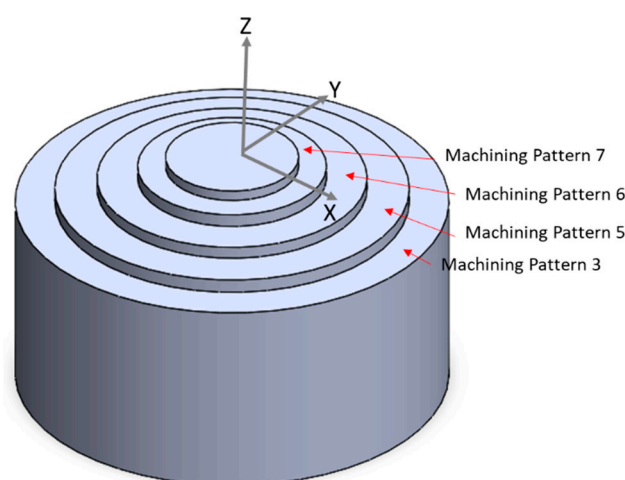


Figure 19. Finished part geometry.

To enhance precision, mathematical models are often employed to optimize the coordinate system settings and evaluate axis errors. For cradle-type five-axis machines, analyzing the tool displacement and surface deviations of the finished workpiece can provide a clear

assessment of rotary axis accuracy. In machines with tilting rotary tables, performing structured machining tests, like cutting a square pyramid, enables detailed identification of positional and directional geometric errors [157].

Alternative methods, such as touch-trigger probing with block test pieces, isolate single-axis errors by measuring reference points at different rotary positions. This approach avoids error coupling between axes and improves measurement accuracy. Optical measurement techniques (OMM) are also used for identifying multiple geometric errors by cutting predefined shallow slots on a workpiece [158], as illustrated in Figure 20. By carefully adjusting the rotary axis rotations, errors in both linear and rotary axes can be decoupled, minimizing the influence of cutting forces and thermal deformation. Additional refinements, such as performing specific machining patterns and analyzing mismatches between slots, further improve measurement reliability [55] (Figure 21).

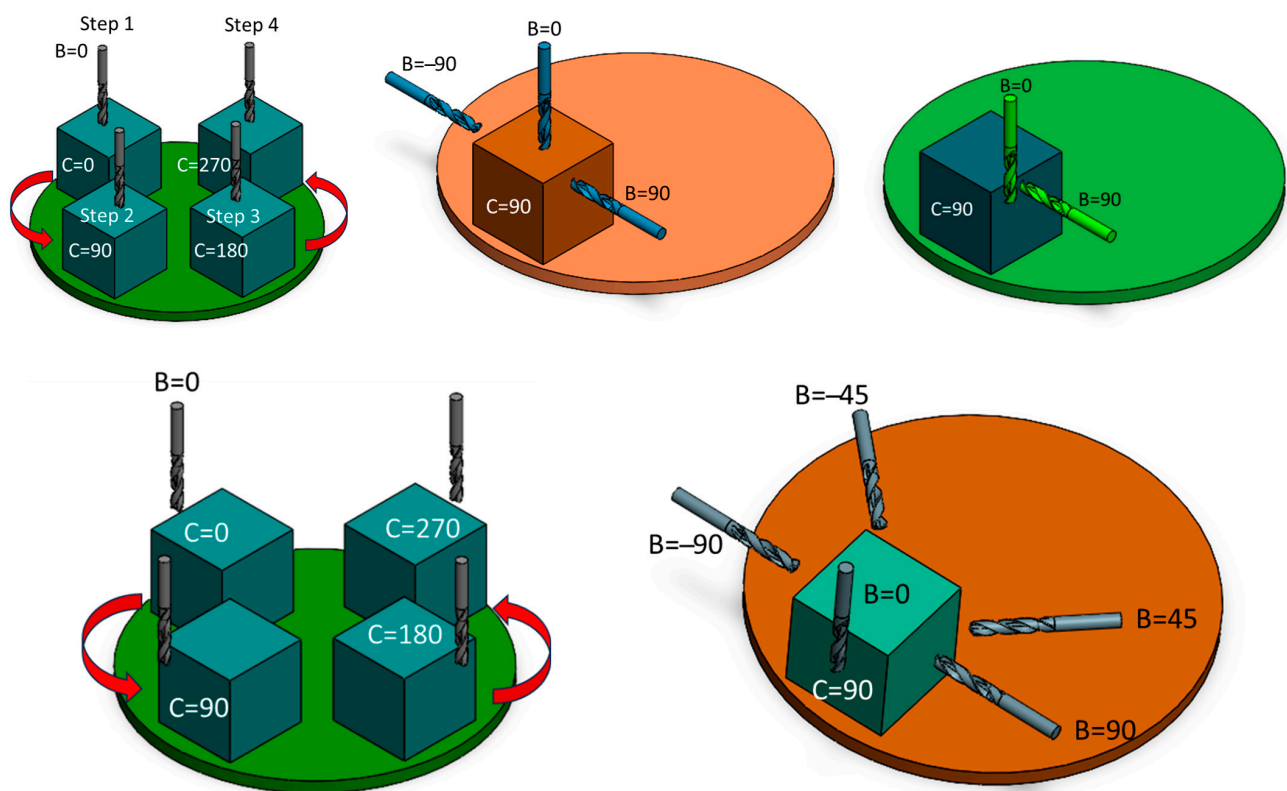


Figure 20. Machining steps on a cubic part.

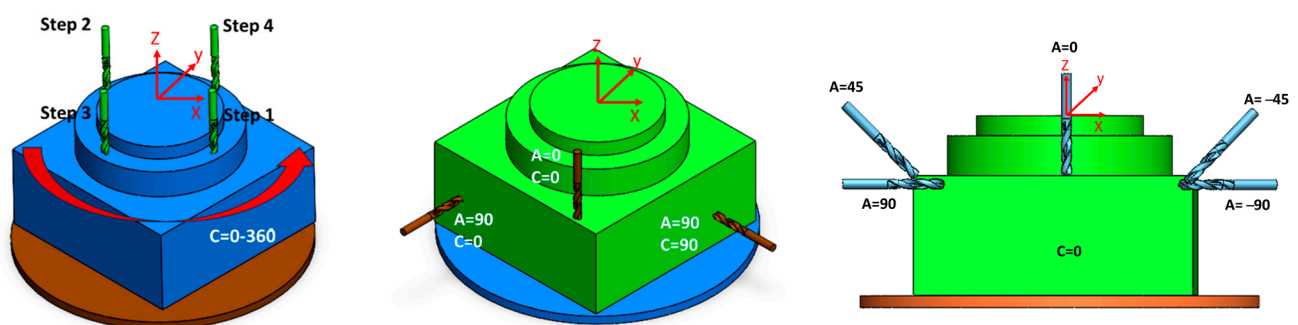


Figure 21. Machining steps on a cylinder-cubic part.

While ball bar (DBB) systems remain widely used for error identification, they face limitations when measuring large-radius circular interpolations [158], similar to challenges

seen in R-tests. Laser trackers, on the other hand, offer advantages like extended measurement ranges and flexible setups, though issues related to measurement uncertainty persist. Optical methods hold significant promise for improving accuracy and efficiency, but separating the contributions of cutting tool errors, servo deviations, and machine vibrations from the overall machining error remains a challenge [158].

In summary, while significant progress has been made in developing methods for identifying rotary axis errors, more robust and user-friendly measurement strategies are still needed. Future efforts should focus on improving precision, reducing uncertainties, and addressing the complexities of real-world machining conditions.

4.2.3. Summary of Techniques

The choice of measurement technique depends on the machine configuration, required precision, and available resources. Table 5 summarizes the key methods discussed in this section.

Table 5. Techniques summary.

Method	Key Features	Advantages	Limitations
Direct Measurement	Optical systems, laser interferometers	High precision, detailed errors	Setup complexity, costly
Laser Trackers	Multilateration, pseudo-multilateration	Large range, flexible setup	Angular error magnification
Double Ball Bar (DBB)	Circular paths, multi-length setups	Simple, cost-effective	1D measurement limitation
R-Test	3D displacement sensing	Fast, automated cycles	Requires sensor calibration
On-Machine Measurement	Touch probes, reference artifacts	Automated, real-time calibration	Setup errors, complex modeling
Machining Experiments	Cone, pyramid, and slot machining tests	Real-world validation	Separating machining influences

By leveraging these techniques, manufacturers can identify, quantify, and compensate for rotary axis errors, ensuring superior precision in multi-axis machining systems. Ongoing advancements in optical sensors, data processing algorithms, and integrated on-machine solutions promise further improvements in rotary axis error measurement and correction.

5. Modeling Strategies for Geometric Errors

Hsu and Wang [159] outlined three key stages for spatial error adjustment: (1) creating a model to identify errors in the machine tool, (2) utilizing appropriate measurement devices to detect discrepancies, and (3) implementing the error model for corrective action. These stages are closely interconnected, as the data obtained from measurement devices play a crucial role in developing effective volumetric error models. However, measurement devices alone cannot fully capture all discrepancies, as they primarily focus on measuring tool center deviations relative to the workpiece. Consequently, robust error models are essential to accurately detect, correct, and compensate for these discrepancies. This section reviews the different spatial error modeling approaches and examines how discrepancies propagate throughout the machine workspace, providing insights into effective error management strategies.

5.1. Error Modeling Approaches

Spatial error modeling [142,160] seeks to connect the origin of spatial inconsistencies with the alignment inaccuracies within the machinery, providing the basis for error compensation. Accurate, reliable, and practical error models ensure precision, especially in CNC machining equipment. These models need to be developed from the perspective

of the entire machining system. Over the past decade, considerable research has focused on developing spatial error modeling methods, with approaches based on homogeneous transformation matrices (HTMs), screw theory, and differential motion matrices (DMMs) being prominent [161].

5.1.1. Homogeneous Transformation Matrix

The HTM, based on multi-body system (MBS) theory, represents the motion configuration of a machine with multiple axes as two connected segments: one linking the part to the main structure and the other connecting the structure to the tool [162,163]. For example, in a CAFXYZ-style five-axis system (illustrated in Figure 22), the HTM offers a standardized method for converting coordinates from the object's local reference to the overall coordinate system using the Denavit–Hartenberg transformation [164].

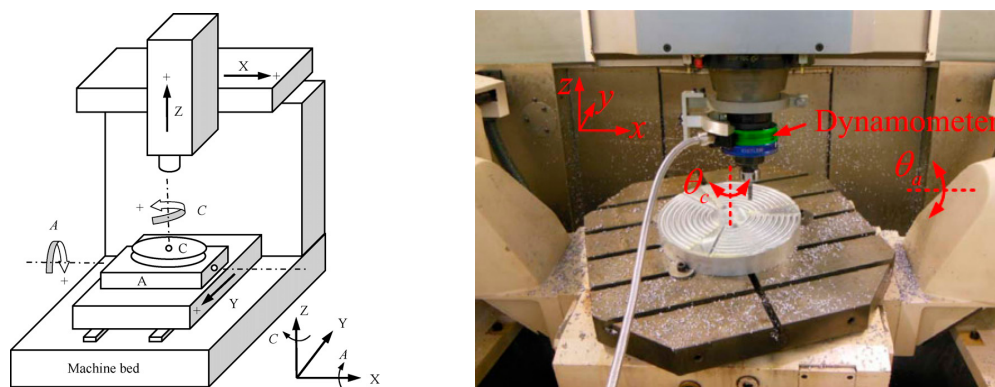


Figure 22. CAFXYZ-style five-axis system [165]. Reproduced with permission from Wu et al. [165].

In this procedure, transformation matrices that represent spatial inaccuracies for each axis are initially created in the coordinate system of the kinematic structure. A forward motion model is then established by successively multiplying all transformation matrices along the kinematic path from the part to the machine. The forward kinematics function is then solved to detect the Position-Independent Geometric Errors (PIGEs) by measuring discrepancies in the tool tip position with suitable instruments [166–168].

Many researchers have demonstrated the efficiency of HTM-based models. For instance, Cheng et al. [169] utilizing MBS theory, developed a three-dimensional error model for three-axis machines in a straightforward matrix representation. Zhong et al. [170] expanded this approach to five-axis systems by incorporating laser interferometers for straight-line movements and laser trackers for rotational motions. Lei et al. [171] proposed using a ball bar testing method to examine dynamic errors in rotary axes using circular positioning techniques.

While HTM-based models are effective, they are often specific to the machine tool's configuration, which can limit their broader applicability. To address this, Fan et al. introduced a generalized function for HTMs, enabling the development of a spatial error model applicable to various three-axis CNC machines. Xiang et al. [172] extended this framework to 12 different machining device types, allowing kinematic models to be generated based on inputting the machine's translational and rotational parameters.

5.1.2. Screw Theory

Screw theory, grounded in Chasles' theorem, provides a mathematical approach to describe rigid body motion [173,174]. Unlike the HTM, which creates local coordinate systems for each axis, screw theory offers a global description of motion and provides explicit solutions for inverse kinematics.

Screw theory [175] is widely used in robotics for tasks such as trajectory planning, motion control, calibration, and error modeling, and it has increasingly been applied to machining systems. Fu et al. [176] developed a POE-based model that systematically integrates spatial discrepancies across machine tool parts. Recent research has expanded screw theory to include both PIGEs and Position-Dependent Geometric Errors (PDGEs) in multi-axis systems. However, the method can be computationally intensive, requiring complex matrix operations. To address this challenge, Qiao et al. [177] introduced the Common Perpendicular Line (CPL) transformation, which aligns spatial error calculations with twist constraints, thereby reducing computational discrepancies.

5.1.3. Differential Motion Matrix

The DMM offers a simpler approach to modeling spatial errors by describing the differential changes in coordinate frames. This method effectively links the discrepancies of individual motion axes to the overall precision of the equipment. The research [178] utilized the DMM to study how discrepancies in individual axes affect tool tip accuracy. The authors demonstrated how errors propagate through the kinematic structure by combining forward kinematics with differential error vectors. Yang and Ding [179] applied the DMM to transform PIGEs between local and global coordinate systems, proposing a least-squares optimization method for error identification.

The clear physical interpretation provided by the DMM [179] makes it a preferred approach in applications where simplicity and clarity are essential. For instance, Tang et al. [180] employed the DMM in multi-axis systems, modeling each axis as a station where discrepancies sequentially propagate (see Figure 23). Liu et al. [162] presented the concept of an Equivalent Rotary Axis (ERA) to define the location and alignment of rotary axes in the global coordinate system (see Figure 24).

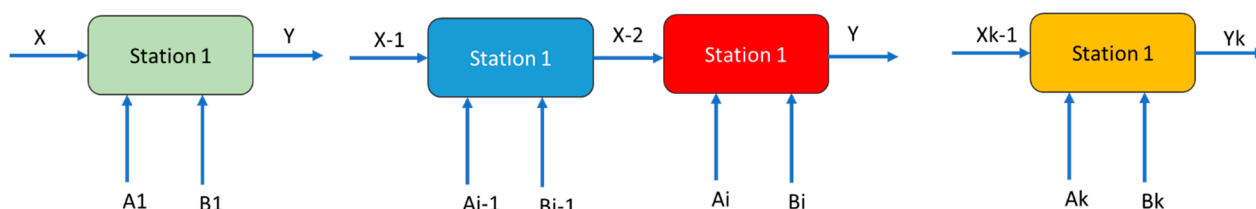


Figure 23. Multi-station system.

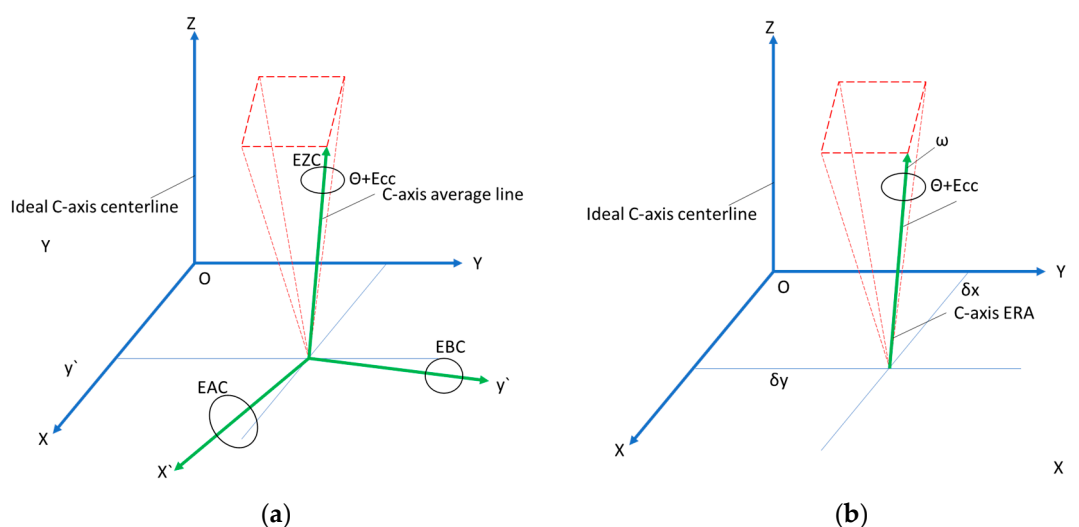


Figure 24. Geometrical errors for (a) PDGEs and PIGEs and (b) ERA.

5.2. Sensitivity Study

Accurate spatial error modeling is essential for error prediction and adjustment. Volumetric discrepancies, which aggregate spatial discrepancies across the machine workspace, are particularly critical. To address this, researchers have developed methods for tracing error propagation and conducting sensitivity analysis.

For instance, Duan et al. [181,182] measured guideway surfaces to analyze straightness discrepancies, while Majda [182] modeled interactions between rolling elements in linear guideways to evaluate their kinematic effects (see Figure 25). Sensitivity analysis, categorized into local (LSA) and global (GSA) methods, has been widely adopted to identify critical error sources. Cheng et al. [183] used LSA to analyze partial variance derivatives, identifying key discrepancies in three-axis machines. Meanwhile, GSA methods, such as Monte Carlo simulations, have been employed to evaluate the influence of spatial discrepancies across broader parameter ranges.

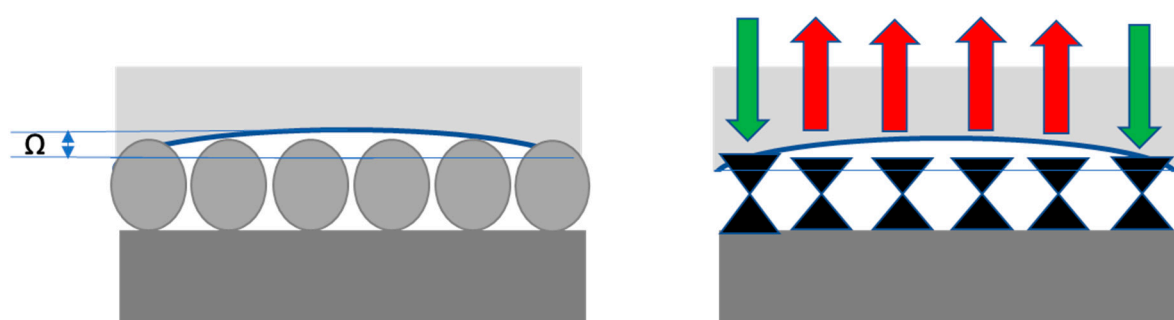


Figure 25. Example diagram of a straightness error model for the roller patch system using contact elements.

Advanced sensitivity analysis techniques have also been integrated with optimization frameworks. Cheng et al. [183] developed an error budget model that balances machining precision and manufacturing costs. Li et al. [184] linked volumetric error models to surface machining profiles, enabling cost-tolerance optimization for five-axis machines. Zhang et al. [150] introduced a multi-error budget framework incorporating spatial, thermal, and tool wear discrepancies, ensuring cost-effective precision machining.

5.3. Sources of Errors in CNC Machines

The errors in CNC machines can originate from several sources, each of which can be represented in the error models. Understanding these sources is crucial to developing effective models for compensation.

- **Geometric Errors**

Geometric errors in CNC machines occur due to imperfections in the machine structure, such as misalignments of the axes, inaccuracies in the machine bed, or wear and tear in the components [55]. These errors are often static and can be modeled as fixed offsets or angular deviations in the machine's kinematic chain [185]. For example, a misalignment in the spindle may cause an error that affects the accuracy of the entire machining process. As discussed earlier, geometric error models [186] typically represent these errors as either position-dependent or position-independent.

- **Thermal Errors**

Temperature changes in the machine components are the causes of thermal errors [34,35]. Such temperature changes can result in expansion or contraction of the materials, hence dimensional inaccuracies. Thermal errors can be complex to model since they vary with respect to the temperature gradients within the machine. Advanced models

make use of sensors and thermal maps to predict how changes in temperature will affect the geometry of the machine [7]. These models often include compensation algorithms that adjust the tool path based on real-time temperature data to mitigate the effects of thermal distortions.

- Mechanical Errors

Most of the mechanical errors are brought about by physical wear in bearings, ball screws, and linear guides [187,188]. With time, the friction in these components might make them less accurate; hence, motion errors arise. These errors can be modeled as gradual changes in the mechanical properties of the components. For example, a worn-out ball screw may introduce backlash that can be quantified and corrected by adjusting the motion commands during machining.

- Dynamic Errors

Dynamic errors are the deviations that occur during machining because of vibrations, cutting forces, or tool deflection. These errors are normally time-dependent and thus difficult to model accurately. However, knowledge of the dynamic behavior of the machine and the involved forces allows for modeling that can predict how the forces will affect the machine's accuracy. Most of these models are performed with advanced simulation tools, which simulate dynamic forces and their interaction with the movement of the machine [189–191].

In general, errors in CNC machining rarely exist in isolation. Instead, they often interact, creating compounded inaccuracies that are far more complex than the sum of individual errors. This phenomenon, known as error coupling, arises when geometric deviations, thermal distortions, and dynamic forces influence one another during machining operations [134,192].

For instance, minor geometric misalignments in a machine's structure can intensify thermal expansion effects. A slight angular error may lead to uneven heat generation, causing inconsistent thermal deformation across the machine. Similarly, vibrations from cutting forces can amplify positional deviations caused by geometric or thermal errors, particularly in high-speed machining or when working on lightweight components. These compounded inaccuracies can significantly affect the quality and precision of the finished workpiece.

The challenge of error coupling becomes even more pronounced in multi-axis systems. Due to mechanical and control system interdependencies, movements in one axis can inadvertently impact the alignment and accuracy of others. For example, position-dependent errors in one axis may induce distortions in the remaining axes, disrupting overall volumetric accuracy [192].

Advanced strategies that consider interdependencies are essential to managing these complexities. Coupled error models, which integrate geometric, thermal, and dynamic factors, provide a more comprehensive understanding of their interactions. Additionally, real-time monitoring systems and hybrid error measurement techniques can help isolate and mitigate coupled errors, ensuring greater machining precision in dynamic environments [193].

Geometric inaccuracies directly influence the quality of machined test workpieces by introducing dimensional deviations and surface irregularities. For example, angular misalignments can result in uneven cuts, while positional errors can cause features to fall out of tolerance [194]. These inaccuracies often interact with thermal and dynamic errors, creating compounded effects that further deteriorate machining precision. For instance, small geometric deviations in the machine's kinematic chain can amplify the effects of thermal expansion, leading to inconsistent tool paths and surface distortions [195,196].

Addressing these challenges requires a multi-faceted approach that includes advanced modeling and compensation techniques. Techniques such as error decoupling algorithms and hybrid measurement systems can effectively identify and isolate geometric errors from other sources [197]. Furthermore, integrating these models with real-time compensation strategies, like those provided by on-machine measurement systems, can significantly enhance the precision and repeatability of machining processes. By tackling geometric accuracy errors holistically, manufacturers can ensure higher-quality outputs and reduce costly rework.

5.4. Error Compensation and Calibration

With the precise development of an error model, compensation in machining is the next task. Error compensation is where a machine's motion is amended for the inaccuracies that were predicted by the error models. This can be achieved in two ways: either in real time or through post-process adjustment.

- **Real-Time Compensation:**

In real-time compensation, the control system of the machine continuously edits the tool path based on an error model during machining. This allows for the mistakes to be corrected as they appear, ensuring that the tool stays on the right track. For instance, if the model of error prediction estimates that the machine will have a deviation on one axis, its control system can make corrections in the motion of other axes to counteract such an error [34,114,198].

- **Offline Calibration:**

Calibration offline means measuring the error after the machine is set up and adjusting the control parameters accordingly. Such compensation is usually conducted once during installation or after major maintenance. In general, calibration involves measurement of the behavior of the machine with precision instruments and comparing it with what is expected. Once the errors are identified, the machine's control system is adjusted to account for these discrepancies, improving the machine's overall accuracy [5,100,144].

5.5. Validation and Verification of Error Models

Once the error model has been developed and compensation strategies are implemented, verification will be necessary to make sure that not only does the model work right, but the methods of compensation do, too. This normally takes the form of validation experiments, which test the machine's performance against known standards. The test results are compared to the values predicted by the error model to verify the effectiveness of the compensation strategies in making the necessary corrections [186]. Verification is a continuous process because errors change over time with wear, thermal effects, and so on. Hence, error models should always be refined and updated from real-world data to sustain high precision during the life cycle of the machine [102].

6. Compensation Techniques for Geometric Error

Geometric error tuning methods seek to enhance the machining accuracy of the CNC machining device by overcoming the inaccuracies in either of two major approaches: error avoidance or error compensation. Error avoidance methods are concerned with the elimination of the sources of spatial inaccuracies by designing and manufacturing machine parts with better accuracy. In general, such methods are followed by the manufacturers of CNC machine tools during the manufacturing stage itself. With higher precision requirements, however, the costs for achieving such a high level of precision go up astronomically. In addition, machines are subject to degradation due to wear, accidental crashes, or misuse during operation [4].

Error compensation has attracted increasing attention in manufacturing research during the last two decades because of its relatively low cost. In this approach, the machining tool paths are adapted to modify the discrepancies without necessarily removing error sources, thus achieving a high level of precision without redesigning and remanufacturing the expensive mechanical structures. This section presents some advances on spatial error adjustment methodologies: hardware and software approaches [90].

6.1. Hardware

Hardware-based adjustment, or so-called real-time adjustment, corrects the relative position of tool and workpiece based on predicted deviation; this approach needs a direct dialogue between the error adjustment system and the CNC controller. For example, Shen et al. presented asynchronous online adjustment methods for discrepancy caused by thermal deformation and spatial inaccuracies. In their approach, volumetric error values calculated by the CNC controller are considered at selected points. In return, it feeds back the adjustment values in every axis into the system [199,200].

Interaction between the adjustment and CNC systems is a crucial element of hardware calibration. Cui et al. [201] suggested an implementation approach that utilizes the handwheel offset feature, employing a pulse-generating device to send an adjustment signal. Similarly, real-time error adjustment has been achieved in Fanuc CNC systems using external mechanical origin offset functions. Fan et al. [202] designed a controller that connects with the CNC system via PMC to modify the external coordinate offset function, enabling precise adjustment.

Recently, Yuen and Altintas [203] introduced a six-degree-of-freedom rotary magnetic table to compensate for spatial discrepancies in gantry-type micro-milling machines. Instead of modifying servo motion commands, their system adjusts the table's position in real time using gradient descent algorithms, simplifying the adjustment process. Although hardware adjustment can achieve high precision even with lower-end machining devices, it requires significant hardware and software, making it a costly option. Moreover, compatibility issues with commercial CNC controllers may limit its accessibility.

6.2. Software

Software-based adjustment, or pre-adjustment, corrects discrepancies by modifying NC codes during post-processing. This approach avoids the need for additional hardware, making it more cost-effective and more accessible to implement. The effectiveness of software adjustment relies heavily on the precision of the modified NC codes, which account for identified spatial discrepancies [204–206].

In initial applications, software calibration was performed by modifying the endpoints of machining path lines in numerical control programs. Although this approach effectively corrected errors at control points, it faced challenges with round and curve-based tool trajectories. To address this limitation, Liu [207] proposed segmenting curved tool paths into linear segments and iteratively refining these segments based on error vectors. This method reduces residual discrepancies by recalculating corrections at each step.

For multi-axis machines, including five-axis CNC systems, spatial error adjustment becomes significantly more complex due to nonlinear kinematic relationships between axes [202]. Bi et al. [208] developed a calibration algorithm that separates the spatial discrepancies of rotary axes, avoiding their coupling effects. Their method corrects tool axis orientation and tool tip position within the Workpiece Coordinate System (WCS). Similarly, Osei et al. [209] proposed decoupling adjustment processes for linear and rotary axes to address their individual discrepancies based on the machine's kinematic chain.

6.3. Iterative and Inverse

Exact decoupling of error parts is often impractical, necessitating iterative adjustment approaches. Peng et al. [210] developed a comprehensive post-processing method for multi-axis machines that iteratively calculates corrected NC data using inverse and forward kinematics solutions. The iterative process continues until the error falls within an acceptable tolerance, as depicted in Figure 26.

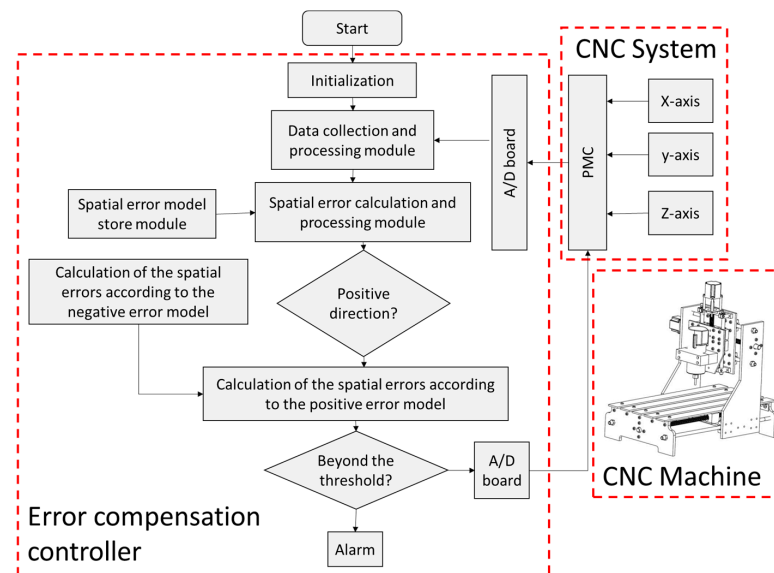


Figure 26. Compensation system flow chart.

While iterative methods are effective, their computational demands can be prohibitive. To improve efficiency, pseudo-inverse algorithms have been explored. Chen et al. [211] proposed using the Jacobian matrix to link tool position discrepancies with adjustment vectors. The pseudo-inverse Jacobian matrix calculates the required corrections, reducing computational complexity. Fu et al. [212] also applied pseudo-inverse calculations to compensate for integrated spatial discrepancies, leveraging differential motion matrices for precise error adjustments. Yang et al. [20] developed an adjustment approach that directly incorporates actual twist coordinates into a generalized inverse kinematics model, enabling accurate error correction in machines with PIGEs. Similarly, Ding et al. [213] analyzed NC code corrections using inverse kinematics algorithms, optimizing adjustment procedures based on HTM properties. Xia et al. [214] introduced a calibration method for the C-axis of gear profile grinding machines, assuming that PIGEs were fully corrected and focusing on the PDGEs of the worktable.

6.4. Challenges in Error Compensation

While error compensation techniques can significantly improve machining accuracy, they are not without challenges. Some of the main difficulties include the following:

- **Complexity of Error Modeling:** Accurate error modeling requires comprehensive data collection and analysis of machine behavior, which can be time-consuming and resource-intensive. Additionally, not all errors are easily quantifiable, especially dynamic errors that result from complex machine interactions.
- **Real-Time Data Processing:** Compensation methods rely on continuous monitoring and adjustment, which requires advanced sensors and high-speed data processing capabilities. The sheer volume of data generated in high-precision machining can be overwhelming and may require sophisticated algorithms and hardware to handle them effectively.

- **Calibration and Maintenance:** Both hardware and software compensation systems need regular calibration and maintenance to ensure long-term performance. Over time, wear and tear on machine components can lead to drift in the compensation system, reducing its effectiveness and requiring recalibration.

6.5. Benefits of Error Compensation

Despite these challenges, the benefits of error compensation are undeniable. By reducing the impact of geometric and thermal errors, manufacturers can achieve higher precision without requiring extensive machine reconfigurations or upgrades. The key benefits include the following:

- **Cost-Effectiveness:** By correcting errors in software rather than making expensive mechanical adjustments, manufacturers can maintain high precision while minimizing downtime and operational costs.
- **Increased Productivity:** Real-time compensation leads to faster machining processes, as adjustments are made automatically without interrupting the operation. This results in higher throughput and reduced cycle times.
- **Improved Product Quality:** Error compensation significantly improves the accuracy of finished products, ensuring that parts meet the required tolerances consistently. This is particularly important in the aerospace, automotive, and medical device manufacturing industries, where even small errors can have significant consequences.

6.6. Economic and Operational Impacts of Compensation Strategies

Error compensation strategies in CNC machining significantly influence the economic and operational aspects of manufacturing. These strategies, whether hardware- or software-based, aim to correct geometric, thermal, and dynamic inaccuracies in machining systems, enhancing precision while maintaining efficiency. The economic and operational implications of implementing compensation strategies extend across multiple domains, including cost, productivity, quality, and sustainability.

6.6.1. Economic Impacts of Compensation Strategies

- Compensation strategies, particularly software-based ones, provide a cost-effective alternative to redesigning or replacing machine components. For instance, Habibi et al. [215] demonstrated that incorporating error compensation algorithms into the CNC controller reduced overall manufacturing costs by 25% compared to hardware-based upgrades. This cost saving arises from the avoidance of expensive equipment overhauls and minimized downtime.
- Manufacturers achieve higher ROI by extending the operational lifespan of existing equipment through compensation techniques. For example, iterative calibration methods and real-time error correction systems can rejuvenate older machines, enabling them to meet contemporary precision requirements without significant capital expenditure [192].
- Accurate compensation minimizes defects, reducing material waste and rework. In industries like aerospace and medical devices, where raw materials are expensive, error compensation systems have demonstrated savings of up to 30% in waste management costs [216].
- Integrating compensation systems with monitoring tools enables predictive maintenance, reducing unexpected machine breakdowns. Real-time detection and correction of errors lower maintenance costs by identifying wear and tear before it escalates into costly repairs [217,218].

6.6.2. Operational Impacts of Compensation Strategies

- Compensation strategies that adjust errors in real time, such as those leveraging digital twins or AI algorithms, reduce machine downtime. For instance, Sa et al. [190] reported a 15% increase in productivity by implementing real-time compensation in multi-axis machining systems.
- Compensation improves the geometric and dimensional accuracy of machined parts, ensuring they meet stringent tolerances. This is critical in sectors like automotive and aerospace, where part reliability directly affects safety and performance. Consistently high-quality output reduces the need for quality control inspections and post-machining adjustments [218].
- Dynamic compensation systems that account for variations in temperature, cutting forces, and tool wear ensure consistent performance across different operating environments. This adaptability allows manufacturers to take on complex machining projects without compromising accuracy or speed [219].
- Modern compensation techniques are designed to integrate seamlessly with existing CNC systems, minimizing disruptions during implementation. For example, open-loop compensation systems can be retrofitted to legacy machines without requiring substantial modifications to their control architecture [220].

6.6.3. Broader Implications for Sustainable Manufacturing

1. By optimizing tool paths and reducing rework, compensation strategies lower energy consumption in machining processes. Studies by Naumann et al. [221] revealed that error-compensated machines reduced energy use by 12%, contributing to greener manufacturing operations.
2. Enhanced machining precision reduces scrap rates, conserving raw materials. In additive manufacturing, for example, compensation strategies mitigate geometric errors in layer deposition, minimizing the need for secondary processes such as surface finishing or structural reinforcement [222].
3. The integration of compensation strategies with smart manufacturing systems supports Industry 4.0 principles by promoting data-driven decision-making and real-time adaptability. This ensures a competitive advantage in a rapidly evolving industrial landscape [223].

7. Emerging Tools and Advanced Instruments

As industries push for higher precision in manufacturing, the role of advanced instruments and emerging tools in ultra-precision machining becomes increasingly important. These tools go beyond traditional measurement and error compensation methods, enabling real-time adjustments and providing deeper insights into machine performance. In ultra-precision machine tools (UPMTs), where even the slightest deviation can result in a significant quality loss, these advanced tools are pivotal for maintaining accuracy, improving productivity, and reducing operational costs.

7.1. Laser Interferometry: A Standard for High-Precision Measurement

Laser interferometry [77,78] has long been a go-to method for measuring linear and angular displacements with extraordinary precision. By utilizing the interference of light waves, this technique measures minute changes in position, achieving resolutions down to nanometers. As shown in Figure 27, its non-contact nature allows it to measure critical components of machine tools without introducing external forces that could alter the system's behavior.

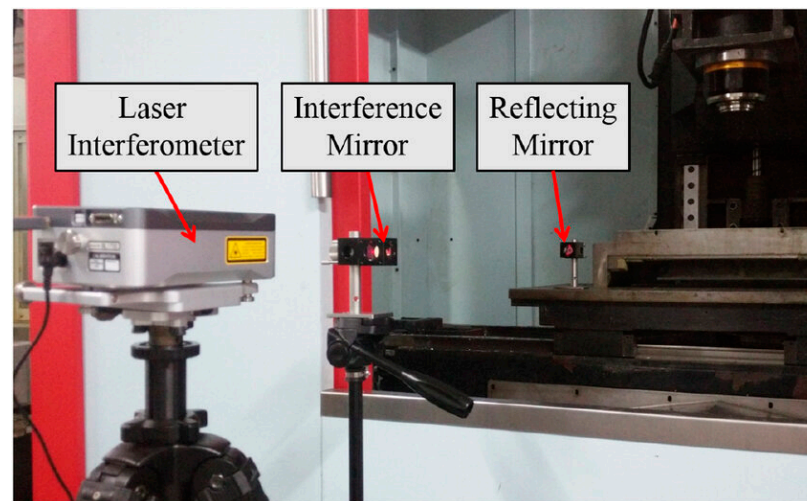


Figure 27. Laser interferometer used for position error [132]. Reproduced with permission from Wu et al. [165].

- Applications in Error Compensation:

Laser interferometry is crucial for detecting and compensating errors in both linear and rotary axes. Continuously measuring deviations in tool position or alignment enables real-time adjustments that help maintain machining accuracy, even in environmental factors such as thermal drift or mechanical wear.

- Advantages:

- Extremely high accuracy (sub-nanometer resolution).
- Non-contact measurement, preserving the integrity of the machining environment.
- Capability to monitor both linear and angular movements with high precision.

- Challenges:

- Susceptible to environmental factors like air currents, temperature changes, and vibrations, which can affect measurement accuracy.
- Setup and alignment require precision, which can be time-consuming.

7.2. Capacitive and Inductive Sensors: Real-Time Feedback for Precision

Capacitive and inductive sensors are increasingly significant in ultra-precision machining, offering real-time feedback on tiny movements and deviations. These sensors are beneficial for measuring small displacements with great sensitivity and can be integrated into linear and rotary axes for continuous error monitoring [224,225].

- Capacitive Sensors:

Capacitive sensors detect small changes in the electric field between the sensor and the machine's surface. These sensors are highly sensitive and capable of measuring sub-nanometer-level deviations. Their high resolution makes them ideal for detecting alignment or positioning errors in ultra-precision machining.

- Inductive Sensors:

Inductive sensors, which use electromagnetic fields to measure the position of a conductive object, offer a robust solution for environments where electromagnetic interference is a concern. These sensors are also useful for monitoring changes in machine components in real time and correcting errors during machining.

- Applications in Error Compensation:

Both capacitive and inductive sensors provide critical data that can be fed into the machine's control system. This allows for automatic adjustments to the tool path or spindle position, correcting errors caused by thermal expansion, tool wear, or mechanical drift.

- Advantages:
 - High resolution and sensitivity for detecting even the smallest errors.
 - Robustness in harsh environments, particularly with inductive sensors.
 - Immediate feedback for real-time adjustments.
- Challenges:
 - Capacitive sensors may be affected by electromagnetic interference or surface conditions.
 - Inductive sensors, while robust, generally provide lower resolution compared to capacitive systems.

7.3. Machine Learning and Artificial Intelligence: Smarter Error Compensation

Geometric errors in CNC machine tools play a significant role in determining manufacturing precision, making their measurement, modeling, and compensation a key focus of research. Advanced techniques, such as AI-driven prediction models and mechanistic approaches, are helping to achieve higher accuracy and reliability, particularly in complex five-axis milling operations. For linear axes, improved measurement models now address error crosstalk and system-level inaccuracies. In five-axis machines, methods like the double ball bar test have proven highly effective in identifying and correcting errors, especially in the rotary axes. Recent developments in volumetric error modeling and compensation techniques are further enhancing the performance of these machines [64,226].

The integration of ML and AI is revolutionizing error measurement and compensation. These systems analyze large volumes of data from sensors and measurement tools, identifying patterns to predict and correct errors in real time, as shown in Figure 28. AI techniques, such as Support Vector Machines, have shown great potential in estimating geometric errors based on machining forces. Similarly, machine learning models enable proactive adjustments, ensuring higher accuracy and efficiency in machining processes. These advancements make machines smarter and more adaptive, driving significant improvements in manufacturing precision and productivity [227,228].

- Machine Learning for Predictive Compensation:

Machine learning algorithms can predict potential errors by analyzing historical data and real-time inputs from sensors. These predictions allow the system to make proactive adjustments to machine parameters, such as tool speed or feed rate, before errors significantly impact the machining process.
- Artificial Intelligence for Adaptive Control:

AI systems enable the CNC machine to continuously adapt to changing conditions. For example, during high-speed machining, AI can adjust parameters such as spindle speed and feed rate in real time, compensating for variations in material properties, tool wear, or environmental conditions.
- Applications in Error Compensation:

By continuously monitoring and adjusting the machining process, ML and AI technologies optimize performance, reduce scrap rates, and improve overall precision. The more data the system collects, the smarter it becomes, leading to increasingly efficient and accurate error compensation.
- Advantages:
 - Continuous learning and adaptation to new machining conditions.

- Ability to predict and prevent errors, reducing the need for reactive adjustments.
- More efficient use of machine resources and reduced human intervention.
- Challenges:
 - Requires large amounts of data for training algorithms, which may not always be available.
 - Complex implementation and the need for specialized expertise in AI and machine learning.

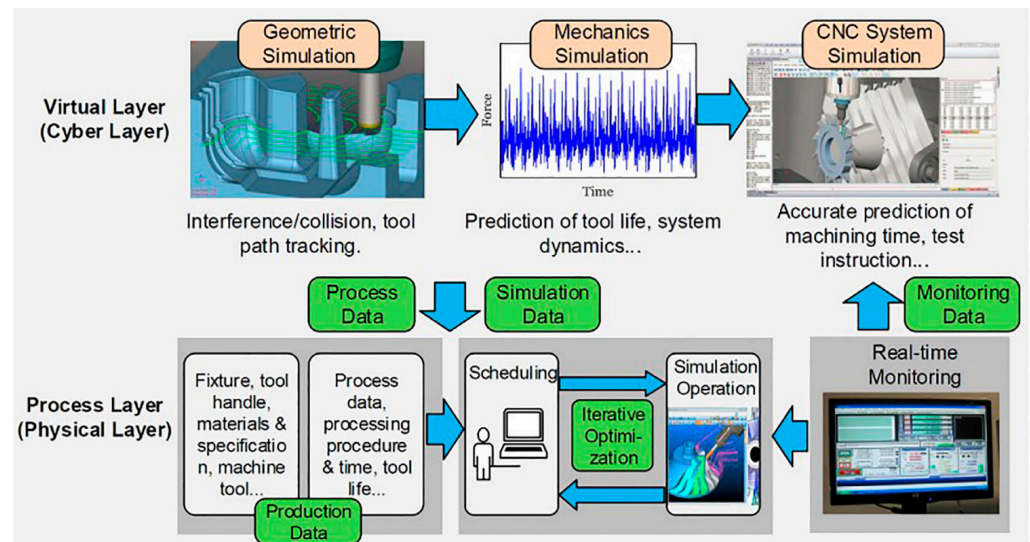


Figure 28. ML model for machine error prediction [229]. Reproduced with permission from Soori et al. [229].

7.4. Digital Twin Technology: Virtual Replicas for Real-Time Monitoring

Digital twin technology involves creating a virtual model of a physical machine that can simulate its behavior in real time [217,218]. By integrating sensor data from the machine, the digital twin provides insights into the machine's performance and allows for real-time predictions of errors, as shown in Figure 29.

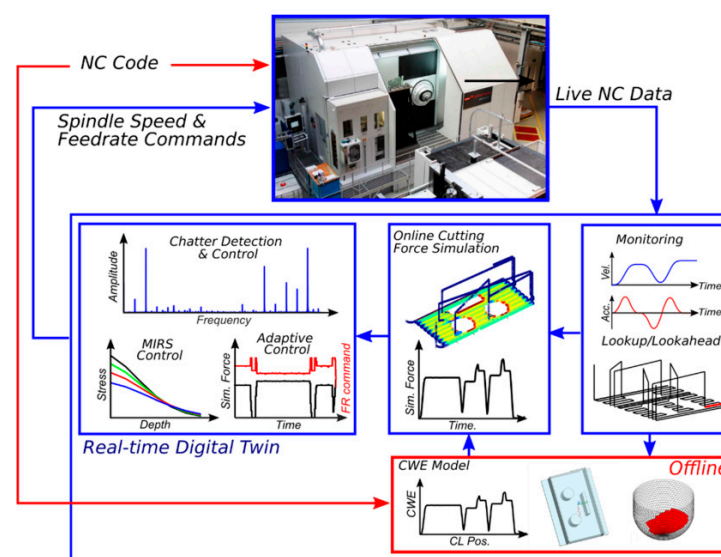


Figure 29. Online DT for machine error prediction [230]. Reproduced with permission from Ward et al. [230].

- Applications in Error Compensation:

The digital twin simulates machine behavior and predicts potential errors before they occur, enabling proactive adjustments to be made. It can also test different machining conditions, identifying optimal parameters without interrupting production. Recent studies have explored the use of digital twins for real-time error prediction and compensation in various industrial applications. Digital twins have been successfully implemented in systems like wound rotor induction machines, providing precise real-time monitoring of electromagnetic parameters [231]. In robotics, combining digital twins with virtual reality has enabled efficient remote programming and trajectory replication with minimal latency [232]. In the machining of thin-wall structures, a digital-twin-in-the-loop method utilizing mass-spring lattice models has significantly enhanced geometric accuracy, reducing errors by 78.96% [233]. Additionally, in CNC machine tools, digital twins have been applied to predict and compensate for thermal-induced time-varying errors, resulting in a 69.19% decrease in hole-pitch error fluctuations [234]. These advancements highlight the transformative potential of digital twins in improving machine accuracy, performance, and real-time error correction across diverse industrial sectors.

- **Advantages:**
 - Provides a real-time, virtual representation of the physical machine, enabling continuous monitoring.
 - Allows for predictive maintenance and preemptive error correction, reducing downtime and improving operational efficiency.
- **Challenges:**
 - Developing a digital twin requires comprehensive data collection and accurate modeling, which can be time-consuming and expensive.
 - Requires integration with existing machine control systems, which may be complex.

7.5. The Future of Advanced Instruments and Tools

The future of ultra-precision machining will likely see further advancements in integrating new technologies. The continuous miniaturization of sensors, improved data analytics, and the rise of AI-driven compensation systems will make machines smarter and more adaptive. Innovations such as integrated, real-time measurement systems and autonomous error correction are set to become more prevalent, further enhancing the capabilities of UPMTs [76,235–238].

- **Smarter Sensors:** the next generation of sensors will be more compact, efficient, and capable of providing continuous, high-precision measurements with minimal impact on the machine's operation.
- **AI-Driven Systems:** AI systems will become more advanced, offering real-time, autonomous error detection and correction, reducing the need for human intervention, and continuously improving machine performance.

8. Conclusions and Future Work

Geometric discrepancies are a primary source of imprecision in machining devices, representing the deviation between the nominal and actual positions and orientations of the tool relative to the workpiece. These discrepancies can be significantly reduced or suppressed using error adjustment strategies. This paper comprehensively reviews current research on measuring, modeling, and adjusting spatial discrepancies in CNC machining devices. With the rapid advancement of measurement technologies, numerous studies have made significant progress in enhancing the spatial precision of CNC tools through

error analysis and mathematical modeling. The key findings and suggested directions for future research are outlined below:

1. Advancements in Geometric Error Identification

Modern measurement technologies allow for the direct and individual identification of spatial discrepancies in linear and rotary axes. Instruments such as laser interferometers have been successfully commercialized for industrial applications. However, these measurement processes are often time-intensive. To address this, multi-degree-of-freedom (MDOF) measurement systems based on optical detection principles have been developed, enabling the simultaneous identification of linear axis discrepancies. Advanced sensor technologies such as capacitive and inductive sensors offer real-time, high-resolution feedback, which can further streamline error detection. Combining these sensors with AI-driven predictive models could enable proactive identification of emerging error trends, reducing the dependency on time-intensive setups.

Future efforts should prioritize the integration of laser interferometry and laser collimation techniques, which offer high precision, extensive measurement ranges, and excellent integration capabilities. Additionally, advanced measurement instruments are needed to identify all 21 spatial discrepancies in three-axis CNC tools through a single installation process. Measurement remains a more complex challenge for rotary axes, as capturing all error parts in a single setup is complicated. Overcoming this limitation will require innovative design of measurement systems and methodologies.

The advancements discussed in this paper are particularly relevant for high-precision industries such as aerospace, medical devices, and automotive manufacturing, where sub-micron accuracy is essential. By integrating these measurement and compensation techniques, manufacturers can achieve greater consistency, reduce rework, and optimize production efficiency.

2. Challenges in Indirect Measurement Techniques

Indirect measurement methods have seen significant industrial adoption, including laser interferometers, laser trackers, DBBs, R-test devices, and touch-trigger probes. Laser interferometers are recognized for their precision but are restricted to straight-line measurements and are less efficient for certain applications. Laser trackers, in contrast, offer flexibility and large measurement ranges but suffer from issues with measurement uncertainty. DBBs are frequently employed for rotary axis discrepancies but are unsuitable for large-radius circular trajectories, while R-tests face similar limitations.

Touch-trigger probes have emerged as a promising alternative, offering compatibility with CNC systems for automated error measurement. However, these measurement techniques share common challenges: they require skilled operators, are expensive, and face limitations in measurement efficiency and precision. Recent developments in lightweight and portable calibration devices, coupled with improved path optimization algorithms for laser-based systems, aim to enhance efficiency while reducing operational costs. Additionally, multi-sensor networks integrated into CNC systems could synchronize data collection across various axes, further improving the accuracy of error compensation.

Future research should improve measurement efficiency, reduce costs, and develop reliable measurement patterns for rotary axis error calibration. Addressing the nonlinear and coupled kinematic relationships of rotary axis discrepancies remains a pressing challenge.

3. Progress and Limitations in Geometric Error Modeling

Significant strides have been made in modeling spatial discrepancies for multi-axis machining devices. These models help link individual component discrepancies to tool-workpiece deviations. However, most models are tailored to specific machine configura-

tions, limiting their applicability to other machine topologies. There is an urgent need for universal error models to accommodate various CNC tool configurations.

Moreover, a holistic modeling approach that integrates spatial, thermal, and servo discrepancies into a unified volumetric error model would be highly valuable. Such models would advance error adjustment technology and enhance the overall precision of multi-axis machining devices. Machine learning and digital twin frameworks could support the development of these comprehensive models by enabling continuous learning and simulation of machine behavior under various operational conditions. In developing these comprehensive models, collaboration between academia and industry would greatly benefit both sectors.

Collaboration between machine tool manufacturers, AI researchers, and sensor technology developers will be crucial in overcoming current limitations. Future studies should also explore the role of quantum sensors and nanotechnology in ultra-precision measurement for next-generation CNC machining systems.

4. The Role of Software Compensation

Software-based adjustment methods, which modify NC codes to correct discrepancies, are widely adopted due to their cost-effectiveness and ease of implementation. Pre-measured error maps are commonly used in commercial CNC systems for linear axes. However, due to their complexity, spatial error adjustment for rotary axes remains a significant challenge. AI-driven algorithms, capable of dynamically adjusting NC code based on real-time sensor data, could revolutionize software compensation by reducing residual errors in linear and rotary axes. These algorithms could also optimize tool paths to minimize tool wear and thermal deformation effects during machining.

Future research should explore adjustment methods that leverage feedback from machined workpieces. This approach could enable targeted adjustment for specific features of the machined part. Additionally, spatial error adjustment can influence the machined surface's quality due to cutter position and orientation adjustments. A valuable research area is developing strategies to smooth tool paths across multi-axis systems after error adjustment.

5. Periodic Detection and Online Calibration for Rotary Axes

Rotary axis discrepancies are a fundamental source of imprecision in multi-axis CNC machines. To maintain positional precision, they require regular monitoring and calibration. However, most existing error measurement and identification methods are offline, necessitating production interruptions. To address this, online error calibration systems integrated with embedded sensors are a promising direction for future development. These systems could be further enhanced with health monitoring capabilities to track machine tool performance over time. The incorporation of IoT-enabled devices for continuous monitoring could further enhance the scope of online calibration. When paired with big data analytics, these systems could offer predictive maintenance solutions, minimizing downtime and extending machine longevity.

In addition, leveraging big data analytics to analyze and trace error behaviors could improve precision and stability. By analyzing trends and patterns in error data, manufacturers could predict and mitigate performance degradation, ensuring longer machine lifespans and more consistent output quality.

Ultimately, the integration of real-time measurement, AI-driven compensation, and predictive analytics is poised to redefine precision machining. As these technologies evolve, CNC machining will transition towards fully autonomous, self-correcting systems, ensuring unparalleled accuracy and efficiency in advanced manufacturing.

6. Future Directions in Geometric Error Detection and Compensation

Despite the significant advancements in geometric error detection and compensation, several challenges require further research and innovation. Emerging technologies such as artificial intelligence, machine learning, and digital twin systems have enhanced error prediction and correction accuracy and efficiency. Additionally, advancements in measurement techniques, including laser interferometry, optical sensors, and capacitive sensing, have improved the precision of geometric error detection. The growing use of on-machine measurement systems has also reduced downtime and streamlined the compensation process, making real-time corrections more practical.

However, many of the existing compensation models are designed for specific machine configurations, making applying them universally across different machining systems challenging. Furthermore, real-time compensation remains challenging due to unpredictable factors such as tool wear, thermal expansion, and dynamic machining conditions. The cost and complexity of implementing advanced measurement and compensation systems also limit their widespread adoption, particularly in small and medium-sized manufacturing enterprises.

To address these challenges, future research should focus on developing more adaptable and generalized compensation strategies that can adjust dynamically to changing machining conditions. AI-driven predictive maintenance, combined with real-time error detection, could be crucial in enhancing long-term reliability and precision. Additionally, exploring cost-effective alternatives to high-end measurement systems could make geometric error compensation more accessible to various industries.

Looking ahead, as CNC machining continues to evolve, integrating intelligent measurement and compensation technologies will be essential in achieving higher precision and efficiency. With AI, automation, and real-time feedback system advancements, the industry is moving towards self-correcting, autonomous machining processes. These innovations will improve accuracy and enhance productivity, reduce waste, and lower operational costs, paving the way for the next generation of ultra-precision manufacturing.

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Nomenclature

DBB	Double Ball Bar
PIGEs	Position-Independent Geometric Errors
PDGEs	Position-Dependent Geometric Errors
UPMT	Ultra-precision Machine Tool
CNC	Computer Numerical Control
HTM	Homogeneous Transformation Matrix
OMM	Optical Measurement Methods
CMM	Coordinate Measuring Machine
AI	Artificial Intelligence
ML	Machine Learning
DT	Digital Twin

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