

Article

Fractional-Order Nonlinear PI Control for Tracking Wind Direction in Large Wind Energy Converters

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Abstract

In this work, the yaw control of large wind turbines is studied. The objective is to analyse how to maximise energy conversion by yawing the rotor in response to wind direction while minimising yaw activity. In order to improve the control performance, three algorithms are used and compared: the classic PI controller, the nonlinear PI controller, and the fractional-order nonlinear PI controller. An adaptive dead-zone and anti-windup procedure for amplitude- and rate-limited actuators are also considered, which helps to reach the main objective. Simulation experiments are carried out on a 20 MW reference wind turbine. The results are very promising, showing clear performance improvements.

Keywords: fractional calculus; fractional-order control; nonlinear PID; wind turbine control; yaw control

1. Introduction

Nowadays, wind turbines are essential contributors in the production of renewable energy in the quantities that are necessary to meet the demands of the contemporary world. In order to meet the continuously increasing demand, not only have these machines had to grow in size, but it is also no longer sufficient to operate them individually but rather to combine them in so-called wind farms.

For the above-mentioned reasons, there is an increasing demanding for a highly optimised control system for wind turbines. On the other hand, the control system is designed to satisfy two main objectives, which often are contradictory: maximum power extraction and active load reduction. The present work is focused on power extraction.

Taking into account the aim expressed above, the most important control loop is the one that allows maximisation of the power extraction under low wind speed [1]. Then, under high wind speed, the pitch control that regulates the rotational speed of the machine becomes indispensable [2]. Moreover, the maximum extraction is reached when the rotor is positioned perpendicular to the wind inflow. Thus, the optimal tracking of wind direction to move the rotor in the correct position, also referred to as yaw control, is the third most significant control loop [3]. In particular, this last control loop is the aim of this paper.

Wind turbine control has been dominated by classic PI or PID controllers (proportional, integral) with an anti-windup mechanism. It is also common to include in the pitch control loop an adaptive procedure, like gain scheduling, to overcome the nonlinear effects that characterise wind turbines. Nevertheless, the flexible structures and nonlinearities of wind turbines cause PI and PID controllers to eventually show limited capacities. In particular, classic PI and PID controllers exhibit weak tracking performance [4], which is the primary objective of yaw control.



Academic Editor: Nicola
Ivan Giannoccaro

Received: 14 November 2025

Revised: 11 December 2025

Accepted: 15 December 2025

Published: 17 December 2025

Citation: Gambier, A.
Fractional-Order Nonlinear PI Control
for Tracking Wind Direction in Large
Wind Energy Converters. *Machines*
2025, 13, 1150. <https://doi.org/10.3390/machines13121150>

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Modern large wind turbines include a complex yaw mechanism consisting of many actuators working in parallel in order to carry out different activities, such as, for example, yawing the rotor out of the wind, sluggishly tracking the wind direction, or affecting the wake steering in case of wind farm control.

In the last years, the importance of yaw control has intensified for individual turbines as well as for groups of turbines within a wind farm that work jointly under wake steering control [5–7]. A review discussing wind direction tracking is available in [8].

Numerous techniques for yaw control are available in the literature. Several of them require wind direction measurement, either by means of a low-cost and inaccurate wind vane or by more-accurate and expensive means such as LiDAR [9,10] or radar [11]. However, although LiDAR and radar measurements provide accurate data about the direction of the wind, they are still expensive to embed into each wind turbine.

It is also possible to substitute the measurements with estimations derived from historical data, prediction models, and prediction algorithms. See, e.g., Bayesian learning [12], fuzzy method [13], and data mining [14]. There also exist approaches that do not rely on wind direction data. This is the case of the rotor speed control loop introduced in [15], while hill climb searching (HCS) control is applied in [16–18]. Moreover, yaw control is also employed to alleviate loads in [15,19,20].

The majority of yaw control methods employ PI or PID controllers. However, as previously mentioned, the classic control approaches are progressively rendered insufficient to maintain an acceptable degree of control performance as a result of the rising complexity of wind energy converters. Therefore, advanced PID control, for example, NPID, has been applied to wind turbines [21–23]. Additionally, a fractional-order PID (FOPID) is utilised to control a permanent magnet synchronous generator (PMSG) as referenced in [24].

In the present work, the yaw control system is implemented by using PI, NPI, and FONPI control laws and studied on a 20 MW reference wind turbine. The objective is to improve the tracking performance of the controller without an excessive increment in actuator activity, by some modification of the standard PI controllers alone, i.e., by using NPI and FONPI. The paper is structured in the following way: Section 2 is dedicated to introducing the necessary fundamentals for PID, NPID, and FONPID control. The formulation of the yaw control problem as well as the proposed solution is the subject of Section 3. Next, the numerical application is presented in Section 4, and the results in Section 5. Finally, Section 6 is devoted to drawing the conclusions.

2. Fractional-Order Nonlinear PID Control

2.1. Classic PID Controller

The PID (proportional–integral–derivative) controller and its reduced form, the PI controller, are well-known and popular classic controllers with copious industrial and academic applications, on which there exists a plethora of literature (see [25–30] just to cite a few examples).

The PID control law includes three parameters, which are often denoted by K_p , K_i , and K_d . Its transfer function with a first-order filter in the derivative part is formulated by

$$G_{PID}(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + T_d s}, \quad (1)$$

where $1/T_d$ is the cut-off frequency, from which amplitudes of the high frequencies in the derivative term are limited. Notice that, when $K_d = 0$, the PID reduces to the PI.

The transfer function (1) can also be expressed by

$$G_{PID}(s) = K_p + K_i (1/s) + \frac{K_d}{(1/s) + T_d}, \quad (2)$$

which is convenient in order to use only integrators. There are also several possible configurations for the controller. In the present work, the automatic reset configuration of Figure 1 is favoured due to its prospective benefits in the context of the later extensions. The controller of Figure 1 can also be reworked in the way described in Figure 2 in order to separate the integrators. The advantage is that different integration techniques can be replaced or combined.

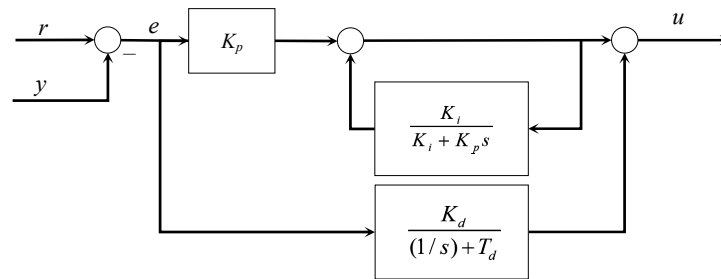


Figure 1. PID controller implemented with an automatic reset configuration.

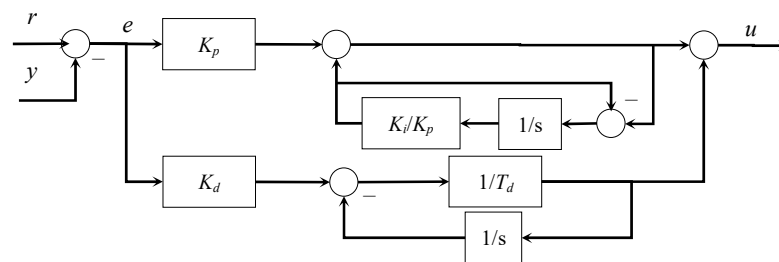


Figure 2. PID controller in automatic reset configuration with separated integrators.

2.2. PID Controller with Nonlinear Gains

The PID controller with nonlinear gains is also known as nonlinear PID (NPID) (see, e.g., [31,32]). The characteristic of the NPID is that its gains are expressed by

$$f(e) = K_0 + K_1[g(e) - 1]. \quad (3)$$

The function $g(e)$ is a nonlinear function of the control error with a range of 1 to 2. Hence, $g(e) = 1$ for $e = 0$ implies $f_{\min} = K_0$, and $g(e) = 2$ for $e = \pm\infty$ corresponds to $f_{\max} = K_0 + K_1$. In other words, K_0 represents the gain when the system is operated at the set point, while $K_0 + K_1$ is the gain when the operation lies distant from the operating point. Notice that, if K_1 is set to 0, the NPID mutates to the classic PID.

Several functions satisfy the above-described conditions for g . For example, the hyperbolic secant function

$$g(e) = 2 - \text{sech}(K_2 e) \quad (4)$$

and the smooth sigmoidal function

$$g(e) = 2 - \text{sigm}(K_2 |e|) \quad (5)$$

are the most prevalent [31]. The additional parameter K_2 is employed to maintain the shape of $g(e)$ and to calibrate its rate of change, as shown in Figure 3.

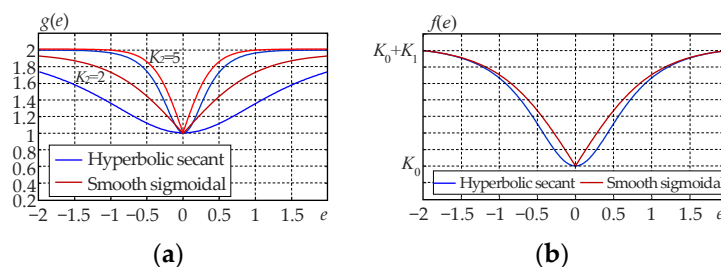


Figure 3. (a) Functions $g(e)$ for values of K_2 . (b) Examples of functions $f(e)$.

The concept described above is now used to set the three gains of the PID controller. Hence, the insertion of (4) in (3) leads to the gains

$$f_p(e) = K_{0p} + K_{1p}[1 - \text{sech}(K_{2p}e)], \quad (6)$$

$$f_i(e) = K_{0i} + K_{1i}[1 - \text{sech}(K_{2i}e)], \text{ and} \quad (7)$$

$$f_d(e) = K_{0d} + K_{1d}[1 - \text{sech}(K_{2d}e)]. \quad (8)$$

Thus, the transfer function of the NPID controller can be expressed according to (2) as

$$G_{NPID}(s) = f_p(e) + f_i(e)(1/s) + \frac{f_d(e)}{(1/s) + T_d}. \quad (9)$$

It is important to remark here that the nonlinear gains can be considered numbers that continuously change. Therefore, they can be included as coefficients in a transfer function (transfer function with variable coefficients).

2.3. Fractional-Order PID Controller

The fractional-order PID (FOPID) was first presented in [33] under the name $PI^\nu D^\mu$, where ν and μ are fractional orders for the integral and derivative terms (see [34–37] for additional information). The continuous integro-differential operator with fractional order is expressed here by

$$X(s) = K s^\gamma, \quad \gamma \in \mathbb{R} \quad (10)$$

with $\gamma > 0$ for the differential and $\gamma < 0$ for the integral case.

The FOPID can be formulated as a generalisation of the classic PID by introducing fractional exponents ν and μ to the Laplace variable s in (2), namely

$$G_{FOPID}(s) = K_p + K_i(1/s^\nu) + \frac{K_d}{(1/s^\mu) + T_d} \quad (11)$$

where $(1/s^\nu)$ and $(1/s^\mu)$ are integrators with ν and μ real and positive (see, e.g., [38] for details).

The operator (10) is a theoretic concept, whose practical implementation requires an approximation. Several approximations are available for (10). However, the Crone approach is applied in this research (as described in [39]). The approximation is expressed as the transfer function

$$X(s) = K_n \prod_{k=1}^n \frac{1 + s/\omega_{z,k}}{1 + s/\omega_{p,k}} \quad (12)$$

with n poles and zeros. Constant K_n is included to set a gain equal to 0 dB at the frequency of 1 rad/s for $K = 1$ in (10). Values $\omega_{z,k}$ and $\omega_{p,k}$ are zeros and poles in the interval $[\omega_l, \omega_h]$, i.e.,

the interval within which the approximation is valid. These poles and zeros are computed by using the recursive formulas

$$\begin{aligned}\omega_{p,k} &= \omega_{z,k-1}\eta, \quad k = 1, \dots, n \\ \omega_{z,k} &= \omega_{p,k-1}\alpha, \quad k = 1, \dots, n\end{aligned}\quad (13)$$

with $\omega_{p,1} = \omega_l\sqrt[n]{\eta}$, $\alpha = (\omega_h/\omega_l)^{\gamma/n}$ and $\eta = (\omega_h/\omega_l)^{(1-\gamma)/n}$.

2.4. Nonlinear PID Controller with Fractional Order

Controllers (2), (9), and (11) can be combined and generalised in order to yield the FONPID, i.e., nonlinear PID with fractional orders, whose transfer function is

$$G_{FONPID}(s) = f_p(e) + f_i(e) (1/s^\nu) + \frac{f_d(e)}{(1/s^\mu) + T_d} \quad (14)$$

where the functional parameters f are calculated by using (6)–(8), and the integrators are approximated by (12) and (13).

Notice that, when setting $\nu = 1$ and $\mu = 1$, the FONPID becomes the NPID, and when setting K_{1p} , K_{1i} , and K_{1d} to 0, the NPID becomes the standard PID.

2.5. Anti-Windup Mechanism for FONPID

The wind-up of the integrator is a recognised drawback of PI and PID controllers and has been extensively investigated (see, for instance, [40–42]). Back calculation, as suggested in [43], is an effective method for overcoming the hurdle, in particular, the magnitude saturation of actuators. However, yaw actuators are limited not only in magnitude but also in rate. Consequently, an anti-windup mechanism for magnitude and rate is proposed in [44] for controllers, which are implemented in the automatic reset configuration. This method is now extended to the FONPID controller, as shown in Figure 4 (see Appendix A for details).

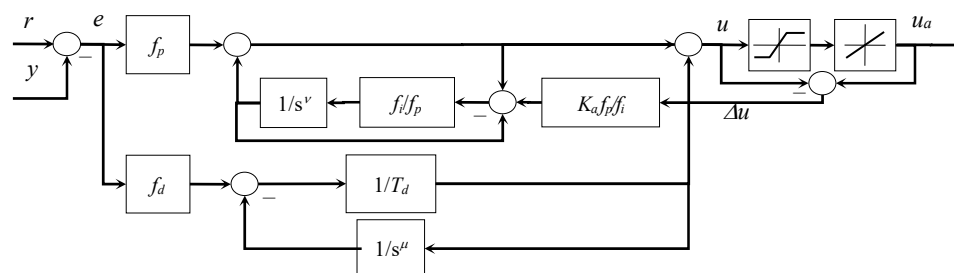


Figure 4. FO-NPID with magnitude and rate anti-windup.

3. Yaw Control of Wind Turbines

The duty of the yaw system is to orient the rotor in the wind direction. The yaw actuators are engaged such that the nacelle moves around the vertical axis of the tower. This process can be accomplished by utilising a wind vane to determine the wind direction. Figure 5 illustrates the procedure of aligning the rotor with the wind direction.

A dead zone [45] is often introduced prior to the conventional controller to complete a classic yaw control loop. Thus, the controller is activated only when the control error exceeds the specified threshold imposed by the dead zone, and it aims to maintain the operation inside a specific band rather than to precisely follow the reference [1].

The reference to follow is obtained by applying weight-averaged filtering to the wind direction signal g_e , which comes from a wind vane placed at the rear of the nacelle. The described control loop is schematised in Figure 6.

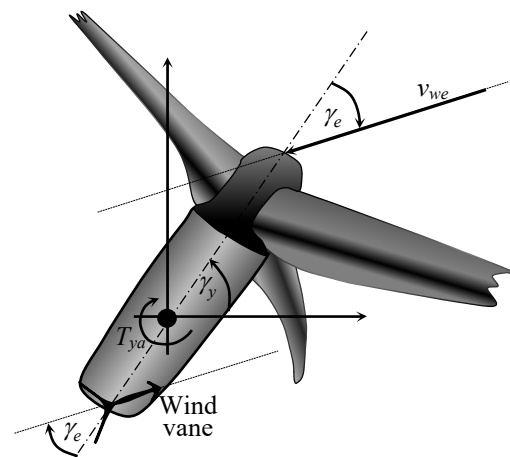


Figure 5. Schematic description of the yaw deviation of a wind turbine.

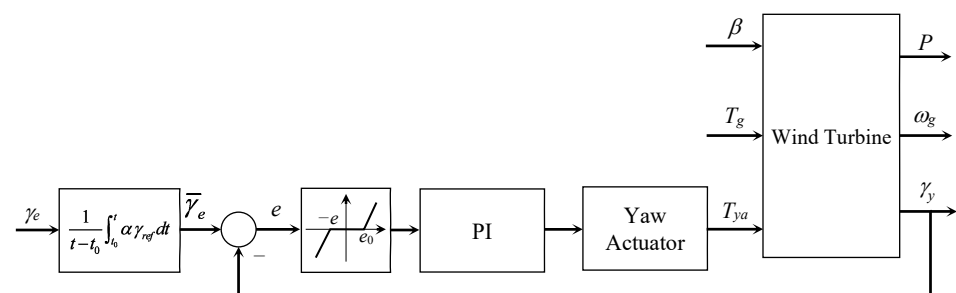


Figure 6. Control system configuration for the yaw control loop.

Typically, the control system is implemented by using a PI controller, which is activated if the control error goes out of the dead zone established by the band $[-e_0, e_0]$, i.e.,

$$e_o(t) = \begin{cases} 0 & |e(t)| \leq |e_0| \\ k e(t) & |e(t)| > |e_0| \end{cases}. \quad (15)$$

The control error is defined by $e = \bar{\gamma}_e - \gamma_y$, where γ_y is the measured yaw angle (see Figure 5). The wind direction measure is currently noisy and inaccurate. Consequently, it is unsuitable to serve as a control system reference. For this reason, it is improved by means of a moving average filter.

On the other hand, permanent reference tracking is unnecessary because the yaw system works at an extremely sluggish pace, such that it can be started once the misalignment is apparent. The dead zone is responsible for this.

The introduction of a dead zone in the yaw control loop can affect the performance, and the definition of the dead band plays an important role. For instance, selecting a large dead zone to reduce yaw action will result in subpar tracking, which causes reduced energy conversion. Conversely, if a small dead zone is chosen, shattering can occur around the reference.

In order to mitigate this problem, an adaptive dead zone is proposed in [46]. The idea is to select a small dead zone when the control error is significant in order to make sure that the output variable approaches closely to the reference, and then the zone is expanded such that the activity of the yaw controllers is lowered after the reference value has been reached.

4. Numerical Study Based on Simulations

4.1. Reference Wind Turbine

The proposed yaw control system has been subjected to an application study on the 20 MW reference wind turbine that was first introduced in [47] and afterwards explored from the control viewpoint in [48]. Thereafter, it was parametrically updated and tuned to guarantee better functionality and total compliance with OpenFAST 4.1.2 (previously referred to as FAST [49]). For the yaw control system, MATLAB/Simulink (R2025b) was used.

The relevant specifications of the reference wind turbine are presented in Table 1.

Table 1. Main parameters of the 20 MW reference wind turbine.

Characteristic	Values
Rated mechanical power	21.191 MW
Rated electrical power	20.000 MW
Rated rotor speed	7.1567 rpm
Rated generator speed	1173.7 rpm
Cut-in, cut-out, and rated wind speed	4.48, 25.0, 10.92 m/s
Rated aerodynamic torque	28,434.70 kNm
Rated generator torque	169.76 kNm
Maximum generator torque	249.81 kNm
Peak power coefficient, optimal TSR	0.4812, 10.115
Gearbox and generator efficiencies	97.8, 96.5%

In order to complete the description of the wind turbine, Figure 7 shows the generator power and rotational speed in Region II with optimal torque control (OTC) [2] and without yaw control.

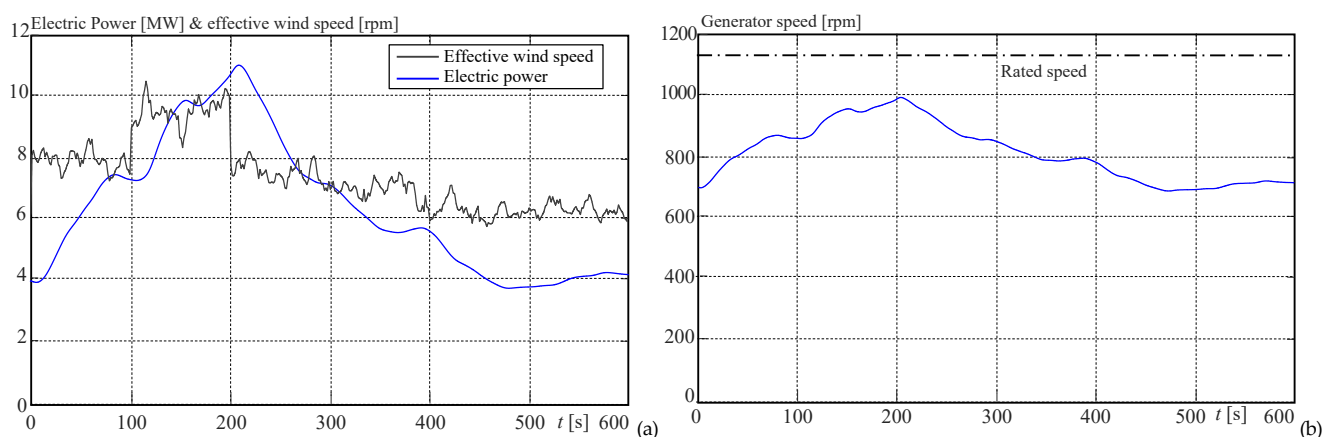


Figure 7. Wind turbine behaviour in Region II under torque control. (a) Power, (b) generator speed.

4.2. Parametrisation of All Controllers

The performance of the PI, NPI, and FOPI controllers is studied in the following sections. Therefore, all three controllers are parametrised by using numerical minimisation of the ITSE objective function (integral time-weighted squared error) expressed by

$$J = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} t \left\{ [e(t)/e_{max}]^2 + \zeta [u(t)/u_{max}]^2 \right\} dt \quad (16)$$

with $\xi = 0.7$. The optimisation is conducted with a simulation-based active set algorithm with the constraints imposed by actuator limits in magnitude and rate. A summary of all the final controller gains is provided in Table 2.

Table 2. Parameters for the yaw controllers.

Parameters	Proportional Gain	Integral Gain	Anti-Windup Gain	Reference Weight	Fractional Order
PI	$K_{py} = 2.3520$	$K_{iy} = 1.7651$	$K_{ay} = 0.9840$	$b = 1.0000$	---
NPI	$K_{0py} = 2.0212$	$K_{0iy} = 3.5332$	$K_{ay} = 6.2975$	$b = 1.0000$	---
	$K_{1py} = 1.2732$	$K_{1iy} = 1.6130$			
	$K_{2py} = 6.9618$	$K_{2iy} = 5.4435$			
FONPI	$K_{0py} = 2.0062$ $K_{1py} = 0.9794$ $K_{2py} = 7.1620$	$K_{0iy} = 3.2396$ $K_{1iy} = 2.2491$ $K_{2iy} = 5.6125$	$K_{ay} = 5.0939$	$b = 1.0000$	$\nu = 1.6$

4.3. Simulation Setup

The simulation of the wind turbine is undertaken with OpenFAST v. 4.1.2. The machine is operated in Region II with a stepwise changing effective wind speed between 5 and 10.5 m/s and 5% turbulence in the x- and y-directions (see Figure 8).

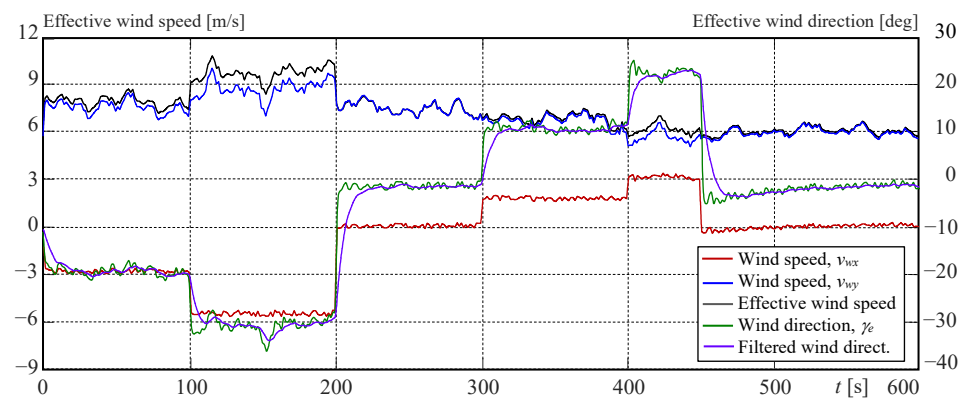


Figure 8. Wind speeds and wind direction for the simulation experiments.

4.4. Simulation Experiments

First, the standard PI is studied for different dead zones. The best found configuration is then used for all controllers. The comparison is carried out by using three objective functions, namely,

$$J_e = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} t e_{\gamma}(t)^2 dt, J_u = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} t u(t)^2 dt, \text{ and } E = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} P_e dt. \quad (17)$$

J_e and J_u are the averaged integral of time-weighted squared control error (ITSE) and the averaged integral time-weighted squared control signal (ITSU), respectively. The third one, E , is the average energy converted, expressed in MWh.

Smaller numbers in the first two functions indicate better performance. J_e provides information regarding the control error. J_u brings information about the actuator activity. The third function quantifies the energy that has been converted. Therefore, a higher value of converted energy is indicative of improved performance.

The second group of experiments is devoted to studying the behaviour of NPI and FONPI controllers in relation to yaw control.

5. Simulation Results and Analysis

5.1. Behaviour of the Performance Indices

In order to correctly evaluate the simulation results, it is important to clearly understand the information that is provided by the performance indices as well as their behaviours. These aspects are specific to the considered application and depend on the simulation time. Therefore, if the wind turbine is changed or if different simulation settings are used, the analysis may vary.

While performance improves with a higher E , ITSE and ITSU exhibit the opposite behaviour. That is, the lower they are, the better the performance. On the other hand, it is not possible to improve ITSE, ITSU, and E simultaneously. For example, if optimisation is conducted to improve ITSE, E may also be better, but ITSU will inevitably deteriorate. Conversely, improving ITSU will lead to worse values for ITSE and E .

Moreover, ITSE presents an important variability in front of ITSU. For example, improving ITSU from 13.3228 to 12.3774 implies a change in ITSE from 284.3880 to 603.5288, and E changes from 1.3014 to 1.2950. The above-mentioned characteristics of the performance indices are shown in the Pareto fronts depicted in Figure 9.

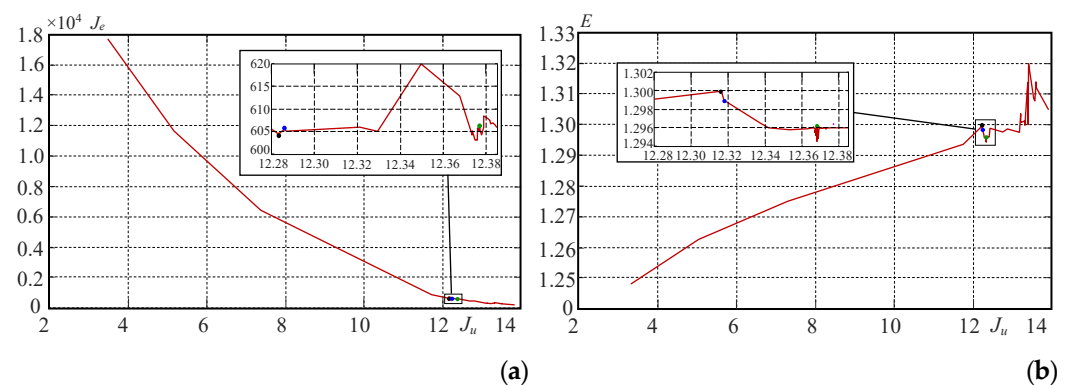


Figure 9. Pareto fronts between J_u and J_e (a) and between J_u and E (b). The points corresponding to PI, NPI, and FONPI are marked green, blue, and black, respectively.

In order to obtain a reduced actuator activity, e.g., ITSU lower than 10, the control error grows substantially such that the ITSE values becomes larger than 3000. Very small control errors require strong active actuators such that ITSU is larger than 14. This condition is illustrated in the convex Pareto front of Figure 9a.

On the other hand, the concave Pareto front between ITSU and E is depicted in Figure 9b. The curve is fairly flat, with E varying between 1.24 and 1.32. It is important to remark that E is expressed in MWh. Hence, a variation of 0.001 of E means that 1 kWh in a 10 min simulation corresponds to a difference of 52.56 MWh per year. From this perspective, the difference is not negligible. This aspect should be taken into consideration when the quantitative analysis is carried out.

5.2. Results Regarding the Study on the Effect of the Dead Zone

In Section 3, the yaw control loop with a dead zone has been explained. The expected result is a reduction in the actuator activity at the expense of a deteriorated performance regarding the power conversion. A way to mitigate this effect is to use an adaptive dead zone of at least two levels: one level for large errors and another level for small errors. The switching-in takes place when the error magnitude enters a zone smaller than 0.1, and the switching-out occurs when the error is larger than 0.5. The evaluation of the performance indices is summarised in Table 3. The yaw angles from the simulations are portrayed in Figure 10.

Table 3. Performance indices for the PI controller with different dead zones.

Dead Zone	J_e	J_u	E (MWh)
± 0.0 deg (control without dead zone)	304.9583	13.1961	1.2977
± 1.0 deg	716.0110	12.3773	1.2990
± 5.0 deg	6336.6523	7.4688	1.2753
± 7.5 deg	11,466.3461	5.2653	1.2630
± 10.0 deg	17,461.8243	3.5857	1.2485
Adaptive dead zone: (± 0.1 , ± 10.0) deg	606.0015	12.3772	1.2961

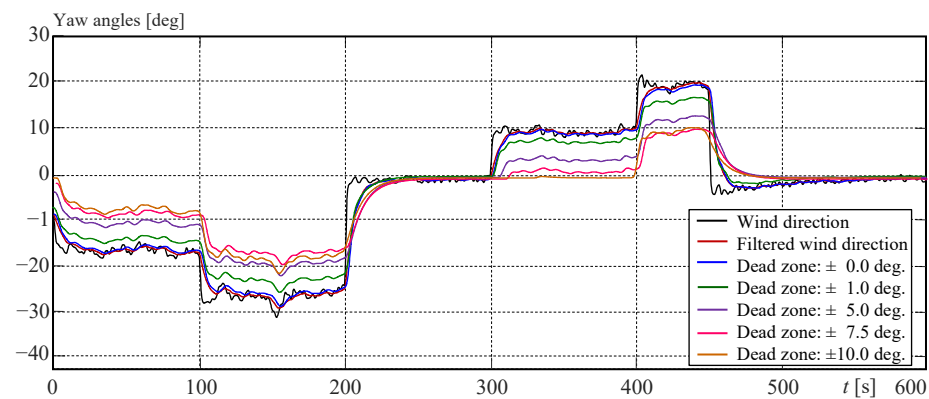
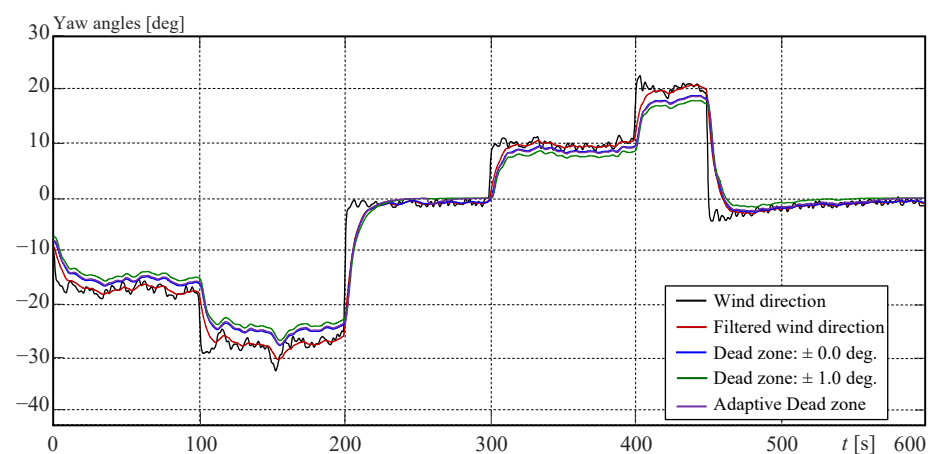
**Figure 10.** Controlled yaw angles for the PI controller and different dead zones.

Figure 10 shows that the actuator activity is reduced as the width of the dead zone increases. However, the cost is a significant degradation in control performance and a reduction in converted energy. On the other hand, the selected adaptive dead zone provides the similar actuator activity and energy conversion as a dead zone of one degree but with an important improvement in control performance. The simulation results are depicted in Figure 11.

**Figure 11.** Controlled yaw angles for the PI controller, including the adaptive dead-zone approach.

5.3. Results of the Study on the PI and NPI Controllers

The next study is to evaluate the performances of the standard PI and the NPI controllers. Both cases are considered in the adaptive dead zone. Figure 12 shows the electric power delivered by the wind turbine operated in open loop, with PI, and with NPI, and Figure 13 shows the controlled yaw angles.

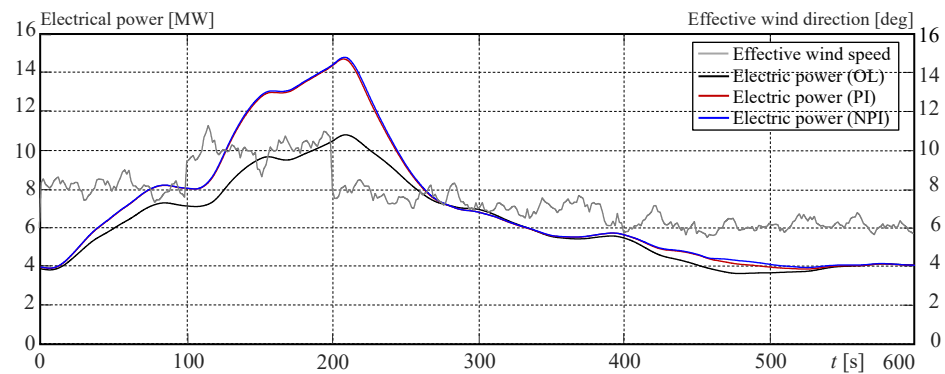


Figure 12. Power output without yaw control, with PI control, and with NPI control.

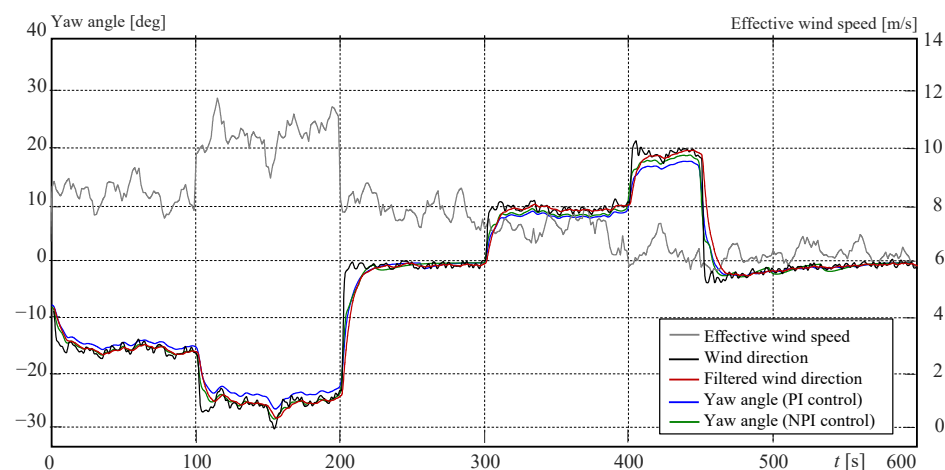


Figure 13. Controlled yaw angle by using PI as well as NPI controllers.

The first observation provided by Figure 12 is the importance of yaw control because the difference between the power curves is remarkable. However, the difference between power curves obtained for each controller does not appear to be significant at first glance. Therefore, the evaluation of the performance indices becomes relevant. These numerical results are presented in Table 4.

Table 4. Performance indices for the PI and NPI controllers.

Controller	J_e	J_u	E (MWh)
Classic PI	606.0019	12.3773	1.2961
Nonlinear PI	605.8192	12.3764	1.2979

It can be observed that the indices are antagonistic. That is, a significant reduction in the error index together with an increase in converted energy comes at the cost of increased yaw activity.

Due to the greater number of parameters available in the NPI, it is possible to assign priorities to the control objective, i.e., lower actuator activity with greater control error and reduced converted energy, or greater converted energy with lower control error but greater actuator activity. The results shown in Table 4 correspond to a balanced solution.

On the other hand, the results corresponding to converted energy seem to indicate that the difference between controllers is not relevant. For a more in-depth analysis, a simple economic calculation is included next to illustrate the results obtained in greater detail.

The NPI yields an 8.9 kWh advantage in energy conversion for a 10 min simulation. That equates to 53.4 kWh in a single hour. Multiplying by 8760 h and a capacity factor

for the location of 0.4, one obtains an annual conversion difference of 187 MWh. Since the current value per kWh for end consumers in Germany averages around EUR 0.35, the change from PI to NPI control law results in a profit of EUR 65,490 for a single machine.

5.4. Results Regarding the Study on FONPI Control

The difference between the NPI and FONPI is given by the fractional exponent included in the integrator instead of the exponent being equal to 1. However, this fractional exponent changes the system dynamics, which can result in the need for a reparametrisation of controller gains. Therefore, this study is carried out in two stages. In the first stage, the controller parameters obtained in the previous experiment are used, and only the exponent is varied from 0.7 to 1.44. Afterwards, the exponent is left constant, and the gains are adjusted again. The results are presented in Table 5 and Figure 14.

Table 5. Performance indices for the FONPI controller with different fractional orders.

Fractional Order	J_e	J_u	E (MWh)
0.70	294.0725	13.3273	1.2978
0.90	283.7354	13.3401	1.2988
1.05	282.2997	13.3354	1.3000
1.07	281.2628	13.3326	1.3001
1.10	281.5985	13.3286	1.3004
1.14	283.0414	13.3235	1.3009
1.20	285.7194	13.3329	1.3016
1.30	295.1614	13.3371	1.3035
1.40	300.7658	13.3733	1.3081
1.44	348.7523	13.4226	1.3200

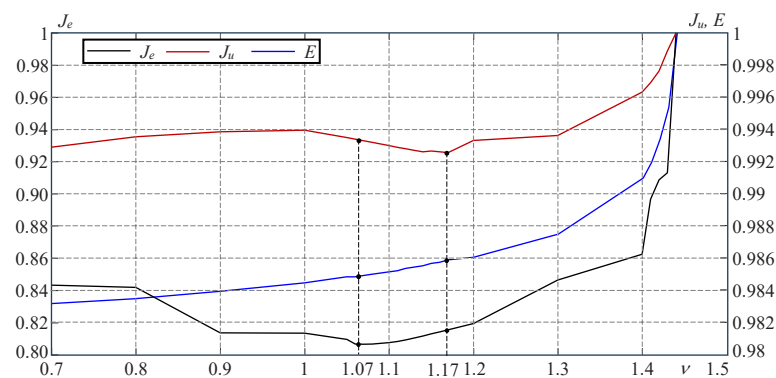


Figure 14. Normalised performance indices in dependence of the fractional order n .

It is important to note that the optimal value for each index is obtained for different values of the fractional order. This means that one index must be prioritised over others. Furthermore, while the range of variation for J_e can be considered significant, this is not the case for J_u and E . In other words, the fractional order does not lead to a substantial improvement in J_u or E . Thus, one strategy for setting up all controller parameters is to first search for them based on J_u or E and then to consider J_e by using the fractional order.

Finally, all parameters have been retuned simultaneously using parametric multiobjective optimisation, with the results of the above-mentioned procedure as initial conditions. The parameters obtained are summarised in the last row of Table 2. To make the compar-

ison easier, the values from Table 4 are replicated in Table 6, which summarises the best performance index values for the FONPI controller.

Table 6. Performance indices for the PI, NPI, and FONPI controllers.

Controller	J_e	J_u	E (MWh)
Classic PI	606.0019	12.3773	1.2961
Nonlinear PI	605.8192	12.3764	1.2979
Fractional-order nonlinear PI	604.5391	12.2835	1.2998

Following the same calculation as in the previous section, the FONPI controller obtains an annual advantage of 194.5 MWh over the PI controller and of 99.86 MWh with respect to the NPI controller. Moreover, this advantage is obtained by also maintaining better performance in control error and control effort. The simulation curves are depicted in Figure 15.

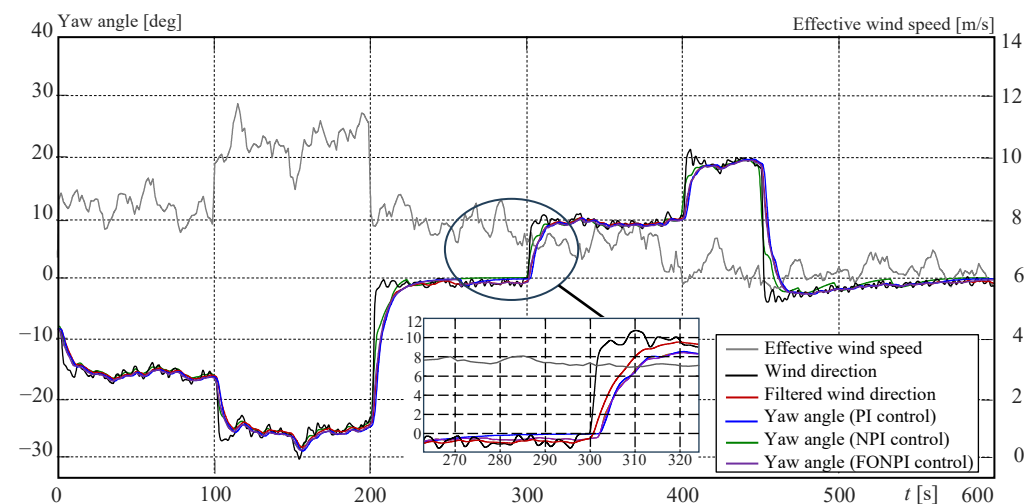


Figure 15. Controlled yaw angles by using PI, NPI, and FONPI controllers.

As previously stated, the simulation curves make it difficult to see how the results provided by the three controllers differ from one another. A better idea is provided by the zoomed-in section, in which we can see that the output variables are quite similar for all three controllers. The difference with respect to the reference (red curve) is due to the effect of the dead zone.

The advantage of using the most advanced controllers is reflected in the quantitative analysis, especially when considering the greater amount of energy converted in annual accumulation. The same applies to the reduction in actuator activity, in which improvement is minimal unless the restrictions on control errors and energy conversions are relaxed. However, the positive effect is reflected in the long-term service life of the actuators.

6. Conclusions

The purpose of this research is to examine the performance of different controllers from the point of view of the yaw control system, i.e., the problem related to the wind direction tracking control. As a control system configuration, a standard tracking control system with a dead zone is used, and three controllers (PI, NPI, and FONPI) are evaluated. The simulation experiments are conducted using a 20 MW reference wind turbine operating in Region II, with turbulent wind speed also changing stepwise and wind direction changing from -30 to 20 degrees. The wind energy system is implemented by using co-simulation, in

which OpenFAST (v. 4.1.2) is used for the machine and MATLAB®/Simulink (R2025b) for the control system.

In order to improve the classic approach, two procedures were designed and implemented in the final control system. On one hand, a special anti-windup mechanism for amplitude and rate limitations was developed. On the other hand, the dead zone was implemented, including an adaptive technique.

The simulation results are assessed quantitatively using performance indices and qualitatively based on the curves that are displayed. While the curves do not always allow the differences between the results to be appreciated, the numerical results of the objective functions show the different performances.

All three controllers show satisfactory performance. However, the NPI exhibits slightly better behaviour with respect to the PI controller, and the FONPI is the best when all parameters are retuned, including the fractional order in the optimisation.

The next step in the research is to study yaw control approaches that do not need the measurement of the wind direction. Hence, they will be compared with the approaches studied in the present work.

Funding: This research received no external funding.

Data Availability Statement: Data are available upon request from the corresponding author if allowed by the affiliated institution.

Conflicts of Interest: The author declares no conflicts of interest. The funders had no role in the design of its study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish these results.

Abbreviations and Nomenclature

Abbreviations

Crone	Commande robuste d'ordre non-entier
FONPI, FONPID	Fractional-order nonlinear PI, fractional-order nonlinear PID control
HCS	Hill climb searching
ITSE	Integral time-weighted squared error
ITSU	Integral time-weighted squared control
LiDAR	Light detection and ranging
NREL	National Renewable Energy Laboratory
P, PI, PID	Proportional, proportional–integral, proportional–integral–derivative control
NP, NPI, NPID	Nonlinear P, nonlinear PI, nonlinear PID control

Nomenclature

Parameters

E	Converted average energy, MWh or kWh
$f(e), g(e)$	Functions for the NPI
f_p, f_i, f_d	Nonlinear functions for the gains of the NPID controller
J_e	Averaged integral of time-weighted squared control error
J_u	Averaged integral of time-weighted squared control signal
K	Gain of the continuous integro-differential operator
$K_p, K_i, K_d, K_a, T_d, b$	Gain of the PID controller
$K_{py}, K_{iy}, K_{ay}, b$	Parameters of the PI yaw controller
$K_{0py}, K_{1py}, K_{2py}, K_{0iy}, K_{1iy}, K_{2iy}, K_{ay}, b$	Parameters of the NPI controller
K_p, K_i, K_d, K_a, b	Parameters of P, PI, PID controllers

Variables

e, e_o, e_0	Control error, output of the dead-zone block, dead-zone limit
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s	Laplace variable
t	Time
T_g	Yaw torque, Nm
T_{ya}	Actuator torque on the yaw axis, Nm
u, u_a, Du	Control variable, actuator output, difference $u_a - u$
y	Output variable
$g_e, g_y,$	Reference yaw angle and measured yaw angle, rad
ξ	Lagrange multiplier for the ITSE
Functions	
$f(e), g(e)$	Nonlinear function of the nonlinear PID controllers
$G_{PID}(s), G_{NPID}(s), G_{FONPID}(s)$	Transfer functions for the controllers
$X(s)$	Continuous integro-differential operator
$U(s), E(s), Y(s), R(s),$	Laplace transformed input, error, output, and reference
n, m	Fractional orders for the FONPID controller
w_z, w_p, w_h, w_l	Zeros and poles in the interval $[\omega_l, \omega_h]$ for the fractional integrator

Appendix A

The aim of this appendix is to verify that the PID scheme of Figure 4 is equivalent to the classic PID with back calculation anti-windup mechanisms, i.e.,

$$U(s) = K_p E(s) + \frac{1}{s} [K_i E(s) + K_a \Delta U(s)] + \frac{K_d s}{1 + T_d s} E(s), \quad (A1)$$

where $E(s) = R(s) - Y(s)$. Hence, the start point is Figure A1, from which the proportional term is obtained as

$$U_p(s) = K_p E(s). \quad (A2)$$

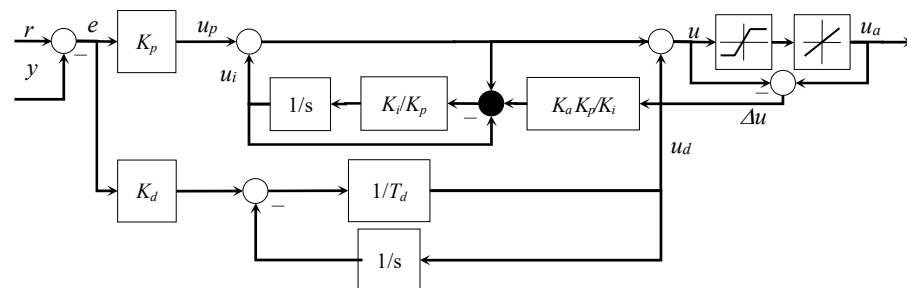


Figure A1. PID in the automatic reset configuration with magnitude and rate anti-windup.

The derivative term is obtained from Figure A1 and expressed by

$$U_d(s) = \frac{1}{T_d} [K_d E(s) - \frac{1}{s} U_d(s)]. \quad (A3)$$

Reordering terms, it follows

$$\frac{[1 + T_d s]}{T_d s} U_d(s) = \frac{K_d}{T_d} E(s), \quad (A4)$$

which leads to

$$U_d(s) = \frac{K_d s}{1 + T_d s} E(s) \quad (A5)$$

i.e., the derivative term of (A1). For the integral term, it is first necessary to move the black summing point in Figure A1 before the block K_i/K_p . The result is shown in Figure A2.

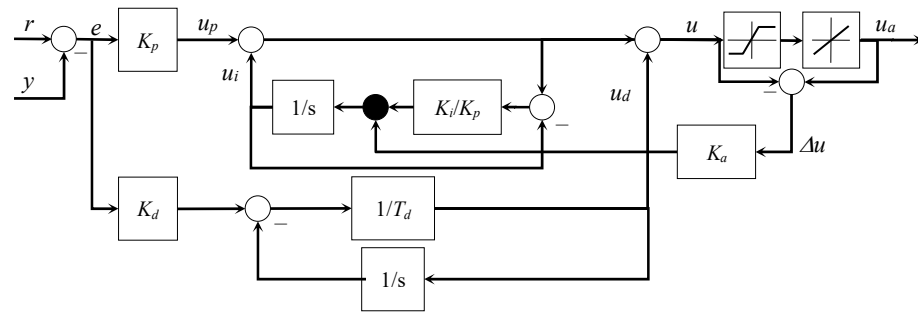


Figure A2. Modified PID in the automatic reset configuration.

The integral part can now be formulated from Figure A2 as

$$U_i(s) = \frac{1}{s} \left[\frac{K_i}{K_p} [U_p(s) + U_i(s) - U_i(s)] + K_a \Delta U(s) \right] \quad (\text{A6})$$

Considering that $U_p(s) = K_p E(s)$, (A6) becomes

$$U_i(s) = \frac{1}{s} [K_i E(s) + K_a \Delta U_i(s)] \quad (\text{A7})$$

Adding (A2), (A7), and (A5), the start Equation (A1) is obtained.

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