

Article

A Fitness Landscape-Based Method for Extreme Point Analysis of Part Surface Morphology

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Abstract: Advancements in Industry 4.0 and smart manufacturing have increased the demand for precise and intricate part surface geometries, making the analysis of surface morphology essential for ensuring assembly precision and product quality. This study presents an innovative fitness landscape-based methodology for extreme point analysis of part surface morphology, effectively addressing the limitations of existing techniques in accurately identifying and analyzing extremum points. The proposed approach integrates adaptive Fitness-Distance Correlation (FDC) with a roughness index to dynamically determine the number and spatial distribution of initial points within the pattern search algorithm, based on variations in surface roughness. By partitioning the feasible domain into subregions according to FDC values, the algorithm significantly reduces optimization complexity. Regions with high ruggedness are further subdivided, facilitating the parallel implementation of the pattern search algorithm within each subregion. This adaptive strategy ensures that areas with intricate surface features are allocated a greater number of initial points, thereby enhancing the probability of locating both regional and global extremum points. To validate the effectiveness and robustness of the proposed method, extensive testing was conducted using five diverse test functions treated as black-box functions. The results demonstrate the method's capability to accurately locate extremum points across varying surface profiles. Additionally, the proposed method was applied to flatness error evaluation. The results indicate that, compared to using only the raw measurement data, the flatness error increases by approximately 3% when extremum points are taken into account.

Keywords: surface morphology; fitness landscape; extreme point analysis; pattern search algorithm



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1. Introduction

In contemporary manufacturing paradigms, the geometric characteristics of a part's surface are key to assembly precision and quality. With Industry 4.0 and smart manufacturing, surface quality standards have risen, especially for high-precision parts. Analyzing surface geometry, including extremal points such as peaks, valleys, and edges, is crucial for assessing machining inaccuracies, predicting assembly precision, and improving product performance and reliability.

The analysis of extreme points in surface morphology relies heavily on surface modeling, which has gained significant attention as products advance in precision, miniaturization, and integration [1]. Recent research in 3D tolerance analysis has introduced several methodologies, including the degree-of-freedom method, skin model shapes, fractal geometry, and data-based reverse modeling. The degree-of-freedom method treats the surface as

an ideal feature within a tolerance zone, useful for representing macroscopic errors, but it cannot capture surface morphology's intricacies [2–4]. The skin model shapes discretize geometric features, generating errors through techniques like modal analysis and mesh deformation; however, challenges remain in preserving error correlations across points and keeping errors within tolerance [5–8]. The fractal method, while effective for characterizing surface roughness, provides excessive geometric detail, limiting its application to assembly accuracy studies [9–12]. Data-based reverse modeling, leveraging high-precision measurement data, enhances accuracy by creating digital representations and integrating real surface errors into propagation analysis [13–19]. These studies have evolved from “ideal geometric variations” to “microscopic geometric expressions”, reflecting a transition from macroscopic to microscopic scales and providing a more accurate depiction of part surface morphology. However, extreme points, as crucial feature information of the part's surface morphology, are not explicitly represented in these models. Taking the evaluation of surface machining errors as an example, the part's surface exhibits complex morphology with alternating valleys and peaks, resulting in multiple extreme points. Current methods for machining error evaluation typically rely on limited measurement data and the least squares method, as demonstrated in references [20–23]. Due to the restricted number of measurement points, these methods may overlook critical extreme points, leading to inaccurate error estimations.

To identify extreme points on part surface morphologies, a common strategy is the Multi-start algorithm, which involves random generation of initial points followed by local searches to locate respective extrema [24]. Wang et al. [25] introduced the Memory Particle Swarm Optimization (MPSO) algorithm, combining Particle Swarm Optimization (PSO) with local search techniques to identify multiple global and local optima in Multi-Objective Optimization Problems (MMOPs). Gnanndt et al. [26] developed a hybrid method for global optimization by combining random search with descent methods, using region control to refine the search process with local minima, similar to step-size control in deterministic optimization. Minaee et al. [27] improved extreme point search efficiency on digital image surfaces by integrating image segmentation and edge detection, extending its applicability to other surface types. Koessler et al. [28] advanced the global extreme point search algorithm for faster identification and established a theoretical foundation for extreme point searches on part surfaces. A review of the existing literature reveals that most studies concentrate on optimizing algorithms to locate all local extrema across the entire surface. However, few investigations address surface ruggedness and attempt to mitigate optimization complexity by partitioning the surface into regions based on ruggedness characteristics.

This paper addresses the challenge of analyzing extreme points on the surface morphology of a part by introducing the concept of the fitness surface. The proposed method decomposes the surface by calculating its roughness and employs a mode-search algorithm to identify all local extreme points. This approach accounts for the geometric characteristics of the part's surface and leverages the advantages of the Multi-start algorithm. The structure of the paper is organized as follows. Section 2 introduces the concept of the fitness surface; Section 3 details the working principles and algorithmic flow for analyzing extreme points on the part's surface morphology; Section 4 validates the proposed method through both theoretical test functions and engineering examples.

2. The Concept of Fitness Landscapes

2.1. Method Overview and Basic Concepts

The concept of a fitness landscape is extensively utilized in fields such as biology, genetics, and optimization theory. It primarily describes the relationship between the solu-

tion space of an optimization problem and its corresponding fitness (or objective function) values. This concept draws inspiration from the undulating terrains found in nature to visually represent the quality of solutions, where each point signifies a potential solution, and the elevation of the point indicates the fitness value of that solution. Fitness Landscapes are particularly significant in genetic algorithms, evolutionary computation, and other heuristic search methods, as they offer an intuitive understanding of the characteristics inherent in a problem's search space. By analyzing the Fitness Landscape, researchers and engineers can more effectively design and refine their optimization strategies to address complex optimization challenges.

The concept of the fitness landscape is also applicable to the analysis of part surface morphology. Fitness landscape analysis has become an effective method for exploring the parameter information of part surface morphology. In the context of part surface morphology, each point on the local surface is treated as a potential solution. Consequently, the part surface morphology can be viewed as a surface with complex fitness values. By systematically searching this fitness landscape, the main features of the part surface morphology can be extracted. The main features of a fitness landscape include:

Height/Depth: In a fitness landscape, the height or depth of a point represents the fitness value of the corresponding solution. In minimization problems, valleys indicate optimal solutions, whereas in maximization problems, peaks signify optimal solutions.

Local Extremes: Local peaks or valleys on the fitness landscape correspond to local optimal solutions. These points can cause the search process to become easily trapped, hindering the discovery of the global optimum.

Global Extremes: The highest or lowest points on the fitness landscape represent the global optimal solutions.

Ruggedness: The ruggedness or irregularity of the fitness landscape reflects the complexity of the optimization problem. A highly rugged fitness landscape typically contains numerous local extremes, increasing the difficulty of locating the global optimal solution.

The analysis of extreme points in part surface morphology can be formulated as an optimization problem, with the solution process resembling strategic exploration on a complex terrain. In this framework, part surface morphology is abstracted as a terrain composed of numerous peaks and valleys, where the elevation of each point represents the fitness of the corresponding solution, i.e., its degree of alignment with the optimization objective. The role of the search algorithm in this terrain is akin to that of an explorer seeking the deepest valleys, aiming to find the optimal solution for a minimization problem. In this process, the optimization algorithm functions as a strategic explorer, continuously navigating the terrain to identify the most efficient path toward its optimization goal—extreme points with the lowest fitness values. The search process is illustrated in Figure 1 below.

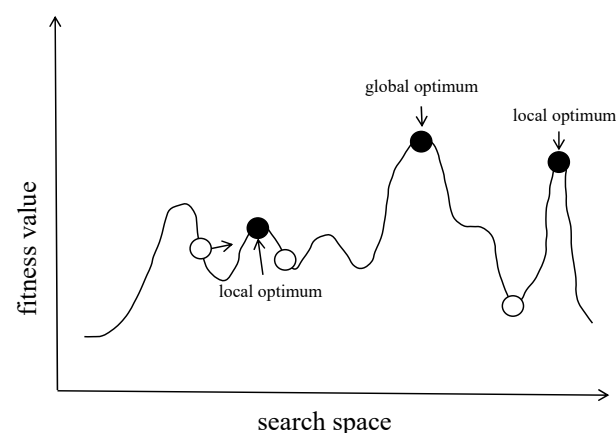


Figure 1. Schematic of extreme point search on the cross-sectional curve of part surface morphology.

2.2. Mathematical Modeling of the Fitness Landscapes

In the context of optimization problems, the fitness landscape offers a visual framework for understanding the complexity of the search space, while heuristic algorithms serve as methods for identifying the optimal solution within this landscape. The definition of the fitness landscape is as follows.

A fitness landscape can be formally defined as a triplet (S, f, d) , where S denotes the solution space of the problem, encompassing all possible solutions. Depending on the specific problem, this space may be either continuous or discrete. The objective function $f(x)$ is a mapping defined on the search space $x \in S$, assigning a metric value to each solution to evaluate its quality. The distance metric $d(x_1, x_2)$ measures the distance between any two solutions x_1 and x_2 within the search space, serving as a function to quantify the distance or similarity between two points. This formalization provides a comprehensive framework for analyzing and navigating the complex morphology of the solution space, facilitating the design and application of effective heuristic algorithms for optimization.

Fitness landscape analysis offers an intuitive approach for optimizing low-dimensional functions. However, for optimization problems involving multi-dimensional variables, the complexity of the fitness landscape becomes challenging to represent through straightforward visualization. In such cases, Fitness-Distance Correlation (FDC) emerges as a crucial tool for quantifying and comprehending the ruggedness of the fitness landscape. FDC provides a systematic method for evaluating the relationship between the fitness of a solution and its distance to other solutions, thereby enhancing our understanding of the landscape's structure and aiding in the development of more effective optimization strategies.

Fitness-Distance Correlation (FDC) is the correlation coefficient between the fitness of a solution and its distance within the solution space. It quantifies the ruggedness of the fitness landscape by calculating the statistical correlation between fitness and distance. In relatively smooth and continuous fitness landscapes, the fitness of a solution and its distance to the optimal point typically exhibit a strong correlation. In such cases, the FDC value tends to be large, usually close to 1 or -1 . This indicates that points near the optimal solution generally possess higher fitness, while those farther from the optimal solution tend to have lower fitness. Conversely, in rugged and highly fluctuating fitness landscapes, the correlation between a solution's fitness and its distance to the optimal point is weaker. In these scenarios, the FDC value is smaller, often approaching 0. This suggests that even solutions in close proximity to the optimal point may not exhibit ideal fitness levels, reflecting the presence of numerous local optima.

In practical scenarios involving complex part surface morphology models, determining the global optimum is often challenging and sometimes even infeasible. This difficulty arises because the exact location of the global optimum may be unknown or difficult to ascertain accurately due to the model's complexity and computational limitations. In such cases, the traditional Fitness-Distance Correlation (FDC) metric, which relies on knowledge of the global optimum, may no longer be applicable. To address this limitation, an improved definition of FDC has been proposed, which utilizes optimal solutions sampled from a dataset constructed from a precise part surface morphology model, thereby replacing the reliance on the traditional global optimum.

In this study, we used an improved fitness distance correlation coefficient (FDC) to quantify the characteristics of the part surface. This improved FDC measures the correlation between the fitness of sample points and their distance to the optimal solution, by considering the optimal points within the sample set, rather than the global optimum. We calculated the distance from each sample point to the optimal points within the sample set and analyzed the correlation between these distances and their corresponding fitness values, in order to assess the complexity and optimization difficulty of the part surface

geometry. It is defined as follows. There is a set of selected sample points on the fitness surface $P = \{x_i | i = 1, 2, \dots, n\}$, where each point x_i represents a possible solution in the solution space. Within this set of samples, we identify the point with the optimal fitness value $S^* = x_{\max}$ (assuming the maximum value $x_{\max}$ as the optimal point), while the point $x_{\max}$ with the minimum fitness value on the set P represents the optimal solution found within the current sample set. The FDC on the sample set P is given by:

$$fdc(s) = \frac{\sum_{i=1}^n (y_i - \bar{y})(d_i - \bar{d})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 \sum_{i=1}^n (d_i - \bar{d})^2}} \quad (1)$$

where y_i represents the corresponding value of point x_i ; the average value of all sample points y_i is represented as $\bar{y}$, that is, $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$; the Euclidean distance from x_i to the optimal point S^* is represented as d_i , that is, $d_i = \|x_i - x_{\max}\|$; and $\bar{d}$ represents the average value of d_i , that is, $\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i$.

FDC is a metric used to describe the relationship between the fitness value y_i of a solution and the distance d_i to the global optimum within the search space S of an optimization problem. The range of values for this metric is $FDC \in [-1, 1]$, where different values reflect different relationships between fitness and distance; when $|FDC| = 1$, it indicates a linear relationship between fitness and distance. For optimization problems, the closer the solution is to the optimal point (the smaller the d_i value), the better the fitness (the smaller the y_i value). This suggests that there is a clear path within the search space leading to the optimal solution. When the absolute value of FDC is small, it signifies the absence of a significant linear relationship between fitness and distance. In such scenarios, fitness does not consistently increase as the distance to the optimal solution decreases. Local peaks or valleys may impede the path toward the optimal solution, thereby diminishing the correlation between fitness and distance to the optimal point. Consequently, smaller absolute values of FDC typically indicate that the fitness landscape in the region is more rugged, rendering the optimization process more complex. The FDC analysis for different intervals of the same function is illustrated in Figures 2–5. The red dots on the curves in the chart represent the lowest points among all the points.

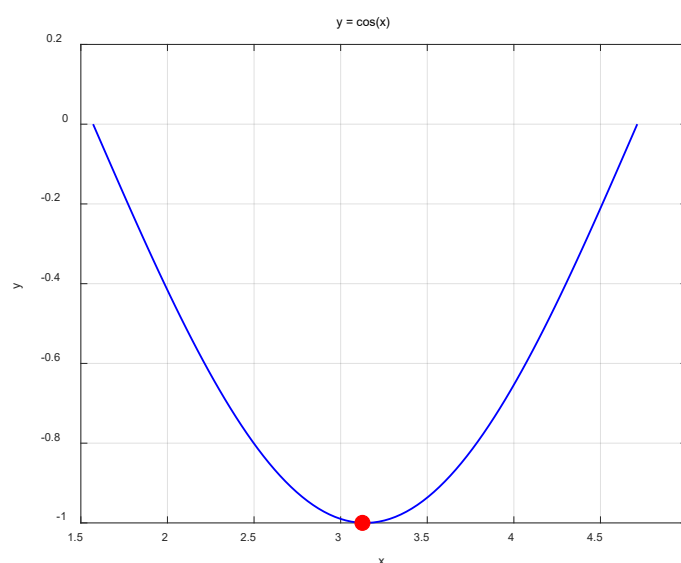


Figure 2. Graph of the function $y = \cos(x)$.

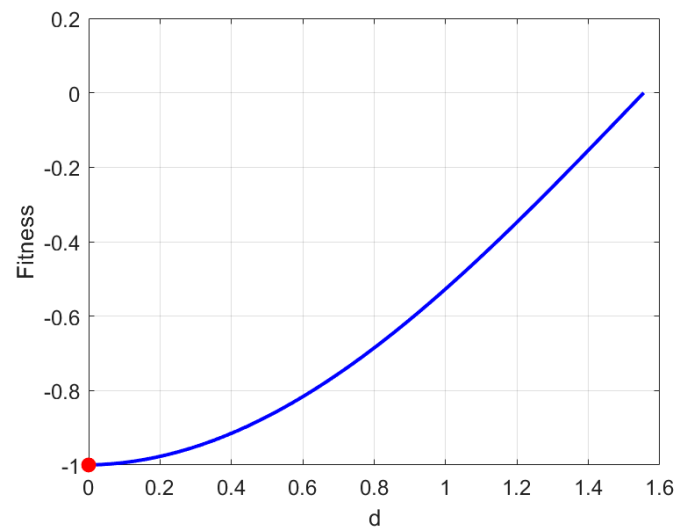


Figure 3. Relationship between fitness value and distance to the optimal point.

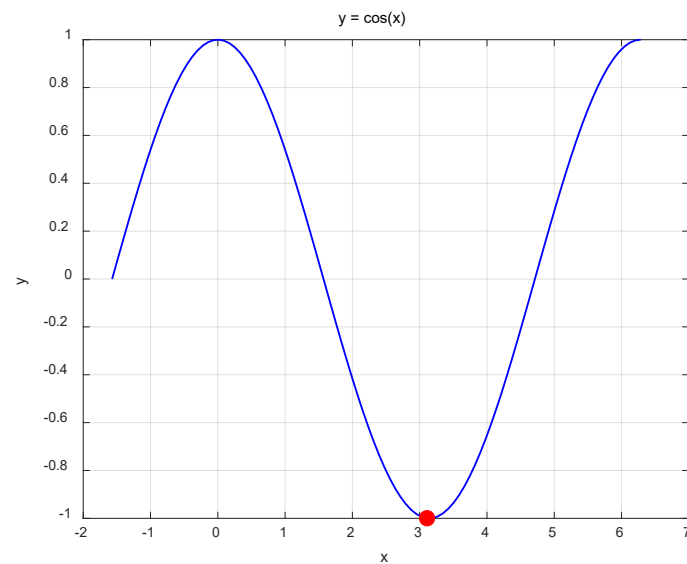


Figure 4. Curve of the function $y = \cos(x)$.

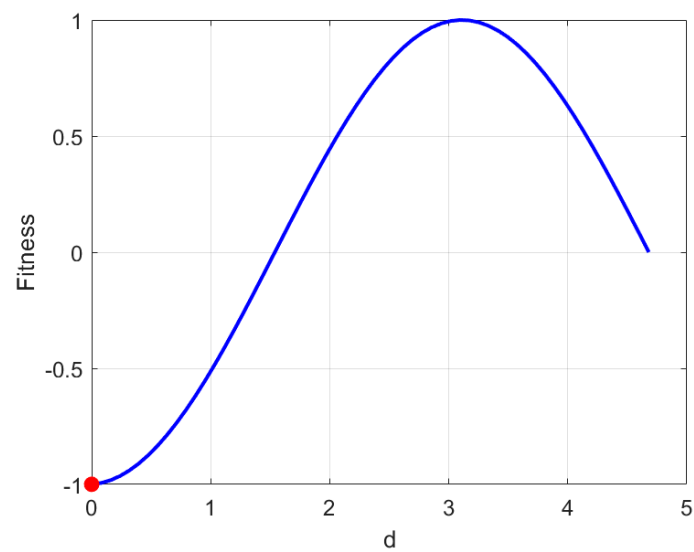


Figure 5. Relationship between fitness value and distance to the optimal point.

Figure 2 shows $y = \cos(x)$, $x \in [\frac{\pi}{2}, \frac{3\pi}{2}]$, where 150 sample points are evenly selected over the entire feasible interval, resulting in $P = \{\frac{\pi}{2}, \frac{1.01\pi}{2}, \dots, \frac{3\pi}{2}\}$, and the optimal point $x_{\min} = \pi$ is found. Figure 3 shows the relationship between the fitness value and the distance to the optimal point within the interval. As can be seen from the graph, the closer the distance to the optimal point, the smaller the fitness value, with $FDC = 0.9786$.

Calculations were performed on different intervals of the same function, and the results indicate that solving this function is relatively straightforward due to its low surface irregularity, facilitating the identification of extreme points. As illustrated in Figure 4, for function $y = \cos(x)$, $x \in [-\frac{\pi}{2}, 2\pi]$, using the same analysis method, we are able to identify the optimal solution point $x_{\min} = \pi$ within the sample range.

In addition, Figure 5 illustrates that the closer a solution is to the optimal point, the lower its fitness value. However, within a certain region, a high-fitness point emerges, disrupting the previously mentioned trend—that is, when the distance from the solution to the optimal point is below a certain threshold, fitness decreases as distance decreases; conversely, when the distance exceeds this threshold, fitness decreases as distance increases. The further calculation result is $FDC = 0.8155$. Compared to Figure 3, the FDC value depicted in Figure 5 has decreased, indicating that as surface ruggedness increases, the FDC value correspondingly decreases. Nevertheless, the surface ruggedness in this range is not highly complex. Although the FDC value is lower than in Figure 3, the correlation coefficient remains relatively high, suggesting that even with slight surface irregularities, the impact on locating extreme points is still limited.

However, there are several limitations associated with using the FDC metric for evaluation. As discussed in the literature, the calculation of FDC relies on a limited sample set, which is often significantly smaller than the overall scale of the search space, potentially leading to biased results. Furthermore, according to Formula (1), the computation of FDC assumes that the position of the global optimum is known, thereby restricting its utility as a predictive tool and rendering it more suitable for post hoc verification. To address these limitations, an improved FDC metric approach is adopted in this study, which divides the search space into subintervals and analyzes the FDC values within each subinterval. Through extensive experiments, it is generally recommended that the FDC value be set between 0.4 and 0.7. Additionally, for black-box function optimization problems of part surface morphology, where gradient information of the unknown function is unavailable, determining the global optimum in advance is challenging. Therefore, this method samples surface information within each subinterval and designates the point with the highest fitness value among all sampled points as the local optimum for that region. This approach not only overcomes the limitations of the traditional FDC metric but also effectively elucidates the ruggedness of the surface.

3. Extreme Point Detection in Part Surface Morphology

3.1. Algorithm Principles

When evaluating the validity of the starting point A within the defined region P, the reference criterion can be set as follows. If point A, as the initial configuration of the search algorithm, leads the algorithm to successfully identify at least one local optimum within region P, then point A is considered a valid initial configuration point within region P. Conversely, if using point A as the starting configuration of the algorithm fails to identify a local optimum within region P, then point A is regarded as an invalid initial configuration point within region P. Figure 6 illustrates the valid initial points within the region. Point A: It is located in region P1 and serves as the starting point for the pattern search. It successfully locates the local optimal solution S1, so it is an effective initial point for region P1.

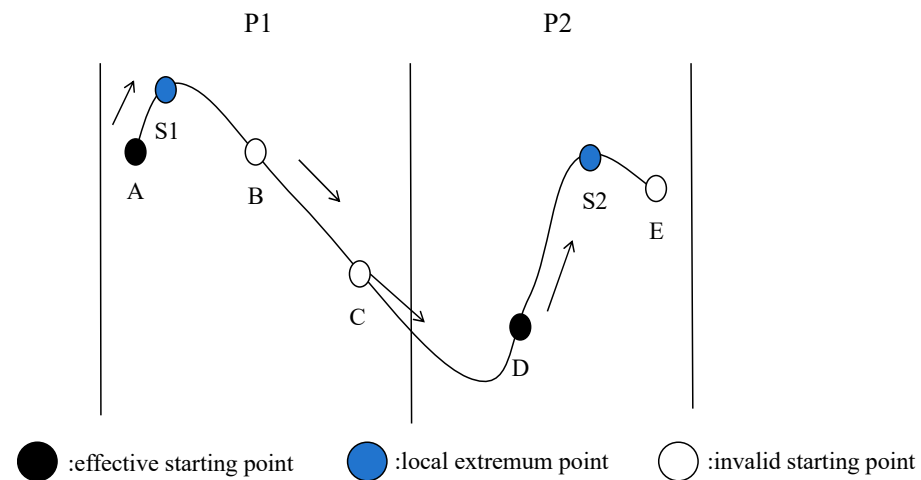


Figure 6. Region of valid initial points.

Points B and C: They are located in region P1, but as starting points, they cannot locate the local optimal solution S1. Therefore, they are invalid initial points for region P1.

Point D: It is located in region P2. As the starting point, it successfully locates the local optimal solution S2, so it is an effective initial point for region P2.

Point E: It is located in a highly rugged region, such as P3. More initial points are required to locate the extreme point. It is either an invalid initial point or a starting point from which the extreme point has not been successfully searched.

According to the definition of local extremum search, if point A, serving as the starting point of the search, successfully locates the local optimum S1 within region P1, then A is considered a valid initial point for P1. In contrast, if points B or C, as starting points, cannot identify the local optimum S1 within P1, then B and C are considered invalid initial points for P1. Similarly, if point D, as the starting point in region P2, is able to successfully locate the local optimum S2, then D is considered a valid initial point for P2.

As shown in Figure 6, the complex terrain of region P1 requires more initial points to increase the probability of finding valid starting points. On the other hand, region P2 has relatively low ruggedness, so a few initial points are sufficient to ensure the identification of extremum points.

In general, for the P1 region shown in Figure 6, which has higher ruggedness, the probability of valid initial points appearing is relatively high. High-ruggedness regions may also contain multiple peaks and valleys, as shown in Figure 7. In this case, regions such as P1 and P3, with higher ruggedness and multiple peaks and valleys, require more initial points, while the P2 region, which is relatively smooth with fewer peaks and valleys, requires fewer initial points to search. Therefore, regions with higher ruggedness require more initial points to search for local extrema.

However, reducing the number of initial points is one of the key conditions for improving the efficiency of extremum search algorithms. Therefore, regions with higher ruggedness can be further divided into smaller sub-regions, which will reduce the number of local extrema in each sub-region. As shown in Figure 7, when the P1 region is divided into two sub-regions, P1.1 and P1.2, the reduction in ruggedness of these sub-regions leads to an increase in their FDC values. Based on this, by continuously dividing the region, the ruggedness of each sub-region can be reduced, and the FDC value can be increased. When the FDC value reaches an appropriate level, the search efficiency for extremum points in each sub-region will improve, and the overall efficiency of the global extremum search algorithm will also increase.

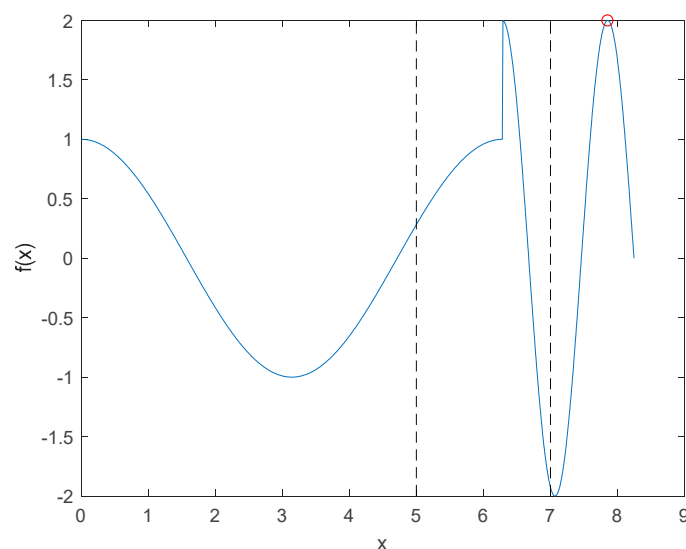


Figure 7. Piecewise function curve graph.

Next, a piecewise function will be used to illustrate the region division method based on FDC:

$$f(x) = \begin{cases} \cos(x), & 0 \leq x \leq 2\pi \\ 2\cos(4(x - 2\pi)), & 2\pi \leq x \leq 2.625\pi \end{cases} \quad (2)$$

As shown in the figure above, the piecewise function exhibits a high degree of ruggedness, and from Figure 8, it can be seen that the pattern of adaptation values for this piecewise function is relatively irregular. Additionally, the computed values of $FDC(s) = -0.091$ for this region show that directly solving the problem is relatively difficult. Due to the presence of multiple peaks and valleys, it is challenging to directly search for the extrema points.

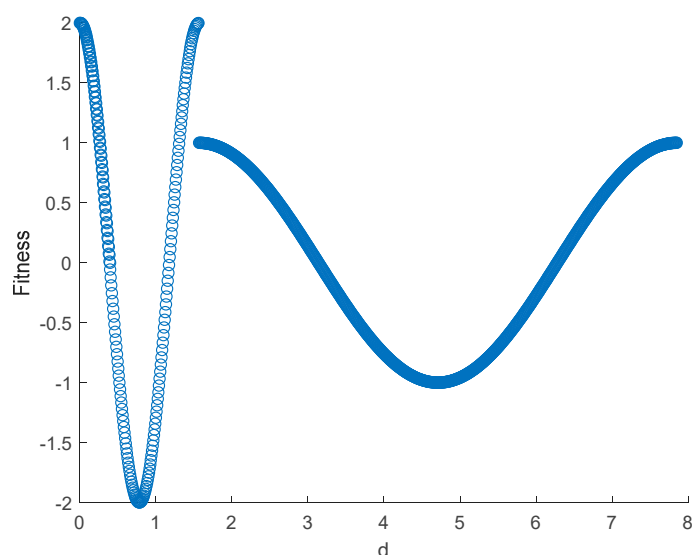


Figure 8. Relationship between fitness value and distance to the optimal point.

Therefore, the region is divided into two equal parts, and sample points are uniformly taken with an interval of 0.01 in the P1 and P2 regions. To refine the analysis of the surface's ruggedness, we split the entire interval into two sections, labeled S1 and S2. For these two intervals, uniform sampling is performed at an interval of 0.01, and the respective FDC values are calculated. The results show that $FDC(P1) = 0.83$ and $FDC(P2) = 0.49$, indicating that the ruggedness of region P2 is significantly greater than that of region P1. Assuming an acceptable FDC threshold of 0.5, the P2 region needs further subdivision.

Next, the P2 region is subdivided into two sub-regions, P2.1 and P2.2. After performing uniform sampling and analysis on these two sub-intervals, the results show that the FDC value in P2.1 is 0.31, while in P2.2, it reaches 0.95. This indicates that the terrain corresponding to S2.2 meets our expected ruggedness standard, while the S2.1 region shows a need for further refinement. Therefore, we subdivide the P2.1 region into P2.1.1 and P2.1.2. After sampling and calculation, $FDC(P2.1.1) = 0.750$ and $FDC(P2.1.2) = 0.870$, the ruggedness indices of both sub-intervals meet the standards, and no further subdivision is necessary.

As a result, the entire interval is effectively divided into three parts: P1, P2.1.1, P2.1.2, and P3. Among these, the regions with higher ruggedness, P2.1.1 and P2.1.2, are accurately identified and effectively subdivided, ensuring that the ruggedness of each sub-region meets our standards. Through this method, we can precisely identify and handle the regions with higher ruggedness, enabling effective analysis and subdivision of the entire surface.

3.2. Algorithm Flow

The pattern search algorithm is a method employed to identify the optimal solution of a function, particularly in scenarios with multiple local extrema. Similar to global extremum search algorithms, such as Multi-start, and gradient descent methods, pattern search algorithms belong to the category of direct search algorithms. A key advantage of pattern search algorithms is that they do not require gradient information of the objective function. The fundamental concept involves iteratively exploring and locating local optimal solutions to an optimization problem. The algorithm utilizes a grid-based search approach rather than relying on gradient information, rendering it suitable for complex functions that are nonlinear, non-differentiable, or noisy. By intelligently adjusting search directions and step sizes, the algorithm can effectively navigate within the feasible domain of the objective function to locate local optimal solutions.

The algorithm starts by randomly selecting a point or determining an initial point based on some heuristic strategy. It then constructs a grid around the current point. This grid is formed by adding scalar multiples of a fixed set of vectors (referred to as patterns) to the current point. These vectors determine the search directions and step sizes. A pattern is a collection of unit vectors, and the number of these vectors is determined by the number of decision variables in the problem. If the number of decision variables is m , the pattern can be represented as $\{x_1, x_2, \dots, x_m\}$, where $x_i (i = 1, 2, \dots, m)$ is a unit vector whose i th element is 1 and the other elements are 0, or $x_i (i = m + 1, m + 2, \dots, 2m)$ can also represent a unit vector where the i th element is -1 and the rest are 0.

The pattern search algorithm operates by systematically exploring points on a grid to identify those that can enhance the objective function value. When such points are discovered on the grid, they are adopted as the new points for the subsequent iteration. If no superior points are identified within the current grid, the algorithm reduces the step size, effectively shrinking the grid scale to perform a more detailed local search. Conversely, if continuous improvements are observed, the compensation parameter may be increased to accelerate the search process. The pattern search algorithm terminates when predefined termination conditions are satisfied.

For the identification of extreme points on a part's surface morphology, the number of initial points in the pattern search algorithm directly influences the number of local extreme points detected. Typically, a single initial point in the pattern search algorithm leads to the discovery of one extreme point. Therefore, to identify all extreme points on the surface, a greater number of initial points is required. By integrating the surface roughness calculation method, the number of initial points can be determined based on the roughness of the surface area. Higher roughness necessitates a larger number of initial points. Consequently,

combining the pattern search algorithm with fitness surface analysis to compute the local extreme points of the part surface morphology constitutes an effective strategy.

For regions exhibiting higher roughness, regional division is implemented, and the FDC values of the subdivided subregions are recalculated. If the FDC values of these subregions are relatively high, it indicates that fewer initial points are required for that area, thereby reducing the search difficulty within the subregion. Conversely, if the FDC values fall below a predetermined threshold, the following Formula (3) is employed for initial point allocation, adding more initial points and thereby increasing the probability of identifying global extreme points.

Initial point allocation:

$$n_i = \left\lceil \frac{n_0}{|f_{dc}(P_i)|} \right\rceil \quad (3)$$

In the formula, n represents the number of initial points, P denotes the partitioned region, and $\lceil \bullet \rceil$ represents the ceiling function (rounding up to the nearest integer).

The steps for identifying the global extreme values of the surface morphology using the improved mode search algorithm are illustrated in Figure 9 below:

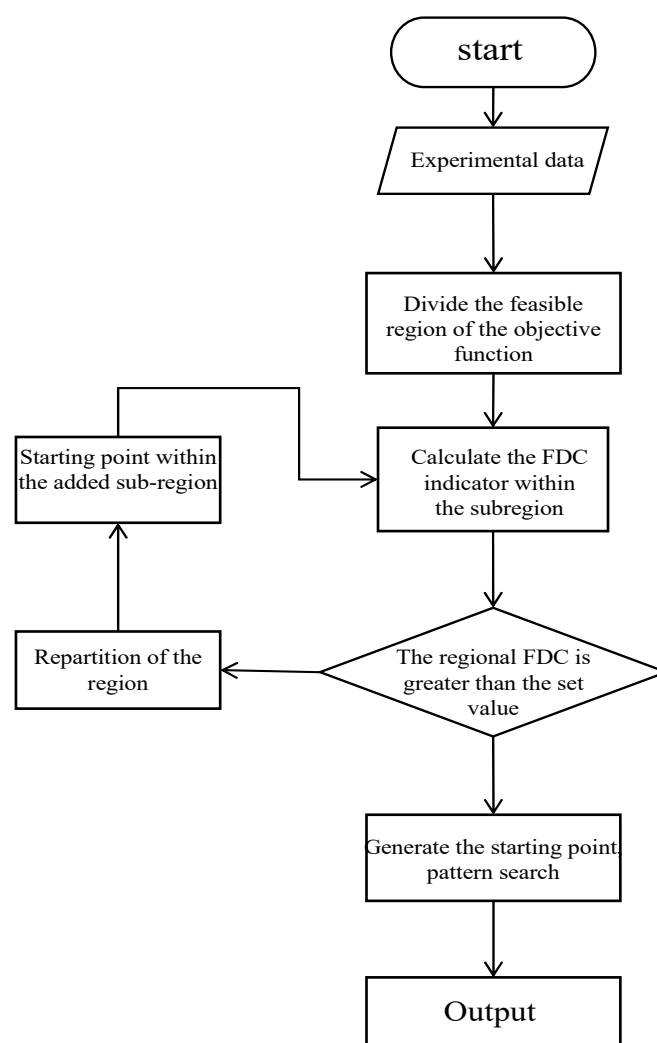


Figure 9. Algorithm flowchart.

Step 1: For the feasible domain of the objective function, the region is initially divided to analyze each subregion. Within each subdivided subregion, a set of sample points is uniformly generated with a fixed sampling interval d , and the extremum points within the

sample set are designated as the optimal points P . Subsequently, the distance from each sample point to the optimal point P is calculated. The FDC index is then computed using Formula (1).

Step 2: For regions where the FDC index is relatively lower than the predefined threshold, further regional division is undertaken. The FDC value for each subregion is recalculated iteratively until the FDC value of a region exceeds the set threshold, at which point no additional division is performed.

Step 3: For each division in region P , N initial exploration points are randomly generated based on the value of FDC according to Formula (3). These points are then used as starting points to execute the pattern search algorithm in pursuit of the optimal solution. The goal is to locate the local optimal solution in each region. The selection of N is based on distributing the total number of initial points across each region according to a certain method, as shown in Formula (4).

$$N = \sum_{i=1}^m n_i \quad (4)$$

Here, N represents the total number of initial points, and m represents the number of sub-regions. The initial point n_0 can be calculated by substituting into Formula (4) from Formula (3).

4. Case Study

This section presents a case study to illustrate the application and effectiveness of the proposed method. Section 4.1 begins with theoretical test functions to assess the method's performance under controlled conditions. Section 4.2 then moves on to a practical engineering case, demonstrating how the method is applied in a real-world setting.

4.1. Theoretical Test Functions

Multiple functions with varying characteristics are employed to generate the surface morphologies of parts. Subsequently, the extremum search method proposed in this study is applied to identify extremum points within these generated morphologies. This process serves to validate the effectiveness of the proposed method by demonstrating its capability to accurately locate extremum points across diverse surface profiles. The proposed method was validated using Matlab R2023. The experiments were conducted on a computer with 32 GB of RAM and an NVIDIA RTX 2000 GPU. All algorithms and visualizations presented in this study were implemented using the Matlab programming language.

Function 1: Peaks function

$$f(x, y) = 3(1 - x)^2 e^{-(x)^2 - (y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) e^{-x^2 - y^2} - \frac{1}{3} e^{-(x+1)^2 - y^2} \quad (5)$$

Function 2: Branin-Hoo function

$$f(x) = \left(x_2 - \frac{5.1x_1^2}{4\pi^2} + \frac{5x_1}{\pi} - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right) \cos(x_1) + 10 \quad (6)$$

Function 3: Six Hump Camel Back function

$$f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4 \quad (7)$$

Function 4: Ackely function

$$f(x) = -20e^{(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^n x_i^2})} - e^{\left(\frac{1}{n}\sum_{i=1}^n \cos(2\pi x_i)\right)} + 20 + e \quad (8)$$

Function 5: Rastrigin function

$$f(x) = 10n + \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i)] \quad (9)$$

Utilizing the improved pattern search algorithm presented in this paper, the surface extremum points for five test functions were calculated. Although Test Functions 1–5 selected in this study are explicit functions, they can still be treated as black-box functions for experimental purposes.

The surface morphology of the Peaks function is presented as Figure 10. The search results for the Peaks function are presented in Table 1 and Figure 11. By applying the improved pattern search algorithm, information regarding regional local extreme points, global extreme points, and regional ruggedness was obtained, as presented in the table above. This demonstrates the effectiveness of the ruggedness index and validates the method used for verifying local extreme points. Figure 10 is a surface morphology diagram. The color represents the height or depth value, with yellow indicating the peak value and blue indicating the trough value. Figure 11 is a distribution diagram of extreme points. The color differentiates different sub-regions or types of extreme values, where yellow represents the peak region and blue represents the trough region. In Figure 11, the local minima of each sub-region are marked with red asterisks.

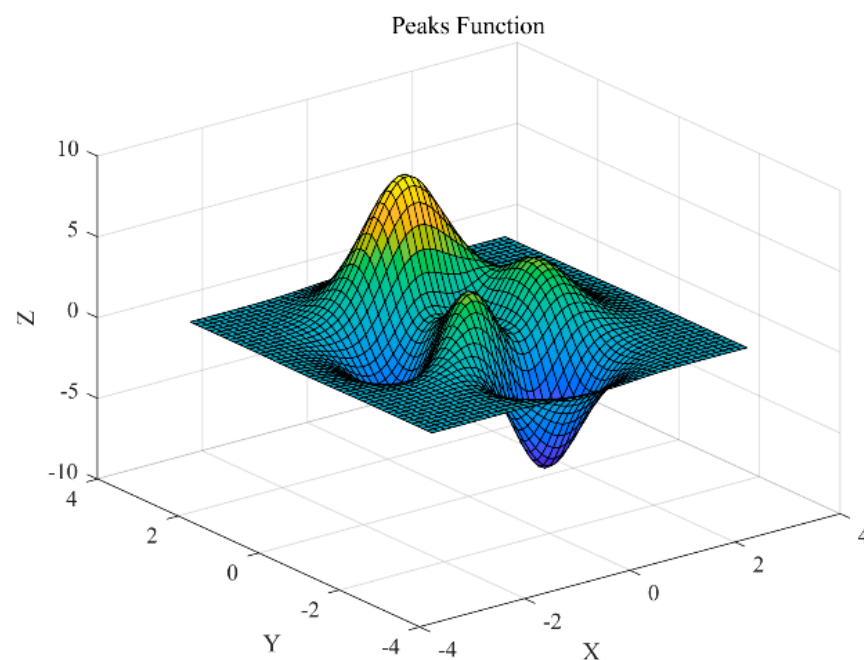


Figure 10. The surface morphology of Peaks function.

Table 1. The extreme points of Peaks function.

Local Minimum of the Region	x	y	f	FDC
P1	−1.58744	0.20134	−2.86052	−0.51618
P2	3.12570	2.20451	−3.04984	0.71009
P4	0.22828	−1.62553	−6.55113	0.62020
P4	3.00000	−0.20000	0.00041	−0.71606

The surface morphology of the Peaks function is presented as Figure 12. The search results for the Peaks function are presented in Table 2 and Figure 13. For the Branin-Hoo function, which has a relatively low local ruggedness, it is divided into four regions for search. The information of all extreme points and the ruggedness index within the feasible

domain of this function are calculated. As shown in Figures 12 and 13, the Branin-Hoo function has a relatively low ruggedness, and the region division is relatively simple. By applying this method, all local extreme points can be directly obtained. Figure 12 is a surface morphology diagram. The color represents the height or depth value, with yellow indicating the peak value and blue indicating the trough value. Figure 13 is a distribution diagram of extreme points. The color differentiates different sub - regions or types of extreme values, where yellow represents the peak region and blue represents the trough region. In Figure 13, the red asterisks represent the extreme points found by the pattern search algorithm.

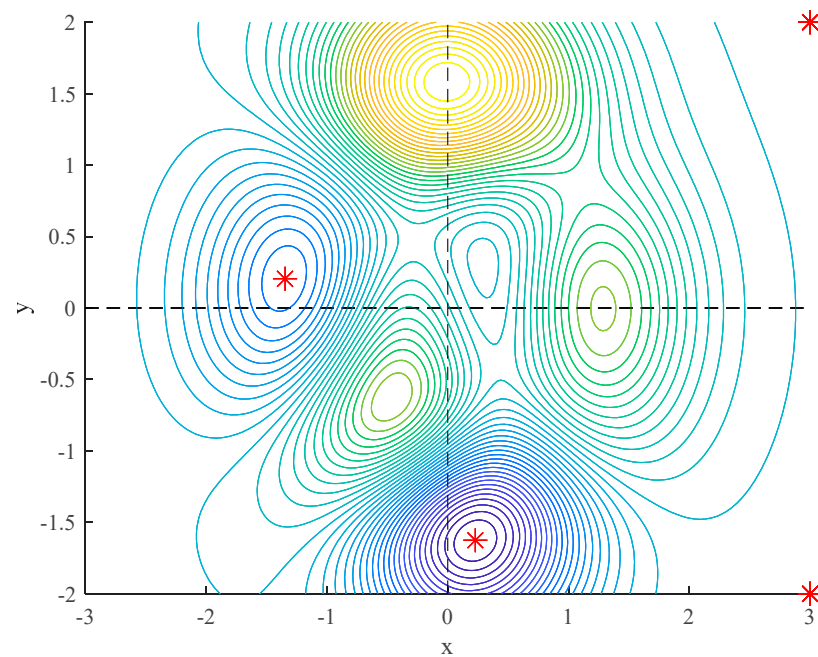


Figure 11. The extreme points of Peaks function.

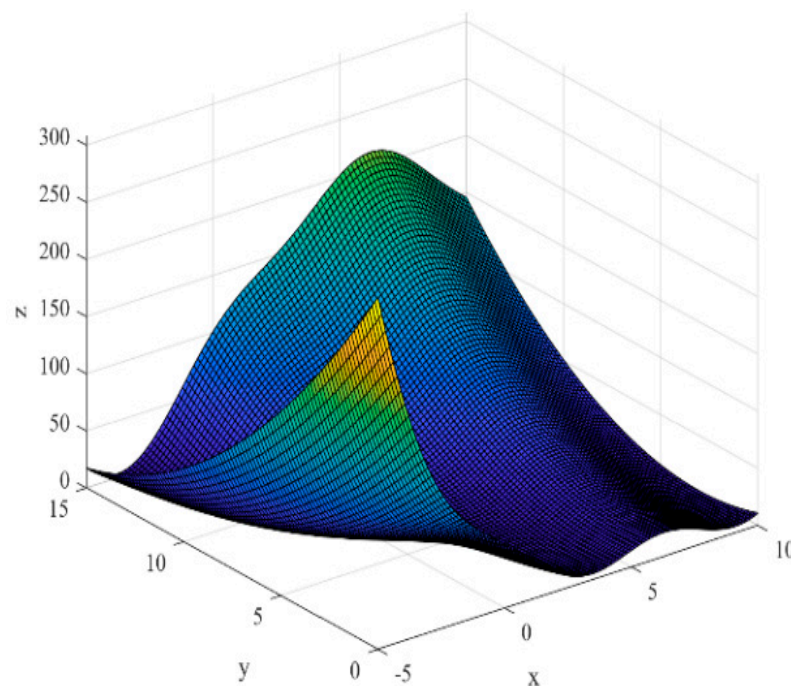
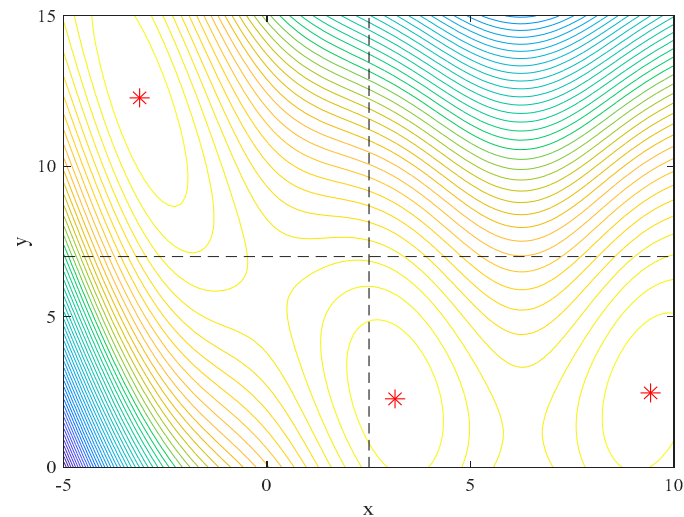


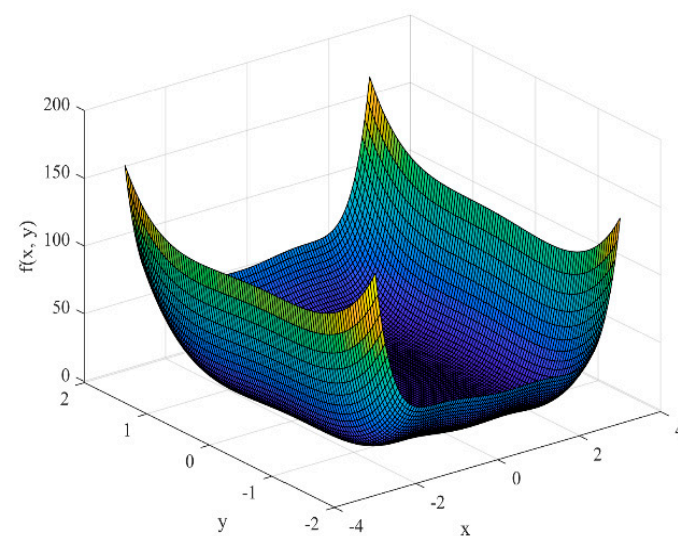
Figure 12. The surface morphology of Branin-Hoo function.

Table 2. The extreme points of Branin-Hoo function.

Local Minimum of the Region	x	y	f	FDC
P1	−4.34093	12.00251	130.34272	−0.93752
P3	9.68231	2.155532	200.15123	−0.92851
P3	2.03201	2.000000	190.00781	−0.88186

**Figure 13.** The extreme points of Branin-Hoo function.

The surface morphology of the Six Hump Camel Back function is presented as Figure 14. The search results for this function are presented in Table 2 and Figure 15. For the Six Hump Camel Back function, the feasible domain of the function is $[-3, 3]^2$, and the function region is divided into nine equal parts. Assuming that the sampling intervals of x_1 and x_2 are both d , the feasible domain FDC index is calculated using Formula (1), as shown in Table 3. Figure 14 is a surface morphology diagram. The color represents the height or depth value, with yellow indicating the peak value and blue indicating the trough value. Figure 15 is a distribution diagram of extreme points. The color differentiates different sub - regions or types of extreme values, where yellow represents the peak region and blue represents the trough region. In Figure 15, red asterisks are also used to mark the extreme points located by the algorithm, verifying the effectiveness of the method.

**Figure 14.** The surface morphology of Six Hump Camel Back function.

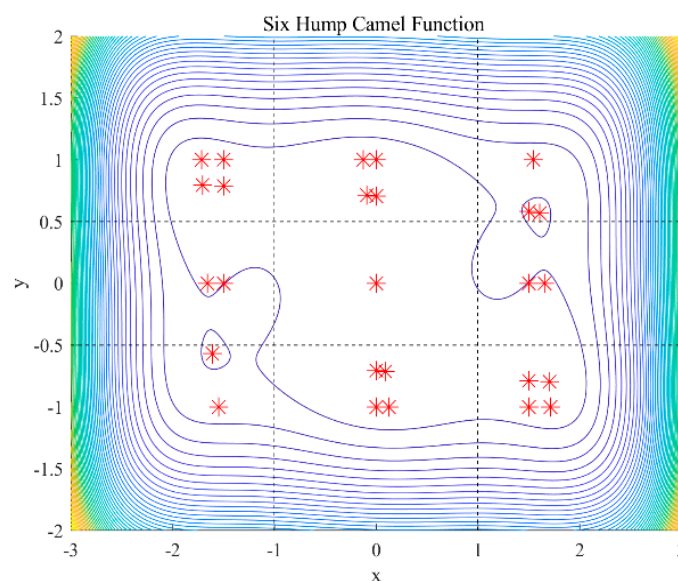


Figure 15. The extreme points of Six Hump Camel Back function.

Table 3. The FDC parameters of Six Hump Camel Back function.

Subinterval FDC	d = 0.1	d = 0.01	Number of Initial Points in Each Region
P1	0.9114	0.9473	10
P2	0.8961	0.9128	10
P3	0.8865	0.9117	10
P4	0.8511	0.9036	10
P5	0.8790	0.8993	10
P6	0.9167	0.9389	10
P7	0.8756	0.9109	10
P8	0.8663	0.8891	10
P9	0.8901	0.9207	10
Sample Size	100	1000	-

After dividing the feasible domain of the test function into nine equal parts and evaluating the FDC calculations with different sampling intervals h within each sub-feasible region, it was observed that the FDC values varied depending on the size of the sampling interval. Notably, smaller sampling intervals led to higher FDC accuracy. However, the analysis regarding the relationship between sample size and interval suggests that even larger sampling intervals are sufficient for a preliminary assessment of the subinterval's morphological complexity. By reducing the sampling interval to 0.1 and selecting a certain number of samples within each sub-region, the FDC values showed relative stability. Further reducing the sampling interval to 0.01, the FDC error range was limited to within 0.05. This indicates that the FDC values are not highly sensitive to changes in the sampling interval, and under normal circumstances, selecting a moderate number of sample points is sufficient for a reasonable FDC estimate. This stability is due to the fact that as long as the sample size is adequate to roughly map the surface complexity, the FDC value remains stable, proving that under the premise of ensuring experimental quality, the chosen sampling interval for $d = 0.1$ is already sufficient.

The surface morphology of the Ackely function is presented as Figure 16. The search results for this function are presented in Table 4 and Figure 17. For this function, due to its high ruggedness, as determined by direct computation, the test function is divided into nine equal segments within the feasible domain. Sampling is performed with a sampling interval of $d = 0.1$. The FDC of each sub-region is computed, and the local extreme

points within each divided region are searched. The FDC calculation results and partial results of the extreme point search for this function are shown in Table 4. Figure 16 is a surface morphology diagram. The color represents the height or depth value, with yellow indicating the peak value and blue indicating the trough value. Figure 17 is a distribution diagram of extreme points. The color differentiates different sub-regions or types of extreme values, where yellow represents the peak region and blue represents the trough region. In Figure 17, red asterisks are also used to mark the extreme points located by the algorithm, verifying the effectiveness of the method.

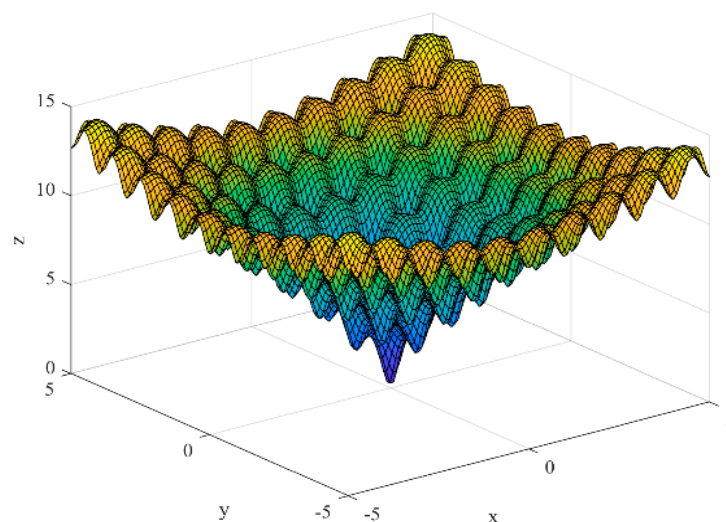


Figure 16. The surface morphology of Ackely function.

Table 4. The extreme points of Ackely function.

Regional Minimum	x	y	f	FDC
P1	−1.9604	−4.5200	10.8326	0.64247
P2	−3.1415	2.2750	0.39788	0.41217
P3	3.14159	1.1173	0.37880	0.67094
P4	0.00001	−2.1000	12.8520	0.94989
P5	−1.4247	−2.4750	0.39788	0.85449
P6	−3.2500	2.7500	13.1665	0.94751
P7	−1.8318	4.4752	0.39788	0.92398
P8	4.2486	2.47500	0.39788	0.91097
P9	4.0663	7.40092	14.19820	0.96738

The algorithm performs a separate search within the nine feasible regions after dividing test function 4. Table 4 only reports the coordinates of some local minimum points from the nine regions. From the FDC index values, it can be seen that with a sampling interval of $d = 0.1$, the FDC values are smaller in the P2 and P1 regions, indicating the presence of multiple local extremes in these regions. By further subdividing these regions and performing pattern search, more local minima can be found. On the other hand, the FDC absolute values in the P3–P9 regions are relatively large, indicating fewer local extrema in these areas. In total, the algorithm identified 60 local minima, validating the effectiveness of the method.

The surface morphology of the Ackely function is presented as Figure 18. The search results for this function are presented in Table 5 and Figure 19. Due to its high surface roughness and the presence of multiple local extremum points, it first defines the initial number of search points. Then, combined with the method proposed in this paper, the total feasible domain of the function is divided into nine sub-feasible regions. Parallel search is

conducted within each sub-feasible region, and the FDC value for each region is calculated. Some of the results are shown in Table 5. Figure 18 is a surface morphology diagram. The color represents the height or depth value, with yellow indicating the peak value and blue indicating the trough value. Figure 19 is a distribution diagram of extreme points. The color differentiates different sub-regions or types of extreme values, where yellow represents the peak region and blue represents the trough region. In Figure 19, red asterisks are also used to mark the extreme points located by the algorithm, verifying the effectiveness of the method.

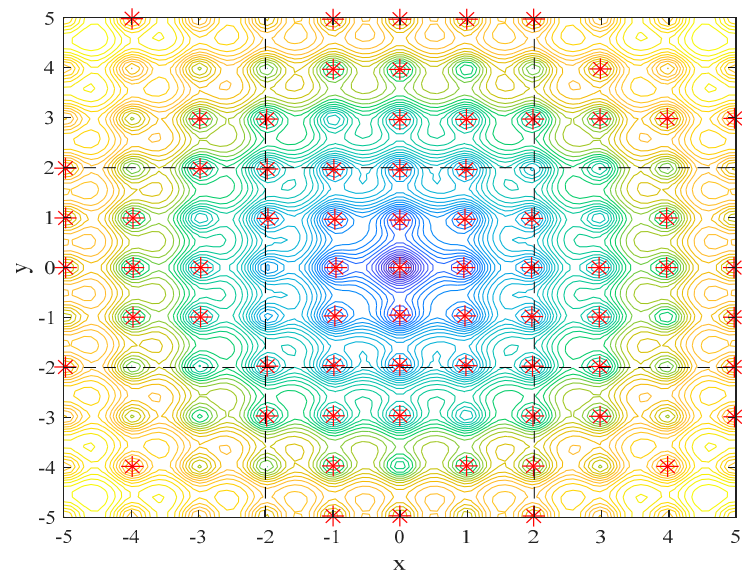


Figure 17. The extreme points of Ackely Function.

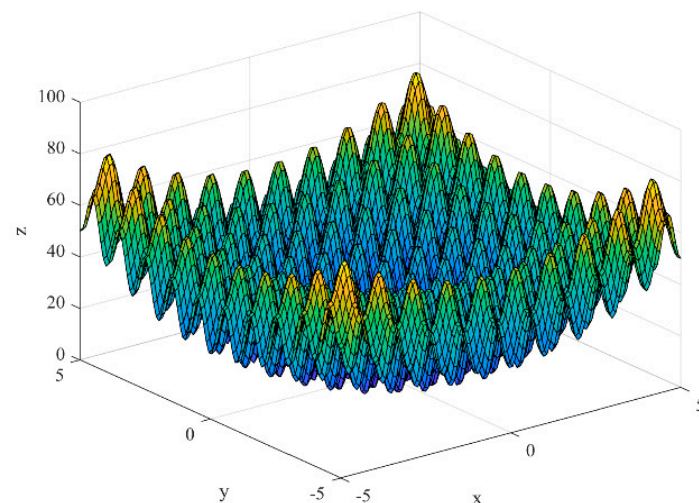


Figure 18. The surface morphology of Rastrigin function.

Due to the high ruggedness of this test function and the need to divide the domain into multiple regions, the method searches for a total of 90 local extremum points for the Rastrigin function. Only a subset of extremum points and some regional FDC index information are shown in the table. For regions P1 to P4, the FDC values are relatively small, so multiple subdivisions of these regions were made, and a parallel search algorithm was implemented to obtain local extremum points within these regions.

For the above test functions, the improved mode search algorithm proposed in this paper was applied. The feasible domains were divided for different functions, and FDC

indices were calculated. A parallel search was conducted to find all local extremum points on the surfaces, which validated the effectiveness and robustness of the method.

Table 5. The extreme points of Ackely function.

Regional Minimum	x	y	f	FDC
P1	−1.58991	−1.77991	15.95966	0.54558
P2	1.52991	1.989912	25.83671	0.53961
P3	4.99561	−0.99496	14.98991	0.70103
P4	−3.71490	2.98485	9.94956	0.35330
P5	−0.51200	−4.5420	10.49309	0.67660
P6	3.53200	0.59360	6.37620	0.46190
P7	−4.99050	−4.9710	4.98991	0.51060
P8	1.58210	−4.5840	3.49600	0.69140
P9	3.53170	−3.67930	6.89030	0.44980

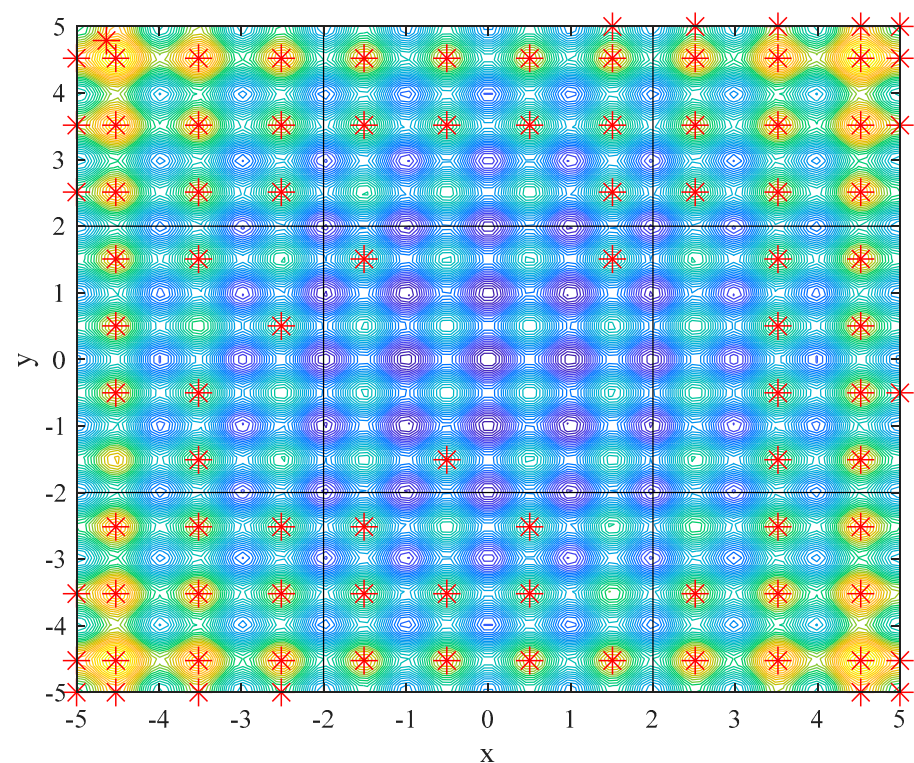


Figure 19. The extreme points of Rastrigin function.

4.2. Practical Engineering Case

We designed a physical experiment to validate the method proposed in this paper. As shown in Figure 20, the test sample is a simulated flat part with sealing requirements. It measures 200 mm × 100 mm, with a flatness tolerance of 0.01 mm. This experiment aims to demonstrate the application of the proposed method in surface quality evaluation.

As shown in Figure 21, the experimental procedure consists of four main steps. First, the NCA8107 Coordinate Measuring Machine (CMM), shown in Figure 22, is used to evenly measure 30 × 20 points on the surface of the test sample. Second, the Kriging model is applied to fit the measurement data and obtain the surface morphology model of the test sample, as shown in Figure 23. Third, the proposed method is used to search for extremum points on the surface morphology model. Finally, both the original measurement data and the extremum points are used for surface quality evaluation of the part.



Figure 20. Physical model of the test sample.

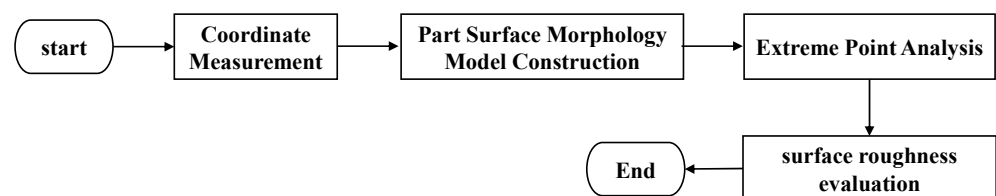


Figure 21. Experimental procedure.



Figure 22. NCA8107 coordinate measuring machine (CMM).

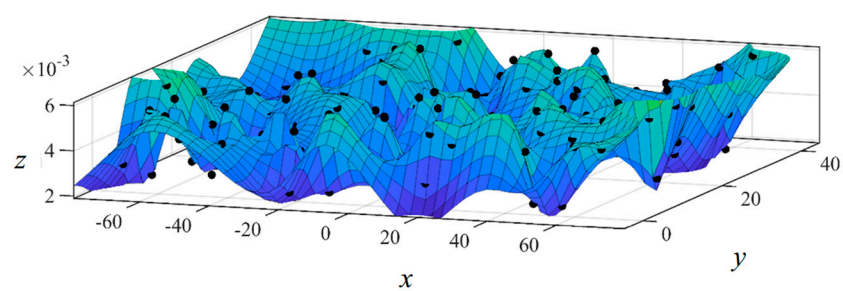


Figure 23. Surface morphology model of the test sample.

To analyze the impact of extremum points on surface quality evaluation, two methods were used for calculation. The first method directly utilizes the measurement data for surface quality evaluation. The second method combines the measurement data with the extremum points before performing the surface quality evaluation. The calculation results of both methods are shown in Table 6. From the results, it can be seen that the flatness evaluation value considering extremum points is 2.84% higher than the one without considering extremum points. This is because when only measurement data are considered, the influence of extremum points is ignored. Extremum points, typically representing peaks or valleys in the surface morphology model, have a direct impact on flatness.

Table 6. Flatness error evaluation results.

Methods	Flatness Error	Flatness Error as a Percentage of Design Tolerance
Without considering extremum points	7.90×10^{-3}	78.94%
Considering extremum points	8.18×10^{-3}	81.81%

5. Conclusions

This study proposes a fitness landscape-based method for extreme point analysis to extract surface morphology characteristic parameters of a part. The method conceptualizes the search for global extremum points as identifying optimal solutions. An adaptive value surface analysis approach, combined with an improved mode search algorithm, is introduced. First, the Fitness-Distance Correlation (FDC) and a roughness index are integrated to analyze surface roughness and determine the number and distribution of initial points based on roughness. The surface morphology's domain is partitioned using the FDC, and initial points are allocated according to the surface roughness, reducing optimization complexity. Finally, the roughness index further subdivides the region, and the mode search algorithm is implemented in parallel to enhance search efficiency. This method enables the extraction of surface parameters and global extremum points without the need for gradient information. Compared to existing methods, the proposed approach improves efficiency by reducing the required number of initial points and adapting the search process based on surface roughness, which addresses challenges in complex surface morphologies. Additionally, it eliminates the need for gradient information, making it suitable for highly intricate surfaces where gradient-based methods may not be effective.

The method is applicable to Industry 4.0 environments, such as coordinate measurement systems (CMSs) for assessing geometric shape or surface roughness. By incorporating roughness and adaptive value surface analysis, it provides precise surface characterization crucial for high-precision manufacturing. At the same time, the method requires the same input as existing surface roughness evaluation methods, making it highly compatible and easy to integrate into current workflows. In conclusion, the proposed method enhances search efficiency and robustness, offering a reliable solution for modern manufacturing applications. Its ability to analyze surface quality aligns with Industry 4.0 objectives, making it an essential tool for smart manufacturing and quality control.

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